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## **Keskusteluaiheita - Discussion papers**

No. 434

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### **HUMAN CAPITAL, EXTERNALITIES, GROWTH AND WELFARE MEASUREMENT**

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**ABSTRACT:** This paper concerns welfare measurement in an economy with human capital, using a Ramsey type growth model. Human capital accumulation is a function of the time consumers spend in education, and will, therefore, depend on the preferences for leisure as well as on the budget constraint. However, human capital is also a separate argument in the production function facing firms, which gives rise to a production externality. In a command optimum, where the externality is internalized, we show that the Hamiltonian along the optimal trajectory (which may be thought of as an augmented NNP measure) is the appropriate welfare indicator. We also discuss the tax - transfer system required for the decentralized economy to reach a command optimum.



## 1. Introduction

The question of how to measure economic welfare has been addressed by several authors during the last decade. In a famous article by Martin Weitzman (1976) it is shown that, under perfect foresight, the net national product (NNP) is a static equivalent of wealth measuring the maximum sustainable consumption level. From the point of view of national accounting, this result is important as it implies that the welfare level only depends on variables that are observed at the time when the measurement is conducted. Much of the more recent research on welfare measurement has focused on how to augment the NNP measure in order to take account of changes in natural resource stocks. In the papers by Dasgupta & Mäler (1991), Mäler (1991) and Hartwick (1990), it is recognized that the traditional NNP measure does not include the impact on future consumption possibilities resulting from current changes in natural resource stocks. The latter means that, in order to derive a welfare indicator related to the NNP, it is necessary to consider net investments (positive or negative) in natural resources.

Another part of the recent literature on welfare measurement deals explicitly with technological and/or environmental change. If there is disembodied technological or environmental change, which is not attributable to the factors of production, the welfare measures containing the value of investments in natural resource stocks would also be incomplete. This result was touched upon by Weitzman and was formally worked out by Aronsson & Löfgren (1992, 1993) and Löfgren (1992). The intuition is that, as disembodied (or non-attributable) technological change affects future consumption possibilities, welfare will depend on time itself. Hence, in this case, we can no longer use an augmented NNP measure as a welfare indicator. As is shown in Aronsson & Löfgren (1992, 1993), this time dependence of welfare can also be made endogenous by replacing the (exogenous) technological change with external economies. In the 1993 paper, the primary focus is on consumption externalities generated by pollution.

Non-attributable technological change or a "Solow residual" is perhaps best viewed as an artifact arising from measurement problems, and is to some extent caused by the value of investments in human capital not being accounted for in the traditional NNP measure. However, investments in human capital may also create positive externalities — endogenous growth theory is built on this notion — and as such they also represent a challenge for welfare measurement. To our knowledge, no previous attempt has been made to take account of these two aspects of human capital simultaneously in connection with the measurement of welfare. The main purposes of this paper are, therefore, to include human capital when measuring welfare and to discuss its role in national accounting by using a Ramsey type growth model.

Following recent studies on economic growth such as Lucas (1988), Razin (1972), Stokey (1991) and Sørensen (1992), we assume that the accumulation of human capital depends on the time consumers spend in education. However, contrary to these studies, we will also include leisure time in the consumers' utility function, which means that labour supply is endogenous here<sup>1</sup>. The incentive to spend time in education follows, in this model, from the assumption that the stock of human capital affects the marginal productivity of labour and, hence, firms' willingness to pay for labour. To formalize this idea, we will consider two types of labour, skilled and unskilled labour, where the stock of human capital affects the marginal productivity of skilled (but not unskilled) labour. The ability to supply skilled labour is bounded by the stock of human capital, which explicitly relates the choice of education to the supply of labour<sup>2</sup>. However, although the stock of human capital is controlled by the consumers, through their choice of education, it is also a production

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<sup>1</sup>Human capital accumulation in a model with endogenous labour supply has previously been studied by Blinder & Weiss (1976), where the primary focus is on consumer behaviour in a decentralized economy.

<sup>2</sup>This approach to describe the incentive to spend time in education is similar to the assumptions made by Grossman & Helpman (1991, p. 123), who also make a distinction between skilled and unskilled labour. As Grossman & Helpman do not include leisure time in the utility function, educational investments will be chosen such as to maximize the present value of lifetime income.

factor from the point of view of firms, which means that human capital accumulation gives rise to an external effect.

The paper is organized as follows. We start by presenting a command optimum, i.e. the solution to the central planner's optimization problem, where the external economies due to human capital accumulation are internalized. It is shown that the Hamiltonian along the optimal trajectory, which may be thought of as an augmented NNP measure, is the appropriate welfare indicator in this case. The command optimum may, therefore, be used as a reference case, with which we can compare the solution in the decentralized economy. The decentralized economy will not, by itself, result in the command optimum, because consumers, who are only concerned with their own utility, do not accumulate enough human capital. If the externality is not internalized, the augmented NNP measure will underestimate the welfare level in the decentralized economy. The final concern of the paper is, therefore, how the command optimum may also be reached in the decentralized economy by designing the tax – transfer system in an optimal way.

## 2. The Ramsey Growth Model With Human Capital

### *Command Optimum*

In each interval of time,  $(t, t+dt)$ , the consumer's utility is assumed to be a function of the consumption of goods and leisure according to  $u(c, z) = u(c, T - \ell_1 - \ell_2 - x)$ , where  $c$  is the consumption of a composite commodity and  $z$  is leisure. Leisure is, in turn, defined as an infinitesimal time endowment,  $T$ , less time spent in market work,  $\ell_1$  and  $\ell_2$ , and time spent in education,  $x$ . We may think of  $\ell_1$  as "unskilled" labour and  $\ell_2$  as "skilled" labour, which will be clear below. The production function of firms is represented by  $f(k, \ell_1 + a(h)\ell_2, h)$ , where  $k$  is the ordinary capital stock,  $h$  is the stock of human capital, while  $a(h)$  is a function such that  $a(0)=1$  and  $a'(h)>0$ . Therefore, in addition to being a separate



argument in the production function,  $h$  will also affect the marginal product of skilled labour. The stock of human capital is assumed to accumulate according to the equation  $dh/dt = g(x) - \gamma h$ , where  $g(x)$  is such that  $g'(x) > 0$ , while  $\gamma$  is the rate of depreciation.

Formally, the central planner's optimization problem is assumed to take the form:

$$\text{Max}_{c, \ell_1, \ell_2, x} \int_0^{\infty} u(c(t), T - \ell_1(t), -\ell_2(t) - x(t)) e^{-\theta t} dt \quad (1)$$

$$\text{s.t.} \quad dk(t)/dt = f(k(t), \ell_1(t) + a(h(t))\ell_2(t), h(t)) - c(t) \quad (2a)$$

$$dh(t)/dt = g(x(t)) - \gamma h(t) \quad (2b)$$

$$0 \leq \ell_1(t) \quad (2c)$$

$$0 \leq \ell_2(t) \leq \bar{\ell}_2(h(t)) \quad (2d)$$

$$0 \leq x(t) \quad (2e)$$

$$k(0) = k_0 \quad (2f)$$

$$h(0) = h_0 \quad (2g)$$

where the twice continuously differentiable functions  $u(\cdot)$  and  $f(\cdot)$  are, as usual, increasing and strictly concave, and  $\theta$  is the rate of time preference. As can be seen, the incentive to spend time in education follows, in this model, from the assumption that the stock of human capital affects the marginal productivity of skilled labour and, according to (2d), that the ability to supply skilled labour is bounded by the stock of human capital. The function  $\bar{\ell}_2(h)$  in (2d) is assumed to be such that  $\bar{\ell}_2(0) = 0$  and  $\bar{\ell}_2'(h) > 0$ . In addition to (2a)–(2g), we also require that  $c \geq 0$  and  $z = T - \ell_1 - \ell_2 - x \geq 0$ , but we will throughout the analysis assume that neither of these two constraints are binding. If we, for notational convenience, neglect the time indicator,  $t$ , the (current value) Lagrangean is written:

$$\begin{aligned} L = & u(c, T - \ell_1 - \ell_2 - x) + \lambda[f(k, \ell_1 + a(h)\ell_2, h) - c] + \mu[g(x) - \gamma h] \\ & + \nu[\bar{\ell}_2(h) - \ell_2]. \end{aligned} \quad (3)$$

Here,  $\lambda$  is the marginal value in utility terms of capital,  $k$ , while  $\mu$  is the marginal value in utility terms of human capital. In addition to the transversality conditions and to (2a), (2b), (2c), (2d) and (2e), the necessary conditions are:

$$\partial L / \partial c = u_c(c, z) - \lambda = 0 \quad (4a)$$

$$\partial L / \partial \ell_1 = -u_z(c, z) + \lambda f_\ell(k, \ell_1 + a(h)\ell_2, h) \leq 0, \text{ and } (\partial L / \partial \ell_1)\ell_1 = 0 \quad (4b)$$

$$\partial L / \partial \ell_2 = -u_z(c, z) + \lambda a(h)f_\ell(k, \ell_1 + a(h)\ell_2, h) - \nu \leq 0, \text{ and } (\partial L / \partial \ell_2)\ell_2 = 0 \quad (4c)$$

$$\partial L / \partial x = -u_x(c, z) + \mu g'(x) \leq 0, \text{ and } (\partial L / \partial x)x = 0 \quad (4d)$$

$$d\lambda/dt = \lambda\theta - \lambda f_k(k, \ell_1 + a(h)\ell_2, h) \quad (4e)$$

$$d\mu/dt = \mu(\theta + \gamma) - \lambda df(k, \ell_1 + a(h)\ell_2, h)/dh - \nu \bar{\ell}_2'(h) \quad (4f)$$

$$\nu \geq 0 \text{ (=0 if } \bar{\ell}_2(h) > \ell_2) \quad (4g)$$

where  $u_c(\cdot) = \partial u(\cdot) / \partial c$ ,  $u_z(\cdot) = \partial u(\cdot) / \partial z$ ,  $f_\ell(\cdot) = \partial f(\cdot) / \partial (\ell_1 + a(h)\ell_2)$ ,  $f_k(\cdot) = \partial f(\cdot) / \partial k$ ,  $f_h(\cdot) = \partial f(\cdot) / \partial h$  and  $df(\cdot) / dh = a'(h)\ell_2 f_\ell(\cdot) + f_h$ .

Although we will only focus attention on cases where  $\ell_2 > 0$  and  $x > 0$ , there are two distinct regimes concerning the labour supply behaviour. First, if  $\ell_2 \in (0, \bar{\ell}_2(h))$ , the first part of (4c) will hold as a strict equality (i.e.  $\partial L / \partial \ell_2 = 0$ ), while at the same time  $\nu = 0$ . In this case, the utility value of education,  $\mu g'(x)$ , will equal the return in utility terms of skilled labour,  $\lambda a(h)f_\ell(\cdot)$ . Moreover, as the first part of (4b) will hold as a strict inequality in this case, it turns out that  $\ell_1 = 0$ . On the other hand, if  $\ell_2 = \bar{\ell}_2(h)$ , which implies that  $\nu > 0$ , the consumer will wish to substitute  $x$  for  $\ell_2$ , although such a substitution is not possible. In this case, there may also be a positive supply of unskilled labour, i.e.  $\ell_1 > 0$ . Therefore, the consumer will choose  $\ell_1 > 0$  only if  $\ell_2 = \bar{\ell}_2(h)$ , as education in this case will not pay off in terms of returns on skilled labour. In other words,  $\ell_2 = \bar{\ell}_2(h)$  implies that the return in utility terms of skilled labour,  $\lambda a(h)f_\ell(\cdot)$ , exceeds the utility value of education,  $\mu g'(x)$ . In the latter case, the consumer may be better off by substituting  $x$  for  $\ell_1$ .

What would be the appropriate measure of welfare in this economy? Let us denote the optimal path of the economy by  $c^*(t)$ ,  $\ell_1^*(t)$ ,  $\ell_2^*(t)$ ,  $x^*(t)$ ,  $k^*(t)$ ,  $h^*(t)$ ,  $\lambda^*(t)$ ,  $\mu^*(t)$  and  $\nu^*(t)$ . Along the optimal path, the Lagrangean is written:

$$L^* = u(c^*, T - \ell_1^* - \ell_2^* - x^*) + \lambda^* [f(k^*, \ell_1^* + a(h^*)\ell_2^*, h^*) - c^*] + \mu^* [g(x^*) - \gamma h^*] + \nu^* [\bar{\ell}_2(h^*) - \ell_2^*] \quad (5)$$

By differentiating (5) totally with respect to time and rearrange we obtain:

$$dL^*/dt = \theta[L^* - u(c^*, T - \ell_1^* - \ell_2^* - x^*)], \quad (6)$$

which has a solution of the form:

$$\theta \int_t^{\infty} u(c^*(s), T - \ell_1^*(s) - \ell_2^*(s) - x^*(s)) e^{-\theta(s-t)} ds = L^*(t) = u(c^*(t), T - \ell_1^*(t) - \ell_2^*(t) - x^*(t)) + \lambda^*(t) dk^*(t)/dt + \mu^*(t) dh^*(t)/dt = H^*(t) \quad (7)$$

where  $H^*(t)$  is the Hamiltonian along the optimal path. Equation (7) means that welfare, here measured as the interest on the present value of future utility, equals the Lagrangean (or Hamiltonian) along the optimal path. The latter may be thought of as an augmented NNP measure, as the welfare at time  $t$  is measured solely by variables observed at time  $t$ .

### *The Decentralized Economy*

In the decentralized economy, firms maximize profit at each instant of time by solving the optimization problem:

$$\text{Max}_{k, \ell_1, \ell_2} \pi = f(k, \ell_1 + a(h)\ell_2, h) - rk - w_1\ell_1 - w_2\ell_2 \quad (8)$$

where  $r$  is the interest rate,  $w_1$  is the wage rate paid to unskilled labour and  $w_2$  is the wage rate paid to skilled labour. The first-order conditions for profit maximization are:

$$f_k(k, \ell_1 + a(h)\ell_2, h) - r = 0 \quad (9a)$$

$$f_{\ell_1}(k, \ell_1 + a(h)\ell_2, h) - w_1 = 0 \quad (9b)$$

$$a(h)f_{\ell_2}(k, \ell_1 + a(h)\ell_2, h) - w_2 = 0 \quad (9c)$$

Turning to consumer behaviour in the decentralized economy, the utility maximization problem is written (if we neglect the time indicator):

$$\text{Max}_{c, \ell_1, \ell_2, x} \int_0^{\infty} u(c, T - \ell_1 - \ell_2 - x) e^{-\theta t} dt \quad (10)$$

$$\text{s.t.} \quad dk/dt = \pi + rk + w_1\ell_1 + w_2\ell_2 - c \quad (11a)$$

$$dh/dt = g(x) - \gamma h \quad (11b)$$

$$0 \leq \ell_1 \quad (11c)$$

$$0 \leq \ell_2 \leq \bar{\ell}_2(h) \quad (11d)$$

$$0 \leq x \quad (11e)$$

$$k(0) = k_0 \quad (11f)$$

$$h(0) = h_0 \quad (11g)$$

where the income of the consumer is the sum of labour income,  $w_1\ell_1 + w_2\ell_2$ , capital income,  $rk$ , and possible pure profit,  $\pi$  (as the consumer is assumed to own the firm). The constraints represented by (11b)–(11g) coincide with (2b)–(2g) in the central planner's optimization problem. Since the consumer takes account of the fact that his/her accumulation of human capital affects the return on skilled labour, i.e.  $w_2 = a(h)w_1$ , the Lagrangean might be written as:

$$L = u(c, T - \ell_1 - \ell_2 - x) + \psi[\pi + rk + w_1 \ell_1 + a(h)w_1 \ell_2 - c] + \phi[g(x) - \gamma h] + \eta[\bar{\ell}_2(h) - \ell_2] \quad (12)$$

where  $\eta$  is the Lagrange multiplier, and  $\psi$  and  $\phi$  are costate variables, i.e.  $\psi$  measures the value in utility terms of a one unit increase in  $k$ , while  $\phi$  measures the value in utility terms of a one unit increase in  $h$ . In addition to the transversality conditions and to (11a), (11b), (11c), (11d) and (11e), the necessary conditions read:

$$\partial L / \partial c = u_c(c, z) - \psi = 0 \quad (13a)$$

$$\partial L / \partial \ell_1 = -u_z(c, z) + \psi w_1 \leq 0, \text{ and } (\partial L / \partial \ell_1) \ell_1 = 0 \quad (13b)$$

$$\partial L / \partial \ell_2 = -u_z(c, z) + \psi a(h)w_1 - \eta \leq 0, \text{ and } (\partial L / \partial \ell_2) \ell_2 = 0 \quad (13c)$$

$$\partial L / \partial x = -u_z(c, z) + \phi g'(x) \leq 0, \text{ and } (\partial L / \partial x)x = 0 \quad (13d)$$

$$d\psi/dt = \psi(\theta - r) \quad (13e)$$

$$d\phi/dt = \phi(\theta + \gamma) - \psi a'(h)w_1 \ell_2 - \eta \bar{\ell}_2'(h) \quad (13f)$$

$$\eta \geq 0 \text{ (=0 if } \bar{\ell}_2(h) > \ell_2) \quad (13g)$$

If we replace  $w_1$ , and  $r$  in (13b), (13c) and (13e) by  $f_\ell(\cdot)$  and  $f_k(\cdot)$ , respectively, from the first-order conditions for the firm, we see that the difference between the decentralized solution and the command optimum has to do with the marginal valuation of the stock of human capital. To be specific, the differential equation for  $\phi$  in (13f) differs from the differential equation for  $\mu$  in (4f), as (13f) does not contain the term  $f_h = \partial f(\cdot) / \partial h$ . The reason is, of course, that the consumer in the decentralized economy does not take account of the fact that the stock of human capital also appears as a separate argument in the firm's production function. As a consequence, the consumer will spend less time in education and, hence, accumulate less human capital than what is socially optimal. This also means that, as the source of the externality is the appearance of  $h$  as a separate argument in the production function, a production function of the type  $f(k, \ell_1 + a(h)\ell_2)$  implies that the decentralized solution coincides with the command optimum. In the latter

case, human capital accumulation gives rise to what is usually referred to as "Harrod — neutral" technological progress, which Razin (1972) has shown to be a both necessary and sufficient condition for the perfect foresight market economy to represent a social optimum.

To measure welfare in the decentralized economy, let  $c^0(t)$ ,  $\ell_1^0(t)$ ,  $\ell_2^0(t)$ ,  $x^0(t)$ ,  $k^0(t)$ ,  $h^0(t)$ ,  $\psi^0(t)$ ,  $\phi^0(t)$  and  $\eta^0(t)$  denote the general equilibrium solution. Along the optimal path, the Lagrangean is written:

$$\begin{aligned} L^0 = & u(c^0, T - \ell_1^0 - \ell_2^0 - x^0) + \psi^0[f(k^0, \ell_1^0 + a(h^0)\ell_2^0, h^0) - c^0] + \phi^0[g(x^0) - \gamma h^0] \\ & + \eta^0[\ell_2^0(h^0) - \ell_2^0] \end{aligned} \quad (14)$$

If we differentiate (14) totally with respect to time and rearrange, we obtain the differential equation:

$$dL^0/dt = \theta[L^0 - u(c^0, T - \ell_1^0 - \ell_2^0 - x^0)] + \psi^0 f_h(k, \ell_1 + a(h)\ell_2, h) \frac{dh^0}{dt} \quad (15)$$

which has a solution corresponding to:

$$\theta \int_t^\infty u(c^0(s), z^0(s)) e^{-\theta(s-t)} ds = L^0(t) + \int_t^\infty \psi^0(s) f_h(s) \frac{dh^0(s)}{dt} e^{-\theta(s-t)} ds \quad (16)$$

where  $z^0 = T - \ell_1^0 - \ell_2^0 - x^0$ . In equation (16), the second term on the right hand side should be interpreted as the present value of the marginal external effect along the optimal path. In this case, when the external effect is not properly internalized, it is evident that the Lagrangean does not measure welfare appropriately. It is also notable that the welfare level given by (16) is smaller in comparison with the command optimum, which represents the first best solution. A natural question is, therefore, whether we can construct a tax — transfer system such that the command optimum is also attainable in the decentralized

economy. This is indeed possible and the argument goes as follows. There is an extra gain,  $\partial f(\cdot)/\partial h = f_h(\cdot) > 0$ , from the stock of human capital, which is not internalized in the consumer's investment decision, i.e.  $f_h(\cdot)$  does not affect the costate variable  $\phi$  corresponding to human capital in the decentralized economy. By taxing the firm for its use of human capital at a rate  $f_h(\cdot)$ , and then redistributing the tax revenues to the consumer, investments in human capital will be valued correctly from society's point of view. Therefore, the best way to think about the tax is, perhaps, in terms of a subsidy from the firm to the consumer. The optimal tax structure is formally described in Proposition 1.

**PROPOSITION 1:** If the firm's use of human capital in production is taxed at a rate  $p_h = f_h(k^*, \ell_1^* + a(h^*)\ell_2^*, h^*)$ , and if the tax revenue,  $p_h h$ , is redistributed to the consumer, the decentralized solution will represent a command optimum.

**PROOF:** Note that if the consumer receives a transfer payment of  $p_h h$ , the necessary conditions in (13a)–(13g) will change to:

$$\partial L / \partial c = u_c(c, z) - \psi = 0 \quad (17a)$$

$$\partial L / \partial \ell_1 = -u_z(c, z) + \psi w_1 \leq 0, \text{ and } (\partial L / \partial \ell_1)\ell_1 = 0 \quad (17b)$$

$$\partial L / \partial \ell_2 = -u_z(c, z) + \psi a(h)w_1 - \eta \leq 0, \text{ and } (\partial L / \partial \ell_2)\ell_2 = 0 \quad (17c)$$

$$\partial L / \partial x = -u_z(c, z) + \phi g'(x) \leq 0, \text{ and } (\partial L / \partial x)x = 0 \quad (17d)$$

$$d\psi/dt = \psi(\theta - r) \quad (17e)$$

$$d\phi/dt = \phi(\theta + \gamma) - \psi(a'(h)w_1\ell_2 + p_h) - \eta\bar{\ell}_2'(h) \quad (17f)$$

$$\eta \geq 0 \text{ (=0 if } \bar{\ell}_2(h) > \ell_2). \quad (17g)$$

Using equations (17) together with the first-order conditions for firm behaviour, which are given by  $f_k(\cdot) = r$ ,  $f_\ell(\cdot) = w_1$ ,  $a(h)f_\ell(\cdot) = w_2$  and (as a consequence of the tax)  $f_h(\cdot) = p_h$ , we see that the necessary conditions in the decentralized economy will coincide with those of the command optimum, Q.E.D.

The reader should note that, as the tax rate,  $p_h$ , varies over time according to the development of  $f_h(\cdot)$ , the government has to determine the command optimum in order to impose the appropriate tax — transfer structure. Therefore, although there exists an optimal tax structure such that the market economy results in the socially optimal solution, the informational requirements are (at the very least) demanding. Another interesting observation is that the consumer receives a stock subsidy, i.e. the subsidy is directed to the stock of human capital, and not to the consumer's decision to spend time in education (which leads to accumulation of human capital). An alternative policy may, therefore, seem to be to subsidize education. However, as will be discussed below, such a subsidization scheme would not bring the economy to a social optimum.

To see this, suppose, for the moment, that we replace the policy suggested in Proposition 1 with a subsidy directed to the time the consumer spends in education. If the consumer receives a transfer payment of  $p_x x$ , where  $p_x$  is a subsidy per unit of education, (13d) will change to  $u_z(\cdot) + \psi p_x + \phi g'(x) \leq 0$  (if we ignore the complementary slackness condition), while the equation for the development over time of  $\phi$  will remain as in (13f). Hence, if the subsidy is directed to the time spent in education, it is not possible to choose a subsidy per unit of education,  $p_x$ , such that the decentralized economy replicates the command optimum. The reason is that the consumer's valuation of the stock of human capital is incorrect in the decentralized economy, and a subsidy directed to education will not eliminate this problem. In other words, as the differential equation for  $\phi$  remains different from the differential equation for  $\mu$ , which is given by (4f),  $\phi$  is not the social value of a one unit increase in the stock of human capital. Therefore, in order to construct a tax — transfer system such that the decentralized economy becomes efficient, it is necessary to use a stock subsidy in order to make the consumer value the stock of human capital appropriately.



### 3. Summary and Conclusions

This paper concerns welfare measurement in a model with externalities due to human capital. As in previous studies of optimal growth, we assume that the accumulation of human capital depends on the time consumers spend in education. Education and human capital accumulation are also related to the labour supply decision made by the consumer. To be specific, the paper considers two types of labour, skilled and unskilled labour, where the stock of human capital affects the marginal productivity of skilled (but not unskilled) labour. The ability to supply skilled labour is bounded by the stock of human capital. However, the stock of human capital also appears as a separate argument in the production function of firms, which means that human capital accumulation leads to a production externality.

In the socially optimal solution, where the externality is properly accounted for, welfare (in terms of the interest on the present value of future utility) is measured by the Hamiltonian along the optimal trajectory, which may be thought of as an augmented NNP measure. The latter result is interesting from the point of view of national accounting, as it implies that welfare can be measured solely by observable variables. On the other hand, the decentralized economy will not, by itself, result in a command optimum. The reason is that, as the consumer is only concerned with his/her own utility, he/she does not take account of the fact that the stock of human capital is a production factor from the point of view of the firm. Hence, the external effect has to do with the fact that the consumer's valuation of the stock of human capital is incorrect from society's point of view, which here means that the consumer will spend less time in education and accumulate less human capital than is socially optimal. When the external effect is not accounted for in the optimization problem, we also find that the Hamiltonian along the resulting (suboptimal) trajectory underestimates welfare by an amount equal to the present value of the marginal external effect. As a final concern of the paper, we determine an optimal tax–transfer

structure, such that the decentralized economy results in the first best optimum. The suggested policy rule means essentially that the firm pays a tax for its use of human capital, and that the resulting tax revenues are redistributed as a stock subsidy to the consumer. As this policy rule implies that the consumer will value human capital in the same way as society, the external effect will be fully internalized.

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