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Designs Balanced For Neighbor Effects in Circular Binary Blocks of Size Ten

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Abstract
Neighbor balanced designs are more useful to remove the neighbor effects in experiments where the performance of a treatment is affected by the treatments applied to its adjacent plots. These designs ensure that treatment comparisons will be less affected by neighbor effects as possible. In literature, these designs are available in circular blocks of size 3, 5, 6, 8, 9. In this article, neighbor balanced designs are constructed in circular binary blocks of size ten. A catalogue of these designs is also compiled.

Keywords: Binary blocks, Circular blocks, Neighbor effects, Neighbor balanced designs.

1. Introduction
Rees (1967) introduced neighbor designs in serology and constructed these designs in complete blocks for all odd number of treatments. A design \((v, k, \lambda')\) in which each pair of distinct treatments appears \(\lambda'\) times as neighbors is called neighbor balanced design, where \(v\) is number of treatments, \(k\) is block size and \(\lambda'\) is number of times each pair of distinct treatments appears as neighbors. Neighbor balanced designs ensure that treatment comparisons will be less affected by neighbor effects because these designs are a tool for local control in biometrics, agriculture, horticulture and forestry. These designs are, therefore, useful for the cases where the performance of a treatment is affected by the treatments applied to its neighboring plots. Neighbor designs were initially used in serology. Rees (1967) presented a technique used in virus research which requires the arrangement in circles of samples from a number of virus preparations such that over the whole set, a sample from each virus preparation appears next to a sample from every other virus preparation. Experiments in agriculture, horticulture and forestry often show neighbor effects, (see Azais et al., 1993). The design strategy of a statistical experiment is influenced, to a large extent, by the nature of dependence that exists among the observations. Neighbor designs are relatively robust to neighbor effects. Rees (1967) generated neighbor designs for \(k \leq 10\) and \(v\) odd up to 41.
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2. Construction of Neighbor Design (ND) for \( k = 10 \)

2.1 ND for \( v = 10i \)

**Theorem 2.1.** ND with \( \lambda' = 2 \) can be generated for \( v = 10i; i \geq 1 \) integer in \( k = 10 \) by developing the following \( i \) initial blocks cyclically mod \((v-1)\).

\[
I_j = (0, 5j-4, 10j-7, 15j-9, 20j-10, 25j-12, 30j-15, \infty) \mod (v-1); \quad j = 1, 2, \ldots, i-1.
\]

\[
I_i = (0, 0, m-3, 2m-5, m-7, 2m-7, m-8, 2m-9, m-12, 2m-15, \infty) \mod (v-1); \quad m = (v-2)/2,
\]

**Proof.** Combined set of forward and backward differences between neighboring elements takes all the values from 1 to \( 2m \) twice. It is, therefore, ND with \( \lambda' = 2 \).

**Example 2.1.** ND for \( v = 30 \) and \( k = 10 \) is generated by developing the following three initial blocks cyclically mod 29.

\[
I_1 = (0, 1, 3, 6, 10, 15, 14, 12, 9, 5), \quad I_2 = (0, 6, 13, 21, 1, 11, 5, 27, 19, 9, 10)
\]

\[
I_3 = (0, 11, 23, 7, 21, 6, 19, 2, 13, \infty)
\]

2.2 ND for \( v = 20i+1; i \) integer

ND can be generated for \( v = 20i+1; i \) integer in \( k = 10 \) by developing \( i \) initial blocks cyclically mod \( v \).

**Example 2.2.** ND for \( v = 41 \) and \( k = 10 \) is generated by developing the following two initial blocks cyclically mod 41.

\[
I_1 = (0, 1, 3, 6, 10, 15, 21, 28, 36, 9), \quad I_2 = (0, 10, 22, 33, 5, 30, 4, 21, 39, 20)
\]

2.3 ND for \( v = 10i+1; i \geq 1 \) odd

ND for \( \lambda' = 2 \) can be generated for \( v = 10i+1; i \geq 1 \) odd in \( k = 10 \) by developing \( i \) initial blocks cyclically mod \((v-1)\).

**Example 2.3.** ND for \( v = 31 \) and \( k = 10 \) is generated by developing the following three initial blocks cyclically mod 31.

\[
I_1 = I_2 = I_3 = (0, 1, 3, 6, 10, 15, 21, 28, 5, 14), \quad I_3 = (0, 11, 23, 3, 16, 1, 13, 26, 5, 15)
\]

2.4 ND when HCF of \( v \) and \( k \) is 5

ND with \( \lambda' = 4 \) can be generated for \( k = 10 \) when HCF of \( v \) and \( k \) is 5 by developing \( v/5 \) initial blocks (two of these blocks contain \( \infty \)) cyclically mod \((v-1)\).

**Example 2.4.** ND is generated for \( v = 35 \) and \( k = 10 \) by developing the following seven initial blocks cyclically mod 34.

\[
I_1 = I_2 = I_3 = (0, 1, 3, 6, 10, 15, 21, 28, 2, 11), \quad I_5 = (0, 10, 22, 1, 15, 30, 12, 29, 5, 17),
\]

\[
I_6 = (0, 10, 22, 1, 15, 30, 12, 25, 5, \infty), \quad I_7 = (0, 10, 22, 1, 15, 30, 12, 27, 9, \infty)
\]

2.5 ND when HCF of \( v-1 \) and \( k \) is 5
ND with $\lambda' = 4$ can be generated for $k = 10$ when HCF of $\nu - 1$ and $k$ is 5 by developing $(\nu - 1)/5$ initial blocks cyclically mod $\nu$.

**Example 2.5.** ND is generated for $\nu = 36$ and $k = 10$ by developing the following seven initial blocks cyclically mod 36.

$$
I_1 = I_2 = I_3 = I_4 = (0,2,3,6,10,15,21,28,1,11), \quad I_5 = I_6 = (0,8,20,33,11,26,6,23,5,13), \\
I_7 = (0,24,2,23,3,20,8,22,1,17)
$$

**2.6 ND when HCF of $\nu$ and $k$ is 2**

ND with $\lambda' = 10$ can be generated for $k = 10$ when HCF of $\nu$ and $k$ is 2 by developing $\nu/2$ initial blocks (five of these blocks contain $\infty$) cyclically mod $(\nu - 1)$.

**Example 2.6.** ND is generated for $\nu = 32$ and $k = 10$ by developing the following 16 initial blocks cyclically mod 31.

$$
I_1 = I_2 = I_3 = I_4 = I_5 = I_6 = I_7 = I_8 = I_9 = I_{10} = (0,1,3,6,10,15,21,28,5,14), \\
I_{11} = (0,10,21,2,15,30,9,20,1,16), \\
I_{12} = I_{13} = I_{14} = I_{15} = (0,10,21,2,15,30,9,20,1,\infty), \\
I_{16} = (0,13,26,8,21,3,18,2,17,\infty)
$$

**2.7 ND when HCF of $\nu - 1$ and $k$ is 2; $(\nu - 1)/2$ odd**

ND with $\lambda' = 5$ can be generated for $k = 10; \ (\nu - 1)/2$ odd when HCF of $\nu - 1$ and $k$ is 2 by developing $(\nu - 1)/4$ initial blocks cyclically mod $\nu$.

**Example 2.7.** ND is generated for $\nu = 33$ and $k = 10$ by developing the following eight initial blocks cyclically mod 33.

$$
I_1 = I_2 = I_3 = I_4 = I_5 = I_6 = I_7 = I_8 = (0,1,3,6,10,15,21,28,3,12), \\
I_9 = I_{10} = (0,10,21,2,15,30,13,23,1,14), \\
I_11 = (0,23,12,25,6,21,4,19,2,17)
$$

**2.8 ND when HCF of $\nu - 1$ and $k$ is 2; $(\nu - 1)/2$ even**

ND with $\lambda' = 10$ can be generated for $k = 10; \ (\nu - 1)/2$ even when HCF of $\nu - 1$ and $k$ is 2 by developing $(\nu - 1)/2$ initial blocks cyclically mod $\nu$.

**Example 2.8.** ND is generated for $\nu = 39$ and $k = 10$ by developing the following 19 initial blocks cyclically mod 39.

$$
I_1 = I_2 = I_3 = I_4 = I_5 = I_6 = I_7 = I_8 = I_9 = I_{10} = (0,1,3,6,10,15,21,28,36,9), \\
I_{11} = I_{12} = I_{13} = I_{14} = I_{15} = (0,10,21,34,9,24,1,18,36,16), \\
I_{16} = I_{17} = (0,10,21,34,9,24,2,20,30,11), \\
I_{18} = (0,10,21,34,9,24,2,20,1,14), \\
I_{19} = (0,26,12,27,10,28,8,23,1,19)
$$

**2.9 ND for $\nu$ odd prime ($\nu > 19$)**

ND can be generated with $\lambda' = 10$ for $\nu$ (prime) $= 10i + 1$ and $k = 10$ by developing the following $(\nu - 1)/2$ initial blocks cyclically mod $\nu$.

$$
I_j = (0, j, 2j, 3j, \ldots, 9j) \mod \nu; \quad j = 1, 2, \ldots, i.
$$
3. Catalog of ND for $k = 10$ for $v = 5i$ and $v = 5i+1$, where $4 \leq i \leq 20$.
(Inclusion of some existing designs in the catalogue is possible)

<table>
<thead>
<tr>
<th>$v$</th>
<th>$\lambda'$</th>
<th>Initial Blocks</th>
<th>Where $\infty$</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>2</td>
<td>(0,6,13,2,11,1,9,16,3,\infty), where $\infty = 19$</td>
<td></td>
</tr>
<tr>
<td>21</td>
<td>1</td>
<td>(0,1,3,6,10,15,8,14,2,10)</td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>4</td>
<td>(0,23,1,22,2,21,3,19,4,11), (0,14,2,16,4,18,6,20,8,\infty), where $\infty = 24$</td>
<td></td>
</tr>
<tr>
<td>26</td>
<td>4</td>
<td>(0,25,1,24,2,23,3,21,4,11), (0,25,1,24,2,23,3,21,4,11), (0,14,1,17,3,19,5,15,25,13)</td>
<td></td>
</tr>
<tr>
<td>40</td>
<td>2</td>
<td>(0,1,3,6,10,15,14,12,9,5), (0,6,13,21,30,1,34,27,19,10), (0,11,23,36,11,26,15,3,29,15), (0,16,33,12,31,11,29,7,23,\infty), where $\infty = 39$</td>
<td></td>
</tr>
<tr>
<td>45</td>
<td>4</td>
<td>(0,43,41,1,4,10,5,12,20,11), (0,43,41,1,4,10,5,12,20,11), (0,43,41,1,4,10,5,12,20,11), (0,43,41,1,4,10,5,12,20,11), (0,34,2,33,3,32,4,21,39,20), (0,34,2,33,3,32,4,21,39,20), (0,34,2,33,3,32,4,21,39,20), (0,34,2,33,3,32,4,21,39,20), (0,19,39,16,38,15,37,14,35,\infty), where $\infty = 44$</td>
<td></td>
</tr>
<tr>
<td>46</td>
<td>4</td>
<td>(0,45,1,44,2,43,3,41,4,11), (0,45,1,44,2,43,3,41,4,11), (0,45,1,44,2,43,3,41,4,11), (0,45,1,44,2,43,3,41,4,11), (0,36,2,35,3,33,4,32,5,20), (0,36,2,35,3,33,4,32,5,20), (0,36,2,35,3,33,4,32,5,20), (0,36,2,35,3,33,4,32,5,20), (0,24,1,26,2,27,3,24,45,23)</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>2</td>
<td>(0,1,3,6,10,15,14,12,9,5), (0,6,13,21,30,35,34,27,19,10), (0,11,23,36,1,11,5,42,29,15), (0,16,33,2,21,36,25,8,39,20), (0,21,43,17,41,16,39,12,33,\infty), where $\infty = 49$</td>
<td></td>
</tr>
<tr>
<td>51</td>
<td>2</td>
<td>(0,50,1,49,2,48,3,46,4,11), (0,50,1,49,2,48,3,46,4,11), (0,41,2,40,3,38,4,37,5,20), (0,41,2,40,3,38,4,37,5,20), (0,29,1,28,2,32,3,27,4,25)</td>
<td></td>
</tr>
<tr>
<td>55</td>
<td>4</td>
<td>(0,53,51,1,4,10,5,12,20,11), (0,53,51,1,4,10,5,12,20,11), (0,53,51,1,4,10,5,12,20,11), (0,53,51,1,4,10,5,12,20,11), (0,44,2,43,3,42,4,21,39,20), (0,44,2,43,3,42,4,21,39,20), (0,44,2,43,3,42,4,21,39,20), (0,44,2,43,3,42,4,21,39,20), (0,33,12,44,22,49,16,37,5,27), (0,23,47,18,44,16,41,11,34,\infty), (0,23,47,18,44,16,41,11,34,\infty), where $\infty = 54$</td>
<td></td>
</tr>
<tr>
<td>56</td>
<td>4</td>
<td>(0,55,1,54,2,53,3,51,4,11), (0,55,1,54,2,53,3,51,4,11), (0,55,1,54,2,53,3,51,4,11), (0,55,1,54,2,53,3,51,4,11), (0,46,2,45,3,43,4,42,5,20), (0,46,2,45,3,43,4,42,5,20), (0,46,2,45,3,43,4,42,5,20), (0,46,2,45,3,43,4,42,5,20), (0,35,1,34,2,33,3,32,4,25), (0,35,1,34,2,33,3,32,4,25), (0,33,1,31,2,34,4,38,5,27)</td>
<td></td>
</tr>
<tr>
<td>60</td>
<td>2</td>
<td>(0,1,3,6,10,15,14,12,9,5), (0,6,13,21,30,40,34,27,19,10), (0,11,23,36,50,6,54,42,29,15), (0,16,33,51,11,31,15,57,39,20), (0,21,43,7,31,56,35,13,49,25), (0,26,53,22,51,21,49,17,43,\infty), where $\infty = 59$</td>
<td></td>
</tr>
<tr>
<td>61</td>
<td>1</td>
<td>(0,1,3,6,10,15,21,28,36,45), (0,51,1,50,2,49,3,47,4,23), (0,41,1,40,3,39,4,31,59,30)</td>
<td></td>
</tr>
<tr>
<td>65</td>
<td>4</td>
<td>(0,63,61,1,4,10,5,12,20,11), (0,63,61,1,4,10,5,12,20,11), (0,63,61,1,4,10,5,12,20,11), (0,63,61,1,4,10,5,12,20,11), (0,54,2,53,3,52,4,21,39,20), (0,54,2,53,3,52,4,21,39,20), (0,54,2,53,3,52,4,21,39,20), (0,54,2,53,3,52,4,21,39,20), (0,54,2,53,3,52,4,21,39,20)</td>
<td></td>
</tr>
</tbody>
</table>
Designs Balanced For Neighbor Effects

<table>
<thead>
<tr>
<th>$v$</th>
<th>$\lambda'$</th>
<th>Initial Blocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>66</td>
<td>4</td>
<td>(0.54, 53.3, 52.4, 21.39, 20), (0.54, 53.3, 52.4, 21.39, 20), (0.43, 21.45, 4.3, 30.5, 32.60, 31), (0.43, 21.45, 4.3, 30.5, 32.60, 31), (0.43, 21.45, 4.3, 30.5, 32.60, $\infty$), where $\infty = 64$</td>
</tr>
<tr>
<td>70</td>
<td>2</td>
<td>(0, 74, 2, 73, 3, 72, 4, 21, 39, 20), (0, 74, 2, 73, 3, 72, 4, 21, 39, 20), (0, 63, 27, 61, 26, 59, 22, 53, $\infty$), where $\infty = 69$</td>
</tr>
<tr>
<td>71</td>
<td>2</td>
<td>(0.70, 1.69, 2.68, 3.66, 4.11), (0.70, 1.69, 2.68, 3.66, 4.11), (0.50, 1.49, 2.48, 3.46, 4.31), (0.50, 1.49, 2.48, 3.46, 4.31), (0.61, 2.60, 3.58, 4.57, 5.20), (0.39, 1.38, 2.43, 3.48, 4.35)</td>
</tr>
<tr>
<td>75</td>
<td>4</td>
<td>(0.73, 71.1, 4.10, 5.12, 20.11), (0.73, 71.1, 4.10, 5.12, 20.11), (0.73, 71.1, 4.10, 5.12, 20.11), (0.64, 2.63, 3.62, 4.21, 39.20), (0.64, 2.63, 3.62, 4.21, 39.20), (0.64, 2.63, 3.62, 4.21, 39.20), (0.53, 55.4, 30.5, 32.60, 31), (0.53, 55.4, 30.5, 32.60, 31), (0.53, 55.4, 30.5, 32.60, 31), (0.30, 60, 18.5, 13.72, 27, 69, 37), (0.33, 67, 28, 64, 26, 61, 21, 54, $\infty$), (0.33, 67, 28, 64, 26, 61, 21, 54, $\infty$), where $\infty = 74$</td>
</tr>
<tr>
<td>76</td>
<td>4</td>
<td>(0.75, 1.74, 2.73, 3.71, 4.11), (0.75, 1.74, 2.73, 3.71, 4.11), (0.55, 1.54, 2.53, 3.51, 4.31), (0.55, 1.54, 2.53, 3.51, 4.31), (0.55, 1.54, 2.53, 3.51, 4.31), (0.55, 1.54, 2.53, 3.51, 4.31), (0.66, 2.65, 3.63, 4.62, 5.20), (0.66, 2.65, 3.63, 4.62, 5.20), (0.66, 2.65, 3.63, 4.62, 5.20), (0.46, 2.45, 3.44, 4.43, 5.41), (0.46, 2.45, 3.44, 4.43, 5.41), (0.43, 1.41, 2.44, 4.48, 5.37)</td>
</tr>
<tr>
<td>80</td>
<td>2</td>
<td>(0.1, 3.6, 10.15, 14.12, 9.5), (0.61, 32, 30.4, 34.27, 19.10), (0.11, 23.36, 50.65, 54.42, 29.15), (0.16, 33.51.71, 21.5, 57, 39.20), (0.21, 43.66, 11.36, 15.72, 49.25), (0.26, 53.23, 61.35, 8.59, 30), (0.31, 63.17, 51.75, 23, 69.35), (0.36, 73.32, 71.31, 69.27, 63, $\infty$), where $\infty = 79$</td>
</tr>
<tr>
<td>81</td>
<td>1</td>
<td>(0.80, 1.79, 2.78, 3.73, 14.13), (0.71, 1.70, 3.69, 4.21, 39.20), (0.60, 1.59, 2.58, 3.57, 4.33)</td>
</tr>
<tr>
<td>85</td>
<td>4</td>
<td>(0.83, 81.1, 4.10, 5.12, 20.11), (0.83, 81.1, 4.10, 5.12, 20.11), (0.83, 81.1, 4.10, 5.12, 20.11), (0.74, 2.73, 3.72, 4.21, 39.20), (0.74, 2.73, 3.72, 4.21, 39.20), (0.74, 2.73, 3.72, 4.21, 39.20), (0.74, 2.73, 3.72, 4.21, 39.20), (0.63, 41.65, 4.30, 5.32, 60.31), (0.63, 41.65, 4.30, 5.32, 60.31), (0.63, 41.65, 4.30, 5.32, 60.31), (0.63, 41.65, 4.30, 5.32, 60.31), (0.63, 41.65, 4.30, 5.32, 60.31), (0.63, 41.65, 4.30, 5.32, 60.31), (0.63, 41.65, 4.30, 5.32, 60.31), (0.63, 41.65, 4.30, 5.32, 60.31)</td>
</tr>
</tbody>
</table>

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### Initial Blocks

<table>
<thead>
<tr>
<th>$\nu$</th>
<th>$\lambda'$</th>
<th>Initial Blocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>86</td>
<td>4</td>
<td>$(0.54, 2.53, 3.52, 4.41, 79, 40), (0.54, 2.53, 3.52, 4.41, 79, 40), (0.54, 2.53, 3.52, 4.41, 79, 40), (0.54, 2.53, 3.52, 4.41, 79, 40), \ldots, (0.39, 79, 36, 78, 35, 77, 34, 75, 4), \text{ where } \infty = 84$</td>
</tr>
<tr>
<td>90</td>
<td>2</td>
<td>$(0.1, 3.6, 10, 15, 14, 12, 9, 5), (0.1, 3.6, 10, 15, 14, 12, 9, 5), (0.1, 3.6, 10, 15, 14, 12, 9, 5), (0.1, 3.6, 10, 15, 14, 12, 9, 5), (0.1, 3.6, 10, 15, 14, 12, 9, 5), \ldots, (0.41, 83, 37, 81, 36, 79, 32, 73, 4), \text{ where } \infty = 89$</td>
</tr>
<tr>
<td>91</td>
<td>2</td>
<td>$(0.90, 1.89, 2.88, 3.86, 4.11), (0.90, 1.89, 2.88, 3.86, 4.11), (0.90, 1.89, 2.88, 3.86, 4.11), (0.90, 1.89, 2.88, 3.86, 4.11), (0.90, 1.89, 2.88, 3.86, 4.11), \ldots, (0.49, 1.48, 2.52, 3.47, 4.45)$</td>
</tr>
<tr>
<td>95</td>
<td>4</td>
<td>$(0.93, 91, 1.4, 10, 5, 12, 20, 11), (0.93, 91, 1.4, 10, 5, 12, 20, 11), (0.93, 91, 1.4, 10, 5, 12, 20, 11), (0.93, 91, 1.4, 10, 5, 12, 20, 11), (0.93, 91, 1.4, 10, 5, 12, 20, 11), \ldots, (0.64, 2.63, 3.62, 4.41, 79, 40), \text{ where } \infty = 94$</td>
</tr>
<tr>
<td>96</td>
<td>4</td>
<td>$(0.95, 1.94, 2.93, 3.91, 4.11), (0.95, 1.94, 2.93, 3.91, 4.11), (0.95, 1.94, 2.93, 3.91, 4.11), (0.95, 1.94, 2.93, 3.91, 4.11), (0.95, 1.94, 2.93, 3.91, 4.11), \ldots, (0.53, 1.51, 2.54, 4.58, 5.47)$</td>
</tr>
<tr>
<td>100</td>
<td>2</td>
<td>$(0.1, 3.6, 10, 15, 14, 12, 9, 5), (0.1, 3.6, 10, 15, 14, 12, 9, 5), (0.1, 3.6, 10, 15, 14, 12, 9, 5), (0.1, 3.6, 10, 15, 14, 12, 9, 5), (0.1, 3.6, 10, 15, 14, 12, 9, 5), \ldots, (0.46, 93, 42, 91, 41, 89, 37, 83, 4), \text{ where } \infty=99$</td>
</tr>
</tbody>
</table>
Designs Balanced For Neighbor Effects

References


