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# ICIR Working Paper Series No. 31/2018

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## The pitfalls of central clearing in the presence of systematic risk

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### Abstract

Through the lens of market participants' objective to minimize counterparty risk, we provide an explanation for the reluctance to clear derivative trades in the absence of a central clearing obligation. We develop a comprehensive understanding of the benefits and potential pitfalls with respect to a single market participant's counterparty risk exposure when moving from a bilateral to a clearing architecture for derivative markets. Previous studies suggest that central clearing is beneficial for single market participants in the presence of a sufficiently large number of clearing members. We show that three elements can render central clearing harmful for a market participant's counterparty risk exposure regardless of the number of its counterparties: 1) correlation across and within derivative classes (i.e., systematic risk), 2) collateralization of derivative claims, and 3) loss sharing among clearing members. Our results have substantial implications for the design of derivatives markets, and highlight that recent central clearing reforms might not incentivize market participants to clear derivatives.

**JEL classification:** G01, G14, G18, G28.

**Keywords:** Central Clearing, Counterparty Risk, Systematic Risk, OTC markets, Derivatives, Loss Sharing, Collateral, Margin.

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Counterparty credit risk is the risk that counterparties do not fulfill their future obligations, e.g., when they default. Counterparty credit risk has emerged as one of the most important factors affecting risk in financial markets and amplifying the 2007-08 financial crisis ([Duffie et al. \(2010\)](#), [Acharya et al. \(2011\)](#), [Arora et al. \(2012\)](#), [Financial Stability Board \(FSB\) \(2017a\)](#)). Lehman Brothers' default during the 2007-08 financial crisis in particular demonstrated that the failure of an entity with large derivative positions can easily result in substantial loss spillovers to its counterparties, creating contagion and externalities to the economy.

Derivatives markets are a natural habitat for counterparty risk. Worldwide over-the-counter (OTC) derivative markets had a notional outstanding amount of \$542 trillion in 2017, according to the Bank for International Settlements (BIS). Before the 2007-08 financial crisis, the derivatives market architecture has been largely dominated by bilateral trades ([Financial Stability Board \(FSB\) \(2017a\)](#)). Bilateral trades are executed directly between two market participants and thus directly expose them to each other's default risk. To mitigate counterparty risk and increase transparency in derivative markets, the G20 leaders initiated a fundamental change in the architecture of these markets, leading to the Dodd-Frank Wall Street Reform and Consumer Protection Act (DFA) in 2010, and the European Market Infrastructure Regulation (EMIR) in 2012. A key element of the new regulation is the mandatory central clearing of standardized OTC derivatives through central clearing counterparties (CCPs). Indeed, the cleared share of Lehman's derivative trades was hedged and closed out by within three weeks after Lehman's failure, suggesting that central clearing stabilizes derivative markets ([Cunliffe \(2018\)](#)).

However, market participants are reluctant to centrally clear derivative contracts in practice, unless forced ([Financial Stability Board \(FSB\) \(2018\)](#)). Central clearing is currently mandatory for standardized interest rate swaps (IRS) contracts and index CDS in the U.S. and EU. Instead, clearing is still optional for single name CDS, foreign exchange forwards, commodity and equity derivatives, which largely remain uncleared (Abad, Aldasorol, Aymanns, D'Errico and Rousová (2016), [Office of the Comptroller of the Currency \(2016\)](#), [Financial Stability Board \(FSB\) \(2017a\)](#)).<sup>1</sup>

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<sup>1</sup>The [Financial Stability Board \(FSB\) \(2017a\)](#) reports that only 28% of outstanding CDS notionals were cleared in December 2016 (compared to 5% in June 2009). The fraction of notionals cleared is even smaller than 20% for foreign exchange, commodity, and equity derivatives in 2016. In contrast, 61% of all interest rate swap notionals outstanding were cleared in December 2016 (compared to 24% in December 2008), and 80% of new index CDS transactions in the U.S. are cleared as of April 2017.

In this paper we provide an intuition for low clearing rates based on the impact of clearing on counterparty risk. We show in a theoretical model that market participants do not necessarily reduce their counterparty risk exposure by moving from bilateral to centrally cleared trades, in particular (a) during market-wide extreme events, (b) if clearing margins are low compared to bilateral margins, or (c) if the exposure is positively correlated with systematic risk.<sup>2</sup> Our results emerge in particular by examining the impact of systematic risk, which we define as co-movement of derivative prices. High systematic risk aggravates the benefits of multilateral netting since it reduces the likelihood that losses to one counterparty can be offset with gains to another counterparty. It also creates *wrong way risk* since entity defaults naturally occur in bad states with low asset prices.

The [Financial Stability Board \(FSB\) \(2018\)](#) stresses that counterparty risk management as well as directional positions are indeed important factors for market participants' decision to centrally clear derivative trades.<sup>3</sup> [Duffie and Zhu \(2011\)](#) argue that counterparty risk exposure is also a reasonable measure for the risk of loss from counterparty defaults, and thus is a first-order consideration for systemic risk analysis. We share this approach and consider our analysis as one of the relevant elements that (1) market participants (should) consider in their evaluation of bilateral vs. multilateral netting, and (2) regulators (should) consider as one of the elements of a cost-benefit analysis of central clearing.<sup>4</sup>

Central clearing has been proposed to reduce counterparty risk exposure especially during the times of extreme events. However, although central clearing might stabilize derivative markets as a whole, we provide theoretical support that CCPs are not a panacea. Instead, during reasonable conditions, counterparty risk exposure with central clearing is actually larger than with bilateral clearing from the viewpoint of a single market participant's counterparty risk exposure. Our analysis therefore supports policymakers' efforts to revise the current implementation of market infrastructure regulation (as put forward, e.g., by the [European Systemic Risk Board \(ESRB\) \(2017\)](#) and the [Financial Stability Board \(FSB\) \(2018\)](#)) and their attempt to carefully evaluate the pros and cons of central clearing.

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<sup>2</sup>A market participant's counterparty risk exposure is defined as the expected loss given default of its counterparties.

<sup>3</sup>Other important factors for the decision to centrally clear are preferential capital treatment of centrally cleared derivatives and market liquidity.

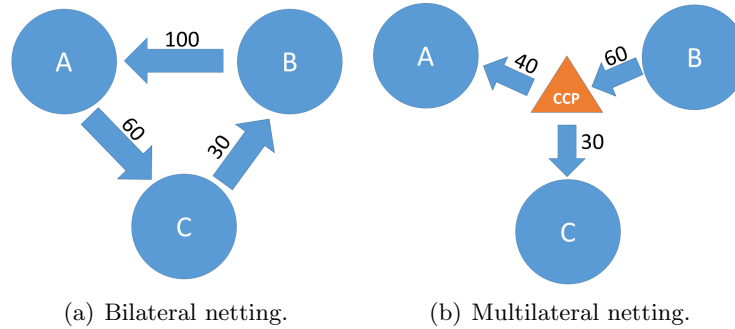
<sup>4</sup>Other important benefits of central clearing are reduced complexity, increased transparency in the derivative market, and reducing payment flows.

Our analysis builds on the model of [Duffie and Zhu \(2011\)](#) who show that central clearing of one derivative class reduces counterparty risk exposure if the number of counterparties is sufficiently large and contract values are uncorrelated. Our contribution is an analysis of (1) systematic risk of derivatives, resulting in correlation across derivative classes (e.g., between interest rate swaps (IRS) and credit default swaps (CDS)) and within derivative classes (e.g., among CDS with different reference entity or different maturity), (2) collateralization of derivative claims, and (3) sharing of CCP losses among non-defaulting clearing members. We show that, from the viewpoint of a single market participant, these three elements can render central clearing harmful for counterparty risk exposure regardless of the number of its counterparties. Hence, in a number of realistic situations, market participants do not lower their counterparty risk exposure with central clearing and thus might prefer bilateral trades. This insight provides an explanation for the reluctance of market participants to clear derivative trades in the absence of a central clearing obligation. It also suggests that central clearing might not always contribute to financial stability but, instead, can amplify financial contagion by increasing counterparty risk.

Central clearing was introduced to mitigate counterparty risk primarily by means of two mechanisms: multilateral netting and loss sharing. Multilateral netting allows market participants to net, i.e., offset gains and losses, across different counterparties at the CCP. Loss sharing is a CCP's main recovery tool and prescribes liquidity injections from non-defaulting clearing members if a CCP's losses exceed the sum of margin and default contribution of the defaulting clearing member as well as the CCP's own funds ([Elliott \(2013\)](#), [Duffie \(2015\)](#), [Financial Stability Board \(FSB\) \(2017b\)](#), [Armakolla and Laurent \(2017\)](#)).

We begin with an analysis of multilateral netting. Netting agreements aggregate outstanding positions into one single claim ([Bergman et al. \(2004\)](#)). Bilateral netting offsets positions across different derivative classes (e.g., IRS and CDS) with a single counterparty. Multilateral netting offsets positions within one derivative class across different counterparties. For example, in [Figure 1](#), A can reduce its total counterparty risk exposure from \$100 to \$40 with multilateral netting, as the exposure of \$100 to B is offset with a loss of \$60 to C. Multilateral netting results from the CCP becoming a counterparty in the middle of each trade.

In our framework, systematic risk reflects market-wide shocks that affect all derivative contracts and thus induces correlation among and within derivative classes. Market-wide shocks are vital to



**Figure 1.** Illustration of bilateral and multilateral netting.

(a) Bilateral netting and (b) multilateral netting across counterparties. Arrows illustrate the flow of profits and losses, e.g., B owes \$ 100 to A.

gauge the effectiveness of central clearing during economic crises. For example, the recent financial crisis of 2007-08 resulted in a sharp price decline in several derivative classes, such as mortgage credit default swaps. However, even during non-crisis periods derivative prices are correlated. For example, we empirically find that index CDS prices are highly correlated: In a single factor model, S&P 500 returns exhibit a correlation of 43% with a basket of U.S. on-the-run index CDS returns, and explain 19% of their variation. This finding is in line with other studies: [Pan and Singleton \(2008\)](#) find that over 96% of the variation in sovereign CDS spreads for one reference country, differing, e.g., by maturity, is explained by a single factor. [Longstaff et al. \(2011\)](#) find that 64% of variation in sovereign CDS spreads for different reference countries is explained by a single global factor.

We show that higher systematic risk results in an increase in counterparty risk exposure with multilateral netting relative to bilateral netting. As a result, with a reasonable calibration of our model, a market with more than 121 clearing members is needed for multilateral netting to reduce counterparty risk exposures compared to a bilateral market. This is substantially more than the typical number of clearing members and dealers in derivative markets in practice.<sup>5</sup>

We also explore times of extreme negative events such as financial crises. These are extremely adverse shocks to the systematic risk component in our model. During such extreme events, counterparty risk exposures substantially increase. We show that, if events are sufficiently extreme,

<sup>5</sup>For example, [Bellia et al. \(2017\)](#) document that there are only 26 clearing members in the sovereign CDS market (in a sample of transactions with at least one European counterparty), accounting for 96.5% of total gross notional amount. [Getmansky et al. \(2016\)](#) find that the largest 5 buyers and sellers in the single-name CDS market (in a sample of transactions with at least one U.S. reference entity or counterparty) account for more than 40% of all bought and sold CDS contracts, respectively.

then multilateral netting leads to larger counterparty risk exposures than bilateral netting for any number of counterparties. The intuition is that large expected portfolio losses or gains dominate potential diversification benefits from netting during extreme events. As a consequence, during extreme events, multilateral netting is not beneficial compared to a bilateral market for any number of clearing members.

The failure of multilateral netting to reduce counterparty risk exposures in all states of the world might be addressed by netting across both counterparties and derivative classes (*cross-netting*). We show that, before considering collateral, only central clearing with a single CCP that clears across all derivative classes and counterparties, i.e., a *Mega CCP*, can unambiguously reduce counterparty risk exposures compared to a bilateral market. Thus, a higher concentration in the CCP market seems beneficial for counterparty risk exposures, which is in line with the result of [Duffie and Zhu \(2011\)](#) that counterparty risk exposure decreases with the total number of CCPs. Derivative market participants already seem to have recognized the benefit of clearing concentration, with the result that a small number of CCPs dominate specific derivative classes.<sup>6</sup> However, the impact of concentration on competition among CCPs and concerns that a single CCP might be too systemically relevant highlight the tension between reducing counterparty risk exposure and reducing systemic risk.

We further introduce margin requirements in our model.<sup>7</sup> Current regulation requires smaller margins for cleared than for non-cleared derivative transactions ([Bank for International Settlements \(BIS\) \(2015, 2014\)](#), [Duffie et al. \(2015\)](#), [Financial Stability Board \(FSB\) \(2018\)](#)). The main reasons for this discrepancy are (1) to increase incentives to clear and (2) the expectation that CCPs are faster in auctioning the portfolios of defaulted clearing members.<sup>8</sup> We show that discrepancies in margin requirements for cleared derivatives (*clearing margin*) and non-cleared derivatives (*bilateral margin*) substantially affect counterparty risk exposures. If clearing margins are sufficiently smaller than bilateral margins, then multilateral netting always results in a higher counterparty

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<sup>6</sup>As of 2018 there are 49 CCPs authorized to offer services in the European Union, of which 32 are authorized to clear equity derivatives, and 24 to clear interest rate derivatives (see [European Securities and Markets Authority \(ESMA\) \(2018a,b\)](#)). However, clearing IRS and CDS concentrates on two CCPs: London Clearing House (LCH) has a market share of 90% in clearing Euro- and USD-denominated interest rate swaps as of February 2018, while ICE Clear Credit clears the vast majority of USD-denominated CDS, and CDX, and Euro-denominated CDS and iTraxx.

<sup>7</sup>We use the terms *margin* and *collateral* interchangeably.

<sup>8</sup>Discrepancies in margin requirements might also result from CCP funding. [Huang \(2018\)](#) links margin requirements to the capitalization of for-profit CCPs, and shows that better-capitalized CCPs require higher margins.

risk exposure than bilateral netting - regardless of the number of clearing members.<sup>9</sup> We derive the corresponding lowest acceptable difference between clearing and bilateral margin such that multilateral netting is beneficial compared to bilateral netting for a sufficiently large number of counterparties - from the viewpoint of market participants. This threshold is primarily driven by systematic risk: The higher the systematic risk, the smaller is the acceptable difference between clearing and bilateral margin. Indeed, we show that the degree of netting is only of minor importance if the clearing margin is not sufficiently large. We show that, as a consequence, current margin practices are unlikely to result in a benefit of multilateral netting from the perspective of a market participant's counterparty risk exposure.

Importantly, even a *Mega CCP* is not able to make up for any discrepancy between clearing and bilateral margin, and has a small impact on the benefit of multilateral netting if margins differ. Therefore, aligning margins for cleared and non-cleared derivatives is of primary importance to achieve a reduction of counterparty risk exposure via central clearing. Otherwise, to reduce counterparty risk exposure from a market participant's perspective, it would be more effective to just increase margin requirements in a bilateral market than to introduce central clearing at all.

As mentioned before, the infrastructure of central clearing does not rely solely on multilateral netting. It also includes loss sharing among surviving clearing members if losses exceed a defaulting clearing member's margin and default fund contributions as well as the CCP's own funds (Elliott (2013), Duffie (2015), Financial Stability Board (FSB) (2017b), Armakolla and Laurent (2017)).<sup>10</sup> For example, in September 2018 a clearing member's default triggered losses of the Swedish clearing house *Nasdaq Clearing AB* in excess of the member's margin and default fund contribution as well as the CCP's own default fund (Finansinspektionen (Financial Supervisory Authority Sweden) (2018)). As a consequence, the excess loss (EUR 107 million) was entirely born by remaining clearing members (Stafford and Sheppard (2018)).

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<sup>9</sup>Note that low margins however result in smaller total margin cost, which can also be beneficial for market participants. Thus, a market participant's decision whether to clear derivatives might ultimately depend on a trade-off between smaller margin cost and higher counterparty risk. In this paper, we, however, entirely focus on the effect on counterparty risk exposure. For an analysis of clearing cost, we refer to Ghamami and Glasserman (2017) and Financial Stability Board (FSB) (2018).

<sup>10</sup>Indeed, there have been several instances of clearinghouse failures in recent decades, for example the failure of the Korean exchange clearinghouse KRX (2014), the French Caisse de Liquidation in Paris (1974), Kuala Lumpur Commodities Clearing House (1983), Hong Kong Futures Exchange (1987), and the New Zealand Futures and Options Exchange (1989) (see Hills et al. (1999), Budding et al. (2016), and Bignon and Vuilleme (2018)). For a detailed discussion of the use of a CCP's funds to cover realized exposure we refer to Armakolla and Laurent (2017) and Elliott (2013).



Loss sharing mutualizes the idiosyncratic part of counterparty risk (Biais et al. (2016)). In the presence of systematic risk, however, buyers and sellers of derivative contracts benefit differently from clearing their trades. We show that, on average, clearing members with net portfolio gains during bad times, e.g., due to a short position in the S&P 500, benefit from loss sharing. Instead, those with net portfolio losses during bad times, e.g., due to a long position in the S&P 500, face a larger counterparty risk exposure when clearing their trades compared to not-clearing. The reason is correlation between market participants' bilateral counterparty risk exposure and default risk. Market participants with gains (losses) in bad (good) times have a high bilateral counterparty risk exposure exactly when counterparties are most likely to default. They benefit from sharing this large exposure with other clearing members. Then, they post *wrong way risk* to the CCP, meaning that their counterparty risk exposure is positively correlated with default risk.<sup>11</sup> In contrast, clearing members with losses (gains) during bad (good) times have a *right way risk*, i.e., a small bilateral counterparty risk exposure when defaults are most likely. Thus, they are worse off with central clearing, as they carry the large exposure of other clearing members with only a small exposure to share themselves.

As a consequence, market participants with a net position that is positively correlated with systematic risk likely do not reduce their counterparty risk by centrally clearing derivative trades. This bifurcation between clearing members with different directions of positions is worsened during extreme negative events. The finding is consistent with the reluctance of asset managers and, particularly, hedge funds to become clearing members at CCPs, as Siriwardane (2018) reports that these are the largest net sellers of CDS protection and, thus, have a positive correlation with systematic risk.

We argue that the only way to reduce the heterogeneity across clearing members' positions is to account for the direction of their positions when distributing a CCP's losses. This can be achieved, e.g., by demanding higher ex-post or ex-ante default fund contributions from clearing members with a negative correlation with systematic risk compared to those with a positive correlation. For example, with variation margin haircutting, a CCP allocates losses by reducing variation margin payments to clearing members whose portfolio values have increased (Elliott (2013)). Then, losses

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<sup>11</sup>See Bank for International Settlements (BIS) and International Organization of Securities Commissions (IOSCO) (2018) for a discussion on wrong way risk.

are mainly allocated to those clearing members with gains in bad times, offsetting their net benefit from loss sharing. Otherwise, the bifurcation among clearing members might lead to distorted incentives to clear derivative transactions.

In summary, our results strongly suggest that central clearing does not have an unambiguously positive effect on counterparty risk exposure. In contrast, we identify a large number of realistic situations in which central clearing does not reduce but increase counterparty risk exposure in comparison to bilateral clearing. This result provides a rationale for the observation that market participants are reluctant to centrally clear derivative contracts, unless forced. Particularly during financial crises, central clearing might lead to higher counterparty risk exposures from the viewpoint of single market participants, particularly for those with a positive exposure to systematic risk. In this study we take the perspective of a single market participant's to develop a comprehensive understanding of the effect of central clearing on counterparty risk exposure. Although this perspective is only partial, i.e., from market participants' point of view conditional on existing trades, it provides important insights that support policymakers in specifying financial market infrastructure regulation to enhance financial stability.<sup>12</sup>

The remainder of this paper is structured as follows. Section 1 describes the related literature. Section 2 presents a stylized model of a derivatives market extending the one from [Duffie and Zhu \(2011\)](#) by introducing systematic risk. In Section 3 we study the impact of multilateral vs. bilateral netting on counterparty risk exposures; while Section 4 focuses on the impact of loss sharing. Section 5 concludes. Propositions and proofs are provided in Appendix A.

## 1 Literature Review

We contribute to a growing strain of research on the role of central clearing for financial stability. [Duffie and Zhu \(2011\)](#) and [Lewandowska \(2015\)](#) study the impact of multilateral vs. bilateral netting on counterparty risk exposure when derivative prices are independently distributed. Their main result is that central clearing decreases counterparty risk exposure if there is a sufficient number of clearing members. [Duffie and Zhu \(2011\)](#) also provide an intuition about the impact

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<sup>12</sup>The ultimate effect of central clearing on financial stability also depends on its contribution to the transparency of derivative markets, as highlighted by [Acharya and Bisin \(2014\)](#), a potential reduction in loss concentration, as highlighted by [Lewandowska \(2015\)](#), and its effect on risk management practices of financial market infrastructures.

of correlation across (but not within) derivative classes: The more correlated different derivative classes are, the lower is the reduction in exposures that is achieved by bilateral netting across these classes. Therefore, with higher correlation across contract classes, bilateral netting becomes relatively less beneficial compared to multilateral netting. [Cont and Kokholm \(2014\)](#) follow this rationale and study the effect of correlation across derivative classes on the benefit of multilateral netting. They conclude that multilateral netting is likely to reduce counterparty risk exposures compared to bilateral netting, in practice.

We extend these two studies on multilateral vs. bilateral netting by (1) the introduction of systematic risk that results in correlation across and within derivative classes,<sup>13</sup> (2) the analysis of central clearing during extreme market events, and (3) the introduction of margin requirements. Our results show that these elements can render central clearing harmful for counterparty risk exposure under very reasonable and empirically justifiable circumstances. Importantly, we show that the presence of systematic risk results in situations in which market participants face higher counterparty risk exposure in centrally cleared than bilateral markets for any number of counterparties, which contrasts previous results.

[Jackson and Manning \(2007\)](#) also study central clearing in the presence of margins and correlated derivative positions. They however focus on the effect of correlation on the counterparty risk exposure with a multi-product CCP compared to that with a CCP that clears only one derivative. We extend this study by examining the effect of correlation on the benefit of clearing (either one or several) derivative classes compared to a bilateral market. We also vary margin levels and show that even a *Mega CCP* is not beneficial compared to a bilateral market if the clearing margin is too small. [Huang and Menkveld \(2016\)](#) and [Menkveld \(2017\)](#) identify concentration of cleared trades in a small number of risk factors as a major risk to the stability of CCPs. We add to their study by showing that such concentration also reduces the benefit of multilateral netting with respect to counterparty risk.

[Ghamami and Glasserman \(2017\)](#) study the capital and collateral costs of central clearing, and

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<sup>13</sup>Correlation across derivative classes results, e.g., when the CDS and interest rate swap price of either one or several market participants are correlated. For example, during the 2007-08 financial both interbank interest rates and CDS spreads for banks increased rapidly ([Brunnermeier \(2009\)](#)), implying correlation between IRS and CDS derivatives. Correlation within derivative classes results, e.g., when CDS with different reference entities or different maturities are correlated. For example, [Longstaff et al. \(2011\)](#) find a high correlation among sovereign CDS spreads for different countries.

find that there is no cost incentive for single market participants to centrally clear derivatives. Their result is contrasted by the [Financial Stability Board \(FSB\) \(2018\)](#)'s assessment that central clearing reforms create an overall incentive to clear. We complement these studies by providing an in-depth analysis of the impact of systematic risk and margins on counterparty risk exposure, and its sensitivity towards margin requirements, the number of counterparties, and systematic risk.

Moreover, we extend the previous studies by considering loss sharing, namely that CCPs provide an implicit insurance against counterparty defaults by allocating non-pre-funded losses to non-defaulting clearing members. [Biais, Heider, and Hoerova \(2016\)](#) study the optimal design of loss sharing and margins in the presence of moral hazard, stressing that loss sharing via central clearing can only provide insurance against idiosyncratic but not against systematic risk. Without considering systematic risk or comparing long and short positions, [Lewandowska \(2015\)](#) shows in a simulation study that loss sharing reduces loss concentration compared to bilateral clearing. [Arnsdorf \(2012\)](#) studies counterparty risk exposure resulting from loss sharing. Similar to our model, his model includes wrong way risk and Value-at-Risk based margin requirements. However, he does neither compare the counterparty risk exposure with central clearing to the case with bilateral netting nor does he study the effect of systematic risk, different margin requirements, or different long and short positions of clearing members. We show that loss sharing is beneficial compared to a bilateral market mostly for market participants that are short in the economy or hedged against systematic risk but not for those that are long in the economy. These differences grow with increasing systematic risk.

Empirical evidence on the impact of central clearing on counterparty risk has been growing only recently, fueled by the increasing availability of granular data. [Loon and Zhong \(2014\)](#) find that central clearing increases CDS spreads and attribute this effect to a reduction in counterparty risk. Their results are contrasted by those of [Du et al. \(2016\)](#) who find no empirical evidence that CDS spreads are positively affected by central clearing. In any case, CDS spreads only reflect market participants expectations and, thus, not necessarily the actual impact of central clearing on counterparty risk exposure. Instead, we characterize situations in which central clearing can actually reduce counterparty risk exposure, and show that its effect might also change over time since it is highly sensitive towards the current market environment as well as margin requirements. [Menkveld et al. \(2015\)](#) provide empirical evidence that the introduction of central clearing reduced

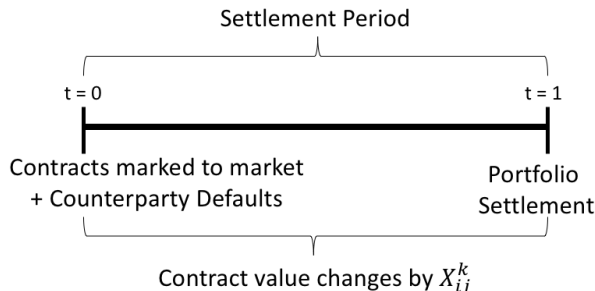
price volatility in equity markets. [Bellia et al. \(2017\)](#) study the determinants for market participants' decision to clear CDS contracts. Their empirical evidence suggests that dealers typically clear contracts with counterparties that would otherwise pose a large counterparty risk exposure. This result highlights the relevance of counterparty credit risk considerations for decisions to centrally clear, which supports the importance of our in-depth analysis of the impact of central clearing on counterparty risk.

## 2 A model of central clearing with systematic risk

Analogously to [Duffie and Zhu \(2011\)](#) and [Cont and Kokholm \(2014\)](#) we compare a central clearing architecture with a bilateral over-the-counter market from a market participant's perspective for a given set of derivative trades. We allow for  $K$  classes of derivative contracts. The classification might result from grouping common derivatives according to the type of underlying, such as interest rate, credit, commodities, or equities. One could also, more granularly, distinguish between derivatives that are sufficiently standardized for central clearing and those that are not. This will be relevant as we will later assume that a central clearing counterparty clears all derivatives within a specific derivative class.

Counterparty credit risk mainly arises from replacement costs during the time between opening and settling a derivative contract ([Bank for International Settlements \(BIS\) \(1998\)](#)). These costs typically result from changes in contract values during the settlement period, which is the time period between the latest exchange of collateral (i.e., variation margin) and the liquidation (i.e., settlement) of a contract portfolio. Clearly, the length of the settlement period depends on the liquidity of the contracts as well as the frequency of margin exchange. It typically ranges from 2 to 5 days for centrally cleared products, as these tend to be very liquid and margins are exchanged daily ([Arnsdorf \(2012\)](#)), but might be larger in non-centrally cleared and less liquid positions. Without loss of generality, we consider a one period model. At time  $t = 0$ , contracts are exchanged (or, equivalently, all contracts are marked to market by the exchange of variation margin) and, subsequently, counterparties might default. At time  $t = 1$ , contracts are settled.

As illustrated in [Figure 2](#), we assume that, during the settlement period, the absolute value change of contracts that market participant  $i$  traded with market participant  $j$  in derivative class



**Figure 2.** Timeline of the model.

Losses due to counterparty default occur between time  $t = 0$ , the most recent date where contracts have been marked to market and counterparties might default, and time  $t = 1$ , at which time the portfolio is settled.

$k$  is given by  $X_{ij}^k = v_{ij}^k r_{ij}^k$ .  $v_{ij}^k$  reflects the contract size, i.e., the quantity traded, and the position of the counterparties. Market participants are called *entities* or *counterparties* hereafter.

$r_{ij}^k$  is the contract return (at market value scaled by the contract size  $v$ ) during the settlement period. By following [Duffie and Zhu \(2011\)](#), for simplicity we assume that all contract returns are normally distributed with zero mean. Symmetry substantially reduces the dimension of our model and seems to be a reasonable assumption, particularly in arbitrage-free and informationally (weakly) efficient markets. The assumption of normally distributed bilateral exposures might not be justified for individual contracts, since these often exhibit heavily skewed and fat-tailed market values. However, due to diversification arising from aggregating across underlying names as well as long and short positions within a specific derivative class, it is reasonable that exposures are substantially less skewed or fat-tailed, particularly for large dealers.

The stochastic return  $r_{ij}^k$  consists of an idiosyncratic and systematic component and is given by

$$r_{ij}^k = \beta_{ij}^k M + \sigma_{ij}^k \varepsilon_{ij}^k, \quad (1)$$

where  $\varepsilon_{ij}^k \sim \mathcal{N}(0, 1)$  is the idiosyncratic risk component and  $\beta_{ij}^k$  is the market exposure of the contract. Due to symmetry, the gain of  $i$  is the loss of  $j$ , i.e.,  $r_{ij}^k = -r_{ji}^k$ . The systematic risk component  $M \sim \mathcal{N}(0, \sigma_M^2)$  serves as a latent variable that reflects the state of the economy and financial market. Large positive (negative) values of  $M$  reflect good (bad) states of the economy with high asset gains. It will be useful to reparametrize  $r_{ij}^k$  in terms of the total contract volatility,  $\sigma_{X,ij}^k = \sqrt{\text{var}(r_{ij}^k)}$ , and correlation with  $M$ ,  $\rho_{X,M,ij} = \text{cor}(r_{ij}^k, M)$ , such that  $\beta_{ij}^k = \rho_{X,M,ij}^k \frac{\sigma_{X,ij}^k}{\sigma_M}$

and  $\sigma_{ij}^k = \sigma_{X,ij}^k \sqrt{1 - (\rho_{X,M,ij}^k)^2}$ . The correlation between two contracts in classes  $k$  and  $m$ , traded between  $i$  and  $j$ , and  $h$  and  $l$ , then equals  $\text{cor}(X_{ij}^k, X_{hl}^m) = \text{sgn}(v_{ij}^k v_{hl}^m) \rho_{X,M,ij}^k \rho_{X,M,hl}^m$ , where  $\text{sgn}(x) = |x|/x$  is the signum function. The correlation is positive if  $i$  and  $h$  have either both long or both short positions, and is negative otherwise. In the following, we will take the viewpoint of one counterparty  $i$ 's contract portfolio  $\{X_{ij}^k : j \in \{1, \dots, \gamma\} \setminus \{i\}, k \in \{1, \dots, K\}\}$ .

Throughout the paper, we assume a positive correlation between contract returns  $r_{ij}^k$  and the state of the economy  $M$ ,  $\beta_{ij}^k > 0$ . This comes without loss of generality, since the final profit and loss of  $X$  ultimately depends on the long and short position of entities. For example, the profit and loss of market participants that are long in the S&P 500 is positively correlated with the economy,  $\text{cor}(X, M) > 0$ . Vice versa, market participants being short in the S&P 500 are negatively correlated with the economy,  $\text{cor}(X, M) < 0$ . Thus, the sign of correlation between  $X$  and  $M$  ultimately depends on an entity's position in the contract.

As we assume symmetric idiosyncratic risk,  $\mathbb{E}[\varepsilon_{ij}^k] = 0$ , the sign of  $v_{ij}^k$  determines an entity's long/short position in the state of the economy:  $v_{ij}^k > 0$  denotes a long-position of entity  $i$ , i.e., the value of the contract for  $i$  increases with  $M$ , and vice versa. For simplicity, we will call entity  $i$  *long in systematic risk* if  $v_{ij}^k > 0$ , and *short in systematic risk* if  $v_{ij}^k < 0$ . The absolute size  $|v_{ij}^k|$  determines the size of the contract and thus reflects the notional. In the absence of systematic risk ( $\rho_{X,M,ij}^k \equiv 0$ ) and with homogeneous positions ( $v_{ij}^k \equiv 1$ ) our model is equivalent to the one of [Duffie and Zhu \(2011\)](#).

First, we begin with the model of a bilateral OTC market. We assume that all entity-pairs have bilateral (close-out) netting agreements with each other. Netting agreements reduce counterparty risk exposures: For example, suppose that counterparty  $i$  trades two contracts with counterparty  $j$  and the value of these contracts is  $X_{ij}^1 = -100$  and  $X_{ij}^2 = 100$ . Without bilateral netting, counterparty  $j$  owes 100 to  $i$  on contract 2 and, thus, counterparty  $i$  loses 100 if  $j$  defaults. Moreover,  $i$  is still obligated to pay 100 to  $j$  for contract 1. With a bilateral netting agreement, the value of the two contracts is canceled out prior to default. In this example, neither counterparty  $i$  or  $j$  would suffer a loss if one of them defaults. Thus, in general, the total counterparty loss of  $i$  given default of  $j$ , i.e., its exposure in all derivative classes  $k = 1, \dots, K$ , equals the positive value of the sum of contract value changes,  $\max\left(\sum_{k=1}^K X_{ij}^k, 0\right)$ .

Second, we introduce central clearing. If derivative class  $K$  is cleared by a central clearing counterparty (CCP), then all positions in this derivative class are netted across counterparties. Thus, the loss of  $i$  given default of the CCP, i.e., its exposure, equals  $\max\left(\sum_{j=1, j \neq i}^{\gamma} X_{ij}^K, 0\right)$ , where  $\gamma$  is the total number of clearing members.

### 3 Bilateral vs. Multilateral Netting

#### 3.1 Systematic risk and counterparty risk exposures

We will stepwise increase the complexity of our model in order to isolate the impact of different components. For this purpose, we distinguish between counterparty risk exposure before considering collateral, called *collateralized counterparty risk exposure*, and counterparty risk exposure exceeding collateral, called *uncollateralized counterparty risk exposure*. We start by studying an entity's collateralized counterparty risk exposure, which corresponds to the metric in [Duffie and Zhu \(2011\)](#) and [Cont and Kokholm \(2014\)](#). For simplicity, we sometimes just refer to it as *exposure*. Our model differs from the previous two studies mainly by the systematic component  $M$  that induces correlation across and within derivative classes.

For simplicity, we consider a market that is as homogeneous as possible, which ensures that our baseline results are not driven by heterogeneity of market participants.<sup>14</sup> For this purpose, we follow [Duffie and Zhu \(2011\)](#) and assume that all contracts are homogeneous in that they exhibit the same distributional properties. We skip entity-specific indices where possible:  $\beta \equiv \beta_{ij}^k$  and  $\sigma \equiv \sigma_{ij}^k$  for all  $i \neq j$  and  $k = 1, \dots, K$ . This assumption substantially reduces the complexity of our model. Moreover, as in [Duffie and Zhu \(2011\)](#) and [Cont and Kokholm \(2014\)](#), all positions are assumed to equal unity,  $v \equiv 1$ .<sup>15</sup> In this case, entities do not hedge systematic risk across or within derivative classes, i.e., are long in systematic risk with each position.<sup>16</sup>

To assess the benefit of multilateral netting (and central clearing in general), we focus on the counterparty risk exposure of a given entity  $i$ . As argued by [Duffie and Zhu \(2011\)](#), counterparty risk exposure is a reasonable measure for the risk of loss from counterparty defaults and thus for

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<sup>14</sup>We conduct sensitivity analyses with regard to the heterogeneity of position sizes  $v$  after establishing our baseline results.

<sup>15</sup>Note that due to the unconditional symmetry of  $X$ , unconditional results also hold if  $v \equiv -1$ .

<sup>16</sup>We will conduct a sensitivity analysis towards this assumption.



a first-order consideration for systemic risk analysis.<sup>17</sup> In a homogeneous market, entity  $i$ 's total counterparty risk exposure with bilateral netting of  $K$  derivative classes with  $\gamma - 1$  counterparties is given by

$$\mathbb{E}[E_i^{BN,K}] = (\gamma - 1)\varphi(0)\sqrt{\sigma_M^2 K^2 \beta^2 + K\sigma^2}. \quad (2)$$

*Proof:* See Proposition 1 in Appendix A.

If derivative class  $K$  is multilaterally netted, then  $i$ 's total counterparty risk exposure is given by

$$\mathbb{E}[E_i^{BN+MN}] = \underbrace{\varphi(0)(\gamma - 1)\sqrt{\sigma_M^2 (K - 1)^2 \beta^2 + (K - 1)\sigma^2}}_{=\mathbb{E}[E_i^{BN,K-1}] \text{ (bilaterally netted)}} + \underbrace{\varphi(0)\sqrt{\sigma_M^2 (\gamma - 1)^2 \beta^2 + (\gamma - 1)\sigma^2}}_{=\mathbb{E}[E_i^{MN}] \text{ (multilaterally netted)}}.$$

*Proof:* See Proposition 2 in Appendix A.

The first term of  $\mathbb{E}[E_i^{BN+MN}]$  gives entity  $i$ 's counterparty risk exposure resulting from bilateral netting agreements with  $\gamma - 1$  counterparties in  $K - 1$  derivative classes, which is  $\mathbb{E}[E_i^{BN,K-1}]$ . The second term is the counterparty risk exposure in the multilaterally netted derivative class  $K$ , which is  $\mathbb{E}[E_i^{MN}]$ .

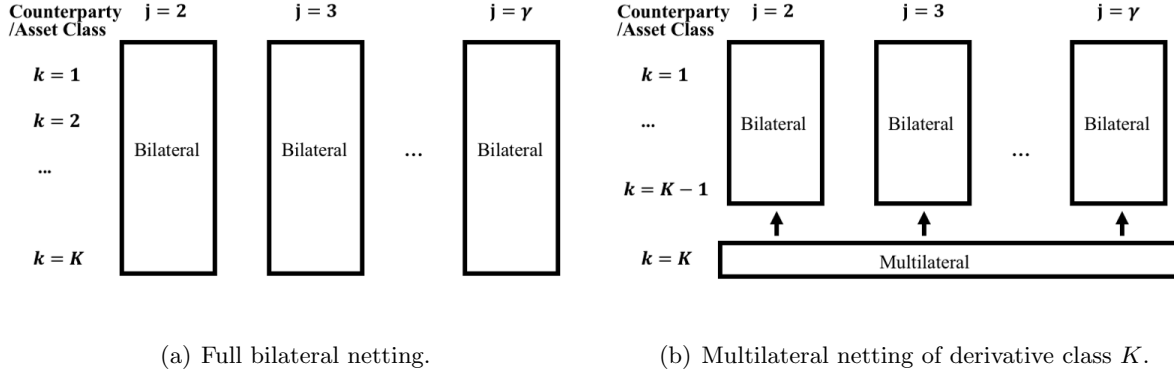
The bilateral and multilateral netting pools are illustrated in Figure 3. Multilateral netting of derivative class  $K$  has two opposing effects: On one hand, it shrinks all bilateral netting pools with different counterparties by taking out derivative class  $K$ . This reduces diversification in these pools. On the other hand, it creates a new pool across all counterparties, i.e., the multilateral netting pool. Clearly, if there is a very large number of counterparties  $\gamma$  compared to the number of derivative classes  $K$ , a high degree of diversification in the multilateral netting pool can offset the reduction in diversification in the bilateral pools.

### 3.1.1 Calibration

In this subsection, we calibrate our model to evaluate the effect of central clearing on counterparty credit risk exposure. The model is calibrated in order to realistically reflect the characteristics of derivative markets. The baseline number of counterparties is  $\gamma = 16$ , which corresponds to the

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<sup>17</sup>The inverse of the collateralized counterparty risk exposure is called *netting efficiency* by Duffie and Zhu (2011). They note that essentially any other risk measure is increasing in counterparty risk exposure under the assumption of normality and symmetry.



**Figure 3.** Bilateral and multilateral netting pools in our model.

$K$  is the number of derivative classes and  $\gamma$  the number of counterparties trading with each other. The illustration is from the perspective of entity  $i$  that trades with  $\gamma - 1$  counterparties.

G16 dealers that trade more than 50% (in terms of outstanding notional) of uncleared interest rate derivatives, 60% of uncleared credit default swaps, and 37% of uncleared foreign exchange derivatives in the European market (Abad, Aldasorol, Aymanns, D’Errico and Rousová (2016)).<sup>18</sup> It is also close to the actual number of clearing members at U.S. and European CCPs.<sup>19</sup> The concentration is even higher among dealers that clear for their clients, as Woodall (2018) reports that only 5 dealers hold 75% of all clients’ positions at the London Clearing House (LCH). We will vary the number of counterparties  $\gamma$  as one of the main parameters of interest. We assume a total number of  $K = 10$  derivative classes. Note that  $K$  mainly reflects the degree of diversification within bilateral pools.

We calibrate the volatility of contract values based on index CDS, since these are already subject to clearing obligations in the U.S. and EU. For this purpose, we retrieve data about the performance of the North American family of CDS indices, the *CDX family*, from January 2006 to 2010 from Markit. We choose this period because it covers the 2007-08 financial crisis. Table 1 reports the names of CDS indices included in our sample. Starting with the assumption of a 5-day settlement period, the descriptive statistics in Table 2 show that the average standard deviation of

<sup>18</sup>According to Abad, Aldasorol, Aymanns, D’Errico and Rousová (2016), the group of G16 dealers includes Bank of America, Barclays, BNP Paribas, Citigroup, Credit Agricole, Credit Suisse, Deutsche Bank, Goldman Sachs, HSBC, JPMorgan Chase, Morgan Stanley, Nomura, Royal Bank of Scotland, Societe Generale, UBS, and Wells Fargo.

<sup>19</sup>ICE Clear Europe has 22 members for CDS clearing, 38 for U.S. futures, 72 for EU futures, and 24 for Endex clearing as of March 2018 (<https://www.theice.com/clear-europe/membership>). ICE Clear U.S. has 38 clearing members in different derivative classes as of March 2018 (<https://www.theice.com/clear-us/membership>). The CDS clearing service of LCH Clearnet (CDSClear) has 13 members, and its cash equities and cash equities equivalent clearing service (EquityClear) has 33 members as of January 2018 (<https://www.lch.com/membership/member-search>).

5-day log returns of index CDS prices roughly equals  $\sigma_X = 0.01$ , which we use as an estimate for total contract volatility. During the same time period, the standard deviation of 5-day log returns of the S&P 500 is roughly  $\sigma_M = 0.03$ , which we use as an estimate for the volatility of the state of the economy, i.e., systematic risk component.

CDX name	Description
CDX NA.HY	North American High Yield CDSs
CDX NA.HY.B	Rating sub-index of CDX NA.HY
CDX NA.HY.BB	Rating sub-index of CDX NA.HY
CDX NA.HY.HB	Sub-index of CDX NA.HY (high beta)
CDX NA.IG	North American investment-grade CDSs
CDX NA.IG.CON	Sub-index of CDX NA.IG (consumer cyclical)
CDX NA.IG.ENRG	Sub-index of CDX NA.IG (energy)
CDX NA.IG.FIN	Sub-index of CDX NA.IG (financials)
CDX NA.IG.TMT	Sub-index of CDX NA.IG (telecom, media and technology)
CDX NA.IG.INDU	Sub-index of CDX NA.IG (industrial)
CDX NA.IG.HVOL	Sub-index of CDX NA.IG (high volatility)
CDX NA.XO	Sub-index of CDX NA.IG (crossover between grade and junk)
CDX.EM	Emerging market CDSs
CDX EM.DIV	Emerging market CDSs (diversified)

**Table 1.** Names, and description of CDX indices included in our data sample. *Source:* [Markit \(2015\)](#).

Statistic	N	Min	Pctl(25)	Median	Pctl(75)	Max	Mean	St. Dev.
S&P 500	1,021	-0.203	-0.013	0.002	0.015	0.175	-0.001	0.031
CDX (all)	590,706	-0.288	-0.002	0.0003	0.004	0.291	0.001	0.012
CDX (CDX.NA.HY)	131,945	-0.096	-0.004	0.002	0.010	0.095	0.003	0.015
CDX (CDX.NA.HY.B)	27,921	-0.090	-0.003	0.0005	0.005	0.146	0.002	0.013
CDX (CDX.NA.HY.BB)	19,474	-0.064	-0.003	0.0004	0.003	0.056	0.0005	0.009
CDX (CDX.NA.HY.HB)	38,254	-0.163	-0.005	0.002	0.011	0.215	0.005	0.024
CDX (CDX.NA.IG)	83,264	-0.288	-0.001	0.0001	0.002	0.291	0.0002	0.006
CDX (CDX.NA.IG.CON)	29,007	-0.046	-0.001	0.000	0.001	0.027	-0.0001	0.005
CDX (CDX.NA.IG.ENRG)	29,007	-0.039	-0.001	-0.00001	0.001	0.032	-0.00003	0.004
CDX (CDX.NA.IG.FIN)	47,653	-0.095	-0.003	0.0003	0.005	0.045	0.0003	0.011
CDX (CDX.NA.IG.TMT)	31,953	-0.056	-0.002	0.00001	0.002	0.078	0.0001	0.006
CDX (CDX.NA.IG.INDU)	35,790	-0.049	-0.002	0.0001	0.002	0.037	0.00002	0.005
CDX (CDX.NA.IG.HVOL)	56,996	-0.073	-0.002	0.0001	0.002	0.048	0.0001	0.008
CDX (CDX.NA.XO)	30,508	-0.081	-0.005	0.001	0.006	0.067	0.001	0.012
CDX (CDX.EM)	14,372	-0.180	-0.003	-0.00001	0.004	0.192	-0.0002	0.018
CDX (CDX.EM.DIV)	14,562	-0.144	-0.002	0.0002	0.003	0.149	0.0002	0.014

**Table 2.** Descriptive statistics of 5-day log returns of CDX indices and the S&P 500. The statistics are based on date-tenor-series-version observations for different index CDX families (see Table 1 for descriptions), all family-date-tenor-series-version observations for CDX (all), and date observations for S&P 500 from January 2006 to December 2009. *Source:* Markit.

To calibrate the correlation between contract returns and the state of the economy, we employ a one-factor model, regressing CDS index returns on 5-day S&P 500 log-returns during 2006 to

	<i>Dependent variable: 5-day CDX return</i>		
	Full (1)	On-the-run (2)	Off-the-run (3)
S&P 500	0.148 t = 370.284***	0.235 t = 23.845***	0.148 t = 369.824***
Observations	590,706	856	589,850
R <sup>2</sup>	0.188	0.400	0.188
Adjusted R <sup>2</sup>	0.188	0.399	0.188
Residual Std. Error	0.011 (df = 590704)	0.007 (df = 854)	0.011 (df = 589848)
Implied correlation $\rho_{X,M}$	0.43	0.63	0.43

*Note:*

\*p<0.1; \*\*p<0.05; \*\*\*p<0.01

**Table 3.** Calibration of the correlation of contract values with the state of the economy. OLS regression of 5-day CDX log returns on 5-day returns of the S&P500 during January 2006 to December 2009:  $CDX_{name,t,tenor,series,version} = \alpha + \beta SP_t + \varepsilon_{name,t,tenor,series,version}$  for all CDX at days  $t$ . The methodology is equivalent to estimating a single-factor model for an equally-weighted basket of all index CDX. The correlation between CDX and S&P 500 returns,  $\rho_{X,M}$ , is implied by the estimated coefficient,  $\rho_{X,M} = \beta \sigma_{S\&P500} / \sigma_{CDX}$ .  
*Source:* Market and own calculations.

2010,

$$CDX_{name,tenor,series,version,t} = \alpha + \beta SP_t + \varepsilon_{name,tenor,series,version,t}, \quad (3)$$

where  $CDX_{name,tenor,series,version,t}$  is the 5-day CDS index log-returns for different family names, tenors, series, and versions at day  $t$  and  $SP_t$  is the 5-day log-return of the S&P 500 at day  $t$ . The estimated OLS coefficients are in Table 3. The implied correlation between CDX and S&P 500 returns roughly equals  $\rho_{X,M} = 0.43$ , which we use as a baseline calibration. It is larger for indices on-the-run (0.63) and slightly smaller for indices being off-the-run (0.4).<sup>20</sup> The methodology is equivalent to estimating the correlation between an equally weighted basket of CDS indices and the S&P 500. We do not allow for different factor loadings  $\beta$  for different indices, since we are interested in only one parameter for the correlation  $\rho_{X,M}$ . The level of correlation is similar when estimating the single-factor model for individual CDS indices for the baseline period from 2006 to 2010 as well as for the period from 2010 to 2018, confirming the robustness of our estimate.<sup>21</sup> Table

<sup>20</sup>CDS indices are frequently updated. The most recently updated index is called *on-the-run* and typically exhibits the highest liquidity. Older versions of the indices are called *off-the-run* and are often still traded but exhibit lower liquidity.

<sup>21</sup>Correlation estimates are available on request. The correlation can be substantially smaller for single reference entities, as these do not diversify across idiosyncratic default risk of entities. For example, the correlation of the S&P 500 with 5-year tenor spreads of Wells Fargo is -0.06, with that of Goldman Sachs -0.12, with that of Deutsche Bank -0.1, with that of General Electric -0.18, with that of AIG -0.16, and with that of Metlife -0.42. The correlation is

4 reports the final baseline calibration.

Variable	Value	Description
$\gamma$	16	Number of counterparties
$K$	10	Number of derivative classes
$\sigma_X$	0.01	Total contract volatility
$\rho_{X,M}$	0.43	Correlation between contract value and state of the economy $M$
$\sigma_M$	0.03	Systematic volatility
$\beta$	0.1433	Implied beta-factor contracts
$\sigma$	0.009	Implied idiosyncratic contract volatility
$v$	1	Initial market value
$\text{cor}\left(r_{ij}^k, r_{hl}^m\right)$	0.185	Implied pair-wise correlation of contracts

**Table 4.** Baseline calibration. We assume the same calibration for each entity and derivative class.

### 3.1.2 What is the minimum number of counterparties that makes multilateral netting preferable to bilateral netting?

In the following we examine the impact of systematic risk on the minimum number of counterparties such that multilateral netting is beneficial compared to bilateral netting, i.e., such that  $\mathbb{E}[E_i^{BN+MN}] < \mathbb{E}[E_i^{BN,K}]$ . In Proposition 2 in Appendix A we show that there exists a positive lower bound for the multilateral netted class-K exposure if, and only if, entities are exposed to systematic risk. Thus, a large number of counterparties cannot guarantee any arbitrarily low level of multilateral exposure in the presence of systematic risk, which is the main distinction from previous models (such as the one from Duffie and Zhu (2011)) and will drive most of our results.

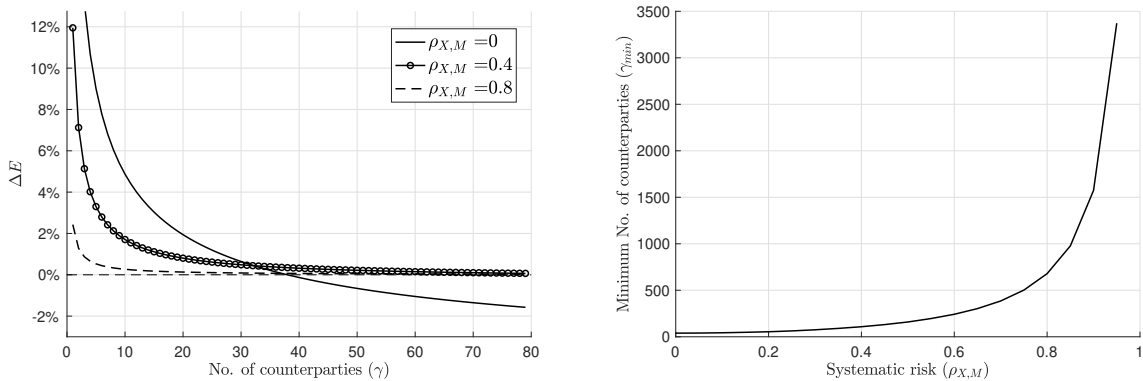
Figure 4 (a) illustrates the relative change in counterparty risk exposures by moving from bilateral to multilateral netting of derivative class  $K$ , which is given by  $\Delta E = \mathbb{E}[E_i^{BN+MN} - E_i^{BN,K}]/\mathbb{E}[E_i^{BN,K}]$ .<sup>22</sup> If  $\Delta E < 0$ , then multilateral netting results in smaller counterparty risk exposure than bilateral netting. In Figure 4 (a),  $\Delta E$  is positive for a small number of counterparties  $\gamma$  and negative for large  $\gamma$ . Thus, multilateral netting reduces counterparty risk only for a large number of counterparties. Indeed, it is straightforward to show that at least  $\gamma = K + 2$  homogeneous entities are needed such that multilateral netting of derivative class  $K$  may reduce counterparty

almost identical with a 3-year tenor. Note that the negative sign of the correlation coefficient reflects the protection buyer's perspective in spreads, while we account for the difference between buyer and seller with the sign of the contract size  $v$ . Thus, we use the absolute value of the correlation.

<sup>22</sup>In the terminology of Duffie and Zhu (2011), multilateral netting *efficiency* relative to bilateral netting *efficiency* is measured by  $-\Delta E$ .

risk exposures in a homogeneous market (see Proposition 3 in Appendix A).

The reason is that a larger number of counterparties leads to more diversification in the multilaterally netted contract pool, i.e., the average volatility decreases, while diversification in the bilateral pools is unaffected by the number of counterparties. The diversification benefit in the multilateral pool is reflected by the average multilaterally netted exposure per counterparty in derivative class  $K$ ,  $\mathbb{E}[E_i^{BN+MN}]/(\gamma - 1) = \varphi(0)\sqrt{\sigma_M^2\beta^2 + \sigma^2}/(\gamma - 1)$ , which is decreasing with the number of clearing members. As a consequence, multilateral netting leads to a reduction in exposures ( $\Delta E < 0$ ) if the number of counterparties  $\gamma$  is sufficiently large, which is also a central insight from Duffie and Zhu (2011).



(a) Change in exposure due to multilateral netting. (b) Minimum number of counterparties such that multilateral netting of derivative class  $K$  reduces exposures.

**Figure 4.** Impact of systematic risk.

(a) Change in collateralized counterparty risk exposure due to multilateral netting of derivative class  $K$ ,  $\Delta E = \mathbb{E}[E_i^{BN+MN} - E_i^{BN,K}]/\mathbb{E}[E_i^{BN,K}]$  with respect to systematic risk, correlation between contract values and the state of the economy  $\rho_{X,M}$ . If  $\Delta E < 0$ , multilateral netting reduces counterparty risk exposure compared to bilateral netting. (b) Minimum number of counterparties  $\gamma_{\min}$  such that multilateral netting of derivative class  $K$  reduces collateralized counterparty risk exposure compared to bilateral netting with respect to systematic risk  $\rho_{X,M}$ . The baseline calibration is described in Table 4.

The minimum number of counterparties such that multilateral netting is preferable to bilateral netting in terms of counterparty risk exposure is given by  $\gamma_{\min} = \inf\{\gamma > 0 : \Delta E < 0\}$ .  $\gamma_{\min}$  is shown in Figure 4 (b). Without systematic risk ( $\rho_{X,M} = 0$ ), multilateral netting is only beneficial when at least 39 counterparties are present. As Figure 4 (b) shows, systematic risk radically changes the minimum number of counterparties:  $\gamma_{\min}$  is steeply increasing with systematic correlation  $\rho_{X,M}$ .<sup>23</sup>

<sup>23</sup>This result differs from previous studies: As Duffie and Zhu (2011) and Cont and Kokholm (2014) examine

Not surprising, systematic correlation reduces the diversification benefit within all netting pools. In the multilateral pool, an additional entity reduces the average multilaterally netted exposure in derivative class  $K$  by

$$\frac{d}{d\gamma} \frac{\mathbb{E}[E_i^{MN}]}{\gamma - 1} = \frac{\varphi(0)\sigma_X (1 - \rho_{X,M}^2)}{2(\gamma - 1)^2 \sqrt{\rho_{X,M}^2(1 - (\gamma - 1)^{-1}) + (\gamma - 1)^{-1}}} < 0, \quad (4)$$

while that in bilateral pools is unaffected by  $\gamma$ . Systematic correlation  $\rho_{X,M}$  reduces the benefit of more counterparties in the multilateral pool, since Equation (4) converges to zero if  $|\rho_{X,M}|$  approaches unity. Thus, it requires a larger number of counterparties  $\gamma$  such that diversification within the multilateral netting pool offsets the reduction in diversification in bilateral pools from removing class  $K$ . Therefore, the higher the systematic exposure is, the smaller is the benefit of multilaterally netting with an additional counterparty and, hence, the minimum number of counterparties  $\gamma_{\min}$  is increasing with multilateral netting.<sup>24</sup>

**RESULT 1.** *Systematic risk increases the minimum number of counterparties  $\gamma_{\min}$  needed such that multilateral netting of one derivative class leads to lower counterparty risk exposure than bilateral netting.*

For our baseline calibration,  $\rho_{X,M} = 0.43$ , multilateral netting only reduces exposures in a market with at least 121 counterparties. This is unrealistically large, compared to the high concentration among a small number of dealers, e.g., in the CDS market (Brunnermeier et al. (2013), Peltonen et al. (2014), Getmansky, Girardi, and Lewis (2016)), and the current number of clearing members at CCPs (see Armakolla and Laurent (2017) and Footnote 19). It also largely exceeds the minimum number of counterparties in the absence of systematic risk (as in Duffie and Zhu (2011)), which is 39 with our calibration. At the most extreme, with perfect correlation across contracts ( $\rho_{X,M} = 1$ ), there is no diversification and thus no difference between bilateral and multilateral netting for any number of counterparties (as proven in Proposition 4 in Appendix A).

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correlation exclusively across derivative classes, more correlation in their models reduces diversification in bilateral but not multilateral netting pools. This reduces the minimum number of counterparties, while correlation across and within derivative classes increases the minimum number of counterparties in our model.

<sup>24</sup>Nonetheless, note that a higher systematic correlation also reduces the inefficiency of multilateral netting, i.e.,  $\Delta E$ , for a small number of counterparties, as Figure 4 (a) shows. The reason is that systematic correlation does not only impact multilateral but also bilateral netting pools. The higher the correlation, the smaller is the difference in diversification between multilateral and bilateral netting and thus the difference between exposures. However, this effect does not make multilateral netting more beneficial than bilateral netting if  $\gamma < \gamma_{\min}$ .

### 3.2 Multilateral netting in extreme events

One primary purpose of central clearing is to enhance financial stability during crisis times (Financial Stability Board (FSB) (2017a)). In these times, where counterparty defaults are more likely than in normal times, central clearing counterparties should ideally absorb losses arising from counterparty defaults and thereby decrease the spillover of losses within the overall financial system. Thus, it is of prevalent importance to examine the impact of central clearing on counterparty risk exposure in exactly these times.

Our model makes it possible to study counterparty risk exposure conditional on specific economic states, i.e., realizations of the state of the economy  $M$ . We are particularly interested in adverse realizations of  $M$  and parametrize  $M = \sigma_M \Phi^{-1}(q)$ , where  $q$  is the quantile (i.e., Value-at-Risk) level and  $\Phi^{-1}$  is the inverse cumulative distribution function of the standard normal distribution. The smaller  $q$ , the more adverse is the economic state.

Conditional on extreme economic states, we compute the counterparty risk exposure with bilateral and multilateral netting. The rationale and approach of the resulting extreme event exposures is similar to the (marginal) expected shortfall of Acharya et al. (2012) and Acharya et al. (2017): While their studies address the capital shortfall of financial institutions during crises, we study the counterparty risk exposure during crises. The total counterparty risk exposure with bilateral netting conditional on a specific state  $M$  is given by

$$\mathbb{E}[E_i^{BN,K} | M] = (\gamma - 1)\sqrt{K} \left( M\sqrt{K}\beta\Phi \left( M\sqrt{K}\frac{\beta}{\sigma} \right) + \sigma\varphi \left( -M\sqrt{K}\frac{\beta}{\sigma} \right) \right) \quad (5)$$

and with multilaterally netting derivative class  $K$  it is given by

$$\begin{aligned} \mathbb{E}[E_i^{BN+MN} | M] &= \mathbb{E}[E_i^{BN,K-1} | M] + M(\gamma - 1)\beta\Phi \left( M\sqrt{\gamma - 1}\frac{\beta}{\sigma} \right) \\ &\quad + \sigma\sqrt{\gamma - 1}\varphi \left( -M\sqrt{\gamma - 1}\frac{\beta}{\sigma} \right). \end{aligned} \quad (6)$$

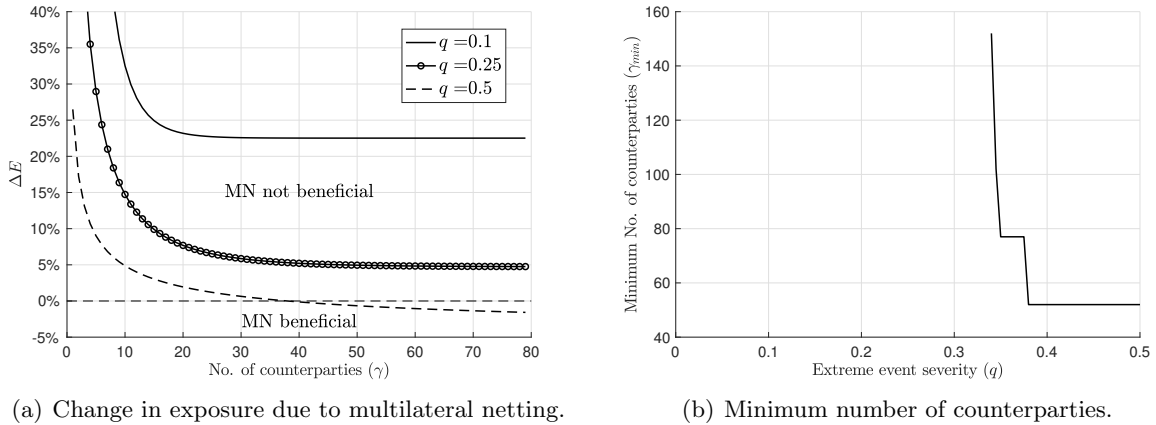
*Proof:* See Proposition 6 in Appendix A.

Most notably, while we assume that the unconditional expected return of individual contracts is zero,  $\mathbb{E}[X_{ij}^k] = 0$ , conditional on a specific state  $M \neq 0$  the expected return is non-zero,  $\mathbb{E}[X_{ij}^k | M] = v\beta M \neq 0$ . The reason is that, in more extreme (good or bad) economic states, one can



expect larger absolute contract values.

As a result, the interplay between the volatility and expected value of netting pools' contracts now governs the effectiveness of multilateral netting. The overall effect crucially depends on the state of the economy. Figure 5 (a) depicts the change in exposure due to moving from bilateral to multilateral netting of class  $K$ . Clearly, in adverse economic states  $M$ , multilateral netting is less beneficial for counterparty risk exposures compared to bilateral netting. If the economic state  $M$  is too extreme, then multilateral netting increases counterparty risk exposure relative to bilateral netting regardless of the number of counterparties. In our example in Figure 5 (b), this already holds for  $q < 0.34$ , i.e., the 34% worst economic states.<sup>25</sup> Our result thus implies that counterparty risk exposures in bad economic states are smaller without multilateral netting for any number of counterparties. Note that this result does not only hold in very extreme states (such as the  $q = 10\%$  worst possible states) but already in relatively moderate states.



**Figure 5.** Impact of extreme events. (a) Change in collateralized counterparty risk exposures due to multilateral netting of derivative class  $K$ ,  $\Delta E = \mathbb{E}[E_i^{BN+MN} - E_i^{BN,K} | M] / \mathbb{E}[E_i^{BN,K} | M]$  conditional on extreme event  $M = \sigma_M \Phi^{-1}(q)$ . The smaller  $q$ , the more adverse is the event. If  $\Delta E < 0$ , multilateral netting reduces counterparty risk exposures compared to bilateral netting. (b) Minimum number of counterparties  $\gamma_{\min}$  such that multilateral netting of derivative class  $K$  reduces collateralized counterparty risk exposure compared to bilateral netting with respect to the severity of extreme events. The baseline calibration is described in Table 4.

The reason for this result is the dominance of extremely large expected contract values during extreme events. By rearranging Equation (5), the counterparty risk exposure with bilateral netting

<sup>25</sup>To produce Figure 5 (b), we calculate whether multilateral netting reduces counterparty risk exposures compared to bilateral netting for any number of counterparties smaller than  $10^8$ .

can be represented as

$$\mathbb{E}[E_i^{BN,K} | M] = (\gamma - 1)\mathbb{E} \left[ \max \left( MK\beta + \sqrt{K}\sigma\tilde{\varepsilon}, 0 \right) \right], \quad (7)$$

with  $\tilde{\varepsilon} \sim \mathcal{N}(0, 1)$ . Clearly,  $\mathbb{E}[E_i^{BN,K} | M]$  is increasing with the number of derivative classes  $K$  if  $M = 0$ , since then it is proportional to  $\sqrt{K}$ . Thus, when one derivative class-K is taken out from bilateral pools, the bilaterally netted counterparty risk exposure decreases due to a smaller volatility in the remaining pool. This leaves room for the total counterparty risk exposure to be smaller after multilaterally netting derivative class K, i.e., that  $\mathbb{E}[E_i^{BN+MN} | M] = \mathbb{E}[E_i^{BN,K-1} | M] + \mathbb{E}[E_i^{MN} | M] < \mathbb{E}[E_i^{BN,K} | M]$ . In contrast, if contracts have sufficiently large negative expected returns in extreme events (i.e., if  $M < 0$ ), then the bilateral exposure in Equation (7) is decreasing with  $K$ . The reason is that the effect of the number of derivative classes  $K$  on the expected value  $MK\beta$  (making it very negative) dominates the effect on total volatility  $\sqrt{K}\sigma$ . In this case, excluding class-K from bilateral pools increases the counterparty risk exposure in these pools, i.e.,  $\mathbb{E}[E_i^{BN,K-1} | M] > \mathbb{E}[E_i^{BN,K} | M]$ . As a result, there is no room for the total counterparty risk exposure to be smaller after additionally multilaterally netting class-K. Thus, counterparty risk exposure is smaller in a bilateral market than with multilateral netting of one derivative class.<sup>26</sup>

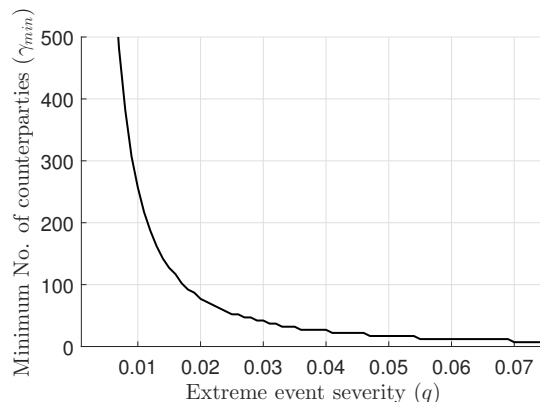
**RESULT 2.** *During sufficiently severe extreme events, multilateral netting of one derivative class does not reduce counterparty risk exposures compared to bilateral netting for any number of counterparties.*

Extreme events make it particularly unfavorable to exclude a derivative class from bilateral netting due to the dominance of large absolute contract  $\gamma$  values. By hedging systematic risk across derivative classes in bilateral pools, entities may thus reduce the unfavorable effect of extreme events. However, hedging across derivative classes seems particularly difficult in practice, as different derivative classes, e.g., CDS and IRS derivatives, exhibit a different exposure to systematic

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<sup>26</sup>A similar rationale holds for market participants that are short in systematic risk, i.e., with  $v\beta < 0$ . In this case, for small  $\gamma$ , the small diversification benefit in the multilateral pool makes multilateral netting less beneficial, analogously to our baseline analysis in Section 3.1.2. For large  $\gamma$  and  $(-M)$ , the expected value of contracts in the multilateral pool is very large, such that there is a negligible benefit of diversification. However, removing class-K contracts from the bilateral pools reduces diversification in these pools (since there are only  $K$  contract classes in bilateral pools compared to  $\gamma \gg K$  contract classes in the multilateral pool), increasing the per contract bilateral counterparty risk exposure. As a result, if  $M$  is sufficiently large and negative and  $v\beta < 0$ , then multilateral netting is also not beneficial compared to bilaterally netting all contracts.

risk, i.e., different levels of  $\beta$ . It seems more likely that market participants hedge within derivative classes (or even the same instruments), thus reducing their exposure to systematic risk within each derivative class, i.e., within the multilateral pool.<sup>27</sup> However, even by perfectly hedging systematic risk within one derivative class, multilateral netting is still not beneficial for a reasonable number of counterparties during extreme economic events, as Figure 6 shows. The reason is that such hedged dealers are still exposed to systematic risk in bilateral pools.



**Figure 6.** Impact of extreme events for dealers.

Minimum number of counterparties  $\gamma_{min}$  such that multilateral netting of derivative class  $K$  reduces collateralized counterparty risk exposure compared to bilateral netting conditional on extreme event  $M = \sigma_M \Phi^{-1}(q)$ . The smaller  $q$ , the more adverse is the event. We assume that  $|v_{ij}^k| = 1$  for all  $i, j, k$ , and that dealers are hedged against systematic risk within derivative classes  $k = 1, \dots, K$ ,  $\sum_j v_{ij}^k = 0$ , and that  $\sum_k v_{ij}^k = K$  with  $(\gamma - 1)/2$  counterparties and  $\sum_k v_{ij}^k = -K$  with  $(\gamma - 1)/2$  counterparties. The baseline calibration is described in Table 4.

### 3.3 Cross-netting and the Mega CCP

To address the failure of multilateral netting to reduce counterparty risk exposures in sufficiently extreme events, one might increase the overall degree of netting. A natural extension is to net across not only one but several derivative classes. We refer to such netting across all  $\gamma - 1$  counterparties and  $\kappa > 1$  derivative classes as *cross-netting*. It occurs when one CCP offers clearing of several derivative classes within one legal entity.<sup>28</sup>

The counterparty risk exposure in  $\kappa$  cross-netted derivative classes with  $\gamma - 1$  counterparties is

<sup>27</sup>For example, Abad, Aldasorol, Aymanns, D’Errico and Rousová (2016) find that the largest 16 dealers in the European derivatives market maintain roughly a net zero position within interest rate swap as well as credit default swap portfolios.

<sup>28</sup>For example, Eurex offers clearing for several derivative classes such as money-market and interest rate derivatives, including margining for a clearing member’s entire portfolio. Cross-netting is promoted by interoperability arrangements that create linkages between different CCPs (Garvin (2012)).

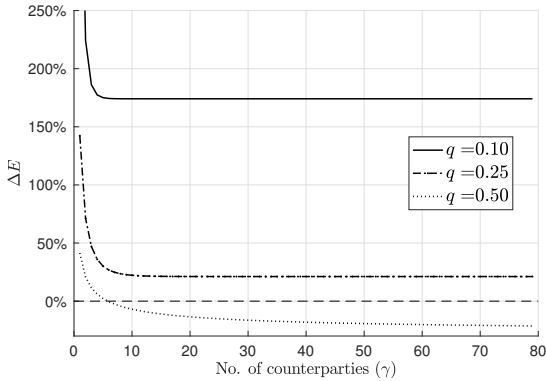
during an extreme event given by

$$\mathbb{E}[E_i^{CN} | M] = M(\gamma - 1)\kappa\beta\Phi\left(\frac{M\sqrt{(\gamma - 1)\kappa\beta}}{\sigma}\right) + \sqrt{(\gamma - 1)\kappa}\sigma\varphi\left(-\frac{M\sqrt{(\gamma - 1)\kappa\beta}}{\sigma}\right). \quad (8)$$

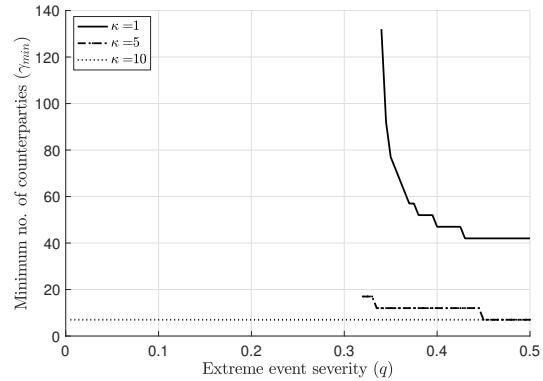
*Proof:* See Proposition 7 in Appendix A.

Figure 7 illustrates the benefit of cross-netting for counterparty risk exposures in extreme events. In Figure 7 (a), the CCP nets across all counterparties and  $\kappa = 5$  derivative classes, where the total number of derivative classes is  $K = 10$ . The figure shows that even with cross-netting of  $\kappa = 5$  derivative classes the counterparty risk exposure is larger in sufficiently extreme states (such as  $q = 0.25$  or  $q = 0.1$ ) than with bilateral netting.

Figure 7 (b) depicts the minimum number of counterparties for cross-netting to be beneficial compared to bilateral netting. We find that cross-netting essentially needs to net across all  $K$  derivative classes and  $\gamma - 1$  counterparties, i.e.,  $\kappa = K$ , in order to be beneficial in all economic environments  $M$ , a case we refer to as *Mega CCP*. In other words, only a *Mega CCP* that clears with all counterparties in all derivative classes can unambiguously reduce counterparty risk exposures in all economic states compared to bilateral netting.



(a) Change in exposure due to cross-netting  $\kappa = 5$  classes.



(b) Minimum number of counterparties.

**Figure 7.** Impact of cross-netting during extreme events.

(a) Effect of netting across counterparties and  $\kappa = 5$  derivative classes on collateralized counterparty risk exposure in an extreme event  $M = \sigma_M\Phi^{-1}(q)$ ,  $\Delta E = \mathbb{E}[E_i^{BN+CN} - E_i^{BN,K} | M] / \mathbb{E}[E_i^{BN,K} | M]$ . The smaller  $q$ , the more adverse is the event. If  $\Delta E < 0$ , cross-netting reduces counterparty risk exposure compared to bilateral netting. (b) Minimum number of counterparties  $\gamma_{\min}$  such that cross-netting of  $\kappa$  derivative classes reduces counterparty risk exposure. In case  $\kappa = K = 10$ , we refer to the CCP as *Mega CCP*. The baseline calibration is described in Table 4.

**RESULT 3.** Only a *Mega CCP*, netting across all derivative classes and counterparties, reduces

*counterparty risk exposures in all economic states compared to bilateral netting.*

### 3.4 Margin requirements and counterparty risk exposures

In the following, we examine the impact of collateral, i.e., margin requirements, on the benefit of multilateral netting. Collateralizing exposures (also called *margining*) is a primary measure to reduce credit risk in derivative transactions ([International Swaps and Derivatives Association \(2017\)](#)). Typically, one distinguishes between initial and variation margins: Initial margin is collateral available to the (central clearing) counterparty and posted at the beginning of a trade to cover potential future counterparty risk exposure. Variation margins are frequently (typically daily) exchanged to compensate for changes in market values. For simplicity, we assume in our model that initial margins were exchanged before the settlement period and contracts are marked to market, i.e., variation margin is exchanged, at the beginning of the settlement period. Then, the remaining collateral available to compensate for losses from counterparty defaults is given by the initial margin.<sup>29</sup>

Regulation for non-centrally cleared derivatives requires initial margins to account for a 99 percent confidence interval over at least a 10-day horizon of market price changes ([Bank for International Settlements \(BIS\) \(2015\)](#)). CCPs are required to establish a single-tailed confidence interval level of at least 99 percent of future exposure, while the margin period is typically 5 days ([Bank for International Settlements \(BIS\) \(2012\)](#), [Bank for International Settlements \(BIS\) \(2014\)](#), [Duffie, Scheicher and Vuillemeys \(2015\)](#), [Ghamami and Glasserman \(2017\)](#)). These requirements result in a smaller margin for cleared than for non-cleared trades, which is intended by policymakers to incentivize market participants to make use of central clearing ([Duffie, Li and Lubke \(2010\)](#)).

In line with recent regulation, we assume that the collateral that  $j$  posts to  $i$  based on a bilateral netting agreement (referred to as *bilateral margin*) is given by the Value at Risk at the

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<sup>29</sup>Note that CCPs also have pre-funded resources that can be employed in case of a loss. However, these are small compared to the collateral posted by clearing members. For example, for CDS clearing, pre-funded resources are 0.5% of initial margins at CME Clearing US, 2.8% at LCH Clearnet SA, and 8% ICE Clear Credit; for IRS clearing, pre-funded resources are 3.2% of initial margin at LCH Ltd. as of March 2016 ([Arnakolla and Laurent \(2017\)](#)). Thus, we do not expect that accounting for pre-funded resources would substantially alter our results.

$\alpha_{BN}$  confidence level of the portfolio value of their trades,

$$C_{ij}^{BN,K} = VaR_{\alpha_{BN}} \left( \sum_{k=1}^K X_{ij}^k \right) = \Phi^{-1}(\alpha_{BN}) \sqrt{\sigma_M^2 K^2 \beta^2 + K \sigma^2}. \quad (9)$$

We refer to  $\alpha_{BN}$  as the *bilateral margin confidence level*.

The *uncollateralized* counterparty risk exposure is the exposure in excess of collateral, and given by

$$\mathbb{E} \left[ \tilde{E}_i^{BN,K} \right] = \mathbb{E} \left[ \sum_{j=1, j \neq i}^{\gamma} \max \left( \sum_{k=1}^K X_{ij}^k - C_j^{BN,K}, 0 \right) \right] \quad (10)$$

$$= (\gamma - 1) \sqrt{\sigma_M^2 K^2 \beta^2 + K \sigma^2} \xi(\alpha_{BN}), \quad (11)$$

where  $\xi(\alpha) = (1 - \alpha) \Phi^{-1}(1 - \alpha) + \varphi(\Phi^{-1}(\alpha))$  adjusts the counterparty risk exposure for collateral.<sup>30</sup>

*Proof: See Proposition 8 in Appendix A.*

If derivative class  $K$  is multilaterally netted, then  $j$  posts collateral (referred to as *clearing margin*) as given by the Value at Risk at the  $\alpha_{MN}$  confidence level,

$$C_j^{MN} = VaR_{\alpha_{MN}} \left( \sum_{i=1, i \neq j}^{\gamma} X_{ij}^k \right) = \Phi^{-1}(\alpha_{MN}) \sqrt{\sigma_M^2 (\gamma - 1)^2 \beta^2 + K (\gamma - 1) \sigma^2}. \quad (12)$$

To compute the uncollateralized counterparty risk exposure of entity  $i$  in the multilaterally netted derivative class  $K$ , we assume that the collateral provided by clearing member  $j$  is available to  $i$  proportionally to the size of  $j$ 's trades with  $i$ . Thus,  $\frac{v_{ij}^K}{\sum_{h=1, h \neq j}^{\gamma} v_{hj}^K} C_j^{MN}$  is assigned to entity  $i$ . With homogeneous entities, the uncollateralized exposure of entity  $i$  is then given by

$$\mathbb{E} \left[ \tilde{E}_i^{BN+MN} \right] = \sqrt{\sigma_M^2 (\gamma - 1)^2 \beta^2 + (\gamma - 1) \sigma^2} \xi(\alpha_{MN}) + \mathbb{E}[\tilde{E}_i^{BN,K-1}] \quad (13)$$

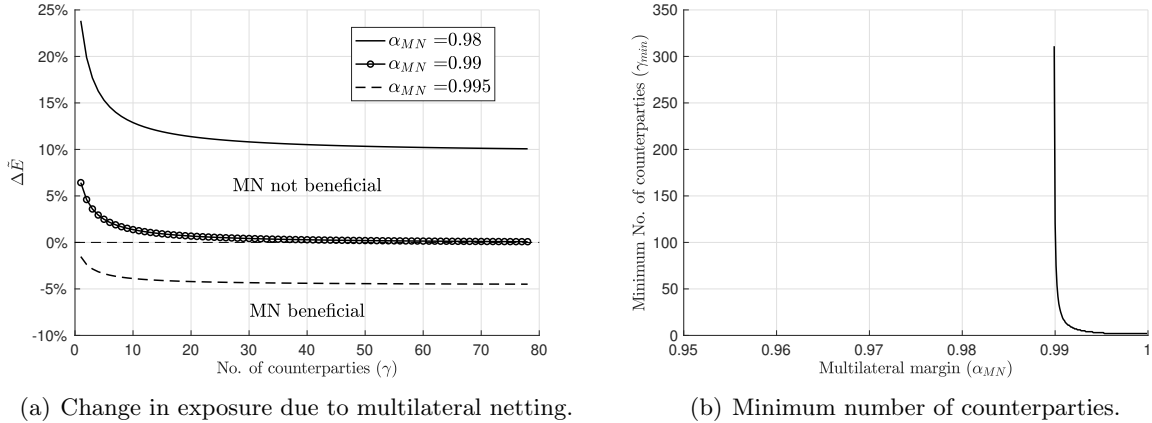
*Proof: See Proposition 8 in Appendix A.*

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<sup>30</sup>If  $\alpha = 0.5$ , then  $\Phi^{-1}(\alpha) = \Phi^{-1}(1 - \alpha) = C^{BN,K} = 0$ , and uncollateralized counterparty risk exposure is equal to collateralized counterparty risk exposure.

Comparing the collateralized and uncollateralized counterparty risk exposure  $\mathbb{E}[E_i^{BN+MN}]$  and  $\mathbb{E}[\tilde{E}_i^{BN+MN}]$  in Equations (6) and (13), respectively, it becomes apparent that the only difference is the adjustment factor  $\xi$ . Hence, margins have an impact on the benefit of multilateral netting only if clearing and bilateral margins differ. The larger (smaller) the confidence level of the clearing margin  $\alpha_{MN}$  relative to that of the bilateral margin  $\alpha_{BN}$ , the larger (smaller) is the reduction of exposures due to multilateral netting of derivative class  $K$ . In other words, with a higher clearing margin it is more likely that multilateral netting is beneficial compared to bilateral netting (for a proof see Proposition 9 in Appendix A).<sup>31</sup> We illustrate this result in Figure 8. Figure 8 (a) depicts the change in uncollateralized exposures due to multilateral netting. Clearly, a small clearing margin confidence level  $\alpha_{MN}$  relative to a bilateral margin confidence level of  $\alpha_{BN} = 0.99$  leads to an increase in uncollateralized counterparty risk exposure.

**RESULT 4.** *The larger the margin for cleared derivatives relative to that for non-cleared derivatives, the lower is the counterparty risk exposure with multilateral netting relative to that with bilateral netting.*



**Figure 8.** Impact of margins.

(a) Change in uncollateralized counterparty risk exposure due to multilateral netting,  $\Delta\tilde{E} = \mathbb{E}[\tilde{E}_i^{BN+MN} - \tilde{E}_i^{BN,K}] / \mathbb{E}[\tilde{E}_i^{BN,K}]$ , of class  $K$  with respect to the multilateral margin confidence level ( $\alpha_{MN}$ ). If  $\Delta\tilde{E} < 0$ , multilateral netting reduces uncollateralized counterparty risk exposure compared to bilateral netting.

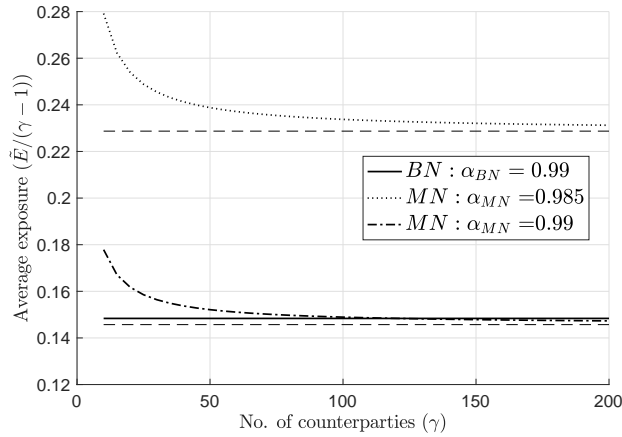
(b) Minimum number of counterparties  $\gamma_{\min}$  such that multilateral netting of derivative class  $K$  reduces uncollateralized exposure with respect to the clearing margin level. The baseline calibration is described in Table 4 and the bilateral margin confidence level is  $\alpha_{BN} = 0.99$ .

Moreover, if the clearing margin is sufficiently small, then multilateral netting does not reduce

<sup>31</sup>Note that the bilateral and clearing margins only differ in the confidence level  $\alpha$ , and that there is a one-to-one and strictly monotone correspondence between confidence level  $\alpha$  and total collateral  $C$ .

counterparty risk exposures for any number of counterparties (e.g., with  $\alpha_{MN} = 0.98$ ). Figure 8 (b) supports this observation, as we do not find any number of counterparties  $\gamma$  that reduces uncollateralized exposures for  $\alpha_{MN} \leq 0.9898$  compared to the bilateral margin confidence level  $\alpha_{BN} = 0.99$ . Hence, uncollateralized exposures are extremely sensitive towards small discrepancies between margins for cleared and non-cleared derivatives.

The reason is systematic risk: The average uncollateralized multilateral exposure per counterparty is bounded from below by  $\mathbb{E}[\tilde{E}_i^{MN}]/(\gamma - 1) > |\rho_{X,M}|\sigma_X\xi(\alpha_{MN})$  (see Proposition 2 in Appendix A). Higher systematic risk ( $\rho_{X,M}$ ) and lower margins ( $\alpha_{MN}$ ) increase the lower bound for the multilateral exposure, which is illustrated in Figure 9. This results resembles the finding of Menkveld (2017) who stresses that current CCP margin practices are inefficient since they do not account for correlation across clearing members. Eventually, if  $|\rho_{X,M}|$  is sufficiently large (or  $\alpha_{MN}$  is too low compared to  $\alpha_{BN}$ ), then the lower bound for the exposure with multilaterally netting  $K$  exceeds the additional exposure from bilaterally netting  $K$ .<sup>32</sup>



**Figure 9.** Impact of margins on average uncollateralized exposure per counterparty.

Average uncollateralized multilaterally netted counterparty risk exposure in class  $K$ ,  $\mathbb{E}[\tilde{E}_i^{MN}]/(\gamma - 1)$ , for different levels of the clearing margin confidence level, and change in the uncollateralized bilateral exposure from including class  $K$ ,  $\mathbb{E}[\tilde{E}_i^{BN,K} - \tilde{E}_i^{BN,K-1}]/(\gamma - 1)$ . If  $\mathbb{E}[\tilde{E}_i^{MN}]/(\gamma - 1) < \mathbb{E}[\tilde{E}_i^{BN,K} - \tilde{E}_i^{BN,K-1}]/(\gamma - 1)$ , multilateral netting reduces uncollateralized counterparty risk exposure compared to bilateral netting. The dashed horizontal lines illustrate the lower bound for the average multilateral exposure,  $|\rho_{X,M}|\sigma_X\xi(\alpha_{MN})$ . The baseline calibration is described in Table 4.

<sup>32</sup>Note that this general result does not change with accounting for the cost of margin. If there is a cost of  $h$  for each dollar of collateral, then the sum of margin cost and total counterparty risk exposure with bilateral netting is  $(\gamma - 1)\sqrt{\sigma_M^2 K^2 \beta^2 + K\sigma^2} [\xi(\alpha_{BN}) + \Phi^{-1}(\alpha_{BN})h]$  and that with multilateral netting is  $\sqrt{\sigma_M^2 (\gamma - 1)^2 \beta^2 + (\gamma - 1)\sigma^2} [\xi(\alpha_{MN}) + \Phi^{-1}(\alpha_{MN})h] + (\gamma - 1)\sqrt{\sigma_M^2 K^2 \beta^2 + K\sigma^2} [\xi(\alpha_{BN}) + \Phi^{-1}(\alpha_{BN})h]$ . Thus, margin cost is mainly a scaling factor that reduces the benefit of higher margins but does not necessarily alter our general results.



Based on this observation, in the following, we derive a condition for the clearing confidence level  $\alpha_{MN}$  such that, for given  $\alpha_{BN}$ , multilateral netting leads to a reduction in uncollateralized counterparty risk exposure compared to bilateral netting. From the lower bound of  $\mathbb{E}[\tilde{E}_i^{MN}]/(\gamma-1)$ , we derive the following threshold for the clearing margin: Multilateral netting does not reduce uncollateralized exposures for any finite number of counterparties  $\gamma < \infty$  if  $\alpha_{MN} \leq \mathcal{H}_{MN}$  with

$$\mathcal{H}_{MN} = \xi^{-1} \left( \frac{\xi(\alpha_{BN})}{|\rho_{X,M}|} \left( \sqrt{K} \sqrt{1 + \rho_{X,M}^2(K-1)} - \sqrt{K-1} \sqrt{1 + \rho_{X,M}^2(K-2)} \right) \right). \quad (14)$$

It is straightforward to show that  $\mathcal{H}_{MN}$  is increasing with systematic correlation  $|\rho_{X,M}|$ ,  $\frac{d\mathcal{H}_{MN}}{d|\rho_{X,M}|} > 0$ , which mainly results from  $\xi$  and thus  $\xi^{-1}$  being monotone decreasing (see Proposition 9 in Appendix A). Hence, the more extreme (positive or negative) the systematic correlation  $\rho_{X,M}$  is, the larger must the clearing margin be for multilateral netting to reduce counterparty risk exposure. Moreover,  $\mathcal{H}_{MN}$  is bounded from above by  $\alpha_{BN}$ ,  $\mathcal{H}_{MN} \leq \alpha_{BN}$ , since

$$\lim_{\rho_{X,M}^2 \rightarrow 1} \mathcal{H}_{MN} = \xi^{-1}(\xi(\alpha_{BN})) = \alpha_{BN}, \quad (15)$$

and bounded from below by zero,  $0 \leq \mathcal{H}_{MN}$ , since

$$\lim_{\rho_{X,M} \rightarrow 0} \mathcal{H}_{MN} = \xi^{-1}(\infty) = 0. \quad (16)$$

Thus, in the case of no systematic risk ( $\rho_{X,M} = 0$ ), for any confidence levels  $\alpha_{BN}$  and  $\alpha_{MN}$  there exists a number of counterparties  $\gamma$  such that multilateral netting is beneficial, which is the result of Duffie and Zhu (2011). However, the larger the systematic correlation, the smaller is the acceptable difference between the margin for cleared and non-cleared derivatives. For example, in our baseline calibration, multilateral netting is not beneficial compared to bilateral netting for any number of counterparties if the bilateral margin is  $\alpha_{BN} = 0.99$  and the clearing margin is below  $\alpha_{MN} \leq 0.98$ , as Figure 8 (a) shows. This is in line with the upper bound we derived above, which is  $\mathcal{H}_{MN} = 0.9897$  for our baseline calibration.

**RESULT 5.** *For every bilateral margin confidence level  $\alpha_{BN} \in (0, 1)$  there exists a threshold  $\mathcal{H}_{MN} \leq \alpha_{BN}$  such that the counterparty risk exposure is larger with multilateral netting than with bilateral netting for any number of counterparties if the clearing margin  $C^{MN} \leq \text{VaR}_{\mathcal{H}_{MN}}(\sum_{i=1, i \neq j}^{\gamma} X_{ij}^k)$ .*

The threshold  $\mathcal{H}_{MN}$  is increasing with the absolute value of systematic correlation  $|\rho_{X,M}|$ , such that a higher clearing margin is necessary for more extreme systematic correlation.

Analogously, one can show that a sufficiently large clearing margin results in an unambiguously smaller counterparty risk exposure with multilateral netting: If  $\alpha_{MN} \geq \mathcal{U}_{MN}$ , then the counterparty risk exposure is smaller with multilateral than bilateral netting for any number of counterparties  $\gamma \geq 2$ , where<sup>33</sup>

$$\mathcal{U}_{MN} = \xi^{-1} \left( \xi(\alpha_{BN}) \left( \sqrt{K} \sqrt{(K-1)\rho_{X,M}^2 + 1} - \sqrt{K-1} \sqrt{(K-2)\rho_{X,M}^2 + 1} \right) \right). \quad (17)$$

This is the case, e.g., with  $\alpha_{BN} = 0.99$  and  $\alpha_{MN} = 0.995$  in Figure 8 (a), since  $\mathcal{U}_{MN} = 0.995$  for our baseline calibration. From Equation (17) it is clear that  $\mathcal{U}_{MN}$  is decreasing with systematic correlation  $|\rho_{X,M}|$  and converging to  $\alpha_{BN}$  for  $|\rho_{X,M}| \rightarrow 1$ . Hence, the larger the absolute value of systematic correlation, the smaller is the necessary clearing margin  $VaR_{\mathcal{U}_{MN}}$  such that multilateral netting is beneficial for any number of counterparties. The necessary clearing margin is always larger than the bilateral margin,  $VaR_{\mathcal{U}_{MN}} > VaR_{\alpha_{BN}}$  for  $|\rho_{X,M}| < 1$ .

**RESULT 6.** For every bilateral margin level  $\alpha_{BN} \in (0, 1)$  there exists a threshold  $\mathcal{U}_{MN} \geq \alpha_{BN}$  such that the counterparty risk exposure is lower with multilateral netting than with bilateral netting for any number of counterparties if the clearing margin  $C^{MN} > VaR_{\mathcal{U}_{MN}}(\sum_{i=1, i \neq j}^{\gamma} X_{ij}^k)$ .

The threshold  $\mathcal{U}_{MN}$  is decreasing with the absolute value of systematic correlation  $\rho_{X,M}$ , such that a smaller clearing margin is sufficient for more extreme systematic correlation.

Eventually, our results divide possible margin confidence levels into three disjunct intervals:

1.  $\alpha_{MN} \in (0, \mathcal{H}_{MN}]$  with  $\mathcal{H}_{MN} \leq \alpha_{BN}$ : Multilateral netting is not beneficial for any number of counterparties  $\gamma$ .
2.  $\alpha_{MN} \in (\mathcal{H}_{MN}, \alpha_{BN}] \cup (\alpha_{BN}, \mathcal{U}_{MN})$ : Multilateral netting is beneficial if the number of counterparties  $\gamma$  is sufficiently large.
3.  $\alpha_{MN} \in [\mathcal{U}_{MN}, 1)$  with  $\mathcal{U}_{MN} > \alpha_{BN}$ : Multilateral netting is beneficial for any number of counterparties  $\gamma \geq 2$ .

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<sup>33</sup>This results from  $\xi(\alpha)$  being strictly positive for any  $\alpha \in (0, 1)$  and  $\xi^{-1}$  having full support on the positive real line.

As outlined above, current regulation requires margins to account for a 99 percent confidence interval over a 10-day margin period for non-cleared contracts and a 5-day margin period for cleared contracts. The difference of 10 and 5 days in calculation horizon for the margin relates to a volatility ratio of  $\sqrt{2}$ , such that  $\sqrt{2}C^{MN} = C^{BN,K}$ , where we assume the same settlement period. Letting  $\alpha_{BN} = 0.99$ , the clearing margin confidence level is  $\alpha_{MN} = \Phi(\Phi^{-1}(\alpha_{BN})/\sqrt{2}) = 0.88$ , i.e.,  $\alpha_{MN} = 0.88$  reflects the 99% Value-at-Risk for a 5-day margin period and  $\alpha_{BN} = 0.99$  that for a 10-day margin period.

In our baseline calibration, multilateral netting with  $\alpha_{MN} = 0.88$  never leads to a reduction in uncollateralized counterparty risk exposures but increases exposures for any number of counterparties  $\gamma$ . Indeed,  $\alpha_{MN}$  is in the first interval,  $\alpha_{MN} \in (0, \mathcal{H}_{MN}]$ , as  $\mathcal{H}_{MN} = 0.9897$  and  $\mathcal{U}_{MN} = 0.995$ . Thus, a confidence level  $\alpha_{MN}$  of more than 98.97% is needed for multilateral netting to be able to achieve a reduction in counterparty risk exposure with a sufficient number of clearing members. If the clearing margin confidence level was at least 99.5%, then multilateral netting would be beneficial for any number of clearing members.

Can a *Mega CCP* compensate for the adverse effect of a small clearing margin? The uncollateralized exposure in cross-netted  $\kappa$  derivative classes is given by

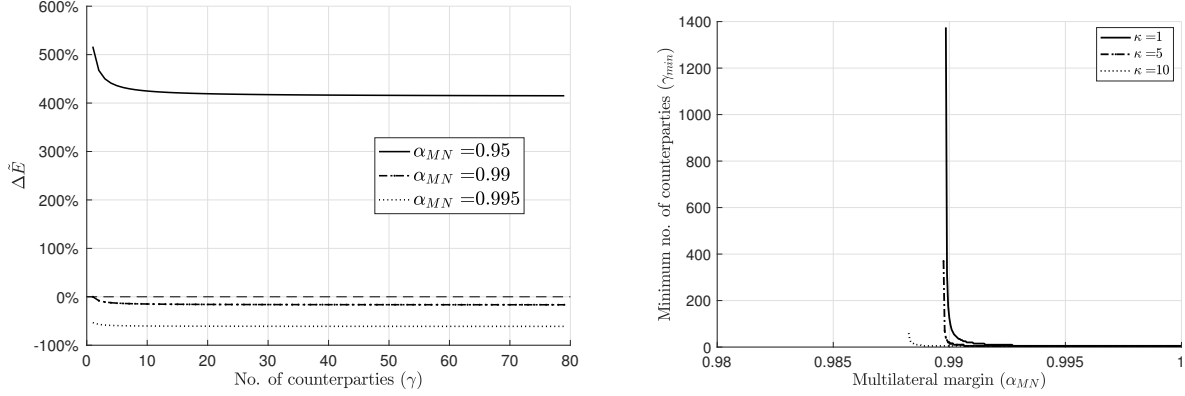
$$\mathbb{E}[\tilde{E}^{CN}] = \sqrt{\sigma_M^2 \kappa^2 (\gamma - 1)^2 \beta^2 + \kappa (\gamma - 1) \sigma^2 \xi(\alpha_{CN})}, \quad (18)$$

where  $\xi(\alpha)$  is defined as above and  $\alpha_{CN}$  is the margin level for cross-netting.

*Proof:* See Proposition 11 in Appendix A.

In Figure 10 (a) we show that a *Mega CCP* underlies the same dynamics as multilateral netting of one derivative class with respect to margins: The smaller (larger) the clearing margin, the larger (smaller) is the uncollateralized exposure. If the clearing margin is sufficiently small, then cross-netting is not beneficial for any number of counterparties, and vice versa.

Analogously to multilateral netting, we derive the smallest acceptable margin confidence level



(a) Change in exposure due to cross-netting across  $\kappa = 10$  derivative classes.

(b) Minimum number of counterparties.

**Figure 10.** Impact of cross-netting on uncollateralized counterparty risk exposure.

(a) Change in uncollateralized counterparty risk exposure due to netting across  $\kappa = 10$  derivative classes and across counterparties on uncollateralized counterparty risk exposure,  $\Delta \tilde{E} = \mathbb{E}[\tilde{E}_i^{BN+CN} - \tilde{E}_i^{BN,K}] / \mathbb{E}[\tilde{E}_i^{BN,K}]$ , for bilateral margin confidence level  $\alpha_{BN} = 0.99$ , and  $\gamma = 16$  entities. If  $\Delta \tilde{E} < 0$ , cross-netting reduces uncollateralized exposure compared to bilateral netting. (b) Minimum number of counterparties  $\gamma_{\min}$  such that cross-netting reduces uncollateralized exposure compared to bilateral netting. The baseline calibration is described in Table 4 and the bilateral margin is  $\alpha_{BN} = 0.99$ .

$\mathcal{H}_{CN}$  such that cross-netting is not beneficial for any number of counterparties if  $\alpha_{CN} \leq \mathcal{H}_{CN}$ :

$$\mathcal{H}_{CN} = \xi^{-1} \left( \frac{\xi(\alpha_{BN})}{|\rho_{X,M}|} \frac{\sqrt{1 + \rho_{X,M}^2 (K - 1)}}{\sqrt{K}} \right). \quad (19)$$

Similarly to  $\mathcal{H}_{MN}$ ,  $\mathcal{H}_{CN}$  is increasing with the absolute value of systematic correlation  $|\rho_{X,M}|$ . A *Mega CCP* is however associated with a larger degree of netting. This reduces the smallest acceptable clearing margin compared to multilateral netting of one derivative class: It is straightforward to show that  $\mathcal{H}_{CN} < \mathcal{H}_{MN}$  for any  $\rho_{X,M} \in (0, 1)$ . As Figure 10 (b) illustrates, this effect however is very small. For example, with  $\alpha_{BN} = 0.99$ , the smallest acceptable confidence level is reduced only by 0.15 percentage points: from  $\mathcal{H}_{MN} = 98.97\%$  with multilateral netting of class  $K$  to  $\mathcal{H}_{CN} = 98.82\%$  with cross-netting of all classes  $k = 1, \dots, K$ .<sup>34</sup> This effect seems still negligible in light of the large difference of 11 percentage points, in practice, between  $\alpha_{BN} = 99\%$  and  $\alpha_{MN} = 88\%$ . We conclude that the degree of netting is only of minor importance if the margin for cleared derivatives is not sufficiently large.

**RESULT 7.** For every bilateral margin confidence level  $\alpha_{BN} \in (0, 1)$  there exists a threshold

<sup>34</sup>The calculation is based on evaluating the counterparty risk exposure for all number of counterparties  $\gamma \leq 10^8$ .

$\mathcal{H}_{CN} \leq \alpha_{BN}$  such that the counterparty risk exposure is larger with a Mega CCP than with bilateral netting for any number of counterparties if the clearing margin  $C^{CN} \leq VaR_{\mathcal{H}_{CN}}(\sum_{i=1, i \neq j}^{\gamma} X_{ij}^k)$ .

The threshold  $\mathcal{H}_{CN}$  is increasing with the absolute value of systematic correlation  $\rho_{X,M}$ , such that a higher clearing margin is necessary for more extreme systematic correlation.

## 4 Loss sharing

In the previous section, we analyzed the counterparty risk exposure of one entity given default of all other entities and of the CCP. However, in practice a CCP is less likely to default than to enter a recovery process that recapitalizes the CCP by exploiting the resources of surviving clearing members (Elliott (2013), Duffie (2015)). Thus, the benefit of central clearing does not depend only on multilateral netting, but also largely on the extent of CCP recovery and resolution procedures and how CCPs allocate losses to clearing members. In the following, we study the resulting realized exposure incurred by each counterparty from both loss sharing and multilateral netting.

We specify a complete network structure among entities' positions, which in particular requires that  $v_{ij}^k = -v_{ji}^k$ .<sup>35</sup> To refrain from further assumptions about the heterogeneity of entities, we assume the same structure for each derivative class  $v_{ij}^k \equiv v_{ij}$  for all  $k = 1, \dots, K$  that includes one entity that is long in systematic risk with each trade, one that is short with each trade, and entities in-between. For example, the network with 5 entities is as follows:

$$(v_{ij})_{i,j \in \{1, \dots, \gamma\}} = \begin{pmatrix} 1 & 1 & 1 & 1 & \text{(long)} \\ -1 & & 1 & 1 & \\ -1 & -1 & & 1 & 1 & \text{(hedged)} \\ -1 & -1 & -1 & & 1 & \\ -1 & -1 & -1 & -1 & & \text{(short)} \end{pmatrix}. \quad (20)$$

In the following, we study whether the combination of loss sharing and multilateral netting

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<sup>35</sup>Duffie and Zhu (2011) argue that notionals (and positions) cannot be known *ex ante* from the perspective of a market designer setting up a clearing infrastructure. Nevertheless, as our results highlight, we find it important to examine the effect of central clearing in the presence of a particular market structure. For this purpose, we choose a network structure of positions that seems to be realistic, for example in the CDS market (Getmansky et al. (2016)). Then, it may well be that it is unknown today which entity will be long or short, while the general market structure is fixed. Moreover, Siriwardane (2018) finds that the positions in the CDS market are very sticky, e.g., that CDS sellers are typically asset managers.

reduces the realized counterparty risk exposure compared to a bilateral market. The impact of central clearing will depend on the direction of a market participant's positions, for which we focus on three specific entities that are illustrated in Equation (20):

- (a) Market participant  $i = 1$  is *long in systematic risk* since the value of all its trades is positively correlated with systematic risk (with  $v_{ij}^k = 1$ ). Thus, its portfolio value is low (and negative) in bad states of the economy, e.g., due to long positions in the S&P 500. In these times, the market participant owes its counterparties, and faces a small counterparty risk exposure itself. This reflects *right way risk* in the sense that the default of counterparties is negatively correlated with counterparty risk exposure.
- (b) Market participant  $i = 5$  is *short in systematic risk* since the value of all its trades are negatively correlated with systematic risk (with  $v_{ij}^k = -1$ ). Thus, its portfolio value increases in bad states of the economy, e.g., due to short positions in the S&P 500. In these times, it faces a large bilateral counterparty risk exposure. This reflects *wrong way risk* in the sense that the default of counterparties are positively correlated with counterparty risk exposure.
- (c) Market participant  $i = 3$  is a *hedged dealer* since it is hedged against systematic risk within each derivative class  $k$  as  $\sum_{j=1, j \neq i}^{\gamma} v_{ij}^k \approx 0$ . Thus, the hedged dealer's counterparty risk exposure in a multilateral pool exhibits a small (absolute) correlation with defaults.

We employ a default model that is based on Merton (1974)'s credit risk model and described in Appendix B in detail. Since default clustering and systematic exposure are important to study the effect of clearing, we adjust Merton (1974)'s original model such that the random value of entity  $i$ 's assets at the settlement period begin is given by

$$A_i = \exp \left( \mu_{A_i} - \frac{\beta_{A_i}^2 \sigma_M^2 + \sigma_{A_i}^2}{2} + \beta_{A_i} M + \sigma_{A_i} W_i \right), \quad (21)$$

where  $(W_1, \dots, W_\gamma)$  are jointly standard normally distributed and correlated with correlation matrix  $(\rho_{A_i, A_j})_{i, j \in \{1, \dots, \gamma\}}$ .  $\beta_{A_i} > 0$  is the exposure of entity  $i$ 's log asset value to the state of the economy  $M$ , i.e., its  $\beta$ -factor.<sup>36</sup>  $\mu_{A_i}$  and  $\sigma_{A_i}$  are the drift and volatility of the asset value process.

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<sup>36</sup>We assume that the state of the economy,  $M$ , impacts both asset values at time  $t = 0$  and contract value changes between  $t = 0$  and  $t = 1$ . Hence, we interpret the state of the economy as a sticky variable that might, e.g., reflect the business cycle.

The pairwise correlation of entity  $i$  and  $j$ 's log assets is given by

$$\tilde{\rho}_{A_i, A_j} = \underbrace{\frac{\beta_{A_i} \beta_{A_j} \sigma_M^2}{\sqrt{\beta_{A_i}^2 \sigma_M^2 + \sigma_{A_i}^2} \sqrt{\beta_{A_j}^2 \sigma_M^2 + \sigma_{A_j}^2}}_{\text{Correlation with } M} + \underbrace{\frac{\sigma_{A_i} \sigma_{A_j} \rho_{A_i, A_j}}{\sqrt{\beta_{A_i}^2 \sigma_M^2 + \sigma_{A_i}^2} \sqrt{\beta_{A_j}^2 \sigma_M^2 + \sigma_{A_j}^2}}_{\text{Default clustering}} \quad (22)$$

and consists of two layers: First, as a small (large negative) value of  $M$  decreases the value of assets, entities are more likely to default in bad states of the economy. This correlation will lead to *wrong way risk* in our model, since it implies correlation between entities' defaults and contract values (Bank for International Settlements (BIS) and International Organization of Securities Commissions (IOSCO) (2018)). It will be useful to reparametrize  $\beta_{A_i}$  in terms of correlation, such that  $\beta_{A_i} = \rho_{A_i, M} \frac{\bar{\sigma}_{A_i}}{\sigma_M}$  and  $\bar{\sigma}_{A_i}^2 = \text{var}(\log(A_i))$ . Then,  $\rho_{A_i, M}$  is the correlation between entity  $i$ 's assets  $A_i$  and the state of the economy  $M$ , and  $\bar{\sigma}_{A_i}^2$  is the log asset value's variance.

Second, given a specific state of the economy, the cross-sectional correlation  $\rho_{A_i, A_j}$  leads to clustered defaults. Clustered defaults might result from interconnectedness between (financial) institutions, like interbanking liabilities, such that the financial distress of one entity spills over to other entities. A prime example has been the default of Lehman Brothers during the 2007-08 financial crisis, that triggered substantial losses at other financial institutions. For simplicity, in the following we will assume that all entities' assets have the same distributional parameters and, thus, drop the parameter indices.

We define by  $D_i$  a binary random variables that equals one if entity  $i$  defaults, i.e., if  $A_i$  breaches an exogenous debt value  $B_i$ . Analogously to Lewandowska (2015), if all derivative classes are bilaterally traded (i.e., non-cleared), then the realized counterparty risk exposure is given by

$$\mathbb{E} \left[ E_i^{*BN, K} \right] = \mathbb{E} \left[ \sum_{j=1, j \neq i}^{\gamma} D_j \max \left( \sum_{k=1}^K X_{ij}^k - C_{ij}^{BN, K}, 0 \right) \right], \quad (23)$$

where the bilateral collateral  $C_{ij}^{BN, K}$  is given as in Section 3.4. Note that a loss realizes only in case a counterparty's default coincides with an adverse price movement in exceedance of collateral.

Now consider the case with derivative class  $K$  being centrally cleared. In line with loss-allocation rules (Arnsdorf (2012), Elliott (2013), Duffie (2015), Lewandowska (2015)), we assume that the

realized counterparty risk exposure at the CCP is shared among all surviving clearing members.<sup>37</sup> Clearing members suffer losses with the CCP only in case at least one clearing member  $j$  defaults and the multilaterally netted contract value of  $j$  exceeds the collateral provided by  $j$ .<sup>38</sup> The aggregate loss of the CCP is given by

$$\bar{L}^{CCP} = \sum_{j=1}^{\gamma} D_j \max \left( \sum_{\substack{g=1, \\ g \neq j}}^{\gamma} X_{gj}^K - C_j^{MN}, 0 \right). \quad (24)$$

As suggested by Duffie (2015), the aggregate loss is shared among surviving clearing members proportionally to the risk of their investments as reflected by their collateral,  $C_i^{MN}$ . Then, the realized counterparty risk exposure of clearing member  $i$  to the CCP is given by<sup>39</sup>

$$\mathbb{E}[E_i^{*MN}] = \mathbb{E} \left[ \frac{(1 - D_i)C_i^{MN}}{\sum_{g=1}^{\gamma} (1 - D_g)C_g^{MN}} \bar{L}^{CCP} \mid \sum_{g=1}^{\gamma} (1 - D_g) > 0 \right]. \quad (25)$$

Clearly, it is driven by a) multilateral netting and b) loss sharing.

If derivative class  $K$  is centrally cleared, then the remaining  $K - 1$  derivative classes are bilaterally netted and the realized exposure is given by Equation (23). The total realized counterparty risk exposure of entity  $i$  is then given by

$$\mathbb{E}[E_i^{*BN+MN}] = \mathbb{E} \left[ E_i^{*MN} + E_i^{*BN, K-1} \right]. \quad (26)$$

We use the baseline calibration from Tables 4 and 5 for all market participants and suppress

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<sup>37</sup>From a financial stability point of view, sharing of CCP losses among clearing members can be seen as a form of contagion of realized exposure. Therefore, as highlighted by the Financial Stability Board (FSB) (2017b), a CCP should settle (part of) its contracts at potential losses (called *partial or full tear up*) only if no other option is likely to result in a better outcome for financial stability. In this sense, the Financial Stability Board (FSB) (2017b) does not consider loss sharing to be contagion if it does not cause adverse financial stability consequences. For the sake of simplicity, we do not include such feedback effects in our model but focus only on the first-order counterparty risk exposure.

<sup>38</sup>In practice, the default of clearing members is absorbed by additional measures of a CCP's so-called *risk waterfall* before additional funds are called from surviving members: After exhausting variation and initial margins, the CCP's equity and clearing members' ex ante contributions to the default fund serve as a buffer against realized exposure (Arnsdorf (2012)). As these additional layers of protection have a similar effect as a higher margin confidence level in our model, we do not model them explicitly. Instead, realized exposures in our model might be interpreted as ex-post default fund contributions.

<sup>39</sup>We condition on at least one entity surviving since 1) it is extremely unlikely that all entities default at the same time, and 2) in practice it seems likely that a government would bail out a CCP in the case that all clearing members default.



entity indices where possible.<sup>40</sup> We assume, in particular, that the CCP and bilateral margin levels both based on a 99% confidence level but will assess the sensitivity of our results towards differences in margins. The correlation between defaults and contract values as well as the loss sharing mechanisms at the CCP does not allow for closed-form solutions for realized counterparty risk exposures. Instead, we are able to derive the bilateral realized counterparty risk exposure conditional on the state of the economy  $M$  and the expected loss at the CCP conditional on the state of the economy  $M$  and defaults  $D$  in closed-forms in Propositions 12 and 13 in Appendix A. The following results are based on a Monte-Carlo analysis with 3.75 million realizations of contract values and defaults, where we smooth the Monte-Carlo estimates for realized counterparty risk exposure by employing the conditional closed-form solutions from Propositions 12 and 13 in Appendix A.<sup>41</sup>

Variable	Value	Description
$pd$	0.05	Individual probability of default
$\rho_{A,A}$	0.05	Correlation of log assets conditional on $M$
$\bar{\sigma}_A$	1	Total log asset volatility
$\rho_{A,M}$	0.1	Correlation between log asset and state of the economy $M$
$\beta_A$	3.33	Implied beta-factor of log assets
$\sigma_A$	0.2	Implied idiosyncratic log asset volatility
$\alpha_{BN}$	0.99	Bilateral margin level
$\alpha_{MN}$	0.99	Multilateral (clearing) margin level

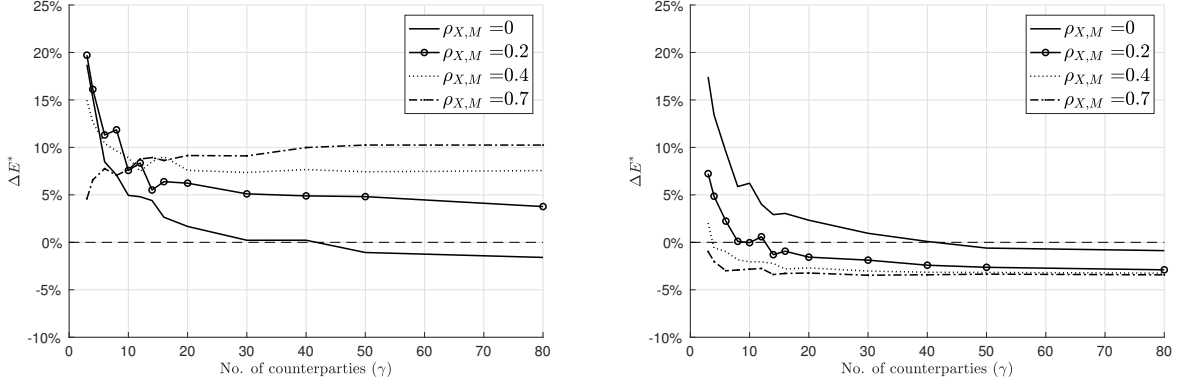
**Table 5.** Baseline calibration of the default model described in Appendix B. We assume the same calibration for each entity and derivative class.

Systematic risk has two effects on realized counterparty risk exposures: On one hand, it reduces the benefit of multilateral netting, as shown in Section 3. On the other hand, it correlates contract values with defaults. A clearing member that is long in systematic risk has a small counterparty risk exposure in times when counterparties are likely to default (and vice versa). Hence, this participant has to bear losses of the CCP in bad states while it actually faces a small bilateral counterparty risk exposure. Consequently, high systematic risk reduces the benefit of central clearing for clearing members that are long in systematic risk. As can be seen from Figure 11 (a), if contract values are sufficiently positively correlated with systematic risk (e.g.,  $\rho_{X,M} \geq 0.4$ ), then central clearing does

<sup>40</sup>In Appendix C, we assess the sensitivity of our results towards heterogeneity of market participants.

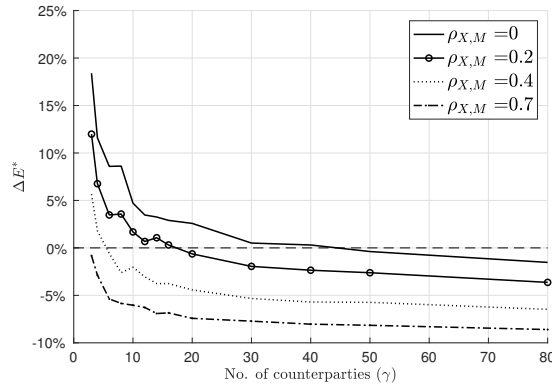
<sup>41</sup>More specifically, we employ the unweighted mean between (1) the pure Monte-Carlo estimate and (2) the Monte-Carlo estimate evaluating the closed-form solutions from Propositions 12 and 13 in Appendix A.

not reduce realized counterparty risk for any reasonable number of counterparties.



(a) Entity that is long in systematic risk ( $v_{ij}^k = 1$ ).

(b) Entity that is short in systematic risk ( $v_{ij}^k = -1$ ).



(c) Hedged dealer ( $\sum_j v_{ij}^k \approx 0$ ).

**Figure 11.** Impact of systematic risk ( $\rho_{X,M}$ ) on realized counterparty risk exposure.

Change in realized counterparty risk exposure due to central clearing of derivative class  $K$ ,  $\Delta E^* = \mathbb{E}[E_i^{*BN+MN} - E_i^{*BN,K}] / \mathbb{E}[E_i^{*BN,K}]$ , for different levels of systematic risk  $\rho_{X,M}$ . If  $\Delta E^* < 0$ , then central clearing reduces the realized exposure compared to bilateral clearing. The baseline calibration is described in Tables 4 and 5.

In contrast, contract values of clearing members that are short in systematic risk are large in bad states of the economy, reflecting positive profits. Thus, such a clearing member's large counterparty risk exposure coincides with a large probability of default of its counterparties. It is then beneficial to share the counterparty exposure with other clearing members (that may have less exposure) particularly if systematic correlation  $\rho_{X,M}$  is large, as Figure 11 (b) shows.

For large levels of systematic risk (e.g.,  $\rho_{X,M} = 0.4$ ), central clearing is even more beneficial for hedged dealers, as Figure 11 (c) shows. The reason is that hedged dealers have a zero net exposure to systematic risk only within but not across derivative classes, i.e., dealers hedge across counter-

parties within the same derivative class, implying  $\sum_j v_{ij}^k \approx 0$  but  $\sum_k v_{ij}^k \neq 0$  (cf. Equation (20)). Thus, hedged dealers are exposed to systematic risk primarily in bilateral netting pools but not multilateral netting pools. Therefore, multilateral netting substantially reduces their counterparty risk exposure compared to bilateral netting. As a consequence, multilateral netting is substantially more beneficial for hedged dealers than for other entities. In summary, these results show that the benefit of central clearing highly depends on the direction of an entity’s position (buy versus sell positions).

**RESULT 8.** *a) Market participants that are long in systematic risk benefit less from central clearing than those that are short.*

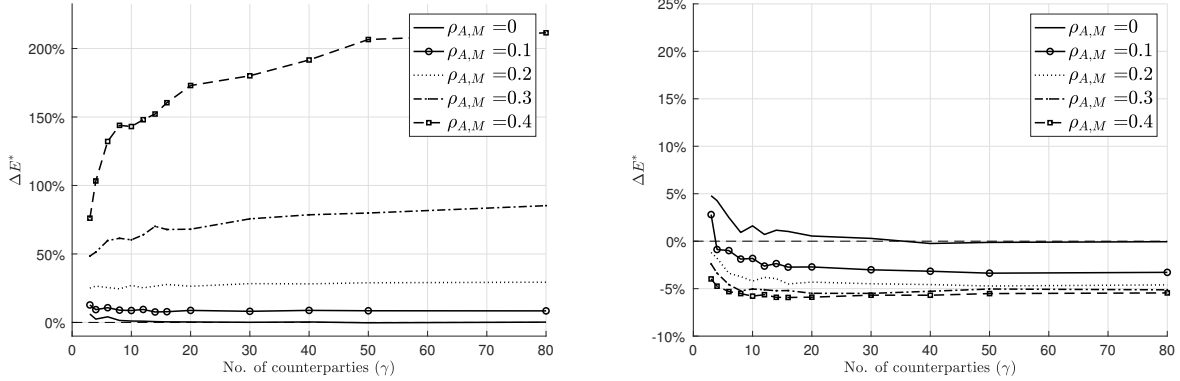
*b) The more a market participant is hedged within one contract class, the more it benefits from central clearing.*

The correlation between defaults (i.e., entities’ asset values) and the state of the economy,  $\rho_{A,M}$ , has a similar effect as  $\rho_{X,M}$ .  $\rho_{A,M}$  essentially controls the *wrong way risk*: With larger positive  $\rho_{A,M}$ , market participants that are long in systematic risk have larger contract values and thus a larger counterparty risk exposure in times with high default probabilities ( $M < 0$ ). The reversed effect occurs for market participants that are short in systematic risk. As Figures 12 (a) and (b) show, particularly for a large correlation between defaults and the state of the economy,  $\rho_{A,M}$ , central clearing then increases realized counterparty risk exposure for market participants that are long in systematic risk but decreases exposure for those that are short. The higher  $\rho_{A,M}$ , the larger is the bifurcation between market participants that are long and short.  $\rho_{A,M}$  does, however, not substantially affect the benefit of central clearing for hedged dealers, as Figure 12 (c) shows. The reason is that dealers diversify their portfolio’s exposure to systematic risk across counterparties (by being long and short within the same derivative class), effectively isolating them from correlation between their portfolio value and counterparty defaults. Consequently,  $\rho_{A,M}$  has a small impact on a dealer’s benefit from central clearing.

Moreover, our simulation results show that default clustering conditional on the state of the economy ( $\rho_{A,A}$ ) has virtually no effect on the benefit of central clearing.<sup>42</sup> Large default clustering ( $\rho_{A,A}$ ) makes it more likely that clearing members default together, resulting in large CCP losses.

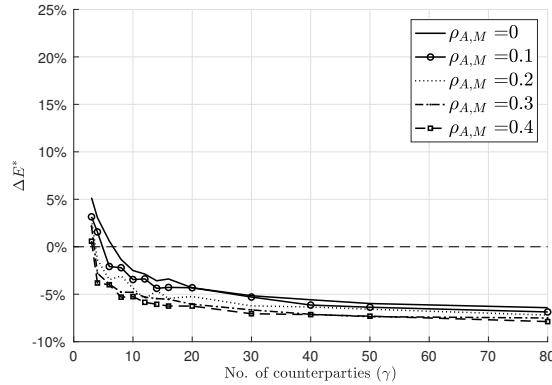
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<sup>42</sup>The results are available on request.



(a) Entity that is long in systematic risk ( $v_{ij}^k = 1$ ).

(b) Entity that is short in systematic risk ( $v_{ij}^k = -1$ ).



(c) Hedged dealer ( $\sum_j v_{ij}^k \approx 0$ ).

**Figure 12.** Impact of default correlation with the state of the economy on realized counterparty risk exposure.

Change in realized counterparty risk exposure due to central clearing of derivative class  $K$ ,  $\Delta E^* = \mathbb{E}[E_i^{*BN+MN} - E_i^{*BN,K}] / \mathbb{E}[E_i^{*BN,K}]$ , for different levels of correlation between entities' asset values and the state of the economy,  $\rho_{A,M}$ . If  $\Delta E^* < 0$ , then central clearing reduces the realized exposure compared to bilateral clearing. The baseline calibration is described in Tables 4 and 5.

At the same time, it becomes more likely that clearing member  $i$  defaults itself in which case  $i$  does not pay a share of the CCP's losses. The first effect offsets the second, resulting in a small effect of  $\rho_{A,A}$  on the benefit of clearing.<sup>43</sup>

Stressing the importance of *wrong way risk*, the previous results are even more pronounced during extreme events. For our baseline calibration, market participants that are short in systematic risk or hedged dealers only benefit from clearing in extreme events occurring with probability less than  $q = 0.05$ , while the counterparty risk exposure of those that are long increases by more

<sup>43</sup>If we instead conditioned on a specific entity surviving, it would always bear part of the CCP's losses. From this perspective, default clustering would substantially increase realized counterparty risk exposure from central clearing and thus reduce its benefit for an entity. The results are available upon request.

than 2000% in these times. Indeed, the latter do not benefit at all from clearing in events more severe than  $q \leq 0.4$ . Strikingly, during sufficiently extreme events (such as the 1% quantile of the state of the economy), the realized counterparty risk exposure of market participants that are long in systematic risk increases with the number of counterparties  $\gamma$ . In these times, the additional exposure shared by additional clearing members outweighs any benefit of central clearing. This underlines that imbalances between clearing members with different position directions manifest particularly in crises.

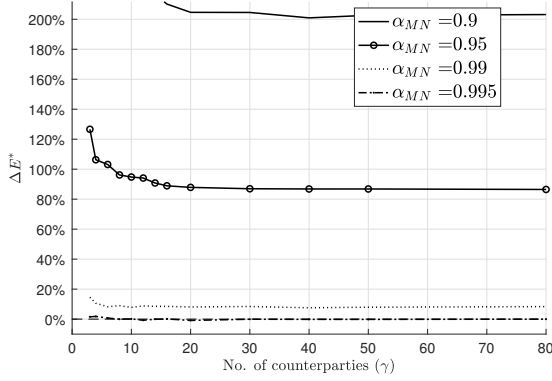
**RESULT 9.** *Differences in the benefit of central clearing between market participants with different directions of net positions are more pronounced (a) when systematic risk  $\rho_{X,M}$  is larger, (b) wrong way risk  $\rho_{A,M}$  is larger, or (c) in extreme adverse economic states.*

The impact of margin requirements on the realized counterparty risk exposure is similar to the impact on uncollateralized exposure: The smaller the clearing margin level ( $\alpha_{MN}$ ) relative to the bilateral one ( $\alpha_{BN}$ ), the larger is the realized exposure with central clearing relative to a bilateral market, implying a smaller (or no) benefit of central clearing. For  $\alpha_{BN} = 0.99$ , if  $\alpha_{MN} < 0.95$ , then central clearing does not lead to a reduction of realized counterparty risk exposures for any market participant and a reasonable number of counterparties  $\gamma \leq 80$  (see Figure 13). As in Section 3.4, the benefit of central clearing is highly sensitive towards the clearing margin level: Counterparty risk exposure increases by roughly 90% (30%) for market participants that are long (short) in systematic risk if moving from bilateral netting with  $\alpha_{BN} = 0.99$  to multilateral netting with  $\alpha_{MN} = 0.95$ . This result suggests that it is unlikely that the current regulation, that imposes a smaller margin for cleared than non-cleared trades, results in a benefit of central clearing for counterparty risk exposure.

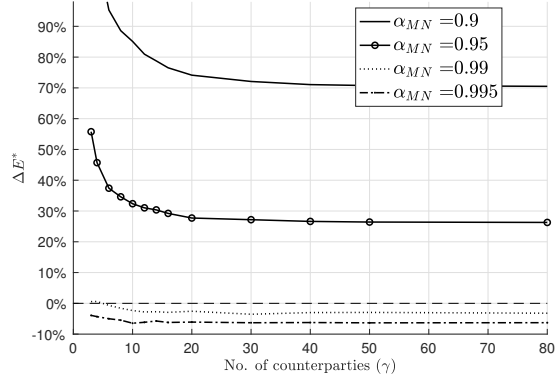
The previous results qualitatively also hold for a *Mega CCP*.<sup>44</sup> While the effect of loss sharing is qualitatively the same with a *Mega CCP*, the additional degree of netting leads to an additional benefit of central clearing. As a result, a market participant that is short in systematic risk or hedged always benefits from central clearing with a *Mega CCP*. A market participant that is long then also benefits from clearing if the systematic correlation  $\rho_{X,M}$  is sufficiently small. With our calibration, such a market participant benefits from a *Mega CCP* for any level of correlation  $\rho_{X,M}$

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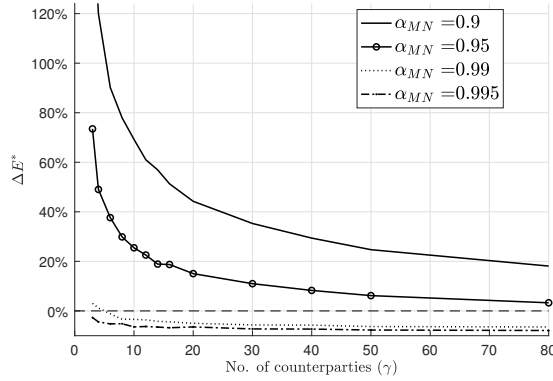
<sup>44</sup>The results are available on request.



(a) Entity that is long in systematic risk ( $v_{ij}^k = 1$ ).



(b) Entity that is short in systematic risk ( $v_{ij}^k = -1$ ).



(c) Hedged dealer ( $\sum_j v_{ij}^k \approx 0$ ).

**Figure 13.** Impact of margins on realized counterparty risk exposure.

Change in realized counterparty risk exposure due to central clearing of derivative class  $K$ ,  $\Delta E^* = \mathbb{E}[E_i^{*BN+MN} - E_i^{*BN,K}] / \mathbb{E}[E_i^{*BN,K}]$ , for different clearing margin confidence levels  $\alpha_{MN}$  and a fixed bilateral confidence level  $\alpha_{BN} = 0.99$ . If  $\Delta E^* < 0$ , then central clearing reduces realized exposure compared to bilateral clearing. The baseline calibration is described in Tables 4 and 5.

lower than 20%.

## 5 Conclusion and policy implications

We present a theoretical analysis of the impact of central clearing on counterparty risk exposure in the presence of systematic risk. Our main result is that the effect of central clearing is highly sensitive towards different levels of systematic risk, margin requirements, extreme market events, and the direction of clearing members' positions. We show that, in many realistic situations, central clearing actually results in counterparty risk exposures that are larger than with bilateral netting.

Central clearing impacts counterparty risk exposure through two channels: multilateral netting

and loss sharing. Multilateral netting allows entities to net counterparty risk exposure across multiple counterparties; while bilateral netting allows netting across multiple derivative classes with each single counterparty. Based on empirical estimates for systematic risk among CDS indices, we show that in the presence of systematic risk it requires at least 121 clearing members for multilateral netting to reduce counterparty risk exposure and provide diversification benefits, which is unrealistically high for CCPs in practice. In the absence of systematic risk (as studied by [Duffie and Zhu \(2011\)](#)), only 39 entities are required. Thus, the presence of systematic risk greatly undermines the benefit of CCPs. Even more concerning, if the collateral provided for cleared contracts is slightly smaller than for non-cleared derivatives, then central clearing does not reduce counterparty risk exposure for any number of counterparties compared to the case of a bilateral market.

The recent financial crises exposed vulnerabilities in the derivatives market architecture which was dominated by bilateral trades. The introduction of mandatory central clearing clearly increased the transparency of derivative markets, but would it increase financial stability in crises? We show, for a realistic calibration, that during extreme (but still reasonable) negative market events, central clearing of some assets is less beneficial compared to a bilateral market. Thus, central clearing increases counterparty risk exposure in times when a reduction is most needed. Following this result suggests to suspend central clearing during crises to reduce counterparty risk exposure - although central clearing was originally proposed to mitigate counterparty risk exposure particularly in crises ([Financial Stability Board \(FSB\) \(2017a\)](#)). Interestingly, the [European Commission \(2016\)](#) and [Council of the European Union \(2017, Article 6b\)](#) have proposed an emergency mechanism to suspend mandatory central clearing of one or more derivative classes in special circumstances. However, it seems unlikely that this regulation will be used to suspend all derivative classes from central clearing when entering a crisis; instead it is more likely that it only affects new derivative trades ([European Systemic Risk Board \(ESRB\) \(2017\)](#)).

We show that, in the absence of margins, only a *Mega CCP* that clears across derivative classes can reduce counterparty risk exposure in extreme events. However, there are substantial operational hurdles to achieving a single *Mega CCP* including competition among current CCPs, political constraints, and country jurisdictions. Also, creating a single *Mega CCP* might lead to concentration of risk in one such entity making it more systemically important and vulnerable,

e.g., to cybersecurity attacks. Derivatives market participants, however, seem to be aware of the beneficial impact of a higher degree of netting at large CCPs. As a consequence, clearing markets are heavily concentrated (e.g., according to an industry analysis, the London Clearing House (LCH) clears more than 90% of the Euro interest rate derivatives market notional as of September 2017).

Nonetheless, even a *Mega CCP* does not unambiguously reduce counterparty risk exposure: We show that, if the margin requirement for cleared derivatives is sufficiently small compared to that for non-cleared derivatives, the degree of netting is only of minor importance. In this case, even a *Mega CCP* does not reduce counterparty risk exposure compared to the bilateral netting case. We conclude that, to reduce counterparty risk, a first-order objective of regulation must be to align margin requirements for cleared and non-cleared derivatives.

We also show that the sharing of a CCP's non pre-funded losses among clearing members creates substantial heterogeneity: Clearing members that are short in systematic risk (e.g., protection buyers) benefit from central clearing, as they build up large counterparty risk exposure exactly when their counterparties exhibit large probabilities of default. In contrast, clearing members that are long in systematic risk (e.g., protection sellers) do not gain any additional benefits from central clearing as they have a small counterparty risk exposure when counterparties are likely to default, and then bear the losses of other clearing members.

Our results thus explain why many market participants choose not to clear contracts if it is not mandatory. For example, less than 20% of single name CDS are cleared as of June 2016 ([Financial Stability Board \(FSB\) \(2017a\)](#)). The heterogeneity in terms of systematic risk exposure also explains why many financial institutions do not become clearing members, e.g., asset managers, that typically exhibit a positive exposure to systematic risk ([Siriwardane \(2018\)](#)).

To circumvent the heterogeneity resulting from loss sharing, CCP recovery tools might account for the direction of positions (long versus short) when allocating losses. One suitable mechanism might be variation margin haircutting, allocating losses mainly to clearing members whose counterparty risk exposure is negatively correlated with systematic risk. However, even with no heterogeneity arising from loss sharing, we describe numerous reasonable situations in which multilateral netting is not beneficial from the viewpoint of a market participant's counterparty risk exposure, compared to bilateral netting. We are not considering in our analysis other benefits regarding central clearing as capital requirements benefits and market liquidity. However, as soon as these



effects are not so large to change our results, our analysis suggests that the ultimate regulatory tool to move to centrally cleared derivative markets is indeed an obligation to clear.

## References

- Acharya, V. and Bisin, A. (2014). Counterparty risk externality: Centralized versus over-the-counter markets. *Journal of Economic Theory*, 149:153–182.
- Acharya, V., Engle, R., and Richardson, M. (2012). Capital shortfall: A new approach to ranking and regulation systemic risk. *American Economic Review: Papers & Proceedings*, 102(3):59–64.
- Acharya, V., Pedersen, L., Philippon, T., and Richardson, M. (2017). Measuring systemic risk. *Review of Financial Studies*, 30(1):2–47.
- Acharya, V., Shachar, O., and Subrahmanyam, M. (2011). Regulating OTC derivatives. In Acharya, V., Cooley, T. F., Richardson, M., and Walter, I., editors, *Regulating Wall Street: The Dodd-Frank Act and the New Architecture of Global Finance*, chapter 13. New York University Stern School of Business.
- Arnakolla, A. and Laurent, J.-P. (2017). CCP resilience and clearing membership. *Working Paper*.
- Arnsdorf, M. (2012). Quantification of central counterparty risk. *Journal of Risk Management in Financial Institutions*, 5(3):273–287.
- Arora, N., Gandhi, P., and Longstaff, F. A. (2012). Counterparty credit risk and the credit default swap market. *Journal of Financial Economics*, 103:280–293.
- Bank for International Settlements (BIS) (1998). OTC Derivatives: Settlement procedures and counterparty risk management. Available at <http://www.bis.org> (accessed September 9, 2018).
- Bank for International Settlements (BIS) (2012). Principles for financial market infrastructures. Available at <http://www.bis.org> (accessed September 9, 2018).
- Bank for International Settlements (BIS) (2014). The standardised approach for measuring counterparty credit risk exposures. Available at <http://www.bis.org> (accessed September 9, 2018).
- Bank for International Settlements (BIS) (2015). Margin requirements for non-centrally cleared derivatives. Available at <http://www.bis.org> (accessed September 9, 2018).

- Bank for International Settlements (BIS) and International Organization of Securities Commissions (IOSCO) (2018). Framework for supervisory stress testing of central counterparties (CCPs). Available at <http://www.bis.org> (accessed September 9, 2018).
- Bellia, M., Panzica, R., Pelizzon, L., and Peltonen, T. (2017). The demand for central clearing: To clear or not to clear, that is the question. *ESRB Working Paper Series*, 62.
- Bergman, W. J., Bliss, R. R., Johnson, C. A., and Kaufman, G. G. (2004). Netting, financial contracts, and banks: The economic implications. *Federal Reserve Bank of Chicago Working Paper*, 2004-02.
- Biais, B., Heider, F., and Hoerova, M. (2016). Risk-sharing or risk-taking? Counterparty risk, incentives and margins. *Journal of Finance*, 71(4):1669–1698.
- Bignon, V. and Vuillemey, G. (2018). The failure of a clearinghouse: Empirical evidence. *Review of Finance*, forthcoming.
- Brunnermeier, M. (2009). Deciphering the liquidity and credit crunch 2007-2008. *Journal of Economic Perspectives*, 23(1):77–100.
- Brunnermeier, M., Clerc, L., Omari, Y. E., Gabrieli, S., Kern, S., Memmel, C., Peltonen, T., Podlich, N., Scheicher, M., and Vuillemey, G. (2013). Assessing contagion risks in the CDS market. *ESRB Occasional Paper*, 4.
- Budding, E., Cox, R., and Murphy, D. (2016). Central counterparties in crisis: International Commodities Clearing House, New Zealand Futures and Options Exchange and the Stephen Francis Affair. *Journal of Financial Market Infrastructures*, 4(3):65–92.
- Cont, R. and Kokholm, T. (2014). Central clearing of OTC derivatives: Bilateral vs multilateral netting. *Statistics & Risk Modeling*, 31(1):3–22.
- Council of the European Union (2017). Proposal for a regulation of the European parliament and of the council amending regulation (EU) No 648/2012 as regards the clearing obligation, the suspension of the clearing obligation, the reporting requirements, the risk-mitigation techniques

- for OTC derivatives contracts not cleared by a central counterparty, the registration and supervision of trade repositories and the requirements for trade repositories. *Interinstitutional File*, 2017/0090.
- Cunliffe, S. J. (2018). Central clearing and resolution - learning some of the lessons of Lehmans. Available at [www.bankofengland.co.uk](http://www.bankofengland.co.uk) (accessed September 9, 2018). *Speech*.
- Du, W., Gadgil, S., Gordy, M. B., and Vega, C. (2016). Counterparty risk and counterparty choice in the credit default swap market. *Federal Reserve Board Working Paper*, 87.
- Duffie, D. (2015). Resolution of failing central counterparties. In Jackson, T., Scott, K., and Taylor, J. E., editors, *Making Failure Feasible: How Bankruptcy Reform Can End 'Too Big To Fail*, chapter 4, pages 87–109. Hoover Institution Press.
- Duffie, D., Li, A., and Lubke, T. (2010). Policy perspectives on OTC derivatives market infrastructure. *Federal Reserve Bank of New York Staff Reports*, 424.
- Duffie, D., Scheicher, M., and Vuillemeys, G. (2015). Central clearing and collateral demand. *Journal of Financial Economics*, 116:237–256.
- Duffie, D. and Zhu, H. (2011). Does a central clearing counterparty reduce counterparty risk? *Review of Asset Pricing Studies*, 1:74–95.
- Elliott, D. (2013). Central counterparty loss-allocation rules. *Bank of England Financial Stability Paper*, 20.
- European Commission (2016). Report from the commission to the European Parliament and the Council under Article 85(1) of Regulation (EU) No 648/2012 of the European Parliament and of the Council of 4 July 2012 on OTC derivatives, central counterparties and trade repositories.
- European Securities and Markets Authority (ESMA) (2018a). List of central counterparties authorised to offer services and activities in the Union. Available at <http://www.esma.europa.eu> (accessed September 9, 2018).
- European Securities and Markets Authority (ESMA) (2018b). List of third-country central

- counterparties recognised to offer services and activities in the Union. Available at <http://www.esma.europa.eu> (accessed September 9, 2018).
- European Systemic Risk Board (ESRB) (2017). Revision of the European Market Infrastructure Regulation. Available at <http://www.esrb.europa.eu> (accessed September 9, 2018).
- Financial Stability Board (FSB) (2017a). Review of OTC derivatives market reforms: Effectiveness and broader effects of the reforms. Available at [www.fsb.org](http://www.fsb.org) (accessed September 9, 2018).
- Financial Stability Board (FSB) (2017b). Guidance on central counterparty resolution and resolution planning. Available at [www.fsb.org](http://www.fsb.org) (accessed September 9, 2018).
- Financial Stability Board (FSB) (2018). Incentives to centrally clear over-the-counter (OTC) derivatives - A post-implementation evaluation of the effects of the G20 financial regulatory reforms. Available at <http://www.fsb.org> (accessed September 9, 2018).
- Finansinspektionen (Financial Supervisory Authority Sweden) (2018). FI's comments on Nasdaq Clearing's decision today. Available at <https://www.fi.se/en/published/news/2018/fis-kommentar-till-nasdaq-clearings-beslut-i-dag/>.
- Garvin, N. (2012). Central counterparty interoperability. *Reserve Bank of Australia Bulletin*, June Quarter.
- Getmansky, M., Girardi, G., and Lewis, C. (2016). Interconnectedness in the CDS Market. *Financial Analysts Journal*, 72:62–82.
- Ghamami, S. and Glasserman, P. (2017). Does OTC derivatives reform incentivize central clearing? *Journal of Financial Intermediation*, 32:76–87.
- Hills, B., Rule, D., Parkinson, S., and Young, C. (1999). Central counterparty clearing houses and financial stability. *Bank of England Financial Stability Review*, 6:122–134.
- Huang, W. (2018). Central counterparty capitalization and misaligned incentives. *Working Paper*.
- Huang, W. and Menkveld, A. J. (2016). Systemic risk in real time: A risk dashboard for central clearing parties (CCPs). *Working Paper*.

- International Swaps and Derivatives Association (2017). Margin Survey 2014. Available at <http://www.isda.org/> (accessed September 9, 2018).
- Jackson, J. P. and Manning, M. J. (2007). Comparing the pre-settlement risk implications of alternative clearing arrangements. *Bank of England Working Paper*, 321.
- Lewandowska, O. (2015). OTC clearing arrangements for bank systemic risk regulation: A simulation approach. *Journal of Money, Credit and Banking*, 47(6):1177–1203.
- Longstaff, F. A., Pan, J., Pedersen, L. H., and Singleton, K. J. (2011). How sovereign is sovereign credit risk? *American Economic Journal: Macroeconomics*, 3(2):75–103.
- Loon, Y. C. and Zhong, Z. K. (2014). The impact of central clearing on counterparty risk, liquidity, and trading: Evidence from the credit default swap market. *Journal of Financial Economics*, 112:91–115.
- Markit (2015). Markit CDX High Yield & Markit CDX Investment Grade Index Rules. Available at <http://www.markit.com/> (accessed September 9, 2018).
- Menkveld, A. J. (2017). Crowded positions: An overlooked systemic risk for central clearing parties. *Review of Asset Pricing Studies*, 7(2):209–242.
- Menkveld, A. J., Pagnotta, E. S., and Zoican, M. A. (2015). Does central clearing affect price stability? Evidence from nordic equity markets. *Working Paper*.
- Merton, R. C. (1974). On the pricing of corporate debt: The risk structure of interest rates. *Journal of Finance*, 29(2):449–470.
- Office of the Comptroller of the Currency (2016). Quarterly report on bank trading and derivatives activities. 4th Quarter 2016.
- Pan, J. and Singleton, K. J. (2008). Default and recovery implicit in the term structure of sovereign CDS spreads. *Journal of Finance*, 63(5):2345–2384.
- Peltonen, T., Scheicher, M., and Vuillemeys, G. (2014). The network structure of the CDS market and its determinants. *Journal of Financial Stability*, 13:118–133.

Siriwardane, E. N. (2018). Limited investment capital and credit spreads. *Journal of Finance*, forthcoming.

Stafford, P. and Sheppard, D. (2018). Trader blows EUR 100m hole in Nasdaq's Nordic power market. Available at <https://www.ft.com/content/43c74e02-b749-11e8-bbc3-ccd7de085ffe>. *Financial Times*, September 13, 2018.

Woodall, L. (2018). Concentration of client positions rises at LCH. Technical report.

## A Proofs

**PROPOSITION 1** (Collateralized bilateral counterparty risk exposure). *The collateralized counterparty risk exposure with bilateral netting is given by*

$$\mathbb{E}[E_i^{BN,K}] = \varphi(0) \sum_{j=1, j \neq i}^{\gamma} \bar{\sigma}_{ij}^{BN,K}, \quad (27)$$

where

$$\left(\bar{\sigma}_{ij}^{BN,K}\right)^2 = \sigma_M^2 \left(\sum_{k=1}^K v_{ij}^k \beta_{ij}^k\right)^2 + \sum_{k=1}^K \left(v_{ij}^k\right)^2 \left(\sigma_{ij}^k\right)^2. \quad (28)$$

If  $v_{ij}^k \equiv 1$  or  $v_{ij}^k \equiv -1$ ,  $\beta_{ij}^k \equiv \beta$ , and  $\sigma_{ij}^k \equiv \sigma$  for all  $j = 1, \dots, \gamma$ ,  $k = 1, \dots, K$ , then the collateralized counterparty risk exposure is  $\mathbb{E}[E_i^{BN,K}] = \varphi(0)(\gamma - 1)\sqrt{\sigma_M^2 \beta^2 K^2 + K\sigma^2}$  or, equivalently,  $\mathbb{E}[E_i^{BN,K}] = \varphi(0)(\gamma - 1)\sigma_X \sqrt{K} \sqrt{1 + (K - 1)\rho_{X,M}^2}$ .

If entity  $i$  is a dealer across derivative classes (i.e.,  $\sum_{k=1}^K v_{ij}^k = 0$ ) and  $|v_{ij}^k| \equiv 1$ ,  $\beta_{ij}^k \equiv \beta$ , and  $\sigma_{ij}^k \equiv \sigma$ , then  $\left(\bar{\sigma}_{ij}^{BN,K}\right)^2 = K\sigma^2$ , and thus  $\mathbb{E}[E_i^{BN,K}] = \varphi(0)(\gamma - 1)\sqrt{K}\sigma^2$ .

### Proof of Proposition 1:

*Proof.* The counterparty risk exposure equals

$$\mathbb{E}\left[E_i^{BN,K}\right] = \sum_{j=1, j \neq i}^{\gamma} \mathbb{E}\left[\underbrace{\max\left(\sum_{k=1}^K X_{ij}^k, 0\right)}_{=E_{ij}^{BN,K}}\right] \quad (29)$$



Define

$$\bar{\mu}_{ij}^{BN,K} = \mathbb{E} \left[ \sum_{k=1}^K X_{ij}^k \right] = \sum_{k=1}^K \mathbb{E} \left[ X_{ij}^k \right] = 0 \quad (30)$$

$$\left( \bar{\sigma}_{ij}^{BN,K} \right)^2 = \text{var} \left( \sum_{k=1}^K X_{ij}^k \right) = \text{var} \left( \sum_{k=1}^K v_{ij}^k (\beta_{ij}^k M + \sigma_{ij}^k \varepsilon_{ij}^k) \right) \quad (31)$$

$$= \text{var} \left( M \sum_{k=1}^K v_{ij}^k \beta_{ij}^k + \sum_{k=1}^K v_{ij}^k \sigma_{ij}^k \varepsilon_{ij}^k \right) \quad (32)$$

$$= \sigma_M^2 \left( \sum_{k=1}^K v_{ij}^k \beta_{ij}^k \right)^2 + \sum_{k=1}^K \left( v_{ij}^k \right)^2 \left( \sigma_{ij}^k \right)^2. \quad (33)$$

For  $Y \sim \mathcal{N}(\mu, \sigma^2)$  we have that  $\mathbb{E}[Y \mid Y > 0] = \mu + \sigma \frac{\varphi(-\mu/\sigma)}{\Phi(\mu/\sigma)}$  and thus  $\mathbb{E}[\max(Y, 0)] = \mathbb{E}[Y \mid Y > 0] \Phi(\mu/\sigma) = \mu \Phi(\mu/\sigma) + \sigma \varphi(-\mu/\sigma)$ . With this in mind, the counterparty risk exposure of  $i$  to  $j$  is given by

$$\mathbb{E}[E_{ij}^{BN,K}] = \bar{\mu}_{ij}^{BN,K} \Phi \left( \bar{\mu}_{ij}^{BN,K} / \bar{\sigma}_{ij}^{BN,K} \right) + \bar{\sigma}_{ij}^{BN,K} \varphi \left( -\bar{\mu}_{ij}^{BN,K} / \bar{\sigma}_{ij}^{BN,K} \right) \quad (34)$$

$$= \bar{\sigma}_{ij}^{BN,K} \varphi(0) = \bar{\sigma}_{ij}^{BN,K} \frac{1}{\sqrt{2\pi}} \quad (35)$$

and the total counterparty risk exposure is given by

$$\mathbb{E}[E_i^{BN,K}] = \varphi(0) \sum_{j=1, j \neq i}^{\gamma} \bar{\sigma}_{ij}^{BN,K}. \quad (36)$$

Assume that  $\beta \equiv \beta_{ij}^k$ , and  $\sigma \equiv \sigma_{ij}^k$ . Then,

$$\left( \bar{\sigma}_{ij}^{BN,K} \right)^2 = \sigma_M^2 \beta^2 \left( \sum_{k=1}^K v_{ij}^k \right)^2 + \sigma^2 \sum_{k=1}^K \left( v_{ij}^k \right)^2 \quad (37)$$

$$= \sigma_M^2 \beta^2 \left( \sum_{k=1}^K v_{ij}^k \right)^2 + K \sigma^2. \quad (38)$$

If  $v_{ij}^k \equiv -1$  or  $v_{ij}^k \equiv 1$ , then  $\mathbb{E}[E_i^{BN,K}] = \varphi(0)(\gamma - 1) \sqrt{\sigma_M^2 \beta^2 K^2 + K \sigma^2}$ .  $\square$

**PROPOSITION 2** (Collateralized multilateral counterparty risk exposure). *The collateralized*

counterparty risk exposure with multilateral netting of derivative class  $K$  is given by

$$\begin{aligned} \mathbb{E}[E_i^{BN+MN}] = \varphi(0) & \left( \sum_{j=1, j \neq i}^{\gamma} \sqrt{\sigma_M^2 \left( \sum_{k=1}^{K-1} v_{ij}^k \beta_{ij}^k \right)^2 + \sum_{k=1}^{K-1} (v_{ij}^k)^2 (\sigma_{ij}^k)^2} \right. \\ & \left. + \sqrt{\sigma_M^2 \left( \sum_{j=1, j \neq i}^{\gamma} v_{ij}^K \beta_{ij}^K \right)^2 + \sum_{j=1, j \neq i}^{\gamma} (v_{ij}^K)^2 (\sigma_{ij}^K)^2} \right). \end{aligned} \quad (39)$$

If  $v_{ij}^k \equiv -1$  or  $v_{ij}^k \equiv 1$ ,  $\sigma_{ij}^k \equiv \sigma$ , and  $\beta_{ij}^k \equiv \beta$  for all  $j = 1, \dots, \gamma$ ,  $k = 1, \dots, K$ , then

$$\mathbb{E}[E_i^{BN+MN}] = \varphi(0) \left( (\gamma - 1) \sqrt{\sigma_M^2 \beta^2 (K - 1)^2 + (K - 1) \sigma^2} \right. \quad (40)$$

$$\left. + \sqrt{\sigma_M^2 \beta^2 (\gamma - 1)^2 + (\gamma - 1) \sigma^2} \right), \quad (41)$$

or, equivalently,

$$\mathbb{E}[E_i^{BN+MN}] = \varphi(0) \left( (\gamma - 1) \sigma_X \sqrt{K - 1} \sqrt{1 + \rho_{X,M}^2 (K - 2)} \right. \quad (42)$$

$$\left. + \sigma_X \sqrt{\gamma - 1} \sqrt{1 + \rho_{X,M}^2 (\gamma - 2)} \right). \quad (43)$$

If  $\rho_{X,M} > 0$ , then there exists a lower bound such that  $\mathbb{E}[E_i^{MN}] > (\gamma - 1) |\rho_{X,M}| \sigma_X \varphi(0)$  for all  $\gamma > 0$ .

If entity  $i$  is a dealer across counterparties and  $|v_{ij}^k| \equiv 1$ ,  $\beta_{ij}^k \equiv \beta$ , and  $\sigma_{ij}^k \equiv \sigma$ , then it holds that  $\sum_{j=1, j \neq i}^{\gamma} v_{ij}^k = 0$  and thus

$$\mathbb{E}[E_i^{BN+MN}] = \varphi(0) \left( (\gamma - 1) \sqrt{\sigma_M^2 \beta^2 (K - 1)^2 + (K - 1) \sigma^2} + \sqrt{(\gamma - 1) \sigma^2} \right), \quad (44)$$

*Proof.* If derivative class  $K$  is multilaterally netted, then the collateralized counterparty risk exposure is

$$\mathbb{E}[E_i^{BN+MN}] = \mathbb{E} \left[ \sum_{j=1, j \neq i}^{\gamma} \max \left( \sum_{k=1}^{K-1} X_{ij}^k, 0 \right) + \max \left( \sum_{j=1, j \neq i}^{\gamma} X_{ij}^K, 0 \right) \right]. \quad (45)$$

The first summand is given by the bilateral exposure for the first  $K - 1$  derivative classes as in

Proposition 1,

$$\mathbb{E}[E_i^{BN,K-1}] = \varphi(0) \sum_{j=1, j \neq i}^{\gamma} \sqrt{\sigma_M^2 \left( \sum_{k=1}^{K-1} v_{ij}^k \beta_{ij}^k \right)^2 + \sum_{k=1}^{K-1} (v_{ij}^k)^2 (\sigma_{ij}^k)^2}. \quad (46)$$

The second summand is the exposure in derivative class  $K$  with multilateral netting, where

$$\bar{\mu}_i^{MN} = \mathbb{E} \left[ \sum_{j=1, j \neq i}^{\gamma} X_{ij}^K \right] = 0 \quad (47)$$

$$(\bar{\sigma}_i^{MN})^2 = \text{var} \left( \sum_{j=1, j \neq i}^{\gamma} X_{ij}^K \right) \quad (48)$$

$$= \text{var} \left( \sum_{j=1, j \neq i}^{\gamma} v_{ij}^K (\beta_{ij}^K M + \sigma_{ij}^K \varepsilon_{ij}^K) \right) \quad (49)$$

$$= \text{var} \left( M \sum_{j=1, j \neq i}^{\gamma} v_{ij}^K \beta_{ij}^K + \sum_{j=1, j \neq i}^{\gamma} v_{ij}^K \sigma_{ij}^K \varepsilon_{ij}^K \right) \quad (50)$$

$$= \sigma_M^2 \left( \sum_{j=1, j \neq i}^{\gamma} v_{ij}^K \beta_{ij}^K \right)^2 + \sum_{j=1, j \neq i}^{\gamma} (v_{ij}^K)^2 (\sigma_{ij}^K)^2. \quad (51)$$

Analogously to Proposition 1, the exposure in the multilaterally netted derivative class  $K$  is given by

$$\mathbb{E} \left[ \max \left( \sum_{j=1, j \neq i}^{\gamma} X_{ij}^K, 0 \right) \right] = \bar{\sigma}_i^{MN} \varphi(0) \quad (52)$$

$$= \varphi(0) \sqrt{\sigma_M^2 \left( \sum_{j=1, j \neq i}^{\gamma} v_{ij}^K \beta_{ij}^K \right)^2 + \sum_{j=1, j \neq i}^{\gamma} (v_{ij}^K)^2 (\sigma_{ij}^K)^2}. \quad (53)$$

Thus, the total counterparty risk exposure is given by

$$\begin{aligned} \mathbb{E}[E_i^{BN+MN}] &= \varphi(0) \left( \sum_{j=1, j \neq i}^{\gamma} \sqrt{\sigma_M^2 \left( \sum_{k=1}^{K-1} v_{ij}^k \beta_{ij}^k \right)^2 + \sum_{k=1}^{K-1} (v_{ij}^k)^2 (\sigma_{ij}^k)^2} \right. \\ &\quad \left. + \sqrt{\sigma_M^2 \left( \sum_{j=1, j \neq i}^{\gamma} v_{ij}^K \beta_{ij}^K \right)^2 + \sum_{j=1, j \neq i}^{\gamma} (v_{ij}^K)^2 (\sigma_{ij}^K)^2} \right). \end{aligned} \quad (54)$$

If  $v_{ij}^k \equiv -1$  or  $v_{ij}^k \equiv 1$ ,  $\sigma_{ij}^k \equiv \sigma$ , and  $\beta_{ij}^k \equiv \beta$ , then

$$\mathbb{E}[E_i^{BN+MN}] = \varphi(0) \left( (\gamma - 1) \sqrt{\sigma_M^2 \beta^2 (K - 1)^2 + (K - 1) \sigma^2} \right. \quad (55)$$

$$\left. + \sqrt{\sigma_M^2 \beta^2 (\gamma - 1)^2 + (\gamma - 1) \sigma^2} \right). \quad (56)$$

Since  $\mathbb{E}[E_i^{MN}]/(\gamma - 1)$  is monotonically decreasing in  $\gamma$  for all  $\gamma > 0$  and

$$\lim_{\gamma \rightarrow \infty} \mathbb{E}[E_i^{MN}]/(\gamma - 1) = \varphi(0) \sigma_M |\beta|, \quad (57)$$

it holds that  $\mathbb{E}[E_i^{MN}]/(\gamma - 1) > \varphi(0) \sigma_M |\beta| = |\rho_{X,M}| \sigma_X \varphi(0)$  for all  $\gamma > 0$ .  $\square$

**PROPOSITION 3** ( $\gamma = K$ ). *Assume that  $v_{ij}^k \equiv 1$ ,  $\beta = \beta_{ij}^k$ , and  $\sigma = \sigma_{ij}^k$  for all  $j = 1, \dots, \gamma$ ,  $k = 1, \dots, K$ . Then,  $\mathbb{E}[E_i^{BN+MN}] > \mathbb{E}[E_i^{BN,K}]$  if  $K + 1 = \gamma$ .*

*Proof.* Assume that  $\gamma = K + 1$ . Then,

$$\mathbb{E}[E_i^{BN,K}] < \mathbb{E}[E_i^{BN+MN}] \quad (58)$$

$$\Leftrightarrow (\gamma - 1) \sqrt{\sigma_M^2 \beta^2 K^2 + K \sigma^2} < (\gamma - 1) \sqrt{\sigma_M^2 \beta^2 (K - 1)^2 + (K - 1) \sigma^2} \quad (59)$$

$$+ \sqrt{\sigma_M^2 \beta^2 (\gamma - 1)^2 + (\gamma - 1) \sigma^2} \quad (60)$$

$$\Leftrightarrow K \sqrt{\sigma_M^2 K^2 \beta^2 + K \sigma^2} < K \sqrt{\sigma_M^2 (K - 1)^2 \beta^2 + (K - 1) \sigma^2} + \sqrt{\sigma_M^2 K^2 \beta^2 + K \sigma^2} \quad (61)$$

$$\Leftrightarrow (K - 1)^2 \sigma_M^2 K^2 \beta^2 + (K - 1)^2 K \sigma^2 < K^2 \sigma_M^2 (K - 1)^2 \beta^2 + K^2 (K - 1) \sigma^2 \quad (62)$$

$$(K - 1)^2 K \sigma^2 < K^2 (K - 1) \sigma^2 \quad (63)$$

$$\Leftrightarrow K - 1 < K. \quad (64)$$

$\square$

**PROPOSITION 4** (Impact of correlation). *Assume that  $v_{ij}^k \equiv 1$ ,  $\sigma_{ij}^k \equiv \sigma$ , and  $\beta_{ij}^k \equiv \beta$  for all  $j = 1, \dots, \gamma$ ,  $k = 1, \dots, K$ .*

a) *If  $\rho_{X,M} = 0$ , then multilateral netting is more beneficial than bilateral netting if, and only if,*

$$K < \frac{\gamma^2}{4(\gamma - 1)}.$$

b) If  $\rho_{X,M} = 1$ , then  $\mathbb{E}[E_i^{BN+MN}] = \mathbb{E}[E_i^{BN,K}]$  for all  $\gamma > 0$  and  $K > 0$ .

*Proof.* a) Assume that  $\rho_{X,M} = 0$ . Then,

$$\mathbb{E}[E_i^{BN+MN}] < \mathbb{E}[E_i^{BN,K}] \quad (65)$$

$$\Leftrightarrow (\gamma - 1)\sigma_X\sqrt{K-1} + \sigma_X\sqrt{\gamma-1} < (\gamma - 1)\sigma_X\sqrt{K} \quad (66)$$

$$\Leftrightarrow \sqrt{\gamma-1}\sqrt{K} > 1 + \sqrt{\gamma-1}\sqrt{K-1} \quad (67)$$

$$\Leftrightarrow (\gamma - 1)K > 1 + 2\sqrt{\gamma-1}\sqrt{K-1} + (\gamma - 1)(K - 1) \quad (68)$$

$$\Leftrightarrow \gamma - 2 > 2\sqrt{\gamma-1}\sqrt{K-1} \quad (69)$$

$$\Leftrightarrow (\gamma - 2)^2 > 4(\gamma - 1)(K - 1) \quad (70)$$

$$\Leftrightarrow K < \frac{\gamma^2}{4(\gamma - 1)}. \quad (71)$$

or, equivalently (for  $\gamma > 1$ ),

$$\mathbb{E}[E_i^{BN+MN}] < \mathbb{E}[E_i^{BN,K}] \quad (72)$$

$$\Leftrightarrow K < \frac{\gamma^2}{4(\gamma - 1)} \quad (73)$$

$$\Leftrightarrow 0 < \gamma^2 - 4\gamma K + 4K \quad (74)$$

$$\Rightarrow \gamma > 2\sqrt{K}(\sqrt{K} + \sqrt{K-1}). \quad (75)$$

b) Assume that  $\rho_{X,M}^2 = 1$ . Then,

$$\mathbb{E}[E_i^{BN+MN}] = \mathbb{E}[E_i^{BN,K}] \quad (76)$$

$$\Leftrightarrow (\gamma - 1)\sigma_X(K - 1) + \sigma_X(\gamma - 1) = (\gamma - 1)\sigma_X K \quad (77)$$

$$\Leftrightarrow (\gamma - 1)(K - 1) + (\gamma - 1) = (\gamma - 1)K \quad (78)$$

$$\Leftrightarrow 0 = 0. \quad (79)$$

□

**PROPOSITION 5** (Impact of volatility). *Assume that  $v_{ij}^k \equiv 1$ ,  $\sigma_{ij}^k \equiv \sigma$ , and  $\beta_{ij}^k \equiv \beta$  for all  $j = 1, \dots, \gamma$ ,  $k = 1, \dots, K$ . The relative reduction of exposures due to multilateral netting of derivative*

class  $K$  is independent from contract volatility  $\sigma_X$  as well as systematic volatility  $\sigma_M$ .

*Proof.* The relative reduction of exposures due to multilateral netting of derivative class  $K$  is given by

$$\frac{\mathbb{E}[E_i^{BN+MN}] - \mathbb{E}[E_i^{BN,K}]}{\mathbb{E}[E_i^{BN,K}]} \quad (80)$$

$$= \frac{\varphi(0) \left( (\gamma - 1)\sigma_X \sqrt{K-1} \sqrt{1 + \rho_{X,M}^2(K-2)} + \sigma_X \sqrt{\gamma-1} \sqrt{1 + \rho_{X,M}^2(\gamma-2)} \right)}{\varphi(0)(\gamma-1)\sigma_X \sqrt{K} \sqrt{1 + \rho_{X,M}^2(K-1)}} - 1 \quad (81)$$

$$= \frac{\sqrt{\gamma-1} \sqrt{K-1} \sqrt{1 + \rho_{X,M}^2(K-2)} + \sqrt{1 + \rho_{X,M}^2(\gamma-2)}}{\sqrt{\gamma-1} \sqrt{K} \sqrt{1 + \rho_{X,M}^2(K-1)}} - 1, \quad (82)$$

which clearly is independent from  $\sigma_X$  and  $\sigma_M$ .  $\square$

**PROPOSITION 6** (Collateralized exposure in extreme events). *If  $v_{ij}^k \equiv 1$ ,  $\sigma_{ij}^k \equiv \sigma$ , and  $\beta_{ij}^k \equiv \beta$  for all  $j = 1, \dots, \gamma$ ,  $k = 1, \dots, K$ , then the collateralized counterparty risk exposure in extreme events with bilateral netting is given by*

$$\mathbb{E}[E_i^{BN,K} | M] = (\gamma - 1) \left( MK\beta\Phi \left( M \frac{\sqrt{K}\beta}{\sigma} \right) + \sqrt{K}\sigma\varphi \left( -M \frac{\sqrt{K}\beta}{\sigma} \right) \right) \quad (83)$$

and if class  $K$  is multilaterally netted, then the collateralized counterparty risk exposure is given by

$$\begin{aligned} & \mathbb{E}[E_i^{BN+MN} | M] \quad (84) \\ & = \mathbb{E}[E_i^{BN,K-1} | M] + M(\gamma-1)\beta\Phi \left( \frac{M\sqrt{\gamma-1}\beta}{\sigma} \right) + \sqrt{(\gamma-1)}\sigma\varphi \left( -\frac{M\sqrt{\gamma-1}\beta}{\sigma} \right). \end{aligned}$$

*Proof.* For the collateralized counterparty risk exposure with bilateral netting in extreme events

we define

$$\bar{\mu}_{ij|M}^{BN} = \mathbb{E} \left[ \sum_{k=1}^K X_{ij}^k \mid M \right] = M \sum_{k=1}^K v_{ij}^k \beta_{ij}^k \quad (85)$$

$$\left( \bar{\sigma}_{ij|M}^{BN} \right)^2 = \text{var} \left( \sum_{k=1}^K X_{ij}^k \right) = \text{var} \left( \sum_{k=1}^K (v_{ij}^k)^2 ((\beta_{ij}^k)^2 M + (\sigma_{ij}^k)^2 (\varepsilon_{ij}^k)^2) \right) \quad (86)$$

$$= \text{var} \left( \sum_{k=1}^K (v_{ij}^k)^2 (\sigma_{ij}^k)^2 (\varepsilon_{ij}^k)^2 \right) = \sum_{k=1}^K (v_{ij}^k)^2 (\sigma_{ij}^k)^2. \quad (87)$$

Then, the collateralized exposure to  $j$  in an extreme event is given by

$$\begin{aligned} \mathbb{E}[E_{ij}^{BN} \mid M] &= \bar{\mu}_{ij|M} \Phi(\bar{\mu}_{ij|M} / \bar{\sigma}_{ij|M}) + \bar{\sigma}_{ij|M} \varphi(-\bar{\mu}_{ij|M} / \bar{\sigma}_{ij|M}) \quad (88) \\ &= M \sum_{k=1}^K v_{ij}^k \beta_{ij}^k \Phi \left( M \frac{\sum_{k=1}^K v_{ij}^k \beta_{ij}^k}{\sqrt{\sum_{k=1}^K (v_{ij}^k)^2 (\sigma_{ij}^k)^2}} \right) + \sqrt{\sum_{k=1}^K (v_{ij}^k)^2 (\sigma_{ij}^k)^2} \varphi \left( -M \frac{\sum_{k=1}^K v_{ij}^k \beta_{ij}^k}{\sqrt{\sum_{k=1}^K (v_{ij}^k)^2 (\sigma_{ij}^k)^2}} \right). \end{aligned}$$

The total counterparty risk exposure is given by

$$\mathbb{E}[E_i^{BN,K} \mid M] = \sum_{j=1, j \neq i}^{\gamma} \mathbb{E}[E_{ij}^{BN} \mid M]. \quad (89)$$

The counterparty risk exposure in extreme events if class  $K$  is multilaterally netted is given by

$$\mathbb{E}[E_i^{BN+MN} \mid M] = \mathbb{E} \left[ \sum_{j=1, j \neq i}^{\gamma} \max \left( \sum_{k=1}^{K-1} X_{ij}^k, 0 \right) + \max \left( \sum_{j=1, j \neq i}^{\gamma} X_{ij}^K, 0 \right) \mid M \right]. \quad (90)$$

The expectation of the first term is given by  $\mathbb{E}[E_i^{BN, K-1} \mid M]$ , as defined above. For the second term we have that

$$\bar{\mu}_{i|M}^{MN} = \mathbb{E} \left[ \sum_{j=1, j \neq i}^{\gamma} X_{ij}^K \right] = M \sum_{j=1, j \neq i}^{\gamma} v_{ij}^K \beta_{ij}^K \quad (91)$$

$$\left( \bar{\sigma}_{i|M}^{MN} \right)^2 = \text{var} \left( \sum_{j=1, j \neq i}^{\gamma} X_{ij}^K \right) = \text{var} \left( \sum_{j=1, j \neq i}^{\gamma} v_{ij}^K \sigma_{ij}^K \varepsilon_{ij}^K \right) = \sum_{j=1, j \neq i}^{\gamma} (v_{ij}^K)^2 (\sigma_{ij}^K)^2. \quad (92)$$

Thus, the expectation of the second term is given by

$$\mathbb{E}[E_i^{MN}] = \bar{\mu}_{i|M}^{MN} \Phi(\bar{\mu}_{i|M}^{MN} / \bar{\sigma}_{i|M}^{MN}) + \bar{\sigma}_{i|M}^{MN} \varphi(-\bar{\mu}_{i|M}^{MN} / \bar{\sigma}_{i|M}^{MN}) \quad (93)$$

$$= M \sum_{j=1, j \neq i}^{\gamma} v_{ij}^K \beta_{ij}^K \Phi \left( \frac{M \sum_{j=1, j \neq i}^{\gamma} v_{ij}^K \beta_{ij}^K}{\sqrt{\sum_{j=1, j \neq i}^{\gamma} (v_{ij}^K)^2 (\sigma_{ij}^K)^2}} \right) \quad (94)$$

$$+ \sqrt{\sum_{j=1, j \neq i}^{\gamma} (v_{ij}^K)^2 (\sigma_{ij}^K)^2} \varphi \left( -\frac{M \sum_{j=1, j \neq i}^{\gamma} v_{ij}^K \beta_{ij}^K}{\sqrt{\sum_{j=1, j \neq i}^{\gamma} (v_{ij}^K)^2 (\sigma_{ij}^K)^2}} \right). \quad (95)$$

If  $v_{ij}^k \equiv 1$ ,  $\sigma_{ij}^k \equiv \sigma$ , and  $\beta_{ij}^k \equiv \beta$ , then

$$\mathbb{E}[E_i^{BN, K} | M] = (\gamma - 1) \left( MK\beta \Phi \left( M \frac{\sqrt{K}\beta}{\sigma} \right) + \sqrt{K}\sigma \varphi \left( -M \frac{\sqrt{K}\beta}{\sigma} \right) \right) \quad (96)$$

and

$$\begin{aligned} \mathbb{E}[E_i^{BN+MN} | M] & \quad (97) \\ &= \mathbb{E}[E_i^{BN, K-1} | M] + M(\gamma - 1)\beta \Phi \left( \frac{M\sqrt{\gamma-1}\beta}{\sigma} \right) + \sqrt{(\gamma-1)}\sigma \varphi \left( -\frac{M\sqrt{\gamma-1}\beta}{\sigma} \right). \end{aligned}$$

□

**PROPOSITION 7** (Cross-netting in extreme events). *If  $v_{ij}^k \equiv 1$ ,  $\sigma_{ij}^k \equiv \sigma$ , and  $\beta_{ij}^k \equiv \beta$  for all  $j = 1, \dots, \gamma$ ,  $k = 1, \dots, K$ , then the collateralized counterparty risk exposure with cross-netting across counterparties and  $\kappa \leq K$  derivative classes is in extreme events given by*

$$\mathbb{E}[E_i^{CN} | M] = M(\gamma - 1)\kappa\beta \Phi \left( \frac{M\sqrt{(\gamma-1)\kappa}\beta}{\sigma} \right) + \sqrt{(\gamma-1)\kappa}\sigma \varphi \left( -\frac{M\sqrt{(\gamma-1)\kappa}\beta}{\sigma} \right). \quad (98)$$

*Proof.* If derivative classes  $K - \kappa + 1, \dots, K$ ,  $\kappa \leq K$ , are netted across classes and counterparties, then the collateralized exposure in an extreme event is given by

$$\mathbb{E}[E_i^{CN} | M] = \mathbb{E} \left[ \max \left( \sum_{j=1, j \neq i}^{\gamma} \sum_{k=K-\kappa+1}^K X_{ij}^k, 0 \right) | M \right]. \quad (99)$$



Let

$$\bar{\mu}_{i|M}^{CN} = \mathbb{E} \left[ \sum_{j=1, j \neq i}^{\gamma} \sum_{k=K-\kappa+1}^K X_{ij}^K \right] = M \sum_{j=1, j \neq i}^{\gamma} \sum_{k=K-\kappa+1}^K v_{ij}^k \beta_{ij}^k \quad (100)$$

$$(\bar{\sigma}_{i|M}^{CN})^2 = \text{var} \left( \sum_{j=1, j \neq i}^{\gamma} \sum_{k=K-\kappa+1}^K X_{ij}^K \right) \quad (101)$$

$$= \text{var} \left( \sum_{j=1, j \neq i}^{\gamma} \sum_{k=K-\kappa+1}^K v_{ij}^k \sigma_{ij}^k \varepsilon_{ij}^K \right) = \sum_{j=1, j \neq i}^{\gamma} \sum_{k=K-\kappa+1}^K (v_{ij}^k)^2 (\sigma_{ij}^k)^2. \quad (102)$$

Thus, the expectation is given by

$$\mathbb{E}[E_i^{CN} | M] = \bar{\mu}_{i|M}^{CN} \Phi(\bar{\mu}_{i|M}^{CN} / \bar{\sigma}_{i|M}^{CN}) + \bar{\sigma}_{i|M}^{CN} \varphi(-\bar{\mu}_{i|M}^{CN} / \bar{\sigma}_{i|M}^{CN}) \quad (103)$$

$$= M \sum_{j=1, j \neq i}^{\gamma} \sum_{k=K-\kappa+1}^K v_{ij}^k \beta_{ij}^k \Phi \left( \frac{M \sum_{j=1, j \neq i}^{\gamma} \sum_{k=K-\kappa+1}^K v_{ij}^k \beta_{ij}^k}{\sqrt{\sum_{j=1, j \neq i}^{\gamma} \sum_{k=K-\kappa+1}^K (v_{ij}^k)^2 (\sigma_{ij}^k)^2}} \right) \quad (104)$$

$$+ \sqrt{\sum_{j=1, j \neq i}^{\gamma} \sum_{k=K-\kappa+1}^K (v_{ij}^k)^2 (\sigma_{ij}^k)^2} \varphi \left( -\frac{M \sum_{j=1, j \neq i}^{\gamma} \sum_{k=K-\kappa+1}^K v_{ij}^k \beta_{ij}^k}{\sqrt{\sum_{j=1, j \neq i}^{\gamma} \sum_{k=K-\kappa+1}^K (v_{ij}^k)^2 (\sigma_{ij}^k)^2}} \right). \quad (105)$$

If  $v_{ij}^k \equiv 1$ ,  $\sigma_{ij}^k \equiv \sigma$ , and  $\beta_{ij}^k \equiv \beta$ , then

$$\mathbb{E}[E_i^{MN} | M] \quad (106)$$

$$= M(\gamma - 1)\kappa\beta\Phi \left( \frac{M\sqrt{(\gamma - 1)\kappa\beta}}{\sigma} \right) + \sqrt{(\gamma - 1)\kappa}\sigma\varphi \left( -\frac{M\sqrt{(\gamma - 1)\kappa\beta}}{\sigma} \right). \quad (107)$$

□

**PROPOSITION 8** (Uncollateralized counterparty risk exposure). *Assume that  $v_{ij}^k \equiv 1$ ,  $\beta \equiv \beta_{ij}^k$ , and  $\sigma \equiv \sigma_{ij}^k$  for all  $j = 1, \dots, \gamma$ ,  $k = 1, \dots, K$ . Then, the uncollateralized counterparty risk exposure with bilateral netting equals*

$$\mathbb{E} \left[ \tilde{E}_i^{BN, K} \right] = (\gamma - 1) \sqrt{\sigma_M^2 K^2 \beta^2 + K \sigma^2 \xi(\alpha_{BN})}, \quad (108)$$

and with class- $K$  multilateral netting it is

$$\mathbb{E} \left[ \tilde{E}_i^{BN+MN} \right] = \sqrt{\sigma_M^2 (\gamma - 1)^2 \beta^2 + (\gamma - 1) \sigma^2 \xi(\alpha_{MN})} \quad (109)$$

$$+ (\gamma - 1) \sqrt{\sigma_M^2 (K - 1)^2 \beta^2 + (K - 1) \sigma^2 \xi(\alpha_{BN})}, \quad (110)$$

where  $\xi(\alpha) = (1 - \alpha) \Phi^{-1}(1 - \alpha) + \varphi(\Phi^{-1}(\alpha))$ .

*Proof.* First, we consider the bilateral case. Define  $\tilde{E}_{ij}^{BN,K} = \max\left(\sum_{k=1}^K X_{ij}^k - C^{BN,K}, 0\right)$ . It is

$$\bar{\sigma}_{ij}^2 = \text{var} \left( \sum_{k=1}^K X_{ij}^k \right) = \sigma_M^2 \left( \sum_{k=1}^K v_{ij}^k \beta_{ij}^k \right)^2 + \sum_{k=1}^K (v_{ij}^k \sigma_{ij}^k)^2, \quad (111)$$

and

$$\bar{\mu}_{ij} = -C_{ij}^{BN,K} = -\Phi^{-1}(\alpha_{BN}) \bar{\sigma}_{ij}, \quad (112)$$

and thus

$$\begin{aligned} \mathbb{E} \left[ \tilde{E}_i^{BN,K} \right] &= \sum_{j=1, j \neq i}^{\gamma} \left( -\Phi^{-1}(\alpha_{BN}) \bar{\sigma}_{ij} \Phi \left( \frac{-\Phi^{-1}(\alpha_{BN}) \bar{\sigma}_{ij}}{\bar{\sigma}_{ij}} \right) + \bar{\sigma}_{ij} \varphi \left( -\frac{-\Phi^{-1}(\alpha_{BN}) \bar{\sigma}_{ij}}{\bar{\sigma}_{ij}} \right) \right) \quad (113) \end{aligned}$$

$$= \sum_{j=1, j \neq i}^{\gamma} \left( -\Phi^{-1}(\alpha_{BN}) \bar{\sigma}_{ij} \Phi(\Phi^{-1}(1 - \alpha_{BN})) + \bar{\sigma}_{ij} \varphi(\Phi^{-1}(\alpha_{BN})) \right). \quad (114)$$

If entities are homogeneous, then the exposure equals

$$\mathbb{E} \left[ \tilde{E}_i^{BN,K} \right] = (\gamma - 1) \bar{\sigma} \left( (1 - \alpha_{BN}) \Phi^{-1}(1 - \alpha_{BN}) + \varphi(\Phi^{-1}(\alpha_{BN})) \right) \quad (115)$$

$$= (\gamma - 1) \sqrt{\sigma_M^2 K^2 \beta^2 + K \sigma^2} \left( (1 - \alpha_{BN}) \Phi^{-1}(1 - \alpha_{BN}) + \varphi(\Phi^{-1}(\alpha_{BN})) \right). \quad (116)$$

Second, we consider the multilateral case, where derivative class  $K$  is multilaterally netted. It

is  $\tilde{E}_i^{BN+MN} = \max \left( \sum_{j=1, j \neq i}^{\gamma} X_{ij}^K - \frac{v_{ij}^K}{\sum_{h=1, h \neq j}^{\gamma} v_{hj}^K} C_j^{MN}, 0 \right)$  the exposure in derivative class  $K$ . Let

$$\bar{\sigma}_i^2 = \text{var} \left( \sum_{j=1}^{\gamma} X_{ij}^K \right) = \sigma_M^2 \left( \sum_{j=1, j \neq i}^{\gamma} v_{ij}^K \beta_{ij}^K \right)^2 + \sum_{j=1, j \neq i}^{\gamma} (v_{ij}^K \sigma_{ij}^K)^2, \quad (117)$$

and

$$\bar{\mu}_i = - \sum_{j=1, j \neq i}^{\gamma} \frac{v_{ij}^K}{\sum_{h=1, h \neq j}^{\gamma} v_{hj}^K} C_j^{MN}, \quad (118)$$

and

$$C_j^{MN} = \Phi^{-1}(\alpha_{MN}) \sqrt{\sigma_M^2 \left( \sum_{i=1, i \neq j}^{\gamma} v_{ij}^K \beta_{ij}^K \right)^2 + \sum_{i=1, i \neq j}^{\gamma} (v_{ij}^K \sigma_{ij}^K)^2}. \quad (119)$$

Assuming that  $v_{ij}^k \equiv 1$ ,  $\beta \equiv \beta_{ij}^k$ , and  $\sigma \equiv \sigma_{ij}^k$ , the collateral is given by

$$C_j^{MN} = \Phi^{-1}(\alpha_{MN}) \sqrt{\sigma_M^2 (\gamma - 1)^2 \beta^2 + (\gamma - 1) \sigma^2} = \Phi^{-1}(\alpha_{MN}) \bar{\sigma}_j, \text{ and the uncollateralized exposure}$$

is

$$\mathbb{E} \left[ \tilde{E}_i^{BN+MN} \right] = \bar{\mu}_i \Phi(\bar{\mu}_i / \bar{\sigma}_i) + \bar{\sigma}_i \varphi(-\bar{\mu}_i / \bar{\sigma}_i) \quad (120)$$

$$= - \sum_{j=1, j \neq i}^{\gamma} \frac{1}{\gamma - 1} \Phi^{-1}(\alpha_{MN}) \bar{\sigma}_j \Phi \left( \frac{-\sum_{j=1, j \neq i}^{\gamma} \frac{1}{\gamma - 1} \Phi^{-1}(\alpha_{MN}) \bar{\sigma}_j}{\bar{\sigma}_i} \right) \quad (121)$$

$$+ \bar{\sigma}_i \varphi \left( -\frac{-\sum_{j=1, j \neq i}^{\gamma} \frac{1}{\gamma - 1} \Phi^{-1}(\alpha_{MN}) \bar{\sigma}_j}{\bar{\sigma}_i} \right) \quad (122)$$

$$= \bar{\sigma} \left( \Phi^{-1}(1 - \alpha_{MN})(1 - \alpha_{MN}) + \varphi(\Phi^{-1}(1 - \alpha_{MN})) \right) \quad (123)$$

$$= \sqrt{\sigma_M^2 (\gamma - 1)^2 \beta^2 + (\gamma - 1) \sigma^2} \left( \Phi^{-1}(1 - \alpha_{MN})(1 - \alpha_{MN}) + \varphi(\Phi^{-1}(1 - \alpha_{MN})) \right). \quad (124)$$

As before  $\mathbb{E}[\tilde{E}_i^{BN+MN}] = \mathbb{E}[\tilde{E}_i^{BN, K-1}] + \mathbb{E}[\tilde{E}_i^{MN}]$ .  $\square$

**PROPOSITION 9.** *Assume that counterparties are homogeneous.*

- 1) *If  $\alpha = \alpha_{BN} = \alpha_{MN}$ , the benefit of multilateral netting is independent from the margin level  $\alpha$ .*

- 2) The smaller margin requirements for multilateral compared to bilateral netting are, the smaller is the benefit of multilateral netting, and vice versa.

*Proof.*

- 1) Assume that  $\alpha = \alpha_{BN} = \alpha_{MN}$ . Then,

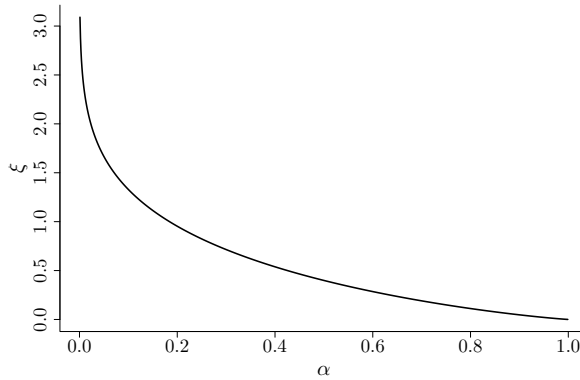
$$\frac{\mathbb{E} \left[ \tilde{E}_i^{BN+MN} \right] - \mathbb{E} \left[ \tilde{E}_i^{BN,K} \right]}{\mathbb{E} \left[ \tilde{E}_i^{BN,K} \right]} \quad (125)$$

$$= \frac{\sqrt{\sigma_M^2(\gamma-1)^2\beta^2 + (\gamma-1)\sigma^2} + (\gamma-1)\sqrt{\sigma_M^2 K^2\beta^2 + K\sigma^2}}{(\gamma-1)\sqrt{\sigma_M^2 K^2\beta^2 + K\sigma^2}} - 1. \quad (126)$$

- 2) As Figure 14 shows,  $\xi(\alpha)$  is decreasing with the margin requirement  $\alpha$ . Thus, the smaller  $\frac{\alpha_{MN}}{\alpha_{BN}}$ , the larger are  $\frac{\xi(\alpha_{MN})}{\xi(\alpha_{BN})}$  and

$$\frac{E \left[ \tilde{E}_i^{BN+MN} \right]}{E \left[ \tilde{E}_i^{BN,K} \right]} = \frac{\sqrt{\sigma_M^2(\gamma-1)^2\beta^2 + (\gamma-1)\sigma^2 \frac{\xi(\alpha_{MN})}{\xi(\alpha_{BN})}} + (\gamma-1)\sqrt{\sigma_M^2 K^2\beta^2 + K\sigma^2}}{(\gamma-1)\sqrt{\sigma_M^2 K^2\beta^2 + K\sigma^2}}, \quad (127)$$

and, thus, the smaller is the benefit of multilateral netting.



**Figure 14.** Margin adjustment factor  $\xi(\alpha) = (1 - \alpha)\Phi^{-1}(1 - \alpha) + \varphi(-\Phi^{-1}(\alpha))$ .

□

**PROPOSITION 10** (Uncollateralized counterparty risk exposure in extreme events.). *If  $v_{ij}^k \equiv 1$ ,  $\sigma_{ij}^k \equiv \sigma$ , and  $\beta_{ij}^k \equiv \beta$  for all  $j = 1, \dots, \gamma$ ,  $k = 1, \dots, K$ , then the uncollateralized counterparty risk*

exposure with bilateral netting extreme event is given by

$$\mathbb{E}[\tilde{E}_i^{BN,K} | M] = (\gamma - 1) \left[ (M\beta K - C^{BN,K}) \Phi \left( \frac{M\beta K - C^{BN,K}}{\sigma\sqrt{K}} \right) \right. \quad (128)$$

$$\left. + \sigma\sqrt{K} \varphi \left( -\frac{M\beta K - C^{BN,K}}{\sigma\sqrt{K}} \right) \right] \quad (129)$$

and with multilateral netting of class  $K$  it is given by

$$\mathbb{E}[\tilde{E}_i^{BN+MN} | M] = \mathbb{E}[\tilde{E}_i^{BN,K-1} | M] \quad (130)$$

$$+ (M(\gamma - 1)\beta - C^{MN}) \Phi \left( \frac{M(\gamma - 1)\beta - C^{MN}}{\sigma\sqrt{\gamma - 1}} \right) + \sigma\sqrt{\gamma - 1} \varphi \left( -\frac{M(\gamma - 1)\beta - C^{MN}}{\sigma\sqrt{\gamma - 1}} \right), \quad (131)$$

where  $C^{BN,K} = \Phi^{-1}(\alpha_{BN})\sqrt{\beta^2\sigma_M^2 + K\sigma^2}$  and  $C^{MN} = \Phi^{-1}(\alpha_{MN})\sqrt{\beta^2\sigma_M^2 + (\gamma - 1)\sigma^2}$ .

*Proof.* Define the bilateral collateral  $C_{ij}^{BN,K} = \Phi^{-1}(\alpha_{BN})\sqrt{\left(\sum_{k=1}^K v_{ij}^k \beta_{ij}^k\right)^2 \sigma_M^2 + \sum_{k=1}^K \left(v_{ij}^k \sigma_{ij}^k\right)^2}$ .

For the uncollateralized exposure in an extreme event with bilateral netting we define

$$\bar{\mu}_{ij|M}^{BN} = \mathbb{E} \left[ \sum_{k=1}^K X_{ij}^k - C_{ij}^{BN,K} | M \right] = M \sum_{k=1}^K v_{ij}^k \beta_{ij}^k - C_{ij}^{BN,K} \quad (132)$$

$$\left(\bar{\sigma}_{ij|M}^{BN}\right)^2 = \text{var} \left( \sum_{k=1}^K X_{ij}^k \right) = \text{var} \left( \sum_{k=1}^K \left( v_{ij}^k \right)^2 \left( \left( \beta_{ij}^k \right)^2 M + \left( \sigma_{ij}^k \right)^2 \left( \varepsilon_{ij}^k \right)^2 \right) \right) \quad (133)$$

$$= \text{var} \left( \sum_{k=1}^K \left( v_{ij}^k \right)^2 \left( \sigma_{ij}^k \right)^2 \left( \varepsilon_{ij}^k \right)^2 \right) = \sum_{k=1}^K \left( v_{ij}^k \right)^2 \left( \sigma_{ij}^k \right)^2. \quad (134)$$

Then, the uncollateralized exposure of entity  $i$  with  $j$  is given by

$$\mathbb{E}[\tilde{E}_{ij}^{BN} | M] = \bar{\mu}_{ij|M} \Phi(\bar{\mu}_{ij|M} / \bar{\sigma}_{ij|M}) + \bar{\sigma}_{ij|M} \varphi(-\bar{\mu}_{ij|M} / \bar{\sigma}_{ij|M}) \quad (135)$$

$$= \left( M \sum_{k=1}^K v_{ij}^k \beta_{ij}^k - C_{ij}^{BN,K} \right) \Phi \left( \frac{M \sum_{k=1}^K v_{ij}^k \beta_{ij}^k - C_{ij}^{BN,K}}{\sqrt{\sum_{k=1}^K \left( v_{ij}^k \right)^2 \left( \sigma_{ij}^k \right)^2}} \right) \quad (136)$$

$$+ \sqrt{\sum_{k=1}^K \left( v_{ij}^k \right)^2 \left( \sigma_{ij}^k \right)^2} \varphi \left( -\frac{M \sum_{k=1}^K v_{ij}^k \beta_{ij}^k - C_{ij}^{BN,K}}{\sqrt{\sum_{k=1}^K \left( v_{ij}^k \right)^2 \left( \sigma_{ij}^k \right)^2}} \right).$$

The total uncollateralized exposure is given by

$$\mathbb{E}[\tilde{E}_i^{BN,K} | M] = \sum_{j=1, j \neq i}^{\gamma} \mathbb{E}[\tilde{E}_{ij}^{BN} | M]. \quad (137)$$

The uncollateralized counterparty risk exposure in derivative class  $K$  if it is multilaterally netted is given by

$$\mathbb{E}[\tilde{E}_i^{MN} | M] = \mathbb{E} \left[ \max \left( \sum_{j=1, j \neq i}^{\gamma} X_{ij}^K - C_j^{MN}, 0 \right) | M \right], \quad (138)$$

where the multilateral collateral is given by

$$C_j^{MN} = \Phi^{-1}(\alpha_{MN}) \sqrt{\sigma_M^2 \left( \sum_{i=1, i \neq j}^{\gamma} v_{ij}^k \beta_{ij}^k \right)^2 + \sum_{i=1, i \neq j}^{\gamma} (v_{ij}^k \sigma_{ij}^k)^2}. \quad (139)$$

Define

$$\bar{\mu}_{i|M}^{MN} = \mathbb{E} \left[ \sum_{j=1, j \neq i}^{\gamma} X_{ij}^K - C_j^{MN} \right] = M \sum_{j=1, j \neq i}^{\gamma} v_{ij}^K \beta_{ij}^K - C_j^{MN} \quad (140)$$

$$(\bar{\sigma}_{i|M}^{MN})^2 = \text{var} \left( \sum_{j=1, j \neq i}^{\gamma} X_{ij}^K \right) = \text{var} \left( \sum_{j=1, j \neq i}^{\gamma} v_{ij}^K \sigma_{ij}^K \varepsilon_{ij}^K \right) = \sum_{j=1, j \neq i}^{\gamma} (v_{ij}^K)^2 (\sigma_{ij}^K)^2. \quad (141)$$

Thus,

$$\mathbb{E}[\tilde{E}_i^{MN} | M] = \bar{\mu}_{i|M}^{MN} \Phi(\bar{\mu}_{i|M}^{MN} / \bar{\sigma}_{i|M}^{MN}) + \bar{\sigma}_{i|M}^{MN} \varphi(-\bar{\mu}_{i|M}^{MN} / \bar{\sigma}_{i|M}^{MN}) \quad (142)$$

$$= \left( M \sum_{j=1, j \neq i}^{\gamma} v_{ij}^K \beta_{ij}^K - C_j^{MN} \right) \Phi \left( \frac{M \sum_{j=1, j \neq i}^{\gamma} v_{ij}^K \beta_{ij}^K - C_j^{MN}}{\sqrt{\sum_{j=1, j \neq i}^{\gamma} (v_{ij}^K)^2 (\sigma_{ij}^K)^2}} \right) \quad (143)$$

$$+ \sqrt{\sum_{j=1, j \neq i}^{\gamma} (v_{ij}^K)^2 (\sigma_{ij}^K)^2} \varphi \left( -\frac{M \sum_{j=1, j \neq i}^{\gamma} v_{ij}^K \beta_{ij}^K - C_j^{MN}}{\sqrt{\sum_{j=1, j \neq i}^{\gamma} (v_{ij}^K)^2 (\sigma_{ij}^K)^2}} \right). \quad (144)$$

For  $v_i^* = \sum_{k=1}^K v_{ij}^k$ ,  $v_{i*}^K = \sum_{j=1, j \neq i}^{\gamma} v_{ij}^K$  and  $(v_{ij}^k)^2 = 1$ ,  $\sigma_{ij}^k \equiv \sigma$ , and  $\beta_{ij}^k \equiv \beta$ , the exposure is given

by

$$\mathbb{E}[\tilde{E}_i^{BN,K} | M] = (\gamma - 1) \left[ (M\beta v_i^* - C^{BN,K}) \Phi \left( \frac{M\beta v_i^* - C^{BN,K}}{\sigma\sqrt{K}} \right) \right. \quad (145)$$

$$\left. + \sigma\sqrt{K} \varphi \left( -\frac{M\beta v_i^* - C^{BN,K}}{\sigma\sqrt{K}} \right) \right] \quad (146)$$

and

$$\begin{aligned} & \mathbb{E}[\tilde{E}_i^{BN+MN} | M] \\ &= \mathbb{E}[\tilde{E}_i^{BN,K-1} | M] \end{aligned} \quad (147)$$

$$+ (Mv_{i*}^K \beta - C^{MN}) \Phi \left( \frac{Mv_{i*}^K \beta - C^{MN}}{\sigma\sqrt{\gamma-1}} \right) + \sigma\sqrt{\gamma-1} \varphi \left( -\frac{Mv_{i*}^K \beta - C^{MN}}{\sigma\sqrt{\gamma-1}} \right). \quad (148)$$

If  $v_{ij}^k \equiv 1$ , then  $v_{i*} = \gamma - 1$  and  $v_i^* = K$ .  $\square$

**PROPOSITION 11** (Uncollateralized exposure with cross-netting.). *Assume that  $v_{ij}^k \equiv 1$ ,  $\beta \equiv \beta_{ij}^k$ , and  $\sigma \equiv \sigma_{ij}^k$  for all  $j = 1, \dots, \gamma$ ,  $k = 1, \dots, K$ . Then, the uncollateralized counterparty risk exposure with cross-netting across  $\gamma - 1$  counterparties and derivative classes  $K - \kappa + 1$  to  $K$  is given by*

$$\mathbb{E}[\tilde{E}_i^{CN,\kappa}] = \sqrt{\sigma_M^2 \kappa^2 (\gamma - 1)^2 \beta^2 + \kappa (\gamma - 1) \sigma^2 \xi(\alpha_{CN})} \quad (149)$$

where  $\xi(\alpha) = (1 - \alpha)\Phi^{-1}(1 - \alpha) + \varphi(\Phi^{-1}(\alpha))$  is the margin adjustment.

*Proof.* The counterparty risk exposure in (cross-netted) derivative classes  $K - \kappa + 1, \dots, K$  is  $\tilde{E}_i^{CN,\kappa} = \max \left( \sum_{j=1, j \neq i}^{\gamma} \sum_{k=K-\kappa+1}^K X_{ij}^k - \frac{\sum_{k=K-\kappa+1}^K v_{ij}^k}{\sum_{k=K-\kappa+1}^K \sum_{h=1, h \neq j}^{\gamma} v_{hj}^k} C_j^{CN,\kappa}, 0 \right)$ , where the collateral is given by

$$C_j^{CN,\kappa} = \Phi^{-1}(\alpha_{CN}) \sqrt{\sigma_M^2 \left( \sum_{k=K-\kappa+1}^K \sum_{i=1, j \neq i}^{\gamma} v_{ij}^k \beta_{ij}^k \right)^2 + \sum_{k=K-\kappa+1}^K \sum_{i=1, j \neq i}^{\gamma} (v_{ij}^k \sigma_{ij}^k)^2} \quad (150)$$

Let

$$\bar{\sigma}_i^2 = \text{var} \left( \sum_{k=K-\kappa+1}^K \sum_{j=1}^{\gamma} X_{ij}^k \right) \quad (151)$$

$$= \sigma_M^2 \left( \sum_{k=K-\kappa+1}^K \sum_{j=1, j \neq i}^{\gamma} v_{ij}^k \beta_{ij}^k \right)^2 + \sum_{k=K-\kappa+1}^K \sum_{j=1, j \neq i}^{\gamma} (v_{ij}^k \sigma_{ij}^k)^2, \quad (152)$$

and

$$\bar{\mu}_i = - \sum_{j=1, j \neq i}^{\gamma} \frac{\sum_{k=K-\kappa+1}^K v_{ij}^k}{\sum_{h=1, h \neq j}^{\gamma} \sum_{k=K-\kappa+1}^K v_{hj}^k} C_j^{CN, \kappa}. \quad (153)$$

Assuming that  $v_{ij}^k \equiv 1$ ,  $\beta \equiv \beta_{ij}^k$ , and  $\sigma \equiv \sigma_{ij}^k$ , the collateral equals

$C_j^{CN, \kappa} = \Phi^{-1}(\alpha_{MN}) \sqrt{\sigma_M^2 \kappa^2 (\gamma - 1)^2 \beta^2 + \kappa (\gamma - 1) \sigma^2} = \Phi^{-1}(\alpha_{CN}) \bar{\sigma}_j$  and the uncollateralized exposure is

$$\mathbb{E} \left[ \tilde{E}_i^{CN, \kappa} \right] = \bar{\mu}_i \Phi(\bar{\mu}_i / \bar{\sigma}_i) + \bar{\sigma}_i \varphi(-\bar{\mu}_i / \bar{\sigma}_i) \quad (154)$$

$$= - \sum_{j=1, j \neq i}^{\gamma} \frac{1}{\gamma - 1} \Phi^{-1}(\alpha_{CN}) \bar{\sigma}_j \Phi \left( \frac{-\sum_{j=1, j \neq i}^{\gamma} \frac{1}{\gamma - 1} \Phi^{-1}(\alpha_{CN}) \bar{\sigma}_j}{\bar{\sigma}_i} \right) \quad (155)$$

$$+ \bar{\sigma}_i \varphi \left( -\frac{-\sum_{j=1, j \neq i}^{\gamma} \frac{1}{\gamma - 1} \Phi^{-1}(\alpha_{CN}) \bar{\sigma}_j}{\bar{\sigma}_i} \right) \quad (156)$$

$$= \bar{\sigma} \left( \Phi^{-1}(1 - \alpha_{CN})(1 - \alpha_{CN}) + \varphi(\Phi^{-1}(1 - \alpha_{CN})) \right) \quad (157)$$

$$= \sqrt{\sigma_M^2 \kappa^2 (\gamma - 1)^2 \beta^2 + \kappa (\gamma - 1) \sigma^2} \left( \Phi^{-1}(1 - \alpha_{CN})(1 - \alpha_{CN}) + \varphi(\Phi^{-1}(\alpha_{CN})) \right). \quad (158)$$

□

**PROPOSITION 12** (Bilateral realized counterparty risk exposure). *Assume that  $\sigma_{ij}^k \equiv \sigma$ ,  $\beta_{ij}^k \equiv \beta$ ,  $\sigma_{A_i} \equiv \sigma_A$ , and  $\beta_{A_i} \equiv \beta_A$ . Conditional on the state of the economy  $M$ , the realized counterparty*



risk exposure of counterparty  $i$  with bilateral netting is given by

$$\begin{aligned} \mathbb{E}[E_i^{*BN,K} | M] &= \sum_{j=1, j \neq i}^{\gamma} \Phi \left( \sqrt{\sigma_A^2 + \beta_A^2 \sigma_M^2} \Phi^{-1}(pd_j) - \beta_A M \right) \\ &\times \left( (M\beta v_{ij}^* - C^{BN,K}) \Phi \left( \frac{\beta M v_{ij}^* - C^{BN,K}}{\sqrt{K}\sigma} \right) + \sqrt{K}\sigma \varphi \left( -\frac{\beta M v_{ij}^* - C^{BN,K}}{\sqrt{K}\sigma} \right) \right), \end{aligned} \quad (159)$$

where  $v_{ij}^* = \sum_{k=1}^K v_{ij}^k$  and total collateral provided by  $j$  to  $i$  is given by

$$C^{BN,K} = \Phi^{-1}(\alpha) \sqrt{\sigma_M^2 K^2 \beta^2 + K\sigma^2}.$$

*Proof.* Conditional on  $M$ ,  $D_j$  and  $\max \left( \sum_{k=1}^K X_{ij}^k - C_{ij}^{BN,K}, 0 \right)$  are independent. The unconditional default intensity is given by

$$\bar{d}_j = \frac{\log B_j}{\sqrt{\sigma_{A_j}^2 + \beta_{A,j}^2 \sigma_M^2}} + \frac{\sqrt{\sigma_{A_j}^2 + \beta_{A,j}^2 \sigma_M^2}}{2} = \Phi^{-1}(pd_j) \quad (160)$$

$$\Leftrightarrow \log B_j = b_j = \sqrt{\sigma_{A_j}^2 + \beta_{A,j}^2 \sigma_M^2} \Phi^{-1}(pd_j) - \frac{\sigma_{A_j}^2 + \beta_{A,j}^2 \sigma_M^2}{2}. \quad (161)$$

The conditional probability of default is given by

$$\mathbb{P}(D_j = 1 | M) = \mathbb{P}(W_j(1) < \bar{d}_{j|M}) = \Phi(\bar{d}_{j|M}), \quad (162)$$

where

$$\bar{d}_{j|M} = \log B_j + \frac{\sigma_{A_j}^2 + \beta_{A,j}^2 \sigma_M^2}{2} - \beta_{A,j} M \quad (163)$$

$$= \sqrt{\sigma_{A_j}^2 + \beta_{A,j}^2 \sigma_M^2} \Phi^{-1}(pd_j) - \beta_{A,j} M. \quad (164)$$

Conditional on  $M$ , it holds that  $\sum_{k=1}^K X_{ij}^k - C_{ij}^{BN,K} \sim \mathcal{N}(\mu_{ij|M}^k, (\sigma_{ij|M}^k)^2)$  with

$$\mu_{ij|M}^k = M \sum_{k=1}^K v_{ij}^k \beta_{ij}^k - C_{ij}^{BN,K} \quad (165)$$

$$(\sigma_{ij|M}^k)^2 = \sum_{k=1}^K (v_{ij}^k \sigma_{ij}^k)^2. \quad (166)$$

Thus,

$$\mathbb{E} \left[ \max \left( X_{ij}^k - C_{ij}^{BN,K}, 0 \right) \right] = \mu_{ij|M}^k \Phi \left( \frac{\mu_{ij|M}^k}{\sigma_{ij|M}^k} \right) + \sigma_{ij|M}^k \varphi \left( -\frac{\mu_{ij|M}^k}{\sigma_{ij|M}^k} \right). \quad (167)$$

Assuming that  $\beta_{ij}^k \equiv \beta$ ,  $\sigma_{A_i} \equiv \sigma_A$ ,  $\beta_{A_i} \equiv \beta_A$ ,  $\sigma_{ij}^k \equiv \sigma$ , and  $|v_{ij}^k| = 1$ , it holds that  $C_{ij}^{BN,K} \equiv C^{BN,K}$ ,  $\mu_{ij|M}^k \equiv M\beta v_{ij}^* - C^{BN,K}$ , and  $(\sigma_{ij|M}^k)^2 \equiv K\sigma^2$ , where  $v_{ij}^* = \sum_{k=1}^K v_{ij}^k$ . Eventually, the realized counterparty risk exposure with bilateral clearing conditional on  $M$  is given by

$$\begin{aligned} \mathbb{E}[E_i^{*BN,K} | M] &= \sum_{j=1, j \neq i}^{\gamma} \Phi \left( \sqrt{\sigma_A^2 + \beta_A^2 \sigma_M^2} \Phi^{-1}(pd_j) - \beta_A M \right) \\ &\times \left( (M\beta v_{ij}^* - C_{ij}^{BN,K}) \Phi \left( \frac{\beta M v_{ij}^* - C_{ij}^{BN,K}}{\sqrt{K}\sigma} \right) + \sqrt{K}\sigma \varphi \left( -\frac{\beta M v_{ij}^* - C_{ij}^{BN,K}}{\sqrt{K}\sigma} \right) \right), \end{aligned} \quad (168)$$

where the total collateral provided by each counterparty  $j$  to counterparty  $i$  is

$$C_{ij}^{BN,K} = \Phi^{-1}(\alpha_{BN}) \sqrt{\beta^2 \sigma_M^2 (v_{ij}^*)^2 + K\sigma}. \quad \square$$

**PROPOSITION 13** (Multilateral realized counterparty risk exposure.). *Assume that  $\sigma_{ij}^k \equiv \sigma$ ,  $\beta_{ij}^k \equiv \beta$ , and  $|v_{ij}| = 1$ . Conditional on the state of the economy  $M$  and default states  $D_1, \dots, D_\gamma$  the realized counterparty risk exposure in the centrally cleared derivative class  $K$  is given by*

$$\mathbb{E}[\bar{L}^{CCP} | M, D] = \sum_{j=1}^{\gamma} D_j \left( \mu_{j|M} \Phi \left( \frac{\mu_{j|M}}{\sigma_{j|M}} \right) + \sigma_{j|M} \varphi \left( -\frac{\mu_{j|M}}{\sigma_{j|M}} \right) \right), \quad (169)$$

where  $\mu_{j|M} = M\beta v_{*j}^K - \Phi^{-1}(\alpha_{MN}) \sqrt{\sigma_M^2 (v_{*j}^K)^2 \beta^2 + (\gamma - 1)\sigma^2}$ ,  $\sigma_{j|M}^2 = \sigma^2(\gamma - 1)$ , and  $v_{*j}^K = \sum_{g=1, g \neq j}^{\gamma} v_{gj}^K$  is the net position of  $j$ 's trades in derivative class  $K$ .

*Proof.* The expected total loss of the CCP given the state of the economy and conditional on defaults  $D = (D_1, \dots, D_\gamma)$  of clearing members is given by

$$\mathbb{E}[\bar{L}^{CCP} | M, D] = \sum_{j=1}^{\gamma} D_j \mathbb{E} \left[ \max \left( \sum_{\substack{g=1, \\ g \neq j}}^{\gamma} X_{gj}^K - C_j^{MN}, 0 \right) | M, D \right], \quad (170)$$

where the collateral is defined as previously. Define by

$$\mu_{j|M} = \mathbb{E} \left[ \sum_{\substack{g=1, \\ g \neq j}}^{\gamma} X_{gj}^K - C_j^{MN} \right] = M \sum_{\substack{g=1, \\ g \neq j}}^{\gamma} v_{gj}^K \beta_{gj}^K - C_j^{MN} \quad (171)$$

$$\sigma_{j|M}^2 = \text{var} \left( \sum_{\substack{g=1, \\ g \neq j}}^{\gamma} X_{gj}^K - C_j^{MN} \right) = \text{var} \left( \sum_{\substack{g=1, \\ g \neq j}}^{\gamma} X_{gj}^K \right) \quad (172)$$

$$= \text{var} \left( \sum_{\substack{g=1, \\ g \neq j}}^{\gamma} v_{gj}^K \sigma_{gj}^k \varepsilon_{gj}^k \right) = \sum_{\substack{g=1, \\ g \neq j}}^{\gamma} \left( v_{gj}^K \sigma_{gj}^k \right)^2. \quad (173)$$

Denote by  $v_{*j}^K = \sum_{g=1, g \neq j}^{\gamma} v_{ij}^K$  and assume that  $\beta_{ij}^k \equiv \beta$ ,  $|v_{ij}| = 1$ , and  $\sigma_{ij}^k \equiv \sigma$ . Then,  $C_j^{MN} = \Phi^{-1}(\alpha_{MN}) \sqrt{\sigma_M^2 (v_{*j}^K)^2 \beta^2 + (\gamma - 1) \sigma^2}$ ,  $\mu_{j|M} = M \beta v_{*j}^K - \Phi^{-1}(\alpha_{MN}) \sqrt{\sigma_M^2 (v_{*j}^K)^2 \beta^2 + (\gamma - 1) \sigma^2}$ ,  $\sigma_{j|M}^2 = \sigma^2 (\gamma - 1)$ , and

$$\mathbb{E}[\bar{L}^{CCP} | M, D] = \sum_{j=1}^{\gamma} D_j \left( \mu_{j|M} \Phi \left( \frac{\mu_{j|M}}{\sigma_{j|M}} \right) + \sigma_{j|M} \varphi \left( -\frac{\mu_{j|M}}{\sigma_{j|M}} \right) \right). \quad (174)$$

□

## B Model for correlated defaults

In order to allow for correlation of entity defaults, we employ a credit model based on the Merton model (Merton (1974)). In particular, we assume that each counterparty  $i$  defaults at the settlement period begin if the random value of its assets is below a given bankruptcy threshold,  $A_i < B_i$ .

The value of assets at settlement period begin is given by

$$A_i = \exp \left( \mu_{A_i} - \frac{\beta_{A_i}^2 \sigma_M^2 + \sigma_{A_i}^2}{2} + \beta_{A_i} M + \sigma_{A_i} W_i \right), \quad (175)$$

where  $(W_1, \dots, W_\gamma)$  are jointly standard normally distributed and correlated with pairwise correlation  $\rho_{A_i, A_j}$ . The log value of assets is normally distributed with  $\log A_i \sim \mathcal{N} \left( \mu_{A_i} - \frac{\beta_{A_i}^2 \sigma_M^2 + \sigma_{A_i}^2}{2}, \sigma_{A_i}^2 + \beta_{A_i}^2 \sigma_M^2 \right)$ . The pairwise correlation of two entities' log assets is given by

$$\tilde{\rho}_{A_i, A_j} = \text{cor}(\log A_i, \log A_j) = \frac{\beta_{A_i} \beta_{A_j} \sigma_M^2 + \sigma_{A_i} \sigma_{A_j} \rho_{A_i, A_j}}{\sqrt{\beta_{A_i}^2 \sigma_M^2 + \sigma_{A_i}^2} \sqrt{\beta_{A_j}^2 \sigma_M^2 + \sigma_{A_j}^2}}. \quad (176)$$

The individual (unconditional) default probability of entity  $i$  is given by

$$pd_i = \mathbb{P}(A_i < B_i) = \Phi \left( \frac{\log B_i - \mu_{A_i} + \frac{\beta_{A_i}^2 \sigma_M^2 + \sigma_{A_i}^2}{2}}{\sqrt{\beta_{A_i}^2 \sigma_M^2 + \sigma_{A_i}^2}} \right). \quad (177)$$

Without loss of generality, we assume that  $\mu_{A_i} \equiv 0$ . Then, the default intensity is given by  $\bar{d}_i = \frac{\log B_i}{\sqrt{\sigma_{A_i}^2 + \beta_{A_i}^2 \sigma_M^2}} + \frac{\sqrt{\sigma_{A_i}^2 + \beta_{A_i}^2 \sigma_M^2}}{2}$ . We define by  $D = (D_1, \dots, D_\gamma)$  a vector of binary random variables  $D_i = \delta_{A_i < B_i}$  that signal the default of entity  $i \in \{1, \dots, \gamma\}$ . The joint distribution of two entities' default state is determined by

$$\mathbb{P}(D_i = 1, D_j = 1) = \mathbb{P}(\bar{Z}_i < \bar{d}_i, \bar{Z}_j < \bar{d}_j) = \Phi_{2, \Sigma}(\bar{d}_i, \bar{d}_j), \quad (178)$$

where  $(Z_i, Z_j)$  are multi-normally distributed with zero mean, unit variance, and correlation matrix

$\Sigma$ , with  $\Sigma_{ij} = \rho$ ,  $i \neq j$ , and  $\Sigma_{ii} = 1$ , and

$$\mathbb{P}(D_i = 1, D_j = 0) = \mathbb{P}(Z_i < \bar{d}_i, Z_j \geq \bar{d}_j) = \mathbb{P}(Z_i < \bar{d}_i, -Z_j < -\bar{d}_j) \quad (179)$$

$$= \mathbb{P}(Z_i < \bar{d}_i, \tilde{Z}_j < -\bar{d}_j) = \Phi_{2, \tilde{\Sigma}}(\bar{d}_i, -\bar{d}_j) \quad (180)$$

where  $(Z_i, \tilde{Z}_j)$  is multi-normally distributed,  $(Z_i, \tilde{Z}_j) \sim \mathcal{N}_2(0, \tilde{\Sigma})$  with correlation matrix  $\tilde{\Sigma}_{ij} = -\tilde{\rho}$ ,  $i \neq j$  and  $\tilde{\Sigma}_{ii} = 1$ ,  $i, j \in \{1, 2\}$ . Iteration yields the general distribution of default states as

$$\mathbb{P}(D = d) = \Phi_{\gamma, \tilde{\Sigma}}(\tilde{d}), \quad (181)$$

where  $\tilde{d}_i = \begin{cases} \bar{d}_i, & d_i = 1 \\ -\bar{d}_i, & d_i = 0 \end{cases}$ ,  $\tilde{\Sigma}_{ii} = 1$ , and  $\tilde{\Sigma}_{ij} = \begin{cases} \tilde{\rho}, & d_i = d_j \\ -\tilde{\rho}, & d_i \neq d_j \end{cases}$ ,  $i \neq j$ . Thus,  $\tilde{\Sigma}$  has a unit diagonal and 4 blocks of  $\tilde{\rho}$  and  $-\tilde{\rho}$ :

$$\tilde{\Sigma} = \begin{pmatrix} 1 & \tilde{\rho} & \dots & \tilde{\rho} & -\tilde{\rho} & \dots & \dots & -\tilde{\rho} \\ \tilde{\rho} & 1 & \tilde{\rho} & \tilde{\rho} & -\tilde{\rho} & \dots & \dots & -\tilde{\rho} \\ & & \ddots & & -\tilde{\rho} & \dots & \dots & -\tilde{\rho} \\ \tilde{\rho} & \dots & \tilde{\rho} & 1 & -\tilde{\rho} & \dots & \dots & -\tilde{\rho} \\ -\tilde{\rho} & \dots & \dots & -\tilde{\rho} & 1 & \tilde{\rho} & \dots & \tilde{\rho} \\ -\tilde{\rho} & \dots & \dots & -\tilde{\rho} & \tilde{\rho} & 1 & \dots & \tilde{\rho} \\ -\tilde{\rho} & \dots & \dots & -\tilde{\rho} & & & \ddots & \\ -\tilde{\rho} & \dots & \dots & -\tilde{\rho} & \tilde{\rho} & \dots & \tilde{\rho} & 1 \end{pmatrix} \quad (182)$$

Assuming homogeneous counterparties (i.e.,  $\bar{d} \equiv \bar{d}_i$ ), the number of defaulting counterparties,  $N_D = \sum_{i=1}^{\gamma} D_i$ , is distributed as

$$\mathbb{P}(N_D = k) = \binom{\gamma}{k} \Phi_{\gamma, \tilde{\Sigma}}(\underbrace{\bar{d}, \dots, \bar{d}}_k, \underbrace{-\bar{d}, \dots, -\bar{d}}_{\gamma-k}), \quad (183)$$

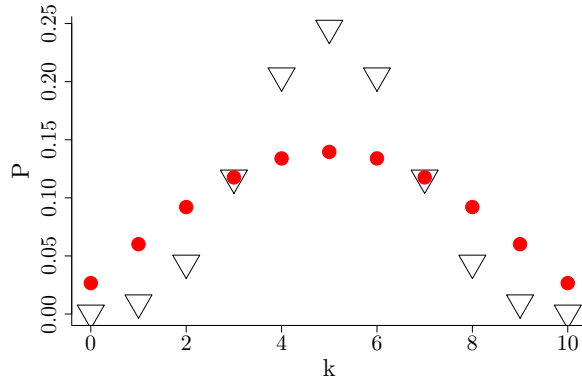
where  $\bar{d} > 0$  is the individual default intensity, and  $\tilde{\Sigma}$  defined as before.

As a benchmark case, consider independent defaults, i.e.,  $\tilde{\rho} = 0$ . Then, the distribution of joint

defaults is given by

$$\Phi_{\gamma, \bar{\Sigma}}(\underbrace{\bar{d}, \dots, \bar{d}}_k, \underbrace{-\bar{d}, \dots, -\bar{d}}_{\gamma-k}) = \Phi(\bar{d})^k \Phi(-\bar{d})^{\gamma-k} = \Phi(\bar{d})^k (1 - \Phi(\bar{d}))^{\gamma-k}. \quad (184)$$

Thus, if defaults are independent, the number of defaults is binomially distributed,  $N_D \sim \text{Binom}(\gamma, \Phi(\bar{d}))$ . As Figure 15 shows, increasing the correlation  $\tilde{\rho}$  yields larger tails of the distribution of  $N_D$ . Then, it is more likely that counterparties default together, i.e., a large or small number of counterparties defaults.



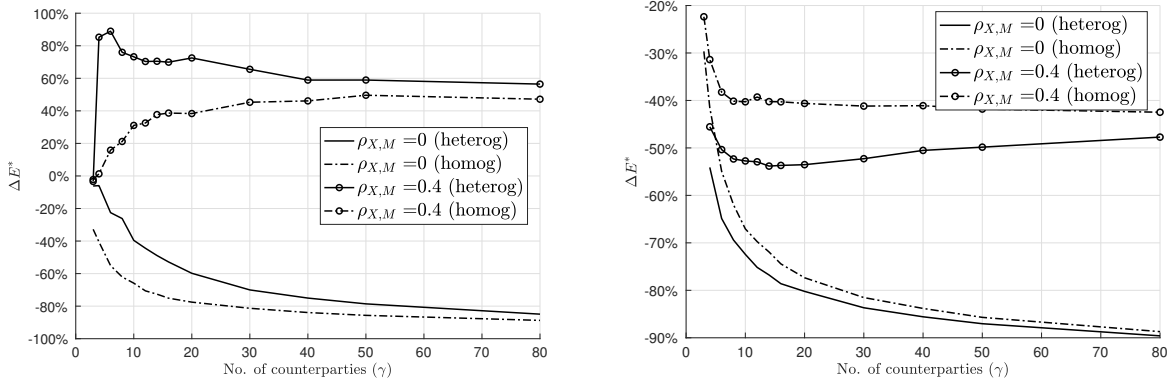
**Figure 15.** Probability distribution of the number of defaults,  $N_D$ , for  $\gamma = 10$  entities and individual probability of default  $pd = 0.5$  if defaults are uncorrelated (triangles) or correlated with  $\tilde{\rho} = 0.25$  (filled dots).

Figure 15 depicts the distribution of  $N_D$  for exemplary parameters. Clearly, increasing the total correlation  $\tilde{\rho}$  yields larger tails of the distribution. Then, it is more likely that entities default in clusters, i.e., that either a small or large number of counterparties defaults together.

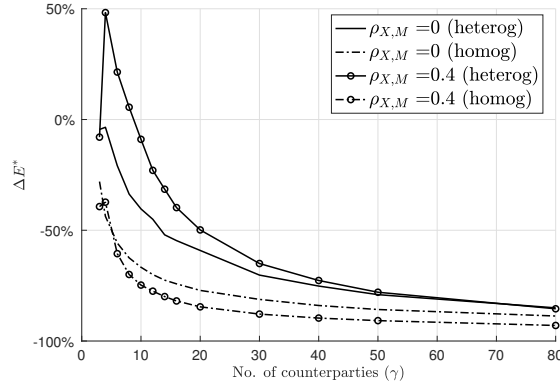
## C Heterogeneity in positions

Our baseline results are based on the assumption of equal trade sizes. Here, we assess the sensitivity of our baseline results towards the case with one particularly large entity. Figure 18 depicts the impact of systematic correlation on realized counterparty risk exposure if the positions of the market participant that is short in systematic risk are ten times larger than those of other entities, i.e.,  $v_{\gamma_j}^k \equiv -10$ . As one might expect, central clearing then becomes slightly more beneficial for this large market participant, as its exposure is larger than that of other clearing members and thus loss sharing is more attractive. For the same reason, central clearing becomes less beneficial for other market participants.

Similarly, in the presence of a large market participant that is long in systematic risk, central clearing is more beneficial for this participant and less beneficial for others, as can be seen in Figure 19. We conclude that larger market participants benefit relatively more from central clearing than in a market with equal trade sizes. These effects are qualitatively the same in the presence of a *Mega CCP*.



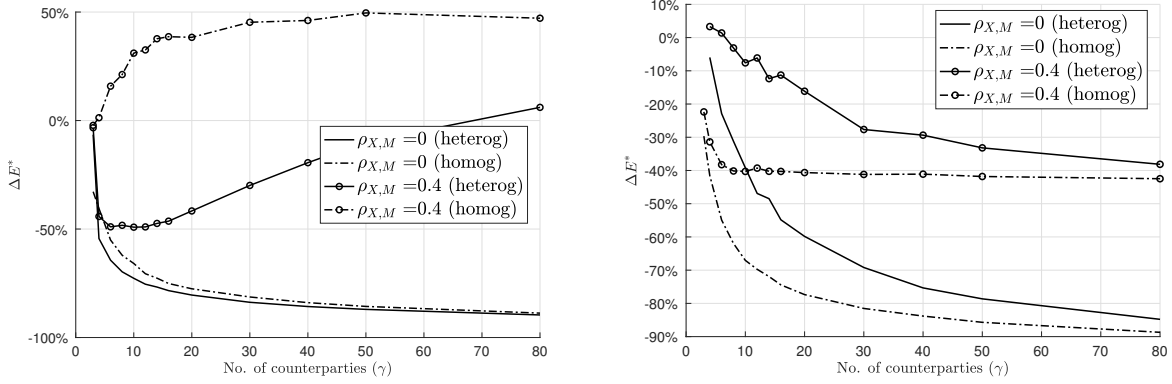
(a) Entity that is long in systematic risk ( $v_{ij}^k = 1$ ). (b) Large entity that is short in systematic risk ( $v_{ij}^k = -10$ ).



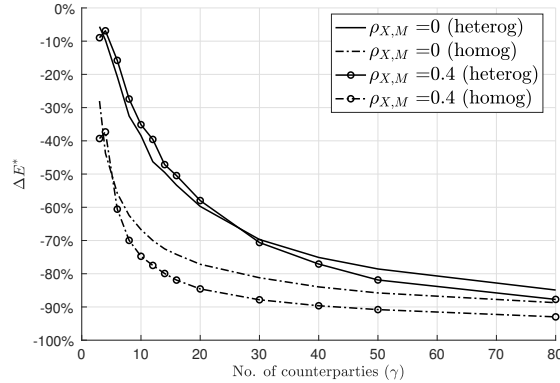
(c) Hedged dealer ( $\sum_j v_{ij}^k \approx 0$ ).

**Figure 16.** Impact of systematic risk on realized counterparty risk exposure in the presence of one large market participant that is short in systematic risk (with  $v_{\gamma j}^k = -10$ ) with a *Mega CCP*. Change in realized counterparty risk exposure due to central clearing of all derivative classes  $1, \dots, K$ , on realized counterparty risk exposure,  $\Delta E^* = \mathbb{E}[E_i^{*CN} - E_i^{*BN,K}] / \mathbb{E}[E_i^{*BN,K}]$ , for different levels of systematic risk  $\rho_{X,M}$ . If  $\Delta E^* < 0$ , then central clearing reduces realized counterparty risk exposure compared to bilateral clearing. The baseline calibration is described in Tables 4 and 5.



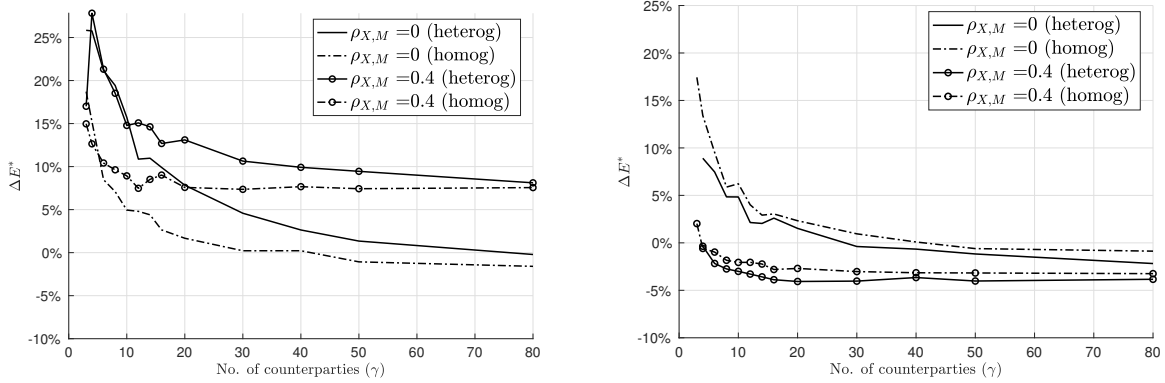


(a) Large entity that is long in systematic risk ( $v_{1j}^k = 10$ ). (b) Entity that is short in systematic risk ( $v_{1j}^k = -1$ ).

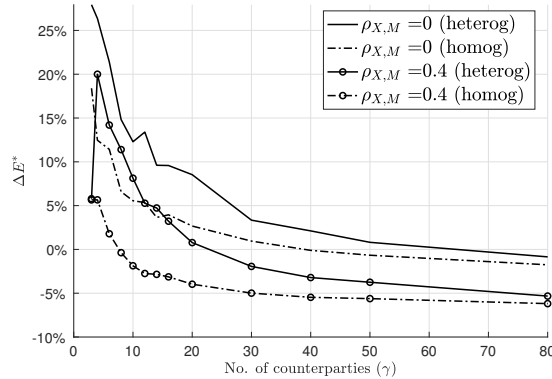


(c) Hedged dealer ( $\sum_j v_{1j}^k \approx 0$ ).

**Figure 17.** Impact of systematic risk on realized counterparty risk exposure in the presence of one large market participant that is long in systematic risk (with  $v_{1j}^k = 10$ ) with a *Mega CCP*. Change in realized counterparty risk exposure due to central clearing of all derivative classes  $1, \dots, K$ , on realized counterparty risk exposure,  $\Delta E^* = \mathbb{E}[E_i^{*CN} - E_i^{*BN,K}] / \mathbb{E}[E_i^{*BN,K}]$ , for different levels of systematic risk  $\rho_{X,M}$ . If  $\Delta E^* < 0$ , then central clearing reduces realized counterparty risk exposure compared to bilateral clearing. The baseline calibration is described in Tables 4 and 5.



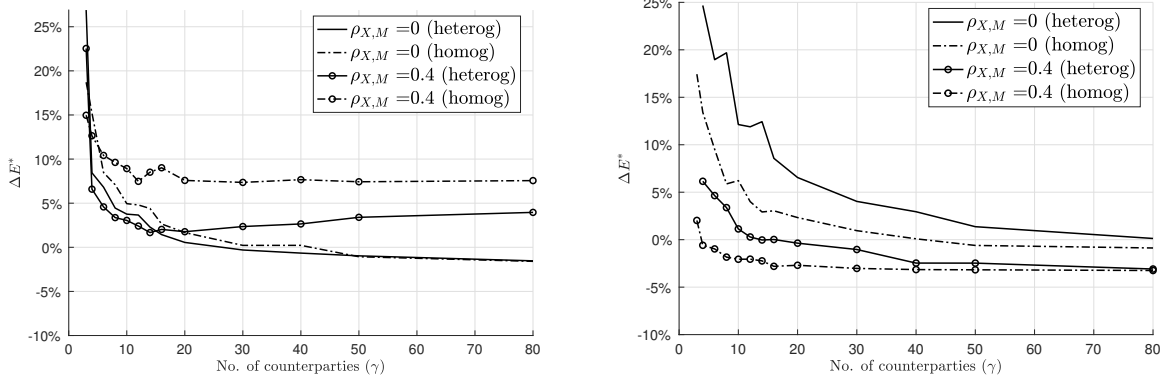
(a) Entity that is long in systematic risk ( $v_{ij}^k = 1$ ). (b) Large entity that is short in systematic risk ( $v_{ij}^k = -10$ ).



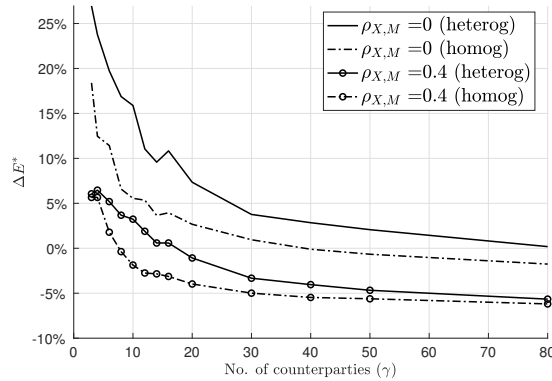
(c) Hedged dealer ( $\sum_j v_{ij}^k \approx 0$ ).

**Figure 18.** Impact of systematic risk on realized counterparty risk exposure in the presence of one large market participant that is short in systematic risk (with  $v_{\gamma j}^k = -10$ ).

Change in realized counterparty risk exposure due to central clearing of derivative class  $K$ ,  $\Delta E^* = \mathbb{E}[E_i^{*BN+MN} - E_i^{*BN,K}] / \mathbb{E}[E_i^{*BN,K}]$ , for different levels of systematic risk  $\rho_{X,M}$ . If  $\Delta E^* < 0$ , then central clearing reduces realized counterparty risk exposure compared to bilateral clearing. The baseline calibration is described in Tables 4 and 5.



(a) Large entity that is long in systematic risk ( $v_{ij}^k = 10$ ). (b) Entity that is short in systematic risk ( $v_{ij}^k = -1$ ).



(c) Hedged dealer ( $\sum_j v_{ij}^k \approx 0$ ).

**Figure 19.** Impact of systematic risk on realized counterparty risk exposure in the presence of one large market participant that is long in systematic risk (with  $v_{1j}^k = 10$ ).

Change in realized counterparty risk exposure due to central clearing of derivative class  $K$ ,  $\Delta E^* = \mathbb{E}[E_i^{*BN+MN} - E_i^{*BN,K}] / \mathbb{E}[E_i^{*BN,K}]$ , for different levels of systematic risk  $\rho_{X,M}$ . If  $\Delta E^* < 0$ , then central clearing reduces realized counterparty risk exposure compared to bilateral clearing. The baseline calibration is described in Tables 4 and 5.