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ANONYMOUS MONEY, MORAL SENTIMENTS AND WELFARE

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Abstract

Some markets are prone to develop shadow transactions for the purpose of tax avoidance. Moral sentiments control the allocation of consumers between the legal and illicit markets. Such sentiments include self-esteem and social disapproval. The market solution leads to fiscal externality resulting from tax avoidance and highlights the conflict between private opportunism and collective values. Shadow markets may, however, enhance consumer welfare by limiting the pricing power of firms and by controlling tax collection. The paper develops a model of endogenous segmentation of markets between moral and immoral behavior. The legal producer can price the self-esteem of honest people, who can blackmail the legal producer with their option of visiting the illicit market. The model has implications for monetary economics: moral sentiments, tax rates, illegal transactions, and probability of being caught become relevant for the demand for money.


Keywords: moral sentiments, shadow economy, welfare.

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1 Introduction

Shadow economy and illicit economic activities may touch deeply moral feelings of people. The inherited economic literature on shadow markets has, however, predominantly been focused on two issues, i.e. surveying payment transactions and providing measures for the size of underground economic activities.\(^1\) It has, however, produced only to a limited extent economic analyses of the mechanisms involved and welfare implications of such activities. In particular, as it is not been conventional to link market activities and moral standards.\(^2\) Our paper addresses these issues. It raises the question what determines the borderline between legal and shadow economies. It asks why moral sentiments can control shadow activities only imperfectly. It therefore introduces preferences for moral standards, determination of morality as social capital and sustainability of social norms when taxes can be avoided by visiting illicit markets. It considers the commodity tax in allocating consumers between the legal and illicit sectors and falling only partly on the owners of the legal firm. The paper then explores the welfare effects are associated. It also suggests that illegal transactions are an important determinant of cash holdings despite the fact that more efficient electronic payment system are at hand.\(^3\)

Some markets are prone to develop shadow transactions for the purpose of tax avoidance especially if the government is viewed as a predatory revenue maximizer. Despite such a view of government, people have intrinsic moral sentiments, with preference for obeying inherited social norms and disapproving deviants. People care what other people think of them and they are subject to self-esteem. In our work, morality is viewed as a net-

\(^1\)A useful classification of this activity is provided by Thomas (1992), subsequently reviewed by Schneider and Enste (2002). In this paper, we consider illegal economic activity which includes market transactions but excludes such activities as exchange of goods and neighborhood help.

\(^2\)Hausman and McPherson (1993) have provided a review of why and how morality influences economic outcomes.

\(^3\)Using cash for payments of illegal anonymous transactions appears as an important motive for why people hold cash in spite of the recent innovations in the payment system. Cash payments are convenient for those who seek to hide their motives. The markets for prostitution, for example, are presumably predominantly based on the use of cash, reducing the enforceability of within family contracts. Modern brand products like Nike or Adidas in sportswear and shoes or Prada on ladies fashion are actively copied and marketed in pirate products. With high legal prices of CDs, people have developed ability to copy music freely from internet. In construction, illegal labor is employed with the aim of tax evasion. Many private services are delivered without receipts. In most economies, there are also well-functioning secondary markets for stolen durable goods. Moreover, the rise of terrorism is largely financed by illegal money.
work good. Apart from consumption opportunities, people value the social approval attached to their behavior within the social network sharing the same moral values. By implication, deviants subject themselves to the risk of being detected and becoming socially stigmatized. Detection is, however, probabilistic and by anonymous transactions immoral people may mimic the moral ones. Morality operates like a particular form of social capital. In our model, it is determined endogenously as the share of people who choose to commit to honest behavior.

A natural way of justifying the moral feelings of social disapproval is to think that those sentiments can be traced to free-riding by the deviants. Abstaining from participation of financing the public goods, those who visit the illicit market exert a negative fiscal externality on the honest consumers. The relation between moral sentiments and market behavior is of particular interest also from the point of view of the interaction of buyers and sellers. When market behavior reflects moral sentiments, the legal firms can opportunistically price the self-esteem of honest people. From the opposite perspective, the consumers in the legal market have the option of visiting the illicit market. This results in an implicit blackmailing option, making the honest consumers earn surplus over the price charged by the legal producer.

We report a number of results. We first show how high tax rate, low expected cost of punishment and low relative cost of production shape the industry equilibrium leading to contraction of the legal sector and a boost in the shadow economy. To qualify, rising tax rate tends to expand the shadow economy through the tax avoidance effect. There is an associated public goods effect, which accelerates or decelerates the expansion of the shadow economy depending on the possibility of the legal firm to make profit when the tax rate is changed under the requirement of balanced government budget. Moral sentiments in terms of self-esteem and disapproval of deviations from social norms operate as mechanisms which limit the shadow market activities. The social disapproval effect, however, tends to be diluted with

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4The reasons for non-opportunistic behavior have been extensively discussed by biologists. Hamilton (1964) introduced the notion of kin selection and Trivers (1971) a more general view of reciprocal altruism. Wilson (1975) represents a comprehensive document of reciprocal behavior among animals extended to human behavior by Binmore (1998). Frank (1988) argued convincingly that the ability of people to behave non-opportunistically serves as a helpful commitment device facilitating beneficial relations like joint ventures.

5Cf. Frank (1987) for a pioneering analysis of honesty and dishonesty.

expansion of shadow economy, providing a further driving force for further shadow market transactions. Even though the moral case against shadow economy is strong, shadow markets are not, however, completely bad. Existence of illicit markets gives rise to a number of conflicting welfare effects. Some of those are welfare enhancing. In particular, shadow markets threat the pricing power of legal producers. In our approach, shadow markets intensify competition between producers. The paper does not, of course, suggest that it would be optimal to have a shadow economy in all industries or that market mechanisms would result in an optimal shadow economy.\(^7\) In industries with intense competition, the social gains from shadow economy are limited. Moreover, the paper suggests that the market solution can hardly be expected to lead to an optimal shadow economy. This view arises from the two externalities explored, fiscal externality and erosion of social capital. In the market solution, the shadow economy results in a welfare loss arising from underprovision of public goods. With shadow markets, moral standards tend to become deteriorated. This is a mechanism which has properties of built-in dynamics. There has also been a popular view that shadow markets tend to operate as a disciplinary mechanism and control wasteful tax collection. We prove that the Laffer curve has a unique maximum. We show, however, that is not necessarily the case that shadow markets enhance welfare by limiting wasteful tax collection. However, for the shadow economy to exist in the social optimum along with the legal sector, it is necessary that people’s moral sentiments do not punish illicit transactions too heavily. Should the legal sector survive with people having incentives for tax evasion, the tax rate cannot too high.

Our paper is structured as follows. After surveying the previous studies, we first explore the market solution and the fiscal externality of tax avoidance, and some welfare effects in the absence of moral sentiments. We introduce public goods. We then introduce moral norms in controlling illicit transactions. Those norms include self-esteem and social punishment. We next consider a Leviathan government’s ability to raise revenue when shadow economy creates an option to avoid taxes. Some further welfare effects are examined in the last section. The final section concludes.

2 Previous Studies

**Shadow Economy and Payment Systems**  It goes without saying that in a fully efficient economy, there is no role for a shadow economy. It arises

\(^7\)For example, informational asymmetries tend to render a shadow economy in medical services inefficient.
exclusively from government intervention, i.e. regulation and taxation. Such a policy intervention has side effects. Many empirical studies referred below show that the size of the shadow economy has been most dramatic in the planned socialistic economies with maximum governmental intervention. However, shadow sectors exist in market economies as well.

As early as in 1958, Cagan proposed that people prefer to use cash in illegal economy, especially in black markets and for tax evasion. Since his seminal paper, a great deal of attention has been devoted to evaluating the magnitude of illegal use of cash. After technological progress with electronic payment systems, use of currency became challenged by e-purse which, interesting enough, seems to have failed. Cash has not disappeared. Consequently, Hancock and Humphrey (1998) ask why cash - with a positive opportunity cost - continues to be used for transactions when other assets earn positive return. They conclude that factors other than the simple opportunity cost must play primary role in determining holdings and the use of cash. After an extensive survey Drehman et al. (2002) conclude that 'bad behavior' creates a demand for anonymous means of payment. Anonymity provides an important distinction between the characteristics of currency and e-money. An important reason for holding cash would be the anonymity and the opportunity to visit illegal economy.8

Magnitude of Shadow Economy Some useful figures for understanding the magnitude of illegal demand for cash are provided, for example, by Rogoff (1998). Accordingly, half of the currency holdings in the OECD countries are in the domestic informal economy. Humphrey et al. (2000) suggest that use of cash in illegal activities is about 67 percent of the total cash use in Norway. Moreover, they predict that the share of cash used in illegal activities continues to rise up to 80 percent by 2005, as in legal use cash will be substituted by other payment mediums. Concluding from these studies it appears that informal economy already is the main holder of cash. In addition, the technological progress will further distort the demand as the share of the legal use will decrease and the share of illegal use will increase. Thus the authorities will face an unpleasant moral dilemma, as the seigniorage revenues are likely to be due to providing the means of payment for illegal activities, for example, tax evasion!

As it has been straightforward to marry use of cash and shadow economy, it is no surprise that first attempts to infer the size of the shadow economy use

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8Drehman et al. (2002) propose that authorities might be able to decrease illegal economic activities by making e-money a legal tender so that e-money could really challenge the currency.
cash data. For a review, we refer to Schneider and Enste (2002). Giles (1999) suggests that the size of the shadow economy has been growing over the past two or three decades in almost all of the countries for which comparative data have been assembled. According to Giles, growth in the underground economy is associated with increases in the actual or perceived tax burden but also with the degree of economic regulation. The view is shared by Thomas (1999) who hints that a growing shadow economy may be an indication of over-taxation and over-regulation, a view which is shared by our model. He also suggests that at least some part of the shadow economy may be social security fraud, making unemployment less bad as it looks. Thus, understanding the economic and social mechanisms in the shadow economy may be much more important than perhaps often thought.

Links to Policy There are several conclusions that have been justified in the light of the above results. Schneider (2000) gives a concerned note by suggesting that under a growing (or substantial) shadow economy, policy is based on mistaken official indicators. In addition, he suggests that a growing shadow economy attracts workers to work in shadow economy and to work less in the official economy. Giles and Caragata (1999) are concerned that unpaid tax in hidden economy or loopholes allowing abusive avoidance in the existing tax system create a deadweight loss on the economy. These will, in turn, undermine taxation equity by shifting tax burden in the direction of honest, socially responsible individuals and corporations. Further, they suggest that growing hidden economy gives rise to an increasing segment of the money supply which is uncontrolled. According to Giles and Caragata, part of the hidden economy is learned response to changing opportunities and constraints in fiscal policy, but on the other hand, there is a threshold level of underground activity.

Theoretical work Theoretical attempts to analyze the shadow economy include Cowell (1989) who asks how far should taxation authorities go in pursuit of the missing income. In a paper which is closest to ours, Acemoglu (1994) analyzes the pecuniary and non-pecuniary aspects of the reward structure on the allocation of talent across different activities with divergent private and social returns. The non-pecuniary aspects of reward structure consist of social status and prestige received for different activities. He presents the following channels via which the prestige and the status are influenced. First, learning by new generations will start from the already established norms and role models. Second, people often adjust their behavior comparing themselves to a particular reference group. Thus, social status
may depend on choices that the rest of the society makes and 'bad behavior' may have less damaging stigma when it is more widespread. Third, individuals feel unhappy living in a highly 'immoral' society while also knowing that 'bad behavior' is bad. Lastly, Acemoglu also notes that the society may lack the social or political will to change the status quo.

The determinants and effects of the informal sector are studied in an endogenous growth model by Loayza (1996). He assumes that the production technology depends on congestable public services. Using data on Latin American countries it is found that the informal sector negatively affects growth and its size depends on tax burden, labor-market restrictions and government institutions. Johnson et al. (1997) consider allocation of labor between the official and the unofficial sectors, its implications for tax revenue, law and order, and efficiency. They suggest that supply of public goods tends to result in increasing returns in private firms, leading to multiple equilibria. Data on the economic transition of formerly communist countries suggests that market-supporting institutions are critical for a successful transition. Friedman et al. (2000) ask to what extent a firm should divert activity to unofficial economy. Using various data sets on Eastern Europe, Former Soviet Union, OECD and Latin America, they suggest that the incentive to go underground to dodge higher tax rates is outweighed by the benefits of remaining official.

Camera (2001) uses the Kiyotaki-Wright random matching model to analyze what would happen if authorities promote the use of e-purse and limit the use of currency. Individuals can engage in illegal and legal production and perform monetary or intermediated cash-less transactions. The government monitors imperfectly the monetary exchange process, whilst the government monitors perfectly the intermediated exchange process. An equilibrium exists with no illegal production if monitoring is sufficiently extensive and money supply is moderate. Transactions may be entirely cash-less when intermediation is sufficiently inexpensive. When enforcement is not too extensive, however, there are monetary equilibria where legal and illicit production co-exist. Moreover, it is possible that an over-provision of currency may induce larger participation in illegal production. Accordingly, the results provide a rationale for limiting the amount of cash in circulation. Nonetheless, using currency as an exogenous discipline on illicit undertakings may have unintended consequences, such as depressing all trading activities and decreasing welfare. Dabla-Norris and Feltenstein (2003) apply an intertemporal general equilibrium model to explore the link between tax rates, access to credit and the size of the underground economy. By assuming that firms can operate partially in the formal and partially in the underground economy they derive entry and exit into underground economy as part of optimizing behavior.
that depends on taxes and interest rates. Simulation results for Pakistan demonstrate that entry into underground economy can have a cyclical nature. Moreover, the share of underground activity will decline over time and sectors gradually move back into legal economy. With low taxes, there is no underground economy, but due to high budget and trade deficit the low tax regime is not sustainable over time. Thus an economy may have to accept some underground activity as part of an otherwise acceptable tax program.

Davidson, Martin and Wilson (2003) suggest that shadow transactions may increase welfare. They argue in the experience goods framework that by allowing agents to self-select into the black market, the government can target tax breaks to transactions involving low-quality goods.

3 Model

3.1 Market Solution Without Moral Sentiments: Fiscal Externality

We consider a shadow economy arising from tax avoidance by consumers. Thereby, it reduces resources available for production of public goods. Our research strategy is to analyze first the industry equilibrium when one sector is taxed and the other is not. Subsequently, we introduce moral standards to examine in which way they control the development of shadow economy.

Anonymous Visits to Shadow Economy We consider a market with a product brand where a producer has market power in pricing. The product is assumed to have some prestige value, determining the basic willingness to pay. In such markets, the entry cost is non-trivial because of the nature of the product or because of barriers to entry. The products are subject to a commodity tax, $\tau > 0$. Because of the market power, the producer is able to shift part of the tax to consumers. The two elements, pricing power and the tax wedge in consumer price create an incentive for illicit production. To avoid social punishment, consumers try to visit the illicit market anonymously.10

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9 Alternatively, we could think of paternalistically regulated industries, like casinos, state monopolies in liquor production etc.

10 To clarify, when we talk about an illicit producer, this should not be understood to refer a registered firm but to an activity which is rival to legal activity. We also notice that we abstract from the possibility that the legal firm operates partly like an illicit producers in the shadow economy. Though such an activity is common in practice, we leave it out if only to keep the model simple enough.
**Consumers**  Consumers are assumed to have preferences over goods and social approval. By their intrinsic preferences, people are assumed to be all alike. They are egoists in the sense of utility maximizers. But they also care for what other people think of them.\textsuperscript{11} Thus, they have subjective preference both for their self-esteem and they care about the expected social punishment of illegal actions.\textsuperscript{12} Thus, morality becomes a \textit{network phenomenon}. However, for tax avoidance reasons, it may become profitable to deviate from the norm of buying only legal products. By anonymous transactions, deviants try to mimic the legal consumers, though risking themselves to social stigma. Hiding deviations from an established social norm becomes attractive, as hiding may help to maintain the status of an honest person.

In this section, we first build the market model without moral sentiments. We assume that consumers differ with respect to their willingness to pay for the prestige of the product brand. We assume that there is continuum of consumers with mass one. They can buy a private product in the legal or illicit market. We denote the producer prices by $p_l, p_i$. The marginal utility from buying the private legal product, $r^k$ for consumer $k$, is assumed to be uniformly distributed over $[0, 1]$. Consumers also value public goods. We assume that private goods and public goods are complementary and that the valuation of public goods is uniform across consumers. Complementarity implies that the marginal utility of a private good, $x$, for consumers is $MU_x = g > 0$. Consumers visiting the illicit market are assumed to be caught with probability $\xi > 0$ and subject to penalty $z > 0$. Consumers $(k, j)$ visiting the legal and illicit market thus are assumed to have net utilities

$$u^k = gr^k - (1 + \tau) p_l, \quad v^j = gr^j - z \xi - p_i.$$  \hspace{1cm} (1)

Non-excludability implies that both honest and dishonest consumers derive utility from public goods though the latter ones free-ride in the financing of those goods.\textsuperscript{13}

In the industry equilibrium, we expect that there will be segmentation of markets, i.e. those consumers with high marginal utility $r^k$ would buy the legal product while those with lower marginal utility buy the illicit product.

\textsuperscript{11}Such a subjective status effect is well-known in psychology, cf. Singh-Manoux, Adler and Marmot (2003). In biology, we refer to Ridley (1996). In economics, Fershtman, Weiss and Hvide (2001) have studied status effects.

\textsuperscript{12}Binmore (1998) has shown that it is rational for people to commit to social norms and social contracts as long as the commitment gains exceed the short-term losses from deviating. Morality arises in the equilibrium of a repeated game as a social contract.

\textsuperscript{13}The public good is more valuable to a consumer with greater willingness to pay for the private good.
Denote the marginal consumers by \((m,n)\) where \(m\) is indifferent between buying the legal and illicit product while \(n\) is indifferent between buying the illicit product and buying none. Thus the marginal utility for any consumer, \(k\), for buying the legal product is \(g(1-k)\). The marginal utility for \(j\) for buying the illicit product is \(g(1-j) - \xi z\). Insert \(k = j = m\) to obtain the expressions for the marginal consumer. For indifference, \(g(1-m) - (1+\tau)p_l = g(1-m) - \xi z - p_i\). Note that the market share of the legal production must be \(x_l = m\).

The immediate observation is that the consumer price in the legal sector has to exceed the price in the illicit sector, \((1+\tau)p_l > p_i\). The price differential is unaffected by the valuation of public goods. Solving for the expression for the willingness to pay by the marginal consumer of the legal product from condition \(\frac{g-r_m}{x_l} = g\), yields \(r_m = (1-x_l)g\). To solve for the marginal willingness to pay for the illicit product, we note that \(\frac{g-r_n}{x_l+x_i} = g\), yielding \(r^n = (1-x_l-x_i)g\). Clearly, \(r^m > r^n\). This implies that \(r^m - (1+\tau)p_l > 0\). By implication, the legal firm cannot exploit the full consumer surplus from its customers. Such a market power of the legal consumer arises from that she can "blackmail" the legal producer with her option to visit the illicit market. The illegal producer, in contrast, is able to exploit the full surplus from its marginal customer.

To summarize, \(r^m > r^n > 0\). Then we know that all those customers with a higher product valuation than the marginal customer \(m\) will buy the legal product. The other active customers buy the illicit product. The third group buys nothing.

Solving for the price differential

\[
(1+\tau)p_l - p_i = \xi z. \tag{2}
\]

What this condition suggests is that a consumer is indifferent between visiting the legal and illicit market if the risk of getting caught and penalized is fully compensated by the price differential. For the marginal consumer \(n\), the net utility from buying the illicit product is zero, \(g(1-n) - \xi z - p_i = 0\). Noting that the \(n\) is the last buyer, i.e. \(n = x_l + x_i\), her net utility is \((1-x_l-x_i)g - \xi z - p_i = 0\). Thus, the valuation of public goods raises the price of the illicit product. Through the first arbitrage condition, this is then reflected in the price of the legal product, too.

The total production thus satisfies

\[
x_l + x_i = 1 - \frac{\xi z + p_i}{g}.
\]
Solving for the prices

\[ p_i = (1 - x_l - x_i)g - z\xi, \quad p_l = \frac{(1 - x_l - x_i)g}{1 + \tau}. \]

**Profits** Market equilibrium can now be analyzed in terms of competition with differentiated products. The legal firm can take opportunistically advantage of honest consumers. On the other hand, the illicit market intensifies competition. This results in fewer consumers in the legal market, exerting a pressure on the legal price.\(^{14}\) Consumers buy the product with greater net utility.\(^{15}\) Under Cournot-competition in differentiated products, the behavior of firms obeys

\[
\max_{x_l} \ (p_l - c_l) x_l, \quad \max_{x_i} \ (p_i - c_i) x_i, \quad (3)
\]

where \(c_l, c_i > 0\) are the production costs.\(^{16}\)

**Nash equilibrium** In Nash equilibrium, market shares satisfy

\[
x_l = \frac{g + z\xi + c_i - 2(1 + \tau)c_l}{3g} \quad (4)
\]
\[
x_i = \frac{g + (1 + \tau)c_l - 2z\xi - 2c_i}{3g} \quad (5)
\]

Similarly, prices are

\[
p_l = \frac{g + (1 + \tau)c_l + z\xi + c_i}{3(1 + \tau)} \quad (6)
\]
\[
p_i = \frac{g - 2z\xi + c_i + (1 + \tau)c_l}{3} \quad (7)
\]

We find

**Lemma 1** *The shadow market exists in the industry equilibrium if*

\[
x_i = \frac{g + (1 + \tau)c_l - 2z\xi - 2c_i}{3g} > 0
\]

\(^{14}\)The firm producing in the legal market obviously has an incentive to capture the consumers also in the illicit market. An example is the illegal production of pirate products of cigarettes in Eastern European countries. The legal firm typically then issues licences to combat the illegal pirate production.

\(^{15}\)One can extend the analysis to quality uncertainty in the illicit market. One of the recent examples is the Estonian vodka. When bought in illicit markets, it has killed some consumers.

\(^{16}\)The decision to become a producer in the illicit market could be analyzed in terms of occupational choice. Such a choice would also be subject to moral considerations which, however, will not be explicitly discussed in the current paper.
High tax rate and low expected cost of punishment and low relative cost of production support the development of shadow markets. In particular, 

**Lemma 2** Increased tax on the legal product shakes the industry equilibrium leading to contraction of the legal sector and a boost in the shadow economy.

These natural conclusions are available from (4) and (5), $\frac{\partial x_l}{\partial \tau} = -\frac{2c_l}{3g} < 0, \frac{\partial x_i}{\partial \tau} = \frac{c_l}{3g} > 0$. When the tax rate is increased, it is natural that the legal price declines, $\frac{\partial p_l}{\partial \tau} = \frac{(c_l - 3p_l)}{3(1 + \tau)} < 0$, and the price in the shadow market increases, $\frac{\partial p_i}{\partial \tau} = \frac{c_l}{3g} > 0$.

However, when tax revenue is used to finance public goods, these natural results no longer hold as their validity is limited to the ceteris paribus case. The role of public goods in the determination of the limits to shadow economy thus deserves attention. Solving

$$\frac{\partial x_l}{\partial g} = \frac{1}{g} \left( \frac{1}{3} - x_l \right), \quad \frac{\partial x_i}{\partial g} = \frac{1}{g} \left( \frac{1}{3} - x_i \right).$$

**Lemma 3** With large market shares ($x_l > \frac{1}{3}, x_i > \frac{1}{3}$), an increase in the supply of public goods reduces both the size of the legal sector and the illicit sector. It is only when the market shares are small that the complementarity effect of public and private goods raises output when more public goods become available.

This somewhat surprising finding follows from the profit maximizing behavior of producers. Realizing that the willingness to pay by consumers for the private products has increased with more public goods available, producers can raise their profits by actually cutting the production and charging higher prices.

### 3.2 Balanced Budget Requirement

Suppose now that the public goods available are constrained by the tax revenue generated by taxation of the legal product. This amounts to considering the industry equilibrium subject to the constraint

$$g = \tau p_l x_l.$$  \hspace{1cm} (9)

With a given tax rate, such a constraint endogenizes the supply of public goods in the economy.\(^\text{17}\) We now have five equations to determine the five

\(^\text{17}\)We consider below the case where the tax rate is determined by revenue maximizing government.
variables \((x_l, x_i, p_l, p_t, g)\) in the industry equilibrium. Dishonest people understand that by free-riding in financing the public goods production they also suffer if the tax revenue is reduced and less public goods are available. Does this understanding restrict the market share of the shadow economy? What does the uncoordinated equilibrium look like?

Suppose that there is a marginal increase in the tax rate \(\tau\). What happens to the shadow markets? From the solution above, the new industry equilibrium satisfies

\[
\frac{dx_i}{d\tau} = \frac{c_l}{3g} + \left(\frac{1/3 - x_i}{g}\right) \left(\frac{dg}{d\tau}\right).
\]

We find that if the tax rate is increased, there are two effects. The first one definitively tends to make the shadow economy larger. This can be called the tax avoidance effect and its magnitude is measured by \(c_l/3g\). High production cost in the legal sector makes the tax avoidance effect large, while large supply of public goods has a negative impact on the expansion of the shadow economy. These effects follow from production decisions by firms. There is a secondary effect which depends on the impact of tax rate on tax revenue and hence on supply of public goods, \(dg/d\tau\) and on the initial market share of the shadow economy, \(x_i\). It is appropriate to call this the public goods effect.

Totally differentiating the budget constraint \(dg = d\tau (p_l x_l) + \tau (dp_l x_l + p_l dx_l)\) and inserting the market reactions, we find

\[
\frac{dg}{d\tau} = \frac{1}{1 + \tau} \left( p_l x_l - \frac{2c_l}{g} \left( x_l + \frac{2(1+\tau)c_l}{g}\right) \right).
\]

The denominator is always positive because \(\frac{1}{3(1+\tau)} x_l < 1\). The sign of numerator is, however, ambiguous. Despite its sign, \(\frac{1/3 - x_i}{g}\) is positive in when shadow economy is small and negative when it is large. The public goods effect therefore generates accelerating or decelerating mechanisms on the expansion of the shadow economy when the tax rate is increased. Those mechanisms depend on the changed possibility of the legal firm to make profit when the tax rate is greater. Consider s small initial shadow economy. A sharp decline in profit \(p_l x_l\) reduces access to public goods, slowing down expansion of the shadow economy. A minor decline in profit, however, makes tax revenue and hence supply of public goods sustainable. Consumers can move to the shadow economy with less concern of what happens to public goods. When the shadow economy has reached more consumers, the sign of \((1/3 - x_i)\) changes, the acclerating secondary effect disappears and the profit effect starts to decelerate. Despite such dynamic mechanisms, the
economy settles down in an equilibrium with positive tax revenue and positive supply of public goods as long as the profit of the legal firm, \( p_t x_t \), does not vanish.

From a dynamic perspective, we have the following result

**Proposition 4**  *Rising tax rate tends to expand the shadow economy through the tax avoidance effect. There is an associated public goods effect under the requirement of balanced budget, which accelerates or decelerates the expansion of the shadow economy, depending on the possibility of the legal firm to sustain profitability when the tax rate is raised.*

### 3.3 Some Welfare Effects

Above we arrived at the market solution with a given tax rate, establishing the condition for existence of the shadow economy in industry equilibrium. It is natural to address the welfare gains and losses associated with the emergence of the shadow economy.

The major welfare loss arises from redistribution of tax burden between honest and dishonest consumers, i.e. the fiscal externality. It is not necessarily the case that the tax burden of each honest consumer increases but they definitively remain the sole contributors to production of public goods. This tends to lead to social disapproval effect to be analyzed in the next section. Moreover, with a given tax rate, the total tax revenue is reduced. This limits the supply of public goods and reduces the well-being of all consumers, both tax-payers and free-riders. Underprovision of public goods arises. Apart from the fiscal externality, there is, however, a welfare gain to the extent that the legal producer faces a more intensive competition in the market. These two effects are opposite, leaving us with the following conclusion

**Proposition 5**  *In the market solution, the shadow economy results in a welfare loss arising from underprovision of public goods; but there is also a welfare gain arising from lower prices.*

With a competitive market and with a limited initial pricing power of the legal firm, it apparently is the welfare loss which dominates. With strong pricing power of a brand producer in circumstances of rather inelastic demand, it cannot be excluded that it may be the welfare gain which dominates. Our next question is whether moral sentiments can help to control the market share of illicit production and whether such a moral-based control is socially desirable. It will turn out that, apart from the two mechanisms studied above, the answer depends also on the view one takes on government as tax collector.
3.4 Moral Norms in Control of Illicit Transactions

**Morality as a Group Phenomenon** We now introduce two mechanisms to control illicit transactions. First, we introduce moral sentiments in the values by consumers. Second, we introduce the cost of holding cash which turns out to operate analogously to the tax on legal transactions.

In addition to their basic willingness to pay, consumers are now assumed also to value self-esteem. Moreover, they are sensitive to social disapproval. In other words, they care much of what they think of themselves and what the other people think of them. The strength of self-esteem effect is measured by parameter $s > 0$ and it is uniform across people. Only deviants are willing to give it up.

The moral sentiments of, say poor and rich are thus equal and independent of, their incomes. The total marginal utility of consumers buying a legal product is then $gr^k + s$ and is uniformly distributed over $[s, g + s]$. Alternatively, consumers can anonymously visit the illicit market with catching-up probability $\xi$. If caught, they suffer from social disapproval cost, $z > 0$.

The model of the previous section is now extended in that the disapproval is assumed to be expressed by those people who adhere to the social norm of visiting the legal market only. The disapproval effect thereby becomes a group phenomenon, eroding with the contraction of the legal market. In terms of consumer valuation, the legal product thus is valued at $gr^k$ while the illegal product is valued at $gr^k - x_l \Delta$, where $\Delta$ is the expected social punishment, $\Delta = E[P]$ with

$$ P = \begin{cases} z & \text{with probability } \xi \\ 0 & \text{with probability } 1 - \xi. \end{cases} $$

We consider fulfilled expectations equilibrium. We assume further that payments in the illicit market are made by cash. More efficient means of payments dominate in the legal market. There is thus an extra cost of making payments in the illicit market in terms of the cost of holding cash. The cost differential is denoted by $\gamma > 0$. It would be possible to interpret the model as a cash-in-advance variety with $p_i$ measuring the amount of cash demanded.

Consumers visiting the legal and illicit markets then have net utilities,

$$ u^k = gr^k + s - (1 + \tau) p_l, \quad u^j = gr^j - x_l z \xi - (1 + \gamma) p_i. $$

---

18 Existence of moral sentiments has been well-known in economics ever since Adam Smith (1966) and studied more recently by Frank (1987, 1988). The origin of those sentiments has been traced both to genetic forces in evolutionary biology or to memes, cultural genes, cf. Dawkins (1976).

19 There is no particular need to introduce any restriction on whether it is the self-esteem or the social disapproval which has greater weight in people’s valuation. The model allows for both cases.
In our model, the mass of people will be endogenously distributed into moral and immoral ones in terms of their behavior. In equilibrium people thus differ by their factual behavior. Moral (honest) and immoral (dishonest) behavior is endogenously determined and so is the strength of the social disapproval effect \( x_i \xi \). Morality as social capital is thus determined by the moral network, the expected relative size of the group of people expressing social disapproval in case of detection. The deviants\(^{20}\) are viewed as those taking the risk of being subject to stigma.\(^{21}\)

Denote again the marginal consumers by \((m, n)\) where \(m\) is indifferent between buying the legal and illicit product while \(n\) is indifferent between buying the illicit product and buying none. Thus the marginal utility for any consumer, say \(k\), for buying the legal product is \(g(1 - k)\). Then the marginal utility for \(j\) for buying the illicit product is \(g(1 - j) - \xi x_i\). For the marginal consumer, her net marginal utilities have to be equal \(g(1 - m) + s - (1 + \tau) p_l = g(1 - m) - \xi x_i - (1 + \gamma)p_i\).

The price differential now satisfies \((1 + \tau)p_l - (1 + \gamma)p_i = s + \xi x_i\). Thus, both the self-esteem effect and the group effect of moral sentiments are reflected in the price differential, as the legal producer can exploit them. For the marginal consumer \(n\), the net utility from buying the illicit product is zero, \((1 - n)g - \xi x_l - (1 + \gamma)p_i = 0\). Noting that the \(n\) is the last buyer, i.e. \(n = x_l + x_i\), her net utility is \((1 - x_l - x_i)g - \xi x_l - (1 + \gamma)p_i = 0\). The total production thus satisfies \(x_l + x_i = g - \xi x_l - (1 + \gamma)p_i\).

Solving for the prices

\[
\begin{align*}
 p_i &= \frac{(1 - x_l - x_i)g - x_l \xi}{(1 + \gamma)}, \\
 p_l &= \frac{(1 - x_l - x_i)g + s}{(1 + \tau)}.
\end{align*}
\]

**Nash equilibrium** With reaction functions of the full model

\[
\begin{align*}
 x_l &= \frac{g + s - (1 + \tau) c_l - x_i}{2}, \\
 x_i &= \frac{g - (1 + \xi) x_l - (1 + \gamma) c_i}{2},
\end{align*}
\]

\(^{20}\)The deviants might value positively other deviants, like members in gangs of subcultures. It would be easy to extend the model in this direction but for simplicity, we abstract from it.

\(^{21}\)The stigma effect has previously been discussed in psychology by Puhl and Brownell (2003) or Schulze and Angermeyer (2003).
it is standard to solve for the Nash equilibrium

\[
x_l = \frac{2s + g + (1 + \gamma) c_i - 2(1 + \tau) c_l}{3g - z\xi} \quad (12)
\]

\[
x_i = \frac{(g - z\xi) g - 2g (1 + \gamma) c_i - (g + z\xi) (s - (1 + \tau) c_l)}{(3g - z\xi) g} \quad (13)
\]

\[
p_l = \frac{(g + 2s + (1 + \gamma) c_i) g + (g - z\xi) (1 + \tau) c_l}{(3g - z\xi) (1 + \tau)} \quad (14)
\]

\[
p_i = \frac{(g - z\xi) (g + (1 + \gamma) c_i) - (g + z\xi) (s - (1 + \tau) c_l)}{(3g - z\xi) (1 + \gamma)} \quad (15)
\]

We comment on equilibrium below.

### 3.5 Competition under Morality

**Comparative Statics** We first develop technically the comparative static effects. Their algebra is subject to condition \( g - \xi z > 0 \). This condition only states the natural requirement that the expected cost of punishment cannot be greater than the maximal willingness to pay. Then, the equilibrium is characterized by the following comparative static results

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These results are to be discussed below.

**Self-Esteem as Conscience and Social Punishment** Comparative static analysis shows the potentially powerful effects of self-esteem and the risk of being caught when deviating and being punished. Those effects tend to maintain the legal sector large. Evaluating we find \( \partial x_l / \partial s = \frac{2}{3g - z\xi} \). This is positive as \( 3g - \xi z > 0 \). Self-esteem operates like a conscience for an individual, supporting the legal production. The mechanism that the social punishment effect depends on the size of the legal sector is confirmed by the result \( \partial x_l / \partial z = \frac{\xi x_l}{3g - z\xi} \).
The price effects are non-trivial. This results from that the self-esteem and social disapproval effects tend to be priced not only in the illicit product. Their effect also spills over to the legal market. It is easy to see that the net surplus of the marginal consumer in the legal market is lower when he is subject to self-esteem. Insert the expressions for the quantity and price from above in the surplus of the marginal consumer, \( g(1 - m) + s - (1 + \tau)p_l \) and develop the partial derivative with respect to \( s \). This suggests that with consumers becoming more moral, they pay a price in terms of reduced surplus. The marginal consumer is now the one who previously was a shadow market visitor. They keep the option of returning to the shadow market. It is easy to see that the greater is the shadow market, the more valuable is this option. Pricing of the shadow market producer, however, reduces this option value.

There is also a self-reinforcing mechanism in the expansion of the shadow economy. To see this, consider two industry equilibria with different social punishment rates, \( z_L < z_H \). From above, we find that in the former case the industry equilibrium is characterized by a greater shadow economy than in the latter case. Thus, when starting with the same initial equilibrium, the expansion of shadow market is in the former case greater than in the latter case. This effect arises from the self-reinforcing element of the diminishing group effect, i.e. erosion of social norms with the expansion of the shadow economy. Even without such a deterioration, the former industry would settle down in larger shadow sector than the latter industry. The group effect makes the industry structure even more differentiated.22

We conclude

**Proposition 6** Moral sentiments in terms of self-esteem and disapproval of deviations from social norms operate as mechanisms which limit the shadow market activities. The social disapproval effect, however, tends to be diluted with expansion of shadow economy, providing a further driving force for further shadow market transactions.

Shadow economies arise from tax avoidance. It is of interest to notice that the cost of holding money operates very much like a commodity tax in our model. To see this, solve for the profits

\[
(p_l - c_l) x_l = \frac{g}{(1 + \tau)} (x_l)^2
\]

22Despite that the equilibrium outcome is characterized by stratification of consumers, the shadow markets do not perform a screening function of people between "honest" and "dishonest" in our model, as all consumers are ex ante identical in terms their preferences for moral sentiments.
\[(p_i - c_i)x_i = \frac{1}{(1 + \gamma)}(x_i)^2.\]

Thus, the tax rate and the cost of holding cash have a systematic impact on the profits of the two producers. Even with uniform cost of transactions, we have

**Proposition 7** Under the norm that legal transactions give rise to self-esteem, the legal output tends to exceed the illicit one.

**Proof.** In the case of duopoly with \(s = z = 0\) and having uniform cost of transactions across means of payment, \(\tau = \gamma\), firms are symmetric and share markets on an equal basis. In the current model, positive self-esteem raises the demand in the legal market, while it reduces the demand in the illegal sector.\]

This result of course continues to hold when \(\tau < \gamma\) but it need not hold if \(\tau > \gamma\). We also point out that the result need not hold in an economy where it is commonplace to avoid taxes and where the self-esteem is not shaken by visits to shadow markets.

4 Shadow Economy and Leviathan Government

4.1 Laffer Curve

The proponents of the view of government as revenue-maximizing Leviathan which uses resources inefficiently obviously welcome the shadow economy.\(^{23}\) This section studies the effects of illicit transactions on an economy’s Laffer-curve. Suppose that the tax revenue, \(T\), is only partly allocated to public goods and that the government is able to extract a fraction, say \(0 < y < 1\) for its own use. Then the resources available for financing public goods are

\[g = (1 - y)T.\]

Assume that the government chooses the tax rate \(\tau\) to maximize its tax revenue collected from the sales of the legal firm, \(T = \tau p_l x_l\). We notice that such a government profile, though a burden on tax paying honest consumers,

\(^{23}\)Waste of tax revenue may result, for example, from influence or bribes by powerful lobbies leading to inefficient public spending. Grossman (2002) shows that if the technology of predation is sufficiently effective in a society, then having a "king" is better for everyone even though the king maximizes the consumption of a ruling elite.
is not necessarily detrimental to public goods production as an increase in tax revenue means also an increase in the supply of public goods. What matters is the magnitude of the fraction, $y$, which the government extracts. We assume that $y$ is constant. Then maximization of $(1 - y)T$ is equivalent to maximizing $T$.

Inserting the legal price and quantity yields

$$T(\tau, s, z, \xi) = \frac{\tau}{(1 + \tau)(3g - z\xi)^2} P(\tau),$$

where

$$P(\tau) = ((g + 2s + (1 + \gamma)c_i)g + (g - z\xi)(1 + \tau)c_i)(2s + g + (1 + \gamma)c_i - 2(1 + \tau)c_i).$$

The self-esteem and social disapproval effects thus interfere with the Leviathan government’s ability to raise tax revenue. The revenue $T(\tau, s, z, \xi)$ is a product of two terms. It is zero when any of those terms is zero, non-zero otherwise. The first of those terms is $\frac{\tau}{(1 + \tau)(3g - z\xi)^2}$ which is zero when the tax rate is zero. The second one $P(\tau)$ is a polynomial of second degree in the tax rate. It has two roots. Recalling that $g - z\xi > 0$, the coefficient of $\tau^2$ is

$$-2(g - z\xi)(c_i)^2 < 0,$$

making $P(\tau)$ a downward-sloping parable. It has two real roots. Thus, the three roots of $T(\tau, s, z, \xi) = 0$ are

$$\tau_1 = 0$$

$$\tau_2 = -\frac{(g + 2s + (1 + \gamma)c_i)g + c_i(g - z\xi)}{c_i(g - z\xi)} < 0$$

$$\tau_3 = \frac{g + 2s - 2c_i + c_i(\gamma + 1)}{2c_i} > 0$$

The positivity of $\tau_3$ follows from that $g + 2s - 2c_i + c_i(\gamma + 1)$ is less than the numerator of $x_i$ which has been recognized to be positive.

Excluding the negative tax rate, we first prove

**Lemma 8** With low (positive) tax rates, tax revenue (Laffer curve) is increasing in the tax rate.

**Proof.** Evaluating the slope of the tax revenue function at origin, we obtain $(\partial T/\partial \tau)_{\tau=0} = p_l x_l + \tau [x_l (\partial p_l/\partial \tau) + p_l (\partial x_l/\partial \tau)]$. With an active legal market, $p_l x_l > 0$ while with $\tau = 0$, the last term is zero. The result follows then by continuity. ■

Thus, the Laffer curve is increasing in the tax rate when the tax rate is low. We also prove that
Lemma 9  The legal sector disappears at tax rate $\tau_3$.

Proof. Inserting the solution $\tau_3$ into the expression for $x_l$, one finds that $x_l = 0$. \hfill \blacksquare

It follows from these two Lemmas that there is unique tax rate $\tau^*$, say $0 < \tau^* < \tau_3$, at which tax revenue has the global maximum.

Proposition 10  The Laffer curve has a unique maximum.

We note that from above, the optimal tax rate $\tau^*$ is close to $\tau_3/2$ but marginally below it because the revenue $\frac{\tau}{(1+\tau)(3g-z\xi)}P(\tau)$ is not a perfectly symmetric parable. As $\tau_3$ positively depends on public goods, self-esteem, the cost of legal production, the cost of holding money and negatively on the cost of legal production, so does presumably the tax rate $\tau^*$.

4.2 Shadow Markets and Tax Revenue

The intuition suggests that having an untaxed sector reduces the resources which the government can extract from consumers. Whether this is socially desirable is another matter and depends on to what extent people value public goods. Moreover, when moral sentiments become important, consumers do not necessarily switch from the taxed sector to the untaxed sector, relaxing the effective constraint on revenue maximizing behavior. It is harder
to predict what the revenue maximizing tax rates are. The government is interested in the tax revenue only.

Let us contrast the cases of one producer and two producers. With one producer, the marginal consumer, say consumer $m$, is indifferent between buying and not buying when the price is $p_M = \frac{(r^m + s)}{(1 + \tau)}$. The willingness to pay can be solved from the condition $\frac{g - r^m}{x_M} = g$. Thus, $r^m = (1 - x_M)g$. Maximizing the monopoly profit gives,

$$x_M = \frac{g + s - (1 + \tau)c_M}{2} = \frac{1}{2}g + \frac{s - (1 + \tau)c_M}{2}$$ \hspace{1cm} (18)

$$p_M = \frac{g + s + c_M(1 + \tau)}{2(1 + \tau)}.$$ \hspace{1cm} (19)

We find that the monopoly profit satisfies

$$\pi_M = \left( \frac{1}{(1 + \tau)} \right) (x_M)^2 = \frac{1}{(1 + \tau)} \left( \frac{g + s - (1 + \tau)c_M}{2} \right)^2.$$ 

The profit for a legal producer in duopoly is $(p_l - c_l)x_l = \frac{1}{(1 + \tau)}(x_l)^2$. Under natural conditions, $x_l < x_M$ as in duopoly, market is shared by two producers. Moreover, $p_l < p_M$. This means that the tax base in smaller in duopoly than in monopoly. The same holds for tax revenue. Untaxed sector reduces the resources which the Leviathan-government can extract from consumers.

Compare next tax revenues with consumers having moral sentiments and without them. We can establish that $\frac{\partial T}{\partial s} = \tau[\frac{\partial p_l}{\partial s}x_l + \frac{\partial x_l}{\partial s}p_l] > 0$. Similarly, $\frac{\partial T}{\partial z} > 0$. These results follow from that $\frac{\partial p_l}{\partial s} > 0$, $\frac{\partial x_l}{\partial s} > 0$, $\frac{\partial p_l}{\partial z} > 0$. What this means is that formation of moral sentiments leads to an increase in the tax base.

5 Further Notes on Welfare Effects

Our model above left us with the conclusion that a shadow economy gives rise not only to welfare losses but also to welfare gains. There is more to be said. Once the government has selfish Leviathan motives for grasping part of the tax revenue, such efforts should be controlled. Shadow markets discipline government spending. They might lead to further welfare gains apart from their role in controlling pricing and monopoly profits. The fiscal externality on tax-payers arising from free-riding in the financing of public goods remains on the cost side. People’s moral attitudes tend to control those who resort to shadow markets. With a small shadow economy, the social punishment
of deviants is severe. With an expanded shadow economy, the punishment is diluted. The private benefit to those obeying social norms is reflected in self-esteem but is priced by the profit maximizing firm. The incidence of moral standards is hence split between the honest consumers and the legal firm.

As there are both welfare gains and losses, it is far from trivial to suggest what the "welfare optimum" might be, as it would depend on number of mechanisms. As it appears that shadow economies perform also a positive function in an economy, we explore the issue by a formal discussion.

**Fixed supply of public goods** Consider an economy with inherited infrastructure, i.e. assume fixed supply of public goods. Introduce \( g = 1 \). Formally, the utilitarian view would explore the maximal aggregated consumer and producer surplus \( W = CS_l + CS_i + \pi_l + \pi_i \) subject to the relevant constraints.\(^{24}\) Here \( CS_l \) and \( CS_i \) denote the after-tax expressions for the consumer surplus and \( \pi_l \) and \( \pi_i \) stand for the profits. Welfare maximum satisfies

\[
\max_{x_l, x_i, x_O} L = W + \lambda_l x_l + \lambda_i x_i + \lambda_O x_O
\]

\[
+ \mu[1 - x_l - x_i - x_O],
\]

where \( x_O \) is the number of consumers who do not buy and where \( \lambda_i \geq 0, \mu \geq 0 \) are the relevant shadow prices. It is immediately clear that the solution to this problem clearly deviates from the market solution because of the externalities involved.

To develop an intuition, consider first the case in the absence of moral sentiments and assume that the cost of production is nil, \( c_l = c_i = 0 \). It is then easy to see that in the market equilibrium, a commodity tax on the legal good operates like a lump-sum tax, borne by the shareholders of the legal firm. The legal price fully accommodates the tax and production efficiency holds in the market (Nash) equilibrium. The tax effects are limited to income effects, with the tax falling on the legal firm’s owners. In the absence of production cost, the optimal profits is low and deviates from zero only to the extent the cost of being caught in the shadow market makes the legal producer charge a positive price. Optimal allocation of consumers into the legal and shadow economy would capture the idea that the marginal utility of the last legal consumer would equal the marginal utility of the first consumer in the shadow market. The cost of being caught and the tax rate would be the decisive determinants of the optimal allocation. With fixed supply of public goods

\(^{24}\)The welfare effect of imperfect competition *per se* is not discussed.
goods, it would be socially desirable to eliminate the wasteful tax collection and allocate the whole production to the shadow economy. All consumers would consume the product.

With \( c_l > 0, c_i > 0 \), however, the desired allocation would be different. The price of the legal product would have to be higher as the price in the shadow market would have be positive. Positive tax revenue would be generated. With \( c_l > 0 \), the commodity tax would not be fully borne by the shareholders becoming distortive and reducing the optimal production of the legal firm and hence resulting in a welfare loss. The shadow economy would clearly remain large in the welfare optimum also of such an economy.

**Moral Sentiments and Welfare** The size of the shadow economy, however, will be controlled if people are subject to moral sentiments. The prediction is that this makes the legal production greater. However, neither in this case is the first-best attainable in the market because of the externalities involved. With expressions for consumer and producer surplus developed in Appendix A, the social first-order conditions for an interior solution can be derived as

\[
x_i = \varphi_o - \varphi_1 x_l, \quad x_l = \phi_0 - \phi_1 x_i,
\]

where

\[
\varphi_o = \frac{1 - (1 + \gamma) c_i}{1 - \gamma}, \quad \varphi_1 = \frac{(1 + z\xi) (1 + \tau) - \tau (1 + \gamma)}{(1 + \tau) (1 - \gamma)}
\]

\[
\phi_0 = \frac{1 + s - (1 + \tau) c_l}{(1 - \tau)}, \quad \phi_1 = \frac{\tau (\gamma + 1) - (1 + z\xi) (1 + \tau)}{\gamma + 1}.
\]

We notice that \( x_l = \phi_0 \) is the socially optimal output if only one firm produces (see Appendix A for derivation with \( g = 1 \) and \( \lambda_g = 0 \)). The second term in the expression for \( x_i \) thus states the effect of the illicit competitor.

To qualify for an interior optimum, the market shares have to fall within \((0, 1)\). Moreover, their sum must also fall within \((0, 1)\). Finally, the lines (21) must cross within the triangle, cf. Figure 2. Formally, the conditions are (i) \( 0 < x_l, x_i < 1, (ii) 0 < x_l + x_i < 1, (iii) \varphi_1 \neq \frac{1}{\phi_1} \).

In terms of \( 0 < \varphi_o < 1 \) and \( 0 < \phi_0 < 1 \), conditions (i) give

\[
0 < \frac{1 + s - (1 + \tau) c_l}{(1 - \tau)} < 1, \quad 0 < \frac{1 - (1 + \gamma) c_i}{1 - \gamma} < 1.
\]

The first of these, respectively the second, gives

\[
\tau < \min \left( \frac{c_i - s}{1 - c_l}, \frac{1 + s - c_l}{1 - c_i}, 1 \right), \quad \gamma < \min \left( \frac{c_i}{1 - c_l}, \frac{1 - c_i}{c_l} \right).
\]
Therefore, for the legal sector to survive, tax rate should not be too high. For the shadow economy to have a chance, the cost of holding cash should not be inflated too much.

What condition (ii) means is that when \( x_i \to 0 \), then \( \varphi_o/\varphi_1 < 1 \), and when \( x_l \to 0 \), then \( \phi_o/\phi_1 < 1 \). Condition (iii) requires that the lines cross. This is equivalent to,

\[
\text{if } \frac{1 + s - (1 + \tau) c_l}{(1 - \tau)} > (1 + \gamma) \frac{(1 + \tau)(1 - (1 + \gamma) c_l)}{(1 + z \xi)(1 + \tau) - \tau (1 + \gamma)} \\
\text{then } \frac{1 - (1 + \gamma) c_i}{1 - \gamma} > (1 + \gamma) \frac{1 + s - (1 + \tau) c_l}{(1 + z \xi)(1 + \tau) - \tau (1 + \gamma)}
\]

Consider these requirements in terms of moral sentiments. We notice that the roles of self-esteem and social disapproval are interrelated. We show that the case (\( < \)) can be excluded. Strong self-esteem and strong moral punishment result in that \( \frac{1 + s - (1 + \tau) c_l}{(1 - \tau)} > (1 + \gamma) \frac{1 - (1 + \gamma) c_l}{(1 + z \xi)(1 + \tau) - \tau (1 + \gamma)} \). Then, the second condition \( \frac{1 - (1 + \gamma) c_i}{1 - \gamma} > (1 + \gamma) \frac{1 + s - (1 + \tau) c_l}{(1 + z \xi)(1 + \tau) - \tau (1 + \gamma)} \) implies that the for the shadow economy to survive as part of social optimum, the self-esteem cannot yet be too great, i.e. there is an upper limit for the magnitude of \( s \). Alternatively, for the legal sector to survive, the expected punishment \( z \xi \) cannot be too small. We thus see that people’s values in terms of moral sentiments will significantly interact with the desirability or undesirability of shadow markets. On the other hand, if people lack from self-esteem, \( s = 0 \), and it is the latter case which becomes relevant. Then for the legal sector to survive, the cost of punishment for deviation \( z \xi \) should not too large. As this tends to lead to strong shadow economy, it is logically incompatible and can be excluded.

Collecting the findings,

**Proposition 11** For the shadow economy to exist in the social optimum along with the legal sector, it is necessary that people’s moral sentiments do not punish illicit transactions too heavily. Should the legal sector survive with people having incentives for tax evasion, the tax rate cannot too high.

In terms of Figure 2 in Appendix B, the equilibrium has to be located in the lower part of the unit square below the 45\(^\circ\) line, say at point \( E \).

### 5.1 Wasteful Tax Collection

One should not only pay attention to people’s moral sentiments in the social first-order conditions, but also to the possibility to finance public goods, threatened by shadow markets. The task would again be to identify the marginal consumer who is indifferent between the legal and illicit consumption
subject to the additional condition that his valuation of the public good coincides with how much he has to pay for public goods. To fix ideas, return to the assumption that the share that the tax collector can waste tax revenue is given by $0 < y < 1$. Public goods available then have to satisfy $g = (1 - y)T$. Denote by $\lambda_g$ the relevant shadow price. The conditions which must hold in the second-best social optimum are

\[
W_{x_l} - \lambda_g (1 - y)T_{x_l} = 0 \\
W_{x_i} - \lambda_g (1 - y)T_{x_i} = 0 \\
W_g + \lambda_g [1 - (1 - y)T_g] = 0 \\
g - (1 - y)T = 0.
\]

We have developed the expressions for the consumer surplus in Appendix A. Depending both on the tax rate and the moral sentiments, the social optimum can be a corner solution or an interior solution. An interior solution is again characterized by $\lambda_l = \lambda_i = \lambda_g = \mu = 0$. When the shadow economy exists in the social optimum, it must hold that the net marginal social utility from keeping the marginal consumer in the legal sector coincides with the marginal social cost in terms of the tax liability. In the internal optimum, $W_{x_l}/T_{x_l} = W_{x_i}/T_{x_i}$. In Appendix A, we derive the social first-order conditions in terms of $(x_l, x_i, \lambda_g, g)$. It is not possible to solve them explicitly because the public goods, $g$, ought to be solved from a second-order equation. Therefore, we resort to characterization of the optimum.

The intuition suggests that there is a case for allocating people to the shadow economy to limit wasteful tax collection. However, the target of financing public goods requires an opposite action and necessitates a smaller shadow economy. To take a position with respect to these conflicting targets necessitates a study of how the government’s share $y$ really is determined. This is an empirical matter. We observe from the social first-order conditions, however, that an increase in the social shadow price of tax revenue $\lambda_g$ does not need to lead to contraction of the shadow market in the welfare optimum. This is unexpected. The model is consistent both with an expansion and contraction. The outcome depends on the magnitudes of the parameters of the model. This conclusion challenges the view that shadow markets necessarily are welfare-enhancing in their ability to limit wasteful tax collection. The reason for ambiguity is that the tax collector only extracts part of the tax revenue, while the rest is used to satisfy people’s demand for public goods.

Proposition 12 It is not necessarily the case that shadow markets enhance welfare by limiting wasteful tax collection.
One way to look into this issue is to note that wasteful tax collection operates much like the fiscal externality caused by consumers in the illicit market. Again, tax-paying consumers lose as they are the sole bearers of the burden of financing public goods while all have access to these goods.

**Lemma 13** Increased wasteful tax collection operates like the fiscal externality of an increase in the number of non-tax paying consumers.

**Optimal Tax** Having shaken the argument that the shadow economy is good to have to control wasteful tax collection, we shake another argument, i.e. why it should not exist. Consider once more an economy with zero production cost \( c_l = c_i = 0 \). This is often the case with many high-tech digital products. As we have stated, a commodity tax on the legal product operates like an income levy on the shareholders of the legal firm. For maximal supply of public goods, it it then optimal to maximize the tax revenue available from the consumers of the legal firm. It falls on the firm’s owners. Even this result does not imply that a shadow economy should not exist. The reason is that competition resulting from having the shadow market reduces the legal price, resulting in a welfare gain. Thus,

**Proposition 14** In the case of products with zero cost of production, the social planner ought to behave like the Leviathan, maximizing the tax revenue from legal consumers but should not necessarily eliminate the shadow economy.

### 6 Final Remarks

We have studied industrial structure when consumers can visit either legal or illicit markets. Contrary to ex ante beliefs, entry of an illegal economic does not necessarily decrease welfare. The welfare gain is obvious as the legal firm is not able to exploit the full consumer surplus. However, the mechanisms are many. When people’s market behavior also reflects their moral sentiments, the legal firm can price the self-esteem of honest people. On the other hand, the buyers in the legal market have the option of visiting the illicit market. Such an implicit blackmailing option makes them earn surplus over the price charged by the legal producer. Those who visit the illicit market benefit from untaxed products, though they subject themselves to moral disapproval. Untaxed products tend also to attract new buyers.

The paper has implications for monetary economics. Ever since Keynes (1936), work on monetary economics has identified the various motives for
why people hold cash in terms of transactions, precautionary and speculative needs. The current paper has explored the microeconomics of a fourth motive, the value of holding cash for anonymous transactions in a shadow economy. Our theorizing leads to a number of new testable implications, not yet analyzed in the monetary economics. By implication and in particular, demand for money depends not only on illegal activities in the economy and on the size of the shadow sectors but also on social norms and morality of people, and the resulting social capital. In addition, it depends on the crime rates, probability of being caught and the resulting punishment from illegal activities. It also depends on the competitiveness and hence excess profits of the legal system and the general level of taxation, and on the industrial structure with industries prone to develop shadow transactions.

A Expressions for consumer surplus

The expressions for consumer surplus are

$$CS_l = \frac{1}{2} g (1 - r^m) x_l + (gr^m + s - (1 + \tau)p_l)$$

$$= \left(\frac{1}{2} x_l + x_i\right) x_l g$$

$$CS_i = \frac{1}{2} (gr^m - x_l z \xi - (1 + \gamma) p_l) x_i = \frac{1}{2} g (x_i)^2.$$  

Thus,

$$CS^D = \frac{1}{2} g (x_l + x_i)^2.$$  (A1)

Under monopoly, consumer surplus is

$$CS_M = \frac{1}{2} g (1 - r^m) x_M = \frac{1}{2} g (x_M)^2.$$  (A2)

The welfare measure with $g = 1$ is provided by

$$WD = \frac{1}{2} (x_l + x_i)^2 + (p_l - c_l) x_l + (p_i - c_i) x_i.$$  (A3)

where the prices can be inserted from the consumer indifference conditions. If only one firm produces the welfare is

$$W_M = CS_M + \pi_M = \frac{1}{2} (x_M)^2 + (p_M - c_M) x_M,$$  (A4)

resulting in optimal output $x_M = \left(\frac{c_l - \frac{s+1}{(1+\tau)}}{1 - \frac{2}{(1+\tau)}}\right) = \phi_0.$

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When public goods are optimized, the social first-order conditions must acknowledge the link between the tax revenue and public goods. Social welfare is linear in $g$ for the reason that consumer surplus and profits (through the prices) are linear in $g$. Maximizing $W + \lambda_g[g - (1 - y)T]$ with respect to $(x_l, x_i, g)$ gives $A5 - A7$. Then substituting in $A8$, gives an equation for optimal $g$. It is a second-order equation.

\begin{align*}
x_l &= \frac{(1 - (1 - y) \tau \lambda_g) (g + s) - (1 + \tau) c_l}{((1 - \tau) - 2\tau \lambda_g (1 - y)) g} - \frac{(1 + \tau) \left(1 + \frac{s \xi}{g}\right) - (1 + \gamma) \tau (1 + (1 - y) \lambda_g)}{((1 - \tau) - 2\tau \lambda_g (1 - y)) (1 + \gamma)} x_i \tag{A5} \\
x_i &= \frac{1 - (1 + \gamma) \frac{c_i}{g} - \frac{(1 + \frac{s \xi}{g}) - \frac{\tau}{1+\tau} (1 + \gamma) (1 + (1 - y) \lambda_g)}{1 - \gamma} x_l}{(1 - \gamma)} \tag{A6} \\
\lambda_g &= \frac{(1 + \tau) (x_l + x_i)^2 + 2(1 - x_l - x_i) \left(x_l + \frac{1+\tau}{1+\gamma} x_i\right)}{2 ((1 + \tau) - (1 - y) \tau x_l (1 - x_i - x_l))} \tag{A7} \\
g &= (1 - y)T \tag{A8}
\end{align*}

\section*{B Optimal industrial structure with shadow economy}

The equilibrium outcome is illustrated in Figure 2. The horizontal and vertical lines at $x_l = 1$ and $x_i = 1$, represent the unit square. The diagonal line, that bisects the unit square, represents the $45^\circ$-line with $x_i + x_l \leq 1$. The optimal outcome must be located below this line. Candidate lines $x_i = \phi_0 - \phi_1 x_l$, $x_l = \phi_0 - \phi_1 x_i$ which satisfy this constraint, are illustrated by a pair of declining lines like $R_1$ and $R_2$ or $R_3$ and $R_4$ that pass through point $E$. However, as they must satisfy the constraints $\phi_0 < 1$ and $\varphi_0 < 1$, lines $R_3$ and $R_4$ cannot represent the optimal choice functions.
Figure 2: Optimal industrial structure with shadow economy

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