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## ABSTRACT

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### **On the Job Search and Business Cycles**

We propose a highly tractable way of analyzing business cycles in an environment with random job search both off- and on-the-job (OJS). Ex post heterogeneity in productivity across jobs generates a job ladder. Firms Bertrand-compete for employed workers, as in the Sequential Auctions protocol of Postel-Vinay and Robin (2002). We identify three channels through which OJS amplifies and propagates aggregate shocks: (i) a higher estimated elasticity of the matching function, when recognizing that at least half of all hires are from other employers; (ii) the differential returns to hiring employed and unemployed job applicants, whose proportions naturally vary over the business cycle; (iii) within employment, the slow reallocation of workers through OJS across rungs of the job ladder, generating endogenous, slowly evolving opportunities for further poaching, which feed back on job creation incentives. Endogenous job destruction, due to either aggregate or idiosyncratic shocks, is countercyclical and thus raises the cyclical volatility of unemployment, closer to its empirical value; but it also stimulates job creation in recessions, to take advantage of the fresh batch of unemployed, and tilts the Beveridge curve up. OJS corrects this tendency and restores a vacancy-unemployment trade-off more in line with empirical observations.

**JEL Classification:** E24, E32

**Keywords:** labor reallocation, business cycles, search frictions

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# 1 Introduction

The currently predominant view of labor markets puts flows of workers and jobs center stage. Worker transitions between employment (E), unemployment (U) and non participation (N) are the key to understand the level and cyclical volatility of (un)employment. While Tobin (1972) called early attention to the importance of on the job search (OJS), for a long time employer-to-employer (EE) transitions were primarily, with very few exceptions, studied by microeconomists to understand turnover and life cycle wage dynamics. EE transitions, after all, do not change the stocks of E, U and N. But it was clear early on that the presence of employed workers in the job search arena generates congestion and thick market effects that directly impact vacancy posting, hence indirectly also UE and NE flows and unemployment. In addition, EE transitions are a natural engine of reallocation towards more productive job matches, hence a possible source of movement in aggregate productivity. Indeed, well over half of new hires each month (thus excluding recalls from unemployment) are workers who come directly from other jobs.

With few exceptions, discussed below, the “Macro-labor” literature formalizes business cycles by introducing aggregate shocks into some variant of the “DMP” (random) search-theoretic framework of Diamond (1982) and Mortensen and Pissarides (1994), where only the unemployed look for jobs and, as a consequence, employers only look forward to hiring unemployed workers. In this paper, we propose a highly tractable model of random OJS and business cycles, and we show that OJS has important qualitative and quantitative consequences for unemployment dynamics.

Our framework follows, by and large, the DMP tradition. To generate a job ladder and meaningful EE transitions in the simplest possible way, we assume ex post heterogeneity in productivity across jobs, as in Pissarides (1985), but now both employed and unemployed workers take part in the same random job search process, albeit with different intensities. Firms post vacancies under free entry. This is the environment studied by Mortensen (1994), under Nash bargaining. Our only significant departure is in wage setting: we assume that

firms commit to wage offers and Bertrand-compete for employed workers, as in the Sequential Auctions protocol of Postel-Vinay and Robin (2002). When contemplating hiring, employers anticipate that they might meet unemployed or employed job applicants. Crucially, from an employer’s vantage point, the value of meeting an employed worker differs from that of meeting an unemployed worker: while the latter essentially tracks TFP, as in the standard DMP model, the value of meeting an employed worker (i) is lower, because employed workers have better outside options than the unemployed, and (ii) depends both on TFP and on the current distribution of match productivity levels amongst employed workers, an endogenous object that varies slowly over the business cycle. The equilibrium of the model is very tractable and easy to compute.

Our main substantive contribution is to show that OJS amplifies and especially propagates the response of the UE probability to an aggregate TFP shock. While a large literature, motivated by Shimer (2005), proposed mechanisms for amplification of aggregate shocks in the DMP model, propagation remains a challenge, due to the forward-looking nature of job creation which prevents any transitional dynamics. To illustrate the issue, the stable recovery of the US civilian unemployment rate from about 10% in 2009 to less than 4% today can only be explained in that model by a sequence of consecutive, favorable, but small aggregate shocks. More plausible appears to be some friction that propagates in time the effects of larger, less frequent aggregate shocks, but the DMP model lacks any such mechanism. In our framework, we show that OJS introduces a natural source of endogenous persistence and propagation through the slow-evolving distribution of employment over the job ladder, a measure of cyclical misallocation, which is also potentially measurable.

We identify three channels through which OJS acts on the transmission mechanism of aggregate shocks. First, due to the procyclical *congestion* created by the employed on the unemployed job searchers, vacancies have to be more “important”, and workers less important, in generating meetings in order to match the observed cyclical volatility of the job finding probability from unemployment. Accordingly, accounting for OJS when estimating a

matching function yields a higher estimated elasticity with respect to vacancies, rising from .32 to .5 in US data. This effect amplifies the impact of aggregate shocks on job creation. Second, employed job searchers are more expensive to hire and less profitable than unemployed ones, and are relatively more prevalent in good economic times; this cyclical *search pool composition* dampens the response of job creation to aggregate shocks.<sup>1</sup> Finally, the *employment composition* by match productivity, which determines the returns to poaching, tracks with a lag job creation, thus dampens and propagates it.

To illustrate these dynamics, suppose a mean-reverting negative TFP shock hits the economy.<sup>2</sup> On impact, the employment distribution does not respond, while vacancy creation declines, as usual. Unemployment rises and reallocation on the job ladder slows down, while workers keep losing jobs. Gradually, the employment distribution deteriorates, making employed workers more “poachable”, which in turn stimulates job creation, dampens the adverse effects of the shock, and accelerates the recovery. As TFP rises back to steady state, reallocation up the ladder resumes, poaching opportunities fade, the recovery in job creation slows down, and the underlying shock propagates in time.

We offer our main quantitative contribution in the version of our model with endogenous separations to unemployment due to idiosyncratic and aggregate TFP shocks. Mortensen and Pissarides (1994) emphasize endogenous job destruction, while the business cycle literature that builds on it almost always assumes only exogenous separations. In a stochastic environment, endogenous separations generate countercyclical layoff rates, with high volatility and little persistence, which is not only in line with the data but also an important contributor to the volatility of unemployment. The main reason why the theory neglected job destruction is that, as pointed out by Mortensen and Nagypal (2007), a burst of en-

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<sup>1</sup>In one of our extensions, human capital acquisition during employment and loss during unemployment can make hiring employed job applicants more profitable and reverse this effect. We conjecture that adverse selection can have similar implications. Eeckhout and Lindelaub (2017) show that this reversed ranking of employed and unemployed job applicants can generate multiple equilibria and sunspot-driven fluctuations.

<sup>2</sup>The focus on TFP as the source of aggregate fluctuations is purely pedagogical and illustrative. Similar effects would result from aggregate demand shocks. As well known, in risk neutral search models, TFP can be reinterpreted as a preference for consumption over leisure.

dogenuous separations makes available for hire a fresh batch of workers, who are perfectly employable because their separations were caused by match idiosyncratic factors that do not carry over to future employment. In turn, this effect stimulates job creation, and can be so strong to overturn the fundamental adverse aggregate shock that generated separations in the first place, and to turn the correlation between the unemployment and vacancy rates positive, while it is close to  $-1$  in the data. We confirm this observation quantitatively in our framework when OJS is not present, so in this respect wage posting or bargaining makes little difference. This counterfactual implication, and Shimer's (2005, 2012) influential emphasis on the UE transition rate as the main driver of unemployment fluctuations, focused the attention on the job creation rate and away from job destruction.

We show that the presence of OJS and EE transitions in the model tends to restore the desired negative comovement between vacancies and unemployed, while preserving the role of layoffs and job destruction over business cycles. Intuitively, the inflow of fresh unemployed workers is diluted by the presence of employed job searchers, whose measure falls, partially offsetting the incentives to create jobs. Fujita and Ramey (2012) highlight this mechanism in a DMP model, where all matches, as in Mortensen and Pissarides (1994), always start from the top level of idiosyncratic productivity, and then deteriorate stochastically. As a result, workers always accept outside offers and reallocation on the job ladder is synchronized with exit from unemployment. We are able to solve the model and to establish this result in the presence of a true job ladder, where the probability of EE (for example) falls in the current wage, as is clearly the case empirically, and the reallocation process is slow and decoupled from unemployment duration.

In order to clearly illustrate how OJS improves the quantitative performance of the business cycle model of unemployment, we first study the model with only exogenous job destruction, and characterize its equilibrium, and then we introduce endogenous separations to unemployment, due to both aggregate and idiosyncratic match-specific productivity shocks. We show the quantitative properties of both versions, with an emphasis on the latter.



Finally, to showcase the tractability and quantitative potential of our model, we pursue two extensions. First, we introduce screening/training costs that the firm needs to pay when selecting a job applicant. As suggested by Pissarides (2009) and exploited by Christiano et al. (2016) in an estimated model without OJS, screening costs raise amplification of aggregate shocks, by insulating part of hiring costs from congestion. The higher volatility of job meeting rates, in turn, brings out more effectively the model’s non linearity stemming from random job search on a ladder. Second, we allow for shocks to the general human capital of the worker, which drifts up during employment and down during unemployment, reflecting learning by doing and skill loss by not doing. This process alters the relative returns to hire employed and unemployed job applicants, in favor of the former, despite their stronger outside option, and provides an additional channel of propagation.

After briefly reviewing in Section 2 the closest references, in Section 3 we describe the baseline model without endogenous job destruction and in Section 4 its equilibrium, in Section 5 we introduce endogenous job destruction, in Section 6 we present quantitative results, and in Section 7 we illustrate the two extensions to screening costs and human capital shocks.

## 2 Related literature

Menzio and Shi (2011) is the best known business cycle model with OJS. Their key assumption of directed, as opposed to our random, job search results in a very tractable Block-Recursive equilibrium, where the employment distribution is not a state variable. In our model, due to wage renegotiation following outside offers, equilibrium remains tractable, despite random job search and the resulting importance of how well current employees are matched to their jobs. Indeed, we consider the relevance of the state of employment allocation to equilibrium dynamics a strength of our analysis, because it improves the business cycle performance of the model on some dimensions, and is potentially measurable. Schaal (2017) exploits Block-Recursivity to introduce idiosyncratic TFP shocks and diminishing

returns at the firm level in the business cycle directed search model. His analysis is entirely quantitative and too complex for characterization. Our random job search framework easily allows for idiosyncratic shocks, that we in fact emphasize. We do not pursue here diminishing returns, which turns firm size into a state variable for the wage posting problem, because they make the job ladder unfold over two dimensions, firm-specific productivity and size. This is less transparent than our one-dimensional, vertical job differentiation, and harder to characterize analytically to uncover the different effects of OJS.

Three previous articles study business cycle models with random OJS. Robin (2011) adopts the same Sequential Auction model of a labor market, i.e. renegotiation, but stresses permanent worker heterogeneity. Firms are identical, thus the job ladder has only two steps: unemployed hires generate profits for firms, while an already employed job applicant extracts all rents from both incumbent and prospective employer. Therefore, employment allocation and poaching opportunities are time invariant. The full stochastic job ladder mechanism, which generates variable misallocation of employment, emerges in two subsequent contributions. Moscarini and Postel-Vinay (2013) assume wage-contract posting without renegotiation, which still allows for a tractable characterization of dynamic equilibrium, but only under additional restrictions. Lise and Robin (2017) allow for ex ante worker and firm heterogeneity and sorting within the more tractable renegotiation framework of the present paper, and is the closest comparison. Here we assume homogeneous workers and a much simpler model of the job ladder, based on ex post match quality draws rather than ex ante firm heterogeneity. This simplification allows a sharp characterization of the effects of OJS and poaching on the amplification and propagation properties of the model, which should also shed light on those predecessors. It also allows us to introduce endogenous separations, which play a critical role in our quantitative analysis, and to embed OJS for the first time in a full-fledged DSGE framework (Moscarini and Postel-Vinay, 2017), with savings, sticky prices, endogenous real interest rate, and monetary policy.

### 3 The economy

Time  $t = 0, 1, 2, \dots$  is discrete. Firms produce a single, homogeneous, non storable good, using only labor. Each unit of labor (“job match”) produces  $z_t y$  units, where  $z_t$  is common to all matches and evolves according to an ergodic Markov process  $z_t = Q(z_{t-1}, \varepsilon_t)$  with white noise innovations  $\varepsilon_t$ , while idiosyncratic productivity  $y \in [\underline{y}, \bar{y}] \subseteq \mathbb{R}_+$  is specific to each match and is set, once and for all when the match forms, equal to a random draw from a cdf  $\Gamma$  with mean  $\mu = \int_{\underline{y}}^{\bar{y}} y d\Gamma(y)$ . Workers and firms are risk neutral and discount per-period utility (quantities of the final good) with factor  $\beta \in (0, 1)$ .

A unit measure of ex-ante homogeneous workers can be employed or unemployed. An unemployed worker receives a value of leisure  $b$  per period, in units of the consumption good, and can search for new jobs with probability one. An employed worker receives a wage  $w_t$ , can separate from his job both endogenously (when the match is no longer profitable, due to aggregate shocks) and exogenously with probability  $\delta \in (0, 1]$ , and become unemployed, in which case he has to wait until next period to search. If he is not separated from this job, he also receives this period, with probability  $s \in [0, 1]$ , an opportunity to search for a new match.

Firms can advertise job vacancies by using  $\kappa$  units of the final good per vacancy, per period. Let effective job market tightness

$$\theta = \frac{v}{u + s(1 - \delta)(1 - u)}$$

be the ratio between vacancies and total search effort by (previously) unemployed  $u$  and (remaining) employed  $(1 - \delta)(1 - u)$ , only a share  $s$  of which may search. A linearly homogeneous meeting function gives rise to a probability  $\phi(\theta) \in [0, 1]$ , increasing in  $\theta$ , for a searching worker of locating an open vacancy, and a probability  $\phi(\theta)/\theta$ , decreasing in  $\theta$ , for an open vacancy of meeting a worker who is searching for jobs. Firms are free to post or withdraw as many vacancies as they like (there is free entry of firms on the search market), and will therefore do so up to the point where the expected value of a vacancy is zero.

Finally, wage setting is modeled following the Sequential Auction model of Postel-Vinay and Robin (2002). A firm can commit to guarantee each worker an expected present value of payoffs in utility terms (a “contract”), including state-contingent wages paid directly to the worker, wages paid by future employers, and value of leisure during any unemployment spells. The contract can be renegotiated only by mutual consent, and is subject to two-sided limited commitment: either party can always unilaterally break up the employment relationship, so firms’ profits cannot be negative (in expected PDV) and the worker’s utility value from staying in the contract cannot fall below the value of unemployment. When an employed worker contacts an open vacancy, the prospective poacher and the incumbent employer observe each other’s match qualities with the worker, and engage in Bertrand competition over contracts. The worker chooses the contract that delivers the larger value.

The timing of events within period  $t$  is as follows:

1. nature draws the  $\varepsilon_t$  innovation to TFP  $z_t = Q(z_{t-1}, \varepsilon_t)$ ;
2. firms and workers produce and exchange wages according to the contracts they are currently committed to; previously unemployed workers receive utility from leisure  $b$ ;
3. existing matches break up, both exogenously with probability  $\delta$  and possibly endogenously if either the firm or the worker wants to irreversibly separate;
4. firms post vacancies;
5. previously unemployed and a share  $s$  of (the still) employed workers search for those vacancies, and random meetings occur;
6. upon meeting, a vacancy and a worker draw a permanent match quality  $y$ , and the firm posting the vacancy offers a contract; if the worker is already employed, his current employer and the firm posting the vacancy observe each other’s match quality and Bertrand-compete in contracts (values promised to the worker);

7. the worker decides whether to accept the new offer and form a new match or remain in his current labor market state, either unemployed or employed in a pre-existing match.

## 4 Equilibrium

### 4.1 Bellman equations and Sequential Auctions

**Value of Unemployment.** There is only one produced good in the economy, which is exchanged for labor. Let  $V_{u,t}$  denote the Bellman value for the worker of being unemployed at time  $t$ , and  $V_{e,t}(w, y)$  the value of being employed in a match of quality  $y$  with a contract that specifies a current wage  $w$ . Then:

$$V_{u,t} = b + \beta \mathbb{E}_t [V_{u,t+1} + \phi(\theta_t) \max(V_{e,t+1}(w_{t+1}, y) - V_{u,t+1}, 0)]$$

where the expectation is taken over realizations of aggregate TFP  $z_{t+1}$ , new match quality  $y$  and associated wage contract  $w_{t+1}$ .

Because firms have all the bargaining power, they extract all the rents from unemployed workers, by making them indifferent between working or remaining unemployed. Therefore, the value of unemployment is time-invariant:

$$V_{u,t} = b + \beta \mathbb{E}_t [V_{u,t+1}] = \frac{b}{1 - \beta} = V_u. \quad (1)$$

**Value of Employment.** In this section, we focus on the simple case where  $b$  is small enough to ensure that no match ever breaks up endogenously, all separations are exogenous and occur with probability  $\delta$ ; we will later relax this assumption. The value of employment  $V_{e,t}(w, y)$  at the beginning of period  $t$  equals the wage plus the discounted expected continuation value, which comprises three terms. With probability  $\delta$  the match separates and the worker joins unemployment and receives a value  $\beta V_u$ ; otherwise, with probability  $1 - s\phi(\theta_t)$  he receives no outside offer and continues with value  $\beta V_{e,t+1}(w_{t+1}, y)$  according to the wage  $w_{t+1}$  in the current contract, while with probability  $s\phi(\theta_t)$  he receives an outside offer from a firm, and an auction takes place. We now analyze the auction.

Let  $\{w_s^*(y)\}_{s=t+1}^\infty$  denote the continuation (state-contingent) contract which delivers to the worker the maximum value  $V_{t+1}(y) = V_{e,t+1}(w_{t+1}^*(y), y)$  that the firm is willing to promise to deliver to the worker at the *beginning* of time  $t + 1$ , after the TFP realization  $z_{t+1}$  is observed but before production takes place in that period. By promising this continuation contract, the firm breaks even, namely its profits equal the value of continuing search (of the vacancy). The auction between an employer and a poacher takes place at the end of period  $t$ , after exogenous separations have unfolded but before observing the TFP realization  $z_{t+1}$ . Therefore,  $\beta\mathbb{E}_t[V_{t+1}(y)] := \bar{V}_t(y)$  is the firm  $y$ 's willingness to pay for the worker in the time- $t$  auction, the maximum time- $t$  expected value that the firm is willing to promise to the worker from  $t + 1$  on. Now consider a firm currently employing a worker at match level  $y$  and promising any continuation contract  $\{w_s(y)\}_{s=t+1}^\infty$ , and confronting an outside offer made to its employee by a firm with match  $y'$ . The second-price auction yields one of three possible outcomes:

1.  $\bar{V}_t(y') < \beta\mathbb{E}_t[V_{e,t+1}(w_{t+1}, y)]$ , in which case the incumbent employer needs to do nothing to retain the worker, the offer is irrelevant, and the promised value at time  $t$ , in case of no separation, is the one delivered with no outside offer and no renegotiation,  $\beta\mathbb{E}_t[V_{e,t+1}(w_{t+1}, y)]$ ;
2.  $\beta\mathbb{E}_t[V_{e,t+1}(w_{t+1}, y)] \leq \bar{V}_t(y') < \bar{V}_t(y)$ , in which case the incumbent employer retains the worker by renegotiating the offer, for a raise to  $\bar{V}_t(y')$ ;
3.  $\bar{V}_t(y) \leq \bar{V}_t(y')$ , in which case the worker is poached with an offer worth  $\bar{V}_t(y)$ .

If the current continuation contract specifies  $\{w_s^*(y)\}_{s=t+1}^\infty$ , the firm is already breaking even, thus will not match any outside offers. In that case, either no threatening outside offers arrive, the promised continuation value is  $V_{t+1}(y)$  ex post, and  $\bar{V}_t(y) = \beta\mathbb{E}_t[V_{t+1}(y)]$  in expectation, or the worker is poached, at value  $\bar{V}_t(y)$ , which is the second price in the auction. Either way, the continuation value of the worker who holds a contract  $\{w_s^*\}_{s=t+1}^\infty$  and survives an exogenous separation is  $\bar{V}_t(y)$ , which is the maximum the firm can deliver.

Under the maintained assumption that the value of leisure  $b$  is low enough never to trigger an endogenous separation (i.e.  $\bar{V}_t(y) \geq \beta V_u$ ), by backward induction, the maximum value that the firm is willing to deliver to the worker at the *beginning* of time  $t$  must solve:

$$V_t(y) = z_t y + \delta \beta V_u + (1 - \delta) \bar{V}_t(y) = z_t y + \delta \beta V_u + (1 - \delta) \beta \mathbb{E}_t [V_{t+1}(y)] \quad (2)$$

In words, the continuation value is already maximized, and the maximum current wage that the firm can pay at time  $t$ , without losing money, is full output. Substituting forward, using the L.I.E. and Transversality, and replacing  $V_u$  from (1),

$$V_t(y) = \sum_{\tau=0}^{+\infty} (1 - \delta)^\tau \beta^\tau \mathbb{E}_t [z_{t+\tau}] y + \frac{\delta \beta}{1 - (1 - \delta) \beta} \frac{b}{1 - \beta}$$

Evaluating at  $t + 1$ , taking expectations at time  $t$ , and using again the L.I.E., the firm's willingness to pay in the auction equals

$$\bar{V}_t(y) = \beta \mathbb{E}_t [V_{t+1}(y)] = \mathcal{Z}(z_t) y + \frac{\delta \beta^2}{1 - (1 - \delta) \beta} \frac{b}{1 - \beta}$$

where

$$\mathcal{Z}(z_t) = \mathbb{E}_t \left[ \sum_{\tau=0}^{+\infty} (1 - \delta)^\tau \beta^{\tau+1} z_{t+\tau+1} \right]$$

is a known function of  $z_t$ , and  $z_t$  only because of the Markov property. So the willingness to pay  $\bar{V}_t(y)$  is affine in  $y$ , and the firm with the higher  $y$  wins the auction. We draw the main conclusion of this subsection: the equilibrium, if it exists, must Rank Preserving (RPE), and the direction of EE reallocation is efficient, always from less to more productive matches.

**Value of a Vacancy.** By the time a firm and a worker who have met on the search market must decide whether to consummate the match or not, they know the quality of the potential match,  $y'$ . The firm's willingness to pay  $\bar{V}_t(y') = \beta \mathbb{E}_t [V_{t+1}(y')]$  is, by definition, the maximum profits that the firm can make, because, when giving this to the worker, the firm breaks even. The threat point of the worker's previous situation is known, too: it is  $\beta V_u$  for an unemployed worker, and the willingness to pay of the existing employer  $\bar{V}_t(y) = \beta \mathbb{E}_t [V_{t+1}(y)]$  for a worker employed in an existing match  $y$ . The firm earns the

difference between its own willingness to pay and the worker's outside option. From an unemployed job applicant this is, if positive,

$$\bar{V}_t(y) - \beta V_u = \mathcal{Z}(z_t)y - \mathcal{B}, \quad \text{where } \mathcal{B} = \frac{\beta b}{1 - (1 - \delta)\beta},$$

and from a job applicant employed in a match of quality  $y$  it is

$$\bar{V}_t(y') - \bar{V}_t(y) = \mathcal{Z}(z_t)(y' - y)$$

again, if positive. The value of a vacancy  $V_{v,t}$  then solves the Bellman equation:

$$\begin{aligned} V_{v,t} = & -\kappa + \beta \mathbb{E}_t [V_{v,t+1}] + \frac{\phi(\theta_t)}{\theta_t} P(u_t) \int_{\underline{y}}^{\bar{y}} \max \langle \mathcal{Z}(z_t)y - \mathcal{B} - \beta \mathbb{E}_t [V_{v,t+1}], 0 \rangle d\Gamma(y) \\ & + \frac{\phi(\theta_t)}{\theta_t} [1 - P(u_t)] \int_{\underline{y}}^{\bar{y}} \int_{\underline{y}}^{\bar{y}} \max \langle \mathcal{Z}(z_t)(y' - y) - \beta \mathbb{E}_t [V_{v,t+1}], 0 \rangle \Gamma(y') dL_t(y) \end{aligned} \quad (3)$$

where the expectation is taken over  $z_{t+1}|z_t$ , and

$$P(u) = \frac{u}{u + (1 - \delta)s(1 - u)} \quad (4)$$

is the probability that a randomly drawn job applicant is unemployed.

## 4.2 Employment (distribution) dynamics

Let  $L_t(y)$  denote the measure of employment at matches  $[y, y]$ , a non-normalized c.d.f. of employment on the job ladder, with domain  $[y, \bar{y}]$ . Due to the RP property of equilibrium, this measure increases with hires from unemployment that draw a match quality below  $y$  and decrease with separations to unemployment and with quits to better matches:

$$L_{t+1}(y) = (1 - \delta) [1 - s\phi(\theta_t)\bar{\Gamma}(y)] L_t(y) + \phi(\theta_t) u_t \Gamma(y) \quad (5)$$

Differentiating both sides, the measure of employment  $\ell_t(y) = L'_t(y)$  at match  $y$  follows

$$\ell_{t+1}(y) = (1 - \delta) \left\{ [1 - s\phi(\theta_t)\bar{\Gamma}(y)] \ell_t(y) + s\phi(\theta_t)\gamma(y) \int_{\underline{y}}^y \ell_t(y') dy' \right\} + \phi(\theta_t)\gamma(y) u_t \quad (6)$$

which nets out flows in and out of the match both from/into unemployment and from/into other matches. The law of motion of the unemployment rate  $u_t = 1 - L_t(\bar{y})$  is familiar:

$$u_{t+1} = [1 - \phi(\theta_t)] u_t + \delta(1 - u_t) \quad (7)$$



### 4.3 Free entry and equilibrium

The free entry condition is  $V_{v,t} = 0$  for all  $t$ . As mentioned, for now we restrict attention to the simple case where  $b$  is small enough to ensure that no match ever breaks up endogenously, namely  $\inf_z \mathcal{Z}(z)y \geq \mathcal{B}$ . Then (3) writes as

$$\kappa \frac{\theta_t}{\phi(\theta_t)} = P(u_t) [\mathcal{Z}(z_t)\mu - \mathcal{B}] + [1 - P(u_t)] \mathcal{Z}(z_t)\Omega_t \quad (8)$$

where the last term:

$$\Omega_t := \int_{\underline{y}}^{\bar{y}} \int_{\underline{y}}^{\bar{y}} \max\langle y' - y, 0 \rangle d\Gamma(y') dL_t(y) = \int_{\underline{y}}^{\bar{y}} \int_{\underline{y}}^{y'} (y' - y) \frac{dL_t(y)}{1 - u_t} d\Gamma(y') \quad (9)$$

is the expected return to the firm from a contact with an employed job applicant, a key object that we will discuss in detail. On the LHS of (8) are vacancy costs times the expected duration of a vacancy, on the RHS the average of the expected discounted profits from hiring an unemployed and an employed job applicant, weighted by the respective shares of the two types of job applicants in the pool of job searchers. Unemployed hires are homogeneous, while employed hires are distributed according to the measure  $dL_t(y)/(1 - u_t)$  of match quality  $y$  in their current jobs, which determines their bargaining power in wage negotiations. Given the predetermined (at time  $t$ ) distribution of employment  $L_t(\cdot)$ , and the resulting unemployment rate  $u_t = 1 - L_t(\bar{y})$ , this equation uniquely pins down equilibrium tightness  $\theta_t$ .

Given initial conditions  $z_0 \in \mathbb{R}_+$  and  $L_0 : [\underline{y}, \bar{y}] \rightarrow [0, 1]$ , a Rational Expectations Equilibrium is a stochastic process for job market tightness  $\theta_t$  solving the free entry condition (8), given:  $P(u)$  in (4),  $u_t = 1 - L_t(\bar{y})$ ,  $z_{t+1} = Q(z_t, \varepsilon_{t+1})$ , and the dynamics of  $L_t(\cdot)$  in (5).

### 4.4 Match surplus and the Labor Wedge

The expected return from an unemployed hire  $\mathcal{Z}(z_t)\mu - \mathcal{B}$  is the (expected, capitalized) difference between Marginal Product of Labor and value of leisure. Due to risk neutrality, the value of leisure also equals the Marginal Rate of Substitution of consumption for leisure. In the Business Cycle accounting literature (Chari, Kehoe and McGrattan, 2007), the ratio between the MRS and the MPL is the “labor wedge”. Measured in the data through the lens

of a neoclassical growth model with balanced growth preferences, this ratio is procyclical (the implicit “tax” rate on labor, equal to one minus this ratio, is countercyclical) and plays a key role in amplifying business cycle fluctuations. Estimated New-Keynesian models (Smets and Wouters, 2007) define the “wage markup” as the ratio between the real wage and the MRS, and find that changes in this mark-up are key to explain inflation and output dynamics. Lacking a mechanism to generate endogenous changes in the wage mark-up, they attribute them to shocks, that they estimate to be procyclical. Gali (2011) calls for a theory of an endogenous wage mark-up. In the business cycle search literature, which typically abstracts from OJS, this wedge corresponds to the firm’s surplus, which compensates for hiring costs. This surplus is procyclical, as long as TFP is persistent, making the returns to hiring unemployed workers, hence labor market tightness, procyclical. In the absence of OJS (setting  $s = 0$ ) our model reduces to the stochastic Nash Bargaining search model of Shimer (2005) where firms have all the bargaining power. In that class of models, Hagedorn and Manovskii (2008) argue that this surplus is small, relative to mean output, providing a rationale for the observed high volatility of unemployment. We now show that OJS substantially changes the terms of this debate.

## 4.5 The Marginal Productivity of Labor Gap

Our model contains an additional, novel wedge, and related transmission mechanism of aggregate shocks to job creation, which is not present in any strands of the literature, and which greatly reduces the importance of the size of the unemployed surplus. Firms, when posting vacancies, also mind the expected return  $\Omega_t$  from an *employed* hire, defined in (9). This is independent of the MRS, hence of the surplus, and depends entirely on the distribution of employment  $L_t(\cdot)$ , which is a slow-moving aggregate state variable.  $\Omega_t$  can be viewed as an index of misallocation relative to the frictionless limit, where workers sample jobs at unbounded rate and thus are always in the best possible match. We call  $\Omega_t$  the “Marginal Productivity of Labor Gap” (MPL Gap, for brevity), because it measures the

expected marginal productivity effect of moving one worker between jobs.

This Gap introduces an additional, time-varying component to labor demand, with a complex cyclical pattern. To illustrate, it is useful to observe that this Gap is larger the worse the normalized employment distribution on the ladder  $L_t(y)/(1 - u_t)$  in a first-order stochastic dominance sense: just integrate (9) by parts and observe that  $\bar{\Gamma}$  is decreasing

$$\Omega_t = \int_y^{\bar{y}} \frac{L_t(y')}{1 - u_t} \bar{\Gamma}(y') dy'.$$

At a cyclical peak, workers have had time and opportunities to climb the ladder, so poaching employees from other firms is both difficult and expensive, and the returns to hiring employed workers are low. After a recession, as the unemployed regain employment, they restart from random rungs on the match quality ladder, which are worse than the employment distribution at the cyclical peak. Hence, early in a recovery many recent hires are easily “poachable”. Moving cheap unemployed job applicants into low-quality jobs makes them only slightly more expensive, and still quite profitable to hire. As time goes by, and unemployment declines, employment reallocation up the ladder through job-to-job quits accumulates, employed workers become more and more expensive to hire, ultimately putting pressure on wages, until we are back to a cyclical peak. Misallocation and the resulting MPL Gap imply a procyclical wage mark-up, or countercyclical labor “tax”, but only as long as employment is still misallocated and vulnerable to poaching.

In the US economy, the transition probability from job to job is fairly small, similar to the separation probability into unemployment, and both are an order of magnitude smaller than the transition probability from unemployment to employment. Therefore, movements in the employment distribution up the job ladder are slow. An important implication is that, in our model, job market tightness, thus the unemployment rate, have significant transitional dynamics. This stands in contrast to the canonical model with only unemployed job search, where job market tightness is a jump variable, the unemployment rate converges very quickly to its new steady state, and both track the current state of TFP essentially one for one.

Before concluding, we show a few more properties of the MPL Gap  $\Omega_t$ , which will illu-

minate quantitative results. Since  $L_t(y)$  is strictly increasing in at least part of its domain, then  $L_t(y) < L_t(\bar{y}) = 1 - u_t$  and the MPL Gap is lower than the unconditional average match quality:

$$\Omega_t = \int_{\underline{y}}^{\bar{y}} \bar{\Gamma}(y) \frac{L_t(y)}{1 - u_t} dy < \int_{\underline{y}}^{\bar{y}} \bar{\Gamma}(y) dy = -\underline{y} + \int_{\underline{y}}^{\bar{y}} y d\Gamma(y) = \mu - \underline{y} < \mu$$

where in the second equality we used, again, integration by parts. Intuitively, employed workers must be compensated for giving up their current job. Furthermore,

$$\mathcal{Z}(z_t)\Omega_t < \mathcal{Z}(z_t)(\mu - \underline{y}) = \mathcal{Z}(z_t)\mu - \mathcal{Z}(z_t)\underline{y} \leq \mathcal{Z}(z_t)\mu - \mathcal{B}$$

where the last inequality follows from the assumption that  $b$  is small enough to make all matches acceptable, namely  $\mathcal{Z}(z_t)\underline{y} \geq \mathcal{B}$  for all  $y$ . We conclude that an employed hire is always less profitable in expectation than an unemployed hire, a property that will play an important role in shaping aggregate equilibrium dynamics.

## 5 Endogenous separations into unemployment

Having characterized equilibrium in the baseline model, we are now ready to take the final step and study our main framework for quantitative analysis, which allows for endogenous job destruction, or separations into unemployment. Let the value of leisure  $b$  be large enough to make some matches, new and old, infeasible when the level of TFP is especially low. We also introduce idiosyncratic shocks: after production begins, match quality  $y_{t+1}$  evolves according to a first-order Markov process, stochastically non-decreasing in  $y_t$  and independent across matches, with transition density  $\pi(y_{t+1}|y_t)$ . So now separations may occur endogenously due to either aggregate or idiosyncratic shocks. The new realization of match quality  $y_t$  occurs at the beginning of period  $t$ , simultaneously with the new realization of aggregate TFP  $z_t$ .

The analysis requires a few modifications to the baseline model of the previous section. Given the firm's bargaining power, the value of unemployment is still constant at  $V_u = b(1 - \beta)^{-1}$ . Since endogenous separations occur after production and exogenous separations,

but before any outside offers arrive, Eq. (2) for the expected value of continuing in a match  $y_t$  changes into

$$V_t(y_t) = z_t y_t + \delta \beta V_u + (1 - \delta) \beta \max \langle V_u, \mathbb{E}_t [V_{t+1}(y_{t+1})] \rangle$$

and the firm's willingness to pay in the auction for a match of current quality  $y_t$  equals  $\bar{V}_t(y_t) = \beta \mathbb{E}_t [V_{t+1}(y_{t+1})]$ , where the expectation is now taken over idiosyncratic shocks too.

$$\begin{aligned} \bar{V}_t(y_t) &= \beta \mathbb{E}_t [z_{t+1} y_{t+1}] + \delta \beta^2 V_u + (1 - \delta) \beta \mathbb{E}_t [\max \langle \beta V_u, \beta \mathbb{E}_{t+1} [V_{t+2}(y_{t+2})] \rangle] \\ &= \beta \mathbb{E}_t [z_{t+1} y_{t+1}] + \delta \beta^2 V_u + (1 - \delta) \beta \mathbb{E}_t [\max \langle \beta V_u, \bar{V}_{t+1}(y_{t+1}) \rangle] \end{aligned}$$

Since  $\mathbb{E}_t [z_{t+1} y_{t+1}]$  is only a function of  $z_t, y_t$  by the Markov property, we can write  $\bar{V}_t(y) = W(z_t, y_t)$  where  $W$  is a time-invariant function of aggregate TFP  $z$  and idiosyncratic match productivity  $y$ , solving

$$W(z, y) = \beta \mathbb{E} [z' y' | z, y] + \delta \beta^2 V_u + (1 - \delta) \beta \mathbb{E} [\max \langle \beta V_u, W(z', y') \rangle | z, y] \quad (10)$$

As long as TFP  $z_t$  (aggregate) and  $y_t$  (idiosyncratic) are both persistent, i.e.  $\mathbb{E} [z' y' | z, y]$  is increasing in  $z$  and  $y$ , and  $\mathbb{E} [z' y' | z, y]$  is bounded (which can be relaxed, but holds in the numerical implementation by discretization), the RHS of (10) maps the set of increasing, bounded, continuous functions into itself, and is a contraction. So there exists a unique increasing, bounded, and continuous function  $W$  which solves (10), hence a continuous, decreasing ‘‘cutoff’’ function  $\hat{y}(\cdot)$  which solves  $W(z, \hat{y}(z)) = \beta V_u$  for every  $z$ . A job match  $y_t$  is formed and then preserved if and only if  $y_t \geq \hat{y}(z_t)$ .

When promising to deliver to the worker value  $W(z, y)$ , a firm is indifferent between employing the worker or not. Then,  $W(z, y)$  also generates the maximum expected profits that the firm can earn from the worker. Given a current employment distribution  $L_t(\cdot)$  and TFP realization  $z_t$ , the free entry condition then reads

$$\kappa \frac{\theta_t}{\phi(\theta_t)} = \int_{\hat{y}(z_t)}^{\bar{y}} \frac{u_t [W(z_t, y) - \beta V_u] + s(1 - \delta) \int_{\hat{y}(z_t)}^y [W(z_t, y) - W(z_t, y')] dL_t(y')}{u_t + s(1 - \delta) [1 - u_t - L_t(\hat{y}(z_t))]} d\Gamma(y) \quad (11)$$

Note the second integral's lower bound  $\hat{y}(z_t)$ : given the timing of events we assumed, where endogenous separations occur after production and before search, a firm posting a vacancy

can meet, and compete for, an employed worker only if the worker's previous employment relationship survived the new TFP realization  $z_t$ , i.e. only if  $W(y', z_t) \geq \beta V_u$ , or  $y' \geq \hat{y}(z_t)$ .

The law of motion of the employment distribution (5) now reads:

$$L_{t+1}(y) = \int_{\underline{y}}^{\bar{y}} \pi(y|y') d\tilde{L}_{t+1}(y') \quad (12)$$

where  $\tilde{L}_t(\cdot)$  is the beginning-of-period- $t$  employment distribution, before the realization of aggregate and idiosyncratic shocks, which solves:  $\tilde{L}_{t+1}(\hat{y}(z_t)) = 0$  for  $y < \hat{y}(z_t)$  and

$$\tilde{L}_{t+1}(y) = (1 - \delta) [1 - s\phi(\theta_t) \bar{\Gamma}(y)] L_t(y) + \phi(\theta_t) u_t [\Gamma(y) - \Gamma(\hat{y}(z_t))] \quad (13)$$

for  $y \geq \hat{y}(z_t)$ .<sup>3</sup> The law of motion of unemployment  $u_t = 1 - L(\bar{y})$  becomes

$$u_{t+1} = \delta(1 - u_t) + L(\hat{y}(z_t)) + [1 - \phi(\theta_t) \bar{\Gamma}(\hat{y}(z_t))] u_t \quad (14)$$

The probability of EU transition now equals  $\delta + \frac{L_t(\hat{y}(z_t))}{L_t(\bar{y})}$ , the probability of UE transition equals  $\phi(\theta_t) \bar{\Gamma}(\hat{y}(z_t))$ , and the probability of EE transition equals  $(1 - \delta) s\phi(\theta_t) \int_{\hat{y}(z_t)}^{\bar{y}} \bar{\Gamma}(y) \frac{dL_t(y)}{L_t(\bar{y})}$ . This completes the description of equilibrium conditions.

## 6 Quantitative analysis

We calibrate the model parameters and simulate its equilibrium outcomes following a history of aggregate shocks. As in the theory, we study first the baseline model with exogenous job destruction only, and then introduce endogenous separations to unemployment, our preferred specification.

The stochastic equilibrium described in Section 4 can be computed exactly in one run, without using any linearization or fixed point algorithms.<sup>4</sup> Equilibrium conditions (5) and

<sup>3</sup>The density  $\ell_t(y) = L'_t(y)$  no longer exists everywhere, because endogenous separations cleanse employment below  $\hat{y}(z_t)$ , "hollow out" the employment distribution  $L_t$  at the bottom  $[y, \hat{y}(z_t)]$ , thus create kinks in the distribution when the economy recovers and hires from unemployment replenish matches of quality in the hollowed out region  $[\hat{y}(z_{t+1}), \hat{y}(z_t)]$ :  $\tilde{L}'_t(\hat{y}(z_t)-) < \tilde{L}'_t(\hat{y}(z_t)+)$ , and  $L'_{t+1}$  inherits this discontinuity.

<sup>4</sup>In the Appendix we study in detail the steady state equilibrium, which provides the basis to calibrate the values of many model parameters, prove its uniqueness, and illustrate comparative statics with respect to changes in aggregate TFP. This exercise sheds some analytical light on the quantitative results from stochastic simulations presented in this section, but is not required to derive them.

(8) are forward-looking only through the term  $\mathcal{Z}_t(z_t)$ , which is exogenous. Therefore, the equilibrium evolution of the economy can be simulated directly. If the support of match quality  $y \sim \Gamma$  is finite,  $\{y_i\}_{i=1\dots R}$ , so is the vector  $L_t(y_i)$ , and it is possible to exactly update this vector and thus compute the path of  $\Omega_t$ , and the whole equilibrium, by just one simulation round, starting from any initial conditions  $z_0$  and  $L_0(\cdot)$  and for any random path of  $z_t$ . If instead the support of  $\Gamma$  is (a subset of) the real line, this strategy is infeasible, because  $L_t(y)$ , which enters the key object  $\Omega_t$ , is infinitely-dimensional. In this case, in order to compute equilibrium, we discretize  $[\underline{y}, \bar{y}]$ , and proceed as above. Alternatively, we could exploit the equilibrium restrictions to derive, by integration by parts, an accurate finite-dimensional approximation algorithm to the dynamics of  $\Omega_t$  (available upon request).

A special case starts the economy from  $L_0(\cdot) = L(\cdot)$  the steady state distribution (Eq. (15)), and studies the effects of an aggregate TFP innovation  $\varepsilon_t$  equal to 1% for  $t = 1$  and zero otherwise. This describes the Impulse Response Function (IRF) of the system from the steady state to a one-off unanticipated TFP shock.

## 6.1 Calibration

To calibrate model parameters we proceed in two steps. First, we choose the values of all parameters except the aggregate TFP process so that the steady state equilibrium matches a few key statistics for the US economy. Second, given this set of parameter values, we calibrate the TFP process so that the stochastic simulation of the model generates an empirically accurate persistence of innovations to and unconditional volatility of Average Labor Productivity (ALP). This second step deserves some discussion.

Job ladder reallocation makes the Solow residual in the model endogenous and different than the underlying exogenous TFP process. In principle, we could estimate an “empirically plausible” TFP process outside of the model and verify what the model then implies for ALP dynamics. We prefer to match the latter by construction, because our emphasis is on job finding probability and unemployment, not on ALP, so we are interested in the model’s

predictions for the former. In fact, we are mostly interested in the model’s potential to amplify and to propagate aggregate shocks. Thus, in principle, we could just study the volatility and persistence of job finding probability *relative* to those of the TFP driving force. Because the model is non linear, however, outcomes are not invariant to the scale and persistence of the shocks, hence we discipline both using the closest empirical analogue to TFP, namely ALP.

We calibrate the model parameters and compute equilibrium at a monthly frequency. We start with preferences. We set the discount factor  $\beta = .95^{\frac{1}{12}}$ , and the value of leisure  $b = 0$  so that no existing job is ever destroyed endogenously. We will later explore the case of endogenous job destruction. Importantly, once we allow for OJS the amplification properties of the model are much less dependent on the value of  $b$ . This value determines the returns to hire unemployed job applicants, while the returns from hiring employed job applicants depend on their current wages, which may have been renegotiated multiple times and thus no longer retain any memory of the opportunity cost  $b$ . So OJS allows to sidestep the debate on the opportunity cost of time that originated from Hagedorn and Manovskii (2008).

Next, we move to transition probabilities between employment and unemployment. Since for now all separations into unemployment are exogenous, we set  $\delta$  equal to the average monthly transition probability from employment into unemployment (EU). Since all new matches are acceptable to the unemployed, we set the job contact probability in steady state equilibrium  $\phi(\theta)$  equal to the average monthly transition probability from unemployment into employment (UE). We estimate these probabilities from unemployment duration stocks (Shimer 2012) in the monthly CPS, respectively the number of workers who report being unemployed for 5 weeks or less divided by employment a month before (EU), which averages 2.4%, and one minus the ratio between the number of workers who report being unemployed for more than 5 weeks and unemployment a month before (UE), which averages 41%. The implied steady-state unemployment rate is  $u = .024/ (.024 + .41) = .055$ .<sup>5</sup>

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<sup>5</sup>This method ignores transitions in and out of non participation, hence overestimates transition probabilities between E and U. Alternatively, we could use gross flows between U and E from the 1990-2018



Given these parameter values, the efficiency of OJS  $s$  is identified by the pace of EE reallocation. To show how, we first find the stationary employment distribution  $\ell(y) = L'(y)$ , which solves the following ordinary linear differential equation:

$$L'(y) = (1 - \delta) [1 - s\phi(\theta) \bar{\Gamma}(y)] L'(y) + s\phi(\theta) \gamma(y) L(y) + \phi(\theta) \gamma(y) [1 - L(\bar{y})]$$

The solution can be found in closed form:

$$\begin{aligned} \ell(y) &= \frac{\phi(\theta) [\delta + (1 - \delta) s\phi(\theta)] \gamma(y) u}{[\delta + (1 - \delta) s\phi(\theta) \bar{\Gamma}(y)]^2} \\ L(y) &= \frac{\phi(\theta) \Gamma(y) u}{\delta + (1 - \delta) s\phi(\theta) \bar{\Gamma}(y)} \\ u = 1 - L(\bar{y}) &= \frac{\delta}{\delta + \phi(\theta)}. \end{aligned} \tag{15}$$

Now we can write the average EE transition probability in steady state equilibrium:

$$\begin{aligned} (1 - \delta) s\phi(\theta) \int_{\underline{y}}^{\bar{y}} \bar{\Gamma}(y) \frac{dL(y)}{1 - u} &= (1 - \delta) s\phi(\theta) \int_{\underline{y}}^{\bar{y}} \frac{L(y)}{1 - u} d\Gamma(y) \\ &= (1 - \delta) s\phi(\theta) \int_{\underline{y}}^{\bar{y}} \frac{u}{1 - u} \frac{\phi(\theta) \Gamma(y)}{\delta + (1 - \delta) s\phi(\theta) \bar{\Gamma}(y)} d\Gamma(y) \\ &= s(1 - \delta) \delta\phi(\theta) \int_0^1 \frac{1 - x}{\delta + (1 - \delta) s\phi(\theta) x} dx \end{aligned}$$

where in the first line we integrate by parts, in the second line we replace the expressions for the steady state unemployment rate and employment distribution from (15), in the third line we change variable to quantiles  $x = \bar{\Gamma}(y)$ . The last expression shows that the steady state EE probability does not directly depend on the specific match quality distribution  $\Gamma$ : when given the opportunity, workers move up the job ladder, no matter how steep it is, at a speed that depends only on  $s$ . Given values of  $\phi(\theta)$  and  $\delta$ , hence  $u$ , we solve for the value of  $s$  that equates the last expression to the average monthly transition probability from job to

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matched files of the monthly CPS, and estimate the average fraction of individuals who switch employment status. This measure suffers from time aggregation from point-in-time observations of employment status, which suppresses short unemployment spells and thus *underestimates* transition probabilities, specifically EU=1.4% and UE=25%, for a steady-state unemployment rate equal to  $u = .014/ (.014 + .25) = .053$ . Because short unemployment spells are more common in expansions, when UE is high and EU low, time aggregation also reduces the volatility of the UE probability and increases that of the EU probability. The quantitative results from the model are, however, similar when we choose this different calibration.

job, which is about 2% in the monthly CPS after its 1994 survey re-design that introduced Dependent Coding.

Next, we calibrate matching frictions. The free entry condition determines the vacancy filling probability. To map it into the the job finding probability from unemployment, which is the main object of interest in the stochastic simulation, we need to specify a functional form for the matching function. We cannot reject empirically the hypothesis of constant returns to scale in matching. Hence, we assume a Cobb-Douglas form  $\phi(\theta) = \min\langle \Phi\theta^\alpha, 1 \rangle$ . We estimate the value of  $\alpha$  by regressing, after HP-filtering with smoothing parameter  $8.1 \times 10^6$  (see Shimer 2012), the log of the monthly job finding probability on the log of vacancies and on the log of total worker search effort  $u_t + (1 - \delta)s(1 - u_t)$ , where the values of  $\delta$  and  $s$  were calibrated before. For the job finding probability we use the unemployment-duration based measure described above, which has standard deviation (in log deviations from HP trend) equal to .147 over the post-war period.<sup>6</sup> For vacancies we use the monthly Composite Help-Wanted Index of Barnichon (2010), updated by the author to cover 1955-2016, and very close to JOLTS vacancies since its 2001 inception. For  $u_t$  we use the civilian unemployment rate from the monthly CPS, 1948-2018. We filter each series separately using the longest time span available for each. We then run regressions with filtered monthly data on the time period where the series overlap, 1955-2016.<sup>7</sup> For our preferred calibration of  $s$  targeting a 2% EE transition probability and  $\delta$  targeting a 2.4% EU separation probability, and the resulting search pool  $u_t + (1 - \delta)s(1 - u_t)$ , the estimated regression coefficient of the log UE job finding probability on log tightness is  $\hat{\alpha} = .5$ .<sup>8</sup> Crucially, as we will soon show, when we estimate a standard matching function ignoring OJS, i.e. when we identify the search pool with just unemployment  $u_t$ , we obtain a lower elasticity  $\hat{\alpha} = .32$ . The reason is simple: this

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<sup>6</sup>The UE measure based on gross flow has the same volatility around .14 over the much shorter 1990-2018 period, when our preferred measure has volatility close to .2 due the correction for time aggregation

<sup>7</sup>To alleviate possible attenuation bias from high-frequency noise, we either take a two-sided moving average of each series with a window of  $\pm 6$  months or we aggregate the data to quarterly frequency by taking averages. The results are similar

<sup>8</sup>Alternatively, to overcome concerns about endogeneity of vacancies, due to shocks to matching efficiency  $\Phi$ , which may be present in the data but not in the model, we could apply the GMM procedure of Borowczyk-Martins et al. (2013) to unfiltered data.

standard method incorporates a term equal to  $\alpha$  times the log of relative search effort by the unemployed vs the employed  $u_t/[u_t + (1 - \delta)s(1 - u_t)]$ , into the residual, which is then negatively correlated with  $\ln(v_t/u_t)$ , creating a downward bias in the estimated elasticity  $\hat{\alpha}$ .

The job finding probability in the data is always less than one at a monthly frequency. To guarantee that this is true in the model also of the vacancy filling probability  $\Phi\theta_t^{\alpha-1}$  at all times, including in steady state, we choose the scale of vacancies so that job market tightness  $\theta_t$  always exceeds the maximum job finding probability observed in the data, that we aim to replicate with  $\Phi\theta_t^\alpha$ . In fact,  $\min_t \theta_t > \max_t \Phi\theta_t^\alpha$  guarantees  $\Phi\theta_t^{\alpha-1} < 1$  for all  $t$ .

Once both probabilities are less than one at all times, the vacancy filling probability can be written as  $\Phi\theta^{\alpha-1} = \Phi^\frac{1}{\alpha} (\Phi\theta^\alpha)^{\frac{\alpha-1}{\alpha}} = \Phi^\frac{1}{\alpha} \phi(\theta)^{\frac{\alpha-1}{\alpha}}$ . Using (15), (4), and  $\mathcal{Z}(z) = \frac{\beta z}{1-\beta(1-\delta)} = \mathcal{B}z/b$  in the free entry condition (8), pins down the steady state equilibrium level of job market tightness  $\theta$ .

$$\kappa \frac{\theta}{\phi(\theta)} = \frac{\beta}{1-\beta(1-\delta)} \frac{u(z\mu - b) + (1-\delta)s(1-u)\Omega}{u + (1-\delta)s(1-u)} \quad (16)$$

Normalizing average steady-state TFP to  $z = 1$  and average match quality to  $\mu = 1$ , this free entry condition simplifies to:

$$\kappa \Phi^{-\frac{1}{\alpha}} \phi(\theta)^{\frac{1-\alpha}{\alpha}} = \frac{\beta}{1-\beta(1-\delta)} \frac{u(1-b) + (1-\delta)s(1-u)\Omega}{u + (1-\delta)s(1-u)} \quad (17)$$

To use this equation, and to solve for expected hiring cost (vacancy cost times vacancy duration) on the LHS, we only need to compute the value of the steady-state MPL Gap  $\Omega$ .

Its expression

$$\Omega = \int_{\underline{y}}^{\bar{y}} \bar{\Gamma}(y) \frac{\delta \Gamma(y)}{\delta + (1-\delta)s\phi(\theta)\bar{\Gamma}(y)} dy \quad (18)$$

shows that, given the calibrated values of transition rates  $\delta, s, \phi(\theta)$ ,  $\Omega$  is uniquely determined by production technology  $\Gamma$ . We assume that the match quality distribution  $\Gamma$  is Pareto with lower bound  $\underline{y}$  and parameter  $\lambda > 1$ . Changing variable in (18) to  $x = \bar{\Gamma}(y) = (y/\underline{y})^{-\lambda}$ , and using the normalization  $\mu = \underline{y}(1 - \lambda^{-1}) = 1$ , we obtain

$$\Omega(\lambda) = \frac{\lambda - 1}{\lambda^2} \int_0^1 \frac{(1-x)x^{-\frac{1}{\lambda}}}{\delta + (1-\delta)s\phi(\theta)x} dx$$

a direct mapping from values of  $\lambda$  to steady-state equilibrium MPL Gap, thus hiring costs. As explained below, we do not specify a value of  $\lambda$ , but will experiment with different values.

Given  $\phi(\theta) = .41$ , the estimated value of  $\hat{\alpha}$ , the value of  $\Omega = \Omega(\lambda)$  and of the parameters, we can use free entry (17) to calibrate the composite parameter  $\psi := \kappa\Phi^{-\frac{1}{\alpha}}$ . Without specifying the scale of vacancies, we cannot separately identify  $\kappa$  from  $\Phi$ , nor do we need to. The free entry condition equates the returns to a contact with a job applicant, which depends on other parameter values and shock realizations, with  $\psi\phi(\theta_t)^{\frac{1-\alpha}{\alpha}}$ . Therefore, knowing the value of  $\psi$  allows computation of the job finding probability  $\phi(\theta_t)$  at each point in time. On the other hand, as illustrated in the Appendix, the steady-state elasticity of the job finding probability  $\phi(\theta)$  to aggregate TFP is independent of the values of the parameters that make up the multiplicative constant  $\psi$ . Hence, we expect the stochastic properties (volatility, persistence, impulse response) of the job finding probability  $\phi(\theta_t)$  to be fairly insensitive to the value of  $\psi$ .

Therefore, the value of  $\lambda$  matters for shock amplification and propagation not through the scale of total returns to hiring and corresponding hiring costs  $\psi$ , but only through the *relative* returns to hire unemployed and employed workers  $\Omega(\lambda)$ , which receive time-varying weights in the free entry condition. Given the exogenous returns  $\mathcal{Z}(z_t)\mu - \mathcal{B}$  to contact a jobless worker, less dispersed match outcomes (higher value of  $\lambda$ ) reduce the MPL Gap and the importance of poaching. In the limit, as  $\lambda \rightarrow \infty$ , all matches are the same, as in Robin (2011), the worker captures all rents from any outside offer, and the free entry condition looks exactly like in the DMP model without OJS; the only difference is the procyclical congestion that employed workers impose on the unemployed in the matching process and that dampens aggregate volatility.

As a final step, we are interested in assessing the size of hiring costs in steady state, a non-targeted moment, as a share of output. These costs equal  $\kappa v = \kappa\theta [u + (1 - \delta)s(1 - u)]$  per period. As mentioned, we cannot separately identify  $\kappa$  and the scale of vacancies  $v$ . But these hiring costs equal, by free entry, the total number of hires, namely the stock of

searchers  $u + (1 - \delta)s(1 - u)$  times the contact probability  $\phi(\theta)$ , multiplied by the expected return from each contact with a randomly drawn job applicant. All of these objects are pinned down by the steady state calibration, therefore we can also compute hiring costs  $\kappa v$ , without having to disentangle its two components.

This completes the steady state calibration. To introduce aggregate shocks and simulate the stochastic equilibrium of the model, we specify the TFP process as an AR(1) in logs:

$$\ln z_t = (1 - \rho) \ln \zeta + \rho \ln z_{t-1} + \varepsilon_t, \quad \varepsilon_t \stackrel{\text{i.i.d.}}{\sim} \mathcal{N}(0, \sigma^2)$$

In the spirit of the steady state normalization  $z = 1$ , we set average log TFP  $\ln \zeta = 0$ , and we calibrate the parameters  $\rho, \sigma$  so that the model's Average Labor Productivity (ALP)  $z_t \bar{Y}_t$ , where  $\bar{Y}_t = \int_{\underline{y}}^{\bar{y}} y \frac{dL_t(y)}{1-L_t(\bar{y})} = \int_{\underline{y}}^{\bar{y}} \frac{1-L_t(y)}{1-L_t(\bar{y})} dy$ , matches time series properties of ALP in the data.  $\bar{Y}_t$  depends on the worker allocation on the job ladder, i.e. by match quality, a slow-moving state variable, hence it has interesting dynamics. To measure ALP in the data, following Shimer (2005) for comparison, we use quarterly Real Output Per Person in the Nonfarm Business Sector (BLS series PRS85006163) in 1947:I-2017:IV, HP filter its log with parameter 100,000 (which corresponds to  $8.1 \times 10^6$  monthly) and trim the first and last two years of data. We assume that the filtered series follows an AR(1) at quarterly frequency and estimate the first order serial correlation (.88) and standard deviation (.093) of innovations. These imply an unconditional standard deviation which almost exactly equals the actual value .0197, providing support to the AR(1) assumption. In each model, we choose values of the monthly log TFP parameters  $\rho$  and  $\sigma$  to target both persistence of innovations (.88) to and unconditional volatility (.0197) of filtered log ALP using simulated log ALP aggregated to quarterly frequency.

Given the AR(1) specification for TFP with Gaussian innovations, we can derive a closed-form expression for  $\mathcal{Z}(z_t)$ . In the Appendix we show

$$\mathcal{Z}(z) = \sum_{\tau=1}^{+\infty} (1 - \delta)^{\tau-1} \beta^\tau z^{\rho^\tau} e^{\frac{\sigma^2}{2} \frac{1-\rho^{2\tau}}{1-\rho^2}} \quad (19)$$

The AR(1) for  $\ln z_t$  can be approximated by a finite Markov chain using Tauchen's method,

and then  $\mathcal{Z}(z)$  can be pre-computed from (19) for all values of  $z$  in its finite support.

To compute equilibrium dynamics, we discretize the Pareto distribution of match quality on a 500-point support  $\underline{y} = y_1 < y_2 < \dots < y_{500} = \bar{y}$ . Given the value of  $\lambda > 1$ , we choose  $\bar{y} = \underline{y}(.001)^{\frac{1}{\lambda}}$  to be the 99.9% percentile of the underlying Pareto distribution, and  $\underline{y}$  to guarantee that the resulting mean of the discrete distribution equals  $\mu = 1$ . We then specify  $z_t$  as a 500-point discrete Markov chain approximation to the AR(1) process described above.

In order to understand the role of OJS, besides the benchmark model just described, we compare it to a model without OJS. We restrict  $s = 0$  to eliminate OJS altogether, and do not target the EE probability, so we also re-estimate the elasticity of the matching function by assuming that the pool of search equals only unemployment, and obtain, as mentioned,  $\hat{\alpha} = .32$ . We also recalibrate the TFP process parameters  $\rho$  and  $\sigma$  and the value of hiring costs  $\psi$ , but do not expect the latter to make a material difference. Note that the model with no OJS has no endogenous productivity component,  $\bar{Y}_t = \mu = 1$  and ALP=TFP, so in this case  $\rho$  and  $\sigma$  can be directly set to the ALP empirical counterparts.

All parameter values for each version of the model are gathered in the upper panel of Table 1. The values of  $\beta = .995$ ,  $b = 0$  and  $\delta = .024$  are maintained in all versions of the model. The value of the parameter  $\lambda$ , which is varied exogenously across our different simulations exercises, is highlighted in gray in the table.

## 6.2 Quantitative Results

We study the unconditional second moments of unemployment, job-finding probability, ALP and other statistics from a stochastic simulation of the model, and their impulse response functions to a one-period negative TFP shock. Table 1 illustrates calibration and results. The first three columns, OJS I-III, refer to three versions of the model with OJS, where the value of the Pareto slope  $\lambda$  of the match quality distribution is allowed to vary. The only material difference this slope makes to the rest of the calibration is in the implied hiring costs: a more dispersed match quality distribution (lower  $\lambda$ ), given the mean, generates a

| <b>Model <math>\Rightarrow</math></b>           | OJS   |       |       |       | NO OJS | Data  |
|---|-------|-------|-------|-------|--------|-------|
| <b>Parameters calibrated externally</b>         | I     | II    | III   | IV    |        |       |
| discount factor $\beta$                         | .995  | .995  | .995  | .995  | .995   |       |
| flow value of leisure $b$                       | 0     | 0     | 0     | 0     | 0      |       |
| exogenous separation prob. $\delta$             | .024  | .024  | .024  | .024  | .024   |       |
| match inequality $\lambda$                      | 1.1   | 3     | 5     | 1.1   | —      |       |
| <b>Parameters calibrated internally</b>         |       |       |       |       |        |       |
| OJS efficiency $s$                              | .176  | .176  | .176  | .176  | 0      |       |
| persistence of log TFP innov's $\rho$           | .947  | .953  | .954  | .947  | .955   |       |
| volatility of log TFP innov's $\sigma$          | .0067 | .0066 | .0066 | .0067 | .0065  |       |
| matching function elasticity $\alpha$           | .50   | .50   | .50   | .32   | .32    |       |
| <b>Targeted moments</b>                         |       |       |       |       |        |       |
| average unemployment rate                       | .055  | .055  | .055  | .055  | .055   | .057  |
| average UE prob.                                | .410  | .410  | .410  | .410  | .410   | .410  |
| average EU prob.                                | .024  | .024  | .024  | .024  | .024   | .024  |
| average EE prob.                                | .020  | .020  | .020  | .020  | 0      | .020  |
| std( <i>Quarterly</i> log ALP)                  | .0198 | .0198 | .0198 | .0193 | .0192  | .0197 |
| persistence of <i>Quarterly</i> log ALP innov's | .884  | .884  | .888  | .877  | .886   | .885  |
| <b>Untargeted moments</b>                       |       |       |       |       |        |       |
| std(log unempl. rate)                           | .0058 | .0048 | .0043 | .0028 | .0032  | .214  |
| std(log UE prob.)                               | .0064 | .0054 | .0050 | .0031 | .0036  | .147  |
| std(log EU prob.)                               | 0     | 0     | 0     | 0     | 0      | .112  |
| elasticity of UE prob w.r.t. TFP                | .332  | .271  | .244  | .160  | .178   |       |
| std(log UE prob.)/std(log ALP)                  | .317  | .268  | .246  | .156  | .178   |       |
| elasticity of UE prob. to vacancies/unempl.     | .37   | .38   | .38   | .27   | .32    | .32   |
| correlation(vacancies,unempl.)                  | -.89  | -.85  | -.83  | -.90  | -.85   | -.86  |

Table 1: Calibration and quantitative results: Baseline model

higher value of climbing the job ladder, hence a larger surplus appropriated by firms that hire unemployed workers, which must then translate into larger hiring costs to match the same, observed job finding probability (JFP) from unemployment of 41% per month. The other parameter values are essentially unchanged.<sup>9</sup>

Given this similarity, we compare this calibration of the OJS model to a version where we shut down OJS ( $s = 0$ ) and recalibrate all parameters (except  $\lambda$  whose value is immaterial without OJS). The results are in the NO OJS column. The comparison reveals that the model with OJS features very modest amplification of aggregate TFP shocks, with the log job finding probability varying between a quarter and a third as much as log ALP, as opposed to about ten times in the data, and yet this is even lower without OJS. This difference is entirely due to the impact of OJS on the estimated elasticity of the matching function  $\hat{\alpha}$ , which is raised from .32 to .5. To corroborate this claim, in column OJS-IV we fix the OJS calibration of column OJS-I and just replace the value of the matching function elasticity with the lower value we estimate under NO OJS in the last column. Amplification reverts to a level below the model without any OJS. This is intuitive: once we allow for procyclical employment to create congestion for the unemployed job seeker, vacancies must be more effective in order to generate the variation in the job finding probability from unemployment that we observe in the data and that we use for this estimation. In turn, a higher estimated elasticity implies that vacancies create less mutual congestion, making the volume of new job creation more responsive to its returns, therefore to aggregate shocks. The other two effects

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<sup>9</sup>Hiring costs are a constant and high (around .8) share of output in each steady state calibration. This is a by-product of the assumption that leisure yields no value,  $b = 0$ , so that all matches are viable. In this case, the firm appropriates all output from unemployed hires, as well as all marginal output from employed hires. This total “new output” times the meeting probability  $\phi(\theta)$  is, by free entry, proportional to hiring costs, through a constant discount factor, and by stationarity equal to the output loss from exogenous separations, namely total output times  $\delta$ . Therefore, hiring costs and total output in steady state are always proportional to each other, with a constant ratio that depends only on  $\beta$  and  $\delta$ , parameters whose values we keep fixed across calibrations. The ratio is high (.8), reflecting modest discounting and separations. When  $b > 0$ , the rents that vacancy-posting firms expect to receive are smaller than total marginal output from new hires, because unemployed hires need to be compensated for their opportunity cost of time. Hence, when output changes across calibrations, and with it proportionally output loss due to separations, in turn equal to output gain from new hires, the expected returns to hiring change less than proportionally, and hiring costs with them. In this case, hiring costs are a decreasing share of output as output increases.



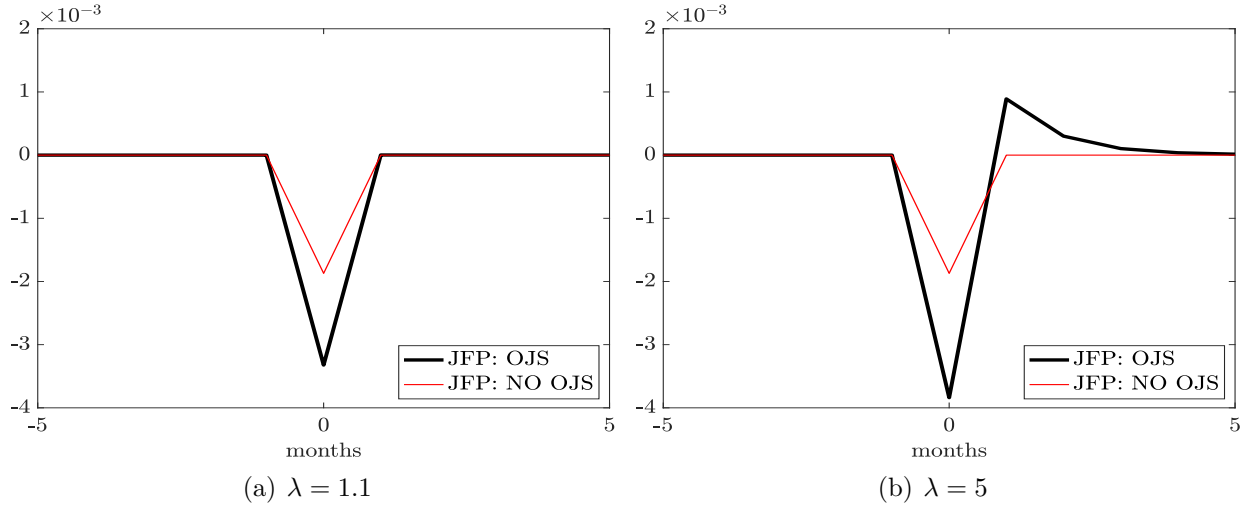


Fig. 1: Response of Job Finding Probability (JFP) to one-period negative TFP shock

of OJS on the calibration, the implied change in hiring costs and moments of the aggregate impulse, explain the remaining, negligible difference in amplification.

The OJS results in the first three columns also show that greater dispersion in match quality (a lower  $\lambda$ ) helps volatility. This is because, as explained earlier, dispersion raises the future expected returns from climbing the job ladder through OJS, more so when the level of TFP is high, because TFP multiplies match quality. Firms appropriate these returns, so that a positive shock raises the returns to hiring both employed and especially unemployed job applicants, more so when match draws have a ticker tight tail. In addition, as firms post more vacancies, they raise the meeting probability for employed job searchers, further raising the future returns to OJS, hence to hiring, which has a multiplier effect.

In Figure 1 we illustrate the impulse response of the (log of the) job finding probability from unemployment to a 1% decline in log TFP that lasts only one period, in calibrations OJS-I ( $\lambda = 1.1$ ), OJS-III ( $\lambda = 5$ ) and NO OJS.<sup>10</sup> The larger amplification warranted by

<sup>10</sup>We prefer this way to illustrate IRFs to persistent shocks vis a vis the more conventional choice of setting in each model the size (one standard deviation) and persistence of innovations to the values estimated for that model. The goal of this unconventional choice, that we maintain from now on, is to facilitate comparison between models, specifically their ability to propagate the *same* aggregate shock. It is important however, to keep in mind that estimated aggregate TFP processes often do differ between models, hence there is no immediate connection between the size of the IRFs of the JFP that we illustrate in the figures and the unconditional volatility of JFP reported in the tables.

OJS, through the estimated elasticity of the matching function, is clear. The canonical model with no OJS has, as is well known, no transitional dynamics in job market tightness and transition rates. The job finding probability returns immediately, like TFP, to its steady state value. In the OJS case, however, we observe some propagation, which is, to the best of our knowledge, a novel result. In this case, the job finding probability overshoots the steady state, and then slowly declines back. We identify the three opposing forces through which OJS affects amplification and propagation.

On the one hand, there are two *composition* effects, one between employment and unemployment and one within employment, which accelerate convergence. First, after a negative TFP shock, the temporarily higher unemployment and lower employment improve the quality of the jobseekers' pool from the viewpoint of firms, that earn higher rents from hiring the unemployed. Second, within employment, the temporary decline in the contact probability slows down reallocation on the job ladder and leads to a temporary deterioration of the employment distribution, or to an increase in misallocation, which makes employed workers more "poachable". Both composition effects raise the returns to job creation, accelerating the recovery. Neither composition effect exists without OJS. On the other hand, OJS also generates a *congestion* effect: as job postings recover and unemployment declines, the employed still create some congestion, making it harder for the unemployed to find jobs. In other words, the pool of job applicants improves, both across employment states and within employment, but locating the better sub-pool (the unemployed) becomes increasingly hard. In this calibration, the first effect dominates on impact, explaining the overshooting, because, after one period of slow job creation, firms benefit from a return to the initial level of TFP, but the composition of job search, both across employment states and within employment, is better than the initial one. The second effect is smaller, and unfolds more gradually. Comparison of the two panels of Figure 1 further shows that the first composition effect, which causes the job finding probability to overshoot, is more potent when match quality is less dispersed (Panel (b),  $\lambda = 5$ ). This is intuitive: with lower dispersion in match quality,

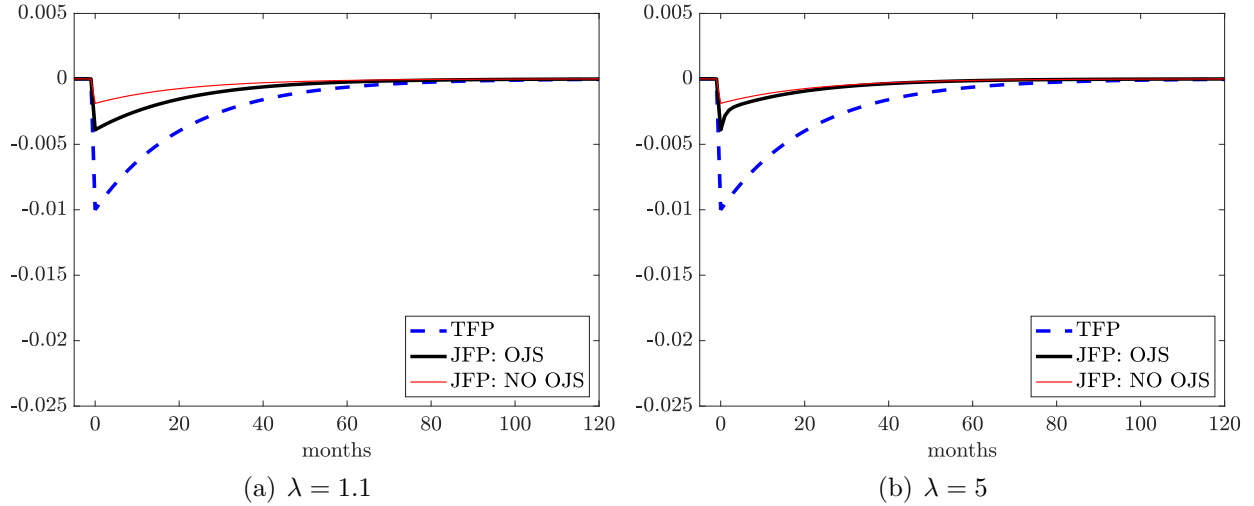


Fig. 2: Response of Job Finding Probability (JFP, solid) to mean-reverting negative Total Factor Productivity shock (TFP, dashed)

competition between employers is more intense, the MPL gap is small, and the returns to hiring an employed worker are low. Thus, the shift of the jobseekers' pool towards more unemployed workers following a negative TFP shock is more beneficial to firms.

In Figure 2 we illustrate the Impulse Response Function of the (log of the) job finding probability from unemployment to a 1% negative innovation in log TFP, which then mean-reverts at the same rate  $\rho$ , the value estimated in the NO OJS model, in the same calibrations as Figure 1. The amplification and propagation of the shock now differs qualitatively from the case analyzed in Figure 1. The impact response is larger with OJS because of the higher estimated matching function elasticity. The distribution of employment of the ladder is a slow-moving variable, which deteriorates as the job contact probability remains lower for a while and can no longer offset exogenous separations. This rising misallocation of employment feeds back, through the composition and congestion effects described above, on job creation itself. With lower match inequality and less scope for reallocation (Panel (b),  $\lambda = 5$ ), convergence is faster, because the distribution of employment moves less, but the impact is slightly stronger, because less muted by composition effects. In the limit as  $\lambda \rightarrow \infty$  all matches are identical and the model reduces to a special case of Robin (2011)

with identical workers, which then features high response on impact, but modest propagation and overall volatility.

### 6.3 Endogenous separations to unemployment

Finally, we turn to the version of the model with endogenous job destruction described and analyzed in Section 5, which is our preferred specification. This version features additional parameters. A conventional choice in the literature, motivated by empirical evidence on the age profile of firm-level productivity and worker separation rates, formalizes idiosyncratic match-specific shocks as a persistent process, typically an AR(1) in logs with mean zero. In our model, this process alone cannot generate our target 2.4% monthly probability of separation to unemployment, unless we assume a very high value of leisure  $b$ , close to mean productivity (normalized to one), thus a tiny match surplus. The reason is that, for plausible size of its innovations, the AR(1) process concentrates much of the mass of its ergodic distribution near its unit mean; therefore, the probability at each point in time that a match hits the equilibrium separation threshold is very small, unless the value of leisure  $b$  is very close to the unit mean productivity. In that case, though, the model with NO OJS is already known to generate large amplification, while our goal is to study the effects of OJS in a more plausible environment. Accordingly, we set the value of leisure  $b$  at .75. We assume that exogenous separation shocks occur with probability  $\delta = 1\%$  in all models. Because this exogenous component is acyclical, however, and the endogenous separation threshold is far from the mass of the match distribution, this formalization cannot reconcile average level and cyclical volatility of the separation rate. To resolve this tension, we introduce an infrequent, “large” idiosyncratic shock, which reduces match productivity by a certain percentage, but does not necessarily cause a separation. This third component makes endogenous separations more likely in recessions, when aggregate TFP drags the productivity distribution closer to the threshold. Accordingly, we choose the parameters of the “small” AR(1) idiosyncratic shock (persistence and standard deviation) and those of the “large” shock (probability of

arrival and proportional size) to match the average level of the EU separation probability and the unconditional volatility of its HP filtered log, which equals .112. To avoid introducing yet additional parameters, we calibrate the sampling distribution  $\Gamma$  to equal the ergodic distribution of the match quality process.

The algorithm to calibrate the model's parameters from steady state equilibrium and to compute a stochastic equilibrium are only slightly more complex than in the exogenous separation case. Given parameter values, the acceptance cutoff function  $\hat{y}(\cdot)$  can be computed beforehand by solving (10) by value function iteration. Steady state equations for the employment distribution now depend on the stationary level of TFP  $z$ , which determines the feasible matching set through the cutoff  $\hat{y}(z)$ . Because of idiosyncratic shocks, endogenous separations exist also in steady state. Thus, EU transition probability in steady state equilibrium contains the endogenous object  $L(\hat{y}(z))/L(\bar{y})$  which depends on unobserved parameters  $s, b, \Gamma, \pi$  etc. So we have to loop over the values of these parameters.

The simulation of a stochastic equilibrium proceeds as before. We draw a path of  $\{z_t\}_{t=0}^T$ . Then, given chosen initial conditions  $L_0$ , thus  $u_0$ , we use the free entry condition (11) to find the value of  $\theta_0$ , and finally use this value in (12), (13) and (14) to update  $L_1$ , thus  $u_1$ , and so on for every  $t \geq 1$ .

Table 2 reports the quantitative results, which now include statistics on the job contact probability, which differs from the job finding probability because some unemployed job applicants are rejected. As well known in the business cycle search literature, endogenous separations greatly amplify the volatility of unemployment through the contribution of countercyclical EU probability. This, however, comes at a dual cost.

First, the volatility of the UE exit probability from unemployment remains a valid empirical target, that the model still needs to explain. Improvement in explaining the volatility of unemployment through job destruction does not resolve the issue of the volatility of job creation. Endogenous separations help also in this respect, because not all new matches are acceptable, especially in recessions, a powerful lever on firms' incentives to post vacancies.

| Model $\Rightarrow$                                | OJS   | NO OJS | Data  |
|--|-------|--------|-------|
| <b>Parameters calibrated externally</b>            |       |        |       |
| discount factor $\beta$                            | .995  | .995   |       |
| flow value of leisure $b$                          | .75   | .75    |       |
| exogenous separation prob. $\delta$                | .010  | .010   |       |
| persistence of log match quality $\rho_y$          | .99   | .99    |       |
| volatility of log match quality innov's $\sigma_y$ | .002  | .002   |       |
| <b>Parameters calibrated internally</b>            |       |        |       |
| OJS efficiency $s$                                 | .086  | 0      |       |
| persistence of log TFP innov's $\rho$              | .962  | .974   |       |
| volatility of log TFP innov's $\sigma$             | .007  | .007   |       |
| prob. of large idiosyncratic shock                 | .074  | .045   |       |
| size of large idiosyncratic shock                  | .124  | .152   |       |
| elasticity of contact prob. to tightness $\alpha$  | .450  | .322   |       |
| <b>Targeted moments</b>                            |       |        |       |
| average unemployment rate                          | .055  | .055   | .057  |
| average UE prob.                                   | .414  | .416   | .410  |
| average EU prob.                                   | .024  | .024   | .024  |
| std(log EU prob.)                                  | .112  | .112   | .112  |
| average EE prob.                                   | .020  | 0      | .020  |
| std( <i>Quarterly</i> log ALP)                     | .0197 | .0196  | .0197 |
| persistence of <i>Quarterly</i> log ALP innov's    | .886  | .888   | .885  |
| <b>Untargeted moments</b>                          |       |        |       |
| std(log unempl. rate)                              | .113  | .117   | .214  |
| std(log UE prob.)                                  | .035  | .040   | .147  |
| elasticity of EU prob w.r.t. TFP                   | -3.63 | -2.80  |       |
| elasticity of UE prob w.r.t. TFP                   | 1.42  | 1.42   |       |
| std(log UE prob.)/std(log ALP)                     | 1.74  | 1.97   |       |
| elasticity of UE prob. to tightness                | .499  | .345   | .49   |
| elasticity of UE prob. to vacancies/unempl.        | .249  | .345   | .32   |
| elasticity of contact prob. to vacancies/unempl.   | .223  | .323   |       |
| correlation(vacancies/unempl.)                     | -.36  | .276   | -.86  |

Table 2: Calibration and quantitative results: Endogenous separations model

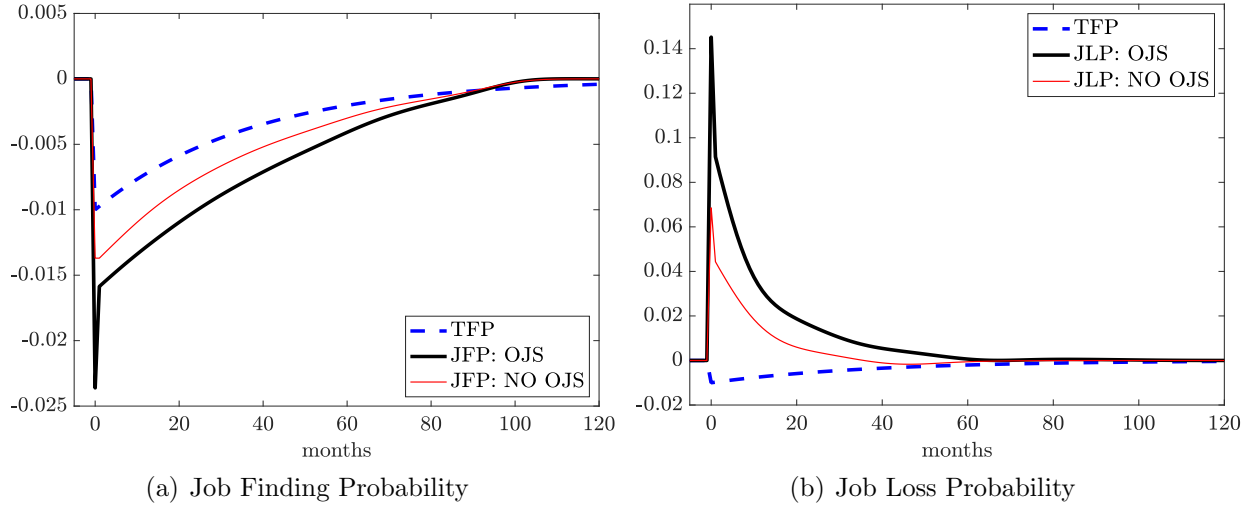


Fig. 3: Responses of Job Finding Probability (JFP) and Job Loss Probability (JLP) to mean-reverting negative Total Factor Productivity shock (TFP): Endogenous Job Destruction model

While still far from the empirical target, the UE probability volatility is greatly increased in this model, relative to the previous version with exogenous separations.

Second, waves of layoffs during recessions raise the pool of workers available for hire, stimulating vacancy creation and even turning the Beveridge curve upward-sloping (Mortensen and Nagypal 2007). For this reason, the business cycle search literature has not pursued this endogenous separation avenue, which was central to the seminal steady state analysis by Mortensen and Pissarides (1994). Fujita and Ramey (2012) show that OJS can help remedy this undesirable byproduct of endogenous separations, while bringing the model closer to the empirical evidence that layoffs do spike in every recession. Intuitively, when search continues on the job, unemployed and employed job searchers are closer substitutes in the eyes of the firm, hence vacancy creation responds less to the sheer size of unemployment, than in the baseline model with unemployed search alone. Consequently, the large inflow of unemployed made available by a wave of endogenous separations does not, per se, stimulate vacancy postings as much.

We find the same result: the Beveridge curve slopes up without OJS, and back down (although not as much as in the data) with it. Fujita and Ramey make their case intro-

ducing OJS in a business cycle version of Mortensen and Pissarides (1994), where wages are determined by Nash Bargaining and all new matches, whether with unemployed and already employed workers, start at the top of the ladder, and deteriorate stochastically over time. As a consequence, all outside offers are accepted, and the allocation of employment on the job ladder is irrelevant to the job creation decision. It has in fact been long thought that tracking the distribution of employment would make the problem of business cycles with random OJS intractable — Menzio and Shi (2011), for example, make this case forcefully. In Moscarini and Postel-Vinay (2013) we showed that equilibrium remains tractable when Rank-Preserving, namely when workers always prefer a more productive match, and that this is the case under commitment by firms to employment contracts without renegotiation. Even more tractable is the case studied here, where we allow firms to renegotiate wage contracts following outside offers. The employment distribution on a job ladder, albeit possibly infinitely-dimensional, is a predetermined state variable that, under the sequential auctions protocol, does not need to be forecasted, but is backward-looking, thus can be computed recursively along the way.

Figure 3 reports the impulse responses of the accession rate from and separation rate to unemployment to the usual 1% TFP shock, returning to its long-run mean at the rate estimated in the NO OJS model. The effects of OJS are, once again, visible, but the main novelty is the size of the response in both models.

We conclude that endogenous separations both to unemployment and to other jobs are very promising joint ingredients of a successful quantitative business cycle model of the US labor market.

## 7 Extensions

To gain further understanding of the role of OJS in shaping cyclical fluctuations in the aggregate labor market, we now extend the model in two different directions. First, we introduce in the hiring technology a new parameter whose value we can fine tune to control



the amplification of aggregate shocks in the model. Next, we introduce worker heterogeneity in human capital arising from learning-by-doing and skill loss during unemployment, which further affects the extent to which aggregate shocks are propagated in the model. In order to isolate the role of these features, we introduce them in the baseline model with exogenous job destruction only.

## 7.1 Screening costs

In this first extension, we introduce an additional source of hiring cost, which we interpret as a screening cost. As before, the firm pays a vacancy posting cost  $\kappa$  to advertise the position, receive applications, and observe the employment status of the applicant. But now, in order to hire a job applicant and to observe the match quality draw  $y'$ , as well as the existing match quality  $y$  of an employed applicant with their current employer, the firm must also pay an additional cost  $\hat{\kappa}$ . This parameter captures all investment into hiring, such as interviewing, screening, and training, that the firm has to make *after* receiving job applications. As pointed out by Pissarides (2009), unlike advertising/vacancy costs, this part of the hiring costs are unaffected by market tightness and congestion, a powerful general equilibrium force that tends to offset aggregate shocks. In an estimated equilibrium search model, Christiano et al. (2016) exploit this insight to resolve the tension highlighted by Shimer (2005).

The firm would not pay the advertising cost  $\kappa$  to post the vacancy in the first place if it was not then willing to pay the screening/training cost  $\hat{\kappa}$  to hire at least some of the job applicants it hopes to come in contact with, at least unemployed job applicants who are homogeneous and most profitable. This property follows from our timing of events: vacancy posting and matching occur in the same period, i.e. under the same information set. Employed job applicants are also homogeneous *ex ante* (i.e. before paying the screening cost), but less profitable. Therefore, it is possible that, for low enough levels of aggregate TFP and the MPL Gap, firms may be unwilling to pay the screening cost to hire an employed job applicant. In order to avoid a complete shutdown of the economy, should this situation

arise, we assume that the screening cost and the value of leisure are still small enough that firms are always willing to post vacancies, because they will encounter sufficiently often some unemployed job applicants, whom they will then be willing to always hire. We also maintain the assumption that the value of leisure is small enough that firms are willing to match with any unemployed job applicant, conditional on screening/training. The free entry condition now writes as:<sup>11</sup>

$$\kappa \frac{\theta_t}{\phi(\theta_t)} = P(u_t) [\mathcal{Z}(z_t)\mu - \mathcal{B} - \hat{\kappa}] + [1 - P(u_t)] \max \langle \mathcal{Z}(z_t)\Omega_t - \hat{\kappa}, 0 \rangle \quad (20)$$

Such a screening/training cost thus allows us to reduce the size of the expected returns to job creation. Accordingly, the economy achieves substantial amplification. The intuition is simple: small average returns are more sensitive to a given aggregate impulse. To leverage this mechanism, Hagedorn and Manovskii (2008) raise the value of leisure  $b$  closer to average output (here equal to one) and thus reduce total match surplus. We do it through additional hiring costs, paid only once, right after meeting. In this sense, our mechanism is formally (albeit not conceptually) similar to Hall and Milgrom (2008)'s cost for the firm of continuing wage negotiations, which is never paid by the firm in equilibrium, but raises the wage and shrinks profits. One advantage of our approach, in terms of computation and understanding of the mechanism, is that a high value of  $b$  would make some jobs infeasible and lead to endogenous separations.

Table 3 repeats the exercises of Table 1. In Table 3, we keep dispersion in match quality constant at  $\lambda = 1.1$ , but vary the magnitude of the screening cost exogenously such that screening costs account for a share of 0%, 50%, and 90% of total hiring costs in OJS columns I, II, and III, respectively (column OJS-I which has a screening cost share of 0% is only a replica of the same column in Table 1, for comparison). Note that the more important hiring costs, the more cyclically volatile the job contact probability, hence reallocation on

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<sup>11</sup>An even simpler case arises when the screening cost is small enough that the firm is always willing to hire any job applicant, whether unemployed or employed, as will be the case in practice in almost all periods in our simulations. Since the screening cost is sunk when match quality is revealed, it still affects the size of the expected returns to hiring, although not the hiring decision itself. The free entry condition further simplifies to  $\hat{\kappa} + \kappa\theta_t/\phi(\theta_t) = P(u_t) [\mathcal{Z}(z_t)\mu - \mathcal{B}] + [1 - P(u_t)] \mathcal{Z}(z_t)\Omega_t$ .

| <b>Model <math>\Rightarrow</math></b>           | <b>OJS</b> |           |            | <b>NO OJS</b> |           | <b>Data</b> |
|---|------------|-----------|------------|---------------|-----------|-------------|
|   | <b>I</b>   | <b>II</b> | <b>III</b> | <b>I</b>      | <b>II</b> |             |
| <b>Parameters calibrated externally</b>         |            |           |            |               |           |             |
| discount factor $\beta$                         | .995       | .995      | .995       | .995          | .995      |             |
| flow value of leisure $b$                       | 0          | 0         | 0          | 0             | 0         |             |
| exogenous separation prob. $\delta$             | .024       | .024      | .024       | .024          | .024      |             |
| match inequality $\lambda$                      | 1.1        | 1.1       | 1.1        | —             | —         |             |
| share of screening in hiring costs              | 0          | .5        | .9         | .5            | .9        |             |
| <b>Parameters calibrated internally</b>         |            |           |            |               |           |             |
| OJS efficiency $s$                              | .176       | .176      | .176       | 0             | 0         |             |
| persistence of log TFP innov's $\rho$           | .947       | .941      | .908       | .955          | .955      |             |
| volatility of log TFP innov's $\sigma$          | .0067      | .0067     | .0067      | .0065         | .0065     |             |
| matching function elasticity $\alpha$           | .50        | .50       | .50        | .32           | .32       |             |
| <b>Targeted moments</b>                         |            |           |            |               |           |             |
| average unemployment rate                       | .055       | .055      | .053       | .055          | .054      | .057        |
| average UE prob.                                | .410       | .410      | .430       | .410          | .420      | .410        |
| average EU prob.                                | .024       | .024      | .024       | .024          | .024      | .024        |
| average EE prob.                                | .020       | .020      | .020       | 0             | 0         | .020        |
| std( <i>Quarterly</i> log ALP)                  | .0198      | .0198     | .0198      | .0197         | .0197     | .0197       |
| persistence of <i>Quarterly</i> log ALP innov's | .884       | .885      | .884       | .886          | .886      | .885        |
| <b>Untargeted moments</b>                       |            |           |            |               |           |             |
| std(log unempl. rate)                           | .0058      | .0101     | .0285      | .0064         | .0312     | .214        |
| std(log UE prob.)                               | .0064      | .0113     | .0323      | .0071         | .0346     | .147        |
| std(log EU prob.)                               | 0          | 0         | 0          | 0             | 0         | .112        |
| elasticity of UE prob w.r.t. TFP                | .332       | .609      | 2.04       | .355          | 1.71      |             |
| std(log UE prob.)/std(log ALP)                  | .317       | .559      | 1.61       | .355          | 1.72      |             |
| elasticity of UE prob. to vacancies/unempl.     | .37        | .37       | .38        | .32           | .32       | .32         |
| correlation(vacancies,unempl.)                  | -.89       | -.88      | -.83       | -.85          | -.85      | -.86        |

Table 3: Calibration and quantitative results: Screening cost model

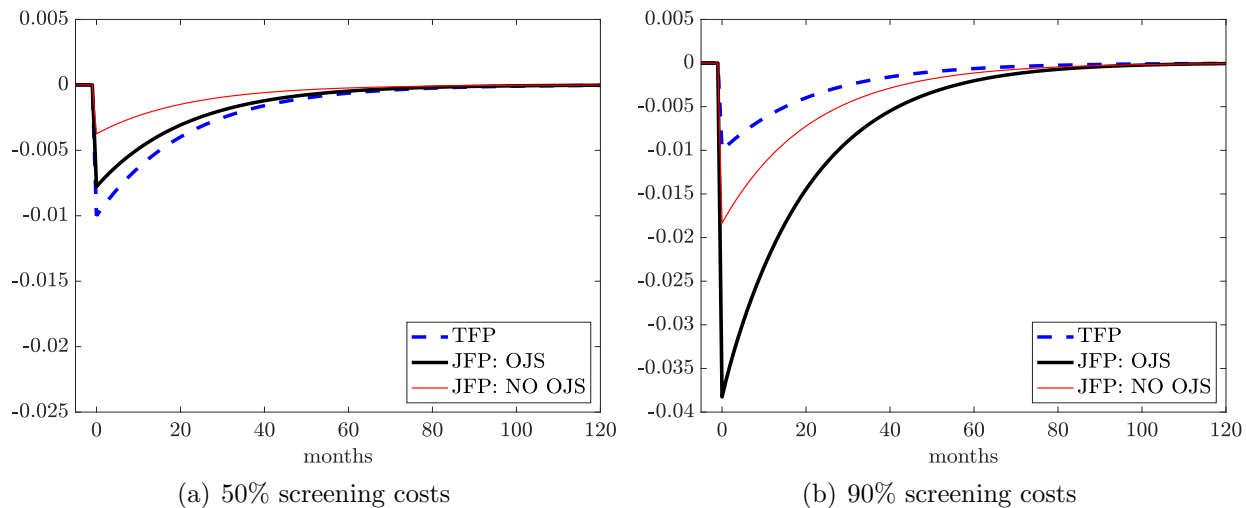


Fig. 4: Response of Job Finding Probability (JFP, solid) to mean-reverting negative Total Factor Productivity shock (TFP, dashed): Screening cost model

the ladder. Given the target volatility of ALP, this requires a much less persistent TFP impulse. Finally, the NO-OJS column shows results from a version of the model where we shut down OJS by setting  $s = 0$  and set the screening cost share to 90% of total hiring costs.

Figure 4 echoes Figure 2 for the new screening costs calibrations. The larger amplification of a given TFP shock guaranteed by screening costs makes the effects of reallocation on the cyclical job ladder even more clearly visible.

## 7.2 Worker heterogeneity

In this second extension, we revert to the case of zero screening cost but assume that each worker has individual-specific, time-varying human capital  $h_t$  evolving stochastically over a finite grid  $0 = h_1 < \dots < h_K = 1$  according to a state-dependent Markov process, similar to Ljungqvist and Sargent (1998). Specifically, the human capital of an employed worker changes from  $h$  to  $h'$  with probability  $\pi_e(h, h')$ , that of an unemployed worker or a laid off worker (on impact of the layoff) with probability  $\pi_u(h, h')$ . Besides having an important tradition in the unemployment literature, this extension allows us to control the relative appeal of hiring unemployed and employed job applicants, thus to fine tune of the composition effects that we highlighted. Intuitively, employed job applicants, while more

expensive to hire, are also on average more productive.

Upon being hired (either from unemployment or from employment), a worker draws an idiosyncratic match quality ‘potential’  $y \in [\underline{y}, \bar{y}]$ ,  $y \sim \Gamma$ . Match output is then:

$$Y(z, y, h) = z \times (\underline{y} + (y - \underline{y}) h)$$

where  $h \in [0, 1]$  is the worker’s human capital and  $z$  is current aggregate TFP. The interpretation is that all matches have the same baseline productivity  $\underline{y}$ , and an idiosyncratic supplementary productivity potential  $y - \underline{y}$ . How much of that added productivity potential is realized depends on the worker’s human capital. Unemployed workers produce  $b$ , regardless of their human capital. We assume that  $b$  and  $\underline{y}$  are such that it is always marginally profitable to hire a worker with human capital  $h = 0$ , even in the worst TFP state.

Similar extensions of the basic search model have been applied elsewhere in the literature, in different contexts and with different purposes: Ljungqvist and Sargent (1998), Kehoe, Midrigan and Pastorino (2017), and, closer to this model, by Walentin and Westermarck (2018) in a model featuring OJS and Sequential Auction bargaining. In the context of our model, heterogeneity in human capital affects the firms’ relative returns to contacting an unemployed vs an employed jobseeker, generically to the detriment of the former, who have lower human capital in equilibrium. Specifically, a TFP shock now perturbs the distribution of human capital amongst employed and unemployed workers, which in turn impacts the returns to vacancy posting, and future hiring. Because the distribution of human capital is another slow-moving state variable, the effect of TFP shocks to future hiring rates is further propagated by that transmission channel.

The firm’s willingness to pay for a given match  $\bar{V}_t(y, h_t)$  now depends on the worker’s human capital as well as on match quality. Following the same reasoning as in Section 4:

$$\bar{V}_t(y, h_t) = \mathbb{E}_t \left[ \sum_{\tau=0}^{+\infty} (1 - \delta)^\tau \beta^{\tau+1} z_{t+\tau+1} \times (\underline{y} + (y - \underline{y}) h_{t+\tau+1}) \right] + \frac{\delta \beta}{1 - \beta(1 - \delta)} \frac{b}{1 - \beta}$$

where the  $t$  subscript on the expectation operator is now shorthand for conditioning on

current TFP  $z_t$  and worker's current human capital  $h_t$ . Note that for any pair  $(y, y')$ :

$$\bar{V}_t(y, h_t) - \bar{V}_t(y', h_t) = (y - y') \mathbb{E}_t \left[ \sum_{\tau=0}^{+\infty} (1 - \delta)^\tau \beta^{\tau+1} z_{t+\tau+1} h_{t+\tau+1} \right]$$

We next turn to the dynamics of the employment distribution. Let  $L_t(y, h)$  denote the measure of workers with human capital  $h$  employed in matches with quality within  $[y, y]$ . Further let  $u_t(h)$  denote the measure of unemployed workers with human capital  $h$ . Then:

$$L_{t+1}(y, h) = (1 - \delta) [1 - s\phi(\theta_t) \bar{\Gamma}(y)] \sum_{h'} \pi_e(h', h) L_t(y, h') + \phi(\theta_t) \Gamma(y) \sum_{h'} \pi_u(h', h) u_t(h')$$

The latter law of motion can be written more compactly in matrix form. Introducing the vector notation  $\mathbf{L}_t(y) = [L_t(y, h_1), \dots, L_t(y, h_K)]^\top$  and  $\mathbf{u}_t = [u_t(h_1), \dots, u_t(h_K)]^\top$ , and, for  $x = e, u$ , defining  $\mathbf{\Pi}_x$  as the  $K \times K$  matrix whose  $(i, j)$  entry is  $\pi_x(h_i, h_j)$ :

$$\mathbf{L}_{t+1}(y) = (1 - \delta) [1 - s\phi(\theta_t) \bar{\Gamma}(y)] \mathbf{\Pi}_e^\top \mathbf{L}_t(y) + \phi(\theta_t) \Gamma(y) \mathbf{\Pi}_u^\top \mathbf{u}_t$$

The (non-normalized) distribution of human capital amongst employed worker thus evolves following:

$$\mathbf{L}_{t+1}(\bar{y}) = (1 - \delta) \mathbf{\Pi}_e^\top \mathbf{L}_t(\bar{y}) + \phi(\theta_t) \mathbf{\Pi}_u^\top \mathbf{u}_t$$

Finally, the (non-normalized) distribution of human capital amongst unemployed worker evolves following:

$$u_{t+1}(h) = [1 - \phi(\theta_t)] \sum_{h'} \pi_u(h', h) u_t(h') + \delta \sum_{h'} \pi_u(h', h) L_t(y, h')$$

or, in matrix form:

$$\mathbf{u}_{t+1} = \mathbf{\Pi}_u^\top \cdot \{[1 - \phi(\theta_t)] \mathbf{u}_t + \delta \mathbf{L}_t(\bar{y})\}$$

Note that the total measure of unemployed workers,  $u_t = \sum_h u_t(h) = 1 - \sum_h L_t(\bar{y}, h)$  obeys a familiar law of motion. Summing the last equation over  $h$  and remembering that  $\sum_h \pi_x(h', h) = 1$  for all  $h'$  and  $x = e, u$ , we obtain  $u_{t+1} = [1 - \phi(\theta_t)] u_t + \delta(1 - u_t)$ . Similarly, the total measure of workers employed in matches with quality  $y$  or less,  $L_t(y) = \sum_h L_t(y, h)$ , follows  $L_{t+1}(y) = (1 - \delta) [1 - s\phi(\theta_t) \bar{\Gamma}(y)] L_t(y) + \phi(\theta_t) \Gamma(y) u_t$ .

In steady state, the laws of motion derived above boil down to:

$$\begin{aligned}\{\mathbf{I}_K - [1 - \phi(\theta)] \mathbf{\Pi}_u^\top\} \mathbf{u} &= \delta \mathbf{\Pi}_u^\top \mathbf{L}(\bar{y}) \\ \{\mathbf{I}_K - (1 - \delta) \mathbf{\Pi}_e^\top\} \mathbf{L}(\bar{y}) &= \phi(\theta) \mathbf{\Pi}_u^\top \mathbf{u}\end{aligned}$$

which implies

$$\left\{ \mathbf{I}_K - [1 - \phi(\theta)] \mathbf{\Pi}_u^\top - \delta \mathbf{\Pi}_u^\top [\mathbf{I}_K - (1 - \delta) \mathbf{\Pi}_e^\top]^{-1} \phi(\theta) \mathbf{\Pi}_u^\top \right\} \mathbf{u} = \mathbf{0}_K$$

so that the steady-state distribution of human capital amongst unemployed workers is the vector  $\mathbf{u}$  in the null space of the matrix in curly brackets whose elements sum to  $u = \frac{\delta}{\delta + \phi(\theta)}$ . Once  $\mathbf{u}$  is known, the joint distribution of match quality and human capital amongst employed workers is given by:

$$\{\mathbf{I}_K - (1 - \delta) [1 - s\phi(\theta)\bar{\Gamma}(y)] \mathbf{\Pi}_e^\top\} \mathbf{L}(y) = \phi(\theta)\Gamma(y)\mathbf{\Pi}_u^\top \mathbf{u}$$

We finally adjust the free-entry condition to this extension of our model. Let:

$$\begin{aligned}\mathcal{Z}(z_t) &= \sum_{\tau=0}^{+\infty} (1 - \delta)^\tau \beta^{\tau+1} \mathbb{E}[z_{t+\tau+1} | z_t] \\ \mathcal{W}_{u,t}(z_t) &= \sum_h \frac{u_t(h)}{u_t} \sum_{h'} \pi_u(h, h') \sum_{\tau=0}^{+\infty} (1 - \delta)^\tau \beta^{\tau+1} \mathbb{E}[z_{t+\tau+1} h_{t+\tau+1} | z_t, h_{t+1} = h'] \\ \text{and } \mathcal{W}_{e,t}(z_t) &= \sum_h \frac{L_t(\bar{y}, h)}{1 - u_t} \sum_{h'} \pi_e(h, h') \sum_{\tau=0}^{+\infty} (1 - \delta)^\tau \beta^{\tau+1} \mathbb{E}[z_{t+\tau+1} h_{t+\tau+1} | z_t, h_{t+1} = h']\end{aligned}$$

Note that the expected returns, per unit of match quality, from hiring an unemployed and an employed worker, (resp.)  $\mathcal{W}_{u,t}(z_t)$  and  $\mathcal{W}_{e,t}(z_t)$ , are functions not only of current TFP  $z_t$ , but also of the current distribution of human capital in, respectively, the populations of unemployed and employed workers. Dependence on those latter state variables is subsumed into the  $t$  subscript. The free entry condition writes as:

$$\kappa \frac{\theta_t}{\phi(\theta_t)} = P(u_t) [\mathcal{Z}(z_t) \underline{y} + \mathcal{W}_{u,t}(z_t) (\mu - \underline{y}) - \mathcal{B}] + [1 - P(u_t)] \mathcal{W}_{e,t}(z_t) \Omega_t$$

where the MPL Gap,  $\Omega_t$ , is defined as in the basic model. Note that, when computing the expected return of a random contact with a worker (the r.h.s. of the free-entry equation

above), the firm must take expectations over the sampling distribution of match qualities, as before, but also over the distribution of worker human capital. The latter is reflected in the variables  $\mathcal{W}_{u,t}(z_t)$  and  $\mathcal{W}_{e,t}(z_t)$ , which involve the current distributions of human capital amongst employed and unemployed workers, respectively, and evolve only as fast as the learning-by-doing process will allow.

The functions  $\mathcal{W}_{u,t}(z_t)$  and  $\mathcal{W}_{e,t}(z_t)$  can be expressed in terms of the matrix notation introduced above, using the assumption that the process of individual human capital is independent of the aggregate TFP process. Defining the function  $\mathbf{Z} : \mathbb{R} \mapsto \mathbb{R}^K$  as:

$$\mathbf{Z}(z) = \left( \sum_{\tau=0}^{+\infty} \beta^{\tau+1} (1-\delta)^\tau \mathbf{\Pi}_e^\tau \mathbb{E}[z_{t+\tau+1} | z_t = z] \right) \cdot \mathbf{h}$$

where  $\mathbf{h} = [h_1, \dots, h_K]^\top$  is the  $K \times 1$  vector of human capital values, we have that:

$$\mathcal{W}_{u,t}(z_t) = \frac{1}{u_t} \mathbf{u}_t^\top \mathbf{\Pi}_u \mathbf{Z}(z_t) \quad \text{and} \quad \mathcal{W}_{e,t}(z_t) = \frac{1}{1-u_t} \mathbf{L}_t(\bar{y})^\top \mathbf{\Pi}_e \mathbf{Z}(z_t)$$

To explore the quantitative implications of this extension, we parameterize learning-by-doing using a slightly amended version of Ljungqvist and Sargent (1998):

$$\mathbf{\Pi}_e = (1 - \pi_e) \begin{pmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ \vdots & & \ddots & & \vdots \\ 0 & \cdots & 0 & 1 & 0 \\ 0 & \cdots & 0 & 0 & 1 \end{pmatrix} + \pi_e \begin{pmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & & \ddots & & \vdots \\ 0 & \cdots & 0 & 0 & 1 \\ 0 & \cdots & 0 & 0 & 0 \end{pmatrix}$$

$$\mathbf{\Pi}_u = (1 - \pi_u) \begin{pmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ \vdots & & \ddots & & \vdots \\ 0 & \cdots & 0 & 1 & 0 \\ 0 & \cdots & 0 & 0 & 1 \end{pmatrix} + \pi_u \begin{pmatrix} 0 & 0 & 0 & \cdots & 0 \\ 1 & 0 & 0 & \cdots & 0 \\ \vdots & & \ddots & & \vdots \\ 0 & \cdots & 1 & 0 & 0 \\ 0 & \cdots & 0 & 1 & 0 \end{pmatrix}$$

In words: in employment, skills appreciate by one notch with probability  $\pi_e$  each period; in unemployment, skills depreciate by one notch with probability  $\pi_u$  each period.

The results reported in Table 4 were produced using  $K = 11$  equally spaced human capital levels,  $\pi_e = .042$ , and  $\pi_u = .28$ . Those numbers imply that it takes on average 20 years of continuous employment for a worker's human capital to rise from the lowest to the



| Model $\Rightarrow$                             | OJS   |       |       | NO OJS | Data  |
|---|-------|-------|-------|--------|-------|
| <b>Parameters calibrated externally</b>         | I     | II    | III   |        |       |
| discount factor $\beta$                         | .995  | .995  | .995  | .995   |       |
| flow value of leisure $b$                       | 0     | 0     | 0     | 0      |       |
| exogenous separation prob. $\delta$             | .024  | .024  | .024  | .024   |       |
| match inequality $\lambda$                      | 1.1   | 3     | 5     | —      |       |
| <b>Parameters calibrated internally</b>         |       |       |       |        |       |
| OJS efficiency $s$                              | .176  | .176  | .176  | 0      |       |
| persistence of log TFP innov's $\rho$           | .947  | .953  | .953  | .955   |       |
| volatility of log TFP innov's $\sigma$          | .0067 | .0066 | .0066 | .0066  |       |
| matching function elasticity $\alpha$           | .50   | .50   | .50   | .32    |       |
| <b>Targeted moments</b>                         |       |       |       |        |       |
| average unemployment rate                       | .055  | .055  | .055  | .055   | .057  |
| average UE prob.                                | .410  | .410  | .410  | .410   | .410  |
| average EU prob.                                | .024  | .024  | .024  | .024   | .024  |
| average EE prob.                                | .020  | .020  | .020  | 0      | .020  |
| std( <i>Quarterly</i> log ALP)                  | .0197 | .0197 | .0198 | .0197  | .0197 |
| persistence of <i>Quarterly</i> log ALP innov's | .885  | .885  | .885  | .885   | .885  |
| <b>Untargeted moments</b>                       |       |       |       |        |       |
| relative returns to contact, empl./unempl.      | 1.07  | .22   | .08   | —      |       |
| std(log unempl. rate)                           | .0060 | .0048 | .0043 | .0033  | .214  |
| std(log UE prob.)                               | .0067 | .0054 | .0050 | .0036  | .147  |
| std(log EU prob.)                               | 0     | 0     | 0     | 0      | .112  |
| elasticity of UE prob w.r.t. TFP                | .346  | .273  | .244  | .180   |       |
| std(log UE prob.)/std(log ALP)                  | .330  | .270  | .245  | .179   |       |
| elasticity of UE prob. to vacancies/unempl.     | .37   | .38   | .38   | .32    | .32   |
| correlation(vacancies,unempl.)                  | -.90  | -.85  | -.83  | -.86   | -.86  |

Table 4: Calibration and quantitative results: Human capital model

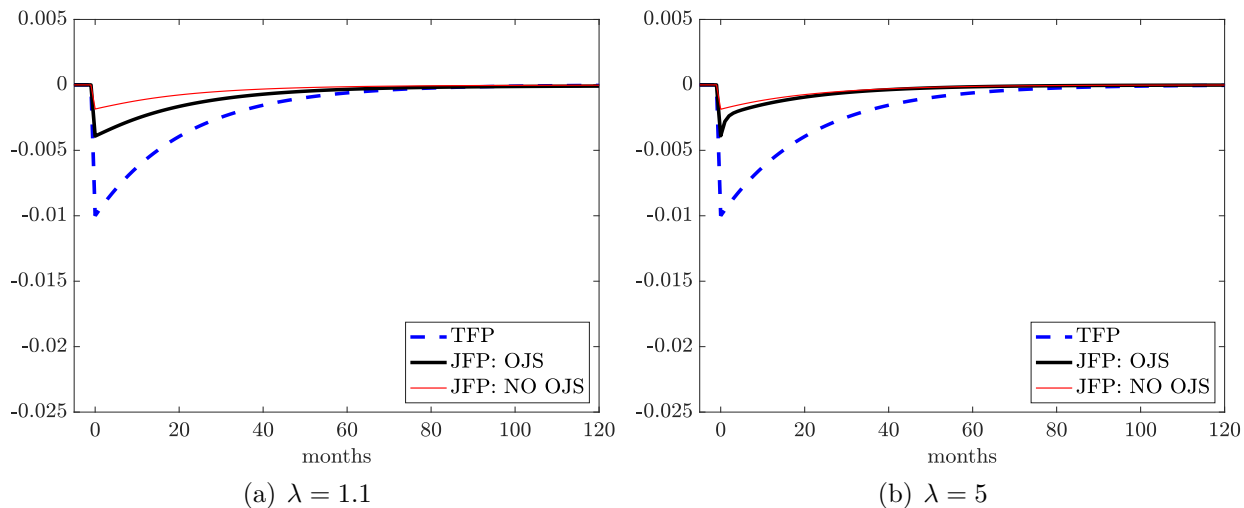


Fig. 5: Response of Job Finding Probability (JFP, solid) to mean-reverting negative Total Factor Productivity shock (TFP, dashed): Human capital model

highest level, and three years to fall from the highest to the lowest level. Even under this calibration, when match quality dispersion is set to  $\lambda = 1.1$ , a contact with the average employed worker is only 7% more valuable to employers than a contact with the average unemployed. Those relative returns drop quickly as the match quality distribution becomes more concentrated (an employed contact is only worth 8% of an unemployed contact when  $\lambda = 5$ ), as the average of a poaching firm becomes very small when all matches are close in quality.<sup>12</sup>

Figure 5 once again echoes Figure 2. While differences between the results of the two models are not immediately evident from a visual comparison of these two Figures, human capital dynamics do add to the propagation of TFP shocks. For example, in the  $\lambda = 1.1$  case, the half-life of the JFR is 26.8 months in the human capital model, compared to only 14.9 months in the baseline case.<sup>13</sup>

<sup>12</sup>For comparison, the value of contacting an employed worker varies between .3% and 3% of the value of an unemployed contact in the baseline model without worker heterogeneity.

<sup>13</sup>Denoting any of the IRF series plotted on Figures 2 and 5 by  $\{\text{IRF}_t\}_{t=1}^T$ , those half-lives are calculated as  $\frac{1}{T} \sum_{t=1}^{T-1} \frac{-\ln 2}{\ln \text{IRF}_{t+1} - \ln \text{IRF}_t}$ . Differences in JFR half-lives thus calculated are 20.0 months (human capital) *vs* 13.4 months (baseline) with  $\lambda = 3$ , and 17.1 months (human capital) *vs* 14.0 months (baseline) with  $\lambda = 5$ .

## 8 Conclusions

We present a tractable business cycle model with random job search both on and off the job and match heterogeneity, which then features a cyclical job ladder. Key to simplicity are ex post heterogeneity in match productivity, revealed only after meeting, wage renegotiation following an outside offer, and no sorting of workers into firms. We identify and quantify three channels through which on the job search affects the dynamic response of the economy's equilibrium to aggregate TFP shocks: congestion, composition of the search pool by employment status, and composition of employment by match quality, summarized by an index of average “poachability” of employed workers, the Marginal Productivity of Labor Gap. Some or all of these three effects are present in existing models, but all of them clearly emerge in our setup. We show that, in a standard calibration, the three effects combined both amplify and propagate aggregate shocks. On the job search offsets the tendency of endogenous separations into unemployment (important to explain the data) to tilt the Beveridge curve upward. We propose this framework as a tractable representation of the labor market for any business cycle model, and in ongoing work we integrate it into a full-fledged DSGE monetary model.

## APPENDIX

### A Comparative statics of Steady State equilibrium

In order to understand the business cycle properties of the model, we inspect analytically the comparative statics properties of its steady state equilibrium in response to changes in the level of TFP. This exercise is well-known to provide, *in the standard model without OJS*, a useful guidance to the quantitative performance of the stochastic model. Specifically, the steady state elasticity of the job finding probability to the level of TFP is an upper bound on their relative volatilities in the model simulation, which is tighter the more persistent TFP

innovations, as comparative statics correspond to fully persistent TFP changes.

Let  $\theta^*$  denote a solution to steady state free entry condition (16). The LHS of this equation is increasing in  $\theta$ , thus in  $\phi(\theta)$ . The RHS is the weighted average of the surplus from hiring an unemployed and (in expectation) an employed worker. We showed that the former surplus is larger, and it is clearly independent of  $\phi(\theta)$ . Its weight is increasing in  $u = \delta/(\delta + \phi(\theta))$ , hence decreasing in  $\phi(\theta)$ . Hence, to show that the RHS of (16) is decreasing in  $\phi(\theta)$ , thus steady state equilibrium ( $\theta^*$ ) is unique, it suffices to show that the MPL Gap term  $\Omega$  is decreasing in  $\phi(\theta)$ . For this last step, observe that  $\Omega = \int_{\underline{y}}^{\bar{y}} \bar{\Gamma}(y) dL(y)/(1-u)$ , where  $\bar{\Gamma}(y)$  is a decreasing function and  $L(y)/(1-u)$  is the normalized c.d.f of employment, which is easily verified to be FSD-increasing in  $\phi(\theta)$ : a higher contact probability reallocates faster employment up the ladder.

In the stochastic model, if the economy is in steady state equilibrium and a TFP innovation occurs at time  $t$ , on impact the employment distribution  $L_t(\cdot)$ , hence the unemployment rate  $u_t$  and the MPL Gap  $\Omega_t$ , all predetermined state variables, do not respond to the shock. Accordingly, to understand the immediate impulse response, we study analytically the steady state partial elasticity,  $\partial \ln \phi(\theta^*) / \partial \ln z$ , keeping  $u$  and  $\Omega$  constant. Over time, these state variables respond, and the economy fully adjusts, converging (barring more shocks) to a new steady state. Accordingly, to understand the evolution of the impulse response function and propagation, we study the total elasticity  $d \ln \phi(\theta^*) / d \ln z$ .

For simplicity, assume a Cobb-Douglas meeting function with elasticity  $\alpha$  of the meeting probability  $\phi(\theta)$ , so that  $\frac{\theta}{\phi(\theta)} \propto \phi(\theta)^{\frac{1-\alpha}{\alpha}}$ . Then, taking logs on both sides of the FEC, Equation (8) becomes:

$$\frac{1-\alpha}{\alpha} \ln \phi(\theta^*) = \text{constant} + \ln [P(u)(z\mu - b) + (1 - P(u))z\Omega]$$

where both the steady state unemployment rate  $u = (1 + \phi(\theta^*)/\delta)^{-1}$  and the MPL Gap (Equation (18)) depend on labor market tightness itself: the higher  $\theta^*$ , the higher contact rates, the better employment allocation, the lower unemployment and the MPL Gap.

To study the elasticity, we start with the canonical case of no OJS,  $s = 0$ :

$$\frac{1 - \alpha}{\alpha} \ln \phi(\theta^*) = \text{constant} + \ln(z\mu - b)$$

The meeting probability does not depend on predetermined, endogenous variables, such as  $u$  or  $\Omega$ , a reflection of the well-known property of the job finding probability in the canonical DMP model, as a jump variable with no transitional dynamics. Therefore, the canonical model features no propagation, and the partial and total elasticities are the same, both proportional to the inverse of the surplus from employment as a share of total revenues:

$$\frac{d \ln \phi(\theta^*)}{d \ln z} = \frac{\alpha}{1 - \alpha} \frac{z\mu}{z\mu - b} \quad (21)$$

For  $\alpha \geq .5$ , typically the empirically relevant range, this elasticity is always larger than one. The closest corresponding empirical magnitude, a regression coefficient of the log probability from unemployment on log Average Labor Productivity, exceeds 10. Hagedorn and Manovskii (2008) calibrate the relative surplus  $\frac{z\mu - b}{z\mu}$  to a small number, generating a large elasticity and large amplification.

Now reintroduce OJS ( $s > 0$ ). Fixing the predetermined variables,  $u$  and  $\Omega$ , the “impulse” partial derivative equals

$$\frac{\partial \ln \phi(\theta^*)}{\partial \ln z} = \frac{\alpha}{1 - \alpha} z \frac{u\mu + (1 - \delta)s(1 - u)\Omega}{u(z\mu - b) + (1 - \delta)s(1 - u)z\Omega}$$

This elasticity is still always larger than one (as long as  $z\mu - b > 0$  and  $\alpha \geq .5$ ) but smaller than in the case without OJS, because, compared to Equation (21), the positive term  $(1 - \delta)s(1 - u)z\Omega$ , measuring the expected returns to hiring an employed worker, is added both to the numerator and the denominator. In fact, as the importance of poaching increases, the value of this elasticity decreases. If there is no unemployment,  $u = 0$ , and  $\alpha = .5$ , clearly the elasticity is equal to one, because  $b$  plays no role in the cost of new hires, whose outside option ( $zy$  at the current job) grows in proportion to  $z$  as much as the new inside option ( $zy'$  at the poacher).

On the other hand, OJS generates interesting propagation. Without going through the algebra, it is easy to see that the difference between the total elasticity (taking into account the the effect of a change in  $\theta^*$  on long-run unemployment  $u$  and MPL Gap  $\Omega$ ) and the impulse response,

$$\frac{d \ln(\phi(\theta^*))}{d \ln z} - \frac{\partial \ln \phi(\theta^*)}{\partial \ln z}$$

is negative. Higher TFP implies, ultimately, lower unemployment. Hence, weight shifts from the expected returns to hiring an unemployed worker ( $z\mu - b$ ) to the lower expected returns to hiring an employed worker ( $z\Omega$ ). In addition, the expected returns to hiring an employed worker per unit of TFP,  $\Omega$ , also eventually decline, as workers match better on the ladder.

Importantly, note that the elasticity of job market tightness w.r.t. aggregate TFP never depends on multiplicative factors that enter the free entry condition (8), most notably vacancy costs  $\kappa$ , the scale of the meeting function, or the discount factor  $\beta$ . This property, common to the entire DMP class of random search models, implies that the business cycle properties of the model are independent of the values of those parameters, as long as they are calibrated to jointly match in steady state the observed average meeting probabilities.

These comparative statics properties suggest the following qualitative features of the impulse response of the job finding probability from unemployment to a positive TFP shock in the stochastic rational expectations equilibrium of the model. Consider first a permanent shock. In the economy without OJS, the job finding probability responds in the same direction and converges immediately to its new long run value, while in the economy with OJS the job finding probability responds less on impact, but overshoots its new steady state, which is higher than before but lower than without OJS. Next, consider a negative, mean reverting shock to TFP. In the economy without OJS, the job finding probability tracks TFP one for one. In contrast, with OJS, the job finding probability converges back to steady state slower than TFP. As unemployment, after increasing at first, falls back, more and more weight shifts to the “less profitable” part of job creation, hiring employed workers. This depresses job creation even while TFP recovers. Employment first reallocates down

the ladder, then slowly converges back to its initial distribution, so the MPL Gap first rises, then declines again, further slowing down the recovery in job creation late in the episode.

## B Derivation of Equation (19)

Given the AR(1) specification for TFP, we can calculate  $\mathcal{Z}(z_t)$ . Substituting forward, taking exponentials and rearranging

$$z_{t+\tau} = z_t^{\rho^\tau} \prod_{j=1}^{\tau} e^{\rho^{\tau-j} \varepsilon_{t+j}}$$

Since  $\rho^{\tau-j} \varepsilon_{t+j} \sim \mathcal{N}(0, \rho^{2(\tau-j)} \sigma^2)$ , we have that  $\mathbb{E}_t [e^{\rho^{\tau-j} \varepsilon_{t+j}}] = e^{\frac{\rho^{2(\tau-j)} \sigma^2}{2}}$ . By the independence of the innovations

$$\mathbb{E}_t \left[ \prod_{j=1}^{\tau} e^{\rho^{\tau-j} \varepsilon_{t+j}} \right] = \prod_{j=1}^{\tau} \mathbb{E}_t [e^{\rho^{\tau-j} \varepsilon_{t+j}}] = \prod_{j=1}^{\tau} e^{\frac{\rho^{2(\tau-j)} \sigma^2}{2}} = e^{\frac{\sigma^2}{2} [\rho^{2(\tau-1)} + \rho^{2(\tau-2)} + \dots + 1]} = e^{\frac{\sigma^2}{2} \frac{1-\rho^{2\tau}}{1-\rho^2}}$$

so

$$\mathbb{E}_t [z_{t+\tau}] = z_t^{\rho^\tau} e^{\frac{\sigma^2}{2} \frac{1-\rho^{2\tau}}{1-\rho^2}}$$

and finally, using the definition of  $Z(z_{t+1})$ , the L.I.E., and the last expression

$$\begin{aligned} \mathcal{Z}(z) &= \sum_{\tau=0}^{+\infty} (1-\delta)^\tau \beta^{\tau+1} \mathbb{E} [z_{t+\tau+1} | z_t = z] \\ &= \sum_{\tau=0}^{+\infty} (1-\delta)^\tau \beta^{\tau+1} z^{\rho^{\tau+1}} e^{\frac{\sigma^2}{2} \frac{1-\rho^{2\tau+1}}{1-\rho^2}} \\ &= \sum_{\tau=1}^{+\infty} (1-\delta)^{\tau-1} \beta^\tau z^{\rho^\tau} e^{\frac{\sigma^2}{2} \frac{1-\rho^{2\tau}}{1-\rho^2}} \end{aligned}$$

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