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Credit Channel and Business Cycle: The Role of Tax Evasion

Bruno Chiarini, Maria Ferrara, Elisabetta Marzano
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Abstract

This paper examines the role of tax evasion in explaining the business cycle in a DSGE model with a financial accelerator. For this purpose, we assume that financially constrained agents are tax evaders, taking advantage of an additional margin of flexibility in coping with adverse shocks. In this setting, we simulate a risk shock that propagates its effects in the credit channel via the financial accelerator mechanism. The results show that tax evasion is pro-cyclical and strengthens the effects of the financial accelerator. Unlike the standard literature, in which tax evasion cushions business cycle fluctuations, here we find that it amplifies macroeconomic fluctuations considerably.

JEL-Codes: E320, E440, H260.

Keywords: tax evasion, financial accelerator, business cycle, risk shocks, DSGE modeling.

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1. Introduction

Shadow economic activities are a common phenomenon\(^1\). In this regard, several contributions in the Dynamic Stochastic General Equilibrium (DSGE) literature have shown that underground production is an important channel for business cycle dynamics (see Conesa Roca et al., 2001; Busato and Chiarini, 2004; Orsi et al., 2012; Annichiarico and Cesaroni, 2016). Moreover, following the recent financial crisis, several studies have emphasized the crucial role of financial factors as well as the prime determinants of business fluctuations (Christiano et al., 2011; Christiano, Motto, Rostagno, 2014 (CMR (2014) hereafter).

In light of that, this paper employs a financial frictions framework to investigate the role of tax evasion, denoting an additional financial source to the credit-constrained agent, namely, the entrepreneur, who is the ultimate provider of capital services. In addition to denoting a tax-free funding channel, tax evasion provides the entrepreneur with additional flexibility to cope with adverse shocks in the business cycle, allocating resources between underground and market production. In this context, where underground economy and financial constraints interplay, our investigation focuses on the impact of tax evasion over the business cycle.

For our purpose, an appropriate reference model is that presented in the seminal paper by CMR (2014), showing that, in addition to financial frictions, risk shocks are important drivers of economic fluctuations. Based on this, we simulate a risk shock for the US economy, taking into account tax evasion aspects. Essentially, we are dealing with a DSGE model including tax evasion and a financial accelerator à la Bernanke, Gertler, and Gilchrist (1999), (BGG (1999), hereafter). This model consists of a “pure” accelerator mechanism, which tends to amplify the cyclical fluctuations of any shock that produces a pro-cyclical impact on economic activity (Christiano et al., 2010). This is actually the case of the risk shock.

This paper features several elements of novelty to contribute to the existing literature. First, as far as we know, no paper has investigated the tax evasion phenomenon in the financial accelerator framework. Second, standard shadow economy literature in general equilibrium settings models tax evasion at the final firm level. However, in our paper, the underground sector affects the intermediate level of the production process. In fact, the entrepreneur, who rents out capital services to intermediate firms, is both the evading agent and the key player of the financial accelerator via her accumulation of risky loans from banks. A third and important contribution is related to the transfer of the entrepreneurs’ assets to households. In CMR (2014), these assets are assumed to be

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\(^1\) In what follows, we treat tax evasion as synonymous with the shadow/underground economy, following the definition of Buehn and Schneider (2012). See also Alm and Embaye, 2013; Schneider and Enste, 2013; Goel and Saunoris, 2014; Hassan and Schneider, 2016.
exogenous; on the contrary, we endogenize these objects assuming that they are directly affected by the cyclical component of the underground economy. The idea is that, by means of tax evasion, entrepreneurs can alter the wealth transfer towards households. This implies a reallocation of assets between consumption and productive uses over the business cycle. Indeed, several papers emphasize the non-negligible distributional impact of tax evasion. However, as far as we know, no study has examined the role of tax evasion in shaping the allocation of resources between consumption and production.

Our main result shows that tax evasion, operating in a financial frictions framework as a “flexibility mechanism” rather than in a “collateral fashion”—which weakens the agent’s ability to borrow—is pro-cyclical. This is a novel result that is at odds with the standard models, in which tax evasion cushions business cycle fluctuations (insurance effect). On the contrary, we show that tax evasion reinforces the financial accelerator’s effects and considerably amplifies macroeconomic fluctuations. Hence, overall, the tax evasion mechanism we model is far from being neutral with respect to the BGG (1999) framework.

The rest of the paper is organized as follows. Section 2 describes the CMR model, emphasizing our new insights regarding tax evasion. Section 3 discusses the calibration of the parameters. Sections 4 and 5 provide our results and some sensitivity analysis, respectively. Finally, Section 6 concludes the paper.

2. Tax evasion in the financial accelerator scheme

This section explains how our insights relative to tax evasion are engaged in the CMR (2014) model. Figure 1 is helpful for our purpose.

The economy is populated by households, intermediate and final firms, a representative retail bank, entrepreneurs, as well as fiscal and monetary policy authorities. Households consume, supply labor services in monopolistic competition to intermediate firms (earning labor income), and deposit savings to banks (receiving interest payments). Importantly, households combine investment goods purchased by final firms and existing undepreciated capital to obtain raw capital, which is then supplied to entrepreneurs.

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2 See Alm et al. 2005; Alm and Finlay, 2012; Barrios et al., 2017; Adam and Kannas, 2011; Doerrenberg and Ducan, 2014.
3 See Dabla Norris and Feltenstein (2005)
4 For further details about the CMR model see CMR (2014).
As shown in Figure 1, monopolistic intermediate firms use labor and capital services rented from entrepreneurs to produce a continuum of differentiated goods. Perfectly competitive final firms buy the intermediate goods and produce the final output, which is converted into consumption goods, investment goods, and public goods. The bank uses household’s deposits to generate liquidity services, which are then provided to entrepreneurs in the form of bank loans.

Entrepreneurs purchase capital from households using both their own net worth and bank loans. Then, they experience an idiosyncratic shock, namely, the risk shock. Next, they optimally choose the capacity utilization of capital, concealing a fraction of it to enjoy greater flexibility. This provides flexibility in terms of costs, in the sense that the underground sector is the cheapest one, and flexibility in terms of wealth, as entrepreneurs can more easily shift their wealth between production and consumption. In making their optimal choice, entrepreneurs consider the risk of tax evasion, maximizing their expected profits in the absence of detection. Then, they rent capital services to intermediate firms and re-sell undepreciated capital to households. Following BGG (1999), entrepreneurs observe the idiosyncratic shock with no charge, while the bank incurs a monitoring cost to do so. To deal with this agency problem, entrepreneurs and banks sign a debt contract. Accordingly, the entrepreneurs commit to pay back the principal plus a non-default interest rate, unless they declare default. In the event of default, the bank conducts a costly verification of the entrepreneurs’ assets, withholding these latter as partial compensation.

Public spending represents a fraction of total output and is financed by lump-sum taxes levied to households. Moreover, it is assumed that government budget constraint is always balanced. The monetary policy authority follows a standard Taylor rule to set the nominal interest rate.

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Fig. 1: The CMR model with tax evasion

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5 Entrepreneurs do not acquire capital from their own households (CMR, 2014).
2.1 The key role of the entrepreneur

Entrepreneurs purchase raw capital $\bar{K}_t$ from households and transform it into effective capital $\omega_t \bar{K}_t$, where $\omega_t$ is the risk shock to which the entrepreneurial investment is subjected. In other words, the realization of the stochastic variable $\omega_t$ determines capital productivity. In this sense, on impact, the risk shock is tantamount to a change in the stochastic technology parameter, although it definitely displays its effects in the credit relationship through the debt contract (see CMR, 2014). To cover capital costs, entrepreneurs combine their net worth $n_t$ with bank loans $B_t$, which by definition are risky, thus generating the financial accelerator mechanism. Moreover, we assume that entrepreneurs shadow part of their activities and evade taxes thereon, receiving extra profits if there is no detection. Therefore, in order to purchase capital, they have three financing sources: their own net worth, bank loans, and the pocketed (concealed) taxation.

At the end of each period $t$, entrepreneurs purchase new capital and face the risk shock. At time $t + 1$, after observing the risk shock, they optimally choose the level of capital utilization and rent out services to the intermediate firm, receiving the rental rate of capital $r_{t+1}^k$. Since entrepreneurial profits are subject to taxation with a marginal tax rate $\tau^k$, entrepreneurs can decide to declare only part of the profits made. As further described in section 3.1, we distinguish between market profits, originating from market capacity utilization, and unreported profits, originating from underground capacity utilization.

We assume that this decomposition matters only for entrepreneurs, as our investigation focuses on the interaction between tax evasion and financial accelerator mechanisms. From the bank perspective, the underground sector does not reduce the entrepreneurial ability to borrow since the bank cares only about the possible default of the entrepreneur. From the household’s point of view, to the extent that the household is interested in the accumulation of the entrepreneurial net worth to carry on the next period, the underground sector does not disturb household behavior. Finally, for the utilization of capital services, the intermediate firm pays the price $r_{t+1}^k$, regardless of the component.

We now detail in a more formalized way how tax evasion is modeled.

2.1.1 Profits and tax evasion

The entrepreneur produces capital services using effective capital $\omega_{t+1} \bar{K}_{t+1}$ and capacity utilization $u_{t+1}$, choosing how to allocate this latter between market $u_{t+1}^m$ and underground $u_{t+1}^u$ production.\footnote{According to US data, we assume that in steady state $u^m = 0.92$ and $u^u = 0.08$. $u = u^m + u^u = 1$} This scheme defines the case of “moonlighting production,” where one part of the productive unit...
operates “above ground” while the other operates “underground.” Moreover, entrepreneurs are assumed to face a convex user cost function:

$$P_{t+1} \Gamma^{-(t+1)} a(u_{t+1}^i)K_{t+1} \quad i=u,m,$$  \hspace{1cm} (1)$$

where $\Gamma$ is the investment growth parameter, and the function $a(u_{t+1}^i)$ is the mechanism driving the convex costs. While we assume that the functional form for the costs of production is the same for both the market and underground components, we tailor the underground production as experiencing a lower degree of convexity to simulate a realistic situation in which the underground production provides a margin of cost flexibility to the entrepreneur. Thus, as in Verona (2012), we adopt the following specification for the $a(u_{t+1}^i)$ function, for the market and underground components, respectively, as follows:

$$a(u_t^m) = \frac{r^k}{\sigma_m^a} \{\exp[\sigma_m^a(u_t^m - u^m)] - 1\}$$  \hspace{1cm} (2)$$

$$a(u_t^u) = \frac{r^k}{\sigma_u^a} \{\exp[\sigma_u^a(u_t^u - u^u)] - 1\},$$  \hspace{1cm} (3)$$

where $r^k$ is the steady state value of the rental rate of capital.\footnote{We assume that $a(u^m) = 0, a'(u^m) = r^k, a''(u^m) = r^k \sigma_m^a > 0; a(u^u) = 0, a'(u^u) = r^k, a''(u^u) = r^k \sigma_u^a > 0.$}

As is clear from equations (2) and (3), for the given functional form of the cost function, the parameter governing the degree of convexity is $\sigma_a^i$; the larger $\sigma_a^i$ is, the more convex the costs of production are.\footnote{It can be easily shown that in either equations (2) or (3) we get: $\frac{a''(u_{t+1}^i)}{a'(u_{t+1}^i)} = \sigma_a^i.$} Here, we introduce flexibility for the shadow activities of entrepreneurs, assuming that $\sigma_m^a > \sigma_u^a$ (flexibility in terms of costs). We are attributing a large technology advantage to the tax-evading agent, which is able to split between market and underground utilization adjustment costs. In fact, we are assuming the following:

$$a(u_{t+1}) = u_{t+1}^m a(u_{t+1}^m) + u_{t+1}^u a(u_{t+1}^u),$$  \hspace{1cm} (4)$$

where the adjustment cost is the weighted sum of the two components and the weight is the instantaneous capital utilization rate of the two sectors. This assumption, although not neutral, is
necessary to allow the market and the underground contribution to profits to be disentangled, as will be clarified below. Moreover, it is consistent with the hypothesis of a separable production function. That said, tax evasion is a risky activity, whereas the engagement in market production necessarily involves tax liabilities. Figure 2 shows that for a given triplet of tax-enforcement parameters \((\rho, \tau^k, s)\), i.e., a proportional profits tax rate \(\tau^k\), a fixed and exogenous probability of being detected and fined \(\rho\), and a fixed surcharge \(s>1\) for income (discovered by the authorities),\(^9\) we read the entrepreneurs’ profits \(\Pi\) as a function of market \((\Pi^m)\) and underground \((\Pi^u)\) profits.

![Fig. 2: Entrepreneurial profits with tax evasion](image)

\[E_t(\Pi_{t+1}) \equiv \Pi^E = E_t(\rho)\Pi^D + E_t(1 - \rho)\Pi^{ND} = (1 - \tau^k)\Pi^m + (1 - \rho \tau^k)\Pi^u.\]  
(5)

To obtain the maximization of expected profits in terms of \(u^m_{t+1}\) and \(u^u_{t+1}\), equation (5) can be written as follows:

\[
\max_{u^m_{t+1}, u^u_{t+1}} (1 - \tau^k)[u^m_{t+1}\tau^k_{t+1} - a(u^m_{t+1})]\tau^{-(t+1)}\omega \bar{K}_{t+1} + (1 - \tau^k \rho s)[u^u_{t+1}\tau^k_{t+1} + \\
-a(u^u_{t+1})]\tau^{-(t+1)}\omega \bar{K}_{t+1},
\]  
(6)

where the first square brackets refer to market profits and the second to underground profits. In equation (6), we assume that \(\rho s < 1\) to ensure the existence of both productions. In other words, in order to avoid corner solutions, it is a necessary and sufficient condition that the expected penalty rate is lower than the regular tax rate. This condition is enough to guarantee that the expected return to a unit of evaded production is positive (Chiarini et al., 2011).

\(^9\) According to US tax laws, a penalty is imposed on the evaded tax (Sandmo, 2005).
Therefore, in their maximization problem (6), entrepreneurs must take into account the probability of being discovered and, in this case, the penalty associated with the pocketed profits. The first order conditions to the problem are as follows:

\[ r^k_{t+1} = a' (u^m_{t+1}) \]  
\[ r^k_{t+1} = a' (u^n_{t+1}), \]  
\[ (7) \]
\[ (8) \]

from which it is clear that the entrepreneurial optimum is independent of the probability of being detected and the penalty function (Sandmo, 2005).

The entrepreneurial average nominal gross rate of return on capital is:

\[ R^k_{t+1} = \left[ \frac{E_t \Pi_{t+1} (1-\delta) q_{t+1}}{T_{dt}} \right] \pi_t + \tau^k \delta - 1 \]  
\[ \text{(9)} \]

where expected profits on capital services, \( E_t \Pi_{t+1} \), include the additional profit component that is in place if tax evasion works, or equivalently, if \( \rho s < 1 \) and \( \pi_t \) represents the inflation rate.

Moreover, after choosing (market and underground) capital utilization, as shown in Figure 1, entrepreneurs re-sell effective capital, net of depreciation \( \delta \), to households at price \( q_{t+1} \), which implicitly defines a source of potential capital gains for the entrepreneur. Finally, \( \tau^k \) denotes the tax rate on capital income, and it is assumed that depreciated capital can be deducted at the historical cost (CMR, 2014).

### 2.1.2 The debt contract

As mentioned above, the bank and the entrepreneur sign a debt contract that determines a loan amount \( B_{t+1} \) and a gross interest rate \( Z_{t+1} \) to be paid if \( \omega \) is high enough. In fact, there is a cutoff level \( \omega_{t+1} \), under which the entrepreneur cannot make the required payment and all his output goes to the bank. The cutoff level expression reads as follows:

\[ \omega_{t+1} (1 + R^k_{t+1}) q_{t+1} K_{t+1} = Z_{t+1} B_{t+1} \]  
\[ \text{(10)} \]

As equation 10 highlights, tax evasion does not directly affect the cutoff level of default. However, an indirect effect works via \( R^k_{t+1} \), as tax evasion impacts profits but not the probability of default.

### 2.1.3 Net worth dynamics

As we have just seen, profits from capital services production and capital gains both contribute to determine the return of capital to the entrepreneur. However, in a context characterized by the resort
to external finance, entrepreneurs have to settle their debt before their net worth is determined at the end of each period. The averaged net worth across entrepreneurs, $n_{t+1}$, evolves according to the following dynamics:

$$n_{t+1} = \gamma_{t} \left[ (1 + R_{t}^{k})q_{t-1}K_{t} - \left[ 1 + R_{t}^{e} + \mu \int_{\omega}^{\omega_{0}} \omega dF_{t}(\omega)(1+R_{t}^{k})q_{t-1}K_{t} \right] (q_{t-1}K_{t} - n_{t}) \right] + w^{e}$$  \hspace{1cm} (11)

The first term in the braces in equation (11) denotes the total revenues entrepreneurs receive from selling capital. The term in square brackets represents the average payments by entrepreneurs to banks\(^{10}\). The variable $\gamma_{t}$ denotes a random fraction of assets in the hands of entrepreneurs, while the relative counterpart $(1 - \gamma_{t})$ is transferred to households; $w^{e}$ are lump-sum transfers to entrepreneurs, and $R_{t}^{e}$ is the risk-free rate.

In the financial accelerator framework, net worth plays a crucial role since, in the presence of asymmetric information and agency problems in financial contracts, banks charge a risk premium. According to the BGG (1999) financial accelerator, the contractual loan rate depends upon the net worth that entrepreneurs can pledge to guarantee the loan. Therefore, the net worth denotes the driving force of the financial accelerator mechanism. As can be seen in equation 11, an important source of net worth is the $\gamma_{t}$ asset of entrepreneurs, which indicates a transfer of funds back and forth between households and their entrepreneurs. In CMR (2014), $\gamma_{t}$ is exogenous and not responsive to economic conditions. We innovate the analysis of the $\gamma_{t}$ variable, assuming that it depends on the cyclical component of the underground sector.

### 2.1.4 Asset transfer and tax evasion

According to equation (11), whenever $\gamma_{t}$ declines, there are two interlinked effects: a) the net worth available in the economy in the next period, $n_{t+1}$, shrinks; b) the fraction of net worth that is held back by entrepreneurs is reduced, which entails an increase of assets $(1 - \gamma_{t})$ in the hands of households. We can refer to the first effect as an extensive margin wealth effect, and it can be measured by the variation in net worth, whereas the latter is an intensive margin wealth effect, which is measured by the pure variation in $\gamma_{t}$.

The rationale for this innovation is twofold. First, with regard to the extensive margin wealth effect, tax evasion is usually a device for concealing wealth. Therefore, when tax evasion declines (increases), entrepreneurs/tax evaders experience a lower (larger) wealth accumulation. As to the intensive margin wealth effect, the $\gamma_{t}$ variable, that is, the share of wealth allocated to capital services production, has a reward given by the profits as well as capital gains (see equation

\(^{10}\) See Christiano, Motto, Rostagno (2010) for further details.
9). When underground production shrinks, it is signal that profitability is diminishing; therefore, it is optimal to reduce the production of capital services, thereby reducing $\gamma_t$.

Then, both the extensive and intensive margin effects would result in a positive relationship between the cyclical fluctuation of tax evasion and the entrepreneurial wealth, $\gamma_t$.

In light of this, we endogenize the $\gamma_t$ variable, assuming that it depends on the underground component as well as on debt leverage. It features the following AR(1) process:

$$\gamma_t = \gamma(1 + \varepsilon_t^Y)exp[\eta_t(u_t^U - u^U)] + \rho^Y (\gamma_{t-1} - \gamma).$$

Equation 12 can be interpreted as an allocation rule, stating how much of the entrepreneur’s wealth is allocated to productive uses, $\gamma_t$, and the quota that is instead devoted to consumption $(1 - \gamma_t)$.

The cyclical deviation of the underground capital utilization from its steady state affects entrepreneurial wealth via the elasticity parameter $\eta_t$ whose expression is given as follows:

$$\eta_t = \frac{B_{t-1}}{K_{t-1}}.\quad (13)$$

We anchor $\eta_t$ to the lagged value of the leverage, meaning that the larger the debt exposure of the entrepreneurs, the larger the intensive margin wealth effect. In other words, entrepreneurs shift assets away from productive use, and as they increasingly do this, their leverage increases (and their exposure to default increases accordingly).

In this way, we are introducing a relevant innovation in the analyses of tax evasion in a general equilibrium framework, that is, the linkage among asset allocation between two alternative uses, leverage, and tax evasion. In other words, as the debt exposure of the entrepreneurs increases, their sensitivity to change asset allocation in responding to exogenous shocks also increases. This is a novel interpretation of the role of tax evasion, suggesting that entrepreneurs’ wealth co-moves with underground production. Once entrepreneurs decide to engage in tax evasion, profit opportunities originating from underground production are a signal to invest/disinvest.

The deviation of the lagged value of $\gamma_t$ from its steady state is amplified by the persistence parameter $\rho^Y$, although, in our model, similarly to CMR (2014), we set it to zero; finally, $\varepsilon_t^Y$ is a stochastic exogenous shock.

Such an innovation in the $\gamma_t$ function also implies that, whenever tax evasion fluctuates over the business cycle, two opposite mechanisms work on households’ consumption. In fact, while households reduce their consumption when the net worth carried on to the next period in the
economy is reduced by the extensive margin wealth effect, they consume more via the intensive margin wealth effect. As we will see in the following analysis, the extensive margin is largely dominant in shaping the response of consumption over the business cycle.

2.2 Firms
Final output $y_t$ is produced by a perfect competitive firm according to the following technology:

$$y_t = \left[ \int_0^1 y_{j,t}^{\lambda_{f,t}} dj \right]^{1/\lambda_{f,t}}, \ 1 \leq \lambda_{f,t} < \infty.$$  

The intermediate goods firm monopolistically produces intermediate time-$t$ input intermediate good $j$, $y_{jt}$, using the following technology:

$$y_{jt} = \epsilon_t (u_{jt}R_{jt})^\alpha (z_{t}l_{jt})^{(1-\alpha)} - \Phi z_t^*,$$  \hspace{1cm} (14)

where $\Phi z_t^*$ denotes fixed costs in production. Equation 14 holds if $(u_{jt}R_{jt})^\alpha (z_{t}l_{jt})^{(1-\alpha)} > \Phi z_t^*$; otherwise, $y_{jt} = 0$. The aggregate services of homogeneous labor are represented by $l_t$. $\epsilon_t$ is a stationary shock to technology, and $z_t^*$ denotes the persistent component of technology, having the following representation: $z_t = \mu z_{t-1}$, where $\mu z_t$ is a stationary stochastic process.

The intermediate good $y_{jt}$ is monopolistically supplied according to the Calvo (1983) sticky price mechanism. Hence, in each period, a fraction $(1 - \xi_p)$ of firms is able to update prices while a fraction $\xi_p$ is not able to adjust and sets the price according to $P_{jt} = \tilde{\pi}_t P_{jt-1}$, where the indexation term $\tilde{\pi}_t$ evolves following $\tilde{\pi}_t = (\pi_t^{target})^t (\pi_{t-1})^{(1-t)}$. In particular, $\pi_t^{target}$ is the inflation rate target in the monetary policy rule.

2.3 Households
There is a continuum of households, indexed by $i \in (0,1)$. Households consume, supply monopolistically labor services to intermediate firms, and allocate savings across assets. Their preferences are defined according to per capita consumption and labor, as follows:

$$E_0 = \sum_{t=0}^{\infty} \beta_t^t \zeta_t \left\{ \log(C_t - bC_{t-1}) - \psi_L \int_0^1 \frac{h_i^{(1+\sigma_L)}}{(1+\sigma_i)} di \right\}; \ b, \sigma_L > 0,$$  \hspace{1cm} (15)

where $\zeta_t$ is a preference shock and $b$ denotes the internal habit parameter. Households’ expenses are devoted to consumption, investment, and existing capital, while revenues originate from differentiated labor and bonds as well as from selling raw capital. Moreover, households own entrepreneurs. Given this assumption (CMR, 2014), each household is interested in instructing its own entrepreneur to maximize the expected net worth. As we have stressed in section 2.1.4,
households experience a wealth effect (intensive and extensive margin wealth effect) through asset transfer. Providing a preview of our results, after a negative risk shock, the wealth channel plays a crucial role in shaping the response of aggregate consumption. Following Erceg et al. (2000), CMR model the labor market as follows. The $i^{th}$ household faces the following labor demand:

$$h_i^t = \left( \frac{W_t^i}{W_t} \right)^{1-\lambda_w} l_t, 1 \leq \lambda_w,$$

where $W_t$ is the aggregate wage index and $W_t^i$ is the $i^{th}$ household wage. In each period, the $i^{th}$ household can optimally choose its wage rate, $W_t^i$, with probability $(1 - \xi_w)$. With probability $\xi_w$, the wage rate is not re-optimized and is set as follows:

$$W_t^i = \pi_w^i (\mu_{zt}^*)^{(1-\vartheta)} (\mu_{zt}^d) W_t^{-1},$$

where $0 \leq \vartheta \leq 1$ and $\pi_w^i = \left( \pi_t^{target} \right)^{l_w} (\pi_{t-1}^{target})^{(1-l_w)}, 0 < l_w < 1$.

2.4 Bank

There is a representative, competitive bank issuing entrepreneurial loans. The amount of credit supplied to entrepreneurs $B_{t+1}$, the related gross interest rate $Z_{t+1}$, and the risk-free interest rate, $R_{t+1}^e$, are chosen to maximize the entrepreneurial net worth subject to the following zero profit condition for the bank:

$$1 - F_t(\tilde{w}_{t+1})Z_{t+1}B_{t+1} + (1 - \mu) \int_{0}^{\tilde{w}_{t+1}} \omega dF_t(\omega)(1 + R_{t+1}^k)q_t K_{t+1} \geq (1 + R_{t+1}^e)B_{t+1}.$$  (16)

On the left-hand side, $1 - F_t(\tilde{w}_{t+1})$ denotes the number of non-bankrupt entrepreneurs multiplied by the interest and principal payment by each entrepreneur. The second term represents the funds received from bankrupt entrepreneurs, net of monitoring costs $\mu$. Therefore, according to equation 16, the funds received in each period $t + 1$ must be no less then the funds paid to households in the same period (see CMR, 2014). As shown in equation 16, the zero profit condition for the bank is written in terms of total capital. Hence, we assume that the decision to evade does not influence the asymmetric information problem between banks and entrepreneurs.

In particular, this assumption not only concerns the probability of default, as mentioned in regard to equation 10, but also the monitoring cost emphasized in equation 1611. Given our hypothesis of a risk neutral entrepreneur, tax evasion works as an additional source of aggregate risk in the rate of return on capital, $R_{t+1}^k$. In the presence of aggregate uncertainty, the financial contract guarantees to the lender, in case of non-default, a state-contingent payment such that the expected (lender) return

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11 We choose to leave the debt contract unchanged with respect to the original model of BGG. For this reason, we leave for future research the assumption of endogenous monitoring cost.
is equal to the risk-free rate. Therefore, from the point of view of the lender, tax evasion is not a matter of concern. In this version of the financial accelerator mechanism, there is no credit rationing issue, and there is no problem of collateral. The external finance premium is motivated by a “costly state verification,” which, according to BGG (1999), is a reasonable and simple enough explanation for the fact that “uncollateralized external finance may be more expansive than internal finance.” Hence, in contrast to some recent literature (Dabla-Norris and Feltenstein, 2005; Blackburn et al., 2012), tax evasion does not expose entrepreneurs to credit rationing originating from a lower level of collateral. Rather, it is a source of additional financial resources, allowing for increased self-financing and mitigating the external finance premium faced by the entrepreneur.

In addition, what we are implicitly assuming is that the agency costs paid to observe the borrower’s realized return on capital allow the lender to observe the true idiosyncratic productivity shock and the full production of the entrepreneur—both the market and underground production. In this sense, we assume that the technology the bank uses for monitoring is more efficient than the technology of the Internal Revenue Service.

2.5 Monetary Policy
According to CMR (2014), monetary authority follows this rule:

\[ R^e_t - R^e = \rho_p(R^e_{t-1} - R^e) + (1 - \rho_p) \left[ \alpha_\pi(\pi_{t+1} - \pi_t) + \alpha_{\Delta y} \frac{1}{4} (g_{y,t} - \mu^*_z) \right] + \frac{1}{400} \varepsilon^P_t \]  

(17)

where \( \varepsilon^P_t \) is the monetary policy shock and \( \rho_p \) is the lagged interest rate parameter; \( (\pi_{t+1} - \pi_t^*) \) is the deviation of anticipated quarterly inflation from the central banks inflation target and \( (g_{y,t} - \mu^*_z) \) is the quarterly growth in GDP.

2.6 Market clearing
Clearing the goods market of the economy, we get:

\[ Y_t = D_t + G_t + C_t + \frac{l_t}{\dd y_{\dd Y_t}} + a(u_t)Y^{-t}\dd K_t, \]

where \( D_t \) denote the aggregate resources used to monitor mutual funds and \( G_t \) is the public spending.
3 Calibration

Our model is calibrated to a quarterly frequency for the US economy. In particular, the calibration of the structural parameters follows the estimates of CMR (2014). In appendix, Table 1 reports the calibration of parameters of interest in this paper.

The empirical evidence shows that the shadow economy estimates for the US account for around 8% of total GDP (Schneider et al., 2010; Enste, 2015). Therefore, we set the steady state values of the market and the underground component of capital utilization at 92% and 8%, respectively.

We depart from CMR (2014) by considering a lower parameter $\sigma^u$, controlling for the degree of convexity of shadow capital utilization costs. The idea is to make the underground sector more flexible in the sense that it is more affordable, thus providing additional profits. Hence, while the parameter controlling for the degree of convexity of capital utilization costs in the market sector $\sigma^m$ is calibrated at 2.54 as in CMR (2014), we set the corresponding parameter in the underground sector at 1.5. Then, we carry out a robustness analysis to investigate the effects of varying the parameter’s value.

In this section, we mainly focus on the enforcement parameters of tax evasion, calibrating them with regard to both IRS estimates and the shadow economy literature.

3.1 The enforcement parameters of tax evasion: Internal Revenue Service (IRS) data

The probability of being inspected in the US varies notably based on different income levels. The IRS has emphasized that in 2014, taxpayers reporting $25,000 to $200,000 in adjusted gross income had a below-average (less than 0.9 percent) chance of getting audited. From there, audit rates rose steadily with income: from 1.75 percent for those making between $200,000 and $500,000 up to 16.22 percent for those making $10 million or more. However, people making more than $200,000 accounted for only 3.6 percent of all tax returns filed in 2013.

Aside from these statistics, the IRS does not disclose what triggers an audit. Therefore, we set the probability of being detected at 5%, following Busato and Chiarini (2004).

Concerning the penalty rate, according to the IRS, if a taxpayer fails to pay the balance due shown on the tax return by the due date (even if the reason for nonpayment is a bounced check), a penalty of 0.5% of the amount of the unpaid tax is levied, up to a maximum of 25%. Hence, in our baseline model, we set the penalty at 25%.
Risk shock and cyclical fluctuations with tax evasion

We simulate the risk shock, comparing the CMR (2014) financial accelerator model (baseline mode, cyan line) with our model augmented with tax evasion (blue line). The results are reported in Figure 3, where dynamics are expressed in percentage deviation from the steady state. The model is numerically solved in DYNARE\textsuperscript{12}.

Risk shocks trigger a credit restriction (Bernanke and Blinder, 1988) to which the entrepreneur is subjected. Starting from the baseline, following a risk shock, entrepreneurs reduce the capacity utilization of capital. In fact, as stressed in CMR (2014), entrepreneurs acquire less capital and their investments decrease. As a consequence, output and consumption fall. Net worth also decreases because the rate of return on entrepreneurs’ capital is reduced, following the slowdown in economic activity (BGG, 1999).

These effects are strongly reinforced in our model, where cyclical fluctuations are sizably amplified and tax evasion results to be pro-cyclical. In the underground economy literature, where sector-specific shocks are assumed (Orsi et al., 2014) or shocks hitting only the formal sector are investigated (Busato and Chiarini, 2004), it is claimed that tax evasion is countercyclical. Contrarily, in our model, entrepreneurs experience a reduction in capital efficiency/productivity in both the market and the underground sector, as the same capital stock is employed. As a result, they mainly downsize underground production, which is more flexible.

\begin{center}
\begin{figure}
\centering
\includegraphics[width=\textwidth]{Fig_3}
\caption{Risk shock}
\end{figure}
\end{center}

\textsuperscript{12} See the following webpage: http://www.dynare.org/ for further details on DYNARE.
This has an immediate and direct effect on the entrepreneurial asset $\gamma_t$ (see equation 11), which, unlike the baseline of CMR (2014), falls down. In our model, where entrepreneurial assets depend upon cyclical fluctuations of the underground component of capacity utilization of capital, tax evasion produces important redistributive effects over the business cycle. In particular, when the shadow economy contracts, we observe a transfer of resources from entrepreneurs to households.

The deeper reduction of net worth, with respect to the baseline, has a twofold cause. First, the reduction of $\gamma_t$ directly impacts the net worth path. This is the extensive margin wealth effect described above. Second, since the risk shock reduces the underground component of capacity utilization, entrepreneurs are subjected to a larger decrease in the rate of their return on capital.

The result is that tax evasion strongly affects the driving force of the financial accelerator mechanism, namely, the entrepreneurial net worth $n_{t+1}$. The smaller credit drop, compared to the baseline, mirrors the greater fall in net worth, suggesting that tax evasion is also an alternative financial source of both net worth and borrowing.

Finally, households’ consumption is affected by tax evasion through both the extensive and intensive margin wealth effects, although in opposite ways. In fact, the reduction of net worth $n_{t+1}$ results in less wealth to carry on to the next period. A negative wealth effect works, and thus forward-looking households reduce their consumption (extensive margin wealth channel). Conversely, due to the wealth redistribution along the business cycle, households hold additional wealth to be allocated for consumption (intensive margin wealth channel). Between the two effects, the former is the prevailing one while consumption falls down more relatively to the baseline.

5 Robustness

In this section we implement a sensitivity analysis to test the robustness of our results.

Our intuition behind the results is that tax evasion provides an important margin of flexibility to the entrepreneur, stemming partly from a different degree of cost convexity (section 3.1).

In light of this, we simulate our model with tax evasion considering lower degrees of convexity of underground utilization adjustment costs $\sigma_u^u$. Figure 4 shows endogenous variables’ dynamics after a risk shock, taking into account different degrees of convexity of the shadow capital utilization cost parameter. In particular, in addition to the baseline (solid line, $\sigma_u^u = 1.5$ ), we also consider lower utilization adjustment costs for underground production: respectively 0.9 (dotted line) and 0.5 (dashed line).

The results show that a lower cost of the underground component leads to wider fluctuations of the business cycle with respect to the baseline model. The idea is that, once
the adverse shock occurs, entrepreneurs are increasingly incentivized to manage the least expensive sector. Greater fluctuations of the underground sector translate into larger reductions of entrepreneurial assets, which, in turn, result in a deeper drop of net worth, further amplifying the financial accelerator effects.

![Fig. 4: Risk shock, different utilization adjustment cost parameters](image)

### 6 Final remarks

This paper contributes to the recent macroeconomic literature examining the contribution of tax evasion to the business cycle in a financial accelerator framework à la BGG (1999).

Our results show that tax evasion significantly strengthens the financial accelerator effects when financially constrained agents have to cope with a risk shock that propagates its effects via the credit channel.

In this setting, although both the market and the underground sectors are involved in the reduction of capital efficiency, the underground sector responds more intensively due to its greater flexibility. As a consequence, the recessionary effect of the shock is amplified. Our results have non-negligible consequences for business cycle analysis in general—and for the contribution of the underground economy to the business cycle in particular. In fact, unlike the standard tax evasion literature, we have demonstrated that the underground economy is procyclical and amplifies economic
fluctuations. Finally, we have also provided a new perspective on the role of tax evasion in shaping resource allocation between different uses, namely, consumption and production.

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References


Appendix: Parameter calibration

Table 1: Parameters Values

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<th>Parameters</th>
<th>Description</th>
<th>Value</th>
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<td>$w^e$</td>
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Steady state tax evasion values

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<th>Parameters</th>
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<td>$u^m$</td>
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<td>$u^u$</td>
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<td>$\sigma^u_\mu$</td>
<td>Underground utilization cost function</td>
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