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**Article**

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# Looking at the Integration of Nations through the Lens of the Merger of Populations: Preliminary Superadditivity and Impossibility Results

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JEL-Classification: D02, D63, F55, P51

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## 1. Introduction

This paper looks at the integration of nations and regions through a somewhat unusual prism. Consider European integration – one of the most interesting social science experiments of recent times. In terms of the variance in the attributes (including the income levels) of the nations that are pooled together, as well as in terms of the scale of the pooling, the process is of little precedence. For these reasons, and since from a historical perspective the process is quite young, it is not all that obvious or certain that what we witness today is here to stay. And, as is often the case, what we observe may differ from what lurks underneath.

In what follows we do not strive to provide a balance sheet of the advantages and disadvantages of integration which, undoubtedly, include various efficiency and productivity gains. Rather, we seek to highlight a particular worrisome aspect of integration.

Integration can be perceived as a merger of populations. Mergers of populations occur in all spheres of life, and in all times and places: conquests bring hitherto disparate populations into one, provinces merge into regions, adjacent villages that experience population growth merge into one town, schools and school classes are merged, and as already noted, European countries have been merging into a union. We employ a particular social index (a statistic), Total Relative

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Deprivation,  $TRD$ , to assess the repercussions of a merger. We first present this index. Following that, we show that in the case of two populations of two persons each with incomes that are all distinct (pairwise different), the  $TRD$  of a merged population is larger than the sum of the  $TRDs$  of the constituent populations when apart. This finding raises the disturbing possibility that in and by itself, integration (for example, European integration) may fail to constitute a panacea of social harmony, or to reward the populace with a sense of improved wellbeing.

We next consider a self-contained, non-publicly financed policy aimed at retaining individuals' levels of wellbeing at their pre-merger magnitudes. Quite surprisingly, a policy of the latter type, which is the staple of public finance (a Pareto neutralizing transfer from the gainers to the losers), cannot be implemented even in a class of simple cases: the loss is more formidable than the gain.

## 2. A Measure of Social (Societal) Dismay

Consider a population  $N$  of  $n$  individuals whose incomes are  $x_1 \leq x_2 \leq \dots \leq x_n$ , where  $n \geq 2$ . The relative deprivation,  $RD$ , of an individual whose income is  $x_i$ ,  $i = 1, \dots, n-1$ , is defined as

$$RD_N(x_i) \equiv \frac{1}{n} \sum_{k=i+1}^n (x_k - x_i), \quad (1)$$

and it is understood that  $RD_N(x_n) = 0$ . Let  $F(x_i)$  be the fraction of those in the population whose incomes are smaller than or equal to  $x_i$ . Then we have the following claim.

**Claim 1:**  $RD_N(x_i) = [1 - F(x_i)] \cdot E(x - x_i | x > x_i)$ . That is, the relative deprivation of an individual whose income is  $x_i$  is the fraction of those in the population whose incomes are higher than  $x_i$  times their mean excess income.

**Proof:** Let us denote by  $k_i$  the smallest  $k \in [i+1, n]$  for which  $x_k > x_i$ . That is,  $k_i$  is the index of the first individual to the right of  $x_i$  in the ordered distribution whose income is strictly higher than  $x_i$ . Since for different  $i$ 's there are different corresponding  $k$ 's, we use the term  $k_i$ .<sup>1</sup> Then we have that

1 For example, let the incomes of a population of five individuals be  $x_1 = 1$ ,  $x_2 = 2$ ,  $x_3 = 2$ ,  $x_4 = 3$ ,  $x_5 = 4$ . Consider  $x_2$ . The next individual with an income higher than  $x_2$  is the individual whose income is  $x_4$ . Consequently,  $k_2 = 4$ .

$$\begin{aligned}
 RD_N(x_i) &\equiv \frac{1}{n} \sum_{k=i+1}^n (x_k - x_i) = \frac{1}{n} \sum_{k=k_i}^n (x_k - x_i) \\
 &= [1 - F(x_i)] \sum_{k=k_i}^n \frac{(1/n)(x_k - x_i)}{1 - F(x_i)} \\
 &= [1 - F(x_i)] E(x - x_i \mid x > x_i). \quad \square
 \end{aligned}$$

The total relative deprivation of the population,  $TRD$ , is naturally the sum of the relative deprivations of all the individuals,

$$TRD_N = \sum_{i=1}^n RD_N(x_i) = \frac{1}{n} \sum_{i=1}^{n-1} \sum_{k=i+1}^n (x_k - x_i). \quad (2)$$

We resort to  $TRD$  as a measure of social dismay.<sup>2</sup>

### 3. Comparing the $TRD$ of a Merged Population with the Sum of the $TRD$ s of Two Constituent Populations of Two-Persons Each

In this section we show that except in the degenerate case in which the incomes of one population are identical to the incomes of the other population, the merger of two populations of two-person each results in the  $TRD$  of the merged population being higher than the sum of the  $TRD$ s of the constituent populations.

- Such a characterization of societal relative deprivation was proposed by YITZHAKI (1979) and axiomatized by BOSSERT and D'AMBROSIO (2006) and EBERT and MOYES (2000) who in turn followed the seminal work of RUNCIMAN (1966). Since the 1960s, a considerable body of research evolved, demonstrating empirically that interpersonal comparisons of income (that is, comparisons of the income of an individual with the incomes of higher income members of his reference group) bear significantly on the perception of well-being, and on behavior. (For a recent review see CLARK, FRIJTERS, and SHIELDS, 2008). One branch of this body of research has dealt with migration. Several studies have shown empirically that a concern for relative deprivation impacts significantly on migration outcomes (STARK and TAYLOR, 1989; STARK and TAYLOR, 1991; QUINN, 2006; STARK, MICEVSKA, and MYCIELSKI, 2009). Theoretical expositions have shown how the very decision to resort to migration and the choice of migration destination (STARK, 1984; STARK and YITZHAKI, 1988; STARK and WANG, 2007), as well as the assimilation behavior of migrants (FAN and STARK, 2007), are modified by a distaste for relative deprivation.

This is not an intuitively obvious result even in the simple case in which the two populations do not overlap and a relatively poor, two-person population merges with a relatively rich, two-person population. In such a case, it is quite clear that upon integration the members of the poorer population are subjected to more relative deprivation, whereas the members of the richer population except the richest are subjected to less relative deprivation. Since one constituent population experiences an increase of its *TRD* while the other constituent population experiences a decrease, whether the *TRD* of the merged population is higher than the sum of the *TRDs* of the constituent populations cannot be ascertained without additional formal analysis. To this end we now state and prove the following claim.

**Claim 2:** Let there be two populations of two-persons each: population *A*, and population *B*. Let the incomes of the four persons be distinct. A merger  $C = A \cup B$  results in an increase of *TRD*, that is,  $TRD_C > TRD_A + TRD_B$ .

**Proof:** With all incomes distinct (pairwise different) we assume, without loss of generality, that the smallest income is 1 and that it is obtained in population *A*. Thus, the incomes in population *A* are

$$1, 1 + \alpha,$$

and the incomes in population *B* are

$$1 + \beta, 1 + \beta + \delta,$$

where  $\alpha, \beta, \delta > 0$  are arbitrary. Clearly,

$$TRD_A = \frac{\alpha}{2}, \text{ and } TRD_B = \frac{\delta}{2}.$$

To evaluate the *TRD* of the four-individual population *C* with incomes

$$1, 1 + a, 1 + a + b, 1 + a + b + c$$

and with arbitrary  $a, b, c > 0$ , we note, referring to the four individuals as (1), (2), (3), and (4), that

$$RD(1) = \frac{1}{4}(a + (a + b) + (a + b + c)), \quad RD(2) = \frac{1}{4}(b + (b + c)),$$

$$RD(3) = \frac{c}{4}, \quad RD(4) = 0.$$

Therefore,

$$TRD_C = RD(1) + RD(2) + RD(3) = \frac{1}{4}(3a + 4b + 3c). \quad (3)$$

We now consider the  $TRD$  of  $C = A \cup B$ . Depending on the relative magnitudes of  $\alpha, \beta, \delta$  we have three cases.

**Case 1.**  $\alpha < \beta$ . Then,  $\beta = \alpha + \varepsilon$  for some  $\varepsilon > 0$ . Then we have incomes

$$1, 1 + \alpha, 1 + (\alpha + \varepsilon), 1 + (\alpha + \varepsilon) + \delta.$$

Using (3),

$$TRD_C = \frac{1}{4}(3\alpha + 4\varepsilon + 3\delta) > \frac{\alpha}{2} + \frac{\delta}{2} = TRD_A + TRD_B.$$

**Case 2.**  $\beta < \alpha < \beta + \delta$ . Then,  $\alpha = \beta + \varepsilon$ . Then we have incomes

$$1, 1 + \beta, 1 + \beta + \varepsilon, 1 + \beta + \varepsilon + (\delta - \varepsilon),$$

and we note, because  $\beta + \delta + \alpha$ , that  $\delta - \varepsilon > 0$  for some  $\varepsilon > 0$ . Using this and (3),

$$TRD_C = \frac{1}{4}(3\beta + 4\varepsilon + 3(\delta - \varepsilon))$$

$$> \frac{1}{4}(3\beta + 2\varepsilon + 2\delta) > \frac{\beta + \varepsilon}{2} + \frac{\delta}{2} = \frac{\alpha}{2} + \frac{\delta}{2} = TRD_A + TRD_B.$$

**Case 3.**  $\alpha > \beta + \delta$ . Then,  $\alpha = \beta + \delta + \varepsilon$  for some  $\varepsilon > 0$ . Then we have incomes

$$1, 1 + \beta, 1 + \beta + \delta, 1 + \beta + \delta + \varepsilon.$$

From (3),

$$TRD_C = \frac{1}{4}(3\beta + 4\delta + 3\varepsilon) > \frac{\beta + \delta + \varepsilon}{2} + \frac{\delta}{2} = \frac{\alpha}{2} + \frac{\delta}{2} = TRD_A + TRD_B.$$

Therefore,

$$TRD_{A \cup B} > TRD_A + TRD_B$$

in all possible cases.  $\square$

**Corollary:** In the degenerate case in which the incomes of population  $A$  are identical to the incomes of population  $B$ ,  $TRD_C = TRD_A + TRD_B$ .

**Proof:** Since without loss of generality the incomes in population  $A$  are  $\{1, 1 + \alpha\}$ , and the incomes in population  $B$  are  $\{1, 1 + \alpha\}$ , we have that  $TRD_A = TRD_B = \alpha/2$ . The  $TRD$  of population  $C$  with incomes  $\{1, 1, 1 + \alpha, 1 + \alpha\}$  is

$$RD(1) + RD(1) = \frac{2}{4} \cdot \frac{\alpha + \alpha}{2} + \frac{2}{4} \cdot \frac{\alpha + \alpha}{2} = \alpha.$$

But  $TRD_A + TRD_B = \alpha$  as well.  $\square$

Another way of seeing this is as follows. When the incomes of population  $A$  are identical to the incomes of population  $B$ , merging the two populations is equivalent to doubling the number of income recipients of each income. Since  $TRD$  is a measure with homogeneity of degree one (increasing the size of every group of income recipients by a factor of  $k$  implies that  $TRD$  also increases by a factor of  $k$ ), it follows that

$$\begin{aligned} TRD_C &= TRD\{1, 1, 1 + \alpha, 1 + \alpha\} = TRD\{2 \cdot 1, 2 \cdot (1 + \alpha)\} \\ &= 2TRD\{1, 1 + \alpha\} = TRD_A + TRD_B. \quad \square \end{aligned}$$

#### 4. A Policy Response to the Post-Merger Increase in *TRD*

The target of a policy response is a derivative of the underlying social welfare function. A policy can be enacted out of a concern that individuals' levels of well-being do not decrease upon a merger.

To ease exposition, we employ in what follows a somewhat modified notation. Consider a population of  $n$  individuals with a vector of incomes  $\mathbf{x} = (x_1, \dots, x_n)^T$ ,  $x_i \geq 0$ ,  $i = 1, \dots, n$ . We measure the  $i$ -th individual's relative deprivation as in (1), which we can rewrite as

$$RD(x_i, \mathbf{x}) = \frac{1}{n} \sum_{j=1}^n \max(x_j - x_i, 0).$$

Correspondingly, total relative deprivation is written as

$$TRD(\mathbf{x}) = \sum_{i=1}^n RD(x_i, \mathbf{x}).$$

Let the individuals' preferences be characterized by a combination of absolute income and relative deprivation:  $u_i = u(x_i, \mathbf{x}) = \alpha_i x_i - (1 - \alpha_i) RD(x_i, \mathbf{x})$  where  $0 < \alpha_i < 1$ ,  $i = 1, 2, \dots, n$ . The underlying idea of the stated policy response is to skim off income from those who reap a gain as a consequence of the merger, and distribute that income to those who experience a loss as a consequence of the merger, such that following the merger no individual will be worse off. There are several problems with such a scheme, however.

*First*, a necessary condition is that there has to be at least one gainer. But as is quite obvious, there may not be any as, for example, when population  $\{1, 2, 3, 4\}$  joins population  $\{5, 5\}$ .

*Second*, for the policy to be applicable, the policy maker would need to know  $\alpha_i$ . If each individual has his own distinct preference structure, the required information is colossal: a policy response that is based on preferences needs to build on invisibles. Two possibilities then come to mind: that all the individuals share the same distaste for relative deprivation, or that they do not. We attend in detail to the former possibility:  $\alpha_i = \alpha \forall i$ ,  $i = 1, 2, \dots, n$ .



That all the individuals share the same distaste for relative deprivation eases drastically the information requirements, allowing working with a single  $\alpha$ . But then, even in the simplest configuration of incomes, impossibility strikes. To see why, consider  $u_i = u(x_i, \mathbf{x}) = \alpha x_i - (1 - \alpha)RD(x_i, \mathbf{x})$  where  $0 < \alpha < 1$ , and let the two income groups be  $A$  with  $\mathbf{x}^A = (1)$ , and  $B$  with  $\mathbf{x}^B = (2, 3)$ . Upon a merger, the relative deprivation of the individual with income 2 is lowered from  $RD(2, (2, 3)) = 1/2$  to  $RD(2, (1, 2, 3)) = 1/3$ . We could reduce this individual's income somewhat and transfer the amount that we skim off to the individual with income 1 whose  $RD$  upon the merger was rising from zero to 1. We know that we cannot take away from the individual with income 2 more than  $1/2$  because if we were, he will have both less income than he had prior to the merger and more  $RD$  than the  $1/2$  that he had prior to the merger. Therefore, we take away less than  $1/2$ , say,  $1/2 - \varepsilon$ , where  $\varepsilon \in (0, 1/2]$  so as to leave the individual no worse off than he were to begin with, ensuring that

$$u\left(2 - \left(\frac{1}{2} - \varepsilon\right), \left(1 + \left(\frac{1}{2} - \varepsilon\right), 2 - \left(\frac{1}{2} - \varepsilon\right), 3\right)\right) \geq u(2, (2, 3))$$

which translates into

$$\alpha\left(\frac{3}{2} + \varepsilon\right) - (1 - \alpha)\frac{1}{3}\left[3 - \left(\frac{3}{2} + \varepsilon\right)\right] \geq \alpha 2 - (1 - \alpha)\frac{1}{2}$$

and which, after simplification, yields the condition

$$\alpha \leq \frac{-2\varepsilon}{4\varepsilon - 3}.$$

We also have that following the transfer, the income of 1 is elevated to  $1 + (1/2 - \varepsilon)$ , and that the  $RD$  of the individual with income 1 (in the income distribution  $\mathbf{x}^{A \cup B} = (1 + (1/2 - \varepsilon), 3/2 - \varepsilon, 3)$ ) is  $1/2 + \varepsilon$ . Seeing to it that the individual with income 1 will not be worse off requires the post-merger, post-transfer wellbeing not to be less than the pre-merger wellbeing  $\alpha \cdot 1$ . That is, we require that

$$\alpha\left(\frac{3}{2} - \varepsilon\right) - (1 - \alpha)\left(\varepsilon + \frac{1}{2}\right) \geq \alpha,$$

or that

$$\alpha \geq \frac{1}{2} + \varepsilon.$$

Upon combining this last condition with the condition for individual with income 2, we get a set of two inequalities

$$\alpha \leq \frac{-2\varepsilon}{4\varepsilon - 3}$$

$$\alpha \geq \frac{1}{2} + \varepsilon.$$

These inequalities cannot, however, be satisfied for  $\varepsilon \in (0, 1/2]$  and  $\alpha \in (0, 1)$ . Here is why.

Consider the function  $g(\varepsilon) = 1/2 + \varepsilon$ . It is a linear function,  $g(1/2) = 1$ , and  $g(0) = 1/2$ . Consider next the function  $f(\varepsilon) = (-2\varepsilon) / (4\varepsilon - 3)$ . We have that  $f(1/2) = 1$ , and  $f(0) = 0$ . Also,  $f'(\varepsilon) = 6 / (4\varepsilon - 3)^2 > 0$ , and  $f''(\varepsilon) = -48 / (4\varepsilon - 3)^3$ . Since  $f''(\varepsilon) > 0$  for  $\varepsilon \in (0, 1/2)$ ,  $f(\varepsilon)$  is a convex function on the interval  $(0, 1/2)$ . It is equal to  $g(\varepsilon)$  in  $\varepsilon = 1/2$  and is lower than  $g(\varepsilon)$  in  $\varepsilon = 0$ . From the convexity property we can be sure that  $f(\varepsilon)$  lies below  $g(\varepsilon)$  in the entire range  $\varepsilon \in (0, 1/2)$ . However, to fulfil the inequalities

$$\alpha \leq \frac{-2\varepsilon}{4\varepsilon - 3}$$

$$\alpha \geq \frac{1}{2} + \varepsilon$$

we would have to find a point where we would be “above”  $g(\varepsilon)$  and, at the same time, “below”  $f(\varepsilon)$ ; from the properties of these functions, this is impossible. The only point in the range of  $\varepsilon$  where these two functions are equal is  $\varepsilon = 1/2$ , but then the solution would be  $\alpha = 1$  which, considering the condition for  $\alpha$ , is not viable.

It is intuitive to see the logic underlying the two inequalities for  $\alpha$ , and to understand the source of the impossibility result. For the individual with income 2 to be content with even a small gain in his *RD* and in spite of a relatively large reduction of his income, his  $\alpha$  must be small. For the individual with income 1

wellbeing of the individual with income  $x_2$  upon the merger, where  $\theta$  is the amount to be transferred to the individual with income  $x_1$ , is

$$\Delta u_2 = \left[ \alpha(x_2 - \theta) - (1 - \alpha) \frac{\max\{(x_1 + \theta) - (x_2 - \theta), 0\} + (x_3 - (x_2 - \theta))}{3} \right] - [\alpha x_2 - (1 - \alpha) \frac{1}{2}(x_3 - x_2)].$$

Since  $\Delta u_2$  is a decreasing function of  $\theta$ , the maximal amount that we could take away from the individual with income  $x_2$  without making him worse off following a merger is the amount  $\theta$  that makes him retain his pre-merger level of wellbeing, that is, an amount such that  $\Delta u_2 = 0$ .

We investigate two possible cases, which correspond to the original ranking being preserved (case 1) or not (case 2).

**Case 1.**  $x_1' = x_1 + \theta \leq x_2 - \theta = x_2'$ , so  $\theta \leq (x_2 - x_1)/2$ . Then:

$$\Delta u_2 = \left[ \alpha(x_2 - \theta) - (1 - \alpha) \frac{(x_3 - (x_2 - \theta))}{3} \right] - [\alpha x_2 - (1 - \alpha) \frac{1}{2}(x_3 - x_2)]$$

and  $\Delta u_2 = 0$  for:

$$\theta = \frac{(1 - \alpha)(x_3 - x_2)}{2 + 4\alpha}.$$

We can then transfer this amount to the individual with income  $x_1$  so his change of wellbeing becomes:

$$\begin{aligned} \Delta u_1 &= \alpha(x_1 + \theta) - (1 - \alpha) \frac{(x_3 - (x_1 + \theta)) + (x_2 - \theta) - (x_1 + \theta)}{3} - \alpha x_1 \\ &= \frac{1 - \alpha}{6 + 12\alpha} (x_1(4 + 8\alpha) - x_2(5 + 4\alpha) - x_3(4\alpha - 1)). \end{aligned}$$

The term  $(1 - \alpha) / (6 + 12\alpha)$  is obviously strictly positive, so we investigate the sign of the term

$$x_1(4 + 8\alpha) - x_2(5 + 4\alpha) - x_3(4\alpha - 1).$$

The term  $(1 - \alpha) / (6 + 12\alpha)$  is obviously strictly positive, so we investigate the sign of the term

$$x_1(4 + 8\alpha) - x_2(5 + 4\alpha) - x_3(4\alpha - 1).$$

Joining the conditions on  $\theta$ :

$$\theta \leq \frac{x_2 - x_1}{2} \quad \text{and} \quad \theta = \frac{(1 - \alpha)(x_3 - x_2)}{2 + 4\alpha}$$

we obtain that

$$x_1 \leq \frac{(2 + \alpha)x_2 + (\alpha - 1)x_3}{1 + 2\alpha}.$$

So we have:

$$\begin{aligned} & x_1(4 + 8\alpha) - x_2(5 + 4\alpha) - (4\alpha - 1)x_3 \\ & \leq \frac{(2 + \alpha)x_2 + (\alpha - 1)x_3}{1 + 2\alpha} (4 + 8\alpha) - x_2(5 + 4\alpha) - (4\alpha - 1)x_3 \\ & = x_2(8 + 4\alpha) + x_3(4\alpha - 4) - x_2(5 + 4\alpha) - (4\alpha - 1)x_3 \\ & = 3(x_2 - x_3) < 0. \end{aligned}$$

We conclude that in this case  $\Delta u_1 < 0$ . Thus, upon a merger, the individual with income  $x_1$  experiences a decrease in his wellbeing even if we transfer to him the largest possible amount from the individual with income  $x_2$ .

**Case 2.**  $x_1' = x_1 + \theta > x_2 - \theta = x_2'$ , so  $\theta > (x_2 - x_1) / 2$ . Then:

$$\begin{aligned} \Delta u_2 = & \left[ \alpha(x_2 - \theta) - (1 - \alpha) \frac{((x_1 + \theta) - (x_2 - \theta)) + (x_3 - (x_2 - \theta))}{3} \right] \\ & - \left[ \alpha x_2 - (1 - \alpha) \frac{(x_3 - x_2)}{2} \right] \end{aligned}$$

and  $\Delta u_2 = 0$  for:

$$\theta = \frac{1}{6}(\alpha - 1)(2x_1 - x_2 - x_3).$$

When we transfer this amount to the individual with income  $x_1$ , we get that

$$\begin{aligned}\Delta u_1 &= \alpha(x_1 + \theta) - (1 - \alpha)\frac{x_3 - (x_1 + \theta)}{3} - \alpha x_1 \\ &= \frac{(1 - \alpha)}{18}(-4x_1(\alpha - 1) + x_2(1 + 2\alpha) - x_3(5 - 2\alpha)).\end{aligned}$$

But from  $x_1 < x_2 < x_3$  we obtain

$$\begin{aligned}(4x_1(1 - \alpha) + x_2(1 + 2\alpha) - x_3(5 - 2\alpha)) \\ < (4x_2(1 - \alpha) + x_2(1 + 2\alpha) - x_2(5 - 2\alpha)) = 0.\end{aligned}$$

So, again, we cannot make  $\Delta u_1$  higher than zero by transferring to the individual with income  $x_1$  an amount which will not harm the individual with income  $x_2$ .

We had implicitly assumed here that  $x_3 > x_1'$ ; otherwise the individual with income  $x_3$  would also have needed compensation, as his deprivation would have increased from zero to a positive value.

This completes the proof that for any populations  $\{x_1\}$  and  $\{x_2, x_3\}$ , such that  $x_1 < x_2 < x_3$ , it is impossible to enact a self-contained tax and transfer policy that will retain post-merger levels of wellbeing at their pre-merger magnitudes.

*Third*, to further see why the implementation of a “tax and transfer” policy will meet hurdles, suppose that 100 individuals with incomes 1 each, that is  $\{1, 1, 1, \dots, 1\}$  join  $\{2, 3\}$ . The gain of the individual with income 2 can, in principle, be “confiscated away”, with the taken income distributed amongst all the individuals with income 1. In order for these individuals to be better off with their extra income, they would need to have a very high  $\alpha$  and a very low disregard for  $RD$ , which is very much against the spirit of our basic presumption that individuals care about relative deprivation considerably.

In sum: in the exhibited cases, a “tax and transfer” scheme cannot achieve its aim because there is not enough of a gain to placate the losers while still keeping the gainers as well off as prior to the merger. In a way, this wellbeing “impossibility result” is akin to the total relative deprivation “superadditivity result”: here as there, aggregate welfare takes a beating.

## 5. Conclusions

As already noted in the Introduction, mergers of populations occur in all spheres of life, and in all times and places. Mergers arise as a result of administrative considerations or naturally, they are imposed or chosen by election. A merger of populations is a far cry from the merger of production lines. The social environment and the social horizons that the individuals who constitute the merged population face change fundamentally upon a merger: others who were previously outside the individuals' social domain are now within. One consequence of this revision of the social landscape, which hitherto appears not to have received attention, is a "built-in" increase in social dismay. Social welfare is affected. Revisiting the European integration example, we contend that in and by itself, this integration can exacerbate social harmony and chip at societal wellbeing in quite unexpected ways. A policy aimed at effectively reversing the deleterious effect of the merger of populations was delineated, illustrated, and evaluated. A tentative conclusion suggested by this assessment is that holding all relevant considerations constant, countering the adverse social welfare effect of the merger of populations may well mandate tapping the government's coffers.

The "superadditivity result" that we derived in this paper is for a specific measure of a population's total relative deprivation (equation (2)). Recalling footnote 2, the appeal of this measure is that it emanates from a solid social-psychological foundation, it rests on a sound axiomatic basis, and it was shown to be empirically significant. Still, a population's total relative deprivation could be measured in a variety of ways and by different indices, and it remains to be checked whether our main claims are yielded by other measures. (For example, it is possible to think about the aggregate of the individual relative deprivations as a weighted rather than as a simple sum, where the weights increase with the extent of the individuals' relative deprivation.) Conversely, the superadditivity property could be considered as an axiom of deprivation indices and if so, incorporating this axiom in the characterization of these indices could yield profound insights about deprivation, and lead to a new class of deprivation indices.

## References

- BOSSERT, WALTER, and CONCHITA D'AMBROSIO (2006), "Reference Groups and Individual Deprivation: An Axiomatic Characterization of Yitzhaki's Index of Individual Deprivation", *Economics Letters* 90 (3), pp. 421–426.
- CLARK, ANDREW E., PAUL FRIJTERS, and MICHAEL A. SHIELDS (2008), "Relative Income, Happiness, and Utility: An Explanation for the Easterlin Paradox and Other Puzzles", *Journal of Economic Literature* 46 (1), pp. 95–144.
- EBERT, UDO, and PATRICK MOYES (2000), "An Axiomatic Characterization of Yitzhaki's Index of Individual Deprivation", *Economics Letters* 68 (3), pp. 263–270.
- FAN, C. SIMON, and ODED STARK (2007), "A Social Proximity Explanation of the Reluctance to Assimilate", *Kyklos* 60 (1), pp. 55–63.
- QUINN, MICHAEL A. (2006), "Relative Deprivation, Wage Differentials and Mexican Migration", *Review of Development Economics* 10 (1), pp. 135–153.
- RUNCIMAN, WALTER G. (1966), *Relative Deprivation and Social Justice*, University of California Press, Berkeley.
- STARK, ODED (1984), "Rural-to-Urban Migration in LDCs: A Relative Deprivation Approach", *Economic Development and Cultural Change* 32 (3), pp. 475–486.
- STARK, ODED, MAJA MICEVSKA, and JERZY MYCIELSKI (2009), "Relative Poverty as a Determinant of Migration: Evidence from Poland", *Economics Letters* 103 (3), pp. 119–122.
- STARK, ODED, and J. EDWARD TAYLOR (1989), "Relative Deprivation and International Migration", *Demography* 26 (1), pp. 1–14.
- STARK, ODED, and J. EDWARD TAYLOR (1991), "Migration Incentives, Migration Types: The Role of Relative Deprivation", *The Economic Journal* 101 (408), pp. 1163–1178.
- STARK, ODED, and QIANG WANG YOU (2007), "Towards a Theory of Self-Segregation as a Response to Relative Deprivation: Steady-State Outcomes and Social Welfare", in Luigino Bruni and Pier Luigi Porta (eds), *Economics and Happiness: Framing the Analysis*, Oxford: Oxford University Press, pp. 223–242.
- STARK, ODED, and SHLOMO YITZHAKI (1988), "Labour Migration as a Response to Relative Deprivation", *Journal of Population Economics* 1 (1), pp. 57–70.
- YITZHAKI, SHLOMO (1979), "Relative Deprivation and the Gini Coefficient", *Quarterly Journal of Economics* 93 (2), pp. 321–324.

## SUMMARY

This paper looks at the integration of regions and nations through the prism of the merger of populations. The paper employs a particular index of social dismay. It presents examples of two of the main results arising from the study of the merger of two populations: that the social dismay of an integrated population is greater than the sum of the social dismay of the constituent populations when apart, and that a self-contained, non-publicly financed policy aimed at retaining the levels of wellbeing of individuals at their pre-merger magnitudes cannot be implemented: there is not enough of a gain to compensate for the loss.