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Working Paper

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CESifo Working Paper, No. 1448

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Suggested Citation: Gersbach, Hans; Haller, Hans (2005) : Bargaining power and equilibrium consumption, CESifo Working Paper, No. 1448, Center for Economic Studies and Ifo Institute (CESifo), Munich

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BARGAINING POWER AND EQUILIBRIUM CONSUMPTION

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CESifo Working Paper No. 1448
Category 10: Empirical and Theoretical Methods
April 2005

An electronic version of the paper may be downloaded
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Abstract

We examine how a shift of bargaining power within households operating in a competitive market environment affects equilibrium allocation and welfare. If price effects are sufficiently small, then typically an individual benefits from an increase of bargaining power, necessarily to the detriment of others. If price effects are drastic the welfare of all household members moves in the same direction when bargaining power shifts, at the expense (or for the benefit) of outside consumers. Typically a shift of bargaining power within a set of households also impacts upon other households. We show that each individual of a sociological group tends to benefit if he can increase his bargaining power, but suffers if others in his group do the same.

JEL Code: D10, D50, D62, D70.

Keywords: household behavior, bargaining power, local and global changes, price effects, general equilibrium.

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We thank Clive Bell, Martin Hellwig, Benny Moldovanu, Till Requate, participants in the Southeast Economic Theory Conference at Georgetown University, and seminar audiences in Basel, Berlin and Heidelberg for helpful comments. Hans Haller gratefully acknowledges the hospitality of the University of Heidelberg and the financial support of the German Science Foundation (DFG) through a Mercator professorship.
1 Introduction

Societies often experience a shift of bargaining power in households. For instance, *ceteris paribus*, the modern heterosexual couple (multi-member household) is distinguished from the traditional heterosexual couple (household) by a shift of bargaining power in favor of the female partner (female parent, woman in the household). Such a shift induces a change in household demand for goods and services. In turn, market clearing might occur at different prices and, consequently, the terms of trade for households might be altered.

It is the consequences, not the causes of shifts in intra-household bargaining power that interest us here. We are concerned with pure economic (positive) effects on the allocation of resources, as well as welfare (normative) effects at both the individual and societal levels. We are going to study those effects in a general equilibrium context. Our study reveals that the magnitude of equilibrium price responses to a shift of intra-household bargaining power matters. If price effects are sufficiently small, then typically an individual benefits from an increase of bargaining power — necessarily to the detriment of others. In particular, the other member(s) of the household will lose. In contrast, if price effects are drastic, then the members of the individual’s household all benefit or are all harmed. Typically a shift of bargaining power within a set of households also impacts upon other households. We show that each individual of a sociological group tends to benefit if he can increase his bargaining power, but suffers if others in his group enjoy more bargaining power. For quasi-linear preferences, however, a change of the bargaining power within a particular household only impacts on the distribution of the numéraire in the household under consideration without affecting the consumption of other commodities. A local change of bargaining power has no price effect and does not affect the utility of individuals in other households.

The underlying model of the household satisfies collective rationality in the sense of Chiappori (1988a, 1992). It departs from traditional economic theory which has, for the most part, treated households as if they were single consumers. The model admits households with several, typically heterogeneous members who have individual preferences. A household takes market prices as given and makes an efficient consumption choice (in terms of the preferences of its members) subject to its budget constraint. Different households may use different collective decision mechanisms. This departure

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1See also the surveys by Bourguignon and Chiappori (1992, 1994).
from the traditional market model enables us to investigate the interplay of dual roles of households: households as collective decision making units on the one hand and as competitive market participants on the other hand.

The current model starts from the general equilibrium model in Haller (2000) where the household structure is fixed.\textsuperscript{2} We specialize by assuming that the efficient collective household decision is the result of (possibly asymmetric) Nash bargaining within the household. This feature allows us to parametrize relative bargaining power, to perform comparative statics and to answer the question at hand, how a shift of bargaining power within households affects equilibrium allocation and welfare.

The model is introduced in the next section. In Section 3, we focus on a two-person household embedded in a larger economy and study how a shift of bargaining power within that household affects the consumption and welfare of its members. We decompose the intra-household effects into two relevant effects, a pure bargaining effect and a price effect. In the presence of negative intra-household externalities, there can be an equilibrium with free disposal where the budget constraint is not binding for the select two-person household and the household is not subject to a price effect. Typically, however, the price effect is non-zero. It can be small (negligible) or large (drastic).

In Section 4, we exemplify the different scenarios suggested by the general comparative statics of Section 2. We go through a sequence of representative examples, with a two-person household and a one-person household, and examine the general equilibrium implications of a shift of bargaining power within the two-person household. We observe that at least one member is always affected by a shift of bargaining power within the two-person household, but that the non-member may be affected as well. We observe further that price effects may be drastic if preferences exhibit little substitutability. We should mention that the findings for these two-household economies are also valid for respective replica economies obtained from the representative examples, provided that each of the two-person households of the replica economy undergoes the same shift of intra-household bargaining power. These shifts constitute a particular instance of a widespread shift of bargaining power in favor of a specific sociological group.

In Section 5, we investigate in more detail shifts of bargaining power in favor of

\textsuperscript{2}See Gersbach and Haller (2001, 2002) for versions with variable household structure.
a specific sociological group, with added emphasis on inter-household or spill-over effects. We distinguish between “first members” and “second members” of households. With particular consumer characteristics, spill-overs are absent: The effects of a change of bargaining power within a household are confined to that household. With different consumer characteristics, spill-overs can occur exactly as described earlier. For instance, a first member of a household benefits from an increase in own bargaining power, but loses if \textit{ceteris paribus} first members of other households gain more bargaining power. In Section 6, we offer concluding remarks.

2 General Equilibrium Model

We consider a finite pure exchange economy. The main departure from the traditional model is that a household can have several members, each with their own preferences.

\textbf{Fixed Household Structure.}

The population is divided into finitely many households \( h = 1, \ldots, n \), with \( n \geq 2 \). Each household \( h \) consists of finitely many members \( i = hm \) with \( m = 1, \ldots, m(h) \), \( m(h) \geq 1 \). Put \( I = \{hm : h = 1, \ldots, n; m = 1, \ldots, m(h)\} \), the finite population of individuals to be considered.

\textbf{Commodities, Endowments, and Individual Preferences.}

The commodity space is \( \mathbb{R}^\ell \) with \( \ell \geq 1 \). Household \( h \) is endowed with a commodity bundle \( \omega_h \in \mathbb{R}^\ell, \omega_h > 0 \). The aggregate or social endowment is \( \omega = \sum_h \omega_h \). A generic individual \( i = hm \in I \) has:

- consumption set \( X_i = \mathbb{R}_+^{\ell} \);
- preferences \( \succ_i \) on the allocation space \( \mathcal{X} \equiv \prod_{j \in I} X_j \) represented by a utility function \( U_i : \mathcal{X} \longrightarrow \mathbb{R} \).

The consumption bundle of a generic individual \( i \) is denoted by \( x_i \). Let \( x = (x_i) \), \( y = (y_i) \) denote generic elements of \( \mathcal{X} \). For \( h = 1, \ldots, n \), define \( \mathcal{X}_h = \prod_{m=1}^{m(h)} X_{hm} \) with generic elements \( x_h = (x_{h1}, \ldots, x_{hm(h)}) \). If \( x \in \mathcal{X} \) is an allocation, then for \( h = 1, \ldots, n \), household consumption is given by \( x_h = (x_{h1}, \ldots, x_{hm(h)}) \in \mathcal{X}_h \).
We will allow for the possibility of consumption externalities. Following Haller (2000), we shall restrict attention to the case where such consumption externalities, if any, exist only between members of the same household. This is captured by the notion of intra-household externalities where utility functions are restricted to the household consumption $x_h$, i.e.:

**E1) Intra-Household Externalities:** $U_i(x) = U_i(x_h)$ for $i = hm$, $x \in X$.

A special case is the absence of externalities to which we sometimes pay particular attention. When there are no externalities, the utility function of an individual $i$ depends only on his consumption bundle $x_i$, i.e.

**E2) Absence of Externalities:** $U_i(x) = U_i(x_i)$ for $i = hm$, $x = (x_i) \in X$.

With a fixed household structure, the latter condition is somewhat less restrictive than it seems. For suppose a consumer $i = hm$ cares about own consumption and household composition, which could be important for household formation. But if household membership, $i \in h$, is a fait accompli, one may omit $h$ as an argument of $i$’s utility function and work with the reduced form E2.

**Budget Constraints:** Now consider a household $h$ and a price system $p \in \mathbb{R}^\ell$. For $x_h = (x_{h1}, \ldots, x_{hm(h)}) \in X_h$, denote total household expenditure

$$p \ast x_h = p \cdot \left( \sum_{m=1}^{m(h)} x_{hm} \right).$$

Then $h$’s budget set is defined as $B_h(p) = \{x_h \in X_h : p \ast x_h \leq p \cdot \omega_h\}$. We define the efficient budget set $EB_h(p)$ by:

$x_h = (x_{h1}, \ldots, x_{hm(h)}) \in EB_h(p)$ if and only if $x_h \in B_h(p)$ and there is no $y_h \in B_h(p)$ such that

$U_{hm}(y_h) \geq U_{hm}(x_h)$ for all $m = 1, \ldots, m(h)$;

$U_{hm}(y_h) > U_{hm}(x_h)$ for some $m = 1, \ldots, m(h)$.

**General Equilibrium:**

A competitive equilibrium (among households) is a price system $p$ together with an allocation $x = (x_i)$ satisfying
(i) \( x_h \in EB_h(p) \) for \( h = 1, \ldots, n \), and

(ii) \( \sum_i x_i = \omega \).

Thus, in a competitive equilibrium among households \((p; x)\), each household makes an efficient choice under its budget constraint and markets clear. Efficient choice by the household refers to the individual consumption and welfare of its members, not merely to the aggregate consumption bundle of the household.

**Nash Bargaining.** An efficient household choice under a budget constraint may be the outcome of maximizing a function of the form

\[
W_h(x_h) = S_h(U_{h1}(x_h), \ldots, U_{hm(h)}(x_h)),
\]

subject to the budget constraint. A special case thereof is a Nash-bargained household decision. In this case, \( S_h \) assumes the form

\[
S_h(U_{h1}, \ldots, U_{m(h)}) = \prod_{m=1}^{m(h)} U_{km}^{\alpha_{hm}},
\]

(1)

with the provision that \( \alpha_{hm} \geq 0 \) and \( U_{hm} \geq 0 \) for \( m = 1, \ldots, m(h) \). The bargaining weight \( \alpha_{hm} \) measures the relative bargaining power of individual \( i = hm \) within household \( h \). In the sequel, we shall concentrate on two-person households, i.e. \( m(h) = 2 \). We assume \( \alpha_{h1}, \alpha_{h2} > 0 \) and \( \alpha_{h1} + \alpha_{h2} = 1 \).

The assumption of Nash-bargained and, hence, efficient household decisions serves us well for the present inquiry into the consequences of shifts of bargaining power. The empirical question of whether collective household decisions are Nash-bargained, indeed, has gotten a fair amount of attention, in particular in the debate between Chiappori (1988b, 1991) on the one side and McElroy and Horney (1981, 1990) on the other side (see Bergstrom (1997) for discussions). There has been a growing number of empirical studies performing empirical tests of the collective rationality approach which nests Nash bargaining models as particular cases (Udry (1996), Fortin and Lacroix (1997), Browning and Chiappori (1998), Chiappori, Fortin and Lacroix (2002), among others).

Two qualifying comments are warranted. First, the interpretation of the maximands of \( S_h \) as Nash-bargained outcomes assumes that for each member of a multi-person
household, the individual’s reservation utility level is zero. The choice of disagreement points for intra-household bargaining is somewhat controversial and depends on the assumed inside or outside options of household members. In Gersbach and Haller (2002), we consider for example an exit option, that is the possibility that a household member leaves, forms a single household and maximizes utility at the going market prices. Such an outside option would complicate notation and the formal analysis, but not alter the qualitative implications. Therefore, we opt here for a price-independent reservation utility which we normalize to zero solely for computational convenience. After a logarithmic transformation of the form (1), this household decision mechanism proves equivalent to the maximization of a utilitarian social welfare function for the household, where the bargaining weights become welfare weights.

Second, although maximization of the Nash product (1) describes the way in which the household reaches an efficient collective decision, it would be a grave mistake to attribute further meaning to the maximal value of (1) and to changes of it. Normative statements always refer to individuals, either one by one, identifying gainers and losers, or as constituents of society. Pareto-optimality and Pareto-improvements are defined in the standard fashion.

For welfare comparisons between societies which differ only with respect to the bargaining power of individuals in households, one can rely on a modified version of the first welfare theorem. With the possibility of multi-person households and intra-household externalities, the crucial property of the classical version of the first welfare theorem, local non-satiation needs to be adapted. The modified property stipulates that each household’s efficient choices under its budget constraint lie on the household’s “budget line”. Haller (2000) calls this property budget exhaustion. He shows the validity of the first welfare theorem for economies with the budget exhaustion property.

Except for subsection 3.2 the economies and corresponding examples in the paper all have unique competitive equilibria and possess the budget exhaustion property. Therefore, equilibrium allocations are Pareto-optimal and comparative statics moves the economy from one Pareto-optimum to another one. Consequently, if a household member gains from a shift in bargaining power, then someone else inside or outside the household must lose.
3 General Comparative Statics for a Two-Person Household

In this section we perform comparative statics with respect to the balance of bargaining power within a two-person household denoted by $h$. We allow for an arbitrary number of commodities and we consider the general case of intra-household externalities. The entire population consists of an arbitrary number, $n$ of households.

Negative intra-household externalities allow for the possibility that a household has a bliss point despite the fact that each household member has monotonic preferences with respect to her individual consumption (see Haller (2000) for examples). If this happens, the corresponding notion of competitive equilibrium among households has to be less demanding. The social feasibility or market clearing condition (ii) has to be replaced by the **free disposal condition**

\[ \sum_i x_i \leq \omega. \]

If in fact an equilibrium with free disposal prevails and the household does not exhaust its budget, then after a small shift of intra-household bargaining power, the resulting equilibrium will most likely be one with free disposal again and the household will still not exhaust its budget. As a consequence, the household’s budget constraint remains non-binding. This means that the household is not exposed to any price effect. In the sequel, we treat first the simpler case of non-binding budget constraint and, hence, zero price effect. We then proceed to the case of a binding budget constraint and typically non-zero price effect. This general comparative statics helps identify two relevant effects, a pure bargaining effect and a price effect.

3.1 Preliminaries

We shall perform comparative statics with respect to the bargaining weights within a select two-person household $h$, with members $h1$ and $h2$. Whenever convenient and unambiguous, we shall drop the household name and simply refer to consumers 1 and 2. Without restriction, we may also assume that our select household has the lowest number, i.e. $h = 1$ and the other households are labelled $k = 2, \ldots, n$. For the sake of convenience, we shall further adopt the notation $\alpha = \alpha_{h1}$ and $1 - \alpha = \alpha_{h2}$ so that comparative statics can be performed with respect to the parameter $\alpha \in (0, 1)$. Finally,
denote $F \equiv \ln S_h$. Explicitly, we obtain

$$F = F(U_1(x_h), U_2(x_h); \alpha) = \alpha \ln U_1(x_h) + (1 - \alpha) \ln U_2(x_h).$$

(2)

While $\alpha$ is treated as variable, the other characteristics of household $h$ as well as all the characteristics of the rest of the households remain fixed. Each household $k \neq h$ is assumed to choose an efficient consumption plan, $x_k \in EB(p)$. It may, but need not, maximize a Nash product.

We assume sufficient regularity in the sense that for each $\alpha \in (0, 1)$ the economy has an equilibrium $(p(\alpha); x(\alpha))$ satisfying:

(iv) local uniqueness and

(v) continuous differentiability in $\alpha$.

For each $\alpha$, at the given price system $p(\alpha)$, household $h$ solves the problem

$$\max F(U_1(x_h), U_2(x_h); \alpha) \text{ s.t. } G(x_h; \alpha) \leq 0$$

(3)

where $G(x_h; \alpha) = p(\alpha)[(x_1 + x_2) - \omega_h]$. The corresponding solution is $x_h(\alpha) = (x_1(\alpha), x_2(\alpha))$. The budget constraint $G(x_h; \alpha) \leq 0$ can be rewritten $x_h \in B_h(p(\alpha))$. In turn the household budget set $B_h(p(\alpha))$ defines a set $\mathcal{V}(\alpha)$ of feasible utility allocations for household $h$, given the price system $p(\alpha)$:

$$\mathcal{V}(\alpha) \equiv \{(V_1, V_2) \in \mathbb{R}^2 : (V_1, V_2) = (U_1(x_h), U_2(x_h)) \text{ for some } x_h \in B_h(p(\alpha))\}$$

In the sequel, the term **Pareto frontier** refers to the Pareto frontier of $\mathcal{V}(\alpha)$ in the space of utility allocations for the household. In particular, $(U_1(x_h(\alpha)), U_2(x_h(\alpha)))$ lies on the Pareto frontier and solves the problem

$$\max F(V_1, V_2; \alpha) \text{ s.t. } (V_1, V_2) \in \mathcal{V}(\alpha).$$

(4)

Finally, for the household under consideration and a given $\alpha$, the term **$\alpha$-indifference curve** refers to a locus in $\mathbb{R}^2$ given by an identity $F(V_1, V_2; \alpha) \equiv \text{const}$. It is instructive to look first at the case $\ell = 1$ of a single good. Assuming that the equilibrium price is positive, the household’s budget set and, therefore, its Pareto frontier is price-independent and the household’s consumption decision is reduced to
the division of a given pie. Consider an increase from \( \alpha \) to \( \alpha + \epsilon \). Then there are only two possibilities. It can happen that

\[
(U_1(x_h(\alpha)), U_2(x_h(\alpha))) = (U_1(x_h(\alpha + \epsilon)), U_2(x_h(\alpha + \epsilon)))
\]
because of a kinked Pareto frontier or a corner solution. But whenever

\[
(U_1(x_h(\alpha)), U_2(x_h(\alpha))) \neq (U_1(x_h(\alpha + \epsilon)), U_2(x_h(\alpha + \epsilon))),
\]
consumer 1 benefits from her increased bargaining power to the detriment of consumer 2. This follows from the fact that an increase in 1’s bargaining power, that is, in \( \alpha \), renders the household’s \( \alpha \)-indifference curves steeper.

### 3.2 Non-Binding Budget Constraint

If the household’s budget constraint is not binding, we have a case of equilibrium with free disposal and the household’s problem can be **locally** described as

\[
\max F(U_1(x_h), U_2(x_h); \alpha).
\] (5)

At the solution \( x_h(\alpha) = (x_1(\alpha), x_2(\alpha)) \), the equation

\[
\frac{\partial F}{\partial U_1} \cdot D_{x_1}U_1 + \frac{\partial F}{\partial U_2} \cdot D_{x_2}U_2 = 0
\] (6)

holds for \( i = 1, 2 \). With \( DU_j = (D_{x_1}U_j, D_{x_2}U_j) \) for \( j = 1, 2 \), equation (6) amounts to

\[
\frac{\alpha}{U_1} \cdot DU_1 = -\frac{1 - \alpha}{U_2} \cdot DU_2,
\] (7)

i.e. in general a small utility gain for one household member is accompanied by a small loss for the other member. For the value function

\[
\Phi(\alpha) \equiv F(U_1(x_h(\alpha)), U_2(x_h(\alpha)); \alpha),
\] (8)

we obtain

\[
\Phi'(\alpha) = \sum_{i=1}^{2} \left[ \frac{\partial F}{\partial U_1} \cdot D_{x_i}U_1 + \frac{\partial F}{\partial U_2} \cdot D_{x_i}U_2 \right] \cdot x'_i(\alpha) + \frac{\partial F}{\partial \alpha}
\] (9)

which by (6) implies a simple case of the envelope theorem:

\[
\Phi'(\alpha) = \frac{\partial F}{\partial \alpha} = \ln U_1(x_h(\alpha)) - \ln U_2(x_h(\alpha))
\] (10)

One is tempted to exploit the following immediate consequence of (10):
Fact 1 The value function (8) increases (decreases) in \( \alpha \), if \( U_1 > U_2 \) (or \( U_1 < U_2 \)).

However, this result alone does not allow the further conclusion that the utility of at least one household member increases (decreases). A look at a more elementary proof of the fact proves instructive. Namely, let without loss of generality \( U_1 > U_2 > 0 \) and consider \( \alpha \) and \( \epsilon \) with \( 0 < \alpha < \alpha + \epsilon < 1 \). Then for sufficiently small \( \epsilon \), \( x_h(\alpha) \in B_h(p(\alpha + \epsilon)) \) and

\[
[U_1(x_h(\alpha + \epsilon))]^{\alpha + \epsilon} \cdot [U_2(x_h(\alpha + \epsilon))]^{1-(\alpha + \epsilon)} \\
\geq [U_1(x_h(\alpha))]^{\alpha + \epsilon} \cdot [U_2(x_h(\alpha))]^{1-(\alpha + \epsilon)} \\
= [U_1(x_h(\alpha))]^{\alpha} \cdot [U_2(x_h(\alpha))]^{1-\alpha} \cdot (U_1/U_2)^\epsilon \\
> [U_1(x_h(\alpha))]^{\alpha} \cdot [U_2(x_h(\alpha))]^{1-\alpha}.
\]

The last inequality shows that the shift in bargaining power has a “nominal effect” on the household’s Nash product even before reoptimization takes place. For this reason, we cannot conclude from a surge of the household’s maximum value of \( F \text{ per se} \) that the utility of at least one household member has increased. The impact of a shift of bargaining power has to be assessed for each household member individually.

When we take a closer look at individual welfare, we encounter the same dichotomy as in the case \( \ell = 1 \):

One possibility is \((U_1(x_h(\alpha)), U_2(x_h(\alpha))) = (U_1(x_h(\alpha + \epsilon)), U_2(x_h(\alpha + \epsilon)))\). For instance, assume (E2), the absence of externalities. Then a non-binding budget constraint for the household requires that both household members be individually locally satiated at their equilibrium consumption. Then for sufficiently small \( \epsilon \), \( x_h(\alpha + \epsilon) \in B_h(p(\alpha + \epsilon)) \), \( x_h(\alpha) \in B_h(p(\alpha + \epsilon)) \), and \( x_h(\alpha) \) and \( x_h(\alpha + \epsilon) \) are close enough so that \( U_i(x_i(\alpha)) \geq U_i(x_i(\alpha + \epsilon)) \) and \( U_i(x_i(\alpha + \epsilon)) \geq U_i(x_i(\alpha)) \), hence \( U_i(x_h(\alpha)) = U_i(x_i(\alpha)) = U_i(x_i(\alpha + \epsilon)) = U_i(x_h(\alpha + \epsilon)) \) for \( i = 1, 2 \).

The second possibility is \((U_1(x_h(\alpha)), U_2(x_h(\alpha)) \neq (U_1(x_h(\alpha + \epsilon)), U_2(x_h(\alpha + \epsilon)))\). Again an increase of \( \alpha \) makes the household’s \( \alpha \)-indifference curves steeper. Hence, as long as \( x_h(\alpha + \epsilon) \in B_h(p(\alpha)) \) and \( x_h(\alpha) \in B_h(p(\alpha + \epsilon)) \), the revised utility allocation \((U_1(x_h(\alpha + \epsilon)), U_2(x_h(\alpha + \epsilon)))\) must lie to the southeast of \((U_1(x_h(\alpha)), U_2(x_h(\alpha)))\). Thus consumer 1 benefits from a small increase of her bargaining power to the detriment of consumer 2.

The foregoing local comparative statics can be easily globalized.
Proposition 1 Suppose that the household’s budget constraint is never binding. If
\(0 < \alpha_s < \alpha^* < 1\), then one of the following two assertions holds:

(i) \(U_1(x_{h}(\alpha_s)) = U_1(x_{h}(\alpha^*))\), \(U_2(x_{h}(\alpha_s)) = U_2(x_{h}(\alpha^*))\).

(ii) \(U_1(x_{h}(\alpha_s)) < U_1(x_{h}(\alpha^*))\), \(U_2(x_{h}(\alpha_s)) > U_2(x_{h}(\alpha^*))\).

The proof of Proposition 1 is given in the appendix. We next examine the case when the budget constraint is binding.

3.3 Binding Budget Constraint

If the budget constraint is binding for household \(h\), then (9) still holds true whereas (6) becomes

\[
\frac{\partial F}{\partial U_1} \cdot D_x U_1 + \frac{\partial F}{\partial U_2} \cdot D_x U_2 = \lambda(\alpha)p(\alpha),
\]

with positive Lagrange multiplier \(\lambda(\alpha)\). Moreover, with binding budget constraints,

\[p(\alpha) \cdot [x_1(\alpha) + x_2(\alpha) - \omega_h] \equiv 0,\]

hence

\[p(\alpha) [x_1'(\alpha) + x_2'(\alpha)] = -p'(\alpha) [x_1(\alpha) + x_2(\alpha) - \omega_h].\]  

Substituting (11) and (12) into (9) yields

\[\Phi'(\alpha) = \frac{\partial F}{\partial \alpha} - \lambda(\alpha)p'(\alpha)[x_1(\alpha) + x_2(\alpha) - \omega_h].\]  

Without further qualification, it is impossible to sign \(\Phi'(\alpha)\). Under additional assumptions, however, one can gain some detailed insights. To this end, let us decompose the effects of a change of consumer 1’s relative bargaining power from \(\alpha\) to \(\alpha + \epsilon\) into two parts:

1. a pure bargaining effect when \(\alpha\) is changed to \(\alpha + \epsilon\) whereas the price system stays at \(p(\alpha)\);

2. a price effect when relative bargaining power remains constant at \(\alpha + \epsilon\) while the price system adjusts from \(p(\alpha)\) to \(p(\alpha + \epsilon)\).

\[\text{Of course, the price effect could be further decomposed into a substitution and an income effect. But that is immaterial to our analysis.}\]
In equation (13), \( p'(\alpha) \) reflects the price effect. If the price effect is negligible, i.e. \( p'(\alpha) \approx 0 \), then \( \Phi'(\alpha) \) can be signed and the conclusion of Fact 1 holds again. As before, this alone does not allow to sign individual utility changes. To achieve the latter, let us assume momentarily that the price effect is negligible and focus on the pure bargaining effect. Equation (11) is the key to the pure bargaining effect. It differs from equation (6) by the right-hand term \( \lambda(\alpha)p(\alpha) \). In analogy to (7), let us rewrite (11) as

\[
\frac{\alpha}{U_1} \cdot DU_1 = -\frac{1 - \alpha}{U_2} \cdot DU_2 + \lambda(\alpha)(p(\alpha), p(\alpha)).
\]  

(14)

Now consider a change \( \Delta x_h \) away from \( x_h(\alpha) \) while maintaining the budget identity, i.e. \( p(\alpha) \cdot (x_h(\alpha) + \Delta x_h) = p(\alpha) \cdot x_h(\alpha) = p(\alpha)\omega_h \). Then \( (p(\alpha), p(\alpha)) \cdot \Delta x_h = p(\alpha) \cdot \Delta x_h = 0 \), hence with (14),

\[
\left[ \frac{\alpha}{U_1} \cdot DU_1 \right] \cdot \Delta x_h = -\left[ \frac{1 - \alpha}{U_2} \cdot DU_2 \right] \cdot \Delta x_h.
\]  

(15)

Thus (7) essentially holds again. Running through the earlier geometric and topological arguments yields

**Proposition 2** Suppose that the household’s budget constraint is always binding while the price effect is negligible. If \( 0 < \alpha_* < \alpha^* < 1 \), then one of the following two assertions holds:

(i) \( U_1(x_h(\alpha_*)) = U_1(x_h(\alpha^*)), U_2(x_h(\alpha_*)) = U_2(x_h(\alpha^*)) \).

(ii) \( U_1(x_h(\alpha_*)) < U_1(x_h(\alpha^*)), U_2(x_h(\alpha_*)) > U_2(x_h(\alpha^*)) \).

Obviously, Propositions 1 and 2 could be combined into one, assuming zero or negligible price effects. If, on the contrary, the price effect is drastic, both utilities may move in the same direction. The magnitude of the price effect — whether it is negligible or drastic or somewhere in between — depends on the size of the household relative to the economy. It also depends on preferences, including the preferences of consumers not belonging to the household, as a comparison of Examples 1 to 3 shows.

The focus on a particular household \( h \) amid many might suggest that shifts of bargaining power are sporadic and therefore price effects are likely to be negligible. Our general analysis provides valuable insights in case the change of bargaining power is a sporadic event, indeed. It helps identify the relevant effects. Drastic price effects
will prevail for instance, if the economy is replicated and the same shift in bargaining power occurs in all households that are replicas of $h$. This brings us back to the facts motivating this inquiry, namely enhanced influence and more specifically increased intra-household bargaining power of women in contemporary industrialized societies as compared with their situation in those societies during the first half of the 20th century or their current situation in “traditional” societies. Such changes occur in many households and, thus, price effects may be drastic.

4 Examples

In this section, we illustrate the propositions and striking effects of the last section by means of examples. The entire population consists of a total of three consumers, two belonging to household $h$ and one forming a one-person household denoted $k$. To capture widespread shifts in bargaining power in a large finite population, one can consider $h$ as a prototype of a two-person household and $k$ as representative of a one-person household. Literally, one can think in terms of replica economies derived from the basic economies under consideration, with an equal number of two-person households like $h$ and one-person households like $k$.

Throughout this section, there are always two goods: $\ell = 2$. The second good serves as numéraire. The symbols $x, x_1, x_2, \ldots, x_i, \ldots$ denote quantities of the first good. The symbols $y, y_1, y_2, \ldots, y_i, \ldots$ denote quantities of the second good. $c^*_i$ stands for the equilibrium consumption bundle of a generic person (individual, consumer) $i$. All consumers fulfill condition E2, i.e., absence of externalities.

To simplify the exposition of the later examples, we consider first an auxiliary example of an economy consisting of two one-person households, $g$ and $k$. The respective consumers are named 0 and 3.

Example 0.

The initial endowments are $\omega_0 = (1, 0)$ and $\omega_3 = (0, 1)$. The utility representations are

\[ u_0 = u_0(x_0, y_0) = x_0^\alpha y_0^{1-\alpha}, \text{ with } 0 < \alpha < 1, \text{ and} \]

\[ u_3 = u_3(x_3, y_3) = x_3^{1/2} y_3^{1/2}. \]

After normalizing the price of the second good, market equilibrium is unique. The
The equilibrium price system is
\[ p^* = \left( \frac{1}{2(1-\alpha)}, 1 \right); \]
the equilibrium consumption bundles are \( c^*_0 = (\alpha, 1/2), c^*_3 = (1-\alpha, 1/2). \)

Now we are prepared to consider the case of three individuals, labelled \( i = 1, 2, 3. \) Consumers 1 and 2 form the two-person household \( h. \) In this household, consumer 1 has bargaining power \( \alpha \) and consumer 2 has bargaining power \( 1-\alpha. \) Consumer 3 constitutes the single household \( k. \) We are going to scrutinize several representative examples which are almost exhaustive in that they exhibit three possible allocative responses to a shift of bargaining power within the two-person household:

(a) Only one member is affected.

(b) The two members are affected in opposite ways.

(c) Both members are affected in the same way.

The examples differ only in individual consumer preferences. The analysis suggests that less substitutability leads to more drastic price effects. We start with the following example of case (a).

**Example 1.**

Here consumer 1 benefits from more bargaining power, to the detriment of consumer 3 while consumer 2 is unaffected. Household \( h \) is endowed with \( \omega_h = (1, 0). \) Its two members, \( i = 1, 2 \) have utility representations
\[ u_1(x_1, y_1) = x_1 \quad \text{and} \quad u_2(x_2, y_2) = y_2. \]

The household maximizes
\[ S_h = u_1^\alpha u_2^{1-\alpha} = x_1^\alpha y_2^{1-\alpha}, \quad 0 < \alpha < 1. \]
The characteristics of household \( k \) are as in the previous example, that is the endowment is \( \omega_3 = (0, 1) \) and the utility representation is 
\[
    u_3(x_3, y_3) = x_3^{1/2} y_3^{1/2}.
\]

Since the aggregate demand function of household \( h \) coincides with the demand function of consumer 0 in Example 0, the equilibrium quantities are 
\[
    p^* = \left( \frac{1}{2(1 - \alpha)}, 1 \right); \\
    c_1^* = (\alpha, 0); \\
    c_2^* = (0, \frac{1}{2}); \\
    c_3^* = (1 - \alpha, \frac{1}{2}).
\]

Hence as asserted consumer 1 benefits from more bargaining power, to the detriment of consumer 3. Consumer 2 is unaffected.

In the example, the first good becomes more valuable to the two-person household as the bargaining power of the first consumer increases. This boosts the equilibrium price of the first good and the income of the two-person household endowed with the first good. The household has become richer both in nominal and real terms. Since the expenditure on the second good remains constant, the second consumer is unaffected. But the increase in the residual income to be spent on the first good more than compensates for the higher price: consumer 1 is better off as a consequence of her increased bargaining power. As for consumer 3, his nominal income derived from the possession of the second resource remains constant. Therefore, he has become poorer, has less purchasing power.

From consumer 2’s perspective, if bargaining power shifts towards her and prices are fixed, then her welfare is increased. But the resulting price variation offsets her gain. That consumer 2 is unaffected by a change in bargaining power seems to be caused by limited substitutability within the two-person household. This is confirmed by the next example where enhanced bargaining power of consumer 1 translates into improved welfare for this consumer and welfare losses for consumers 2 and 3.
Example 2.

Here consumer 1 benefits from more bargaining power to the detriment of consumer 2. Consumer 3 either gains or loses. Household $h$ is still endowed with $\omega_h = (1, 0)$. But now each member $i = 1, 2$ has Cobb-Douglas preferences with utility representation

$$u_i(x_i, y_i) = x_i^{\gamma_i} y_i^{1-\gamma_i}, 0 < \gamma_i < 1.$$  

The household maximizes

$$u_1^\alpha u_2^{1-\alpha} = (x_1^{\gamma_1} y_1^{1-\gamma_1})^\alpha (x_2^{\gamma_2} y_2^{1-\gamma_2})^{1-\alpha}$$

$$= x_1^{\alpha \gamma_1} x_2^{\alpha (1-\gamma_2)} y_1^{\alpha (1-\gamma_1)} y_2^{(1-\alpha)(1-\gamma_2)}.$$  

Again, $\alpha$ and $1-\alpha$ lend themselves as measures of relative bargaining power of consumer 1 and consumer 2, respectively.

Household $k$ has the single member 3, with the same consumer characteristics as before.

We obtain:

**Fact 2** A shift of bargaining power from consumer 2 to consumer 1 benefits consumer 1 and harms consumer 2, who ends up consuming less of both commodities.

In Example 2 there is more substitutability in the economy than in Example 1. Example 3 exhibits less substitutability than Example 1, because the preferences of consumer 3 will be altered from Cobb-Douglas to Leontieff. It turns out that the lack of substitution by consumer 3 necessitates a major price adjustment to re-equilibrate the market after bargaining power within household $h$ has shifted. As a result, we observe a drastic price effect: When bargaining power within their household changes, the equilibrium utilities of consumers 1 and 2 are moving in the same direction.

Example 3.

Here a shift of bargaining power from consumer 2 to consumer 1 benefits both consumers to the detriment of consumer 3. This example is identical with Example 1, except that consumer 3 now has Leontieff preferences with utility representation

$$u_3(x, y) = \min(x, y).$$
After setting \( s = \min(x_3, y_3) \), the utility maximization problem for consumer 3 can be rewritten as

\[
\max s \quad \text{s.t.} \quad (p_1 + 1)s = 1
\]

with solution \( s = 1/(p_1 + 1) \).

Household \( h \)'s demand is \((\alpha, (1 - \alpha)p_1)\). Therefore, market clearing for the first good requires \(1/(p_1 + 1) = 1 - \alpha\). Thus in equilibrium,

\[
\begin{align*}
p^* &= (\alpha/(1 - \alpha), 1); \\
c_1^* &= (\alpha, 0), \\
c_2^* &= (0, \alpha), \\
c_3^* &= (1 - \alpha, 1 - \alpha).
\end{align*}
\]

Thus a shift of bargaining power from consumer 2 to consumer 1 benefits both members of the household to the detriment of consumer 3. A reverse shift harms 1 and 2, and leaves 3 better off.

The examples suggest that comparative statics is sensitive to the degree of substitutability in the economy. Enhanced substitutability appears to mitigate price effects. Indeed, if in a further variation of Example 1, one assumes linear preferences (perfect substitutability) for consumer 3, with utility representation \( u_3(x_3, y_3) = x_3 + y_3 \), then the price effect is zero. Moreover, for two-good economies exhibiting CES-utility functions for all individuals with the same elasticity of substitution, the magnitude of the price effect can be parameterized by the elasticity of substitution in the economy. The price effect depends negatively on the elasticity of substitution.

The next section will lend additional support to the conclusion that there exists a negative relationship between substitutability and the price effect. We will examine societies where all individuals have quasi-linear utilities. In that case, the price effect is zero. A gain in bargaining power benefits the consumer at the detriment of the household member who has less power. Other households, however, are not effected since the price effect is zero. This indicates that sufficient substitutability can completely eliminate the price effect, confirming the informal conclusion that enhanced substitutability tends to mitigate price effects.
5 Comparative Statics Across Households

Until now we have focused primarily on intra-household effects, that is, on the utility changes in a particular household when bargaining power shifts within that household. Via a series of examples, we have demonstrated that such a shift of bargaining power can affect the members of the corresponding two-person household in three different ways: Only one member is affected; the two members are affected in opposite ways; both members are affected the same way. We have argued earlier that the above examples can be readily reinterpreted as instances of widespread shifts of bargaining power in a replica economy. In the resulting replica economy, the main focus remains on intra-household effects, on the repercussions on the members of those households in which a shift in bargaining power has occurred. However, we have also seen that third parties can be affected. In this section, we redirect our attention to such inter-household or spill-over effects. We start with a neutrality result that can serve as a benchmark.

5.1 A Neutrality Result

We consider a society with \( n > 1 \) identical households. Household \( h (h = 1, \ldots, n) \) has members \( h1 \) and \( h2 \), called the first member and the second member, respectively. There are \( \ell \) goods \( (\ell > 1) \). The consumption of good \( k \) \((k = 1, \ldots, \ell) \) by individual \( hi \) \((i = 1, 2) \) is denoted by \( x_{hi}^k \). Each household \( h \) is endowed with \( w_h = (w_{1h}, \ldots, w_{\ell h}) \).

The two members of household \( h \) have quasi-linear utility representations of the form

\[
U_{h1}(x_{h1}) = u_{h1}(x_{h1}^1, \ldots, x_{h1}^{\ell-1}) + x_{h1}^\ell
\]

\[
U_{h2}(x_{h2}) = u_{h2}(x_{h2}^1, \ldots, x_{h2}^{\ell-1}) + x_{h2}^\ell
\]

where \( u_{hi} \) is assumed to be strictly concave, strictly increasing and differentiable. Household \( h \) maximizes

\[
S_h = U_{h1}^{\alpha_h} U_{h2}^{1-\alpha_h} \text{ or } ln S_h = \alpha_h ln U_{h1} + (1 - \alpha_h)ln U_{h2}
\]

where \( 0 < \alpha_h < 1 \) is the bargaining power of individual \( h1 \) in household \( h \). We denote equilibrium values by \( \hat{x}_{hi}^k \) and equilibrium utilities by \( \hat{U}_{hi} \) and \( \hat{u}_{hi} \). For the following we assume that for any array of bargaining power parameters \((\alpha_1, \ldots, \alpha_n)\) under consideration, each individual consumes a non-negative amount of the natural numéraire.
good \( \ell \) in every market equilibrium. We also assume that for any array \((\alpha_1, \ldots, \alpha_n)\), the corresponding economy has a unique market equilibrium, up to price normalization. These two assumptions are inessential for our argumentation but simplify the exposition considerably. We shall indicate below which modifications are necessary if the two assumptions are removed. We consider a market equilibrium and parametric changes of the bargaining power in household \( h \) and obtain:

**Proposition 3 (No Spill-overs)** With quasi-linear preferences:

(i) A change of \( \alpha_h \) in a particular household \( h \) does not impact on non-members.

(ii) \( \frac{\partial \hat{x}_{h1}}{\partial \alpha_h} = \frac{\partial \hat{x}_{h2}}{\partial \alpha_h} = 0 \) for all \( k = 1, \ldots, \ell - 1 \).

(iii) \( \frac{\partial \hat{x}_{h1}}{\partial \alpha_h} > 0, \frac{\partial \hat{x}_{h2}}{\partial \alpha_h} < 0 \).

(iv) Suppose that households are homogeneous with respect to individual utility representations and household endowments with \( w_h = \bar{w}, \forall h = 1, \ldots, n \). Then:

\[
\hat{x}_{h1} = \alpha_h \bar{w} + \alpha_h \hat{u}_{h2} - (1 - \alpha_h) \hat{u}_{h1} \\
\hat{x}_{h2} = (1 - \alpha_h) \bar{w} + (1 - \alpha_h) \hat{u}_{h1} - \alpha_h \hat{u}_{h2}
\]

The proof of Proposition 3 is given in the appendix. Proposition 3 illustrates that with quasi-linear preferences, a change of the bargaining power within a particular household only impacts on the distribution of the numéraire in household \( h \) without affecting the consumption of the first \( \ell - 1 \) commodities. A local change of bargaining power has no price effect and does not affect the utility of individuals in other households. This also means that a household \( h \) cannot manipulate outcomes and possibly improve utility of household members at the expense of outsiders by misrepresenting internal bargaining power.

The result is another example of an important line of research that examines in which circumstances individuals have an incentive to misrepresent their preferences in the market place. Recently, Makowski, Ostroy and Segal (1999) have comprehensively characterized continuous, efficient and anonymous incentive compatible mechanisms and have shown that such mechanisms must be perfectly competitive, i.e. no agent can change the Walrasian equilibrium price vector by changing his announced preferences. Quasi-linear preferences are one of the examples that can allow for incentive compatible mechanisms or perfect competition. Our investigation shows that with quasi-linear
preferences a multi-person household has no incentive to misrepresent the internal bargaining power.

Regarding our simplifying assumptions for the neutrality result, interiority and uniqueness of equilibrium, giving up the first assumption requires to work with Kuhn-Tucker conditions instead of first order conditions. Without the second assumption, multiple equilibria cannot be ruled out. But a market clearing price system \((p_1, \ldots, p_{\ell-1}, 1)\) with respect to some array of bargaining power parameters is also market clearing with respect to all other arrays. Given any such market clearing price system and the associated equilibrium selection, the conclusion of Proposition 3 continues to hold.

### 5.2 Separate Sphere Consumption

We next turn to situations where internal bargaining power changes in a particular household have spill-over effects on other households. In particular, we examine how individuals are affected if similar (dissimilar) persons in other households can increase their bargaining power. We examine an economy like in the last subsection, but with different individual preferences. We assume households which are homogeneous at the beginning but undergo large sociological changes thereafter. We assume \(\ell = 2\) and that all households have the same endowment \(w_h = \bar{w} = (\bar{w}_1, \bar{w}_2)\).

Individuals have separate spheres of consumption, i.e.

\[
U_{h1}(x_{1h1}, x_{2h1}) = U_{h1}(x_{1h1}^1),
\]
\[
U_{h2}(x_{1h2}, x_{2h2}) = U_{h2}(x_{2h2}^2).
\]

The utility functions are assumed to be strictly increasing, strictly concave and differentiable. The assumption of separate sphere consumption is one convenient way to divide the society into different sociological groups where individuals are similar within a group and dissimilar across groups. Here we have two groups, “first members” (denoted \(h1\)) and “second members” (denoted \(h2\)) of households. Again household \(h\) maximizes

\[
S_h = U_{h1}^{\alpha_h} U_{h2}^{1-\alpha_h}
\]

where \(0 < \alpha_h < 1\). We obtain, with \(^\hat{\cdot}\) denoting again equilibrium values:
Proposition 4 (Spill-overs) Under separate spheres consumption, there exists a unique market equilibrium (up to price normalization) for each array \((\alpha_1, \ldots, \alpha_n)\) of bargaining power parameters. Moreover, for any two households \(g \neq h:\)

(i) \(\alpha_h > \alpha_g \Rightarrow \hat{x}_{h1}^1 > \hat{x}_{g1}^1.\)

(ii) \(\alpha_h = \alpha_g \Rightarrow \hat{x}_{h1}^1 = \hat{x}_{g1}^1.\)

(iii) \(\partial \hat{x}_{h1}^1 / \partial \alpha_h > 0, \partial \hat{x}_{g1}^1 / \partial \alpha_h < 0.\)

(iv) \(\partial \hat{x}_{h2}^2 / \partial \alpha_h < 0, \partial \hat{x}_{g2}^2 / \partial \alpha_h > 0.\)

The proof of Proposition 4 is given in the appendix. Proposition 4 has clear-cut implications. Consider the sociological groups “first-members” and “second-members”, defined by similarities with respect to preferences. If all individuals in the first sociological group have the same bargaining power (and as a consequence all “second-members” as well), all households consume their endowments since we are in an equilibrium with no active trade. An identical shift of bargaining power across all households has no effect on utilities of any individual either since we will again arrive at an equilibrium with no trade. The situation is completely different when only some members of a sociological group enjoy higher bargaining power. For instance, a “first-member” suffers when only other “first-members” gain more bargaining power in their respective households. Conversely, the “first-member” benefits from higher own bargaining power as long as other “first-members” do not experience a change of bargaining power. The analogue holds for the other sociological group. Therefore, the main thrust of Proposition 4 is that each individual of a sociological group tends to benefit if he can increase his bargaining power but tends to suffer if others in his group are able to do the same.

For separate sphere economies of the type discussed above we obtain as an immediate consequence a power illusion phenomenon. Consider two separate sphere economies denoted by \(E_1(\{\alpha_h^1\}_{n_1})\) and \(E_2(\{\alpha_h^2\}_{n_1})\) with households that are homogeneous with respect to individual utility functions and endowments. Equilibrium utilities are denoted by \(\hat{U}_{h1}^1, \hat{U}_{h2}^1\) and \(\hat{U}_{h1}^2, \hat{U}_{h2}^2\), respectively. Then the following holds:
Corollary 1 (Power Illusion)

(i) If $\alpha_1^h = \bar{\alpha}_1^1$ for all $h$ and $\alpha_1^2 > \max_{h \neq 1} \{\alpha_2^h\}$, then $\hat{U}_1^{11} < \hat{U}_2^{11}$.

(ii) If $\alpha_1^h = \bar{\alpha}_1^1$ for all $h$ and $\alpha_1^2 < \min_{h \neq 1} \{\alpha_2^h\}$, then $\hat{U}_1^{11} > \hat{U}_2^{11}$.

The corollary illustrates that a member of a sociological group is better off if he has the highest internal bargaining power even if the level of his power is much smaller than in another economy where all individuals of the group have the same bargaining power, that is $\bar{\alpha}_1^1 > \alpha_1^2$. The underlying intuition runs as follows: Diversity across households opens trade opportunities. The gains from trade will, as a rule, accrue primarily to the members of a sociological group who have relatively higher bargaining power than other members of the group. The absolute level of bargaining power is not important. When, however, the bargaining power of other individuals in the same sociological group is enhanced as well and all individuals of the sociological group enjoy an identical level in bargaining power, the original gain is totally eroded.\textsuperscript{4}

5.3 An Example

To illustrate the preceding proposition by solving explicitly for the market equilibria, we consider again a society with $n > 1$ identical two-member households.

To simplify notation, we use the symbols $x_{h1}$ and $x_{h2}$ to denote quantities of the first good consumed by household member $h1$ and $h2$, respectively. The symbols $y_{h1}$ and $y_{h2}$ denote quantities of the second commodity consumed by household member $h1$ and $h2$, respectively.

Each household $h$ is endowed with $\omega_h = (1, 2)$. The two members of household $h$ have utility representations

$$U_{h1}(x_{h1}, y_{h1}) = x_{h1} \quad \text{and} \quad U_{h2}(x_{h2}, y_{h2}) = y_{h2}.$$ 

The household $h$ maximizes

$$S_h = U_{h1}^{\alpha} U_{h2}^{1-\alpha} = x_{h1}^{\alpha} y_{h2}^{1-\alpha} \quad \text{or} \quad \ln S_h = \alpha \ln x_{h1} + (1 - \alpha) \ln y_{h2}.$$ 

\textsuperscript{4}When separate sphere consumption does not apply in the strict way postulated above, only partial erosion will occur, e.g. when all individuals have Cobb-Douglas utility functions.
where $0 < \alpha < 1$.

The aggregate demand function of household $h$ ($x_h = x_{h1} + x_{h2}$, $y_h = y_{h1} + y_{h2}$) is given by

\[
x_h = \alpha(2 + p_1)/p_1, \\
y_h = (1 - \alpha)(2 + p_1).
\]

where good 2 has been used as the numéraire. If $\alpha$ is the same value across households, market equilibrium does exhibit zero net trades since excess demands are identical for all households. Thus, market equilibrium is given by

\[
x^*_h = x^*_{h1} = 1, \\
y^*_h = y^*_{h2} = 2, \\
p^*_1 = (2\alpha)/(1 - \alpha).
\]

The utilities of the members of each household are $U^*_{h1} = 1$, $U^*_{h2} = 2$.

Next consider $0 < \alpha < \alpha + \varepsilon < 1$ and $1 \leq \hat{h} \leq n$. Suppose that in the first $\hat{h}$ households, bargaining power shifts by $\varepsilon$ from consumer 2 to consumer 1.

Market equilibrium for the first commodity obtains if

\[
(n - \hat{h})(\alpha(2 + p_1)) + \hat{h}(\alpha + \varepsilon)(2 + p_1) = np_1, \tag{19}
\]

\[
p_1(\varepsilon, \hat{h}) = \frac{2n\alpha + 2\hat{h}\varepsilon}{n(1 - \alpha) - \hat{h}\varepsilon}. \tag{20}
\]

The equilibrium allocation is given by

\[
x^*_h = x^*_{h1} = \frac{2n(\alpha + \varepsilon)}{2n\alpha + 2\hat{h}\varepsilon} \quad \text{for} \; h = 1, \ldots, \hat{h}; \\
y^*_h = y^*_{h2} = \frac{2n(1 - \alpha - \varepsilon)}{n(1 - \alpha) - \hat{h}\varepsilon} \quad \text{for} \; h = 1, \ldots, \hat{h}; \\
x^*_h = x^*_{h1} = \frac{2n\alpha}{2n\alpha + 2\hat{h}\varepsilon} \quad \text{for} \; h = \hat{h} + 1, \ldots, n; \\
y^*_h = y^*_{h2} = \frac{2n(1 - \alpha)}{n(1 - \alpha) - \hat{h}\varepsilon} \quad \text{for} \; h = \hat{h} + 1, \ldots, n.
\]
Although the actual $\hat{h}$ is a natural number we can treat equilibrium consumption levels as functions of real-valued parameters and obtain

$$\frac{\partial x_{h1}^*}{\partial \hat{h}} < 0 \text{ for } h = 1, \ldots, \hat{h};$$

$$\frac{\partial y_{h2}^*}{\partial \hat{h}} > 0 \text{ for } h = 1, \ldots, \hat{h};$$

$$\frac{\partial x_{h1}^*}{\partial \hat{h}} < 0 \text{ for } h = \hat{h} + 1, \ldots, n;$$

$$\frac{\partial y_{h2}^*}{\partial \hat{h}} > 0 \text{ for } h = \hat{h} + 1, \ldots, n.$$

Since $x_{h1}^* > 1, y_{h2}^* < 2$ for $h = 1, \ldots, \hat{h}$ and $x_{h1}^* < 1, y_{h2}^* > 2$ for $h = \hat{h} + 1, \ldots, n$, we obtain the following utility changes:

- The first-members of households with bargaining power $\alpha + \varepsilon$ suffer a utility loss if the same bargaining power shift occurs in other households as well. Each member of the sociological group “first-members” benefits from an increase in his own bargaining power but is harmed if others gain more bargaining power as well.

- The second-members in households with bargaining power $1 - \alpha - \varepsilon$ suffer a utility loss but less so if other individuals of his sociological group experience the same. The second member in households with power $1 - \alpha$ benefits if the bargaining power of other “second-members” decreases.

In sum, each individual of a sociological group benefits if he can increase his bargaining power, but suffers if others in his group achieve the same. Each individual of a sociological group is harmed by a decrease in its bargaining power, but less if other individuals of his group experience the same decrease.

A complete shift of bargaining power has no effect on utilities of any individuals since we are again in an equilibrium with no trade. Bargaining power changes are completely offset by the corresponding shifts in equilibrium prices.
6 Concluding Remarks

The current analysis is confined to a general equilibrium model of a pure exchange economy with a fixed household structure and Nash-bargained household decisions for a select two-person household. General comparative statics as well as numerical examples lend support to the following conclusions. As a rule, a consumer benefits from more bargaining power at the expense of her fellow household member and the other consumer(s). However, in a closed economy, a shift of bargaining power within a significant number of two-person households may cause drastic price effects. As a consequence, both members of such a household may benefit from or both members may be harmed by a shift of internal bargaining power. In exceptional cases, it can happen that a household member is unaffected.

The current analysis further shows that the aggregate equilibrium consumption of a household can be positively affected by a shift of internal bargaining power. This suggests the possibility that a sophisticated household might succeed in an attempt to manipulate the market outcome, not by misrepresenting endowments or individual preferences, but by misrepresenting the internal bargaining power. To illustrate this novel way of manipulation, which is not yet documented in the literature, let us reconsider Example 2. Suppose the household pretends that the bargaining power of the first consumer is higher than it actually is and they submit the corresponding excess demands to the market. If $\gamma_1 > \gamma_2$, i.e. if the first good is relatively more important to the first consumer, they will end up with a higher aggregate amount of the first good and the same amount of the second good in equilibrium. Whether or not both gain from a successful manipulation depends on the internal distribution of aggregate consumption. If they divided the goods in accordance with their pretended bargaining power, put their money where their mouth is, then consumer 1 would gain and consumer 2 would lose from manipulation. If they divide the goods according to the true bargaining power — which fixes a proportional sharing rule for each of the goods — then both gain from manipulation. As noted before, quasi-linear preferences rule out spill-overs and, consequently, this kind of manipulation.

To reiterate, the current model assumes a fixed household structure and pure exchange. Removing any of these restrictions leads to a host of new important issues, which are left to future research.
7 Appendix

Proof of Proposition 1

Suppose that the household’s budget constraint is never binding as hypothesized. For every $\alpha \in (0, 1)$, we can choose an $\epsilon(\alpha) > 0$ so that the local comparative statics prevail in the open neighborhood $N(\alpha) \equiv (\alpha - \epsilon(\alpha), \alpha + \epsilon(\alpha))$. Each set $C(\alpha) = N(\alpha) \cap [\alpha_*, \alpha^*]$ is relatively open in the interval $[\alpha_*, \alpha^*]$. The family $C(\alpha), \alpha \in [\alpha_*, \alpha^*]$, is an open covering of the compact set $[\alpha_*, \alpha^*]$. It has a finite subcovering. Let us fix a minimal finite subcovering $C(\alpha_k), k = 1, \ldots, K$. Without loss of generality, assume $\alpha_1 < \alpha_2 < \ldots < \alpha_K$. We claim that:

(A) If $\alpha_* < \alpha_1$, then $\alpha_* \in C(\alpha_1)$.

(B) If $\alpha_K < \alpha^*$, then $\alpha^* \in C(\alpha_K)$.

(C) For each $k \leq K - 1$, there exists $\beta_k$ with $\alpha_k < \beta_k < \alpha_{k+1}$ and $\beta_k \in C(\alpha_k) \cap C(\alpha_{k+1})$.

To show (A), suppose it were false, i.e. $\alpha_* < \alpha_1$ and $\alpha_* \notin C(\alpha_1)$. Then there exists $k > 1$ with $\alpha_* \in C(\alpha_k)$ and, consequently, $C(\alpha_1) \subset C(\alpha_k)$, contradicting the minimality of the covering. Claims (B) and (C) are shown by similar reasoning.

Now fix $\beta_1, \ldots, \beta_{K-1}$ according to (C) and let us go from $\alpha_*$ to $\alpha^*$ taking small steps, namely

- from $\alpha_*$ to $\alpha_1$, from $\alpha_1$ to $\beta_1$,
- from $\beta_1$ to $\alpha_2$, from $\alpha_2$ to $\beta_2$,
- \ldots \ldots \ldots \ldots \ldots
- from $\beta_{K-1}$ to $\alpha_K$, and $\alpha_K$ to $\alpha^*$.

During each step, either the utilities remain unchanged or consumer 1’s utility goes up and consumer 2’s utility goes down. Hence the assertion.

\[\square\]
For convenient reference, we state an obvious auxiliary result before proceeding to the proof of Fact 2.

**Lemma 1** Let real numbers $\sigma, \tau, z > 0$ be given. The solution of the problem

$$\max z_1^\sigma z_2^\tau \text{ s.t. } z_1 \geq 0, z_2 \geq 0, z_1 + z_2 = z$$

is $z_1 = \frac{\sigma}{\sigma+\tau} \cdot z$, $z_2 = \frac{\tau}{\sigma+\tau} \cdot z$, with value

$$\left(\frac{\sigma}{\sigma+\tau}\right)^\sigma \cdot \left(\frac{\tau}{\sigma+\tau}\right)^\tau \cdot z^{\sigma+\tau}.$$ 

**Proof of Fact 2**

Let $x = x_1 + x_2$ and $y = y_1 + y_2$ denote the total amounts purchased by household $h$. By Lemma 1, maximization of the Nash product $u_1^\alpha u_2^{1-\alpha}$ requires

$$x_1 = \frac{\sigma}{\sigma+\tau} \cdot x,$$
$$x_2 = \frac{\tau}{\sigma+\tau} \cdot x,$$
$$y_1 = \frac{\sigma^*}{\sigma^*+\tau^*} \cdot y,$$
$$y_2 = \frac{\tau^*}{\sigma^*+\tau^*} \cdot y$$

where

$$\sigma = \alpha \gamma_1,$$
$$\tau = (1-\alpha) \gamma_2,$$
$$\sigma^* = \alpha (1-\gamma_1),$$
$$\tau^* = (1-\alpha)(1-\gamma_2).$$

Moreover, at the maximum,

$$u_1^\alpha u_2^{1-\alpha} = \left(\frac{\sigma}{\sigma+\tau}\right)^\sigma \left(\frac{\tau}{\sigma+\tau}\right)^\tau \left(\frac{\sigma^*}{\sigma^*+\tau^*}\right)^{\sigma^*} \left(\frac{\tau^*}{\sigma^*+\tau^*}\right)^{\tau^*} x^\delta y^{1-\delta}$$

with

$$\delta = \sigma + \tau = \alpha \gamma_1 + (1-\alpha) \gamma_2 = \gamma_2 + \alpha (\gamma_1 - \gamma_2);$$
$$1 - \delta = \sigma^* + \tau^* = \alpha (1-\gamma_1) + (1-\alpha) (1-\gamma_2) = 1 - \gamma_2 - \alpha (\gamma_1 - \gamma_2).$$
Therefore, in equilibrium, the aggregate consumption for household $h$ is $(x, y) = (\delta, \frac{1}{2})$. The associated individual shares are

$$x_1 = \frac{\sigma}{\sigma + \tau} x = \sigma;$$

$$x_2 = \frac{\tau}{\sigma + \tau} x = \tau;$$

$$y_1 = \frac{\sigma}{\sigma + \tau^*} y = \frac{1}{2} \frac{\sigma^*}{\sigma + \tau^*};$$

$$y_2 = \frac{\tau}{\sigma + \tau^*} y = \frac{1}{2} \frac{\tau^*}{\sigma + \tau^*}.$$

As a function of $\alpha$, consumer 1 achieves

$$u_1 = \sigma_1 \left( \frac{1}{2} \right)^{1-\gamma_1} \left( \frac{\sigma^*}{\sigma + \tau^*} \right)^{1-\gamma_1} \left( \frac{\alpha}{1 - \gamma_2 - (\gamma_1 - \gamma_2) \alpha} \right)^{1-\gamma_1}$$

which is increasing in $\alpha$. Consumer 2 achieves

$$u_2 = \tau_2 \left( \frac{1}{2} \right)^{1-\gamma_2} \left( \frac{\tau^*}{\sigma + \tau^*} \right)^{1-\gamma_2} \left( \frac{1 - \alpha}{1 - \gamma_1 - (\gamma_2 - \gamma_1)(1 - \alpha)} \right)^{1-\gamma_2}$$

which is decreasing in $\alpha$. Hence a shift of bargaining power from consumer 2 to consumer 1 benefits consumer 1 and harms consumer 2, who ends up consuming less of both commodities.

Proof of Proposition 3

Good $\ell$ serves as a numéraire so that the price system assumes the form $(p_1, \ldots, p_{\ell-1}, 1)$. We consider the first-order conditions of maximizing $S_h$ in household $h$:

$$\alpha_h \frac{1}{U_{h1}} \frac{\partial u_{h1}}{\partial x_{h1}^k} - \lambda_h p_k = 0, \ k = 1, \ldots, \ell - 1;$$

$$\alpha_h \frac{1}{U_{h1}} - \lambda_h = 0;$$

$$(1 - \alpha_h) \frac{1}{U_{h2}} \frac{\partial u_{h2}}{\partial x_{h2}^k} - \lambda_h p_k = 0, k = 1, \ldots, \ell - 1;$$

$$(1 - \alpha_h) \frac{1}{U_{h2}} - \lambda_h = 0.$$

Note that our assumption of sufficient endowments with the numéraire good in all households allows us to work with the entire set of first-order conditions.
Therefore:
\[
\lambda_h = \alpha_h \frac{1}{U_{h1}} = (1 - \alpha_h) \frac{1}{U_{h2}}. \tag{21}
\]
\[
\frac{\partial u_{h1}}{\partial x_{h1}^k} = \frac{\partial u_{h2}}{\partial x_{h2}^k} = p_k, \ k = 1, \ldots, \ell - 1. \tag{22}
\]

Because of differentiability and strict concavity, the demand of household \( h \) for commodities \( k = 1, \ldots, \ell - 1 \) is independent of the bargaining power \( \alpha_h \) and \( 1 - \alpha_h \) of individual \( h_1 \) and \( h_2 \), respectively. Hence, by the budget constraint and budget exhaustion also the aggregate household demand for commodity \( \ell \) is independent of \( \alpha_h \). Therefore, the market clearing price system \( (p_1, \ldots, p_{\ell-1}, 1) \) does not depend on internal bargaining power of households and, hence, changes of bargaining power in household \( h \) have no effect on other households. This establishes assertions (i) and (ii).

In contrast to all other goods, a shift of power in household \( h \) affects the distribution of the numéraire good in household \( h \), as we shall establish next. Using the notation for equilibrium values we obtain from equation (21):
\[
\frac{\alpha_h}{\hat{u}_{h1} + \hat{x}_h^\ell} = \frac{1 - \alpha_h}{\hat{u}_{h2} + \hat{x}_h^\ell}. \tag{23}
\]

Since \( \hat{u}_{h1} \) and \( \hat{u}_{h2} \) are independent of \( \alpha_h \) and \( \hat{x}_h^\ell \) does not depend on \( \alpha_h \) either, assertion (iii) follows. Using again the fact that variations in \( \alpha_h \) have no effect on aggregate excess demand, we conclude that if households are completely homogeneous with respect to \( U_{hi} \) and \( w_h \), then a market equilibrium does not exhibit any positive net trades. Therefore, \( \hat{x}_{h1}^\ell + \hat{x}_{h2}^\ell = w_h^\ell \) and via equation (23) we obtain the expressions in (iv).

\[\Box\]

**Proof of Proposition 4**

We normalize prices by \( p_2 = 1 \). Then the problem of household \( h \) is given by:
\[
\max \{ \ln S_h = \alpha_h \ln U_{h1}(x_{h1}^1) + (1 - \alpha_h) \ln U_{h2}(x_{h2}^2) \} \\
s.t. \ x_{h1}^1 p_1 + x_{h2}^2 = w_h^1 p_1 + w_h^2
\]

The first-order conditions amount to:
\[
\alpha_h \frac{1}{U_{h1}(x_{h1}^1)} U'_{h1}(x_{h1}^1) - \lambda_h p_1 = 0;
\]
(1 - αₜ) \frac{1}{U_{h2}(x_{h2}^2)} U_{h2}'(x_{h2}^2) - \lambda_h = 0.

Using the budget constraint and the first-order conditions yields

\alpha_h \frac{1}{U_{h1}(x_{h1}^1)} U_{h1}'(x_{h1}^1) - (1 - \alpha_h) \frac{U_{h2}'(w_{h1}^1 p_1 + w_{h1}^2 - x_{h1}^1 p_1)}{U_{h2}(w_{h1}^1 p_1 + w_{h1}^2 - x_{h1}^1 p_1)} p_1 = 0

or

\frac{\alpha_h}{1 - \alpha_h} F_1'(x_{h1}^1) = F_2'(w_{h1}^1 p_1 + w_{h1}^2 - x_{h1}^1 p_1) \cdot p_1 \tag{24}

where \( F_1 \equiv \ln U_{h1} \) and \( F_2 \equiv \ln U_{h2} \). \( F_1' \) and \( F_2' \) are strictly decreasing functions. Hence, for a given \( p_1 \), a higher (equal) \( \alpha_h \) requires a higher (identical) consumption of good 1 to preserve (24). This shows (i) and (ii).

By the same argument, an increase of \( \alpha_h \) raises \textit{ceteris paribus} the aggregate demand for good 1. Further examination of (24) shows that for fixed bargaining power parameters, aggregate demand for good 1 is a decreasing function of \( p_1 \). Consequently, if only \( \alpha_h \) is increased, then the equilibrium price \( \hat{p}_1 \) rises and the equilibrium consumption of all first members except \( h1 \) is reduced. Finally, market clearing implies that the equilibrium consumption of \( h1 \) goes up. This shows (iii) and, by symmetry, (iv).
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