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Heterogeneous Workers, Trade, and Migration

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Heterogeneous Workers, Trade, and Migration

Abstract

We develop a model that combines monopolistic competition on goods markets with skill-type heterogeneity on the labor market to analyze the effects of trade and migration on welfare and inequality. Skill-type heterogeneity and partial specificity to firms' endogenously chosen skill requirements lead to endogenous worker-firm match quality, endogenous wage markups, and within-firm wage inequality. We identify novel effects of trade and migration. Trade enhances firms' monopsony power on the labor market and worsens the average quality of worker-firm matches, but the gains from trade theorem survives. Integration of labor markets leads to two-way migration between symmetric countries. Migration enhances competitiveness on the labor market and tends to increase the average quality of worker-firm matches. Trade and migration are complements. Our model clearly advocates opening up labor markets simultaneously with trade liberalization.

JEL-Codes: F120, F160, F220, J240.

Keywords: two-way migration, gains from trade, heterogeneous workers.

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1 Introduction

Economic well-being crucially depends on how well workers' abilities are matched with firms' skill requirements, determined by the tasks to be performed in production. In this paper, we use a general equilibrium model to show that the quality of worker-firm matches in an economy with horizontal skill heterogeneity depends upon how open the economy is to trade and migration. By horizontal skill heterogeneity, we mean that, controlling for skill *levels*, workers have different *types* of skills. We show that under plausible conditions, this leads to monopsony power on the labor market and to imperfect matching between workers and firms. Our central contribution is to demonstrate that economic globalization has important normative consequences by altering the equilibrium wage-markups as well as the equilibrium quality of worker-firm matching. We show that trade increases firms' monopsony power on the labor market while worsening the average quality of skill-type matches, yet the gains from trade theorem survives. Migration enhances the competitiveness of labor markets and tends to increase the average quality of skill-type matches in both countries. We also show that imperfect worker-firm matching constitutes an incentive for two-way migration between symmetric countries.

Heterogeneity of workers has recently gained substantial attention in trade theory. Grossman and Maggi (2000) and Ohnsorge and Trefler (2007) were the first to analyze, in different model environments, how differences in the skill distribution between countries shape the pattern of trade in sorting models with skill heterogeneity on the part of workers and submodularity or supermodularity of production techniques on the part of sectors and firms. Two-sided heterogeneity coupled with log-supermodularity assumptions are also at the core of a new, general formulation of the theory of comparative advantage developed by Costinot (2009). Costinot and Vogel (2010) develop an application of Costinot (2009) where workers with different skill levels are assigned to tasks (intermediate inputs) differing by skill intensity. Log-supermodularity with respect to the skill levels of workers and the skill intensity of tasks leads to an equilibrium assignment featuring positive assortative matching.¹ The matching function and the associated wage schedule allow the authors to analyze the distributional consequences of endowment changes, technological change, and trade.

¹ A survey of assignment models of this kind in trade is found in Costinot and Vogel (2015)

These assignment models of international trade leave two important issues to be addressed. First, two-sided heterogeneity may not be amenable to ranking, in which case the concept of supermodularity cannot be applied. The obvious case in point is horizontal skill heterogeneity (skill types). Empirically, horizontal heterogeneity appears no less important than vertical skill heterogeneity. Consider an architect and a fashion designer who have undergone the same amount of training. They might be regarded as having the same level of skills, yet they obviously embody different types of skills. The same might be said when comparing a bridge building engineer with an architect or with a fashion designer, whereby it is equally obvious that the bridge building engineer embodies a skill type closer to that of an architect than that of a fashion designer. Moreover, the architect might be considered a skill type which is at an equal “distance” to the skill types, respectively, of the bridge building engineer and the fashion designer. These examples illustrate what we mean by horizontal, or skill-type heterogeneity of workers (as opposed to heterogeneity in skill levels), and they clearly demonstrate that it is a pervasive phenomenon.² Assignment based on log-supermodularity is often described as reflecting comparative advantage of workers for different industries, occupations, or tasks. Assignment based on horizontal skill heterogeneity as considered in this paper may be described as reflecting absolute advantage.

The second issue is that in assignment models that feature a continuum of tasks as well as a continuum of skills the matching is always *perfect*: each task is assigned a unique skill level and each skill level gets assigned to only one task (see, e.g., Ohnsorge and Treffer, 2007; Costinot and Vogel, 2010). But arguably, perfect matching is an extreme outcome. We often observe imperfect matching where any one industry or occupation employs workers with different levels or types of skill, with some workers better suited to the relevant tasks than others. Our model features imperfect matching, whereby the average quality of matching is determined endogenously through entry decisions by firms as well as through self-assignment of workers according to absolute advantage based on their skill types.

² The above types of assignment models could, in principle, be reformulated for horizontal skill heterogeneity. In particular, with multi-dimensional heterogeneity it is possible to arrive at skill types that, while horizontal in one sense, may still be ranked in some other sense. This is the case in Ohnsorge and Treffer (2007) where workers who are ranked according to different *relative* skill levels might be considered as different skill types, conditional on having the same *absolute* level of any one of the two skills. But this is a special case. In many cases, horizontal skill heterogeneity defies a clear ranking of skill types.

A key tenet of our approach is that horizontal skill heterogeneity among workers begets endogenous heterogeneity among firms regarding the skill requirements for production. This is analogous to endogenous product differentiation in goods markets. Vogel (2008) argues that heterogeneity of consumers prompts each firm to choose an idiosyncratic combination of product characteristics putting its product at a certain distance to products of competing firms, looked at from consumers’ point of view. By complete analogy, we argue that a firm facing a labor market with skill-type heterogeneity will choose a combination of skill requirements, called the *ideal* skill type, placing it at a certain distance from those of its competitors. Choosing skill requirements may relate to organizing production as a certain combination of different tasks, but also to the choice of the particular good to be produced.

Following Amiti and Pissarides (2005), we use a circular representation of skill-type heterogeneity among workers to model firms’ skill-type choices as spatial competition. Thus, we model skill-type heterogeneity of workers on a single dimension, although the underlying abilities surely have many dimensions, say because they relate to more than just one task. The big advantage of this approach is that it allows us to discuss the extent to which firms employ workers deviating from their respective ideal skill types—we speak of the “skill reach” of firms. The underlying assumption is that workers with skill types that differ from a certain firm’s ideal type don’t find all of their skills entirely useless for this firm, but will simply be less productive when working for this firm than workers with skill types closer to the firm’s ideal type. A firm wanting to increase its employment will have to reach out to workers with skill types farther away from its ideal skill type, and closer to other firms’ ideal types. This is in line with empirical evidence that workers’ skills are transferrable, or portable, across jobs, but imperfectly so, as presented in Gathmann and Schönberg (2010). It is also consistent with evidence from German employer-employee data provided by Gulyas (2018), who finds that the worker-firm match quality is below average for newly hired workers in growing firms. We provide a complete description of the labor supply curve of individual firms facing such worker heterogeneity, which allows us to determine wage markups. Moreover, we solve for a firm’s equilibrium “skill reach”, determined by the condition that the marginal worker is indifferent between working in this firm and competing firms with the closest ideal skill types. The larger the skill reach of a firm, the lower the average productivity of its workers and thus the quality of its worker-matches.

Our model combines this type of spatial competition on the labor market with monopolistic competition on goods markets, based on translog preferences featuring love-of-variety. This is in the spirit of Krugman (1979) and gives our model a modern trade theory flavor. We consider a two-stage game for symmetric firms where stage one involves entry, including firms' choice of an ideal skill type, and stage two involves markup pricing and wage setting, based on love-of-variety-preferences and the labor supply curves as determined by the entry and skill-type choice of stage one.

In a nutshell, our contributions are as follows. First, we establish conditions under which a unique, symmetric subgame-perfect equilibrium of this game exists, treating the size of the labor force as well as the degree of horizontal skill-type differentiation and the distribution of workers over skill types as exogenous primitives of the economy.

Secondly, we apply the model to identify two novel effects of trade that derive from skill-type heterogeneity of workers. The familiar trade-induced exit of firms causes a loss in aggregate productivity through a lower average quality of worker-firm matches. Moreover, it reduces competition in the labor market, leading to higher markups between wages and the marginal productivity of workers. However, comparing free trade with autarky, we prove that the conventional pro-competitive and variety effects of trade dominate these adverse labor market effects. Hence, the gains from trade theorem survives. But the trade-induced firm exit aggravates inequality, because workers at the bottom end of the income distribution will see their skill type becoming less suitable in production. Piecemeal trade liberalization involves a non-monotonicity: When gradually reducing the real trade cost from a prohibitive level to zero, aggregate welfare is rising (falling) for high (low) initial levels of trade costs.

Thirdly, we show that horizontal worker heterogeneity gives rise to a novel explanation for two-way migration between symmetric countries. With borders open for migration, in each country there are some workers who find some foreign firms better suited for their skill types than domestic firms, and there exists a subset of firms finding some foreign workers who are better suited for their skill requirements than domestic ones. We demonstrate that any trade-cum-migration equilibrium always delivers higher aggregate welfare than an equilibrium with free trade alone, because it lowers wage markups in all countries. But the effects of migration on income inequality and on the average quality of worker-firm matches are ambiguous. Moreover, in contrast to piecemeal integration of goods markets, piecemeal integration of labor markets is unambiguously welfare increasing

for all countries, whenever it leads to more migration in equilibrium.

The remainder of the paper is organized as follows. In Section 2, we briefly discuss three different strands of literature that we contribute to with this paper. In Section 3 we describe the general model framework and characterize the autarky equilibrium. In Section 4, we then discuss the effects of a transition from autarky to free trade and the scenario of piecemeal trade liberalization. In Section 5, we introduce labor mobility and analyze the effects of migration, comparing “trade cum migration” with trade alone. Section 6 concludes.

2 Related literature

2.1 Worker heterogeneity and trade

Our paper is closely related to Ohnsorge and Trefler (2007) and Costinot and Vogel (2010), who develop continuous assignment models of trade based on worker heterogeneity coupled with supermodularity. These models feature perfect competition, and assignment is perfect in that there is a unique and monotonic matching function mapping worker skills to sectors. Our paper expands this stream of literature by considering horizontal skill heterogeneity which defies supermodularity. In addition, we allow for increasing returns to scale and imperfect competition. Assignment is between workers and firms (instead of industries), and it is driven by absolute, rather than comparative advantage.³ Crucially, the number as well as types of firms are endogenous. Matching is imperfect, which provides for globalization to affect aggregate productivity through the average quality of matches.

Two-sided heterogeneity on the labor market is also analyzed in Davidson et al. (2008). As in this paper, assignment is between workers and firms, but heterogeneity is verti-

³ There is a long strand of literature, originating in Roy (1951), that considers assignment of skill types defined as incorporating certain bundles of different skills that need to be matched with multi-dimensional skill requirements of jobs; see Mandelbrot (1962); Rosen (1978); Moscarini (2001); Lazear (2009), and Lindenlaub (2017). Welch (1969) compares this approach to the ideal variety approach to consumption theory proposed by Lancaster (1966), and applied to trade in Lancaster (1980). Moscarini (2001) is particularly noteworthy in the present context since he is similarly concerned with the quality of worker-firm matches in economies with two-sided heterogeneity. In his model, macroeconomic conditions of the labor market determine the average quality of matches through search incentives while skill-type heterogeneity drives Roy-type selection on the level of the individual worker. In this paper, we focus on the influence of a country’s openness on goods and labor markets on the average quality of matches.

cal; workers (managers) are low-skilled or high-skilled. When entering the market, firms choose whether to adopt a “low-tech” or a “high-tech” production technology. Two-sided heterogeneity comes with a technological feature that is somewhat similar to the log-supermodularity assumption in the above assignment models: conditional on other inputs, a high-skilled worker is more productive than a low-skilled one in a “low-tech” firm, but more productive still in a “high-tech” firm where low-skilled workers have zero productivity. Matching is subject to costly search with bilateral wage bargaining. The outcome potentially involves imperfect matching, meaning that not all high-skilled workers are matched to “high-tech” firms, and globalization potentially affects aggregate productivity by reallocating high-skilled workers to “high-tech” firms.⁴

There is also a strand of literature in the spirit of Melitz (2003), dealing with efficient assignment of heterogeneous workers to firms differing in productivity. In Helpman et al. (2010) there is vertical skill heterogeneity among workers, but although the skill distribution among the labor force is known to the firm, an individual worker’s skill level is not directly observable. Firms invest into screening activities in order to identify workers above a desired minimum ability. All workers are *ex ante* identical, but become heterogeneous *ex post*, once hired by a specific firm. Importantly, the workers’ skills are completely specific to the firm, which generates firm-worker bargaining and wage heterogeneity across firms, with more productive firms paying a higher wage. There is a similarity to the above assignment models in that more productive firms (acting under monopolistic competition on goods markets) have a stronger incentive to screen and will employ workers with a larger average ability level. In this sense, there is positive assortative matching, but the matching will not be perfect. Globalization has a productivity effect, over and above the familiar selection effect from Melitz (2003) models, running through improved matching: The reallocation to more productive firms gets reinforced through the more productive (exporting) firms increasing their screening investment, thus realizing a higher average ability of employed workers. The screening investment that firms determine upon entering bears a resemblance to our firms’ decision about the ideal worker characteristics in production, which in turn determines the firm’s skill reach, comparable to the firm’s minimum ability level determined by the firm’s screening effort in Helpman et al. (2010).⁵

⁴ For an empirical demonstration of this effect, see Davidson et al. (2014).

⁵ Wage dispersion among *ex ante* identical workers is also analyzed by Egger and Kreickemeier (2009), the reason there being fair wage preferences of workers, coupled with Melitz-type firm heterogeneity. In their

A recent paper by Becker et al. (2018) is similar to ours in using a circular representation of horizontal worker heterogeneity. The circle represents a continuum of tasks, and a point on the circle marks the task a worker is best at, called her core ability. Production is organized in terms of occupations. An occupation specifies a certain subrange of tasks, all of which are performed by each of the workers hired for that particular occupation, independently of a worker's core ability. A worker's efficiency when working in a certain occupation is falling in the average distance between the occupation-specific tasks and the worker's core ability, which in turn depends on the length of the occupation's task interval. More productive firms choose a larger number of occupations, so their workers perform tasks closer to their core abilities, thus enjoying a better matching. However, the relative importance of skill mismatch also depends on firm performance, rendering the relationship between firm size and wage inequality ambiguous. This implies that the reallocation and reorganization induced by trade has an a priori ambiguous effect on wage inequality. Our model isolates the positive relationship between skill reach and wage dispersion, leading to an unambiguously positive relationship between trade and inequality.

Grossman et al. (2017) propose a model featuring three-fold heterogeneity to describe simultaneous matching of heterogeneous workers to heterogeneous managers and sorting of these types into one of two industries. Industries differ in labor intensity, that is, the optimal number of workers of a given type that are paired with a manager of a given type in equilibrium. A change in the relative price of the two industries' goods affects the income distribution between occupations in accordance with the Stolper-Samuelson theorem. It also affects the between-industries wage distribution because sorting based on comparative advantage generates factor specificity. Moreover, inequality within occupation and industry is affected because workers and managers are re-matched. The aggregate effect of trade on inequality is ambiguous, depending on the equilibrium matching and sorting pattern. In their model, wages vary between workers (managers) due to their inherent skill-level differences, due to the fact that they are matched with differently skilled managers (workers) and different measures of workers of the same type within a firm. Worker

model, the fair wage preferences are a vehicle for rent sharing. Wage inequality among identical workers then obtains simply because workers are matched with firms differing in productivity: high productivity firms have high profits, leading to high wages through rent sharing. In Egger and Kreickemeier (2012) this idea is extended by including ex ante heterogeneity among individuals in terms of their managerial ability. Individuals self-select into managing firms employing the rest of the population as (homogeneous) workers.

types are uniquely matched with manager types and each type is assigned to only one industry. Hence, there is no notion of mismatch and no wage inequality between workers with identical skill levels, which are the essential aspects of our model.

More broadly, our paper also contributes to a voluminous modern literature on gains from trade in the spirit of Krugman (1979) and Melitz (2003). Arkolakis et al. (2012) have reinvigorated the discussion of gains from trade in these new trade models; for a recent survey of this literature, see Costinot and Rodriguez-Clare (2014). Our contribution here is to emphasize two additional welfare channels that derive from horizontal skill heterogeneity among workers through endogenous wage markups as well as through the endogenous quality of worker-firm matching. To achieve these contributions, we simplify in assuming away firm-heterogeneity in productivity analyzed by Melitz (2003). We assume a translog expenditure function, which is nested in Arkolakis et al. (2018) and implies sub-convex demand, as shown by Mrázová and Neary (2017). Thus, our model falls into the category of recent trade models delivering the familiar pro-competitive effects of trade.⁶

Our analysis of trade liberalization is also related to the recent work by Bykadorov et al. (2016) on the welfare effects of trade liberalization in monopolistic competition models with preferences that allow for a description of autarky as a result of prohibitive, but finite trade cost. They find that in such models lower trade cost are always harmful in the neighborhood of autarky due to firms' neglect of consumers' love for variety. In our setting this effect is embedded in an environment featuring wage markups and imperfect matching.

2.2 Two-way migration

Trade models highlighting endowment-based comparative advantage imply that trade and migration are substitutes, but if trade is driven by other forces they may be complements, as first emphasized by Markusen (1983). Empirical evidence strongly favors the view

⁶ This categorization of trade models has recently emerged from attempts to move away from CES demand structures to allow for endogenous markups. Subconvexity of demand functions imply that any scenario leading to lower firm-sales in a certain market, such as entry of foreign firms into the domestic market, also leads to lower price markups on goods markets. For a detailed discussion, see Zhelobodko et al. (2012) and Mrázová and Neary (2014, 2017).

that trade and migration are “non-substitutes”; see Felbermayr et al. (2015). Our model identifies a novel cause of strong complementarity between trade and migration. We identify an incentive for migration which is present absent trade, and which even increases with trade. Moreover, we demonstrate that this type of migration has effects opposite to those of trade.

Our model is able to explain two-way migration between similar countries. There is ample evidence that this type of migration is important empirically. Yet, it proves difficult to explain. Much of the migration literature on sorting across destinations has invoked location-specific preferences to rationalize these flows.⁷ More specific theoretical models derive two-way migration incentives from social stigma attached to employment in low social status occupations, as in Fan and Stark (2011), or from migration serving as signaling device for high skilled individuals when skills are unobservable, as in Kreickemeier and Wrona (2017). In Schmitt and Soubeyran (2006), two-way migration arises within occupations but across skill levels, and only if countries’ skill distributions are sufficiently different. In our model, two-way migration incentives between similar countries derive from the firm-specificity of skills and firms’ endogenous location choices in the skill space.

2.3 Circular models of heterogeneity

In employing a circular representation of continuous skill heterogeneity as in Amiti and Pissarides (2005), our model may also be seen in a broader tradition of other spatial competition models, such as the circular city model developed by Vickrey (1964) and Vickrey et al. (1999) or models of product differentiation in the spirit of Lancaster (1966) and Salop (1979). For trade applications, see Helpman (1981), Grossman and Helpman (2005), Eckel (2009a,b), who formulate the Lancaster model in circular space. In a labor market context, a circular model of worker heterogeneity has been used to analyze unemployment (Bhaskar and To, 1999, 2003; Marimon and Zilibotti, 1999; Thisse and Zenou, 2000), wage competition (Fiorillo et al., 2000), and the distribution of profits between workers and firms (Hamilton et al., 2000).

In all of these models the existence and uniqueness of the location equilibrium is as-

⁷ See the strand of empirical literature building on Grogger and Hanson (2011). Caliendo et al. (2015) develop a general equilibrium model with trade and migration involving exogenously given location preferences.

sumed, but not discussed. Yet, it is well known that models of spatial competition are often plagued by problems of existence and/or uniqueness of a pure strategy, subgame-perfect equilibrium, the reason being that maximum profits in post-entry stages of the game are not globally quasi-concave. Economides (1989) demonstrates the existence of a symmetric subgame-perfect equilibrium in a three-stage game of entry, variety choice and pricing. Vogel (2008) develops a similar model of three stages of decision making, but with firm heterogeneity in productivity and proves existence and uniqueness of the equilibrium. Our model of spatial competition in the labor market exhibits crucial differences regarding the maximum-profit function, requiring a different approach to the question of existence and uniqueness. To the best of our knowledge, we are the first to provide a detailed description of conditions under which a circular model of skill-type heterogeneity allowing for a flexible functional relationship between skill productivity and skill distance has a unique and symmetric subgame-perfect equilibrium.

3 The Modeling Framework

Our model economy is endowed with a mass L of workers, which are differentiated by the *types* of skills they possess. We assume that the space of skill types may be characterized by a circle with circumference $2H$, henceforth called the skill circle. Each location on the circle represents a skill type, and types that are more similar are located closer to each other. The circumference of this circle H measures the degree of horizontal skill differentiation present in the labor force. The labor force L is uniformly distributed over the circle, so that a mass of $\frac{L}{2H}ds$ workers is located within an interval of length ds on the circle. When setting up production, a firm chooses a certain location on the skill circle, which then determines that firm's ideal skill type. When working for this firm, workers are differently productive, depending on the distance between their skills and the firm's ideal skill type.

Consumer preferences are described by a translog expenditure function, which implies love of variety. Firms are fully symmetric in terms of technology. In *stage one*, firms decide on whether to enter and, if so, where to locate on the skill circle. Setting up production requires a fixed labor input α , defined in terms of efficiency units of the corresponding ideal skill type. In addition, production requires β units of this input per unit of the good

produced. In *stage two*, firms set profit maximizing goods prices as well as wage rates, based on their market power on the goods as well as the labor market. Stage two thus leads to a Bertrand-Nash equilibrium in prices and wage rates, conditional on the number and skill positions of firms determined in stage one. Stage-one decisions anticipate the Bertrand-Nash equilibria of stage two (subgame perfection). We assume free entry of an infinite number of potential entrepreneurs with zero outside options. Hence, equilibrium is characterized by zero profits.

The remainder of this section first looks at price setting in stage two. This is followed by a detailed analysis of the entry and location decision in stage one and a proof of existence of a unique equilibrium characterized by a symmetric location pattern of firms. The section will close by demonstrating that the equilibrium features excess entry of firms.

3.1 Price and wage setting with worker heterogeneity

3.1.1 Labor supply

The more a worker's skills deviate from the firm's ideal skill type the less productive she is when working for this firm. We model this through a function $f[d]$ which gives the number of efficiency units of labor delivered per physical unit of labor by a worker whose skills are at distance d from the ideal type.⁸ We assume that $f'[d] < 0$, $f'[0] = 0$, $f''[d] < 0$, and $f[d] = f[-d]$. This last property states that distance in either direction on the circle has the same effect. Without loss of generality, we set $f[0] = 1$.⁹

We assume enforceable contracts between firms and workers, specifying the quantity of, and price for, efficiency units of labor. Each worker inelastically supplies one unit of physical labor and knows her skill distance from all firms positioned on the skill circle as well as the productivity schedule $f[d]$. Thus, she knows the income she will earn per physical unit of labor when working for a certain firm offering a certain wage rate

⁸ We use brackets $[\cdot]$ to collect arguments of a function and parentheses to collect algebraic expressions.

⁹ One might ask why a firm should not always be able to secure the optimal combination of skills by employing convex combinations of workers embodying different combinations of skills. The answer is that doing so would entail a cost of communication between workers of different skill types. Therefore, other things equal, having the ideal skill type embodied in each worker is always less costly than combining different types of workers. The function $f[d]$ above may be interpreted as representing this cost of combining different skill types embodied in different workers.

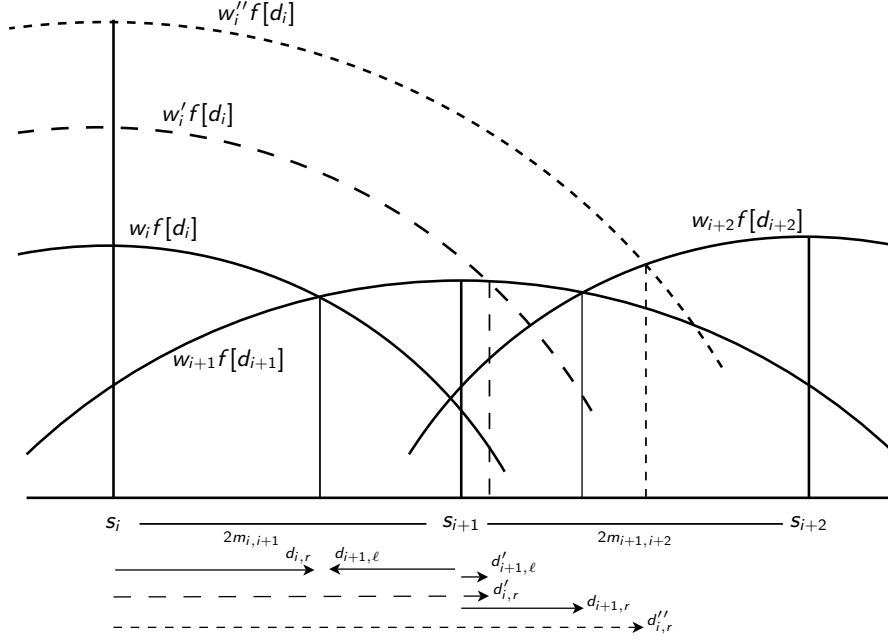


Figure 1: Sorting of workers

per efficiency unit. All workers sort themselves into employment by different firms so as to maximize their individual incomes, given firm-specific wage offers as well as their skill distance to these firms. For any pair of wage rates between two neighboring firms, there is a marginal worker who is indifferent between the two firms as their wage offers amount to an equal income per physical unit of labor. All inframarginal workers earn wages above their respective outside options which are the wages paid by the closest neighboring firm. This implies that the entire employment surplus is appropriated by workers, which is consistent with a zero-profit equilibrium. The reason for why firms are unable to appropriate any employment surplus through wage discrimination is that they are unable to observe an individual worker's skill type. This effectively rules out paying each worker a wage rate equal to her outside option.

Figure 1 illustrates this type of worker sorting. It looks at a range on the skill circle encompassing the location of three neighboring firms with optimal skill types s_i , s_{i+1} and s_{i+2} , which are at distances $2m_{i,i+1}$ and $2m_{i+1,i+2}$ from each other. The concave curves depict firm-specific schedules $w_g f[d_g]$, $g = i, i + 1, i + 2$, giving the income that a worker at distance d_g from firm g 's position may expect to earn per physical unit of labor when working for this firm, given that it offers a wage rate per efficiency unit equal to w_g . The skill distance d_g is measured both to the left and the right from s_g . We define $d_{i,r}$

such that all workers in the interval $[s_i, d_{i,r}]$ prefer working for firm i to working for firms $i + 1$ or $i + 2$, and similarly for the interval $[s_i - d_{i,\ell}, s_i]$ to the left. In other words, $d_{i,r}$ and $d_{i,\ell}$ measure the skill distance between firm i 's ideal skill type and the marginal worker to the right and left, respectively, who is indifferent between working for firm i and its two neighboring firms. We shall refer to this distance as the *skill reach* of firm i . The assignment that we obtain is one governed by absolute advantage, as opposed to the comparative advantage assignments considered in Ohnsorge and Treffer (2007) and Costinot and Vogel (2010). Comparing any two workers working for different firms, it will always be true that either worker is more productive in the firm he ends up working for than the other worker would be if both were to work in the same firm.

If firms i and $i + 1$ set wages equal to w_i and w_{i+1} , then firm i 's skill reach to the right is implicitly determined by the condition $w_i f[d_{i,r}] = w_{i+1} f[2m_{i,i+1} - d_{i,r}]$; see the solid arrow at the bottom of Figure 1.¹⁰ For a higher wage rate w'_i , a completely analogous condition $w'_i f[d'_{i,r}] = w_{i+1} f[2m_{i,i+1} - d'_{i,r}]$ leads to a greater skill reach $d'_{i,r}$, indicated by the long-dashed arrow.¹¹ With firm-specific wages w'_i and w_{i+1} firm i 's skill reach extends beyond s_{i+1} . Firm i would thus be able to hire firm $i + 1$'s ideal skill type, plus some workers to the right of s_{i+1} , up to point $s_i + d'_{i,r}$. In turn, firm $i + 1$ would be left employing workers in the interval $[s_i + d'_{i,r}, s_{i+1} + d'_{i+1,r}]$, with workers in the interval $[s_{i+1} + d_{i+1,r}, s_{i+2} + d_{i+2,r}]$ being employed by firm $i + 2$.¹² Increasing its wage rate further to w''_i would allow firm i to out-compete firm $i + 1$ and start attracting workers from firm $i + 2$. The skill reach covered by w''_i is implicitly determined by $w''_i f[d''_{i,r}] = w_{i+2} f[2m_{i,i+1} + 2m_{i+1,i+2} - d''_{i,r}]$; see the short-dashed arrow in Figure 1. As will become evident below, out-competing neighboring firms via high enough wages will never occur in equilibrium.

In what follows we use \mathbf{w}_{-i} to denote the $N - 1$ vector of wage rates set by all firms other than i , such that the first element is the wage set by the first neighbor to its right, and so on until the N -th element which is the wage set by the first neighbor to its left.¹³

¹⁰ Note that we have replaced d_{i+1} by $2m_{i,i+1} - d_i$, in line with the aforementioned sorting of workers.

¹¹ Due to symmetry as assumed, we have $f[2m_{i,i+1} - d'_i] = f[d'_i - 2m_{i,i+1}]$. This allows us to use the same condition determining marginal workers to the left and the right of s_{i+1} . Workers at a distance $d'_{i,r} - 2m_{i,i+1}$ to the left of s_{i+1} would earn the marginal worker's income if working for firm $i + 1$, but they are better off working for firm i . The marginal worker is thus uniquely determined by the above condition.

¹² The distance $d_{i+2,r}$ lies to the right of the range covered by Figure 1.

¹³ A well-defined labor supply function as derived in this subsection requires $N \geq 2$. We shall assume below

Accordingly, \mathbf{m}_i denotes the N -dimensional vector $(2m_{i-1,i}, 2m_{i,i+1}, \dots, 2m_{i-2,i-1})$, where we set $i-1 = N$ if $i = 1$, and $i+1 = 1$ if $i = N$. We shall henceforth refer to \mathbf{m}_i as the distance vector viewed at from firm i 's perspective, where $2m_{i-1,i}$ and $2m_{i,i+1}$ must be interpreted, respectively, as the distance between firm i and its first left-hand and first right-hand neighbor, and so on. It now follows that the right-hand skill reach from firm i 's location on the skill circle may be written as $d_{i,r} = d_r[w_i, \mathbf{w}_{-i}, \mathbf{m}_i]$ and analogously for the skill reach to its left, $d_{i,\ell} = d_\ell[w_i, \mathbf{w}_{-i}, \mathbf{m}_i]$.¹⁴ Clearly, the skill reaches $d_{i,r}$ and $d_{i,\ell}$ are increasing in w_i and weakly increasing in \mathbf{m}_i , but weakly decreasing in \mathbf{w}_{-i} .

The amount of efficiency units firm i is able to attract by setting a wage rate w_i is the integral over all efficiency units $f[d]$ from distance zero up to distance $d_{i,r}$ plus the corresponding integral from zero to $d_{i,\ell}$. Writing

$$L^{S,\ell} = \int_0^{d_\ell[w_i, \mathbf{w}_{-i}, \mathbf{m}_i]} f[d] \frac{L}{2H} dd \quad \text{and} \quad L^{S,r} = \int_0^{d_r[w_i, \mathbf{w}_{-i}, \mathbf{m}_i]} f[d] \frac{L}{2H} dd,$$

firm i 's labor supply schedule now emerges as

$$L^S[w_i, \mathbf{w}_{-i}, \mathbf{m}_i] = \begin{cases} L^{S,\ell} + L^{S,r} & \text{if } d_{i,\ell} \leq -d_{i,r} \\ 0 & \text{otherwise.} \end{cases} \quad (2)$$

It should be noted that \mathbf{w}_{-i} as well as \mathbf{m}_i carry information about the number of firms (the dimension of \mathbf{m}_i), hence we abstain from listing N as an argument in the labor

that N is "large".

¹⁴Note that the function $d_r[\cdot]$ is uniform across firms, but the value of this function is firm-specific. The function $d_r[w_i, \mathbf{w}_{-i}, \mathbf{m}_i]$ is implicitly defined as the solution to the following condition:

$$w_i f[d_{i,r}] = w_{i+j} f \left[\sum_{k=1}^j 2m_{i+k-1,i+k} - d_{i,r} \right] \quad \text{where} \quad (1)$$

$$j = \operatorname{argmax}_j \left\{ w_{i+j} f \left[\sum_{k=1}^j 2m_{i+k-1,i+k} - d_{i,r} \right] \mid 1 \leq j \leq N-1 \right\}.$$

This condition includes combinations of wage rates where firm i out-competes some of its nearest neighbors. In these expressions, $i+j$ indicates firm i 's relevant competitor employing the marginal worker at distance $d_{i,r}$ from firm i 's ideal type. The second line identifies the relevant competitor as the firm which is the first to meet firm i 's wage offer as the skill distance increases. A completely analogous condition determines the left-hand skill reach $d_{i,\ell}$.

supply function. Intuitively, for a low enough wage rate w_i firm i 's labor supply will fall down to zero. This happens if the condition $d_{i,\ell} \leq -d_{i,r}$ is violated, in which case the overall distance covered by firm i then has zero measure. For wages above this threshold level, the firm faces a labor supply function for efficiency units that is increasing in its own wage. Moreover, this function is continuous in w_i , except for points where a further increase in w_i reduces labor supply to the nearest competitor down to zero. But, as we have emphasized above, such out-competing of neighbors will never arise in the scenarios considered below. In what follows we shall use η_i to denote the elasticity of firm i 's labor supply function (2). Obviously, this elasticity is a function of w_i , \mathbf{w}_{-i} , and \mathbf{m}_i .

3.1.2 Goods demand

Individual k derives utility from consumption of a bundle of N differentiated varieties according to an indirect utility function of the form

$$\ln V_k = \ln y_k - \ln P[\mathbf{p}], \quad (3)$$

where $P[\mathbf{p}] = P[p_1, \dots, p_i, \dots, p_N]$ is the minimum unit expenditure function and y_k denotes income. Following Diewert (1974) and Bergin and Feenstra (2000), we assume that preferences are characterized by a symmetric translog expenditure function¹⁵

$$\ln P[\mathbf{p}] = \frac{1}{2\gamma N} + \frac{1}{N} \sum_{i=1}^N \ln p_i + \frac{\gamma}{2N} \sum_{i=1}^N \sum_{j=1}^N \ln p_i (\ln p_j - \ln p_i), \quad (4)$$

which is homogeneous of degree one in prices \mathbf{p} . The parameter $\gamma > 0$ controls the degree of substitutability between varieties, a larger γ implying higher substitutability. Using Roy's identity, the Marshallian demand function for variety i can be derived as

$$q_{ik}[\mathbf{p}, y_k] = \frac{\partial \ln P[\mathbf{p}]}{\partial \ln p_i} \frac{y_k}{p_i} = \delta_i \frac{y_k}{p_i}, \quad (5)$$

¹⁵Recent applications of the symmetric translog expenditure system are Feenstra and Weinstein (2010), Arkolakis et al. (2010) and Rodríguez-López (2011). This expenditure system constitutes a second-order Taylor approximation of any symmetric expenditure function (see Feenstra and Weinstein, 2010).

where

$$\delta_i = \frac{1}{N} + \gamma (\overline{\ln p} - \ln p_i) \quad (6)$$

is the expenditure share for variety i and $\overline{\ln p} := \sum_i \ln p_i / N$. Thus, the preferences underlying the above expenditure function are homothetic. Inserting (6) into (5) and using Y to denote aggregate income, revenue r_i then follows as

$$r_i = \delta_i Y \quad \text{with} \quad \delta_i = \gamma \mathcal{W} \left[\exp \left\{ \frac{1}{\gamma N} + \overline{\ln p} \right\} \frac{q_i}{\gamma Y} \right], \quad (7)$$

where $\mathcal{W}[\cdot]$ denotes the *Lambert function*.¹⁶ While (6) expresses the expenditure share as a function of $\ln p_i$, in (7) this share is expressed as a function of the quantity q_i ; Appendix A.1 has the details. Given consumers' love for variety, no two firms will produce the same variety, so that we may use i to indicate firms.

3.1.3 Pricing equilibrium

Armed with these representations of goods demand and labor supply, firm behavior in stage two may now be characterized by the following profit maximization problem:

$$\begin{aligned} \max_{w_i} \quad & r_i - w_i L_i \\ \text{s.t.:} \quad & q_i = \frac{L_i - \alpha}{\beta} \quad \text{with} \quad L_i = L^S[w_i, \mathbf{w}_{-i}, \mathbf{m}_i], \text{ and } q_i \geq 0, \end{aligned} \quad (8)$$

where r_i is given by (7). The restriction ensures that the firm is on its labor supply function and produces a positive quantity. We proceed under the assumption that the non-negativity constraint is non-binding. The corresponding restrictions on the parameter space are discussed in Appendix A.3. Note that the problem (8) is conditional on N and \mathbf{m}_i , which are determined in stage one to be discussed below.

We assume that firms pursue Bertrand strategies on both the goods and the labor market, taking the average log price $\overline{\ln p}$ and aggregate income Y as being beyond their

¹⁶The Lambert function $\mathcal{W}[z]$ defines the implicit solution to $xe^x = z$ for $z > 0$. Furthermore, it satisfies $\mathcal{W}'[z] = \frac{\mathcal{W}[z]}{(\mathcal{W}[z]+1)z} > 0$, $\mathcal{W}''[z] < 0$, $\mathcal{W}[0] = 0$ and $\mathcal{W}[e] = 1$.

own influence. The perceived price elasticity of demand for variety i emerges as

$$\varepsilon_i[p_i, \overline{\ln p}, N] := -\frac{d \ln q_i}{d \ln p_i} = 1 - \frac{d \ln \delta_i}{d \ln p_i} = 1 + \frac{\gamma}{\delta_i} > 0, \quad (9)$$

where δ_i is given in (7).

The first-order condition implies

$$p_i = \frac{\varepsilon_i}{\varepsilon_i - 1} \frac{\eta_i + 1}{\eta_i} w_i \beta. \quad (10)$$

Pricing thus involves a double markup over marginal cost.¹⁷ From (9) and (6), we may write the first markup which derives from product differentiation as

$$\frac{\varepsilon_i}{\varepsilon_i - 1} = \left(1 + \frac{\delta_i}{\gamma}\right) = \mathcal{W} \left[\frac{\eta_i}{w_i(\eta_i + 1)} \exp \left\{ 1 + \frac{1}{\gamma N} + \overline{\ln p} \right\} \right]. \quad (11)$$

See Appendix A.1 for details. In this equation, the argument of the Lambert function \mathcal{W} is a “summary measure” of the conditions that firm i faces on the labor market as well as the goods market. Given $\mathcal{W}'[Z] > 0$, a higher average log-price of the firm’s competitors and a lower degree of substitutability γ both lead to a higher markup. The same holds true for a smaller number of firms, whereas the markup is falling in perceived marginal cost. The second markup in (10) derives from the firm’s monopsony power on the labor market, where the firm faces a finite elasticity of supply $\eta_i < \infty$. Remember that η_i is a function of w_i , \mathbf{w}_{-i} and \mathbf{m}_i ; see (2) above.

Combining markup pricing with the constraint in (8) gives rise to a best response function $w_i = w[\mathbf{w}_{-i}, \mathbf{m}_i, N]$. For ease of exposition, we drop $\overline{\ln p}$ and Y as arguments of the best response function. Equilibrium wages then follow as the fixed point of all firms’ best response functions:

$$w_i^e = w^e[\mathbf{m}_i, N] \quad \text{for } i = 1, \dots, N. \quad (12)$$

Given these wage rates, (10) determines equilibrium prices $p_i^e = p^e[\mathbf{m}_i, N]$ and thus equilibrium profits, which we write as $\pi_i^e = \pi^e[\mathbf{m}_i, N]$. The conditions under which such

¹⁷ It is easy to verify that under the assumptions made, the second-order condition is satisfied.

an equilibrium exists and is unique may be summarized by the following lemma.

Lemma 1. *The Bertrand game of wage and price setting in stage two has a unique equilibrium with $q_i > 0$, if (i) marginal profits w.r.t. output are positive in the neighborhood of $q_i = 0$, and (ii) if the profit function is quasiconcave in w_i . A sufficient condition for the profit function to be quasiconcave is that the firm's labor supply function is concave in w_i . If the marginal cost β is sufficiently low, then quasiconcavity of profits obtains independently of the curvature of labor supply.*

The proof of this lemma follows in Appendix A.2. The first condition rules out corner solutions in which some firms find it optimal, conditional on entry, not to produce at all. For a given labor market environment $(H, f[d], \mathbf{m}_i \text{ and } L)$, marginal profits are high if the marginal cost β is low, and if the degree of substitutability in demand (captured by γ) is low. A low enough value of γ (low substitutability) ensures that the choke price for a new good is high enough, so that firms that set high prices due to tight labor market conditions $(\mathbf{m}_i, \mathbf{w}_{-i})$ still face some demand for their goods. However, independently of the choke price, a low enough value of β will always ensure positive marginal profits. Quasiconcavity in w_i depends on the curvature of the revenue function in q_i as well as on the curvature of labor supply in w_i . But the lower the marginal cost β , the less important the curvature of labor supply. Hence, even if labor supply is convex in w_i , concavity of the revenue function still generates quasiconcavity profits in w_i . In what follows, we assume that the sufficient conditions for a unique equilibrium as described in Lemma 1 are satisfied.

We want to stress that the above Lemma allows for a firm-specific distance vector \mathbf{m}_i , even though firms are symmetric as regards technology and demand. Structural symmetry implies that we have uniform functions $w^e[\cdot], p^e[\cdot]$ and $\pi^e[\cdot]$ that describe the stage-two equilibrium. But we do not impose symmetry on the distance vector, since at this stage of the analysis we simply do not know whether the entry game among symmetric firms will in fact generate a symmetric distance pattern on the skill circle.

3.2 Entry decision and the equilibrium distance pattern

The key challenge here is to characterize the stage-two pricing game in a way that allows us to examine whether deviations from the symmetric location pattern are profitable. One way to think of the location choice in stage one is to view a firm's strategy space as a set of addresses it can choose from, taking as given the addresses of other firms, and where the firm's pay-off is given by equilibrium profits as determined in the subsequent pricing game. This setup is chosen, for instance, in Economides (1989) and Vogel (2008) who analyze product differentiation, with consumers evenly distributed over a circle characterizing ideal product characteristics, and with utility quadratic or linear, respectively, in the distance between a consumer's and the firm's position on this circle. In our circular model of the labor market the address of a firm on the circle is not informative. We have shown above that equilibrium profits in the pricing game depend only on the distances to other firms, described by \mathbf{m}_i , and not on their positions as such. Knowing about this, a firm is unlikely to consider alternative addresses for itself while assuming that all other firms keep their addresses fixed. This would imply that the firm assumes it can influence the overall pattern of distances, which seems questionable. We therefore reduce the firm's choice of entry and the choice of its position on the skill circle to the decision whether to enter, given the firm's *beliefs* about the type of distance patterns that it may *rationally* expect to face upon entry. In turn, beliefs relate to two sets: the set of conceivable distance vectors, given that a certain number of firms have entered, and the set of conceivable numbers of firms that may enter. Our approach allows for more general functional forms for a worker's productivity, $f[d]$, than the quadratic and linear functions considered, respectively by Economides (1989) and Vogel (2008).¹⁸

We assume that there is an infinite number \bar{N} of *potential* entrants. Given the circumference H , any given number of *actual* entrants N renders a set of infinitely many possible N -dimensional distance vectors \mathbf{m} between these N firms. In the following, we use \mathcal{M}_N to denote this set, which is an exhaustive description of possible labor market

¹⁸ There is a further noteworthy difference between our labor market circle and the product circle in Economides (1989) and Vogel (2008). In their setup, the utility function determines the cut-off between consumers served by different firms, but demand per consumer is normalized to one. In our model, workers deliver different efficiency amounts of labor. Therefore, the relationship between the firm's labor supply curve and the skill reach, which is the equivalent to their consumer cut-off, is somewhat more involved than the relationship between the firm's demand function and the consumer cut-off.

environments that a firm may face, conditional on N . Our solution strategy for the entry game rests on the assumption that any firm i views possible realizations of distances as random variables, forming beliefs about conceivable distance patterns \mathbf{m}_i . We describe these beliefs by a joint pdf $\mu_i[\mathbf{m}_i|N]$.¹⁹ The function μ_i assigns a unique probability to any $\mathbf{m}_i \in \mathcal{M}_N$.

The strategy space for a firm is characterized by a binary decision variable \mathcal{I}_i , where $\mathcal{I}_i = 1$ indicates entry, and $\mathcal{I}_i = 0$ indicates non-entry. Firm i 's expected payoff, conditional on N , is $\mathbb{E}_i[\pi^e[\mathbf{m}_i, N]]$, where \mathbb{E}_i denotes the expected value formed over all distance vectors viewed from firm i 's perspective, using firm i 's set of conditional beliefs $\mu_i[\mathbf{m}_i|N]$, and $\pi^e[\mathbf{m}_i, N]$ is the equilibrium profit in the second-stage pricing game; see above. The decision rule, conditional on N , is as follows:

$$\mathcal{I}_i = \begin{cases} 1 & \text{if } \mathbb{E}_i[\pi^e[\mathbf{m}_i, N]] \geq 0 \text{ and } \nu_i[N] > 0 \\ 0 & \text{otherwise} \end{cases} \quad \text{for all } i = 1 \dots \bar{N} \quad (13)$$

This rule invokes the firm's beliefs about possible values of N , characterized by the pdf $\nu_i[N]$. Since $N = \sum_{j \neq i} \mathcal{I}_j + 1$, (13) is readily interpreted as a best response function. We shall use $\mathcal{I}_i[N]$ to denote the outcome of decision rule (13).

Given that firms are symmetric, it seems natural to assume uniform beliefs, $\mu_i = \mu$ and $\nu_i = \nu$ for all i . This implies that the outcome of the decision rule will be the same across all firms as well. Hence, we either have $\sum_{i=1}^{\bar{N}} \mathcal{I}_i[N] = 0$ or $\sum_{i=1}^{\bar{N}} \mathcal{I}_i[N] = \bar{N}$. Using N^e to denote the equilibrium number of firms, we can now draw on fixed-point logic to describe an equilibrium of the entry game by the following conditions:

$$\sum_{i=1}^{\bar{N}} \mathcal{I}_i[N^e] = \bar{N} \geq N^e \quad \text{and for any } \tilde{N} > N^e: \quad \sum_{i=1}^{\bar{N}} \mathcal{I}_i[\tilde{N}] = 0 \quad (14)$$

The first is a condition on entry; assuming N^e firms will be in the market, at least N^e firms must in fact decide to enter. The second is a complementary non-entry condition; assuming the number of firms will be $\tilde{N} > N^e$, no one wants to enter. Clearly, the two conditions jointly determine a unique equilibrium value $N^e > 0$.

¹⁹In the terminology of dynamic games with incomplete information, this set of possible labor market environments corresponds to an information set; see Mas-Colell et al., (1995, ch. 9).

To determine the equilibrium distance vector \mathbf{m}_i , we now impose two consistency requirements on the sets of beliefs $\mu[\mathbf{m}_i, N]$ and $\nu[N]$ that we argue are implied by the assumption of structural symmetry of firms, coupled with the assumption that firms are fully informed about the characteristics and logic of the circle. These requirements are based on the observation that any one of the distance vectors $\mathbf{m}_i \in \mathcal{M}_N$ completely describes the pattern of distances between *all* firms on the circle, which implies unique distance vectors $\mathbf{m}_j \in \mathcal{M}_N$ for all $j = 1, \dots, N$, $j \neq i$. We collect these distance vectors corresponding to any one \mathbf{m}_i in the set $\{\mathbf{m}_{-i}\} \subset \mathcal{M}_N$. The first consistency requirement relates to $\nu[N]$, and the second relates to $\mu[\mathbf{m}_i|N]$:

$$\nu[N] = 0, \text{ if } \pi^*[\mathbf{m}_j, N] < 0 \text{ for at least one } \mathbf{m}_j \in \{\mathbf{m}_{-i}\} \text{ for each } \mathbf{m}_i \in \mathcal{M}_N \quad (15)$$

$$\mu[\mathbf{m}_i|N] = 0, \text{ if } \nu[N] > 0 \text{ and } \pi^*[\mathbf{m}_j, N] < 0 \text{ for at least one } \mathbf{m}_j \in \{\mathbf{m}_{-i}\} \quad (16)$$

Requirement (15) states that any firm i attaches a zero belief to N , if *for each conceivable* distance vector the implied distance vectors for its competing firms are such that at least one of its competitors makes zero maximum profits in the pricing game. It would clearly be irrational to maintain a positive likelihood for such a number of firms in any one firm's set of beliefs $\nu[N]$. Requirement (16) states that a specific distance vector $\mathbf{m}_i|N$ receives a zero likelihood in any firm i 's set of beliefs $\mu[\mathbf{m}_i|N]$, if the implied distance vectors for firm i 's competitors are such that for at least one of the competing firms maximum profits in the second-stage pricing game are negative.

Given consistent beliefs, we can prove that under plausible restrictions on the parameter space discussed below there exists a *unique* equilibrium with (\mathbf{m}^e, N^e) , where \mathbf{m}^e denotes the *symmetric* distance vector with $m_{i,i-1} = m_{i,i+1} = m = H/N^e$ for all i indicating entering firms.

Lemma 2. *Given that firms play entry strategies as described in (13), there exists a subgame-perfect entry equilibrium as defined in (14), with a finite number of entering firms symmetrically positioned on the skill circle, and this equilibrium is unique, provided that the following conditions are met: (i) Firms' beliefs about conceivable distance vectors and the number of entrants are consistent, (ii) the fixed cost of production is not too large relative to the size of the labor force and relative to the degree of product differentiation, and (iii) the marginal cost is small enough, relative to the fixed cost.*

The proof, details of which are found in Appendix A.3, involves two steps. The first is to show that for *symmetric* distance patterns $\pi^e[\mathbf{m}, N]$ is strictly decreasing in N . Ignoring the integer constraint on N , this ensures existence of an equilibrium candidate $(\mathbf{m}^{ce}, N^{ce})$ where $\pi^e[\mathbf{m}^{ce}, N^{ce}] = 0$. The logic of the entry equilibrium as defined in (13) and (14), coupled with consistent beliefs as defined in (15) and (16), then implies that any outcome with $N < N^e$ cannot arise as an equilibrium. The reason is that with any $N < N^{ce}$ non-entrants will know that more entry (with $N = N^{ce}$) is possible without losses, provided the distance pattern is symmetric. Importantly, this holds true for symmetric as well as asymmetric distance patterns corresponding to $N < N^{ce}$.

The second step of the proof then looks at *asymmetric* distance patterns for $N \geq N^{ce}$, demonstrating that this would involve negative profits for at least one entrant. Therefore, the exact same logic of (13) and (14) implies that such an outcome cannot be an equilibrium either. Taken together, all of this implies that the symmetric distance pattern with $(\mathbf{m}^{ce}, N^{ce})$ is the only equilibrium, $(\mathbf{m}^{ce}, N^{ce}) = (\mathbf{m}^e, N^e)$. A key element of this second step is how moving from a symmetric to an asymmetric distance pattern affects firms' maximum profits. Clearly, moving to asymmetry reduces the average productivity of workers, thus reducing effective aggregate labor supply. Therefore, in any asymmetric pattern the smallest firm faces a lower effective labor supply for a notionally unchanged wage rate and will suffer from moving up its average cost curve as a result of moving to asymmetry. At the same time, a move to asymmetry changes firms' labor supply elasticities, and will do so differently across firms. Appendix A.3 proves that with any asymmetric pattern the smallest firm unambiguously ends up with lower profits than in the symmetric case, provided that the degree of economies of scale is large enough, i.e., that β is low enough relative to α .

3.3 Autarky equilibrium

We now turn to the determination of m , the symmetric equilibrium half-distance between any two representative firms, as well as the solution for the goods price and the wage rate for the representative firm. In this section, we do this for the closed economy, thus paving the way for comparative analysis of various opening up scenarios in subsequent sections.

The number of firms N is related to m through the circumference of the skill circle: $m = H/N$. In a symmetric equilibrium we have $p_i = p$, with $\overline{\ln p} = \ln p$, as well as $w_i = w$.

Under symmetry, the elasticity of labor supply in (2) may be written as

$$\eta[m] := \left. \frac{\partial L_i^S}{\partial w_i} \frac{w_i}{L^S} \right|_{w_i=w} = -\frac{f[m]^2}{2F[m]f'[m]}, \quad (17)$$

where $F[m] := \int_0^m f[d]dd$. Our assumption that $f''[m] < 0$ ensures the labor supply elasticity is falling in m . Invoking symmetry in (6) simplifies the expressions for ε and δ , allowing us to write the profit maximizing price (10) as

$$p[m] = \rho[m]\psi[m]\beta, \quad (18)$$

$$\text{where } \rho[m] := 1 + \frac{1}{\gamma N[m]} \quad \text{and} \quad \psi[m] := \frac{\eta[m] + 1}{\eta[m]}. \quad (19)$$

In (18), we have normalized the wage per efficiency unit to one.²⁰ Note that $\rho'[m] > 0$ as well as $\psi'[m] > 0$. Firms' market power in the either market increases as firms become larger and the number of firms falls.

Given a uniform distribution of the workforce around the circle, the average productivity of workers is

$$\theta[m] = \frac{1}{m} \int_0^m f[d]dd. \quad (20)$$

Notice that we have $\theta'[m] = (f[m] - \theta[m])/m < 0$ since $f'[m] < 0$. Given our wage normalization, $\theta[m]$ represents average income per worker. Aggregate income emerges as $Y = L\theta[m]$, and output per firm is

$$q[m] = \frac{1}{N[m]} \frac{L\theta[m]}{p[m]}. \quad (21)$$

The zero-profit condition requires

$$p[m] = \frac{\alpha + \beta q[m]}{q[m]}. \quad (22)$$

²⁰ We are free to do so, since our equilibrium is homogeneous of degree zero in nominal prices. This can easily be seen from substituting (9) and (6) in (10), which yields $p_i = \left(1 + \frac{1}{\gamma N} + \overline{\ln p} - \ln p_i\right) \frac{\eta_i + 1}{\eta_i} w_i$.

Without loss of generality, we may choose units such that $\beta = 1$. The labor market clearing condition may then be written as $\alpha + q[m] = \frac{L}{N[m]}\theta[m]$, and $N[m]q[m] = L\theta[m] - \alpha N[m]$. Substituting these expressions in (22), we obtain the following representation of the zero-profit condition:

$$p[m] = g[m] := \frac{L\theta[m]}{L\theta[m] - \alpha N[m]}. \quad (23)$$

Note that $g[m] > 1$ is the usual measure of the degree of economies of scale, i.e., the ratio of average to marginal cost, applied to the economy at large. We have $g'[m] < 0$. With zero profits, this ratio must be equal to the price relative to marginal cost. With $w\beta = 1$ from scaling and normalization, this leads to (23). Intuitively, with a higher distance between firms zero profits require a lower price. Combining the zero-profit condition (23) with the Bertrand pricing equation in (18), we finally arrive at the following condition determining m :

$$g[m] = \rho[m]\psi[m]. \quad (24)$$

If we let H converge to zero, then this equilibrium converges to the equilibrium in the model considered by Krugman (1979) for the special case of translog preferences; see Appendix A.4.

When deriving welfare results below, we take an ex ante view, assuming that workers regard each point on the circle as being equally likely to become an ideal type. Given a symmetric equilibrium, expected utility of a worker is then equal to

$$\ln V = \ln \theta[m] - \left(\frac{1}{2\gamma N[m]} + \ln p[m] \right). \quad (25)$$

Intuitively, this welfare measure is rising in income and the number of firms in the market while falling in the price of a typical variety of goods.

3.4 Distortions

The equilibrium described above involves four distortions. (i) When considering market entry, firms fail to take into account the positive effect of their entry on welfare through

a larger number of varieties. Following Dixit and Stiglitz (1977), this is often referred to as “consumer-surplus distortion.” (ii) Potential entrants ignore the positive effect on average productivity arising from a better quality of matches in the labor market. This is novel in the present model, relative to standard models of monopolistic competition, and we call it the “productivity distortion.” (iii) Potential entrants anticipate both, a goods price markup as well as a wage markup, but fail to see that they receive operating profits on such markups only at the expense of incumbent firms, due to the overall resource constraint. Following Mankiw and Whinston (1986), this may be called the “business-stealing” effect. (iv) Potential entrants fail to anticipate that their entry will reduce the magnitudes of these same markups, due to enhanced competition.

Distortions (i) and (ii) constitute positive externalities, working towards insufficient entry in a laissez faire equilibrium, while distortions (iii) and (iv) work towards excessive entry. As is well known, in the standard CES version of the monopolistic competition model distortions (i) and (iii) offset each other and firm entry is efficient. In Appendix A.5 we show that in this model the net result of distortions (i)-(iv) is excess entry. Thus, our model inherits the “excess entry” result established by Salop (1979) for the circular city model. Moreover, the result is in line with Bilbiie et al. (2008), who find that in a monopolistic competition equilibrium with symmetric translog preferences the business-stealing effect dominates the consumer-surplus effect, giving rise to excess entry. The excess-entry result plays a crucial role in the determination of the gains from globalization below, since these partly unfold through a mitigation of distortions.

4 Symmetric trading equilibrium

In this section, we explore the gains from trade as well as the effect of trade on income inequality. The first subsection compares autarky with free trade, where we introduce trade simply by allowing for the number of countries to increase beyond one (which is autarky) and allowing for firms in all countries to sell on all national markets without any border frictions. Mrázová and Neary (2014) call this the extensive margin of globalization. In the second subsection we then turn to the intensive margin of globalization by holding the number of countries fixed at two but allowing for trade to be costly, and by looking at a marginal reduction of the trading cost. We assume countries to be fully symmetric,

including the extent of worker heterogeneity, so as to clearly isolate the channels that emanate from horizontal worker heterogeneity as such.

4.1 Free trade

We assume that there are k symmetric countries and denote the total number of firms worldwide by $N^T = kN$. Absent all barriers, prices for domestic and imported goods are equal, given by

$$p[m] = \left(1 + \frac{1}{\gamma k N[m]}\right) \psi[m]. \quad (26)$$

This expression reflects the fact that firms now take into account foreign competitors, but it keeps the simplified form familiar from the autarky equilibrium; see (19). Absent all trade barriers, prices of imported and domestic varieties are fully symmetric, whence the price of any variety consumed is equal to the average price. In what follows, we define $\rho^T[m] := 1 + \frac{1}{k\gamma N[m]}$ as the goods price markup under free trade. It is obvious that $\rho^T[m] < \rho[m]$.

Total demand per variety remains unchanged, since the lower domestic demand is compensated by the larger number of countries:

$$q[m] = \frac{kL\theta[m]}{kN[m]p[m]} = \frac{L\theta[m]}{N[m]p[m]}. \quad (27)$$

The labor market clearing condition similarly remains unaffected. The equilibrium condition determining m then follows as

$$g[m] = \rho^T[m]\psi[m]. \quad (28)$$

The following proposition summarizes the comparison between autarky, $k = 1$, and free trade among $k > 1$ countries.

Proposition 1. *Opening up to free trade among $k > 1$ symmetric countries has the following effects, relative to an autarky equilibrium (with $k = 1$): (i) There is exit of firms in each country, with an increase in the total number of varieties available. (ii)*

There is a higher wage markup, coupled with a lower price markup, but goods prices are unambiguously lower. (iii) Each country's labor market suffers from a fall in the average matching quality, implying lower average income. (iv) Each country enjoys a higher aggregate welfare, measured by a higher real income. (v) Income inequality increases.

Proof: A formal proof is relegated to Appendix A.6.1.

The increase in variety and the pro-competitive effect on the goods market are standard results in trade models with monopolistic competition and endogenous markups. The novel insight here relates to adverse labor market effects: A lower number of domestic firms lowers the degree of competition on labor markets, increasing the wage markup, but the pro-competitive effect on the goods market dominates. In addition, the exit of firms makes it more difficult for workers to find firms matching well with their skills, causing a reduction in the productivity of the average worker. However, the variety and pro-competitive effects more than compensate for this negative productivity effect, making the economy better off under free trade than under autarky. The positive welfare effect involves two channels. The first runs through higher variety and lower goods prices. In addition, there is a positive first-order effect deriving from firm exit since the autarky equilibrium features excess entry. On account of $f'[m] < 0$ exit of some firms will reduce the lower bound of wages. Since the upper bound of wages is fixed at $f[0] = 1$, and given a uniform distribution of workers over the skill circle, this entails an increase in income inequality.

4.2 Costly trade and piecemeal trade liberalization

The superiority of free trade to autarky does not imply that any piecemeal liberalization in a world with costly trade is always beneficial. We stick to the symmetric case, but for simplicity reduce the number of countries to $k = 2$, using an asterisk to denote the foreign country. Suppose that firms face iceberg transport cost $\tau > 1$ for exports. A domestic firm selling q_i units on the domestic market and q_i^* units on the export market then needs a labor input equal to $\alpha + q_i + \tau q_i^*$.²¹ We assume that markets are segmented, and therefore, firms can set market-specific quantities independently. The firm thus maximizes

²¹ Remember that we have scaled units such that the marginal production cost β is equal to unity.

profits with respect to the wage, which determines its labor supply and hence total output $\bar{q}_i = q_i + \tau q_i^*$, and with respect to the quantity sold on the domestic market, observing $q^* = \frac{1}{\tau}(\bar{q}_i - q_i)$. Hence, it solves the following maximization problem:

$$\begin{aligned} & \max_{w_i, q_i} \{r_i + r_i^* - w_i(\alpha + \bar{q}_i)\} \\ \text{s.t.:} \quad & r_i = \delta_i Y, \quad r_i^* = \delta_i^* Y^* \\ & \bar{q}_i = q_i + \tau q_i^* \quad \text{with} \quad q_i \geq 0 \text{ and } q_i^* \geq 0 \\ & \alpha + \bar{q}_i = L^S[w_i, \mathbf{w}_{-i}, \mathbf{m}_i], \end{aligned} \tag{29}$$

where

$$\delta_i = \frac{1}{N^T} + \gamma (\overline{\ln p} - \ln p_i) = \gamma \mathcal{W} \left[\exp \left\{ \frac{1}{\gamma N} + \overline{\ln p} \right\} \frac{q_i}{\gamma Y} \right] \quad \text{and} \tag{30}$$

$$\delta_i^* = \frac{1}{N^T} + \gamma (\overline{\ln p} - \ln p_i^*) = \gamma \mathcal{W} \left[\exp \left\{ \frac{1}{\gamma N} + \overline{\ln p} \right\} \frac{q_i^*}{\gamma Y} \right]. \tag{31}$$

In these equations, $\overline{\ln p} = \frac{1}{N} \sum_{j=1}^N \ln p_j + \frac{1}{N^*} \sum_{j^*=1}^{N^*} \ln p_{j^*}$ denotes the log average price of competitors, where j and j^* index firm i 's domestic and foreign competitors. Due to symmetry, the average log price is the same across markets.²² The first-order condition with respect to q_i requires that marginal revenue be equalized across markets, whence $p_i \left(\frac{\varepsilon-1}{\varepsilon} \right) = \frac{p_i^*}{\tau} \left(\frac{\varepsilon^*-1}{\varepsilon^*} \right)$. The first-order condition with respect to w_i requires that marginal revenue is equal to perceived marginal cost; see Appendix A.6.2 for details. A symmetric equilibrium with all wages normalized to unity then implies the following optimal pricing conditions:

$$p = \frac{\varepsilon}{\varepsilon - 1} \psi[m] \quad \text{with} \quad \frac{\varepsilon}{\varepsilon - 1} = 1 + \frac{\delta}{\gamma} \tag{32}$$

$$p^* = \frac{\varepsilon^*}{\varepsilon^* - 1} \psi[m] \tau \quad \text{with} \quad \frac{\varepsilon^*}{\varepsilon^* - 1} = 1 + \frac{\delta^*}{\gamma} \tag{33}$$

²² Due to symmetry, the expenditure functions are the same in both countries, but expenditure shares for domestic and imported goods are different. Expenditure shares are obtained by differentiation of the log expenditure function, i.e. $\delta_i := \frac{\partial \ln P}{\partial \ln p_i}$ and $\delta_i^* := \frac{\partial \ln P}{\partial \ln p_i^*}$, and then applying the same logic as outlined in Appendix A.1 to express them in terms of q_i and q_i^* , respectively.

The labor market clearing condition is

$$N[m] (\alpha + q[p, p^*, m] + \tau q^*[p, p^*, m]) = L\theta[m]. \quad (34)$$

In contrast to the autarky and the free trade case, the pricing conditions cannot be simplified further because individual firms' prices in (30) are not equal to average prices in the economy. The equilibrium skill reach of the representative firm, m , as well as domestic and export prices are determined by the system of equations (32), (33) and (34). This system is the analogue to the free trade equilibrium condition (28) above.

Our preferences imply the existence of a prohibitive level of the trade cost. We denote this prohibitive level by $\bar{\tau}$, and it is determined implicitly by $\delta_i^* = 0$ in (30). Note that with $\delta_i^* = 0$ the price elasticity of demand for foreign goods becomes infinite; see (9). Note also that high values of γ imply low values of $\bar{\tau}$. We can now state the following proposition on piecemeal trade liberalization.

Proposition 2. *For two identical countries in a trading equilibrium, a decrease in trade costs τ within the non-prohibitive range, $\tau \in [1, \bar{\tau})$, has the following effects: (i) There is exit of firms in each country. (ii) The price of imported varieties falls, but the change in the price of domestically produced goods is ambiguous: it falls at low initial levels of τ , and it increases at high initial levels of τ . (iii) Aggregate welfare rises for sufficiently low initial levels of τ , and it falls for sufficiently high initial levels of τ . (iv) Income inequality increases.*

Proof: A formal proof is relegated to Appendix A.6.3.

To see the intuition for the ambiguity in domestic prices and welfare, first note that $d(\tau q^*) = q^* d\tau + \tau dq^*$. With $\tau = \bar{\tau}$, we have $q^* = 0$ so that a lowering of trade cost is devoid of any first-order cost savings effect while an increase in q^* causes resource use in the form of trade cost. At the other extreme, as $\tau \rightarrow 1$, we have $q^* > 0$ so that trade liberalization entails resource savings. To understand what this means for welfare, consider the total differential of the indirect utility function, using $\hat{x} := \frac{dx}{x}$ to denote a

relative change in x :

$$\widehat{V} = \left(\frac{\partial \ln \theta}{\partial \ln m} - \frac{\partial \ln P}{\partial \ln m} \right) \widehat{m} - N\delta\hat{p} - N\delta^*\hat{p}^*. \quad (35)$$

Additional firm output \bar{q} leads to firm exit, $\widehat{m} > 0$, which lowers welfare in the first term on the right-hand side. In addition, firm exit leads to a higher wage markup, contributing to a higher domestic price p . For high values of τ , this is magnified by the resource use effect from trade liberalization, for low values of τ it gets moderated by a resource savings effect. A lower price for imported varieties leads firms to lower their price markup on domestic goods, which counteracts the higher wage markup and dominates for low levels of τ .²³ In (35), the effects of changes in prices of domestic and imported goods are weighted by the respective expenditure shares. For a high initial level of trade costs the expenditure share for imported goods is small, so that consumers hardly benefit from the decrease in the price of imports, while being much affected by the change in the price of domestic goods, which is positive for a high initial level of trade costs. The opposite holds for low levels of τ . Hence, we find a U-shaped relationship between welfare and the level of trade costs.²⁴ We know from Proposition 1 that free trade, $\tau = 1$, is better than autarky, $\tau = \bar{\tau}$. Hence there is a threshold value $\tilde{\tau} < \bar{\tau}$, such that trade, though costly, is unambiguously better than autarky for $1 \leq \tau < \tilde{\tau}$.

5 Migration

Consider two perfectly symmetric countries in a free trade equilibrium of the type characterized above, symmetry also meaning that the labor force in both countries is distributed over the exact same skill circle. Interpreting average wage income as expected wage in-

²³Note that this effect arises even at the prohibitive margin with $\tau = \bar{\tau}$ where no imports take place in the initial equilibrium. For $\tau = \bar{\tau}$ the trading equilibrium is quantitatively identical to the autarky equilibrium considered in Section 3.3. This can be shown by inserting the implicit solution for $\bar{\tau}$, obtained by setting $\delta^* = 0$, into the pricing condition (32). Yet, the disciplinary effect of a decrease in import prices works through $\overline{\ln p}$ in (32), even if $\delta^* = 0$.

²⁴The U-shaped welfare curve does not hinge upon worker heterogeneity in the labor market, nor on translog preferences. Bykadorov et al. (2016) demonstrate that welfare losses for high initial trade cost obtain in any model of monopolistic competition with an additive utility function featuring a variable elasticity of substitution and a finite prohibitive real trade cost.

come for potential migrants, there is no incentive for international migration based on comparison of average incomes. However, except for an unlikely knife-edge case, in both countries some workers will find firms in the other country which provide a better match for their skill type than their present firm.²⁵ Hence, opening up the two labor markets will lead to a new sorting of workers between firms, domestic and foreign. Moreover, it will lead to a new entry and location game of domestic and foreign firms on the skill circle.

To analyze this type of labor market integration, we augment the above two-stage game of entry and pricing by allowing for workers to move across countries.²⁶ We assume that migration is costly, and we shall look at piecemeal integration of labor markets, including the polar cases where the migration cost is prohibitively high and zero, respectively. In an equilibrium with non-prohibitive cost of migration, there will be two-way migration between the two countries, although on the macro level these countries appear perfectly symmetric. In principle, this type of migration will occur independently of the degree of goods market integration, but for simplicity we assume free trade. As will become evident below, the migration incentive is even higher in a free trade equilibrium than under autarky. In this sense, then, trade and migration are complements.

Given what we have seen above, we expect this type of migration to be gainful for both economies, for two reasons. First, cross-border hiring increases competition in both countries' labor markets, thus lowering the welfare cost from the double markup on goods prices. And secondly, in both countries, cross-border hiring lowers the skill-type distance between (domestic and foreign) firms, thus increasing the average quality of matches between firms and workers.

5.1 Labor supply with integrated labor markets

We model the cost of migration as reducing the productivity of a worker to a fraction $1 - \lambda$ if she moves to the other country. When working for a domestic firm at distance d , a domestic worker delivers $f[d]$ efficiency units while delivering only $f[d](1 - \lambda)$ efficiency

²⁵ The knife edge case features firms in both countries positioned on identical points on the skill circle.

²⁶ Given perfect symmetry across countries as well as free trade between them, all of what we shall demonstrate below is equally valid for the case where workers actually move from one country to the other and the case where there is cross-border hiring, with workers staying in their home country.

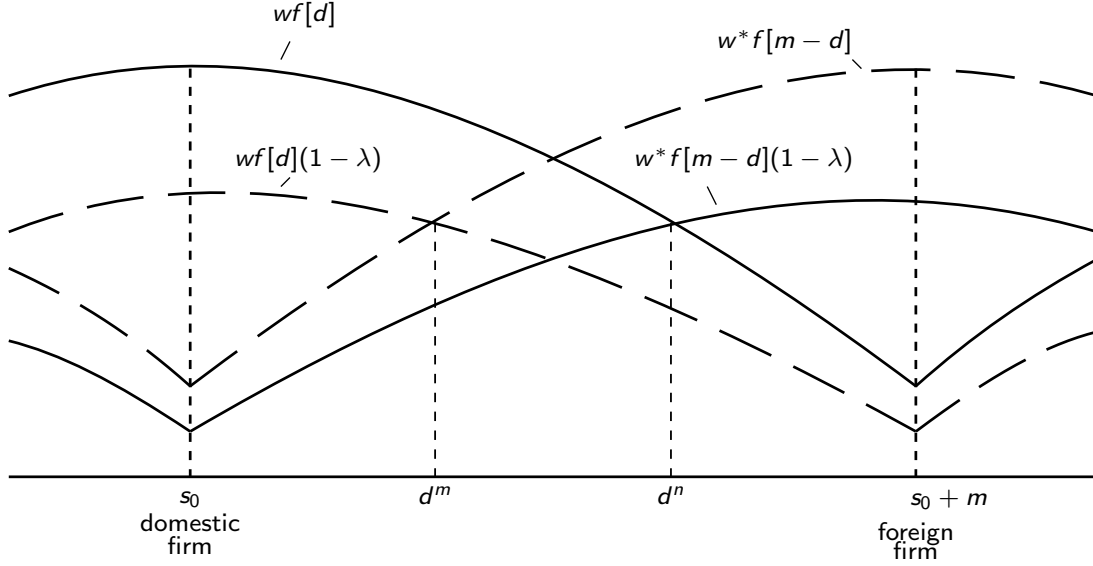


Figure 2: Sorting of workers with migration

units when working for a foreign firm at the same skill distance d .²⁷ Allowing for cross-border employment, this modifies the sorting of workers.

Suppose that the number of domestic and foreign firms is as in the free trade equilibrium with national labor markets, but labor markets are now open and firms are positioned such that any one firm faces two neighboring firms from the other country. We call this an alternating location pattern. Consider a representative domestic firm posting a wage rate equal to w , with the neighboring foreign firm posting a wage rate equal to w^* . We continue using $2m$ to denote the symmetric skill distance between two firms located in the same country. In Figure 2, the domestic firm is located at s_0 on the skill circle, and we use d^n to denote the skill reach of the domestic firm for native workers to its right and its left, and d^m to denote this firm's skill reach for migrants from the other country, again symmetrically in both directions. The two skill reaches are determined by the following

²⁷ With positive trade cost, a migration equilibrium like the one considered here would potentially be subject to instability, since by moving one way into one of the two countries workers could avoid all trade costs, and this gain might potentially outweigh the migration cost. Assuming a zero trade cost equilibrium to start with, any positive cost of migration makes our equilibrium immune to this type of agglomeration force. In a world with zero costs of both trade and migration the equilibrium outcomes in the dispersed and the agglomeration equilibrium are the same in terms of prices and welfare. For agglomeration equilibria, see Amiti and Pissarides (2005). The proportionality assumption for the cost of migration is convenient but not crucial. A general characterization of the specifications generating the results derived in this section is found in Appendix A.7.6.

conditions:

$$wf[d^n] = w^*f[m - d^n](1 - \lambda) \quad \text{and} \quad wf[d^m] = w^*f[m - d^m]\frac{1}{1 - \lambda}. \quad (36)$$

As the level of migration costs falls, the two skill reaches converge; with $\lambda = 0$ they coincide at $m/2$. The domestic firm at s_0 employs domestic workers with skill types in the interval $(s_0 - d^n, s_0 + d^n)$, and foreign workers (migrants) located in the interval $(s_0 - d^m, s_0 + d^m)$, while the foreign firm located at $s_0 + m$ employs foreign workers located in the interval $(s_0 + m - d^n, s_0 + m + d^n)$ and domestic workers (migrants) with skill types in the interval $(s_0 + m - d^m, s_0 + m + d^m)$. Notice that $d^n + d^m = m$.

Labor supply as a function of the firm's wage now emerges as

$$L^{S,M}[w, w^*, m, \lambda, L, H] = \frac{L}{H} \left(\int_0^{d^n[w, w^*, m, \lambda]} f[d]dd + \int_0^{d^m[w, w^*, m, \lambda]} f[d](1 - \lambda)dd \right) \quad (37)$$

where a superscript M indicates the case of migration, as opposed to closed labor markets. In the above equation, $d^n[w, w^*, m, \lambda]$ and $d^m[w, w^*, m, \lambda]$ are implicitly determined by (36). Symmetry across countries implies $d^n = d^n[m, \lambda]$ and $d^m = d^m[m, \lambda] := m - d^n[m, \lambda]$. As before, m may be interpreted as a mismatch indicator for domestic firms and workers. In a symmetric equilibrium, the average productivity of workers then emerges as

$$\theta^M[m, \lambda] := \frac{1}{m} \left(\int_0^{d^n} f[d]dd + \int_0^{d^m} f[d](1 - \lambda)dd \right). \quad (38)$$

By complete analogy to (17), the perceived elasticity of labor supply (37), evaluated at the symmetric equilibrium, can be derived as

$$\eta^M[m, \lambda] \Big|_{w=w^*} = \frac{2f[d^n]^2}{f'[d^n] + (1 - \lambda)f'[m - d^n]} \frac{-1}{\int_0^{d^n} f[d]dd + (1 - \lambda) \int_0^{d^m} f[d]dd}. \quad (39)$$

Details of this derivation are found in Appendix A.7.1. Note that the labor supply function is subject to the constraint $d^m[m, \lambda] \geq 0$, which ensures that both skill reaches lie in between the positions of the domestic and the foreign firm. This is equivalent to the

condition that the migration cost λ is below its prohibitive level.²⁸ As the migration cost approaches the prohibitive level, the supply of efficiency units of labor becomes equal to the supply under autarky. This is readily verified by inserting $d^n = m$ and $d^m = 0$ into (37).

Interestingly, even if the level of migration costs is prohibitive, firm behavior is influenced by the mere potential of migration through the perceived elasticity of labor supply.²⁹ The possibility of attracting migrants by setting higher wages and thus increasing the supply of efficiency units implies that firms perceive a higher elasticity of labor supply, even if they do not employ any migrant in equilibrium. Let $\bar{\lambda}$ denote the prohibitive level of migration costs, determined by setting $d^m[m, \lambda] = 0$. The perceived wage elasticity of labor supply evaluated at $\bar{\lambda}$ is given by

$$\eta^M[m, \bar{\lambda}] = -\frac{2f[m]^2}{f'[m] + (1 - \bar{\lambda})f'[0]} \frac{1}{F[m]}. \quad (40)$$

Note that concavity of $f[d]$ is sufficient to ensure that $\eta^M[m, \bar{\lambda}]$ is larger than the elasticity of supply under autarky as given in (17). However, as we shall see below, *with prohibitive* λ the symmetric alternating location pattern does not constitute an equilibrium as described in Section 3.2. Nevertheless, the stage-two pricing equilibrium evaluated at the symmetric alternating location pattern and prohibitive cost of migration will prove useful as a reference point.

We show in Appendix A.7.1 that $\eta^M[m, \lambda]$ is decreasing in λ , provided that $f'''[d]$ is not too large. In what follows, we assume that this condition holds.³⁰ By analogy to (19), we now use $\psi^M[m, \lambda] := (\eta^M[m, \lambda] + 1)/\eta^M[m, \lambda]$ to denote the wage markup under migration. For a given level of m , the markup is unambiguously lower with migration and $\lambda \in [0, \bar{\lambda}]$ than without.

²⁸ Otherwise, if migration costs are too large relative to firm size, firms cannot attract any migrants in the first place and the supply curve looks different since they then compete again only with firms from the same country.

²⁹ We thank Vitor Trindade for pointing this out to us.

³⁰ The reasoning behind this condition is as follows: A higher λ leads firms to increase the share of migrants employed by shifting d^n outwards and d^m inwards. If the curvature of $f[d]$ falls (in absolute terms) as the skill reaches move to the right, an increase in λ helps firms to avoid competition by employing more native workers in the range where the curvature of $f[d]$ is lower and fewer migrants in the range where the curvature of $f[d]$ is strong. We rule this out by assuming that the curvature does not decrease too much (in absolute terms) as the skill reach moves to the right.

In addition to the wage markup, migration also affects the average quality of skill matches between workers and firms. It is obvious that for prohibitively high migration costs, $\lambda = \bar{\lambda}$, the average matching quality, as given in (38), is the same function of m as under autarky, given in (20): $\theta^M[m, \bar{\lambda}] = \theta[m]$. Moreover, as shown in Appendix A.7.4, θ^M is falling in λ , reaching $\theta^M[m, 0] = \theta[m/2]$ for frictionless migration where $\lambda = 0$. It is instructive to see how effective labor supply to a representative firm is affected by the cost of migration, holding m constant. Under frictionless migration, $\lambda = 0$, labor supply (37) emerges as

$$L^{S,M} = 2 \frac{L}{H} \int_0^{\frac{m}{2}} f[d] dd = \frac{2L}{N^M} \theta^M[m, 0] = \frac{L}{N} \theta[m/2]. \quad (41)$$

Note that $N^M = \frac{2H}{m} = 2N$, where N is the number of firms in each country. Comparing this to the autarky case, both the number of firms and mass of workers are doubled. However, we know from above that for $\lambda < \bar{\lambda}$ we have $\theta^M > \theta$. Hence, firms face a larger supply of efficiency units of labor with migration than under national labor markets. The reason is that, while employing the same mass of workers in either case, with migration each firm finds workers with skills closer to its optimal type; the skill reach has fallen to $m/2$, compared to m in the case of closed labor markets. We note in passing that the productivity gains from migration, $\theta^M[m, \lambda] - \theta[m]$, are increasing in m ; see Appendix A.7.2 for details. This implies that trade and migration are complements in the sense that firm exit brought along by trade (cp. Propositions 1 and 2) enhances the productivity gains from migration.

5.2 Equilibrium with open labor markets

Importantly, all of the above is conditional upon a given level of m , as implied by the assumption that the number of firms per country is the same as in the free trade equilibrium with national labor markets. To examine whether an alternating location pattern is an equilibrium in our two-stage game of entry and pricing, we first look at the corresponding zero-profit equilibrium in stage two.

5.2.1 Stage two: pricing

By complete analogy to (28), a stage-two zero-profit equilibrium requires

$$g^M[m, \lambda] = \rho^T[m] \psi^M[m, \lambda]. \quad (42)$$

In this equation, $\rho^T[m]$ denotes the free trade price markup over perceived marginal cost obtaining in a free trade equilibrium without migration.³¹ Under free trade, this markup simplifies to $1 + 1/(\gamma N^M)$, where N^M is the number of firms world-wide; see (26). Unlike the wage markup $\psi^M[m, \lambda]$, the price markup is affected by migration only through the number of firms. The term $g^M[m, \lambda]$ on the left measures the degree of scale economies, taking into account the labor market clearing condition, which now reads as $\alpha + q = (m/H)L\theta^M[m, \lambda]$, as well as goods market clearing, which requires $q = L\theta^M[m, \lambda]/(pN)$. This measure thus reads as

$$g^M[m, \lambda] := \frac{L\theta^M[m, \lambda]}{L\theta^M[m, \lambda] - \alpha H/m}. \quad (43)$$

In order to understand the effects of labor market integration, we proceed in two steps. In the first step we look at a situation where migration is allowed in principle, but where the cost of migration is prohibitively large, $\lambda = \bar{\lambda}$, and compare this case with the equilibrium outcome under national labor markets. In the second step we look at situations with $\lambda < \bar{\lambda}$.

Proposition 3. *(i) For two symmetric countries with free trade and open labor markets, a stage-two zero-profit equilibrium with an alternating pattern of firm location but prohibitive migration cost, $\lambda = \bar{\lambda}$, features a welfare level which is unambiguously higher and a number of firms which is lower than in a free trade equilibrium with national labor markets. (ii) For $\lambda \in [0, \bar{\lambda})$, a piecemeal integration of labor markets, $d\lambda < 0$, has an ambiguous effect on the number of firms, but it unambiguously leads to lower prices and an increase in welfare in both countries. At $\lambda = \bar{\lambda}$, $d\lambda < 0$ lowers the number of firms.*

³¹ Throughout our analysis of migration, we use a superscript T to denote functions that take the same form under migration and free trade alone, while using a superscript M to denote functions that are fundamentally different under integrated labor markets compared with free trade alone.

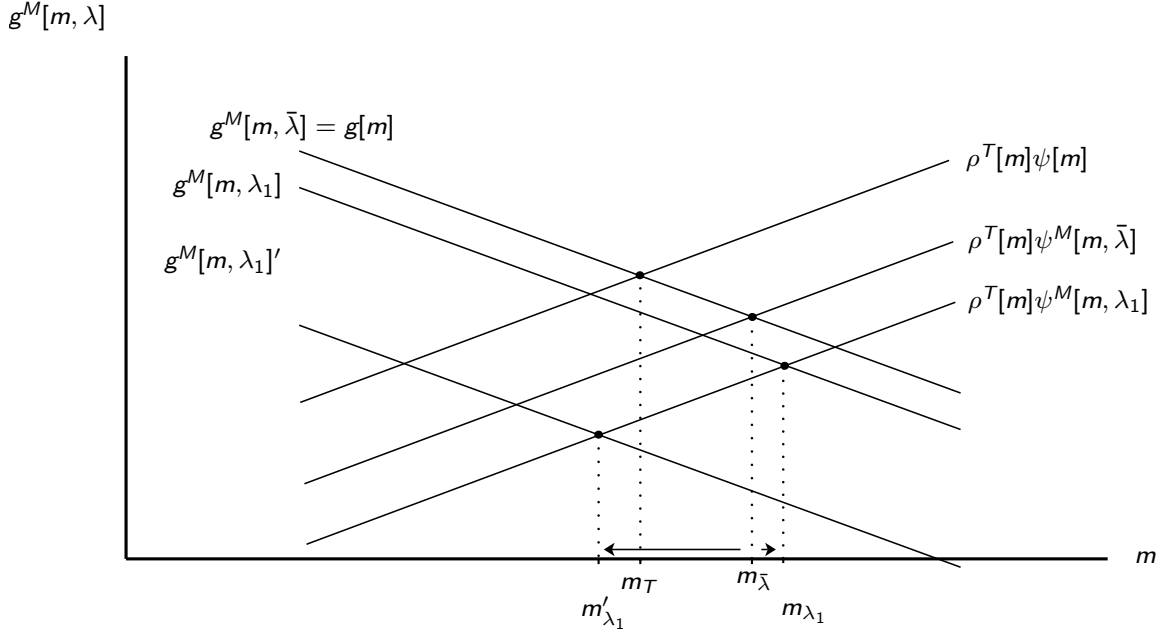


Figure 3: Comparative statics of the skill reach m

Proof: The analytical details of the proof are relegated to Appendix A.7.3 for part (i) and to Appendix A.7.4 for part (ii).

The intuition for part (i) is as follows. In the situation considered, the degree of labor market competition is higher than in autarkic labor markets, while labor supply is the same. Hence, maximum profits in the stage-two pricing equilibrium with the same number of domestic and foreign firms as in a trading equilibrium with national labor markets are negative. A zero-profit equilibrium therefore requires a lower number of firms. A key point to understand the welfare increase is that the excess entry property of the autarky equilibrium demonstrated in Section 3.4 is inherited by the stage-two zero-profit equilibrium with symmetric alternating location patterns for *any* $\lambda \in [0, \bar{\lambda}]$. While the productivity distortion is not affected as long as no one migrates, the wage markup is lowered because firms perceive a larger elasticity of labor supply. With a lower wage markup distortion relative to the productivity distortion, the allocation is now closer to the social optimum. Therefore, firm exit following the opening up of labor markets entails a first-order welfare gain, even if the cost of migration is prohibitive at $\bar{\lambda}$.

The intuition for part (ii) is best grasped from Figure 3, which depicts the schedules $g^M[m, \lambda]$ and $\rho^T[m]\psi^M[m, \lambda]$, identifying the equilibrium value of m at the intersection, in line with the zero-profit equilibrium condition (43). The vertical axis of Figure 3

may be interpreted as measuring goods prices. Remember that $g^M[m, \lambda]$ measures the inverse degree of scale economies, which is equivalent to the markup required for zero profits. An increase in m makes firms larger, but it also lowers the productivity of the average worker. Appendix A.7.4 shows that the size effect always dominates, whence the g^M -line is downward-sloping. The $\rho^T[m]\psi^M[m, \lambda]$ -line, depicting the double markup, is upward-sloping since a lower number of firms (higher m) reduces both the perceived price elasticity of goods demand as well the perceived labor supply elasticity. We know from part (i) of Proposition 3 that the intersection point for $\lambda = \bar{\lambda}$ involves a lower value of m than the free trade equilibrium with national labor markets, which is determined by $g[m] = \rho^T[m]\psi[m]$.

Now consider a reduction in λ from $\bar{\lambda}$ to $\lambda_1 \in [0, \bar{\lambda})$. For a notionally unchanged value of m , this improves the productivity of the average worker through savings in migration cost as well as through a resorting of workers from native employment into migration. This means that the g^M -line is shifted down by a reduction in λ . We have shown above that the perceived elasticity of labor supply increases with a lower cost of migration, meaning that for a notionally unchanged m firms charge a lower wage markup $\psi^M[m, \lambda]$. Thus, the markup schedule shifts down as well, rendering an ambiguous effect on m . In the figure, the case $g^M[m, \lambda_1]$ ($g^M[m, \lambda_1]'$) depicts a relatively weak (strong) shift in the g^M -line, leading to an increase (a decrease) in m . The ambiguity in the adjustment of m implies that income inequality under migration can generally be lower or higher than in the free trade equilibrium. Goods prices, however, are unambiguously lower.

The welfare effect is determined by the change in real income and the number of varieties. Real income is given by $\theta^M[m, \lambda]/p[m]$. Invoking the indirect utility function, the welfare effect of our scenario may be described as

$$\hat{V} = \frac{\partial \ln [\theta^M/p]}{\partial \lambda} d\lambda + \frac{\partial \ln [\theta^M/p]}{\partial m} dm - \frac{1}{4\gamma H} dm. \quad (44)$$

The first term describes the direct effect of lower migration costs, $d\lambda < 0$, on real income. From the above it follows that this term is unambiguously positive. The remaining terms involving dm are ambiguous in their entirety, because dm as caused by $d\lambda < 0$ is ambiguous. Since the second-stage “trade cum migration” equilibrium with symmetric alternating firm locations inherits the excess entry property, the positive real income effect of firm exit in the second term must dominate the negative variety effect in the third

term. In other words, if the equilibrium adjustment depicted in Figure 3 leads to $dm > 0$, then the overall effect of $d\lambda < 0$ on welfare is positive. If $dm < 0$, then the welfare effect is less straightforward. While the final term of this expression is unambiguously positive in this case, the first two terms seem ambiguous. However, Appendix A.7.4 shows that inserting $dm = (\partial m / \partial \lambda) d\lambda$ renders the first two terms of (44) unambiguously positive for any initial $\lambda \in [0, \bar{\lambda}]$.

Proposition 3 implies the following corollary:

Corollary 1. *Any stage-two zero-profit equilibrium with open labor markets and alternating patterns of location delivers a level of welfare that is strictly larger for both countries, regardless of the value of $\lambda \in [0, \bar{\lambda}]$, than the level of welfare in the trading equilibrium with national labor markets.*

5.2.2 Stage one: entry

It still remains to be shown whether a stage-two zero-profit equilibrium with an alternating pattern of location constitutes an equilibrium of the stage-one game of entry and location as considered in Section 3.2 above. Towards this end, we return to Figure 2 depicting worker sorting with migration. It is relatively easy to see that for any level of migration cost $\lambda \in (0, \bar{\lambda}]$ and number of firms (equal in both countries) there exists a location pattern where the sorting of workers implies strictly national hiring. In Figure 2, consider an alternative position of the domestic firm further to the right, say s'_0 , such that $wf[s'_0 - s_0](1 - \lambda) < w^*f[m]$. Consider all domestic firms similarly locating themselves closer to their right-hand foreign neighbors. Then, no foreign worker would sort into employment in a domestic firm, and no domestic worker would sort into employment in a foreign firm. Of course, this type of location behavior is possible only for a strictly positive level of the migration cost. Adjusting the number of firms in any such no-migration-situation so as to satisfy the zero-profit condition restores a stage-two equilibrium with de facto national labor markets.

From Proposition 3 we know that in the neighborhood below $\bar{\lambda}$ the number of firms in such an equilibrium is lower than under national labor markets, but reacts ambiguously to reductions in λ within the interval $\lambda \in [0, \bar{\lambda})$; see Figure 3. Therefore, for any value of $\lambda \in [0, \bar{\lambda})$ the number of firms in a stage-two zero-profit equilibrium with an alternating

location pattern may be larger, equal to, or lower than under national labor markets. Applying logic familiar from (14) and Lemma 2, we can now state that if the number of firms is different it will always be the stage-two equilibrium with the higher number firms that prevails as an equilibrium also in the stage-one game of entry and location.

The ambiguous reaction of the number of firms to $d\lambda < 0$ means that a reduction in the cost of migration may lead firms to escape the pro-competitive effect of more integrated labor markets by suitably adjusting their locations when entering so as to restore de facto national labor markets. Concentrating on full equilibria of our two-stage game of entry and pricing, our model delivers the following prediction about the welfare effect of migration.

Proposition 4. *(i) For two symmetric countries, any full equilibrium of the two-stage game featuring free trade and positive levels of migration involves a higher level of welfare for both countries than a trading equilibrium with national labor markets. (ii) Any piecemeal integration of labor markets, $d\lambda > 0$, is welfare-increasing for both countries, provided that it increases the equilibrium level of migration. (iii) A zero-level of the migration cost leads to a unique full equilibrium with positive migration delivering the maximum possible level of welfare for both countries over the entire range of $\lambda \in [0, \bar{\lambda}]$. (iv) Any full equilibrium with positive levels of migration involves less income inequality than the equilibrium with national labor markets, and piecemeal labor market integration lowers the degree of income inequality, provided that it increases the level of migration.*

Proof: As we demonstrate in Appendix A.7.5, the logic of Lemma 2 implies that the symmetric alternating pattern with zero profits is the only stage-two equilibrium that satisfies consistent beliefs in the entry game *and* involves migration. Together with Corollary 1, this proves part (i) of the proposition. Part (ii) of Proposition 3 implies that $d\lambda < 0$ raises welfare for both countries, provided that we are, and remain, in an equilibrium with positive migration. And it is obvious that in such a situation $d\lambda < 0$ also increases the level of migration. If we are in a full equilibrium of de facto national labor markets, then $d\lambda < 0$ may or may not lead to migration, but if it does, then Part (i) of Proposition 3 tells us that this will be welfare-increasing. As we have emphasized above, a zero-level of migration cost precludes firms choosing a location pattern that leads to an equilibrium

with de facto national labor markets. Therefore, $\lambda = 0$ necessarily leads to an equilibrium with positive migration. Part (iii) of the proposition then follows from the welfare effect of piecemeal integration described in part (ii) of Proposition 3. Part (iv) of the proposition follows from the result that any increase in the number of firms reduces income inequality.

6 Conclusion

We have readdressed the common narrative of variety-based gains from trade. Traditional models of monopolistic competition stress the importance of a large resource base for a high degree of product differentiation, if production is subject to a non-convex technology. By opening up to trade, even small countries may enjoy the benefits of a large resource base. Domestic firms may be driven out of the market, but this has no adverse effect. If anything, it increases the average productivity level through a positive selection effect. This view neglects an important aspect of the labor market: If the labor force is heterogeneous in terms of skill types that are specific to the production of certain goods, then firms have monopsony power on the labor market and some workers will be employed in less than ideal matches. As a result, productivity gains from specializing on a coarser set of goods come at the expense of a less competitive labor market and more workers being employed in less than ideal matches.

We have developed a model that combines monopolistic competition on goods markets with skill-type heterogeneity on the labor market. The average quality of skill-type matches is endogenous through a two-stage game. In stage one, firms choose whether to enter and where to locate in a circular skill space. In stage two, firms play pricing strategies in the goods market as well as the labor market, and workers sort themselves into firms based on absolute advantage of their skill types. In this environment, opening up to trade is a less benign force than portrayed in conventional models of monopolistic competition. In particular, trade-induced firm exit worsens the average quality of matches between the types of skills that workers bring to their firms and the specific skill requirements of the goods produced by these firms, and it increases the distortion between the marginal productivity of labor and the wage rate. This latter effect works against the conventional pro-competitive effect of trade on the goods markets where trade lowers the markup between marginal cost and prices. However, comparing free trade with autarky in

a symmetric many-country world, we find that the variety and pro-competitive effects on goods markets unambiguously dominate the adverse effects from a lower average quality of worker-firm matches and from higher markups on the labor market. The gains from trade theorem survives. Looking at piecemeal trade liberalization between two symmetric countries, we find an ambiguity: Trade liberalization is only beneficial if initial trade cost are below a certain threshold. Above the threshold, decreasing trade cost lead to lower welfare.

Labor market integration gives rise to a migration incentive, whereby firms engage in cross-border hiring even under complete symmetry between countries. This type of two-way migration looms large in the data, but has so far lacked convincing explanation in standard models of migration. In our model environment of skill-differentiation on the labor market, migration has effects that are opposite to those of trade. It tends to improve the quality of matches while at the same time lowering firms' monopsony power on labor markets. In contrast to trade liberalization, a piecemeal reduction in the cost of migration is unambiguously welfare increasing whenever it leads to more migration. Trade and migration are complements, rather than substitutes, since trade-induced specialization increases migration incentives. Our model clearly advocates opening up labor markets simultaneously with trade liberalization.

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Appendix

A.1 Expenditure share and markup

A.1.1 Proof of Equation (7)

Starting out from (6), inserting $p_i = \frac{\delta_i Y}{q_i}$ gives

$$\delta_i = \frac{1}{N} + \gamma \overline{\ln p} - \gamma \ln \frac{\delta_i Y}{q_i}. \quad (\text{A.1})$$

This can be rewritten as

$$\frac{\delta_i}{\gamma} + \ln \frac{\delta_i}{\gamma} = \frac{1}{\gamma N} + \overline{\ln p} - \ln \frac{Y}{q_i} - \ln \gamma. \quad (\text{A.2})$$

Applying the Lambert function $\mathcal{W}[z]$, defined as the solution to $\ln x + x = \ln z$, we obtain $\delta_i = \delta[q_i, \overline{\ln p}, N, Y]$ as given in (7).

A.1.2 Proof of Equation (11)

Similar logic can be applied to obtain an explicit solution for the optimal price determined by the first-order condition (10). Defining perceived marginal cost as $\tilde{w}_i := [(\eta_i + 1)/\eta_i]w_i\beta$ and observing (6) and (9), this condition can be written as

$$\frac{p_i}{\tilde{w}_i} + \ln p_i = 1 + \frac{1}{\gamma N} + \overline{\ln p}. \quad (\text{A.3})$$

The left-hand side is an implicit function of the profit maximizing price p_i . Rewriting (A.3) as

$$\frac{p_i}{\tilde{w}_i} + \ln p_i - \ln \tilde{w}_i = 1 + \frac{1}{\gamma N} + \overline{\ln p} - \ln \tilde{w}_i \quad (\text{A.4})$$

and applying the Lambert function to the left-hand side, we obtain the following explicit solution for p_i

$$p_i = \mathcal{W} \left[\tilde{w}_i^{-1} \exp \left\{ 1 + \frac{1}{\gamma N} + \overline{\ln p} \right\} \right] \tilde{w}_i. \quad (\text{A.5})$$

which implies the price markup as given in (11).

A.2 Existence and uniqueness of the pricing equilibrium (Lemma 1)

We invoke the Index Theory approach outlined in Vives (2001) to proof that under certain restrictions on the parameter space there is a unique solution to the second-stage game. The Index Theorem approach is based on the Poincaré-Hopf Index Theorem which implies that a solution to a system of reaction functions is unique, if

- i) payoff functions are quasiconcave in firms' own strategies, i.e., wages,
- ii) the strategy space is convex and compact and all equilibria are interior,
- iii) the Hessian is negative definite at the equilibrium point.

We first show that condition i) holds if the elasticity of marginal labor supply is not too large and condition ii) holds if marginal revenue is positive for output levels arbitrarily close to zero. Then, we show that condition iii) is always fulfilled in a transformed game where firms' strategies are *log* wages. Since conditions i) and ii) also hold in the transformed game, the Index Theorem implies that the transformed game has a unique solution. Since $\ln w$ is a positive monotone transformation of w for $w > 0$, this implies uniqueness of the solution to the original game.

i) Quasiconcavity of the profit function. In the second stage, firm i takes the distance pattern \mathbf{m}_i as well as aggregate income Y and the average log price $\overline{\ln p}$ as given and determines its optimal wage as the best response to other firms' wage choices \mathbf{w}_{-i} by maximizing profits as given in (8). We first describe the range of wages firm i can choose from and then analyze the conditions under which its profit function is quasiconcave in the wage. The set of permissible wages is bounded from below by $w_{\alpha i} := L^{-1}[\alpha, \mathbf{w}_{-i}, \mathbf{m}_i] > 0$, which denotes the wage level where the second constraint in (8) binds.³² Moreover, firms never set wages above the *choke price* divided by the marginal labor requirement β . The choke price is defined as the limit of marginal revenue with respect to q_i as q_i converges to zero: $p_{\text{choke}} := \lim_{q_i \rightarrow 0} \frac{\partial r_i}{\partial q_i}$, where r_i is given in (7). In the sequel, we shall simplify by writing r'_i for $\frac{\partial r_i}{\partial q_i}$. We may write

$$r_i = \gamma Y \mathcal{W}[W_i^q] \quad \text{where} \quad W_i^q := \left[\exp \left\{ \frac{1}{\gamma N} + \overline{\ln p} \right\} \frac{q_i}{\gamma Y} \right]. \quad (\text{A.6})$$

³² Note that $w_{\alpha i}$ approaches zero if all firms lower their wages towards zero. We assume that $w_{\alpha i}$ is positive because otherwise $\ln w$ (which we will be working with below) is not defined. This assumption has no bearing on the equilibrium outcomes. Moreover, excluding the possibility of zero wages can be justified by assuming that at a zero wage workers prefer not to work and hence firms need to pay at least the reservation wage.

We have $\mathcal{W}'[W_i^q] = \mathcal{W}[W_i^q] / ((\mathcal{W}[W_i^q] + 1)W_i^q)$, whence

$$r'_i = \gamma Y \frac{\mathcal{W}[W_i^q]}{(\mathcal{W}[W_i^q] + 1)W_i^q} \left[\exp \left\{ \frac{1}{\gamma N} + \overline{\ln p} \right\} \frac{1}{\gamma Y} \right] = \frac{\gamma Y \mathcal{W}[W_i^q]}{(\mathcal{W}[W_i^q] + 1)q_i}. \quad (\text{A.7})$$

Applying L'Hôpital's rule, we obtain

$$p_{\text{choke}} = \exp \left[\frac{1}{\gamma N} + \overline{\ln p} \right]. \quad (\text{A.8})$$

For easier notation, we use $L_i := L^S[w_i, \mathbf{w}_{-i}, \mathbf{m}_i]$ to denote firm i 's labor supply.

A sufficient condition for $\pi[w_i, \mathbf{w}_{-i}, \mathbf{m}_i, N]$ to be quasiconcave in the firm's wage rate is that $\frac{\partial^2 \pi_i}{\partial w_i^2} < 0$ whenever $\frac{\partial \pi_i}{\partial w_i} \geq 0$. Marginal profits are given by

$$\frac{\partial \pi_i}{\partial w_i} = \left(\frac{r'_i}{\beta} - w_i \right) \frac{\partial L_i}{\partial w_i} - L_i \quad \text{if } \frac{\partial L_i}{\partial w_i} \text{ exists,}$$

where

$$\frac{\partial L_i}{\partial w_i} = \frac{L}{2H} \sum_{c=\ell, r} f[d_{i,c}] \frac{\partial d_{i,c}}{\partial w_i} > 0 \quad \text{with} \quad \frac{\partial d_{i,c}}{\partial w_i} = \frac{f[d_{i,c}]}{-w_i f'[d_{i,c}] - w_c f'[2m_{i,c} - d_{i,c}]} > 0. \quad (\text{A.9})$$

Marginal profits change in the firm's wage according to

$$\frac{\partial^2 \pi_i}{\partial w_i^2} = \frac{r''_i}{\beta^2} \left(\frac{\partial L_i}{\partial w_i} \right)^2 - 2 \frac{\partial L_i}{\partial w_i} + \left(\frac{r'_i}{\beta} - w_i \right) \frac{\partial^2 L_i}{\partial w_i^2} \quad \text{if } \frac{\partial L_i}{\partial w_i} \text{ exists,} \quad (\text{A.10})$$

where

$$\frac{\partial^2 L_i}{\partial w_i^2} = \frac{L}{2H} \left(\sum_{c=\ell, r} 3f'[d_{i,c}] \left(\frac{\partial d_{i,c}}{\partial w_i} \right)^2 + (w_i f''[d_{i,c}] - w_c f''[2m_{i,c} - d_{i,c}]) \left(\frac{\partial d_{i,c}}{\partial w_i} \right)^3 \right) \leq 0 \quad (\text{A.11})$$

and $r''_i = \frac{\partial r'_i}{\partial q_i} = -\frac{r'_i}{q_i} \frac{W_i^q(W_i^q+2)}{(W_i^q+1)^2}$. Rewriting (A.10), we obtain

$$\frac{\alpha + \beta q_i}{\beta q_i} |\varepsilon_{r'_i}| \geq \frac{\eta_{L'_i}}{\eta_i} \quad (\text{A.12})$$

as a sufficient condition for quasiconcavity, where

$$\varepsilon_{r'_i} = \frac{r''_i}{r'_i} q_i = -\frac{W_i^q(W_i^q + 2)}{(W_i^q + 1)^2} \leq 0 \quad \text{and}$$

$$\eta_{L'_i} = \frac{\partial^2 L_i}{\partial w_i^2} w_i \bigg/ \frac{\partial L_i}{\partial w_i} \leq 0.$$

With revenue concave in q_i , concavity of the labor supply function is a sufficient condition for quasiconcavity of the profit function in w_i since it implies that revenue is also concave in w_i and that the cost function $w_i L_i$ is convex in w_i . Condition (A.12) clearly holds in this case since concavity of the labor supply function implies that the wage elasticity of marginal labor supply $\eta_{L'_i}$ is negative. More generally, condition (A.12) holds if $\eta_{L'_i}$ is not too large relative to η . Moreover, condition (A.12) holds irrespective of the curvature of the labor supply function if $\frac{L_i}{L_i - \alpha} = \frac{\alpha + \beta q}{\beta q}$, the inverse degree of economies of scale, is sufficiently large. This is always the case for low levels of the marginal cost β . To gain more intuition for the role of β , note that $1/\beta$ is the derivative of q_i with respect to L_i , hence for any given level, slope, and curvature of L_i , β determines the relative weight of the curvature of revenue for the curvature of the profit function. Hence, whenever $\frac{\partial L_i}{\partial w_i}$ exists, quasiconcavity obtains if β is sufficiently small, or if the labor supply function is concave.

Whenever firm i chooses a wage so that it out-competes neighbor $i + j$ and i starts competing with the next relevant competitor $i + j'$, $j' > j$, the labor supply function and thus the profit function exhibits a kink and $\frac{\partial L_i}{\partial w_i}$ does not exist. However, we can show that the labor supply function is always flatter after the kink and hence, the kinks do not impair the concavity of the profit function. Let $\tilde{w}_{i,c} := \tilde{w}_{i,c}[\mathbf{w}_{-i}, \mathbf{m}_i]$ denote the wage where the relevant competitor on side $c = \ell, r$ is just overbid. Then, (A.9) implies that

$$\lim_{w_i \rightarrow \tilde{w}_{i,c}^-} \frac{\partial L_i^c}{\partial w_i} = \frac{L}{2H - \tilde{w}_{i,c} f'[d_{i,c}[\tilde{w}_{i,c}]] - w_{i+j} f'[2m_{i,i+j} - d_{i,c}[\tilde{w}_{i,c}]]} \frac{f[d_{i,c}[\tilde{w}_{i,c}]]^2}{\quad} \quad \text{and} \quad (\text{A.13})$$

$$\lim_{w_i \rightarrow \tilde{w}_{i,c}^+} \frac{\partial L_i^c}{\partial w_i} = \frac{L}{2H - \tilde{w}_i f'[d_{i,c}[\tilde{w}_{i,c}]] - w_{i+j'} f'[2m_{i,i+j'} - d_{i,c}[\tilde{w}_{i,c}]]} \frac{f[d_{i,c}[\tilde{w}_{i,c}]]^2}{\quad}. \quad (\text{A.14})$$

If $i + j$ was overbid by $i + j'$ at its own location, $\lim_{w_i \rightarrow \tilde{w}_{i,c}^-} \frac{\partial L_i^c}{\partial w_i} \geq \lim_{w_i \rightarrow \tilde{w}_{i,c}^+} \frac{\partial L_i^c}{\partial w_i}$ follows from $-f'[2m_{i,i+j'} - d_{i,c}[\tilde{w}_{i,c}]] > -f'[2m_{i,i+j} - d_{i,c}[\tilde{w}_{i,c}]] > 0$ and $w_{i+j'} > w_{i+j}$. If $i + j$ was overbid by i at its own location, $\lim_{w_i \rightarrow \tilde{w}_{i,c}^-} \frac{\partial L_i^c}{\partial w_i} \geq \lim_{w_i \rightarrow \tilde{w}_{i,c}^+} \frac{\partial L_i^c}{\partial w_i}$ follows from $2m_{i,i+j'} - d_{i,c}[\tilde{w}_{i,c}] < 0$ and $-f'[2m_{i,i+j'} - d_{i,c}[\tilde{w}_{i,c}]] > 0 > -f'[2m_{i,i+j} - d_{i,c}[\tilde{w}_{i,c}]] > 0$. A similar argument applies to the slope of the labor supply on firm i 's other side if it also exhibits a kink at $\tilde{w}_{i,c}$. Otherwise, the derivative of the labor supply function on that side exists. It follows that $\lim_{w_i \rightarrow \tilde{w}_{i,c}^-} \frac{\partial \pi_i}{\partial w_i} \geq \lim_{w_i \rightarrow \tilde{w}_{i,c}^+} \frac{\partial \pi_i}{\partial w_i}$ if $\left(\frac{r'_i}{\beta} - w_i\right) > 0$. If $\left(\frac{r'_i}{\beta} - w_i\right) < 0$, then

$\lim_{w_i \rightarrow \bar{w}_{i,c}^-} \frac{\partial \pi_i}{\partial w_i}, \lim_{w_i \rightarrow \bar{w}^+} \frac{\partial \pi_i}{\partial w_i} < 0$. This proves that under the conditions specified above, profits are globally quasiconcave.

ii) The strategy space is convex and compact, and all solutions are interior if γ is sufficiently small. Firm i 's strategy space is given by the interval $S_i = [\underline{w}_{\alpha,i}, \bar{w}]$ and hence it is convex, closed and bounded. Interior solutions require that the slopes of the profit functions at the boundaries of the strategy space point inwards. At the lower bound, this condition holds if marginal revenue at \bar{w}_i , that is, at $q_i = 0$, is sufficiently large. Sufficiently small values of γ for any fixed number of firms, a given average price, fixed and variable cost, and labor market conditions $\mathbf{w}_{-i}, \mathbf{m}_i$, ensure that $\left. \frac{\partial \pi_i}{\partial w_i} \right|_{w_i = \underline{w}_i} = \left(\frac{p_{choke}}{\beta} - \underline{w}_i \right) \left. \frac{\partial L_i}{\partial w_i} \right|_{w_i = \underline{w}_i} - \alpha > 0$. $L_i[\bar{w}, \mathbf{w}_{-i}, \mathbf{m}_i] > \alpha$ if $\bar{w} > w_{\alpha,i}$ implies $\left. \frac{\partial \pi_i}{\partial w_i} \right|_{w_i = \bar{w}_i} < 0$.

iii) The Hessian of the log-transformed game is negative definite at the equilibrium point. We prove negative definiteness of the Hessian by showing that the game in transformed strategies $\ln w_i \in \tilde{S}_i$, with $\tilde{S}_i = [\ln w_{\alpha,i}, \ln \bar{w}]$, exhibits diagonal dominance at the equilibrium point where

$$\frac{\partial \pi_i}{\partial \ln w_i} = 0 \quad \forall i.$$

Diagonal dominance at the equilibrium point requires that

$$\left| \frac{\partial^2 \pi_i}{\partial \ln w_i^2} \right| - \sum_{j \neq i}^N \left| \frac{\partial^2 \pi_i}{\partial \ln w_i \partial \ln w_j} \right| > 0, \quad (\text{A.15})$$

where

$$\begin{aligned} \frac{\partial^2 \pi_i}{\partial \ln w_i^2} &= w_i \frac{\partial \pi_i}{\partial w_i} + w_i^2 \frac{\partial^2 \pi_i}{\partial w_i^2} = w_i^2 \frac{\partial^2 \pi_i}{\partial w_i^2} \quad \text{since} \quad \frac{w_i \partial \pi_i}{\partial w_i} = 0 \\ \frac{\partial^2 \pi_i}{\partial \ln w_i \partial \ln w_j} &= \begin{cases} w_i w_j \frac{\partial^2 \pi_i}{\partial w_i \partial w_j} & \text{for } j = \ell, r \\ 0 & \text{for } j \neq i, \ell, r. \end{cases} \end{aligned}$$

In an interior equilibrium no firm is overbid. This implies that around the equilibrium point the labor supply function is smooth and we do not need to worry about the kinks. Moreover, it implies that firm i 's relevant competitors are its immediate neighbors, that is, $\ell = i - 1$ and $r = i + 1$.

Firm i 's marginal profits change in its neighbors' log wages according to

$$\frac{\partial^2 \pi_i}{\partial w_i \partial w_c} w_i w_c = \frac{r_i''}{\beta^2} \frac{\partial L_i}{\partial w_i} \frac{\partial L_i}{\partial w_c} w_i w_c - \frac{\partial L_i}{\partial w_c} w_i w_c + \left(\frac{r_i'}{\beta} - w_i \right) \frac{\partial^2 L_i}{\partial w_i \partial w_c} w_i w_c \leq 0, \quad (\text{A.16})$$

where

$$\begin{aligned}
\frac{\partial L_i}{\partial w_c} w_c &= \frac{L}{2H} f[d_{i,c}] \frac{\partial d_{i,c}}{\partial w_c} w_c < 0 \quad \text{since} \quad \frac{\partial d_{i,c}}{\partial w_c} = \frac{f[2m - d_{i,c}]}{w_i f'[d_{i,c}] + w_c f'[2m_{i,c} - d_{i,c}]} < 0. \\
\frac{\partial^2 L_i}{\partial w_i \partial w_c} w_i w_c &= \frac{L}{2H} \left(2f'[d_{i,c}] \frac{\partial d_{i,c}}{\partial w_i} w_i \frac{\partial d_{i,c}}{\partial w_c} w_c + f'[2m_{i,c} - d_{i,c}] \frac{\partial d_{i,c}}{\partial w_i} w_i \frac{\partial d_{i,c}}{\partial w_i} w_c \right. \\
&\quad \left. + (w_i f''[d_{i,c}] - w_c f''[2m_{i,c} - d_{i,c}]) \frac{\partial d_{i,c}}{\partial w_i} \frac{\partial d_{i,c}}{\partial w_i} w_i \frac{\partial d_{i,c}}{\partial w_c} w_c \right) \\
&= \frac{L}{2H} \left(-2f'[d_{i,c}] \left(\frac{\partial d_{i,c}}{\partial w_i} w_i \right)^2 + f'[2m_{i,c} - d_{i,c}] \frac{\partial d_{i,c}}{\partial w_i} w_i \frac{\partial d_{i,c}}{\partial w_i} w_c \right. \\
&\quad \left. - (w_i f''[d_{i,c}] - w_c f''[2m_{i,c} - d_{i,c}]) \frac{\partial d_{i,c}}{\partial w_i} \left(\frac{\partial d_{i,c}}{\partial w_i} w_i \right)^2 \right) \leq 0. \tag{A.17}
\end{aligned}$$

The second step follows from $\frac{\partial d_{i,c}}{\partial w_c} w_c = \frac{\partial d_{i,c}}{\partial w_i} w_i$. Since the first two terms in (A.17) are always positive, it holds that

$$\left| \frac{\partial^2 \pi_i}{\partial w_i \partial w_c} w_i w_c \right| \leq \frac{r_i''}{\beta^2} \frac{\partial L_i}{\partial w_i} \frac{\partial L_i}{\partial w_c} w_i w_c - \frac{\partial L_i}{\partial w_c} w_i w_c + \left| \left(\frac{r_i'}{\beta} - w_i \right) \frac{\partial^2 L_i}{\partial w_i \partial w_c} w_i w_c \right|.$$

Provided that condition i) holds, diagonal dominance as defined in (A.15) at the equilibrium point obtains if

$$\begin{aligned}
& -\frac{r_i''}{\beta^2} \left(\frac{\partial L_i}{\partial w_i} w_i \right)^2 + 2 \frac{\partial L_i}{\partial w_i} w_i^2 - \left(\frac{r_i'}{\beta} - w_i \right) \frac{\partial^2 L_i}{\partial w_i^2} w_i^2 \\
& \geq \sum_{c=\ell, r} \left(\frac{r_i''}{\beta^2} \frac{\partial L_i}{\partial w_i} \frac{\partial L_i}{\partial w_c} w_i w_c - \frac{\partial L_i}{\partial w_c} w_i w_c + \left(\frac{r_i'}{\beta} - w_i \right) \frac{\partial^2 L_i}{\partial w_i \partial w_c} w_i w_c \right) \\
\Leftrightarrow & -\frac{r_i''}{\beta^2} \left(\frac{\partial L_i}{\partial w_i} w_i \right) \left(\frac{\partial L_i}{\partial w_i} w_i + \sum_{c=\ell, r} \frac{\partial L_i}{\partial w_c} w_c \right) + \frac{\partial L_i}{\partial w_i} w_i^2 + w_i \left(\frac{\partial L_i}{\partial w_i} w_i + \sum_{c=\ell, r} \frac{\partial L_i}{\partial w_c} w_c \right) \\
& - \left(\frac{r_i'}{\beta} - w_i \right) \left(\frac{\partial^2 L_i}{\partial w_i^2} w_i^2 - \sum_{c=\ell, r} \frac{\partial^2 L_i}{\partial w_i \partial w_c} w_i w_c \right) \geq 0 \\
\Leftrightarrow & \frac{r_i'}{\beta} \frac{\partial L_i}{\partial w_i} w_i \geq 0. \tag{A.18}
\end{aligned}$$

The last step follows from

$$\frac{\partial L_i}{\partial w_i} w_i = - \sum_{c=\ell, r} \frac{\partial L_i}{\partial w_c} w_c \quad \text{and} \quad \frac{\partial^2 L_i}{\partial w_i^2} w_i^2 - \sum_{c=\ell, r} \frac{\partial^2 L_i}{\partial w_i \partial w_c} w_i w_c = - \frac{\partial L_i}{\partial w_i} w_i. \tag{A.19}$$

This proves that the game in log transformed strategies exhibits diagonal dominance at

the equilibrium point.

Since conditions i) and ii) clearly also hold for the transformed game, there exists a unique solution to this game. Since $\ln \mathbf{w}$ is a monotone transformation of \mathbf{w} for $\mathbf{w} > 0$, uniqueness in the transformed game implies uniqueness of equilibrium in the original game. This completes the proof.

A.3 Proof of existence and uniqueness of the entry equilibrium (Lemma 2)

We first prove that under condition ii) of Lemma 2 there is a unique number of firms $N_{\text{sym}}^e \geq 1$ corresponding to a symmetric second-stage equilibrium that yields $\pi^e[\mathbf{m}_{\text{sym}}, N_{\text{sym}}^e] = 0$. Moreover, we show that $\pi^e[\mathbf{m}_{\text{sym}}, N] = 0$ is decreasing in N .

Existence and uniqueness of the second-stage zero-profit equilibrium for symmetric distance patterns. As described in Section 3.3, for symmetric distance patterns a second-stage zero-profit equilibrium is given by a root of the function

$$G[m] := \pi^e[\mathbf{m}_{\text{sym}}, N] = \beta \rho[m] \psi[m] - \beta g[m], \quad (\text{A.20})$$

where $g[m] > 1$ is the inverse of an aggregate version of the familiar measure of the degree of scale economies. We expect this to be falling in m : The larger firm size m , and the smaller the number of firms, the closer average cost to marginal cost. In turn, $\rho[m] := 1 + \frac{1}{\gamma N[m]}$ and $\psi[m] := \frac{\eta[m]+1}{\eta[m]}$ are the two markups on the goods and the labor market, respectively. Given that a symmetric equilibrium has $N = H/m$, we have $\rho_m = 1/(\gamma H) > 0$. As shown in Section 3.3, $\eta_m < 0$, whence we have $\psi'[m] = -\eta'[m]/\eta[m]^2 > 0$. As expected from intuition, both markups are falling in the number of firms and thus rising in the half-distance between two neighboring firms, m . Note that $G[m] > 0$ implies positive profits, while $G[m] < 0$ implies that firms make losses.

The following conditions are *sufficient* for a symmetric zero-profit equilibrium to *exist* and to be *unique*: a) $G[H] > 0$, b) $G[m]$ is continuous and $G'[m] > 0$ in the interval $(\tilde{m}, H]$, where \tilde{m} is defined by $\frac{L}{N[\tilde{m}]} \theta[\tilde{m}] = \alpha$.

Condition a) requires that a single firm in the market makes non-negative profits, that is,

$$\frac{L\theta[H]}{L\theta[H] - \alpha} \leq \left(1 + \frac{1}{\gamma}\right) \psi[H]. \quad (\text{A.21})$$

Observing that $\psi[m]$ increases in m , we can set $\psi[H]$ on the right-hand side to its minimum

level of unity to obtain

$$\frac{\alpha}{L\beta}(1 + \gamma) \leq \frac{F[H]}{H}, \quad (\text{A.22})$$

which is a sufficient condition for (A.21). It shows, that given α, β, L and H , the degree of substitutability of goods in the utility function γ must not be too large. Relating back to (A.21) in its original form, these restrictions imply that the price markup over marginal cost that a single firm can choose exceeds its average cost.³³

Condition b) requires that firm entry associated with a decrease in the skill reach m lowers profits in the relevant range where firms produce positive output, that is, for $m \in (\tilde{m}, H]$. Since we know from above that $\rho'[m] > 0$ as well as $\psi'[m] > 0$, condition b) is satisfied if $g_m < 0$. It is straightforward to show that

$$g'[m] = \frac{\frac{L}{H}f[m]}{\frac{mL}{H}\theta[m] - \alpha} \left(1 - \frac{\frac{mL}{H}\theta[m]}{\frac{mL}{H}\theta[m] - \alpha} \right) < 0 \quad \text{for } m \in (\tilde{m}, H]. \quad (\text{A.23})$$

Hence, there exists a unique $N_{\text{sym}}^e \geq 1$ satisfying $G[m] = \pi^e[\mathbf{m}_{\text{sym}}, N_{\text{sym}}^e] = 0$. Condition b) and $m = \frac{2H}{N}$ imply $\frac{\partial \pi^e[\mathbf{m}_{\text{sym}}, N]}{\partial N} < 0$.

Existence and uniqueness of the symmetric subgame-perfect equilibrium. With firm entry determined by (13) and consistent beliefs as described in Section (3.2), the existence of N_{sym}^e satisfying $\pi^e[\mathbf{m}_{\text{sym}}, N_{\text{sym}}^e] = 0$ implies that no (symmetric or asymmetric) distance pattern involving a number of firms smaller N_{sym}^e can be an equilibrium according to (14).

To prove that $\mathbf{m}_{\text{sym}}|N_{\text{sym}}^e$ is an equilibrium, and, in fact, the only equilibrium, it remains to show that there is no other distance pattern with $N \geq N_{\text{sym}}^e$ consistent with (13) and (14). We do so by showing that every *asymmetric* distance pattern with $N \geq N_{\text{sym}}^e$ and every *symmetric* distance pattern with $N > N_{\text{sym}}^e$ implies negative profits for at least one firm, which implies $\nu(N) = 0$ for all $N > N_{\text{sym}}^e$. The result for symmetric distance vectors follows readily from $\frac{\partial \pi^e[\mathbf{m}_{\text{sym}}, N]}{\partial N} < 0$.

The proof for asymmetric location patterns is slightly more involved and requires restrictions on the parameter space. It runs along the following line of argument. We conjecture that the symmetric zero-profit solution characterized by $\pi^e[\mathbf{m}_{\text{sym}}, N_{\text{sym}}^e] = 0$ is an equilibrium and then consider any possible change towards an asymmetric location pattern featuring the same, or a larger number of firms. Since the symmetric location

³³This condition is well known from the standard New Trade Theory model with homogeneous workers (cp. Equation (10) in Krugman, 1980).

pattern maximizes labor supply per firm, such a change must bring about a decrease in q for at least one firm. Let j be the firm that produces the smallest quantity in any arbitrarily chosen asymmetric location pattern with N_{sym}^e firms. Then, we can show that if the marginal cost β are sufficiently small, firm j 's profits must be negative in the asymmetric pattern. Hence, no asymmetric location pattern can be an equilibrium consistent with optimal entry choices of entrants and non-entrants.

From the point of view of any firm i , a zero-profit equilibrium is characterized by

$$\rho[N, q_i, \mathbf{q}_{-i}] \psi[\mathbf{m}_i, N] = g[q_i] \quad \text{where} \quad g[q_i] = \frac{L_i}{L_i - \alpha} = \frac{\alpha + \beta q_i}{\beta q_i} \quad \forall i,$$

which states that the product of markups equals the inverse of the degree of economies of scale (cp. (24)).³⁴ Note that $q_i = \frac{L_i[\mathbf{m}_i, N] - \alpha}{\beta}$ is also a function of the location pattern and so are $\rho[N, q_i, \mathbf{q}_{-i}]$ and $g[q_i]$.³⁵ However, it will prove important that $g[\cdot]$ and $\rho[\cdot]$ depend on \mathbf{m}_i *only* through output quantities, since this allows us to pin down the changes in $g[\cdot]$ and $\rho[\cdot]$ for firm j for an arbitrary change in the location pattern, observing that, by definition, q_j decreases whenever we move away from symmetry.

The change towards an asymmetric location pattern can be described in terms of changes in potentially all elements of the distance vector \mathbf{m}_i . Profits are affected by corresponding changes in the markups and the degree of economies of scale. In the new location pattern, firm i 's profits will be negative if and only if the total markup ($\rho_i \psi_i$) increases by less (falls by more) than the inverse of the degree of economies of scale, g_i . That is, a sufficient condition for $\pi^e[\mathbf{m}_i, N_{\text{sym}}^e] < 0$ for all $\mathbf{m}_i \neq \mathbf{m}_{\text{sym}}$ is that

$$d(\rho_i \psi_i) = \rho_i d\psi_i + \psi_i d\rho_i < dg_i, \quad \text{where} \quad (\text{A.24})$$

$$dg_i = \frac{\partial g[q_i]}{\partial q_i} \frac{\partial q_i}{\partial L_i} dL_i = -\frac{\alpha}{(\beta q_i)^2} dL_i \quad \text{with} \quad dL_i = \sum_k^N \frac{\partial L_i}{\partial m_{k,k+1}} dm_{k,k+1} \quad (\text{A.25})$$

$$d\rho_i = \sum_k^N \frac{\partial \rho_i}{\partial q_k} \frac{\partial q_k}{\partial L_k} dL_k \quad (\text{A.26})$$

$$d\psi_i = \frac{1}{\eta_i} \frac{dL_i}{L_i} - \frac{1}{\eta_i} \sum_k^N \frac{\frac{\partial^2 L_i}{\partial w_i \partial m_{k,k+1}}}{\frac{\partial L_i}{\partial w_i}} dm_{k,k+1}. \quad (\text{A.27})$$

Note that L_i depends on the distances between *all* firms and not just firm i 's neighbors, because it is a function of firm i 's own and its neighbors' equilibrium wages in the second

³⁴ Using the first-order condition (10), we can write optimum profits $\pi^e[\mathbf{m}_i, N]$ of any firm as $\pi_i^e = w_i^e L_i \left(\psi_i \rho_i \frac{L_i - \alpha}{L_i} - 1 \right)$. Then, for $w_i^e, L_i > 0$, $\pi_i^e = 0$ iff $\rho_i \psi_i = g_i$.

³⁵ The dependence of ρ_i on \mathbf{q}_{-i} derives from the dependence of ρ_i on total expenditure $Y = \sum_k^N r_i[q_k]$.

stage, which jointly solve all firms' first-order conditions and therefore depend on the complete distance vector.

Consider the problem of firm j , defined as the firm that produces the smallest amount of output in the asymmetric allocation. Then it is true that $dL_j < 0$ and $dg_j > 0$. Moreover, in the asymmetric allocation it is true that $q_j \leq \frac{1}{N} \sum_k q_k$, which, by concavity of the revenue function, implies $\ln p_j \geq \overline{\ln p}$. Therefore, it follows from $\rho_j = 1 + \frac{\delta_j}{\gamma}$ that

$$d\rho_j = \frac{1}{\gamma} (\delta_j^{asym} - \delta_j^{sym}) = \overline{\ln p} - \ln p_j \leq 0.$$

Firm j 's price markup weakly decreases because its expenditure share is weakly smaller in the asymmetric location pattern. Note that an asymmetric location pattern where all firms produce the same quantity is conceivable. In these situations the above statements hold with equality. The decrease in the degree of economies of scale $dg_j > 0$ and the decrease in the price markup work towards lowering firm j 's optimum profits. However, the effect of the change in the location pattern on the wage markup is ambiguous. The first term in (A.27) is strictly negative for $dL_j < 0$, but the second term, reflecting the sum of the elasticities of marginal labor supply with respect to the change in the location pattern, is difficult to sign. It represents the change in competitiveness of firm j 's labor market environment due to changes in the distances to its neighbors and the equilibrium wage adjustments to the change in the overall distance pattern. Therefore, according to (A.25), it holds that firm j 's optimum profits decrease whenever the sum of the effects on the degree of economies of scale, the price markup, and negative effect on the wage markup due to $dL_j < 0$ overcompensate a potentially positive effect on the wage markup due to a lower degree of competitiveness in firm j 's labor market environment. This is always true if the marginal cost β are small relative to the fixed cost α , in which case the effect on average cost is large compared to the adjustment in the wage markup, which is independent of β . Hence, for sufficiently small β it holds that every departure from symmetry (holding fixed N) leads to a decrease in firm j 's profits. Since we are starting from the zero-profit equilibrium, firm j 's profits will be negative in any asymmetric location pattern featuring the same number of firms as the symmetric starting point.

We may now conclude that no asymmetric location pattern with $N = N_{\text{sym}}^e$ exists where all firms make positive profits. Moreover, the exact same rationale implies that no asymmetric pattern with a number of firms larger N_{sym}^e exists where all firms make positive profits. This completes the proof.

A.4 The limiting case of $H \rightarrow 0$

As we let the degree of skill heterogeneity approach zero, our equilibrium converges to the equilibrium of a standard monopolistic competition model with translog preferences. From the previous appendix it follows that if an equilibrium exists with some \bar{H} , it also exists for $H < \bar{H}$. In all of these equilibria, m will be smaller than \bar{H} , ensuring $H/m = N > 1$. Consider an exogenous decrease in the degree of skill differentiation $\hat{H} < 0$ within the interval $(0, \bar{H}]$. A smaller circumference means that the mass of labor on any interval of the skill circle increases. Holding m constant for a moment, this would allow firms to expand output without having to rely on workers with less suitable types of skills, thus increasing the degree of scale economies and decreasing $g[m, H]$. Moreover, from $N = H/m$ a smaller H means a lower number of firms, which implies a higher goods price markup. But this, together with the size effect, implies positive profits. Hence, $\hat{N} = \hat{H}$ with $\hat{m} = 0$ is not an equilibrium adjustment. Totally differentiating (24), we obtain³⁶

$$\hat{m} = \frac{g_H - \psi[m]\rho_H}{-g_m + \psi[m]\rho_m + \rho[m, H]\psi_m} \frac{H}{m} \hat{H} = \frac{g[m, H](g[m, H] - 1) + \psi[m]\frac{m}{\gamma H}}{\frac{f[m]}{\theta[m]}g[m, H](g[m, H] - 1) + \psi[m]\frac{m}{\gamma H} + \frac{\psi_m m}{\psi[m]}} \hat{H}.$$

The “multiplier” in front of \hat{H} is positive, meaning that m falls as H decreases, but $f[m]/\theta[m] < 1$ and $\psi_m m/\psi[m] \geq 0$ imply that the multiplier can be greater or smaller one. Thus, the net effect on $N = H/m$ is generally ambiguous. Now, let $H \rightarrow 0$, whence $m = H/N$ must approach zero as well. Therefore, $f[m]/\theta[m]$ goes to unity and $\psi_m m/\psi[m] \geq 0$ goes to zero, so that the multiplier approaches unity and N converges to a constant \underline{N} . Returning to the equilibrium condition (24) and letting $m \rightarrow 0$ ($\theta[m] \rightarrow 1, \psi[m] \rightarrow 1$) and $H/m = N \rightarrow \underline{N}$. We finally obtain that \underline{N} must satisfy

$$\frac{L}{L - \alpha \underline{N}} = 1 + \frac{1}{\gamma \underline{N}} \quad (\text{A.28})$$

which is the equilibrium condition for the number of firms in a Krugman (1979)-type model with homogeneous workers and translog preferences.

A.5 The constrained social optimum

The social planner maximizes log utility with respect to m and subject to the condition that price equals average cost and to the endowment constraint, which we can combine

³⁶ To ease notation, in what follows we use subscripts to denote partial derivatives of functions with multiple arguments.

to $p = \frac{L\theta[m]}{L\theta[m] - \alpha N[m]}$:

$$\max_m \ln V = \ln \theta[m] - \left(\frac{1}{2\gamma N[m]} + \ln p[m] \right) \quad \text{s.t.} \quad p[m] = \frac{L\theta[m]}{L\theta[m] - \alpha N[m]}.$$

The first-order condition is

$$\frac{Lf[m]}{L\theta[m] - \frac{\alpha H}{m}} = 1 + \frac{m}{2\gamma H}. \quad (\text{A.29})$$

The second-order condition requires

$$\frac{d^2 \ln V}{dm^2} = -\frac{(L\theta'[m] + \frac{\alpha H}{m^2})^2}{(L\theta[m] - \frac{\alpha H}{m})^2} + \frac{L\theta''[m] - 2\frac{\alpha H}{m^3}}{L\theta[m] - \frac{\alpha H}{m}} < 0. \quad (\text{A.30})$$

A sufficient condition for this to hold is

$$\theta''[m] := \frac{\partial^2 \theta[m]}{\partial m^2} = \frac{1}{m} \left(f'[m] - \frac{2}{m}f[m] + \frac{2}{m}\theta[m] \right) \leq 0 \quad (\text{A.31})$$

which requires $f[m] \geq \theta[m] + \frac{m}{2}f'[m]$. Since concavity of $f[\cdot]$ implies $f[m] \geq f[\frac{m}{2}] + \frac{m}{2}f'[\frac{m}{2}]$ and (by Jensen's inequality) $f[\frac{m}{2}] \geq \theta[m]$, it follows that $f[m] \geq f[\frac{m}{2}] + \frac{m}{2}f'[\frac{m}{2}] \geq \theta[m] + \frac{m}{2}f'[m]$ and therefore $\theta''[m] \leq 0$ and $\frac{\partial^2 \ln V}{\partial m^2} < 0$ always hold. To compare the planner's solution with the laissez faire equilibrium determined by (24) we rewrite (A.29) as

$$g[m] = \frac{\theta[m]}{f[m]} \frac{1}{\psi[m]} \psi[m] \rho[m/2]. \quad (\text{A.32})$$

The difference between the two conditions appears on the right-hand side of this equation. Since $g'[m] < 0$, the social planner's solution implies a larger m than the market equilibrium, if the right-hand side is smaller than $\psi[m]\rho[m]$ for all values of m . Since $\rho'[m] > 0$,

$$\frac{\theta[m]}{f[m]} \frac{1}{\psi[m]} < 1 \quad (\text{A.33})$$

is a sufficient condition for this to hold. Rearranging (A.33) and inserting $\psi[m] = \frac{f[m]^2 - 2f'[m]m\theta[m]}{f[m]^2}$ yields $\frac{1 + \frac{2}{f[m]}f'[m]m}{f[m]} < \frac{1}{\theta}$, which holds a fortiori because concavity of $f[\cdot]$ implies that $\frac{1 + f'[m]m}{f[m]} < 1$. Hence, condition (A.33) is fulfilled and it follows that the market equilibrium firm size is too small compared to the socially optimal allocation.

A.6 Further details of the trading equilibrium

A.6.1 Proof of Proposition 1

(i) Log-differentiating the equilibrium condition (28) and setting $k = 1$, we obtain

$$\hat{m} = A\hat{k} \quad \text{with} \quad A := \frac{\psi[m]\frac{1}{\gamma H}}{-g'[m] + \psi[m]\frac{1}{\gamma H} + \rho^T[m]\psi'[m]}. \quad (\text{A.34})$$

Since $g'[m] < 0$ and $\psi'[m] > 0$ (see Appendix A.3 for details) we find that $0 < A < 1$, which implies $0 < \hat{m} = A\hat{k} < \hat{k}$. Hence, m increases and the number of firms in each country falls. However, $A < 1$ implies that the total number of available varieties $N^T = kN > N^A$ is still larger with trade than under autarky.

(ii) As the price markup depends negatively on the number of available varieties kN , it follows directly from the previous result that it must fall. Furthermore, we know from above that the wage markup increases. Log-differentiating (26) and setting $k = 1$ yields

$$\hat{p} = B\hat{k} \quad \text{with} \quad B = \frac{\frac{m}{\gamma H}}{\left(1 + \frac{m}{\gamma H}\right) \left(-g'[m] + \psi[m]\frac{1}{\gamma H} + \rho^T[m]\psi'[m]\right)}. \quad (\text{A.35})$$

Since $-1 < B < 0$, it follows that $\hat{p} < 0$.

(iii) This follows from $\theta'[m] = \frac{1}{m} (f[m] - \theta[m]) < 0$.

(iv) We know from above that goods prices are lower in the free trade equilibrium, which contributes to higher real incomes. At the same time, exit increases m and thus average productivity $\theta[m]$, which has a negative effect on real income. The logic of A.5 implies that the free trade equilibrium, like autarky, is characterized by excess entry. Hence, the net effect of an increase in m must be positive. With higher real income and a larger variety available for consumption as established in (i), it follows from (25) that welfare of the worker earning average income increases.

A.6.2 The first-order conditions with two symmetric countries and positive trade cost

Under the assumption that the constraints $q_i, q_i^* \geq 0$ never bind, we may write (29) as

$$\max_{w_i, q_i} \left\{ r_i[q_i, N, \overline{\ln p}, Y] + r_i^* \left[\frac{\bar{q}_i - q_i}{\tau}, N, \overline{\ln p}, Y \right] - w_i L_i \right\}.$$

The first-order condition with respect to w_i then obtains as

$$\frac{p^*}{\tau} \left(\frac{\partial \ln p^*}{\partial \ln \frac{\bar{q}_i - q_i}{\tau}} + 1 \right) \frac{\partial L_i}{\partial w_i} = w_i \frac{\partial L_i}{\partial w_i} + L_i \quad \Leftrightarrow \quad p^* = \frac{\varepsilon_i^*}{\varepsilon_i^* - 1} \frac{\eta_i + 1}{\eta_i} w_i \tau,$$

and the first-order condition with respect to q_i reads

$$p \left(\frac{\partial \ln p}{\partial \ln q_i} + 1 \right) \frac{\partial L_i}{\partial w_i} = \frac{p^*}{\tau} \left(\frac{\partial \ln p^*}{\partial \ln \frac{\bar{q}_i - q_i}{\tau}} + 1 \right) \frac{\partial L_i}{\partial w_i} \quad \Leftrightarrow \quad p \frac{\varepsilon_i - 1}{\varepsilon_i} = \frac{p^*}{\tau} \frac{\varepsilon_i^* - 1}{\varepsilon_i^*}.$$

Both first-order conditions together imply (32) and (33).

A.6.3 Proof of Proposition 2

In the symmetric equilibrium with identical countries the average price in the domestic and the foreign market is the same and given by $\overline{\ln p} = \overline{\ln p^*} = 1/2 \ln p + 1/2 \ln p^*$. Inserting $\overline{\ln p}$ and $\overline{\ln p^*}$ into the Z-terms in (32), (33), we can use the same logic as in A.1 to obtain explicit solutions for p and p^* , where the price markups no longer depend on the own price, but only on the respective other price and the number of firms:

$$p = \frac{\mathcal{W}[\tilde{Z}]}{2} \psi \quad \text{with } \tilde{Z} = \frac{2}{\psi} \exp \left\{ 2 + \frac{m}{\gamma H} + \ln p^* \right\} \quad (\text{A.36})$$

$$p^* = \frac{\mathcal{W}[\tilde{Z}^*]}{2} \psi \tau \quad \text{with } \tilde{Z}^* = \frac{2}{\psi \tau} \exp \left\{ 2 + \frac{m}{\gamma H} + \ln p \right\}. \quad (\text{A.37})$$

Inserting $p = \frac{\mathcal{W}[\tilde{Z}]}{2} \psi$ and $p^* = \frac{\mathcal{W}[\tilde{Z}^*]}{2} \psi \tau$ into the \tilde{Z} -terms, we obtain

$$p = \mathcal{W} \left[\mathcal{W}[\tilde{Z}^*] \tau \exp \left\{ 2 + \frac{m}{\gamma H} \right\} \right] \frac{\psi}{2} \quad (\text{A.38})$$

$$p^* = \mathcal{W} \left[\frac{\mathcal{W}[\tilde{Z}]}{\tau} \exp \left\{ 2 + \frac{m}{\gamma H} \right\} \right] \frac{\psi}{2} \tau. \quad (\text{A.39})$$

It proves convenient to focus on the price markup values $W := \mathcal{W}[\tilde{Z}]$ and $W^* := \mathcal{W}[\tilde{Z}^*]$ instead of prices. The corresponding system of equations determining these values emerges as

$$W := W[W^*, m] = \mathcal{W} \left[W^* \tau \exp \left\{ 2 + \frac{m}{\gamma H} \right\} \right] \quad (\text{A.40})$$

$$W^* := W^*[W, m] = \mathcal{W} \left[\frac{W}{\tau} \exp \left\{ 2 + \frac{m}{\gamma H} \right\} \right]. \quad (\text{A.41})$$

Note that for zero trade costs ($\tau = 1$) the price markups are identical. While the markup on domestic varieties increases in τ , the markup on foreign varieties falls in the level of trade costs. For any $\tau > 1$, it must therefore be true that $W > W^*$.

The two-country version of (A.3) can be written as $p = \left(1 + \frac{1}{\gamma N^T} + \frac{1}{2}(\ln p^* - \ln p) \right) \tilde{w}$ and analogously for p^* . In view of (A.36) and (A.37) it follows that $\frac{W}{2} = 1 + \frac{1}{\gamma N^T} + \frac{1}{2}(\ln p^* - \ln p)$ and $\frac{W^*}{2} = 1 + \frac{1}{\gamma N^T} + \frac{1}{2}(\ln p - \ln p^*)$. The expenditure shares in (30) can therefore be written as

$$\delta = \left(\frac{W}{2} - 1 \right) \gamma \quad \text{and} \quad \delta^* = \left(\frac{W^*}{2} - 1 \right) \gamma. \quad (\text{A.42})$$

Direct demand functions for foreign varieties in terms of W^* obtain as $q^* = \frac{\delta^* Y}{p^*} \left(1 - \frac{2}{W^*} \right) \frac{\gamma Y}{\psi}$. This implies that the prohibitive level of trade costs $\bar{\tau}$ for which $q^* = 0$ satisfies $\mathcal{W} \left[\frac{W}{\bar{\tau}} \exp \left\{ 2 + \frac{2}{\gamma N^T} \right\} \right] \equiv 2$. It follows that for non-prohibitive trade costs $W \geq W^* \geq 2$. Inserting demand and income $Y = L\theta$ into the labor market clearing condition (34), and rearranging terms gives

$$\begin{aligned} \gamma \left(2 - \frac{2}{W} - \frac{2}{W^*} \right) &= \frac{\frac{L\theta[m]}{N[m]} - \alpha}{L\theta[m]} \psi[m] \\ \gamma h[W, W^*] &= \frac{\psi[m]}{g[m]N[m]}. \end{aligned} \quad (\text{A.43})$$

For easier reference the second line introduces $h[W, W^*] := \left(2 - \frac{2}{W} - \frac{2}{W^*} \right)$. (A.43), (A.40) and (A.41) form our system of equations in W, W^* and m .

(i) Comparative statics of firm size and markups. The proof of Proposition 2 requires that we solve this system for an exogenous change in τ . Doing so by log-linearization, we write the solution as $\widehat{W} = \omega \hat{\tau}$, $\widehat{W}^* = \omega^* \hat{\tau}$ and $\widehat{m} = \mu \hat{\tau}$. We next explore the sign of the elasticities ω, ω^* and μ . For notational convenience we suppress the functional dependence of N and ψ on m in the following, whenever it is not crucial. Log-differentiating (A.43),

(A.40), (A.41) leads to

$$\begin{aligned} & \begin{bmatrix} -\frac{\partial \ln h}{\partial \ln W} & -\frac{\partial \ln h}{\partial \ln W^*} & \frac{\partial \ln \psi}{\partial \ln m} - \frac{\partial \ln g}{\partial \ln m} - \frac{\partial \ln N}{\partial \ln m} \\ -1 & \frac{\partial \ln W}{\partial \ln W^*} & \frac{\partial \ln m}{\partial \ln W^*} \\ \frac{\partial \ln W}{\partial \ln W} & -1 & \frac{\partial \ln W^*}{\partial \ln m} \end{bmatrix} \begin{bmatrix} \widehat{W} \\ \widehat{W^*} \\ \widehat{m} \end{bmatrix} = \begin{bmatrix} 0 \\ -\frac{\partial \ln W}{\partial \ln \tau} \hat{\tau} \\ -\frac{\partial \ln W^*}{\partial \ln \tau} \hat{\tau} \end{bmatrix} \\ & \begin{bmatrix} -\frac{1}{W-1-\frac{W}{W^*}} & -\frac{1}{W^*-1-\frac{W^*}{W}} & \frac{\frac{Lf[m]}{N}}{\frac{L\theta[m]}{N}-\alpha} - \frac{f[m]}{\theta[m]} + 1 + \frac{\partial \ln \psi}{\partial \ln m} \\ -1 & \frac{1}{W+1} & \frac{1}{\gamma N} \frac{1}{W+1} \\ \frac{1}{W^*+1} & -1 & \frac{1}{\gamma N} \frac{1}{W^*+1} \end{bmatrix} \begin{bmatrix} \widehat{W} \\ \widehat{W^*} \\ \widehat{m} \end{bmatrix} = \begin{bmatrix} 0 \\ -\frac{1}{W+1} \hat{\tau} \\ \frac{1}{W^*+1} \hat{\tau} \end{bmatrix}. \end{aligned} \quad (\text{A.44})$$

Denoting the 3×3 -matrix of derivatives by D , it follows that

$$\omega = \frac{1}{(W+1)(W^*+1)} \left[\left(\frac{\frac{Lf[m]}{N}}{\frac{L\theta[m]}{N}-\alpha} - \frac{f[m]}{\theta[m]} + 1 + \frac{\partial \ln \psi}{\partial \ln m} \right) W^* - \frac{1}{\gamma N h[W, W^*]} \frac{4}{W^*} \right] \frac{1}{\det[D]} \quad (\text{A.45})$$

$$\omega^* = \frac{1}{(W+1)(W^*+1)} \left[- \left(\frac{\frac{Lf[m]}{N}}{\frac{L\theta[m]}{N}-\alpha} - \frac{f[m]}{\theta[m]} + 1 + \frac{\partial \ln \psi}{\partial \ln m} \right) W + \frac{1}{\gamma N h[W, W^*]} \frac{4}{W} \right] \frac{1}{\det[D]} \quad (\text{A.46})$$

$$\mu = \frac{2W^*/W - 2W/W^*}{h[W, W^*](W+1)(W^*+1)} \frac{1}{\det[D]}. \quad (\text{A.47})$$

The signs of the elasticities hinge upon the sign of the determinant which is given by

$$\begin{aligned} \det[D] &= \left(\frac{\frac{Lf[m]}{N}}{\frac{L\theta[m]}{N}-\alpha} - \frac{f[m]}{\theta[m]} + 1 + \frac{\partial \ln \psi}{\partial \ln m} \right) \frac{WW^* + W + W^*}{(W+1)(W^*+1)} \\ &\quad - \frac{1}{\gamma N h[W, W^*]} \frac{(2+W^*)\frac{2}{W} + (2+W)\frac{2}{W^*}}{(W+1)(W^*+1)}. \end{aligned} \quad (\text{A.48})$$

Since $WW^* > 2$ and $W \geq W^*$, we have $WW^* + W + W^* > (2+W^*)\frac{2}{W} + (2+W)\frac{2}{W^*}$. This implies that $\det[D] > 0$ if

$$\frac{\frac{Lf[m]}{N}}{\frac{L\theta[m]}{N}-\alpha} - \frac{f[m]}{\theta[m]} + 1 + \frac{\partial \ln \psi}{\partial \ln m} > \frac{1}{\gamma N h[W, W^*]}. \quad (\text{A.49})$$

We know from above that $\frac{f[m]}{\theta[m]} < 1$ and $\frac{\partial \ln \psi}{\partial \ln m} > 0$, and therefore, inequality (A.49) holds if

$$\frac{\frac{Lf[m]}{N}}{\frac{L\theta[m]}{N}-\alpha} > \frac{1}{\gamma N h[W, W^*]}. \quad (\text{A.50})$$

Using the equilibrium condition (A.43), we can rewrite this as $\psi[m] \geq \theta[m]/f[m]$. We have demonstrated in Appendix A.5 that this inequality always holds. Hence, it follows that $\det[D] > 0$.

Returning to our elasticity ω , we note that $W^* \geq \frac{4}{W^*}$, $\det[D] > 0$ and (A.49) jointly imply $\omega > 0$. By analogy, it follows that $\omega^* < 0$. And finally, $W \geq W^*$ implies that $\mu \leq 0$. For reasons pointed out in the text, μ is monotonic in the initial level of trade costs, converging to zero as τ approaches one. In view of A.47, the level of τ enters through W and W^* . The lower the trade cost level, the smaller the difference between W and W^* . At $\tau = 1$, price markups are identical and $m = 0$. This proves part (i) of the proposition.

(ii) Changes in prices. The proposition states that for $\hat{\tau} < 0$, $\hat{p}^* < 0$ while \hat{p} is ambiguous. The price of imported varieties is affected by the change in τ and the changes in both markups

$$\hat{p}^* = \left(\omega^* + \frac{\partial \ln \psi}{\partial \ln m} \mu + 1 \right) \hat{\tau}, \quad (\text{A.51})$$

where $\frac{\partial \ln \psi}{\partial \ln m} = \frac{-2mf''[m]F[m]}{f[m]^2\psi[m]} - \frac{2mf'[m]}{f[m]} > 0$. Inserting (A.46) and (A.47) shows that \hat{p}^* is positive if and only if

$$-\frac{d_{13}W - \frac{2}{W} \frac{2}{\gamma h N} + \frac{\partial \ln \psi}{\partial \ln m} \frac{1}{h[W, W^*]} \left(\frac{2W}{W^*} - \frac{2W^*}{W} \right)}{d_{13}(WW^* + W + W^*) - \frac{2}{\gamma h[W, W^*]N} \left(\frac{2+W^*}{W} + \frac{2+W}{W^*} \right)} + 1 > 0 \quad (\text{A.52})$$

where d_{13} is the element in row 1 and column 3 of D . Canceling identical terms in the denominator and the numerator shows that this is true if

$$\frac{\frac{\partial \ln \psi}{\partial \ln m} \frac{1}{h[W, W^*]} \left(\frac{2W}{W^*} - \frac{2W^*}{W} \right)}{d_{13}(WW^* + W^*) - \frac{2}{\gamma h[W, W^*]N} \left(\frac{W^*}{W} + \frac{2+W}{W^*} \right)} < 1.$$

Noting that $d_{13} = \frac{\frac{L f[m]}{N} - \frac{f[m]}{\theta[m]}}{\frac{L \theta[m]}{N} - \alpha} + 1 + \frac{\partial \ln \psi}{\partial \ln m}$ and observing the inequality in (A.50), it follows that $WW^* + W^* \geq \frac{2W^*}{W} + \frac{4+2W}{W^*}$ and $WW^* + W^* \geq \frac{1}{h[W, W^*]} \left(\frac{2W}{W^*} - \frac{2W^*}{W} \right)$ is sufficient for the inequality in (A.52) to hold. Remembering from above that $W \geq W^* \geq 2$, it is straightforward to show that these two conditions are fulfilled.

The change in the domestic price obtains as

$$\hat{p} = \left(\omega + \frac{\partial \ln \psi}{\partial \ln m} \mu \right) \hat{\tau}. \quad (\text{A.53})$$

We know from above that $\omega > 0$; the pro-competitive effect of lower trade costs on the

goods market. This is potentially offset by an increase in the wage markup. For τ close to one, the goods market effect clearly dominates as μ is close to zero.

Conversely, at $\bar{\tau}$ (prohibitive trade cost level), the labor market effect dominates. Inserting (A.45) and (A.47) gives

$$\hat{p} = \left[W^* \left(\frac{\frac{Lf[m]}{N}}{\frac{L\theta[m]}{N} - \alpha} - \frac{f[m]}{\theta[m]} + 1 + \frac{\partial \ln \psi}{\partial \ln m} \right) - \frac{2}{\gamma N h[W, W^*]} \frac{2}{W^*} - \frac{\partial \ln \psi}{\partial \ln m} \frac{1}{h[W, W^*]} \left(\frac{2W}{W^*} - \frac{2W^*}{W} \right) \right] \times \frac{1}{(W+1)(W^*+1)} \frac{\hat{\tau}}{\det[D]}. \quad (\text{A.54})$$

Remember that prohibitive trade costs imply an infinite price elasticity and therefore a price markup of zero, whence $W^* = 2$. To see whether $\hat{p} > 0$ for $\tau = \bar{\tau}$, as stated in Proposition 2, we must therefore evaluate the bracketed term at $W^* = 2$. We obtain

$$-2 \frac{\frac{Lf[m]}{N}}{\frac{L\theta[m]}{N} - \alpha} + 2 \frac{f[m]}{\theta[m]} - 2 - 2 \frac{\partial \ln \psi}{\partial \ln m} + \frac{2}{\gamma N h[W, W^*]} \frac{2}{W^*} + \frac{\partial \ln \psi}{\partial \ln m} (W+2). \quad (\text{A.55})$$

Inserting the equilibrium condition (A.43), which reduces to $\gamma h[W, W^*] = \frac{L\theta[m]/N - \alpha}{L\theta[m]} \psi = \frac{2}{W} \frac{1}{N}$ at $\tau = \bar{\tau}$, shows that the expression is negative, if

$$\psi W \frac{f[m]}{\theta[m]} < 2 \frac{f[m]}{\theta[m]} + W - 2 + W \frac{\partial \ln \psi}{\partial \ln m}. \quad (\text{A.56})$$

Inserting the explicit expressions for ψ and $\frac{d \ln \psi}{d \ln m}$ leads to

$$\frac{W}{\theta[m]} \frac{f[m]^2 - 2f'[m]F[m]}{f[m]} < W - 2 + \frac{2f[m]}{\theta[m]} + W \left(\frac{-2f''[m]\theta}{f[m]^2\psi} - \frac{2mf'[m]}{f[m]} \right). \quad (\text{A.57})$$

Since $f''[m] \leq 0$, the inequality holds if

$$\frac{W}{\theta[m]} \frac{f[m]^2 - 2f'[m]F[m]}{f[m]} < W - 2 + \frac{2f[m]}{\theta[m]} - W \frac{2mf'[m]}{f[m]}. \quad (\text{A.58})$$

Rearranging terms shows that this inequality holds if $f[m] < \theta[m]$, which is true given $f'[m] < 0$. This completes the proof of part (ii) of Proposition 2.

(iii) **Welfare.** Indirect utility of the worker receiving average income in the equilibrium with trade costs is given by $\ln V = \ln \theta[m] - \ln P^T[p, p^*, m]$, where

$$\ln P^T[p, p^*, m] = \frac{1}{2\gamma N^T} + \frac{1}{N^T} \sum_{i=1}^{N^T} \ln p_i + \frac{\gamma}{2N^T} \sum_{i=1}^{N^T} \sum_{j=1}^{N^T} \ln p_i (\ln p_j - \ln p_i), \quad (\text{A.59})$$

with $N^T = N + N^*$ and i, j indexing domestic *and* foreign varieties. Under symmetry, $N^* = N = N^T/2$ and the price index simplifies to

$$\ln P^T[p, p^*, m] = \frac{1}{4\gamma N} + \frac{1}{2} \ln p + \frac{1}{2} \ln p^* - \frac{\gamma N}{4} (\ln p - \ln p^*)^2. \quad (\text{A.60})$$

The change in indirect utility is then

$$\widehat{V} = \left(\frac{\partial \ln \theta}{\partial \ln m} - \frac{\partial \ln P}{\partial \ln m} \right) \widehat{m} - \frac{\partial \ln P}{\partial \ln p} \widehat{p} - \frac{\partial \ln P}{\partial \ln p^*} \widehat{p}^*, \quad (\text{A.61})$$

with $\frac{\partial \ln \theta}{\partial \ln m} = \frac{f[m] - \theta[m]}{\theta[m]} < 0$, $\frac{\partial \ln P}{\partial \ln m} = \frac{1}{4\gamma N} + \frac{\gamma N}{4} (\ln p - \ln p^*)^2 > 0$, $\frac{\partial \ln P}{\partial \ln p} = \frac{1}{2} + \frac{\gamma N}{2} (\ln p - \ln p^*) \geq 0$ and $\frac{\partial \ln P}{\partial \ln p^*} = N\delta^* \geq 0$. Inserting yields (35).

Using the results that at the prohibitive level of trade costs $\delta^* = 0$, $\widehat{p} > 0$ and $\widehat{m} > 0$, it follows from (35) that $\widehat{V} < 0$ at $\tau = \bar{\tau}$. At $\tau = 1$ it holds that $\widehat{m} = 0$, $\widehat{p} < 0$ and $\widehat{p}^* < 0$. Hence, $\widehat{V} > 0$ at $\tau = 1$.

A.7 Additional details of the trade and migration equilibrium

A.7.1 The elasticity of labor supply

The elasticity of labor supply in the symmetric alternating equilibrium is defined as $\frac{\partial L^{S,M}}{\partial w_i} \frac{w_i}{L^{S,M}}$. From (37) and (36), we obtain

$$\frac{\partial L^{S,M}}{\partial w_i} = \frac{L}{H} \frac{\partial d_i^n}{\partial w_i} f[d_i^n] + (1 - \lambda) \frac{L}{H} \frac{\partial d_i^m}{\partial w_i} f[d_i^m] \quad \text{with} \quad (\text{A.62})$$

$$\frac{\partial d_i^n}{\partial w_i} = \frac{f[d_i^n]}{-w_i f'[d_i^n] - w^*(1 - \lambda) f'[m - d_i^n]} \quad (\text{A.63})$$

$$\frac{\partial d_i^m}{\partial w_i} = \frac{(1 - \lambda) f[d_i^m]}{-w_i (1 - \lambda) f'[d_i^m] - w^* f'[m - d_i^m]}. \quad (\text{A.64})$$

Evaluating $\frac{\partial L^{S,M}}{\partial w_i} \frac{w_i}{L^{S,M}}$ at the symmetric equilibrium, where it holds that $w_i = w^* \equiv 1$, $d_i^n = d^n$, $d_i^m = d^m = m - d^n$ and $f[d^n] = (1 - \lambda)f[d^m]$, we obtain

$$\begin{aligned} \eta^M &= \frac{\partial L^{S,M}}{\partial w_i} \frac{w_i}{L^S} \Big|_{w_i=w} = \frac{L}{H} \left(\frac{f[d^n]^2}{-f'[d^n] - (1 - \lambda)f'[m - d^n]} + \frac{(1 - \lambda)^2 f[d^m]^2}{-(1 - \lambda)f'[d^m] - f'[m - d^m]} \right) \\ &\quad \times \frac{1}{\frac{L}{H} \left(\int_0^{d^n} f[d]dd + (1 - \lambda) \int_0^{d^m} f[d]dd \right)} \\ &= \frac{2f[d^n]^2}{f'[d^n] + (1 - \lambda)f'[d^m]} \frac{-1}{\int_0^{d^n} f[d]dd + (1 - \lambda) \int_0^{d^m} f[d]dd} \end{aligned} \quad (\text{A.65})$$

as displayed in (39). The elasticity of labor supply decreases in m :

$$\eta_m^M = \eta^M \left[\frac{2f'[d^n]}{f[d^n]} \frac{\partial d^n}{\partial m} - \frac{-f''[d^n] \frac{\partial d^n}{\partial m} - (1 - \lambda)f''[d^m] \frac{\partial d^m}{\partial m}}{-f'[d^n] - (1 - \lambda)f'[d^m]} - \frac{f[d^n]}{m\theta^M} \right] < 0, \quad (\text{A.66})$$

where $\frac{\partial d^n}{\partial m} = \frac{(1 - \lambda)f'[d^m]}{f'[d^n] + (1 - \lambda)f'[d^m]} > 0$ and $\frac{\partial d^m}{\partial m} = \frac{f'[d^n]}{f'[d^n] + (1 - \lambda)f'[d^m]} > 0$. Furthermore, provided that $f'''[\cdot]$ is not too positive, η^M decreases in λ :

$$\eta_\lambda^M = \eta^M \left[\underbrace{\frac{2f'[d^n]}{f[d^n]} \frac{\partial d^n}{\partial \lambda}}_{<0} + \underbrace{\frac{f''[d^n] \frac{\partial d^n}{\partial \lambda} + (1 - \lambda)f''[d^m] \frac{\partial d^m}{\partial \lambda} + f'[d^m]}{-f'[d^n] - (1 - \lambda)f'[d^m]}}_{<0} + \underbrace{\frac{F[d^m]}{F[d^n] + (1 - \lambda)F[d^m]}}_{>0} \right] < 0 \quad (\text{A.67})$$

with $\frac{\partial d^n}{\partial \lambda} = \frac{f[d^m]}{-f'[d^n] - (1 - \lambda)f'[d^m]} > 0$ and $\frac{\partial d^m}{\partial \lambda} = -\frac{\partial d^n}{\partial \lambda} < 0$. $\eta_\lambda^M < 0$ follows from the fact that the first term in the brackets (in absolute terms) exceeds the third, since

$$\frac{2f'[d^n]}{f[d^n]} \frac{\partial d^n}{\partial \lambda} = 2 \underbrace{\frac{f[d^m]}{f[d^n]} \frac{f'[d^n]}{f'[d^n] + (1 - \lambda)f'[d^m]}}_{\geq 1} \geq \underbrace{\frac{F[d^m]}{F[d^n] + (1 - \lambda)F[d^m]}}_{< 1}. \quad (\text{A.68})$$

A.7.2 The productivity gains from migration

For $\lambda \in [0, \bar{\lambda})$, the productivity gains from migration $\theta^M[m, \lambda] - \theta[m]$ increase in m , since $\frac{\partial(\theta^M[m, \lambda] - \theta[m])}{\partial m} = \frac{1}{m} (f[d^n] - \theta^M[m, \lambda] - f[m] + \theta[m]) > 0$, where the sign follows from

$$\begin{aligned} f[d^n] - f[m] &> \frac{1}{d^m} (-F[m] + F[d^n] + (1 - \lambda)F[d^n]) \\ &= \frac{1}{d^m} \left((1 - \lambda) \int_0^{d^m} f[d]dd - \int_{d^n}^m f[d]dd \right) = \frac{1}{d^m} \int_0^{d^m} ((1 - \lambda)f[d] - f[d^n + d])dd \end{aligned}$$

$$> \frac{1}{m} (-F[m] + F[d^n] + (1 - \lambda)F[d^m]) = \theta^M[m, \lambda] - \theta[m].$$

The first inequality is due to the condition $(1 - \lambda)f[d^m] = f[d^n]$ and the concavity of $f[\cdot]$, which implies $(1 - \lambda)f[d] - f[d^n + d] \leq (1 - \lambda)f[d^m] - f[m]$ for $d \in [0, d^m]$.

A.7.3 Analytical details of the proof of Proposition 3, part (i)

The number of firms is too large in the migration equilibrium. The social planner solves the same maximization problem as in Appendix A.5, additionally taking into account the integrated labor market.³⁷ The first-order condition of the planner then obtains as

$$\frac{Lf[d^n]}{L\theta^M - \frac{\alpha H}{m}} = 1 + \frac{m}{4\gamma H}. \quad (\text{A.69})$$

where d^n, θ^M are shorthands for $d^n[m, \lambda], \theta^M[m, \lambda]$, respectively. A comparison with the market solution (42) shows that, as before, the number of firms in the market equilibrium is too large, if the markup distortion is larger than the productivity distortion. This is the case in the migration equilibrium with non-prohibitive λ . The relevant condition is $\psi^M > \frac{\theta^M}{f[d^n]}$. Inserting for ψ^M this is equivalent to $1 - \frac{m\theta^M(f'[d^n] + (1 - \lambda)f'[d^m])}{2f[d^n]^2} > \frac{\theta^M}{f[d^n]}$. This, in turn, holds if $1 - \frac{m\theta^M f'[d^n]}{2f[d^n]^2} > \frac{\theta^M}{f[d^n]}$, since $-f'[d^n](1 - \lambda)/(2f[d^n]^2) \geq 0$. Rewriting the condition leads to $f[d^n] > \theta^M + \frac{m}{2} \frac{f'[d^n]}{f[d^n]} \theta^M$. We show below that $f[\frac{d^n}{2}] \geq \theta^M$. Then, this inequality holds if

$$f[d^n] > f\left[\frac{d^n}{2}\right] + \frac{m}{2} \frac{f'[d^n]}{f[d^n]} \theta^M. \quad (\text{A.70})$$

Concavity of $f[\cdot]$ implies that $f[d^n] \geq f[\frac{d^n}{2}] + f'[d^n]\frac{d^n}{2}$. Moreover, we have $f[\frac{d^n}{2}] + f'[d^n]\frac{d^n}{2} > f[\frac{d^n}{2}] + f'[d^n]\frac{m}{2} \frac{\theta^M}{f[d^n]}$ because $m \geq d^n$ and $\theta^M > f[d^n]$. Therefore, (A.70) holds a fortiori. Hence, the markup distortion exceeds the productivity distortion and consequently, the number of firms in the market equilibrium with migration is too large.³⁸

³⁷Note that this assumes that either the planner maximizes welfare for both countries or takes as given that a planner in the foreign country solves the exact same problem.

³⁸There is a subtle point to this proof in that $\theta^M[m, \lambda]$ is not necessarily concave in m , if there is migration. As a result, the social welfare function is not globally concave. However, it can be shown that the first order condition in A.69 still describes a global maximum and that the social welfare function is monotonically increasing in the relevant range. Details of the proof are available upon request.

Proof that $\theta^M \leq f\left[\frac{d^n}{2}\right]$. Using the expression for θ^M in (38) and Jensen's inequality which states that $f[E[x]] \geq E[f[x]]$ for concave functions $f[x]$, we can state

$$\theta^M = \frac{1}{m} \int_0^{d^n} f[d]dd + (1-\lambda) \frac{1}{m} \int_0^{d^m} f[d]dd \leq \frac{d^n}{m} f\left[\frac{d^n}{2}\right] + (1-\lambda) \frac{d^m}{m} f\left[\frac{d^m}{2}\right]. \quad (\text{A.71})$$

Since $d^n + d^m = m$, we have $\theta^M \leq \frac{d^n}{m} f\left[\frac{d^n}{2}\right] + (1-\lambda) \frac{d^m}{m} f\left[\frac{d^m}{2}\right]$. This reduces to $\theta^M \leq f\left[\frac{d^n}{2}\right]$, provided that $(1-\lambda) f\left[\frac{d^m}{2}\right] \leq f\left[\frac{d^n}{2}\right]$. From (36) it follows that a symmetric equilibrium is characterized by $(1-\lambda) = f[d^n]/f[d^m]$, so the condition becomes $\frac{f\left[\frac{d^m}{2}\right]}{f\left[\frac{d^n}{2}\right]} \leq \frac{f[d^m]}{f[d^n]}$, which is implied by $d^m \leq d^n$ and $f''[\cdot] \leq 0$. This completes the proof.

A.7.4 Proof of Proposition 3, part (ii)

Totally differentiating (42) yields $\hat{m} = C\hat{\lambda}$ where C is given by³⁹

$$C = \frac{g_\lambda^M - \rho^T \psi_\lambda^M}{-g_m^M + \rho^T \psi_m^M + \psi^M \rho_m^T} \frac{\lambda}{m} \leq 0 \quad \text{with} \quad (\text{A.72})$$

$$g_\lambda^M = \frac{L\theta_\lambda^M}{L\theta^M - \alpha N} - \frac{L\theta^M}{(L\theta^M - \alpha N)^2} L\theta_\lambda^M > 0 \quad \text{and} \quad \theta_\lambda^M = -\frac{1}{m} \int_0^{d^m} f[d]dd < 0 \quad (\text{A.73})$$

$$g_m^M = \frac{L\theta_m^M}{L\theta^M - \alpha N} - \frac{L\theta^M}{(L\theta^M - \alpha N)^2} \left(L\theta_m^M + \frac{\alpha N}{m} \right) < 0 \quad \text{and} \quad \theta_m^M = \frac{1}{m} (f[d^n] - \theta^M) < 0 \quad (\text{A.74})$$

$$\psi_\lambda^M = -\frac{1}{(\eta^M)^2} \eta_\lambda^M > 0 \quad \text{with } \eta_\lambda^M \text{ as in (A.67)} \quad (\text{A.75})$$

$$\psi_m^M = -\frac{1}{(\eta^M)^2} \eta_m^M > 0 \quad \text{with } \eta_m^M \text{ as in (A.66)} \quad (\text{A.76})$$

$$\rho_m^T = \frac{1}{2\gamma H} > 0. \quad (\text{A.77})$$

While the denominator of C is always positive (a larger firm size m decreases the markup needed for zero profits g^M and increases both the price markup and the wage markup), the sign of the numerator depends on whether the effect of λ on g^M (which is positive) is stronger than the effect on the wage markup (which is also positive). In either case, prices fall as migration costs fall.

The effect on average income is ambiguous. While the partial effect of lower migration costs is positive, there is a countervailing effect when the general equilibrium adjustments

³⁹Note that for notational convenience here and in the following we omit the functional dependence of $g^M, \psi^M, \rho^M, \theta^M, d^n$ on m and, where relevant, on λ .

lead to firm exit. In either case, however, real income increases when migration costs fall, as the decrease in prices overcompensates the potential decrease in average income. We show this by log-differentiating real income $\frac{\theta^M}{p} = \frac{L\theta^M - \frac{\alpha H}{m}}{L}$ using (42):

$$d \ln \left[\frac{\theta^M}{p} \right] = \frac{\partial \ln \left[\frac{\theta^M}{p} \right]}{\partial \lambda} \hat{\lambda} + \frac{\partial \ln \left[\frac{\theta^M}{p} \right]}{\partial m} \hat{m} \quad (\text{A.78})$$

with

$$\frac{\partial \ln \left[\frac{\theta^M}{p} \right]}{\partial \lambda} = \frac{L\theta_\lambda^M}{L\theta^M - \frac{\alpha H}{m}} < 0 \quad \text{and} \quad \frac{\partial \ln \left[\frac{\theta^M}{p} \right]}{\partial m} = \frac{L\theta_m^M + \frac{\alpha H}{m^2}}{L\theta^M - \frac{\alpha H}{m}} > 0. \quad (\text{A.79})$$

In these equations $\theta_\lambda^M = -\frac{1}{m} \int_0^{d^m} f[d] dd < 0$ and $\theta_m^M = \frac{1}{m} (f[d^m] - \theta^M) < 0$. It follows from (A.69) that $\frac{\partial \ln[\theta^M/p]}{\partial m} > 0$ in the relevant range. Hence, the log-change in real income induced by a decrease in λ is clearly positive, if \hat{m} is also positive. To show that real income also increases if \hat{m} is negative, we use (A.72) and (A.79) to rewrite (A.78) as

$$d \ln \left[\frac{\theta^M}{p} \right] = \frac{\lambda}{(L\theta^M - \alpha N)(-g_m^M + \rho\psi_m^M + \psi^M \rho_m)} \times \left[\left(L\theta_m^M + \frac{\alpha N}{m} \right) (g_\lambda^M - \rho\psi_\lambda^M) + (-g_m^M + \rho\psi_m^M + \psi^M \rho_m) L\theta_\lambda^M \right] \hat{\lambda}. \quad (\text{A.80})$$

We know that the first fraction on the right-hand side above is positive, hence we must show that the square-bracketed term is negative. Using

$$\left(L\theta_m^M + \frac{\alpha N}{m} \right) g_\lambda^M = \left[\frac{L\theta_m^M + \frac{\alpha N}{m}}{L\theta^M - \alpha N} - \frac{L\theta^M (L\theta_m^M + \frac{\alpha N}{m})}{(L\theta^M - \alpha N)^2} \right] L\theta_\lambda^M \quad (\text{A.81})$$

and

$$L\theta_\lambda^M g_m^M = \left[\frac{L\theta_m^M}{L\theta^M - \alpha N} - \frac{L\theta^M (L\theta_m^M + \frac{\alpha N}{m})}{(L\theta^M - \alpha N)^2} \right] L\theta_\lambda^M, \quad (\text{A.82})$$

we can reduce the expression in squared brackets on the right-hand side of (A.80) to

$$L\theta_\lambda^M \left(\frac{\frac{\alpha N}{m}}{L\theta^M - \alpha N} + \psi_m^M \rho + \rho_m \psi^M \right) - \left(L\theta_m^M + \frac{\alpha N}{m} \right) \rho \psi_\lambda^M. \quad (\text{A.83})$$

This is negative since $\theta_\lambda^M < 0$ and $\psi_\lambda^M > 0$. Hence, a decrease in λ raises real income also if it leads to exit of firms. This completes the proof.

A.7.5 Conditions for existence and uniqueness of the symmetric equilibrium with trade and migration

In this section we briefly show that with free trade and migration, qualitatively similar restrictions on the parameter space and the shape of $f[\cdot]$ ensure existence and uniqueness of the symmetric equilibrium. In analogy to Section A.3, we show that qualitatively similar conditions are needed for quasiconcavity of the profit function and for the existence of an interior solution. Log diagonal dominance is shown to hold at the equilibrium point, guaranteeing uniqueness of the second-stage equilibrium. Then, we describe conditions under which the symmetric alternating pattern is the only pattern consistent with free entry when migration cost are non-prohibitive.

Quasiconcavity of profits. Firm i 's labor supply function with integrated labor markets in the general case (asymmetric location pattern and domestic or foreign identity of neighbors) is given by

$$L_i^M = \sum_{c=\ell,r} (L_{i,c}^n + L_{i,c}^m) \quad \text{with} \quad (\text{A.84})$$

$$L_{i,c}^n = \begin{cases} \frac{L}{H} F[d_{i,c}^n] & \text{if } d_{i,c}^n > -d_{i,c'}^n \\ 0 & \text{otherwise} \end{cases} \quad \text{and} \quad L_{i,c}^m = \begin{cases} \frac{L}{H} (1 - \lambda) F[d_{i,c}^m] & \text{if } d_{i,c}^m > -d_{i,c'}^m \\ 0 & \text{otherwise} \end{cases}$$

for $c, c' = \ell, r, c \neq c'$. We now denote with c, c' the *relevant* competitor of firm i on either side.⁴⁰ Note that with integrated labor markets and a positive migration cost, firm i 's relevant competitor for natives on a given side may be different from firm i 's relevant competitor for migrant labor *on that same side*.⁴¹

The cutoff for native workers on side $c = \ell, r$, $d_{i,c}^n$, is determined by $w_i f[d_{i,c}^n] = w_c f[m_c^n - d_{i,c}^n]$ if c is a domestic firm, and by $w_i f[d_{i,c}^m] = w_c^* f[m_c^n - d_{i,c}^m](1 - \lambda)$ if c is a foreign firm. The cutoff for native workers, $d_{i,c}^m$, is determined by $w_i f[d_{i,c}^m] = w_c f[m_c^m - d_{i,c}^m]$ if c is a domestic firm and by $w_i f[d_{i,c}^m](1 - \lambda) = w_c^* f[m_c^m - d_{i,c}^m]$ if c is a foreign firm. We denote with m_c^n, m_c^m the distance to the respective relevant competitor. The slope of a firm's supply of native labor when competing with a firm in the other country is

$$\frac{\partial L_{i,c}^n}{\partial w_i} = \frac{L}{H} f[d_{i,c}^n] \frac{\partial d_{i,c}^n}{\partial w_i} = \frac{L}{H} \frac{f[d_{i,c}^n]^2}{-w_i f'[d_{i,c}^n] - w_c^* (1 - \lambda) f'[m_c^n - d_{i,c}^n]}, \quad (\text{A.85})$$

⁴⁰ The relevant competitor can be identified in similar way as explained in footnote ¹⁴.

⁴¹ With asymmetric locations and positive λ , it is conceivable that the competitor for natives, firm $i + 1$, is overbid by a foreign firm $i + 2$ with regard to migrants but not natives.

and when competing with a firm from the same country it is

$$\frac{\partial L_{i,c}^n}{\partial w_i} = \frac{L}{H} f[d_{i,c}^n] \frac{\partial d_{i,c}^n}{\partial w_i} = \frac{L}{H} \frac{f[d_{i,c}^n]^2}{-w_i f'[d_{i,c}^n] - w_c f'[m_{i,c}^n - d_{i,c}^n]}. \quad (\text{A.86})$$

Analogously, the slope of the supply of migrant labor when the competitor is foreign is

$$\frac{\partial L_{i,c}^m}{\partial w_i} = \frac{L}{H} (1 - \lambda) f[d_{i,c}^m] \frac{\partial d_{i,c}^m}{\partial w_i} = \frac{L}{H} \frac{(1 - \lambda) f[d_{i,c}^m]^2}{-w_i f'[d_{i,c}^m] - \frac{w_c^*}{(1 - \lambda)} f'[m_{i,c}^m - d_{i,c}^m]}. \quad (\text{A.87})$$

When the competitor is in the same country, it is

$$\frac{\partial L_{i,c}^m}{\partial w_i} = \frac{L}{H} (1 - \lambda) f[d_{i,c}^m] \frac{\partial d_{i,c}^m}{\partial w_i} = \frac{L}{H} \frac{(1 - \lambda) f[d_{i,c}^m]^2}{-w_i f'[d_{i,c}^m] - w_c f'[m_{i,c}^m - d_{i,c}^m]}. \quad (\text{A.88})$$

As above, quasiconcavity of profits holds if condition (A.12) is fulfilled, and if the labor supply function becomes flatter at the kinks. Using the defining equations for the cutoffs with a foreign neighbor and equations (A.85)-(A.88), it is straightforward to show for all possible cases (the competitor who is overbid is foreign or domestic, the next competitor is foreign or domestic) that the respective labor supply schedule for natives and migrants becomes flatter at the kinks. Hence, quasiconcavity obtains under the restriction that the elasticity of marginal labor supply is not too large if positive. A similar condition on the choke price as above ensures that all solutions are interior.

Diagonal dominance at the equilibrium point. First note that

$$\frac{\partial^2 L_i}{\partial w_i^2} = \sum_{c=\ell, r} \left(\frac{\partial^2 L_{i,c}^n}{\partial w_i^2} + \frac{\partial^2 L_{i,c}^m}{\partial w_i^2} \right) \quad \text{and} \quad \frac{\partial^2 L_i}{\partial w_i \partial w_c} = \frac{\partial^2 L_{i,c}^n}{\partial w_i \partial w_c} + \frac{\partial^2 L_{i,c}^m}{\partial w_i \partial w_c}, \quad (\text{A.89})$$

where, if c is a foreign competitor,

$$\begin{aligned} \frac{\partial^2 L_{i,c}^n}{\partial w_i^2} &= \frac{L}{2H} \left(3f'[d_{i,c}^n] \left(\frac{\partial d_{i,c}^n}{\partial w_i} \right)^2 + (w_i f''[d_{i,c}^n] - w_c^* (1 - \lambda) f''[m_{i,c}^n - d_{i,c}^n]) \left(\frac{\partial d_{i,c}^n}{\partial w_i} \right)^3 \right) \\ \frac{\partial^2 L_{i,c}^m}{\partial w_i^2} &= \frac{L}{2H} \left(3(1 - \lambda) f'[d_{i,c}^m] \left(\frac{\partial d_{i,c}^m}{\partial w_i} \right)^2 + (w_i (1 - \lambda) f''[d_{i,c}^m] - w_c^* f''[m_{i,c}^m - d_{i,c}^m]) \left(\frac{\partial d_{i,c}^m}{\partial w_i} \right)^3 \right) \\ \frac{\partial^2 L_{i,c}^n}{\partial w_i \partial w_c} &= \frac{L}{2H} \left(2f'[d_{i,c}^n] \frac{\partial d_{i,c}^n}{\partial w_i} \frac{\partial d_{i,c}^n}{\partial w_c^*} + (1 - \lambda) f'[m_{i,c}^n - d_{i,c}^n] \left(\frac{\partial d_{i,c}^n}{\partial w_i} \right)^2 \right. \\ &\quad \left. + (w_i f''[d_{i,c}^n] - w_c^* (1 - \lambda) f''[m_{i,c}^n - d_{i,c}^n]) \left(\frac{\partial d_{i,c}^n}{\partial w_i} \right)^2 \frac{\partial d_{i,c}^n}{\partial w_c^*} \right) \end{aligned}$$

$$\begin{aligned}\frac{\partial^2 L_{i,c}^m}{\partial w_i \partial w_c} &= \frac{L}{2H} \left(2(1-\lambda) f'[d_{i,c}^m] \frac{\partial d_{i,c}^m}{\partial w_i} \frac{\partial d_{i,c}^n}{\partial w_c^*} + f'[m_{i,c}^m - d_{i,c}^m] \left(\frac{\partial d_{i,c}^m}{\partial w_i} \right)^2 \right. \\ &\quad \left. + (w_i(1-\lambda) f''[d_{i,c}^m] - w_c^* f''[m_{i,c}^m - d_{i,c}^m]) \left(\frac{\partial d_{i,c}^m}{\partial w_i} \right)^2 \frac{\partial d_{i,c}^m}{\partial w_c^*} \right)\end{aligned}$$

and, if c is a domestic competitor,

$$\begin{aligned}\frac{\partial^2 L_{i,c}^n}{\partial w_i^2} &= \frac{L}{2H} \left(3f'[d_{i,c}^n] \left(\frac{\partial d_{i,c}^n}{\partial w_i} \right)^2 + (w_i f''[d_{i,c}^n] - w_c f''[m_{i,c}^n - d_{i,c}^n]) \left(\frac{\partial d_{i,c}^n}{\partial w_i} \right)^3 \right) \\ \frac{\partial^2 L_{i,c}^m}{\partial w_i^2} &= \frac{L}{2H} \left(3(1-\lambda) f'[d_{i,c}^m] \left(\frac{\partial d_{i,c}^m}{\partial w_i} \right)^2 + (w_i f''[d_{i,c}^m] - w_c f''[m_{i,c}^m - d_{i,c}^m]) \left(\frac{\partial d_{i,c}^m}{\partial w_i} \right)^3 \right) \\ \frac{\partial^2 L_{i,c}^n}{\partial w_i \partial w_c} &= \frac{L}{2H} \left(2f'[d_{i,c}^n] \frac{\partial d_{i,c}^n}{\partial w_i} \frac{\partial d_{i,c}^n}{\partial w_c} + f'[m_{i,c}^n - d_{i,c}^n] \left(\frac{\partial d_{i,c}^n}{\partial w_i} \right)^2 \right. \\ &\quad \left. + (w_i f''[d_{i,c}^n] - w_c f''[m_{i,c}^n - d_{i,c}^n]) \left(\frac{\partial d_{i,c}^n}{\partial w_i} \right)^2 \frac{\partial d_{i,c}^n}{\partial w_c} \right) \\ \frac{\partial^2 L_{i,c}^m}{\partial w_i \partial w_c} &= \frac{L}{2H} \left(2(1-\lambda) f'[d_{i,c}^m] \frac{\partial d_{i,c}^m}{\partial w_i} \frac{\partial d_{i,c}^n}{\partial w_c} + f'[m_{i,c}^m - d_{i,c}^m] \left(\frac{\partial d_{i,c}^m}{\partial w_i} \right)^2 \right. \\ &\quad \left. + (w_i f''[d_{i,c}^m] - w_c^* f''[m_{i,c}^m - d_{i,c}^m]) \left(\frac{\partial d_{i,c}^m}{\partial w_i} \right)^2 \frac{\partial d_{i,c}^m}{\partial w_c} \right).\end{aligned}$$

It now follows that (A.19) also holds for the case of migration. Moreover, we see that log diagonal dominance in accordance with (A.15) holds here as well. Hence, the second-stage wage equilibrium with migration is unique.

Existence and uniqueness of the symmetric alternating location equilibrium. A condition on the fixed cost relative to the size of the labor force similar to (A.21) can be derived that ensures the existence of a symmetric second-stage equilibrium with symmetric distance pattern and zero profits. Moreover, it holds that $G^M[m] := \rho^T[m, \lambda] \psi^T[m] - g^M[m, \lambda]$ is monotonically increasing in m , hence the symmetric zero-profit solution is unique and second-stage profits for symmetric distance vectors are decreasing in N . To show that under the same assumption on consistency of beliefs as described in Section 3.2 the symmetric alternating distance pattern is the unique equilibrium as defined in (14), we again need a restriction on the magnitude of the change in the wage markup relative to the change in $g^M[m, \lambda]$. Analogously to the proof in Section A.3, a small enough level of β always assures that this condition holds. By the same logic as outlined in Section A.3,

uniqueness of the symmetric equilibrium can be proven by showing that moving from the symmetric alternating equilibrium to any asymmetric pattern with the same or a larger number of firms implies negative profits for at least one firm.

Note that in addition to the alternating pattern another fully symmetric location structure is conceivable, namely, one where each firm has one domestic neighbor and one foreign neighbor. However, the alternating pattern is the one that maximizes labor supply per firm. With equal wages and one relevant domestic neighbor at a distance $m_{i,c}$, labor supply from the side where the domestic neighbor is located is given by

$$L_i^{M,D} = \frac{L}{2H} (2 - \lambda) \int_0^{m_{i,c}/2} f[d] dd \quad (\text{A.90})$$

for $\lambda \in [0, \bar{\lambda}]$. If, instead, the competitor at distance $m_{i,c}$ is foreign, then the labor supply is

$$L^{M,F} = \frac{L}{2H} \int_0^{d_{i,c}^n} f[d] dd + \frac{L}{2H} (1 - \lambda) \int_0^{d_{i,c}^m} f[d] dd, \quad (\text{A.91})$$

where $d_{i,c}^m \leq \frac{m_{i,c}}{2}$, $\frac{m_{i,c}}{2} \leq d_{i,c}^n < m$ and $d_{i,c}^m + d_{i,c}^n = m_{i,c}$. For notational convenience, we henceforth set $L/(2H) = 1$. Then, the difference in supply of efficiency units for a given wage results as

$$L^{M,F} - L^{M,D} = \int_{m_{i,c}/2}^{d_{i,c}^n} f[d] dd - (1 - \lambda) \int_{d_{i,c}^m}^{m_{i,c}/2} f[d] dd. \quad (\text{A.92})$$

Using the fact that with symmetric wages $\int_{d_{i,c}^m}^{m_{i,c}/2} f[d] dd = \int_{m_{i,c}/2}^{d_{i,c}^m} f[m_{i,c} - d] dd$, this can be rewritten as

$$L^{M,F} - L^{M,D} = \int_{m/2}^{d_{i,c}^n} (f[d] - (1 - \lambda)f[m_{i,c} - d]) dd \geq 0. \quad (\text{A.93})$$

The inequality follows from $f[d] - (1 - \lambda)f[m_{i,c} - d] \geq 0 \forall \frac{m_{i,c}}{2} \leq d_{i,c}^n < m_{i,c}$. Hence, in the symmetric equilibrium the labor supply for a given wage is (weakly) larger if the neighbor is foreign. If $\lambda = 0$, labor supply is identical in both cases. Hence, by the same logic that rules out asymmetric distance patterns with a number of firms larger or equal to the number of firms in the symmetric alternating zero-profit solution, non-alternating symmetric distance patterns cannot constitute an equilibrium as defined in (14), unless migration cost are zero. In the last case, the symmetric alternating and non-alternating equilibrium are indistinguishable.

A.7.6 Robustness with respect to the specification of migration costs

The proof of Proposition 3 reveals that our results are valid for more general specifications of migration costs. The positive welfare effect of the potential of migration established in Proposition 3 part (i) stems from a first-order welfare gain due the reduction of the markup distortion. Hence, the validity of Proposition 3 part (i) is maintained, provided that the excess-entry property of the autarky equilibrium is preserved. The proof of Proposition 3 part (ii) shows that positive welfare gains from lower migration costs occur, provided that $\theta_\lambda^M < 0$ and $\eta_\lambda^M < 0$, and that the excess-entry result holds. It is relatively straightforward that this holds for a wide range of migration cost specifications.