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**Working Paper**

## Savings, asset scarcity, and monetary policy

WWZ Working Paper, No. 2018/13

**Provided in Cooperation with:**

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*Suggested Citation:* Altermatt, Lukas (2018) : Savings, asset scarcity, and monetary policy, WWZ Working Paper, No. 2018/13, University of Basel, Center of Business and Economics (WWZ), Basel, <https://doi.org/10.5451/unibas-ep64166>

This Version is available at:

<https://hdl.handle.net/10419/185815>

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May 2018

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# Savings, asset scarcity, and monetary policy\*

Lukas Altermatt<sup>†</sup>

May 7, 2018

## Abstract

This paper analyzes optimal monetary and fiscal policy in a model where money and savings are essential and asset markets matter. The model is able to match some stylized facts about the correlation of real interest rates and stock price-dividend ratios. The results show that fiscal policy can improve welfare by increasing the amount of outstanding government debt. If the fiscal authority is not willing or able to increase debt, the monetary authority can improve welfare of current generations by reacting procyclically to asset return shocks; however, this policy affects welfare of future generations if it is not coordinated with fiscal policy measures. The model also shows that policies like QE reduce welfare of future generations.

Keywords: *New monetarism, overlapping generations, zero lower bound, optimal stabilization*

JEL codes: E43, E44, E52, G12, G18

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\*I am grateful to my adviser Aleksander Berentsen for his useful comments, which greatly improved the paper, and to my colleagues Mohammed Ait Lahcen, Florian Madison, and Romina Ruprecht for many insightful discussions. I thank Randall Wright, Cyril Monnet, Tim Kroencke, Sarah Lein, Francesca Carapella, Pedro Gomis-Porqueras, Simon Straumann, and seminar participants at the 2018 T2M Conference at Université Paris-Dauphine, the 11th RGS Doctoral Conference in Essen, the Cadi Ayyad University of Marrakech, the University of Bern, and the Alumni Conference in Gerzensee for valuable comments and suggestions.

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# 1 Introduction

In recent years, economists and central bankers have begun to focus their attention much more sharply on asset markets. While there was general consensus before the financial crisis of 2007-2009 that monetary policy should not take asset markets into account<sup>1</sup>, the events of the last decade have altered this opinion. On the one hand, there were claims that monetary policy had in fact caused the financial crisis<sup>2</sup>, and that improvements in monetary policy would be sufficient to prevent similar events from occurring in future. On the other hand, the central banks had started interfering in asset markets with policies such as quantitative easing, so that the effect of such policies on asset markets had to be studied.

There is also another reason why asset markets have become more important for policymakers in recent years. Already before the financial crisis occurred, a global savings glut was being discussed<sup>3</sup>; i.e., people argued that global savings were increasing, while the amount of investment opportunities could not keep pace with the increase. This development can be attributed to demographic changes in developed countries, on one hand, and, on the other hand, to the increased access of residents from developing countries, such as China, to global asset markets.

In the context of these developments, the question whether monetary policy can and should interfere with asset markets became relevant again. In this paper, I want to answer this question. To do so, I build a model in which the two most important features regarding this question are essential: Savings and money. To study the role of savings in this paper, I use the overlapping-generations (OLG) model based on Wallace (1980), as it is the most natural framework. While there are many OLG papers in which money plays a role, money is used only as a savings instrument in most, if not all, of them. Since I want to build a model where money can be used not only as a savings instrument, but also as a medium of exchange instrument for transactions, I combine the OLG structure with the Lagos and Wright (2005) (LW) framework, as money is essential for transactions in this class of models<sup>4</sup>. Combining the OLG and LW frameworks allows me to create a model in which prices for government bonds and risky assets are determined endogenously. In the model, agents can use government bonds (nominal, safe assets), equity (real, risky assets), or fiat money to save. Fiat money is essential for intra-period trade, but it is typically dominated in

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<sup>1</sup>As formulated by, e.g., Bernanke and Gertler (2001)

<sup>2</sup>e.g., by Taylor (2014).

<sup>3</sup>e.g., by Bernanke (2005).

<sup>4</sup>There have been a few other papers that combined the OLG structure with the money search environment of LW, such as Zhu (2008), Jacquet and Tan (2011), Waller (2009), or Hiraguchi (2017). However, none of them studied questions similar to the ones that I study in this paper, and the exact details regarding how the models are combined also differ compared to my model.

terms of the rate of return by the other assets, so it is not used for savings in equilibrium, except at the zero lower bound. Although bonds cannot yield a liquidity premium in the model (they cannot be used for intra-period trade), a zero lower bound (i.e., a situation where the interest rate on bonds is equal to zero) can occur. In the model, an economy hits the zero lower bound solely due to an increase in the stochastic discount factor (SDF), which is driven by a scarcity of savings instruments. Similarly, such a scarcity of savings instruments can lead to an increase in asset prices. Such an asset price increase might look like a bubble, but it simply reflects a shortage in the supply of savings instruments relative to demand. In fact, this can be understood as a portfolio balance effect, as an agents' optimal portfolio would consist of only government bonds, but once the supply of bonds is too low and hence the SDF (and thus the price of bonds) increases, agents start demanding risky assets, which in turn increases their price as well.

In this paper, I first study an economy with only fiat money and government bonds available, and then an economy with only fiat money and risky assets available, in order to understand some basic concepts. Most of the important effects, e.g., the zero lower bound or an increase in the price of risky assets, already occur in these simple versions of the model. After that, I analyze an economy where all three assets are available and study optimal monetary and fiscal policies in that context.

Since I want to answer questions that are relevant for policymakers, it is important that the model I use is not only logically sound, but also able to match some stylized facts with regard to the topic at hand, i.e., the effects of monetary policy on asset markets. Analyzing the data shows that since at least 1980, the dividend-price ratios of equity and real interest rates were negatively correlated in the United States. Around the beginning of the financial crisis however, this correlation ceased to exist. The regime change occurred more or less at the same time as the zero lower bound was hit. In my model, there are several equilibrium regimes. In some of them, asset prices and real interest rates are negatively correlated, while in the other regimes they are uncorrelated. Some of these regimes are only able to occur at the zero lower bound. There are two reasons in the model why an economy can move from a regime with negative correlation away from the zero lower bound to a regime without correlation at the zero lower bound: (1) a decrease in inflation, or (2) a simultaneous decrease in the supply of safe assets accompanied by an increase in the supply of risky assets. I argue that both of these situations occurred during and after the financial crisis. Inflation decreased in the United States as well as in most other developed economies, and many assets that were considered safe before the financial crisis, such as mortgage-backed securities or sovereign bonds from southern European countries, turned out to be risky at that time. Thus, the model is able to match the transition from a world where dividend-price ratios and real interest

rates were strongly correlated to one where they are uncorrelated.

The results I find for optimal monetary and fiscal policy are novel and interesting: As it is standard in this type of models, the Friedman rule (i.e., setting the opportunity cost of holding money to zero) allows the first best to be achieved. Away from the Friedman rule, the fiscal authority can increase welfare by issuing a sufficient number of government bonds. However, in reality it may not be feasible politically to run the Friedman rule or to issue a large amount of public debt. If that is the case, then the monetary authority can use an optimal stabilization policy which applies different inflation rates after equity market shocks in order to increase welfare. The optimal policy is procyclical, i.e., inflation should be set higher when stock market returns are high and lower when stock market returns are low. Such a policy is able to minimize or even completely eliminate the risk that savers face by creating a negative correlation between the returns of nominal government bonds and real assets. However, if the monetary authority runs this policy on its own, then it will also affect the welfare of future generations by changing the bonds-to-money ratio. If the fiscal authority is willing to cooperate with the monetary authority, the effects on the welfare of future generations can be mitigated. To do this, the fiscal authority has to set the growth rate of government bonds such that it equals the fiat money growth rate in every period, as this keeps the bonds-to-money ratio constant.

Besides this main result, there are two other interesting findings regarding monetary policy: The model shows that (1), quantitative easing reduces the welfare of future generations, because it reduces the returns on all savings instruments; and (2), increasing the inflation target reduces the risk of hitting the zero lower bound, but still decreases welfare overall.

## 1.1 Related Literature

This paper is mainly related to the literature on monetary policy and asset prices. As already mentioned above, there was a general consensus before the financial crisis that monetary policy should not directly react to asset markets. After the crisis, this view was challenged by several authors. In an empirical analysis with long time series, Schularick and Taylor (2012) show that periods of financial instability are often caused by credit booms that have gone wrong, and they advocate that monetary policy be employed to control such booms. However, Ajello et al. (2015) conclude in a recent empirical analysis that the optimal monetary policy response to credit conditions is only marginally different (namely 10 basis points in their baseline specification of the model) from a response that exclusively takes price level stability and output into account. To find that result, they used a two-period New Keynesian model to which they added an equation describing credit

conditions, in addition to the standard equations for the output gap and inflation. In another study, Stein (2012) creates a model in which private money creation by banks leads to an externality. In some environments which Stein studies, conventional monetary policy is not enough to solve the problem, and instead additional regulation is needed. In Stein's model, agents are risk-neutral, but derive additional monetary services from holding riskless assets. Stein then shows that private banks are able to create such assets to a certain extent, and that they will create more of them than the socially optimal amount. This can then lead to fire sales in some states of the world, which in turn creates the externality. Nistico (2012) found that an interest-rate rule which responds to deviations in stock prices could lead to additional instability. Gali (2014, 2017) uses an OLG framework to study the link between monetary policy and rational asset price bubbles. He finds that a stronger interest rate response to bubble fluctuations can, surprisingly, increase the volatility of the bubble component, and that stabilization of the bubble itself calls for a negative interest rate response. In the more recent paper, he finds that an interest rate rule which responds directly to the bubble can succeed in fine-tuning the economy, but only if measurement is precise and the rule's parameters are exactly calibrated. In contrast, a policy that directly targets inflation attains the same stabilization effects without some of the destabilizing risks of the interest rate rule.

My paper is also similar in some ways to the work of Caballero et al. (2017) and especially Caballero and Farhi (2017). In their paper, Caballero and Farhi show that a shortage of safe assets can lead to a situation that they label the safety trap, which is similar to a liquidity trap but has even more severe effects. Such a safety trap is deflationary and leads to sharp decreases in output in their model. To obtain these results, Caballero and Farhi add nominal rigidities, two types of agents and financial frictions to a perpetual youth OLG model. Although the model I present here shares similarities with Caballero and Farhi (2017), the focus is different, as I am more interested in the behavior of asset prices and what consequences these have for agents, and subsequently how monetary policy can improve the outcome.

Other papers studying the effects of shortages of safe assets are Caballero et al. (2008), Caballero and Krishnamurthy (2009), Bernanke et al. (2011), Barro and Mollerus (2014), and He et al. (2015). The macroeconomic effects caused by a shortage of safe assets found in these studies mostly correspond to the results from Caballero and Farhi (2017).

My paper is also related to the literature on liquidity traps, e.g., Krugman et al. (1998), Eggertsson and Woodford (2003, 2004), Eggertsson and Krugman (2012), Williamson (2012, 2016), Rocheteau et al. (2016), Guerrieri and Lorenzoni (2017), Cochrane (2017), Geromichalos and Herrenbrueck (2017), or Altermatt (2017). While some of these papers (e.g., Williamson (2012, 2016)) focus on

the liquidity services provided by bonds, others such as Eggertsson and Krugman (2012) explain the liquidity trap by financial frictions such as tightened borrowing constraints. In the model presented here, economies do not become stuck at the zero lower bound for these reasons, but simply because certain assets are essential for savings and thus agents are willing to pay a premium for them.

## 1.2 Data

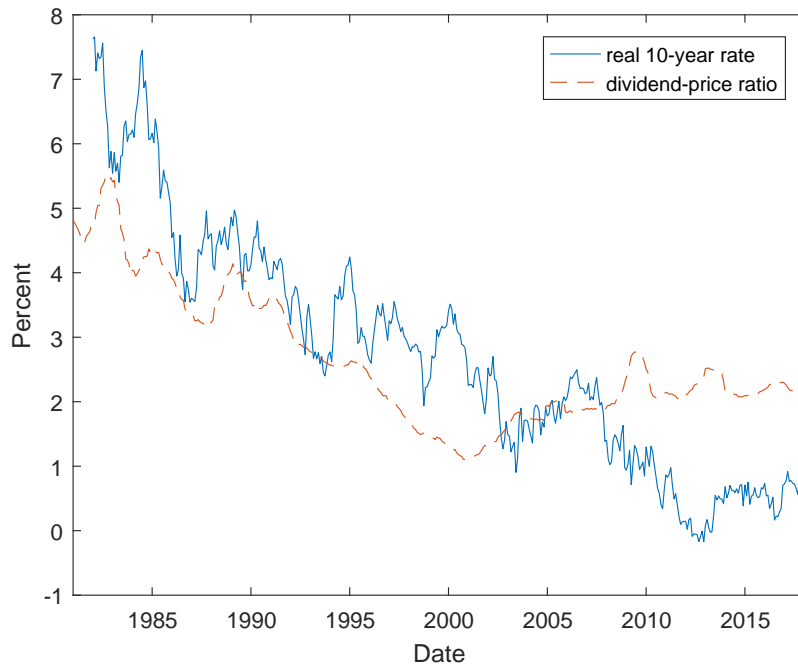


Figure 1: Real ten-year interest rates and the dividend-price ratio for US equity.

As mentioned above, the model is able to match stylized facts about the correlation between the dividend-price ratio<sup>5</sup> of stocks and the real interest rate. Figure 1 shows the real ten-year interest rate for the United States and the dividend-price ratio for US-traded equity from 1982 until 2017<sup>6</sup>. It is clearly visible from the graph that there is a strong correlation between these

<sup>5</sup>While the price-dividend ratio is the more common measure, I use the dividend-price ratio here, because the correlation is more clearly evident in this case. Given that both the real interest rates and the price-dividend ratio depend negatively on the respective asset prices, a positive correlation between these variables therefore also indicates a positive correlation between bond and asset prices, other things equal.

<sup>6</sup>Data on real interest rates is taken from the Federal Reserve Bank of Cleveland. The data on the dividend-price ratio is based on the Center for Research in Security Prices' (CRSP) measures of value-weighted return including and excluding dividends for all equities traded in the U.S. The calculation of the dividend-price ratio follows John Cochrane: <https://johncochrane.blogspot.ch/2018/02/stock-gyrations.html>. His blog post also contains the graph presented here.



two time series. The correlations for the whole sample as well as for two subsamples are listed in table 1.

<b>Time period</b>	<b>correlation</b>
1982-2017	0.738
1982-2007	0.870
2008-2017	-0.029

Table 1: Correlations between real interest rates and dividend-price ratios.

Table 1 shows that there is a strong positive correlation for the whole sample. However, a closer inspection shows that the correlation was even stronger from 1982 until 2007, but that it vanished afterwards, approximately at the time the financial crisis occurred. Since then, the dividend-price ratio has been quite stable, while the real interest rate decreased further until approximately 2013, and has subsequently recovered. This suggests that there was a structural break in the relationship between (expected) inflation, bond prices, and asset prices during or after the financial crisis. As already explained above, my model is well suited to explain such structural breaks, as it exhibits several different equilibrium regimes.

The rest of the paper is organised as follows. In Section 2, the basic environment is explained. Section 3 presents the economy with only government bonds and fiat money, while Section 4 presents an economy with only risky assets and fiat money. In Section 5, an economy with all three assets is analyzed. In Section 6, I analyze the optimal monetary and fiscal policy in the model with all assets present. Finally, Section 7 concludes.

## 2 The basic environment

Time is discrete and continues forever. Each period is divided into two subperiods, called centralized market (CM) and decentralized market (DM). At the beginning of a period, the CM takes place, and after the CM closes, the DM opens and remains open until the period ends. In any period  $t$ , there is a measure  $N_t$  of buyers born. Buyers live for three subperiods, i.e., generation  $t$  buyers are born at the beginning of the CM of period  $t$ , continue to the DM of period  $t$ , and then to the CM of period  $t + 1$ , at the end of which they die. There is also a measure  $N_t$  of infinitely lived sellers alive in period  $t$ . The population growth rate is assumed to be constant over time and is defined as  $\frac{N_t}{N_{t-1}} = n$ . Young buyers and all sellers are able to produce a general good  $x$  in the

CM. Sellers can produce a special good  $q$  in the DM that gives utility to buyers. In the DM, young buyers and sellers are matched bilaterally, and buyers can make take-it-or-leave-it offers. In the CM, a centralized market exists for the general goods produced by young buyers and sellers, and sellers as well as old buyers gain utility from consuming them. Neither general goods nor special goods can be stored by agents. The preferences of buyers are given by

$$\mathbb{E}_t \left\{ -h_t^y + u(q_t^y) + \beta U(x_{t+1}^o) \right\}. \quad (1)$$

Equation (1) states that buyers obtain disutility  $h$  from producing in the CM, obtain utility  $u(q)$  from consuming in the DM and  $U(x)$  from consuming in the CM when they are old, and that they discount the second period of their life by a factor  $\beta \in (0, 1)$ . Actions occurring in the first period of a buyer's life (when young) are indicated by a superscript  $y$ , while actions occurring in the second period of a buyer's life (when old) are indicated by a superscript  $o$ . Assumptions on the utility functions are that  $U(0) = u(0) = 0$ ,  $U'(0) = u'(0) = \infty$ ,  $u'(q) > 0$ ,  $u''(q) < 0$ ,  $U'(x) > 0$ , and  $U''(x) < 0$ .

The preferences of the sellers are given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (x_t^s - h_t^s - c(q_t)). \quad (2)$$

Equation (2) states that sellers discount future periods by a factor  $\beta \in (0, 1)$ , obtain linear utility from consuming  $x$  in the CM, a linear disutility  $h$  from producing in the CM, and disutility  $c(q)$  from producing in the DM, with  $c(0) = 0$ ,  $c'(0) = 0$ ,  $c'(q) > 0$ ,  $c''(q) > 0$ , and  $c(\bar{q}) = u(\bar{q})$  for some  $\bar{q} > 0$ . Furthermore, I define  $q^*$  as  $u'(q^*) = c'(q^*)$  and  $x^*$  as  $U'(x^*) = 1$ , i.e., as the socially efficient quantities. The variables relating to the sellers are indicated by a superscript  $s$ . I assume that sellers cannot commit to any future payments.<sup>7</sup>

There is also a monetary and a fiscal authority. The monetary authority issues fiat money  $M_t$ , which it can costlessly produce. Actions of the monetary authority always take place at the beginning of the period. The amount of general goods that one unit of fiat money can buy in the CM of period  $t$  is denoted by  $\phi_t$ , the inflation rate is defined as  $\phi_t/\phi_{t+1} - 1 = \pi_{t+1}$ , and the growth

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<sup>7</sup>While the concept of having these two types of agents is borrowed from the LW framework, there are a couple of interpretations for them in the context of this model. One is that buyers can be considered as households with a finite lifespan, while sellers are akin to firms. Another interpretation is that young buyers are middle-aged agents who need to save for retirement, and old buyers are retired agents. In this interpretation, sellers can be considered young agents who still have a long investment horizon and are thus able to focus on expected returns only. One could even add a transition (e.g., some probability each period) for sellers to become buyers. Since the preferences of a young buyer and a seller in the CM are similar, this would not change the model.

rate of fiat money from period  $t - 1$  to  $t$  is  $\frac{M_t}{M_{t-1}} = \gamma_t$ . Newly-printed money is distributed as a lump-sum transfer to young buyers. The real value of these transfers is denoted by  $\Delta_t$ . Agents' money holdings are denoted as  $m_t$ .

The fiscal authority has to finance some spending  $g_t$  in each period, and can do so by levying lump-sum, per capita taxes  $T_t$  (raised on all agents, i.e., young and old buyers as well as sellers) or by issuing one-period bonds. If the government issues bonds, they are sold for the market-clearing price  $\rho_{b,t}$  and redeemed for one unit of fiat money in the next period. This gives rise to the following government budget constraint:

$$\phi_t \rho_{b,t} B_t + \left( \frac{1}{n} + 2 \right) N_t T_t = \phi_t B_{t-1} + g_t. \quad (3)$$

It is assumed that the government exogenously decides whether to finance its expenditures through debt or taxes. Specifically, I will assume in some sections that the supply of bonds is either zero or strictly positive in all periods. I define the net real lump-sum tax to agents as  $\tau_t = T_t - \Delta_t$ . The growth rate of bonds is defined as  $\frac{B_t}{B_{t-1}} = \gamma_t^B$ , and  $\mathcal{B}_t = \frac{B_t}{M_t}$  denotes the bonds-to-money ratio.

In Section 5, I assume that there is also an endowment of risky assets with aggregate value  $A_t$  available in the economy. For simplicity, I will assume that the sellers are endowed with the risky assets at the beginning of the CM of each period. These risky assets are perfectly divisible. In the following period, the assets pay a high return  $\kappa^H$  with probability  $\chi$  and a low return  $\kappa^L$  with probability  $1 - \chi$ . The return is an aggregate shock; i.e., when a return is realized in a given period, it is the same for all the assets in the economy. Thus, there is also no private information about the return on an asset. After the realization of the shock, the assets pay out the real return and cease to exist; i.e., they are replaced by a new set of assets that is independent of the old set in each period.

These assets are intended to represent the aggregate stock market; i.e.,  $A_t$  represents the unconstrained value of all outstanding equities in period  $t$ . In other words,  $A_t$  is the universe of risky investments in the world, and thus already perfectly diversified.

In the next section, I will analyze an economy where only bonds and fiat money are available for agents to save. After that, I will look at an economy where risky assets and fiat money are present, and then I will finally allow agents to use all three assets.

### 3 The economy with bonds

In this section, I will assume that the fiscal authority finances some share of its expenditure according to equation (3) by issuing bonds, and that the bonds are sold for the price that clears the market. Furthermore, I assume that there are no risky assets available in this economy; i.e.  $A_t = 0 \forall t$ , so that the only available assets are bonds and fiat money.

#### 3.1 The buyer's problem

A buyer has to decide how many goods he wants to produce in the first CM, in order to acquire fiat money and bonds for consumption in the later phases of his life. A buyer's value function when he is young  $W^{y,b}$  is

$$W^{y,b} = \max_{h_t^y, m_t^y \geq 0, b_t^y, d_t^y} -h_t^y + u(q(\phi_{t+1}d_t^y)) + \beta W^{o,b}(b_t^y, m_t^y)$$

$$s.t. \quad h_t^y - \tau_t = \phi_t(m_t^y + d_t^y + \rho_{b,t}b_t^y).$$

Here,  $h_t$  denotes the goods produced in the CM,  $d_t^y$  denotes the money holdings that a young buyer plans to take to the DM,  $m_t^y$  denotes the money holdings that he saves for the next period, and  $b_t^y$  denotes his bond holdings. Note that the DM money and the savings money are quantities of the same object, but I use two different variables for them instead of the sum, which makes it simpler to solve the problem. Note also that all decision variables of the buyer's problem are subject to a nonnegativity constraint, but only the one on money as a means of saving might be binding in equilibrium, which is why I only made this constraint explicit. The function  $q(\phi_{t+1}d_t^y)$  depends on the terms of trade in the DM and will be made explicit later. Finally,  $W^{o,b}(b_t)$  denotes the value function of an old buyer with bond holdings  $b_t$ , which is simply

$$W^{o,b}(b_t^y) = \max_{x_{t+1}^o} U(x_{t+1}^o)$$

$$s.t. \quad \phi_{t+1}(m_t^y + b_t^y) = x_{t+1}^o.$$

Here,  $x_{t+1}^o$  denotes the CM consumption of an old buyer. The old buyer sells all his assets in the CM and consumes the rewards.

By substituting in the constraints and the value function of the old buyer, we obtain the lifetime value function of a buyer:

$$W^b = \max_{d_t^y, m_t^y \geq 0, x_{t+1}^o} - \left( \phi_t d_t^y + \phi_t (1 - \rho_{b,t}) m_t^y + \frac{\phi_t}{\phi_{t+1}} \rho_{b,t} x_{t+1}^o \right) + u(q(\phi_t m_t^y)) + \beta U(x_{t+1}^o). \quad (4)$$

### 3.1.1 The DM problem

To find the first-order conditions for the buyer's problem, we need to know the terms of trade in the DM. As stated above, it is assumed that buyers can make take-it-or-leave-it offers to sellers. Their offer has to satisfy the sellers' participation constraint. This gives rise to the following maximization problem:

$$\begin{aligned} \max_{q_t, d_t} \quad & u(q_t) - \beta \phi_{t+1} d_t \\ \text{s.t.} \quad & -c(q) + \beta \phi_{t+1} d_t \geq 0. \end{aligned}$$

The sellers' participation constraint will always be binding, so that the solution to this problem is

$$q = c^{-1}(\beta \phi_{t+1} d_t). \quad (5)$$

### 3.1.2 Solution to the buyer's problem

Now, we can substitute equation (5) in the buyer's lifetime value function given by (4) and solve the maximization problem to obtain the following first-order conditions:

$$\begin{aligned} d_t^y : \quad & 1 = \beta \frac{\phi_{t+1}}{\phi_t} \frac{u' \circ c^{-1}(\beta \phi_{t+1} d_t^y)}{c' \circ c^{-1}(\beta \phi_{t+1} d_t^y)} \\ m_t^y : \quad & 1 \geq \rho_{b,t} \\ x_{t+1}^o : \quad & 1 = \frac{\beta}{\rho_{b,t}} \frac{\phi_{t+1}}{\phi_t} U'(x_{t+1}^o). \end{aligned}$$

The first-order condition for DM money is the standard result from Lagos and Wright (2005) and captures the trade-off between consumption and the inflation tax. The first-order condition for money as a means of saving shows that agents only use it if the bond price equals 1. The first-order condition on consumption for an old buyer yields a pricing formula for bonds that says that the bond price depends on inflation, and on a stochastic discount factor that consists of the actual discount factor  $\beta$  and the marginal utility of old-age consumption<sup>8</sup>.

<sup>8</sup>By defining the stochastic discount factor (SDF) as  $\Lambda_t \equiv \beta U'(x_{t+1}^o)$ , one can write the price of nominal bonds

### 3.2 Bond market clearing

For the bond market to clear, the price of bonds has to adjust such that agents are willing to hold all bonds. The demand for bonds  $b_t^y$  by buyers is given by

$$b_t^y = \frac{x_{t+1}^o}{\phi_{t+1}} - m_t^y.$$

This can be seen directly in the constraint on the value function for old buyers. Since only young buyers demand bonds, total demand for bonds by buyers is given by  $N_t b_t^y$ .

Sellers will only hold bonds if there is no cost to hold them. This means that sellers only hold bonds if  $\rho_{b,t} \leq \frac{\beta}{1+\pi_{t+1}}$ . However, if  $\rho_{b,t} < \frac{\beta}{1+\pi_{t+1}}$ , sellers want to hold an infinite amount of bonds. Since the supply of bonds is finite, the price of bonds will be driven up until  $\rho_{b,t} = \frac{\beta}{1+\pi_{t+1}}$ , which if we interpret  $1/\beta$  as the real interest rate, can be called the Fisher equation (Fisher, 1930). This creates a lower bound on the price of bonds. The amount of bonds held by an individual seller is denoted as  $b_t^s$ , so that the total demand for bonds by sellers is  $N_t b_t^s$ . Now, we can add up the total demand for bonds to determine the market clearing condition:

$$x_{t+1}^o - \phi_{t+1} m_t^y + \phi_{t+1} b_t^s = \frac{\phi_{t+1} B_t}{N_t} \quad (6)$$

with  $b_t^s = 0$  if  $\rho_{b,t} > \frac{\beta}{1+\pi_{t+1}}$ , and  $b_t^s = \left( \frac{B_t}{N_t} + m_t^y - \frac{x_{t+1}^o}{\phi_{t+1}} \right)$  otherwise.

Equation (6) shows that an increase in the supply of bonds  $B_t$  has to be offset by an increase in  $x_{t+1}^o$  if  $1 > \rho_{b,t} > \frac{\beta}{1+\pi_{t+1}}$ . From the solution to the buyer's problem, we know that  $x_{t+1}^o$  is decreasing in the price of bonds. Thus, an increase in the supply of bonds will result in a decrease in the price of bonds. This mechanism is at work until the supply of bonds is high enough for the bond price to fall to  $\rho_{b,t} = \frac{\beta}{1+\pi_{t+1}}$ . From that point onwards, a further increase in the supply of bonds will only lead to an increase in  $b_t^s$ . I therefore label this lower bound for bond prices as the unconstrained bond price, and it is defined by  $\rho_b^* \equiv \frac{\beta}{1+\pi_{t+1}}$ . The corresponding minimal amount of bonds required to reach  $\rho_b^*$  is denoted as  $\mathcal{B}^*$ . From the first-order condition for  $x_{t+1}^o$ , it can be seen that old agents consume the efficient quantity of CM goods if bonds are priced at  $\rho_b^*$ , which means that if the supply of bonds is at least  $\mathcal{B}^*$ , CM consumption is efficient.

While an increase in the supply of bonds leads to a decrease in the price of bonds if that price is not yet at the lower bound, a decrease in the supply of bonds leads to an increase in the price of bonds as a function of the SDF and inflation only, as is standard practice in finance. All price changes in this model occur through changes in the SDF, but because the SDF is an endogenous variable, I will not use this notation in the remainder of the paper.

bonds. However, the price of bonds is also bounded above, namely by  $\rho_{b,t} = 1$ . This is because at that price, holding bonds and fiat money is equally costly. Since fiat money and bonds are equally suitable savings instruments as long as their prices are equal, agents are never willing to pay a higher price for bonds than this. Instead, they would start using fiat money to save if the supply of bonds is not high enough for the bond market to clear at a price  $\rho_{b,t} = 1$ . I will denote the maximal amount of bonds that leads to a bond price of  $\rho_{b,t} = 1$  as  $\underline{\mathcal{B}}$ . This upper bound in bond prices corresponds to a lower bound in the bond interest rate, i.e., the zero lower bound.

Note that at the Friedman rule ( $1 + \pi_{t+1} = \beta$ ), the upper and lower bound of the bond price collapse into one, leaving only  $\rho_{b,t} = 1$  as a possibility.

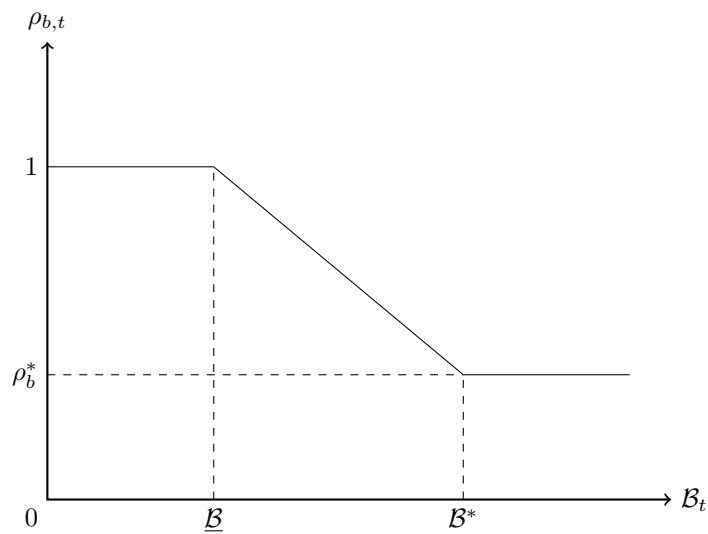


Figure 2: Bond price as a function of supply.

Figure 2 shows the bond price as a function of the supply of bonds for a given inflation rate. between  $\underline{\mathcal{B}}$  and  $\mathcal{B}^*$ , the price of bonds decreases with increases in the supply of bonds, while the bond price is equal to its upper (lower) bound if supply is lower than  $\underline{\mathcal{B}}$  (higher than  $\mathcal{B}^*$ ). If inflation decreases,  $\underline{\mathcal{B}}$  shifts to the right while  $\rho_b^*$  increases, meaning that the range where bond prices change after changes in supply becomes smaller.

### 3.3 Real interest rates

As I want to look at the correlation between asset prices and real interest rates later in the model, it is important to define the real interest rate. I consider

$$1 + r^n = \frac{1}{\beta} \tag{7}$$

to be the natural real interest rate.  $r^n$  is a benchmark that is given purely by agents' preferences. The actual real interest rate in the model is given by

$$1 + r_t = \frac{1}{\rho_{b,t}} \frac{\phi_{t+1}}{\phi_t}. \quad (8)$$

At  $\rho_b^*$ ,  $r_t = r^n$ .

### 3.4 Money market clearing

Next, I want to state the money market clearing condition:

$$\phi_t M_t = N_t z_t^y. \quad (9)$$

Here,  $z_t^y = \phi_t(m_t^y + d_t^y)$  is total real demand for fiat money, given by the real balances of young buyers. Sellers acquire no money in the CM.  $\phi_t M_t$  denotes the supply of fiat money.

#### 3.4.1 Steady-state inflation

In a steady state, the rate of return on money is constant over time and equals

$$\frac{1}{1 + \pi_{t+1}} = \frac{\phi_{t+1}}{\phi_t} = \frac{\frac{N_{t+1} z_{t+1}^y}{M_{t+1}}}{\frac{N_t z_t^y}{M_t}} = \frac{n}{\gamma}. \quad (10)$$

Thus, inflation only depends on the growth rates of money and the population in steady state.

### 3.5 Steady-state equilibrium

In a steady state, all real per-capita variables are constant, which means that time subscripts can be dropped. Additionally, we can also plug in the steady-state value for the rate of return on money. Thus, the relevant conditions are:

$$1 = \beta \frac{n}{\gamma} \frac{u' \circ c^{-1}(\beta \frac{n}{\gamma} z_d^y)}{c' \circ c^{-1}(\beta \frac{n}{\gamma} z_d^y)} \quad (11)$$

$$1 \geq \rho_b \quad (12)$$

$$1 = \frac{\beta}{\rho_b} \frac{n}{\gamma} U'(x^o), \quad (13)$$

where  $z_d^y = \phi_t d_t^y$ , i.e., real balances held for transaction purposes.

Now we can define an equilibrium in this economy:



**Definition 1.** *An equilibrium is a sequence of prices  $\rho_{b,t}$ , and quantities  $\phi_t m_t^y$ ,  $\phi_t d_t^y$ ,  $x_t^o$ , and  $b_t^s$ , that simultaneously solve the equations (11) and (13), as well as the inequalities (12) and (6) and the corresponding complementary slackness conditions  $\forall t$ .*

Since the left-hand side of the bond-market clearing condition (equation (6)) consists only of real variables, its right-hand side has to be constant over time in a steady state. This implies:

$$\frac{B_{t+1}}{B_t} = \frac{N_{t+1}}{N_t} \frac{\phi_{t+1}}{\phi_{t+2}} = n \cdot \frac{\gamma}{n} = \gamma.$$

This shows that the growth rate of bonds has to equal the growth rate of fiat money for a steady-state to exist, which corresponds to a constant bonds-to-money ratio,  $\mathcal{B}_t = \mathcal{B} \forall t$ .

As can easily be seen in equation (11), the real balances that are chosen by young buyers only depend on the inflation rate that is prevalent in the economy. Old buyers' CM consumption, however, also depends on the price of bonds and thus also depends on the bond market clearing. If the bond supply in this economy is plentiful, the demand for bonds cannot be absorbed by buyers only, and thus the bond price will be at its unconstrained level, which in turn allows for first-best consumption in the CM according to equation (13). If bonds become scarce and their price increases above the Fisher equation, the CM consumption of old buyers decreases.

As just shown, the version of the model that only contains bonds and fiat money is relatively simple and straightforward. However, a zero lower bound can occur even in this simple model, and the concepts developed about bond market clearing still hold in the full model. In the next section, I want to analyze an economy which only has risky assets and fiat money in order to, once again, present some basic concepts. After that, I will analyze the full model with both bonds and risky assets present.

## 4 The economy with risky assets

In this section, I will assume that the fiscal authority finances all its expenditures by raising taxes and does not issue any bonds. However, there is some endowment  $A_t > 0$  of risky assets held by sellers each period. Thus, buyers can only use risky assets or fiat money to save in this version of the model. Before analyzing the buyers' problem, I want to analyze the nature of the risky assets a little further.

I first want to define an unconstrained price of the risky assets, which is the price at which a

risk-neutral agent is indifferent to holding these assets. This is the case if the price of the asset exactly equals its discounted expected return, so that

$$\rho_{a,t} = \beta (\chi \kappa^H + (1 - \chi) \kappa^L). \quad (14)$$

For the remainder of this paper, I will assume that  $1 = \beta(\chi \kappa^H + (1 - \chi) \kappa^L)$ . This normalizes the unconstrained price of the risky asset to 1, so whenever a price  $\rho_{a,t} > 1$  is observed, the asset is traded above its unconstrained value. Note that this also implies  $\kappa^H \geq \frac{1}{\beta}$  and  $\kappa^L \leq \frac{1}{\beta}$ .

At the unconstrained price  $\rho_{a,t} = 1$ , a seller is indifferent between holding the asset and selling it. Thus whenever the assets are priced at their unconstrained value, sellers absorb any risky assets that are not demanded by buyers. At  $\rho_{a,t} > 1$ , sellers strictly prefer to sell the assets, and this implies that to observe such prices, all risky assets must be held by buyers.

#### 4.1 The buyers' problem

The buyers' problem here is pretty similar to the one analyzed in the previous section, except that we now also have to take the riskiness of the assets into account. A young buyer solves the problem

$$\begin{aligned} W^{y,a} &= \max_{h_t^y, d_t^y, m_t^y \geq 0, a_t^y} -h_t^y + u(q(\phi_{t+1} d_t^y)) + \beta [\chi W^{o,a}(\kappa^H a_t^y, m_t^y) + (1 - \chi) W^{o,a}(\kappa^L a_t^y, m_t^y)] \\ \text{s.t.} \quad & h_t^y - \tau_t = \phi_t(m_t^y + d_t^y) + \rho_{a,t} a_t^y. \end{aligned}$$

For old buyers, the problem is simply

$$\begin{aligned} W^{o,a}(\kappa_{t+1} a_t^y, m_t^y) &= \max_{x_{t+1}^o} U(x_{t+1}^o) \\ \text{s.t.} \quad & x_{t+1}^o = \kappa_{t+1} a_t^y + \phi_{t+1} m_t^y. \end{aligned}$$

Now, we can combine the two value functions in a lifetime problem by substituting the assets<sup>9</sup> in the two budget constraints:

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<sup>9</sup>The problem of determining what to substitute to combine the value functions is not trivial, as depending on the rates of return, buyers might choose to only save one type of asset. As will be shown later, buyers are willing to use a positive amount of risky assets for their savings as long as the expected return on the risky assets is higher than the return on fiat money. For any inflation rate above the Friedman rule, this condition is fulfilled at the unconstrained asset price. But since prices above the unconstrained level can only be observed if buyers hold all risky assets, we can conclude that buyers demand at least some share of risky assets for any inflation rate above the Friedman rule. This analysis thus does not hold at the Friedman rule, but at any other feasible rate of inflation.

$$\begin{aligned}
W^a = & \max_{\substack{h_t^y, d_t^y, \\ x_{t+1}^H, x_{t+1}^L, m_t^y \geq 0}} - h_t^y + u(q(\phi_{t+1}d_t^y)) + \beta \left[ \chi U(x_{t+1}^H) + (1 - \chi)U(x_{t+1}^L) \right] \\
\text{s.t. } & h_t^y - \tau_t = \phi_t(m_t^y + d_t^y) + \frac{\rho_{a,t}}{\kappa^H} (x_{t+1}^H - \phi_{t+1}m_{t+1}) \quad (\lambda^H) \\
\text{s.t. } & h_t^y - \tau_t = \phi_t(m_t^y + d_t^y) + \frac{\rho_{a,t}}{\kappa^L} (x_{t+1}^L - \phi_{t+1}m_{t+1}) \quad (\lambda^L).
\end{aligned}$$

The buyer's maximization problem gives rise to the following first-order conditions:

$$\begin{aligned}
h_t : & \quad 1 = \lambda^H + \lambda^L \\
d_t^y : & \quad \phi_t(\lambda^H + \lambda^L) = \beta \phi_{t+1} \frac{u' \circ c^{-1}(\beta \phi_{t+1} d_t^y)}{c' \circ c^{-1}(\beta \phi_{t+1} d_t^y)} \\
m_t^y : & \quad \frac{\phi_t}{\phi_{t+1}} (\lambda^H + \lambda^L) \geq \frac{\rho_{a,t}}{\kappa^H} \lambda^H + \frac{\rho_{a,t}}{\kappa^L} \lambda^L \\
x_{t+1}^H : & \quad \lambda^H \frac{\rho_{a,t}}{\kappa^H} = \beta \chi U'(x_{t+1}^H) \\
x_{t+1}^L : & \quad \lambda^L \frac{\rho_{a,t}}{\kappa^L} = \beta (1 - \chi) U'(x_{t+1}^L).
\end{aligned}$$

I made use of the solution to the DM problem as derived in Section 3.1.1, since that problem is not affected by the type of assets present in the economy. By replacing the Lagrange multipliers, the following five equilibrium conditions are obtained:

$$1 = \beta \frac{\phi_{t+1}}{\phi_t} \frac{u' \circ c^{-1}(\beta \phi_{t+1} d_t^y)}{c' \circ c^{-1}(\beta \phi_{t+1} d_t^y)} \quad (15)$$

$$\frac{1}{\beta} \frac{\phi_t}{\phi_{t+1}} \geq \chi U'(x_{t+1}^H) + (1 - \chi) U'(x_{t+1}^L) \quad (16)$$

$$\frac{\rho_{a,t}}{\beta} = \kappa^H \chi U'(x_{t+1}^H) + \kappa^L (1 - \chi) U'(x_{t+1}^L) \quad (17)$$

$$\phi_{t+1} m_t^y = \frac{\kappa^L x_{t+1}^H - \kappa^H x_{t+1}^L}{\kappa^L - \kappa^H}. \quad (18)$$

As in the previous section, the choice of DM money holdings is independent of all other endogenous variables and determined entirely by equation (15). Equation (16) is related to the use of money as a means of savings. If it is slack, money holdings are zero, and thus equation (18) simplifies to  $\kappa^L x_{t+1}^H = \kappa^H x_{t+1}^L$ , which jointly with equation (17) determines consumption levels in  $t + 1$  in that case. If, however, equation (16) holds at equality, consumption levels in  $t + 1$  are jointly determined by equations (16) and (17), and then equation (18) solely determines the amount of money holdings used for savings.

Note that  $m_t^y$  is increasing in the price of the risky assets  $\rho_{a,t}$ . If  $\rho_{a,t}$  is sufficiently small, equation

(16) cannot hold at equality for nonnegative money balances. For some price  $\rho_{a,t}$ , equation (16) holds at equality with  $m_t^y = 0$ . I will label this price as  $\tilde{\rho}_a$ . For any  $\rho_a \leq \tilde{\rho}_a$ , buyers choose not to hold any money balances for savings, while they hold positive money balances for any  $\rho_a > \tilde{\rho}_a$ . At  $\bar{\rho}_a \equiv (1 + \pi_{t+1})(\chi\kappa^H + (1 - \chi)\kappa^L) = \frac{1 + \pi_{t+1}}{\beta}$ , expected returns on risky assets and fiat money are equal. Since buyers are risk-averse, they will strictly prefer to save with money at this price, and thus  $x_{t+1}^H = x_{t+1}^L$  and  $a_t^y = 0$  at  $\bar{\rho}_a$ . For any  $A_t > 0$ ,  $\bar{\rho}_a$  cannot occur in equilibrium.

## 4.2 Asset market clearing

To close the model, an asset market clearing condition is needed. Demand for assets by buyers can be found by rearranging the budget constraint of old buyers and is thus given by

$$a_t^y = \frac{x_{t+1}^H - \phi_{t+1}m_t^y}{\kappa^H} = \frac{x_{t+1}^L - \phi_{t+1}m_t^y}{\kappa^L}.$$

Because I normalized the expected return on a risky asset to  $\frac{1}{\beta}$ , sellers are willing to hold any amount of risky assets at the price  $\rho_{a,t} = 1$ . At any price lower than that, sellers would demand an infinite amount of the assets, thus pushing up the price. Therefore,  $\rho_{a,t} = 1$  is the lower bound for the asset price. At any price higher than that, sellers are not willing to hold any risky assets. Thus, market clearing for risky assets is given by

$$\frac{A_t}{N_t} = \frac{x_{t+1}^H - \phi_{t+1}m_t^y}{\kappa^H} + a_t^s \quad (19)$$

with  $a_t^s = 0$  if  $\rho_{a,t} > 1$ , and  $a_t^s = \frac{A_t}{N_t} - \frac{x_{t+1}^H - \phi_{t+1}m_t^y}{\kappa^H}$  otherwise.

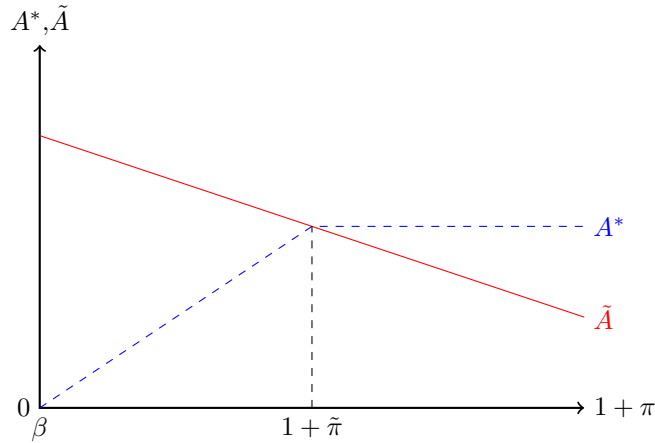


Figure 3:  $A^*$  and  $\tilde{A}$  as a function of inflation.

Equation (19) shows that an increase in the supply of assets leads to either an increase in  $x_{t+1}^o$  or a decrease in money savings  $m_t^y$  as long as  $\rho_{a,t} > 1$ . It can be seen directly from equation

(17) that an increase in CM consumption has to lead to a decrease in the asset price  $\rho_{a,t}$ , and we already established that  $m_t^y$  increases as the price of risky assets rises, so that a decrease in money savings also forces the asset price to go down. Therefore, an increase in the asset supply unambiguously leads to a decrease in asset prices, and vice versa. However, if the price hits the lower bound, a further increase in asset supply does not have an effect on prices, as the sellers absorb any additional risky assets at a price  $\rho_{a,t} = 1$ . Thus, I denote the corresponding quantity of assets that is at least required to reach  $\rho_{a,t} = 1$  as  $A^*$ .

The price  $\bar{\rho}_a$  corresponds to  $A_t = 0$ . At this price, the expected return on risky assets is equal to the return on money, and thus risk-averse buyers are not willing to hold any risky assets. Therefore,  $\rho_{a,t} < \bar{\rho}_a$  for any positive supply of risky assets, as otherwise the market can not clear. Finally, there is the threshold  $\tilde{\rho}_a$  at which buyers start using both savings instruments. I denote the corresponding quantity of assets that leads to  $\tilde{\rho}_a$  as  $\tilde{A}$ . Note that  $\tilde{\rho}_a \leq 1$  in principle, so that it is not clear whether  $\tilde{\rho}_a$  is a feasible price. If  $\tilde{\rho}_a < 1$ , buyers use money to save even if the asset price is unconstrained. Higher inflation and less variance  $\kappa^H - \kappa^L$  both make it more likely that  $\tilde{\rho}_a > 1$ . At some inflation rate  $\tilde{\pi}$ , equation (16) holds at equality for  $m_t^y = 0$  and  $\rho_{a,t} = 1$ . At any inflation rate lower (higher) than  $\tilde{\pi}$ ,  $\tilde{\rho}_a < 1$  ( $\tilde{\rho}_a > 1$ ). In other words, this means that there is some inflation rate at which buyers are exactly indifferent between using and not using money to save if the asset price is at its unconstrained value. For inflation levels below  $\tilde{\pi}$ ,  $A^*$  is a function of inflation, while it is determined entirely by preference parameters if inflation is above  $\tilde{\pi}$ . This is depicted in Figure 3. To the left of the crossing with the blue dashed line, the red line symbolizes the supply of risky assets required to reach a risky asset price that would make buyers indifferent to using money to save *if there were no sellers*, i.e., if  $\rho_a < 1$  were feasible.

Figure 4 shows the price of assets as a function of asset supply for relatively low inflation rates, i.e.,  $\pi < \tilde{\pi}$ . In this case, buyers hold a savings portfolio consisting of assets and money for all possible equilibrium realizations of asset prices. The price of assets is downward sloping in the asset supply until  $\rho_{a,t} = 1$ . The dotted line in Figure 4 depicts how the asset prices would develop for  $A_t > A^*$  if there were no sellers.

Figure 5 shows the price of assets as a function of the asset supply for  $\pi > \tilde{\pi}$ . High inflation means that  $\tilde{\rho}_a > 1$ , so that at  $\tilde{A}$ , the slope changes as agents stop using fiat money to save for any  $A_t > \tilde{A}$ . If inflation decreases, both  $\bar{\rho}_a$  and  $\tilde{\rho}_a$  decrease, as money becomes relatively more attractive as a means of saving, causing agents to start using money to save and stop demanding risky assets at lower prices.

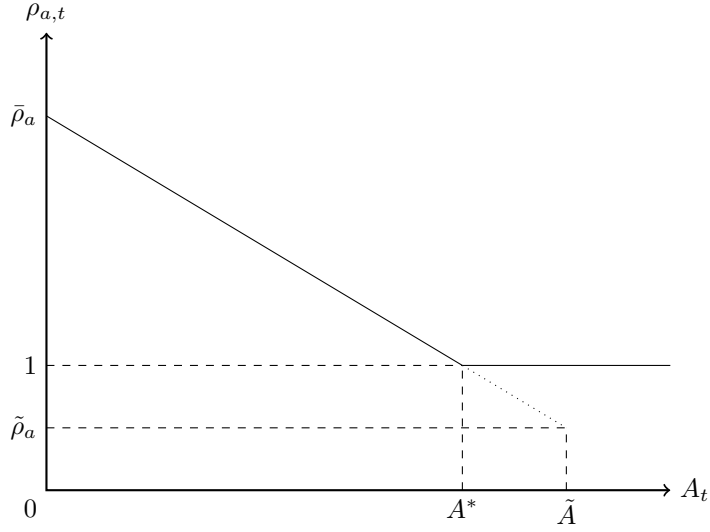


Figure 4: Asset price as a function of supply for relatively high inflation rates.

Similar to the previous section on an economy with only bonds and fiat money, this section mainly serves the purpose of illustrating some functional features of risky assets as a means of saving and especially the market clearing conditions and the existing thresholds on the price of risky assets. The next section presents the full model where buyers can use all three assets to save.

## 5 The economy with government bonds and risky assets

In this section, both government bonds and risky assets are present, as is fiat money. The optimal portfolio decision of any agent thus involves three different assets now.

### 5.1 The buyer's problem

With all assets present, the buyer's decision becomes more complex, but is similar to the two previous cases. The buyers' first-period problem is as follows:

$$\begin{aligned}
 W^{y,ab} = & \max_{h_t^y, a_t^y, m_t^y \geq 0, a_t^y \geq 0, b_t^y} -h_t^y + u(q(\phi_{t+1}d_t^y)) + \beta [\chi W^{o,ab}(\kappa^H a_t^y, m_t^y, b_t^y) + (1 - \chi)W^{o,ab}(\kappa^L a_t^y, m_t^y, b_t^y)] \\
 \text{s.t. } & h_t^y - \tau_t = \phi_t(m_t^y + d_t^y + \rho_{b,t}b_t^y) + \rho_{a,t}a_t^y.
 \end{aligned}$$

The notation follows that of the previous two sections. The old buyer's value function is

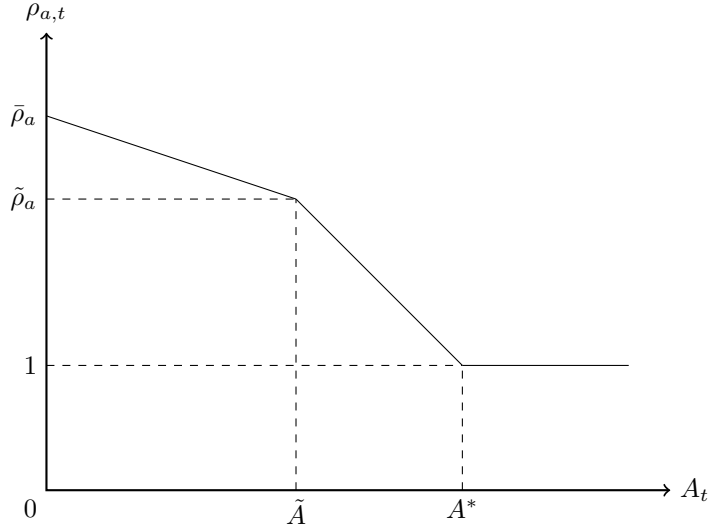


Figure 5: Asset price as a function of supply for relatively high inflation rates.

$$W^{o,ab}(\kappa_{t+1}a_t^y, m_t^y, b_t^y) = \max_{x_{t+1}^o} U(x_{t+1}^o)$$

$$s.t. \quad x_{t+1}^o = \kappa_{t+1}a_t^y + \phi_{t+1}m_t^y + \phi_{t+1}b_t^y.$$

Again, the value functions can be combined. It is best to combine them by replacing bonds, because agents prefer bonds as a savings instrument to money and risky assets, so they will always hold a positive amount of bonds.

$$W^{ab} = \max_{\substack{h_t^y, d_t^y, x_{t+1}^H, x_{t+1}^L, \\ m_t^y \geq 0, a_t^y \geq 0}} -h_t^y + u(q(\phi_{t+1}d_t^y)) + \beta \left[ \chi U(x_{t+1}^H) + (1-\chi)U(x_{t+1}^L) \right]$$

$$s.t. \quad h_t^y - \tau_t = \rho_{a,t}a_t^y + \phi_t(m_t^y + d_t^y) + \frac{\phi_t}{\phi_{t+1}}\rho_{b,t}(x_{t+1}^H - \kappa^H a_t^y - \phi_{t+1}m_t^y) \quad (\lambda^H)$$

$$s.t. \quad h_t^y - \tau_t = \rho_{a,t}a_t^y + \phi_t(m_t^y + d_t^y) + \frac{\phi_t}{\phi_{t+1}}\rho_{b,t}(x_{t+1}^L - \kappa^L a_t^y - \phi_{t+1}m_t^y) \quad (\lambda^L).$$

The buyer's maximization problem gives rise to the following first-order conditions:

$$\begin{aligned}
h_t : & & 1 &= \lambda^H + \lambda^L \\
d_t^y : & & \phi_t(\lambda^H + \lambda^L) &= \beta\phi_{t+1} \frac{u' \circ c^{-1}(\beta\phi_{t+1}d_t^y)}{c' \circ c^{-1}(\beta\phi_{t+1}d_t^y)} \\
m_t^y : & & \phi_t(\lambda^H + \lambda^L) &\geq \rho_{b,t}\phi_t(\lambda^H + \lambda^L) \\
a_t^y : & & \rho_{a,t}(\lambda^H + \lambda^L) &\geq \frac{\phi_t}{\phi_{t+1}}\rho_b(\lambda^H\kappa^H + \lambda^L\kappa^L) \\
x_{t+1}^H : & & \lambda^H \frac{\phi_t}{\phi_{t+1}}\rho_{b,t} &= \beta\chi U'(x_{t+1}^H) \\
x_{t+1}^L : & & \lambda^L \frac{\phi_t}{\phi_{t+1}}\rho_{b,t} &= \beta(1 - \chi)U'(x_{t+1}^L).
\end{aligned}$$

I made use of the solution to the DM problem as derived in Section 3.1.1, since that problem is not affected by the type of assets present in the economy. By replacing the Lagrange multipliers, the following five equilibrium conditions are obtained:

$$1 = \beta \frac{\phi_{t+1}}{\phi_t} \frac{u' \circ c^{-1}(\beta\phi_{t+1}d_t^y)}{c' \circ c^{-1}(\beta\phi_{t+1}d_t^y)} \quad (20)$$

$$1 \geq \rho_{b,t} \quad (21)$$

$$\frac{\phi_t}{\phi_{t+1}} \frac{\rho_{b,t}}{\beta} = \chi U'(x_{t+1}^H) + (1 - \chi)U'(x_{t+1}^L) \quad (22)$$

$$\frac{\rho_{a,t}}{\beta} \geq \kappa^H \chi U'(x_{t+1}^H) + \kappa^L (1 - \chi)U'(x_{t+1}^L) \quad (23)$$

$$x_{t+1}^H - x_{t+1}^L = a_t^y(\kappa^H - \kappa^L). \quad (24)$$

Equation (20) shows that the choice of money holdings used in DM meetings is independent of other decisions and depends only on the terms of trade and inflation rates. Condition (21) shows that agents only want to save with money if the price of bonds equals one. Equation (22) sets the cost of acquiring and holding bonds equal to the benefit of holding more bonds, namely more consumption in both the high and the low state. If condition (23) holds with equality, agents acquire risky assets such that the cost of acquiring them is equal to the benefit that they can derive from them. If asset prices are too high, condition (23) will not hold at equality, and agents thus acquire no risky assets. Finally, equation (24) states that any difference in consumption levels in the second period is caused by asset holdings. Consequently, consumption in the low and the high state will be equal if the return on the asset is not risky or if agents do not hold any risky assets.

## 5.2 Bond market clearing

The bond market clearing works similarly to what I showed in Section 3.2, but some variables change due to the presence of risky assets. Thus I will briefly summarize the main takeaways from



the analysis performed earlier, but now updated with the new variables:

$$b_t^y = \frac{x_{t+1}^H - \kappa^H a_t^y - \phi_{t+1} m_t^y}{\phi_{t+1}} = \frac{B_t}{N_t} - b_t^s \quad (25)$$

$$\text{with } b_t^s = 0 \text{ if } \rho_{b,t} > \frac{\beta}{1 + \pi_{t+1}}, \text{ and } b_t^s = \frac{B_t}{N_t} - \frac{x_{t+1}^H - \kappa^H a_t^y - \phi_{t+1} m_t^y}{\phi_{t+1}} \text{ otherwise.}$$

Equation (25) states that all bonds have to be held by young buyers unless the bonds are priced at the Fisher equation. Further, it shows that an increase in the supply of bonds leads to a decrease in their price. From condition (21) and the analysis performed in Section 3.2, we know that the bond price is bounded above by one, which means that the bond price can lie in the range

$$\beta \frac{\phi_{t+1}}{\phi_t} \leq \rho_{b,t} \leq 1.$$

### 5.3 Asset market clearing

Asset market clearing functions just like in Section 4.2, only with some slight changes to the variable that determines the buyer's asset holdings. From equation (24), we know these are given by

$$a_t^y = \frac{x_{t+1}^H - x_{t+1}^L}{\kappa^H - \kappa^L}.$$

Thus, for the asset market to clear, the following condition needs to hold:

$$\frac{A_t}{N_t} = \frac{x_{t+1}^H - x_{t+1}^L}{\kappa^H - \kappa^L} + a_t^s \quad (26)$$

$$\text{with } a_t^s = 0 \text{ if } \rho_{a,t} > 1, \text{ and } a_t^s = \frac{A_t}{N_t} - \frac{x_{t+1}^H - x_{t+1}^L}{\kappa^H - \kappa^L} \text{ otherwise.}$$

Buyers are only willing to hold risky assets if bonds are priced above their unconstrained price  $\rho_b^*$ , as otherwise bonds and risky assets have the same rate of return. Once the price of bonds increases above  $\rho_b^*$ , the ensuing demand for risky assets can be interpreted as a portfolio rebalancing effect.

### 5.4 Equilibrium

In the following, I will define the equilibrium for the model that has all the assets<sup>10</sup>:

<sup>10</sup>I will not formally define a steady-state equilibrium, as it is not required for the results I want to show. However, one could easily obtain the steady-state equilibrium by using the steady-state inflation rate defined in Section 3.4.1 and dropping time subscripts from all variables. Although I derived the steady-state inflation rate in the model with bonds only, it applies here as well, as money market clearing and thus steady-state inflation do not depend on the types of assets that are present in the model.

**Definition 2.** An equilibrium is a sequence of prices  $\rho_{b,t}$ ,  $\rho_{a,t}$ , and quantities  $d_t^y$ ,  $x_{t+1}^H$ ,  $x_{t+1}^L$ ,  $m_t^y$ ,  $a_t^y$ ,  $b_t^s$ , and  $a_t^s$ , that simultaneously solve the equations (20), (22), (24), and the following inequalities with complementary slackness conditions: (21), (23), (25), and (26)  $\forall t$ .

The equilibrium conditions give rise to a number of different regions in the parameter space for which the resulting equilibrium differs. In the following, I will characterize the different possible equilibria. One aspect in the characterization will be the correlation between asset prices and the real interest rate, which is helpful in linking the equilibrium regions with the observed data. Note that expected dividends, given by  $\kappa^H$  and  $\kappa^L$ , are constant in the model, which means that asset price changes are equivalent to changes in the dividend-price ratio. For a given inflation rate, the selection of an equilibrium region depends on the supply of risky assets  $A_t$  and the bonds-to-money ratio  $\mathcal{B}_t$ . Figures 6 and 7 depict the equilibrium regions for high and low inflation rates, respectively.

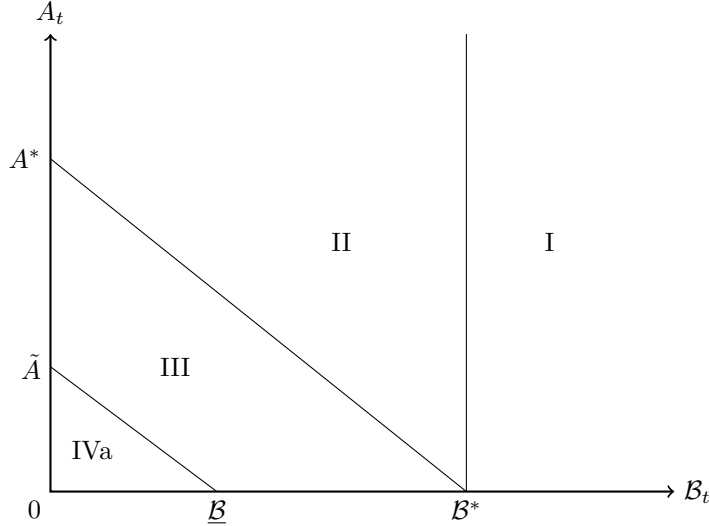


Figure 6: Equilibrium regions for  $\tilde{\rho}_a > 1$ .

**Proposition 1.** If the supply of government bonds exceeds  $\mathcal{B}^*$ , buyers will only hold bonds, while sellers hold all the risky assets and the remaining government bonds. Risky asset prices and real interest rates are both constant.

$\mathcal{B}^*$  is the amount of bonds that allows buyers to only use bonds for their savings, as defined in Section 3. If the supply of government bonds is at least that large, the bonds will be priced at their lower bound, i.e.,  $\rho_{b,t} = \rho_b^*$ . This means that a bond pays the same expected return as a risky asset, but since buyers are risk-averse, they will only hold the riskless asset, i.e., the bond. It can be shown that at this bond price, condition (23) can only hold at equality for  $x_{t+1}^H = x_{t+1}^L$ ,

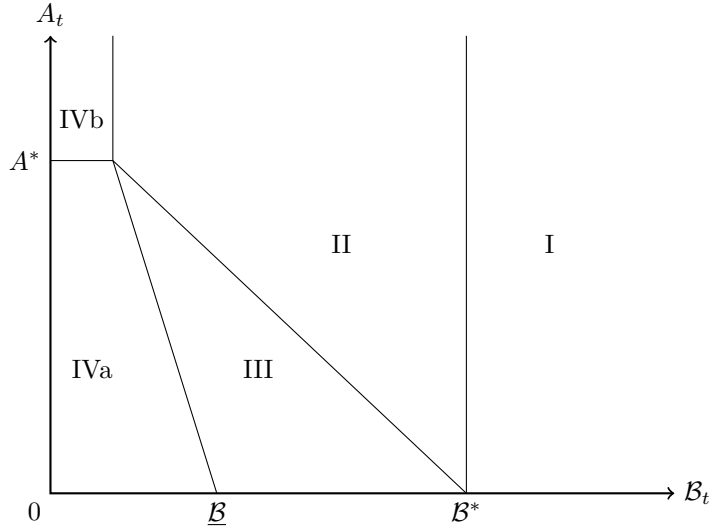


Figure 7: Equilibrium regions for  $\tilde{\rho}_a \leq 1$ .

which is only possible for  $a_t^y = 0$  according to equation (24), thus proving the proposition.

I will denote the parameter region for which Proposition 1 holds as region I. It is defined by  $\rho_{b,t} = \rho_b^*$ , while  $\rho_{a,t} = 1$ . In this case,  $r_t = r^n \forall t$ , so that both the price of risky assets as well as the real interest rate are constant over time. As both of these variables are time-varying in reality, equilibrium region I does not seem to be prevalent in today's economies.

**Proposition 2.** *If  $B_t < B^*$ , but the combined supply of  $B_t$  and  $A_t$  is sufficiently large, buyers will hold all the bonds and some risky assets. Sellers hold the remaining risky assets. There is a risk premium paid on risky assets compared to government bonds, but risky assets are still priced at their unconstrained value. Real interest rates and asset prices are uncorrelated.*

Once the price of bonds is lifted above the lower bound, the expected return on bonds becomes lower than the expected return on a risky asset priced at its unconstrained value. Buyers can thus increase their utility by holding some risky assets. The amount of risky assets they are willing to hold depends on their risk aversion. Whenever  $\rho_{b,t} > \rho_b^*$  and  $\rho_{a,t} = 1$ , for condition (23) to be a strict inequality,  $x_{t+1}^H > x_{t+1}^L$  is required. However, this leads to a contradiction with equation (24), as  $x_{t+1}^H > x_{t+1}^L$  is only possible for  $a_t^y > 0$ , which in turn requires equation (23) to hold at equality. This shows that buyers will always hold some risky assets in this case. Note that in this region, we will typically observe  $x^H > x^* > x^L$ .  $r_t < r^n$  in this region and varies with inflation and the bonds-to-money ratio. However, because the price of risky assets is always equal to 1, real interest rates and asset prices are uncorrelated.

I will denote the parameter region for which Proposition 2 holds as region II. It is defined by  $1 > \rho_{b,t} > \rho_b^*$ , while  $\rho_{a,t} = 1$ .

**Proposition 3.** *If the combined supply of bonds and risky assets is scarce, the price of both assets will be above its lower bound, and buyers will hold the entire supply of both assets. Real interest rates and asset prices are negatively correlated.*

In region II, buyers demand some risky assets. The lower the buyer's risk aversion and the higher the price of government bonds, the more risky assets they demand. At some point, demand exceeds supply at the unconstrained price, thus raising the price of risky assets above its unconstrained level. This discourages sellers from holding risky assets. Once risky assets and bonds are both priced above their unconstrained value, their prices are correlated, which in turn means that real interest rates and asset prices are negatively correlated.

I will denote the parameter region for which Proposition 3 holds as region III. It is defined by  $1 > \rho_{b,t} > \rho_b^*$ , while  $\rho_{a,t} > 1$ , so that both assets are priced above their unconstrained value. This region can be considered an *asset shortage equilibrium*.

**Proposition 4.** *If the combined supply of bonds and risky assets is severely scarce, bonds will yield the same return as fiat money, and buyers will also use fiat money to save. Real interest rates and asset prices can be negatively correlated or uncorrelated.*

Once the bond price is driven up to  $\rho_{b,t} = 1$ , buyers will cease to demand more bonds or risky assets, but will start to save with fiat money instead. Depending on the risk aversion parameters and the supply of risky assets, the price of risky assets can be at its unconstrained value, or above it. In an economy without risky assets,  $\rho_{b,t} = 1$  if  $\mathcal{B}_t \leq \underline{\mathcal{B}}$ . With risky assets available, the amount of bonds required to step away from the zero lower bound becomes smaller as buyers also use risky assets to save.

I will denote the parameter region for which Proposition 4 holds as region IV. It is defined by  $\rho_{b,t} = 1$  and can therefore be considered a *zero lower bound equilibrium*. Note that it can exist with either  $\rho_{a,t} > 1$  or  $\rho_{a,t} = 1$ , depending on the inflation rate. I denote the parameter space where we simultaneously observe an *asset shortage* and a *zero lower bound* as region IVa, while a zero lower bound without an asset shortage is denoted as a IVb equilibrium. In region IVa, asset prices and real interest rates are negatively correlated (nominal interest rates are constant at zero, but an increase in inflation makes real assets relatively more attractive to hold and thus increases

their price). In region IVb, real interest rates and asset prices are uncorrelated, as assets are priced at their unconstrained value regardless of the inflation rate.

A IVb equilibrium can only exist if buyers use money to save even when they are satiated with risky assets. Thus, this equilibrium exists only if  $\pi < \tilde{\pi}$  and thus  $\tilde{\rho}_a \leq 1$ . This situation is depicted in Figure 7.

## 5.5 Risk premium

The risk premium is typically defined as the difference in expected return between a risky and a safe asset that both deliver the same services. Thus, the difference in expected return between the risky asset and the government bond is a natural candidate for the risk premium in this model, which I therefore define as

$$\mathcal{R}_t = \frac{1}{\beta} / \rho_{a,t} - \frac{\phi_{t+1}}{\phi_t} / \rho_{b,t}.$$

Note that if both assets are priced at their unconstrained value, i.e., in region I,  $\mathcal{R}_t = 0$ . However, the risk premium measures the difference in expected returns that makes a single agent indifferent between the two assets. Since buyers are not willing to hold risky assets in region I, the risk premium cannot be measured in that region. In all other regions, however, buyers hold a positive amount of both assets and are thus indifferent between the two at the margin. Interestingly,  $\mathcal{R}_t$  is not a constant: In region II,  $\rho_{a,t} = 1$  everywhere, while  $1 > \rho_{b,t} > \beta \frac{\phi_{t+1}}{\phi_t}$ . Thus, the risk premium can also vary within these bounds on  $\rho_{b,t}$ . In region III,  $\mathcal{R}_t$  can increase or decrease depending on whether  $\rho_{a,t}$  or  $\rho_{b,t}$  increase faster, but in region IVa, the risk premium decreases again because  $\rho_{b,t} = 1$ , whereas  $\rho_{a,t}$  increases the further the economy advances into region IVa. In region IVb,  $\rho_{b,t} = \rho_{a,t} = 1$ , so the risk premium is constant at  $\mathcal{R}_t = \frac{1}{\beta} - \frac{\phi_{t+1}}{\phi_t}$ .

## 5.6 Comparative statics

In this subsection, I want to analyze the comparative statics of inflation, the supply of both risky assets and bonds, as well as the riskiness of the assets in all equilibrium regions.

### 5.6.1 Inflation

From equation (20), it is obvious that inflation reduces  $\phi_t d_t^y$  and thus also the DM consumption of young agents. Since this is true for all regions, it is clear that inflation always has some negative effects. As it turns out, inflation does not have positive effects on the CM consumption either, thus making the Friedman rule optimal.

In region I, because the bond price compensates fully for inflation, an increase in inflation leads to a decrease in bond prices in this region, without affecting second-period consumption or other real variables. In region II, the bond price does not fully compensate for inflation because of the scarcity of bonds. Without scarcity, the bond price would drop exactly as much as inflation increases, allowing buyers to obtain the same level of consumption after the change, just like in region 1. However, to obtain the same level of consumption as before the increase in inflation, buyers would need to hold more bonds, which puts upward pressure on the bond price. As a result, the bond price drops somewhat (at the previous bond price, agents would demand fewer bonds than before), but not enough to fully compensate for inflation. Asset prices stay at their lower bound in this region, because there is no asset scarcity. This means that an increase in inflation leads to buyers holding more assets (assets become relatively more attractive, because the difference in returns increases), and thus also the difference  $x_{t+1}^H - x_{t+1}^L$  increases. All in all, buyers are worse off.

In region III, both assets are scarce, and thus buyers hold all bonds and all risky assets. The effect of an increase in inflation on bonds is thus similar to what happens in region II, but since buyers cannot compensate by holding more risky assets now, the price of risky assets also increases. Thus, buyers have to work more to purchase the assets, and they obtain less consumption in both states, because the real rate of return on bonds decreases, making them clearly worse off. In region IV, bonds are so scarce that fiat money is a perfect substitute for bonds as a means of saving. Thus, an increase in inflation does not affect bond prices, which remain at  $\rho_{b,t} = 1$ , meaning that the real return on bonds decreases even more than in regions II and III. Additionally, buyers will reduce their money savings somewhat, because the real return on money also decreases. In region IVb, buyers can substitute money and bonds for risky assets, so that the overall effects are similar to region II, i.e., consumption variance increases, but expected consumption remains constant. In region IVa, buyers would also like to purchase more risky assets, but because they are scarce in this region, they merely drive up asset prices, making the overall effects in this region similar to those in region III. Clearly, buyers are again made worse off by an increase in inflation in region IV.

### 5.6.2 Decrease in the supply of risky assets

In regions I, II, and IVb, such a change has no real effect, as at least some risky assets are held by sellers, and the sellers are indifferent to holding them. In region III, a decrease in the supply of risky assets leads to an increase in the price of risky assets. As this makes bonds relatively more attractive, bond prices also increase. Thus, second-period consumption falls in both states,

because less risky assets can be held by buyers, but also the difference  $x_{t+1}^H - x_{t+1}^L$  decreases as  $a_t^y$  falls.

In region IVa, the effect is similar, except that instead of increasing bond prices, money savings increase. This increase reduces the difference  $x_{t+1}^H - x_{t+1}^L$ . However, expected consumption also falls somewhat, as buyers are not willing to fully replace the missing assets with money savings due to the inflation tax. Thus, buyers are worse off.

### 5.6.3 Decrease in the supply of government bonds

As bonds are not scarce in region I, a marginal decrease in their supply has no real effect. In region II, bonds are scarce, thus a further decrease in supply leads to an increase in the bond price. This makes risky assets relatively more attractive, and since they are not scarce, buyers can easily invest in them as substitutes. Thus, their consumption variance increases, but their expected consumption does not decrease. In region III, similar mechanisms are at play as in region II, but since risky assets are also scarce, buyers cannot hold more of them and instead drive up their price. This makes purchasing the assets more expensive, and drives down CM consumption in both states as fewer assets are available. In region IV, a decrease in bonds has no real effect again, as bonds are already priced at their upper bound, and a further decrease in supply thus only increases money savings.

### 5.6.4 Riskiness of the asset

In general, it is clear that the less risky the asset is, i.e., the smaller  $\frac{\kappa^H}{\kappa^L}$  is, the better risky assets become as a substitute for bonds.

In region I, the riskiness of the asset does not matter as only risk-neutral agents (sellers) are willing to hold them. In region II, less riskiness makes the assets more attractive, thus increasing  $a_t^y$ , which lowers pressure on bond prices and thus leads to lower  $\rho_{b,t}$ . Agents consume more in the low state and welfare increases. In region III, the greater attractiveness of the assets increases their scarcity, thus increasing asset prices. However, the pressure on bond prices is reduced, which leads to a decrease in bond prices. Overall, agents are better off, because the variance of their consumption decreases. In region IVb, the effects are similar to region 2, but without a change in bond prices. In region IVa, asset prices increase, but welfare still increases as buyers experience less variance in their second-period consumption.

## 5.7 Transitions from different equilibrium regimes

Figures 6 and 7 show how the supply of risky assets or the bonds-to-money ratio can induce transitions from one equilibrium region to another. With the help of the comparative statics, we can also assess how changes in inflation can lead to such transitions. If inflation is relatively high (i.e., Figure 6 applies), a decrease in inflation could lead to a transition from equilibrium region III to region IVa. Other transitions induced by changes in inflation are not possible, as inflation only affects  $\tilde{A}$  and  $\underline{\mathcal{B}}$ . If inflation is relatively low however (Figure 7 applies), inflation affects  $A^*$  (increasing with inflation) and  $\underline{\mathcal{B}}$  (decreasing with inflation). This means that all transitions among equilibrium regions II, III, IVa, and IVb could be induced by changes in inflation.

Given the data on the correlation of the dividend-price ratio of US stocks and the real interest rate presented in Section 1.2, a transition from region III to region IVb is of particular relevance, as such a transition seems to have occurred after the financial crisis; i.e., we moved from a regime with a negative correlation between asset prices and real interest rates to a regime with no correlation, and simultaneously the zero lower bound was hit. In the model, such a transition can occur for two reasons: (1) A simultaneous reduction in the supply of safe assets and an increase in the supply of risky assets, as can be seen in Figure 7, or (2) a decrease in inflation, as this causes an increase in  $\underline{\mathcal{B}}$  and a decrease in  $A^*$ . Interestingly, both scenarios seem to apply: Inflation rates were lower in the years during and after the financial crisis than before, and many assets that were considered safe (e.g., mortgage-backed securities, European sovereign debt) turned out to be risky in fact. Thus, the model is able to explain the transition observed in reality.

## 6 Policy

In this section, I analyze how monetary or fiscal policy can improve market outcomes in the model presented in Section 5, i.e., the one with all assets present. In fact, it is easy for the authorities to achieve the first-best: Running the Friedman rule ensures both  $q^*$  and  $x^*$ . If this is not an option, it is still possible to achieve  $x^*$  by issuing enough bonds; i.e., enough to reach a bonds-to-money ratio of  $\mathcal{B}^*$ . However, neither of these policies seems realistic. First, there are political limits to the amount of debt governments can issue, and second, the Friedman rule has never been implemented by a central bank. Therefore, in this section I want to analyze what other policies could be used to improve welfare. Specifically, I will show that the monetary authority can improve welfare through stabilization policy for the current generation, but that this will also affect the welfare of future generations if the fiscal authority does not coordinate its actions with the monetary authority. But



first, I want to quickly discuss the optimal inflation target.

## 6.1 Optimal inflation target

As explained above, the first-best can be achieved by running the Friedman rule, i.e., by setting  $1 + \pi = \beta$ . This ensures that first-best quantities can be consumed even if agents save by using money, thereby making scarcities of bonds or risky assets irrelevant. On top of this, Section 5.6.1 showed that any increase in inflation is welfare-reducing. This is true even though the zero lower bound is more likely to occur at low inflation rates, because the zero lower bound is also less harmful if inflation is low. This shows that it is not a good idea to increase inflation targets in order to reduce the probability of hitting the zero lower bound. Even though the policy change achieves this goal, total welfare is reduced.

## 6.2 Optimal stabilization policy

To analyze the optimal stabilization policy, I will assume that monetary authority commits to a long-term expected inflation rate, but that the monetary authority can inject a larger or smaller money supply depending on the asset return, thus creating above or below average inflation temporarily. In fact, we will see that it is optimal to inject more money when asset returns are high and less money when asset returns are low, so we can write the expected price level of next period as

$$\mathbb{E}_t[\phi_{t+1}] = \chi\phi_{t+1}^L + (1 - \chi)\phi_{t+1}^H. \quad (27)$$

Thus, the value of money is low when asset returns are high, because the monetary authority injected more money than expected, and vice versa. This is a similar policy to the one studied in Berentsen and Waller (2011), but the mechanism here is different. In equation (27),  $\mathbb{E}_t[\phi_{t+1}]$  is given by the monetary authority's inflation target and the current value of money.

By adopting such a policy, the monetary authority makes bonds (and also fiat money) a savings instrument that is negatively correlated with the risky asset. Thus, if the monetary authority chooses the right values, the overall portfolio of buyers becomes risk-free.

The DM choices of buyers are not affected by this policy, as the expected inflation rate during the DM is equal to the long-run inflation rate. For sellers, this means that their CM consumption becomes random, as it now depends on the realization of the inflation rate. However, because sellers are risk-neutral, this does not reduce their welfare<sup>11</sup>.

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<sup>11</sup>While the risk-neutrality of the sellers allows for Pareto improvements through stabilization policy in this model,

To make the buyers' portfolio risk-free, the condition

$$\kappa^H a_t^y + \phi_{t+1}^L b_t^y = \kappa^L a_t^y + \phi_{t+1}^H b_t^y \quad (28)$$

has to hold, as then buyers obtain the same amount of consumption in both states. If equation (28) is satisfied for the portfolio that the buyers choose to hold, they are indifferent at the margin between the two kinds of assets. Thus, prices will adjust, such that the expected returns on the two assets will be equal, which is given by

$$\frac{1}{\rho_{a,t}} = \frac{\beta \mathbb{E}_t[\phi_{t+1}]}{\phi_t \rho_{b,t}}. \quad (29)$$

Now for the monetary authority to pick the correct values of money in period  $t + 1$ , it needs to know which portfolio the agents are going to choose. In equilibrium regions III and IVa, buyers hold all assets available, so  $N_t a_t^y = A_t$  and  $N_t b_t^y = B_t$ . Thus, the monetary authority knows the buyers' portfolio and can choose the value of money in the next period given equation (27), the current value of money, and its inflation target such that (28) holds. Prices will then adjust such that (29) is satisfied<sup>12</sup>.

In equilibrium regions II and IVb, buyers do not hold all assets, and thus it becomes more complex for the monetary authority to know which portfolio buyers will choose to hold. However, bonds are still in short supply, so  $N_t b_t^y = B_t$  still holds. This means that  $a_t^y$  also becomes a decision variable for the central bank, and it can choose this variable such that

$$\kappa^H a_t^y + \phi_{t+1}^L \frac{B_t}{N_t} = \kappa^L a_t^y + \phi_{t+1}^H \frac{B_t}{N_t} = x^* \quad (30)$$

holds, given that  $N_t a_t^y \leq A_t$  and its choices for the value of money given equation (27). This policy will lead to the equilibrium prices  $\rho_{b,t} = \rho_b^*$  and  $\rho_{a,t} = 1$ , which also satisfy (29).

So to sum up, this policy is able to eliminate consumption volatility completely in all equilibrium regions. As this happens without reducing expected consumption, it is welfare improving. In regions II and IVb, the implementation of this policy even leads to the first best. These results are true as long as the central bank has access to lump-sum taxes as well as transfers. If taxes are

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this result does not hold for more general utility functions of sellers. But even then, stabilization policy can make some agents better off, so it can still be a beneficial policy if these agents are particularly important. At the very least, this shows that a countercyclical monetary policy has clear negative effects on some agents in the economy.

<sup>12</sup>In equilibrium region IVa, buyers might still want to save with all three assets, i.e., they also hold some money. This is the case if  $\rho_{b,t} = 1$ . In this case, (28) becomes  $\kappa^H a_t^y + \phi_{t+1}^L (b_t^y + m_t^y) = \kappa^L a_t^y + \phi_{t+1}^H (b_t^y + m_t^y)$ , and the monetary authority has to adjust its policy accordingly.

not feasible, there is an upper bound on the next period's value of money, namely  $\phi_{t+1} \leq \phi_t$ . This means that the optimal  $\phi_{t+1}^H$  might not be available. In this case,  $x^L = x^H = x^*$  is not achievable even in region II, but by setting  $\phi_{t+1}^H = \phi_t$  and  $\phi_{t+1}^L$  according to equation (27) the policy will still increase welfare by reducing consumption variance to a certain extent.

This analysis shows that reacting to asset return shocks can be welfare improving. Bernanke and Gertler (2001) argue that asset returns should only matter for monetary policy if they affect the inflation rate. That is not the case here, as inflation remains constant as long as the central bank does not change its policy. Still, monetary policy that takes asset returns into account improves welfare, because it allows agents to smooth consumption across different states. This shows that there is a role for stabilization policy when aggregate asset return shocks affect the consumption of savers. Additionally, note that to achieve the optimal stabilization policy, the monetary authority increases inflation when asset returns are high, and lowers inflation when asset returns are low. This also goes against the conventional wisdom that inflation should be countercyclical to prevent the economy from 'overheating' in good times or to 'kickstart' it when it is in a slump. While Berentsen and Waller (2011) also find a procyclical policy to be optimal in their model, the problems that this policy solves are different. They have underconsumption in good states and overconsumption in bad states without intervention, while in my model, there is overconsumption in good states and underconsumption in bad states without intervention.

### 6.2.1 Effect on future generations

The analysis above focused on how the policy affects the welfare of current generations. However, making the future money supply a function of the asset return shock also has effects on future generations. As the analysis of the model showed, the bonds-to-money ratio is an important variable for young buyers who choose their savings portfolio. The higher the bonds-to-money ratio is, the lower the bond prices are, and the more old age consumption the current young generation can afford. But if the monetary authority implements the policy described above, money growth will be relatively high after high asset return shocks and relatively low after low asset return shocks. If the fiscal authority keeps the growth rate of bonds constant, this means that the bonds-to-money ratio is high after bad shocks and low after good shocks, which in turn affects the welfare of the current young generation accordingly. However, the fiscal authority can easily mitigate this effect by setting the bond growth rate equal to the money growth rate not only on average, but in every period. In this way, the bonds-to-money ratio remains constant. As this policy requires a much lower average bonds-to-money ratio than  $\mathcal{B}^*$ , it might be feasible politically.

### 6.3 Quantitative easing

While quantitative easing is not optimal in this model, the framework still lends itself well to an analysis of the effect of quantitative easing on the welfare of future generations. Quantitative easing, i.e., printing money to buy bonds, is a policy that reduces the bonds-to-money ratio. As I have shown in this paper, a reduction in the bonds-to-money ratio is bad for the welfare of young buyers. Thus, while quantitative easing might have some positive effects on current output, policymakers should keep in mind that it negatively affects the current young generation.

## 7 Conclusion

This paper shows that, depending on inflation and the supply of risky assets and government bonds, there are equilibrium regions where the prices of risky assets and bonds are positively correlated, and regions where they are uncorrelated. The model shows that, away from the Friedman rule, welfare increases with the amount of government debt up to a certain level, while it remains constant for higher levels of government debt. If it is politically infeasible for the fiscal authority to provide enough government debt, the monetary authority can improve welfare by running a procyclical monetary policy, i.e., by increasing inflation when asset returns are high and decreasing inflation when asset returns are low. If this policy is implemented, the aggregate savings portfolio of old agents is risk-free. However, without interventions of the fiscal authority, this policy affects the bonds-to-money ratio. As the welfare of young agents increases with the bonds-to-money ratio, young agents are worse off when asset returns are high if this policy is implemented, and better off when asset returns are high. By also adapting the growth rate of bonds depending on asset returns, the fiscal authority can prevent these effects on young agents.

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