ECONSTOR
Make Your Publications Visible.

A Service of

ZBW Leibniz-Informationszentrum Wirtschaft
Leibniz Information Centre for Economics

Jeanjean, François

Conference Paper

Impact of Technical Progress on the relationship between Competition and Investment

29th European Regional Conference of the International Telecommunications Society (ITS): "Towards a Digital Future: Turning Technology into Markets?", Trento, Italy, 1st - 4th August, 2018

Provided in Cooperation with:
International Telecommunications Society (ITS)

Suggested Citation: Jeanjean, François (2018) : Impact of Technical Progress on the relationship between Competition and Investment, 29th European Regional Conference of the International Telecommunications Society (ITS): "Towards a Digital Future: Turning Technology into Markets?", Trento, Italy, 1st - 4th August, 2018, International Telecommunications Society (ITS), Calgary

This Version is available at:
https://hdl.handle.net/10419/184948

Standard-Nutzungsbedingungen:

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

Terms of use:

Documents in EconStor may be saved and copied for your personal and scholarly purposes.

You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.

WWW.ECONSTOR.EU

Mitglied der Leibniz-Gemeinschaft

Impact of Technical Progress on the relationship between Competition and Investment

François Jeanjean*
Orange, 78 rue Olivier de Serres, Paris

July 20, 2018
Working Paper Version, comments are welcome

Abstract

This paper investigates the impact of technical progress on the relationship between competition an investment. Using a model of oligopoly competition with differentiated products where firms invest to reduce their marginal cost of production, I find that technical progress, which increases the impact of investment on cost reduction, decreases the level of competition that maximizes investment of the industry. This feature holds also for consumer surplus and Welfare. In the model, competition is measured either by the number of competitors or by the degree of substitutability between offers. Result holds for both measures.Two parametric examples illustrate these features.

Keywords: Market structure, Investment, technical progress, competition

JEL Classification: D21, D43, D92, L13, O31.

1 Introduction

The relationship between competition and investment is a long lasting debate in industrial organization. However, no clear overall conclusions have been reached so far. Theoretically, Schmutzler (2013) showed that the relationship can go in any direction. Those different shapes depend on the characteristics of the industry. Many parameters may matter: Market structure, type of competition, consumers demand, cost patterns and technical progress. To my knowledge, the specific influence of the latter parameter of technical progress on this relation has not been thoroughly studied, despite the essential role of technical progress for dynamic efficiencies and for economic growth.

This paper attempts to fill the gap, focusing on the impact of technical progress on the relationship between competition and investment. In the paper,

*Orange, francois.jeanjean@orange.com

technical progress is understood as a technological opportunity in cost reducing innovation (process innovation) or quality improving innovation (product innovation), and competition can be measured either by the number of competitors or by the degree of product substitutability.

Using a simple oligopoly model of price competition, as in Motta and Tarantino (2017), it is shown that a higher level of technical progress reduces the degree of competition (number of competitors or degree of product substitutability) that maximizes investment at the industry level. As a result, industries experiencing a low technical progress are more likely to exhibit an increasing shaped relationship between competition and investment while industries experiencing a higher technical progress are more likely to exhibit an inverted-U or a decreasing one. The higher the technical progress, the lower the number of competitors or the degree of product substitutability corresponding to the maximum investment.

More precisely, this paper shows that, on the one hand, in a symmetric market, a higher degree of technical progress reduces the number of firms for which the investment at industry level is maximised and, on the other hand, in an asymmetric market, technical progress reduces the degree of substitutability for which the level of investment is maximum at industry level.

More precisely, I show that technical progress is a kind of competition in the sense that an increase in technical progress reallocates output from less efficient firms to most efficient ones, which is, according to Boone (2008), a general feature of more intense competition. This means that technical progress is actually a form of competition, a dynamic form of competition. Since a rise in technical progress increases competition, it is not surprising that the other kinds of competition, the static forms of competition, as the number of firms or the degree of substitutability have to decrease to maintain the optimal level of competition that maximizes investment.

To illustrate the results, this paper provides parametric examples using demand functions in the manner of Shubik and Levithan (1980) or Singh and Vives (1984).

The following of the paper is organized as follow: Section 2 is a literature review, section 3 describes a general model of price competition with investment. Section 4 derives this model for symmetric markets in order to study the impact of technological opportunity on the number of firms and shows that the maximum investment occurs for a lower number of firms when the technological opportunity increases. Section 5 shows that the decrease in the level of competition maximizing investment with technical progress still holds when competition is measured as the degree of substitutability and Section 6 concludes.

2 Literature review

The economic literature about the relationship between competition and investment (or innovation) is already quite developped and started a long time ago. Insofar as innovation requires investment, we can consider in the rest of the pa-

per that the incentives to innovate and the incentives to invest in cost reduction or in quality improvement go hand in hand.

Schumpeter (1942) insisted on the ability of large firms to innovate which suggests that concentration fosters innovation and thus competition hampers it. Arrow (1962) showed that competition tends to foster innovation because firms in competition have more incentives to innovate than a monopolist whose innovation sould cannibalize the profit. However, Gilbert and Newberry (1982) highlighted that a monopolist could have more incentives to innovate than a firm under competition. Indeed, a monopolist loses more than a firm under competition by not investing in innovation because the rent of a monopoly is higher than the joint profit of a duopoly.

Aghion et al (2005) reconcile those two views. They distinguish between two effects, the "scumpeterian effect", where competition reduces investment incentives and "escape competition effect" where competition fosters investment. They show that the relationship between competition and investment is governed by those two opposing effects and finally they found an inverted U relationship for United Kingdom economy.

More precisely, Aghion et al (2014) developed a model that details the mecanism of the two effects. In the escape competition effect, competition reduces the pre innovation profit, which fosters innovation while in the schumpeterian effect, competition decreases rather the post innovation profit which impacts adversely innovation. In their model, both effects coexist. The escape competition effect is represented by symmetric duopolies engaged in a neck to neck competition, both competitors using the same technology. Schumpeterian effect is represented by asymmetric duopolies with a leader and an outsider. The leader exerts a monopoly power because it uses a higher level of technology and the follower sells nothing but is willing to innovate to catch up with the leader. When a competitor innovates, in the symmetric competition, it acquires a monopoly power and the duopoly becomes asymmetric and then represents, the shumpeterian effect. When the outsider innovates in the asymmetric duopoly, it catches up with the leader and the duopoly becomes symmetric and then represents the escape competition effect. Duopolies thus go back and forth between the symmetrical and asymmetrical form. When the symmetrical form prevails, escape competition effect dominates and the global effect is that competition fosters innovation. When the asymmetrical form prevails, schumpeterian effect dominates and the global effect is that competition hampers innovation. Aghion et al showed that an increase in competition intensity tends to destabilize the symmetrical form and thus fosters the asymmetrical form. As a result, an increase in competition intensity tends to decrease the slope of the competition innovation curve. A low intensity of competition is more likely to provide a increasing slope while a high intensity of competition is more likely to provide a decreasing one.

This model is particularly usefull for this paper to understand the role of technical progress. Indeed, technical progress or the size of innovation impacts the post-innovation profit rather than the pre-innovation profit. Therefore, a higher size of innovation, which means a higher technical progress, increases the

shumpeterian effect in the asymmetrical duopolies and does not impact the escape competition effect. As a result, a higher size of innovation reduces the slope of the competition-innovation relationship and thereby, the level of competition that maximizes investment. It is noteworthy that technical progress, exactly like competition tends to decrease the slope of the competition-innovation relationship. Finally, this is not surprising if technical progress is a form of competition.

Schmutzler (2013) noticed that the relationship between competition and investment could take any shape, increasing decreasing, U shaped or inverted U shaped. Indeed, this depends on the relative influences of shumpeterian and escape competition effect.

For symmetric markets, Vives (2008) presents a benchmaking analysis of different models in which several examples point out that an increase in technological opportunity leads to more concentrated markets in free entry regime which means that technological progress reduces the number of firms so that competition is sustainable. This may seem counterintuitive, a higher technological opportunity should increase total output and attract more firms, but Tandon (1984) explains this apparent paradox by the fact that technological opportunity entails more investment in cost reduction that act as entry barriers. The latter effect outweighs the first. This increased investment due to technological opportunity, in some way, strengthens the Schumpeterian effect more than the escape competition effect. This is consistent with empirical findings of Kamien and Schwartz (1982) or Scherer and Ross (1990). Jeanjean and Houngbonon (2017) empirically showed that for the mobile telecommunication markets, characterized by a high level of technical progress, investment per firm decreases with the number of firms and investment of the industry is ambiguous (it tends to increase in the short run but eventually falls in the long run). They also noticed the increasing impact of asymmetry on investment.

The literature on technological diffusion is also usefull to understand the relationship between competition and innovation. In this literature, a new technology is announced. The cost of adoption is supposed to decrease over time, firms choose when to adopt. The earlier they adopt, the earlier they benefit from the new technology but the higher is the adoption cost. The adoption date is a trade-off between growth of profit and cost of adoption. Reinganum (1981) shows that firms adopt at different times even if they are initially identical. Fudenberg and Tirole (1985) studied the case where firms attempt to preempt innovation because the leader earns more than the follower. In that case, there is still a leader and a follower, but they earn the same actualized value. In both cases, Jeanjean (2017) investigated the impact of technical progress on the technological adoption. The higher the technical progress, the larger the size of innovation at equal cost of adoption or the lower the cost of adoption for an equal size of innovation. The higher the technical progress, the earlier the firms adopt in average. However, there is a limit because firms can not adopt before the innovation is performed. Technical progress, stimulating the impact of competitive pressure tends to advance the date of adoption of the leader and to delay that of the follower with a positive overall effect on the investment of the industry because the former effect is larger than the latter. However, the

limit stops this dynamic. Beyond the degree of substitutability where the leader adopts immediately, an increase in competitive pressure can no longer advance the adoption of the leader but still delay that of the follower. As a result, investment of the industry decreases. Technical progress increases competition and thus reduces the degree of substitutability for which this occurs.

Furthermore, financial constraints may also impede to achieve the investment required by the "Escape competition effect" as noticed by Houngbonon & Jeanjean (2016).

3 The model

In this section, we consider, as in Motta and Tarentino (2017), a model of oligopoly with differentiated goods where N firms compete in price. Demand for the good produced by firm i is given by $q_i(p_i, p_{-i})$ where p_i is the price of firm i and p_{-i}, the vector of price of the $N-1$ firms different from firm i.

Firms set simultaneously their price and their cost reducing investment. firm i set price p_i and cost reducing investment x_i to maximize its profit π_i. The cost of investment is denoted by $F(x_i)$ where F is increasing and convex $F'(x_i) > 0$ and $F"(x_i) > 0$. Moreover, $F(0) = 0$, without investment there is no cost of investment. In our model, technological progress decreases the cost of investment through the function F. The marginal cost of firm i depends on the cost reducing investment: $c_i(x_i) \leq c_i(0)$. $c_i(x_i)$ is decreasing and convex $c_i^{'}(x_i) < 0$ and $c_i"(x_i) \geq 0$ and $c_i(0) = c_{0i} \geq 0$. Technical progress, denoted τ, reduces the cost of the investment $x_i.\partial F(x_i)/\partial\tau < 0$

Profit of firm i writes:

$$\pi_i(p_i, p_{-i}, x_i) = (p_i - c_i(x_i))q_i(p_i, p_{-i}) - F(x_i) \tag{1}$$

The solution of the maximization problem leads to two first-order conditions, one for the price and the other for the investment:

$$\frac{\partial \pi_i}{\partial p_i} = q_i(p_i, p_{-i}) + \frac{\partial q_i(p_i, p_{-i})}{\partial p_i}(p_i - c_i(x_i)) = 0 \tag{2}$$

$$\frac{\partial \pi_i}{\partial x_i} = -c_i'(x_i)q_i(p_i, p_{-i}) - F'(x_i) = 0 \tag{3}$$

The assumption of simultaneous choice guarantee that Investment of firm i, x_i is independent of competitors' investment, as a result, x_i increases with q_i.

c_i depends only on x_i and c_{0i}, the marginal cost of firm i without investment. x_i depends only on q_i and the technical progress, τ. q_i depends only on p_i and p_{-i} according to demand function. Yet p_i depend on c_i and the marginal cost of the rivals c_{-i}. As a result, given the demand function and the level of technical progress, all the variables: prices, outputs, marginal costs, investments and profits depend only on the initial marginal costs of the firms c_{0i}.

A firm with no output is excluded from the market. This means that the number of firms tends to decrease for a higher degree of substitutability.

An increase in technical progress entails an increase in investment x_i which decreases marginal cost c_i. Technical progress, thus improves firms efficiency. More efficient firms tends to decrease prices while increasing the price cost margin and the output.

3.1 Technical progress can be considered as a form of competition.

Proposition 1 *Technical progress reallocates output from less efficient firms to most efficient ones. In other words, it increases market share of most efficient firms at the expense of the less efficient ones.*

Proof. The growth of efficiency involved by technical progress is all the higher as the firm is already efficient, wich means that differences in efficiency among firms increase with technical progress and thus, the differences in output accelerate. As a result, if firm i is strictely more efficient than firm j, then the ratio $\frac{\partial q_i/\partial \tau}{\partial q_j/\partial \tau} > \frac{q_i}{q_j}$. We denote Q, the total output at industry level, $Q = \sum_{i=1}^{N} q_i$. This leads to $\frac{\partial q_i/\partial \tau}{q_i} > \frac{\partial q_j/\partial \tau}{q_j}$. Average efficiency is $\frac{1}{N}\sum_{i=1}^{N}\left(\partial q_i/\partial \tau\right)$, thus if firm i is more efficient than the average, then $\frac{\partial q_i/\partial \tau}{q_i} > \frac{1}{Q}\sum_{i=1}^{N}\left(\partial q_i/\partial \tau\right)$. This inequation yields $\frac{\partial q_i}{\partial \tau}Q - q_i\sum_{i=1}^{N}\left(\partial q_i/\partial \tau\right) > 0$. Market share of firm i is written:$\sigma_i = q_i/Q$.

An increase in technical progress yields ■

$$\frac{\partial\left(\sigma_i\right)}{\partial \tau} = \frac{\left(\frac{\partial q_i}{\partial \tau}Q - q_i\sum_{i=1}^{N}\left(\partial q_i/\partial \tau\right)\right)}{Q^2} > 0 \tag{4}$$

This ratio is positive because the numerator and the denominator are both positive. This means that an increase in technical progress increases the market share of the most efficient firms to the expense of the less efficient ones. As more efficient firms have already a larger market share, technical progress increases the market share of firms all the more they have already a large one.

As mentionned by Boone (2008) the output reallocation effect is a general feature of more intense competition. As aconsequence, technical progress can be considered as an actual form of competition. This is a dynamic form of competition that is involved by investment while the number of firms or the degree of substitutability are considered as static form of competition.

It is therefore not surprising, insofar as there is an optimal level of competition which maximizes investment, that technical progress as a dynamic form of competition tends to replace the static form of competition to achieve this optimal level of competition consisting of the static and dynamic forms of competition.

3.2 Parametric model

To illustrate the impact of technological opportunity on investment, I choose a parametric model for which $c_i(x_i) = c_{0i} - x_i$ and $F(x_i) = x_i^2/2\tau$. The higher τ, the parameter representing technical progress, the less expensive the investment for a given impact on marginal cost x_i. I choose the traditional utility function from Shubik and Levitan (1980).

$$U(q_1, ..., q_N) = \sum_{i=1}^{N} \alpha_i q_i - \frac{1}{2}\left(\sum_{i=1}^{N} q_i^2 + 2\gamma \sum_{j \neq i} q_i q_j\right) - \sum_{i=1}^{N} p_i q_i \tag{5}$$

where $\alpha_i > 0$ and $\gamma \in (0, 1)$ represents the product substitutability. $\gamma = 0$ means product are independent and firms act as monopolists and $\gamma = 1$ means product are perfect substitutes.

Maximization of consumer utility leads to:

$$p_i = \alpha_i - q_i - \gamma \sum_{j \neq i} q_j \tag{6}$$

And finally, firm i's demand is given by:

$$q_i = \frac{(\alpha_i - p_i)\left[\gamma\left(N-2\right)+1\right] - \gamma \sum_{j \neq i}\left(\alpha_j - p_j\right)}{\left(1-\gamma\right)\left[\gamma\left(N-1\right)+1\right]} \tag{7}$$

The general expression of q_i function of initial marginal costs can be derived using equations (2), (3) and (7):

$$q_i = \frac{A\left[\left(A^2 + \left(1-\gamma\right)\left(A+\gamma\right) - A\tau\right)\left(\alpha_i - c_{0i}\right) - \gamma A \sum_{j \neq i}\left(\alpha_j - c_{0j}\right)\right]}{\left(1-\gamma\right)\left(A+\gamma\right)A^2 + \left(\left(1-\gamma\right)\left(A+\gamma\right) - A\tau\right)\left(\left(1-\gamma\right)\left(A+\gamma\right) + A\left(1+A-\tau\right)\right)} \tag{8}$$

with $A = 1 + \gamma\left(N-2\right)$

As an alternative, we can use the Singh and Vives (1984) demand function (in the appendix).

We can check, with the parametrical model that technical progress reallocates output from less efficient firms to most efficient ones. Indeed, technical

progress increases the market share of firms with above average efficiency and decreases the market share of firms with below average efficiency. Denoting $\sigma_i = q_i / \sum_j q_j$, the market share of firm i, if $(\alpha_i - c_{0i}) > \sum_j (\alpha_j - c_{0j}) / N$ then $\partial\sigma_i/\partial\tau > 0$ and if $(\alpha_i - c_{0i}) < \sum_j (\alpha_j - c_{0j}) / N$ then $\partial\sigma_i/\partial\tau < 0$ and if $(\alpha_i - c_{0i}) = \sum_j (\alpha_j - c_{0j}) / N$ then $\partial\sigma_i/\partial\tau = 0$. (see proof in the annexes)

In the next sections 3 and 4, I will respectively study how the general model applies in the symmetric case, where the intensity of competition derives from the number of identical competitors, and in the asymmetric case where the intensity of competition derives from the degree of substituability that affects differently the competitors having different efficiencies.

4 Symmetrical market

In this part, we consider a symmetrical market with N firms. In a symmetrical market, $\forall (i,j) \in \mathbb{R}^2$, $c_{0i} = c_{0j} = c_0$ and $\alpha_i = \alpha_j = \alpha$. This leads to $p_i = p_j = p$; $q_i = q_j = q$; $x_i = x_j = x$; $c_i(x) = c_j(x) = c(x)$ and $\pi_i = \pi_j = \pi$.

Investment per firm is F and investment of the industry is $I(N) = NF$. We denote ε, the elasticity of investment per firm according to the number of firms: $\varepsilon = \frac{\partial F}{\partial N}\frac{N}{F}$.

4.1 Number of firms maximizing investment of the industry

The number of firms maximizing investment of the industry is $N_{\max} = \underset{N\in\mathbb{N}^+}{\arg\max} I(N)$ which can be rewritten: $N_{\max} = \max\left\{1, \underset{N\in\{\lfloor N^*\rfloor, \lfloor N^*\rfloor+1\}}{\arg\max} I(N)\right\}$ with $N^* = \underset{N\in\mathbb{R}^+}{\arg\max} I(N)$

$N^* \in \mathbb{R}^+$, this allow us to get rid of the integer problem for N while maintaining the model realistic with $N_{\max} \in \mathbb{N}^+$.

I assume that $F(N)$ and $\varepsilon(N)$ are definite and continuous for all $N \in \mathbb{R}^+$.

Notice that $\frac{\partial I}{\partial N} = (1+\varepsilon)F$. For $F > 0$, $sign(\frac{\partial I}{\partial N}) = sign(1+\varepsilon)$, and $\frac{\partial^2 I}{\partial N^2} = \frac{\partial \varepsilon}{\partial N}F + (1+\varepsilon)\frac{\partial F}{\partial N}$

Proposition 2 *If* $\exists\ N \in \mathbb{R}^+$ *such that* $\varepsilon = -1$, $\frac{\partial \varepsilon}{\partial N} < 0$, $\varepsilon(0) < \varepsilon(N)$ *and* $\lim_{N\to+\infty} \varepsilon < \varepsilon(N)$ *then there is an absolute maximum,* N^* *which is finite and different from* 0 *and* $N_{\max}$ *is finite,* $N_{\max} \geq 1$

Proof. Indeed, in that case, the first order condition $\frac{\partial I}{\partial N} = 0$ and the second order condition $\frac{\partial^2 I}{\partial N^2} < 0$ are both fulfilled. N is a maximum. the conditions $\varepsilon(0) < \varepsilon(N)$ and $\lim_{N\to+\infty} \varepsilon < \varepsilon(N)$ ensure that if there is only one maximum this is a absolute maximum and if there are several maximum for different finite

values of $N > 0$, one of them is an absolute maximum. If $N_{\max}$ is the unic absolute maximum, and $N_{\max} > 1$, the relationship between the number of firms and the investment of the industry is inverted U shaped. ■

Otherwise three cases can occur:

- First case: $\exists\ N \in \mathbb{R}^+$ such that $\varepsilon = -1$, $\frac{\partial \varepsilon}{\partial N} < 0$, but $\varepsilon(0) \geq \varepsilon(N)$ or $\lim_{N \to +\infty} \varepsilon \geq \varepsilon(N)$. In that case, N is a relative but not an absolute maximum.

- Second case: $\forall\ N \in \mathbb{R}^+$, $\varepsilon < -1$. In this case, $\frac{\partial I}{\partial N} < 0$, investment of the industry always decreases with the number of firms, thus, $N^* = 0$ and $N_{\max} = 1$. The investment of the industry is maximum under monopoly.

- Third case: $\forall\ N \in \mathbb{R}^+$, $\varepsilon > -1$. In this case, $\frac{\partial I}{\partial N} > 0$, investment of the industry always increases with the number of firms, thus N^* and $N_{\max}$ tends toward infinity. The higher the number of firms, the higher the investment of the industry.

4.2 Impact of technical progress on the relationship between the number of firms and investment per firm

Now, we are focusing on the impact of technical progress on the relationship between the number of firms and investment per firm F.

Lemma 3 *The impact of technical progress on the elasticity of investment according to the number of firms is in the same sense as the impact of the number of firms . In other words, sign* $\left(\frac{\partial \varepsilon}{\partial \tau}\right) = sign\left(\frac{\partial F}{\partial N}\right)$. *Or sign* $\left(\frac{\partial \varepsilon}{\partial \tau}\right) = sign\,(\varepsilon)$ *since sign* $\left(\frac{\partial F}{\partial N}\right) = sign\,(\varepsilon)$

Proof. *Proposition 1 showed that technical progress increases the market share of firms all the more they are efficient and have already a large market share. Investment per firm F is linked to output by equation (3), thus sign* $\left(\frac{\partial F}{\partial N}\right) = sign\left(\frac{\partial q}{\partial N}\right)$. *Consequently, if* $\frac{\partial F}{\partial N} < 0$, *then* $\frac{\partial q}{\partial N} < 0$ *and* $\varepsilon < 0$. *Moreover,* $\frac{\partial q}{\partial N} < 0$ *implies the ratio* $\frac{q(N)}{q(N+1)} > 1$ *and, therfore,* $\frac{\frac{\partial q}{\partial \tau}(N)}{\frac{\partial q}{\partial \tau}(N+1)} > \frac{q(N)}{q(N+1)}$ *or, as F is linked to q,* $\frac{\frac{\partial F}{\partial \tau}(N)}{\frac{\partial F}{\partial \tau}(N+1)} > \frac{F(N)}{F(N+1)}$ *and* $\frac{\frac{\partial F}{\partial \tau}(N)}{F(N)} > \frac{\frac{\partial F}{\partial \tau}(N+1)}{F(N+1)}$ *which means that* $\frac{\partial\left(\frac{1}{F}\frac{\partial F}{\partial \tau}\right)}{\partial N} < 0$. *This inequation can be rewritten* $\frac{\partial^2 F}{\partial N \partial \tau}\frac{1}{F} - \frac{1}{F^2}\frac{\partial F}{\partial \tau}\frac{\partial F}{\partial N} < 0$ *and it turns out that* $\frac{\partial^2 F}{\partial N \partial \tau}\frac{1}{F} - \frac{1}{F^2}\frac{\partial F}{\partial \tau}\frac{\partial F}{\partial N} = \frac{1}{N}\frac{\partial \varepsilon}{\partial \tau}$. *As a consequence* $\frac{\partial \varepsilon}{\partial \tau} < 0$. *Same manner, if* $\frac{\partial F}{\partial N} > 0$ *then* $\frac{\partial \varepsilon}{\partial \tau} > 0$ *and* $\varepsilon > 0$ *thus sign* $\left(\frac{\partial \varepsilon}{\partial \tau}\right) = sign\left(\frac{\partial F}{\partial N}\right) = sign(\varepsilon)$ ■

Proposition 4 *Technical progress tends to reduce the number of firms that maximizes investment of the industry.*

Proof. If $N^* \in \mathbb{R}^+$ is the number of firms maximizing investment of the industry, then first order condition and second order condition are both fulfilled,

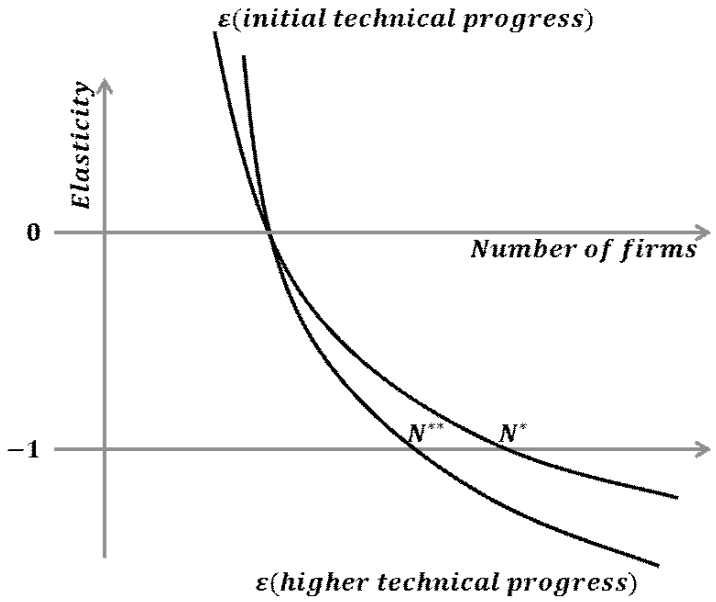
$\varepsilon(N^*) = -1$ and $\frac{\partial^2 I}{\partial N^2}(N^*) = \frac{\partial \varepsilon}{\partial N}(N^*) F(N^*) + (1 + \varepsilon(N^*)) \frac{\partial F}{\partial N}(N^*) < 0$. replacing $\varepsilon(N^*)$ by -1 yields $\frac{\partial \varepsilon}{\partial N}(N^*) F(N^*) < 0$, thus $\frac{\partial \varepsilon}{\partial N}(N^*) < 0$. From Lemma 1, since ε is negative, we know that $\frac{\partial \varepsilon}{\partial \tau}(N^*) < 0$. As a result, an increase in technical progress reduces $\varepsilon(N^*)$. Under increased technical progress $\varepsilon(N^*) < -1$. Let us denote N^{**}, the number of firm maximizing investment of the industry under increased technical progress. Same manner, $\varepsilon(N^{**}) = -1$ and $\frac{\partial \varepsilon}{\partial N}(N^{**}) < 0$. The number of firms has to decrease to increase ε such that $\varepsilon(N^{**}) = -1$ because $\frac{\partial \varepsilon}{\partial N}(N^{**}) < 0$, thus $N^{**} < N^*$.

If $\forall N \in \mathbb{R}^+$, $\varepsilon(N) < -1$, investment of the industry is maximized under monopoly. An increase in technical progress has no chance to change this because technical progress decreases ε and, therefore ε remains under -1.

If $\forall N \in \mathbb{R}^+$, $\varepsilon(N) > -1$, the higher the number of firms, the higher investment of the industry. An increase in technical progress can make the number of firms maximizing investment of the industry finite. If $0 > \varepsilon(N) > -1$ then $\frac{\partial \varepsilon}{\partial \tau} < 0$ and it is possible that under increased technical progress $\varepsilon(N) = -1$. If $\varepsilon(N) > 0$, it is not possible because $\frac{\partial \varepsilon}{\partial \tau} > 0$ and technical progress increases ε which remains above -1.

In all cases, a higher technical progress maintain or decreases the number of firms maximizing investment of the industry. ■

The graph below (figure 4.2) represents the fall in the maximising number of firms under a higher technical progress.



Fall in the maximising number of firms

4.3 Impact on consumer surplus and Welfare:

As for investment, in symmetric market, consumer surplus or welfare at industry level equals N times the consumer surplus or the welfare brought by each firm. we denote respectively CSI and WI, Consumer surplus and welfare at industry level. $CSI = N\ CS$ and $WI = N\ W$. We denote, respectively the elasticities of consumer surplus and welfare according to the number of firms: $\varepsilon_{CS} = \frac{\partial CS}{\partial N}\frac{N}{CS}$ and $\varepsilon_W = \frac{\partial W}{\partial N}\frac{N}{W}$. However, the link with output may be different for the consumer surplus compared to investment. A decrease in consumer surplus per firm, CS, following an increase in the number of firms implies a decrease in output per firm, q. Indeed, if q increased, total output increased then price decreased and in this case consumer surplus would increase. But the reverse is not necessarily true, it is possible that, following an increase in the number of firm, output per firm and price both decrease. In particular, if q decrease but total output Nq increases, in this case price decreases. It is then possible that consumer surplus decreases. In other words: If $\frac{\partial CS}{\partial N} < 0$ then $\frac{\partial q}{\partial N} < 0$. But $\frac{\partial q}{\partial N} < 0$ does not necessarily cause $\frac{\partial CS}{\partial N} < 0$.

Lemma 5 *If the impact of the number of firms on consumer surplus is decreasing, then the impact of technical progress on the elasticity of consumer surplus according to the number of firms is also decreasing . In other words: If* $\frac{\partial CS}{\partial N} < 0$ *then* $\frac{\partial \varepsilon_{CS}}{\partial \tau} < 0$ *. Or if* $\varepsilon_{CS} < 0$ *then* $\frac{\partial \varepsilon_{CS}}{\partial \tau} < 0$ *because* $sign(\varepsilon_{CS}) = sign\left(\frac{\partial CS}{\partial N}\right)$.

Proof. An increase in output induced by technical progress increases consumer surplus. If $\frac{\partial q}{\partial \tau} > 0$ then $\frac{\partial CS}{\partial \tau} > 0$.The proof is the same as Lemma 1 replacing F by CS. However, the reverse is not necessarily true. If $\varepsilon_{CS} > 0$ then $\frac{\partial \varepsilon_{CS}}{\partial \tau}$ is not necessarily positive. This is the difference between consumer surplus and investment. ■

Proposition 6 *Technical progress tends to reduce the number of firms that maximizes consumer surplus and welfare.*

Proof. The proof is the same as Proposition 1 replacing F by CS. Indeed, the proof uses only the sense If $\varepsilon < 0$ then $\frac{\partial \varepsilon}{\partial \tau} < 0$ because the first order condition entails $\varepsilon = -1$. ■

Welfare is the sum of Consumer surplus and profit. Profit is linked to output and, as a result, a decrease in profit following an increase in the number of firms implies a decrease in output. As a result, a decrease in welfare implies a decrease in output. If $\frac{\partial W}{\partial N} < 0$ then $\frac{\partial q}{\partial N} < 0$.

Using the same reasonning than for consumer surplus leads to a similar conclusion: If $\frac{\partial W}{\partial N} < 0$ then $\frac{\partial \varepsilon_W}{\partial \tau} < 0$. Or if $\varepsilon_W < 0$ then $\frac{\partial \varepsilon_W}{\partial \tau} < 0$ because $sign(\varepsilon_W) = sign\left(\frac{\partial W}{\partial N}\right)$.

4.4 Impact of technical progress in the parametric model:

We derive the expression of demand per firm in the symmetric case from equation (8), (results for Singh & Vives demand function are in the annexes), using the fact that $\forall i, j\ \alpha_i = \alpha_j = \alpha$ and $c_{oi} = c_{oj}$[1]:

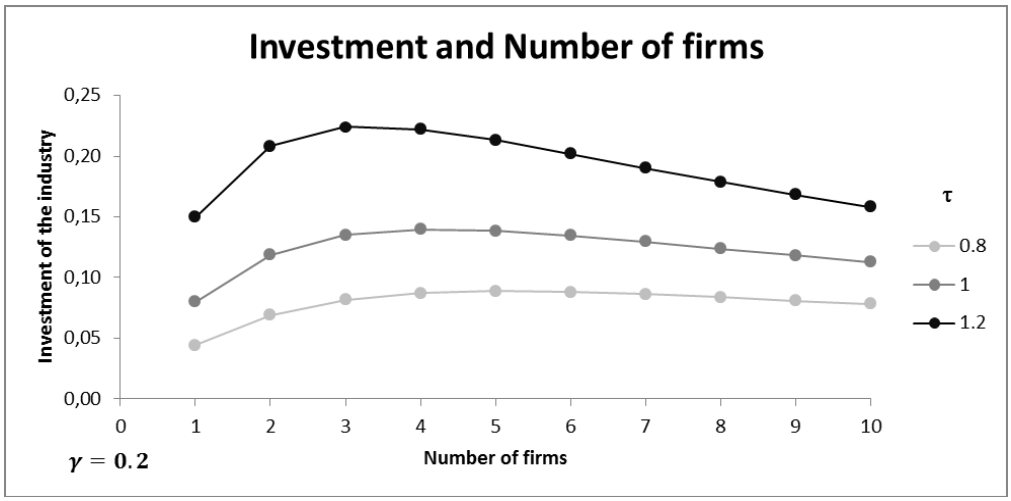
$$q = \frac{(\gamma(N-2)+1)(\alpha - c_0)}{(1-\gamma)(\gamma(N-1)+1) - (\gamma(N-2)+1)\tau + (\gamma(N-2)+1)(\gamma(N-1)+1)} \tag{9}$$

Equation (3) yields $x = \tau q$, equation (7) yields $p = \alpha - (\gamma(N-1)+1)q$.

Investment per firm is written: $F = \tau q^2/2$ and thus investment at the industry level is given by $I(N) = \tau N q^2/2$

We verify that for the parametric model $sign(\varepsilon) = sign(\partial\varepsilon/\partial\tau)$ (in the appendix). This causes, by Lemma 1 and Proposition 2 that the number of firms which maximizes investment decrease with technical progress.

The graph below (4.4) illustrates the negative impact of technical progress, τ, on the number of firms maximizing investment.



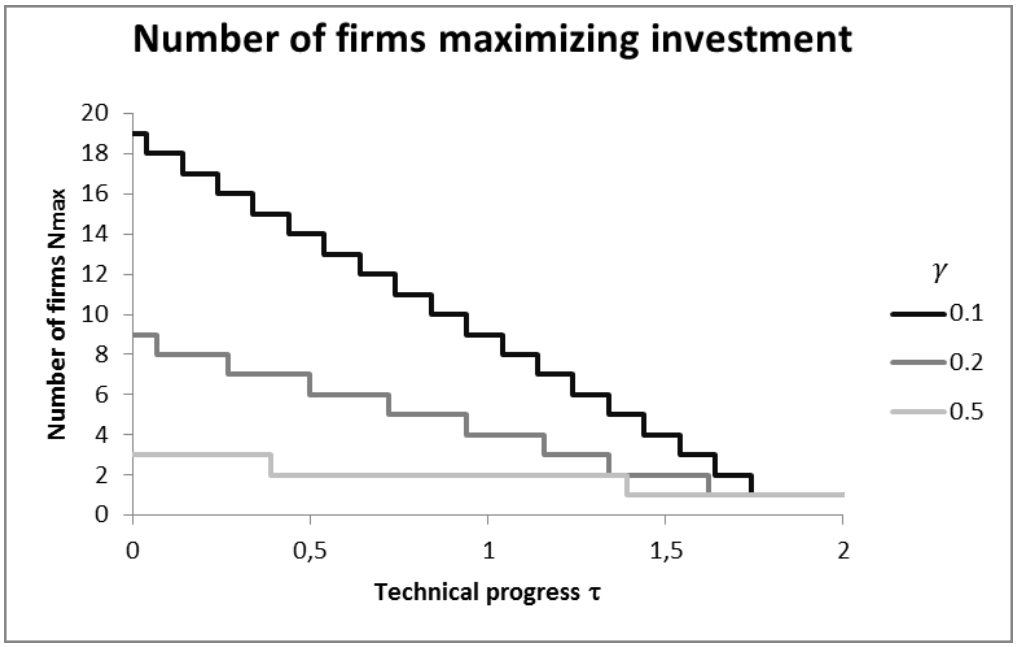
Technical Progress, Competition and Investment

The graph above represents the investment of the industry in function of the number of firms in the parametric model for Schubik and Levitan demand function where $\gamma = 0.2$ and $\alpha - c_0 = 0.4$. (The Singh and Vives demand function provides similar results). Different series are represented with varying shades of gray corresponding to different level of technical progress τ. The darker the

[1] There is a constraints on the value of marginal cost which can not be negative, thus $x \leq c_0$. A higher α allows to choose a higher c_0 with a constant efficiency $(\alpha - c_0)$. This reduces the constraint.

color, the higher the technical progress. The graph shows that the maximum investment occurs for 5 firms when $\tau = 0.8$, for 4 firms when $\tau = 1$ and for 3 firms, when $\tau = 1.2$. We can see that, as expected, the number of firm maximizing investment of the industry decreases with the number of firms.

The graph below represents $N_{\max}$, the number of firm that maximizes investment according to different values of technical progress, τ and substitutability, γ.



$N_{\max}$ decreases with technical progress

This graph shows how the number of firm decreases with technical progress. Moreover, $N_{\max}$ tends to fall with the degree of substitutability. It is not surprising as the degree of substitutability tends to increase static competition. For an equal level of technical progress, a higher degree of substitutability tends to decrease the other forms of static competition, the number of firms.

Consumer Surplus and Welfare:

Consumer surplus at industry level can be calculated replacing $p = \alpha - (1 + \gamma (N-1)) q$ in equation (5):

$$CSI(N) = \frac{N (1 + \gamma (N-1)) q^2}{2} = \frac{I(N) (1 + \gamma (N-1))}{\tau} \tag{10}$$

At firm level, for symmetric market, $CSI(N) = N\ CS(N)$. Thus $CS(N) = \frac{F(N)(1+\gamma(N-1))}{\tau}$.

We can verify that if $\varepsilon_{CS} < 0$ then $\frac{\partial \varepsilon_{CS}}{\partial \tau} < 0$ but if $\varepsilon_{CS} > 0$, $\frac{\partial \varepsilon_{CS}}{\partial \tau}$ is not necessarily positive (in the annexes). Consequently, an increase in technical progress tends to decrease the number of firms maximizing consumer surplus.

Moreover, if N^* maximizes investment of the industry and N^{**} maximizes consumer surplus, then $N^{**} \geq N^*$ (in the annexes)

Welfare is written $WI(N) = CSI(N) + N\pi(N)$, using equations (10), (1), $x = \tau q$ and $p = \alpha - (1 + \gamma (N-1)) q$ we can write:

$$WI(N) = (\alpha - c_0) Nq + I(N) - CSI(N) \tag{11}$$

Or at firm level: $W = (\alpha - c_0) q + F - CS$

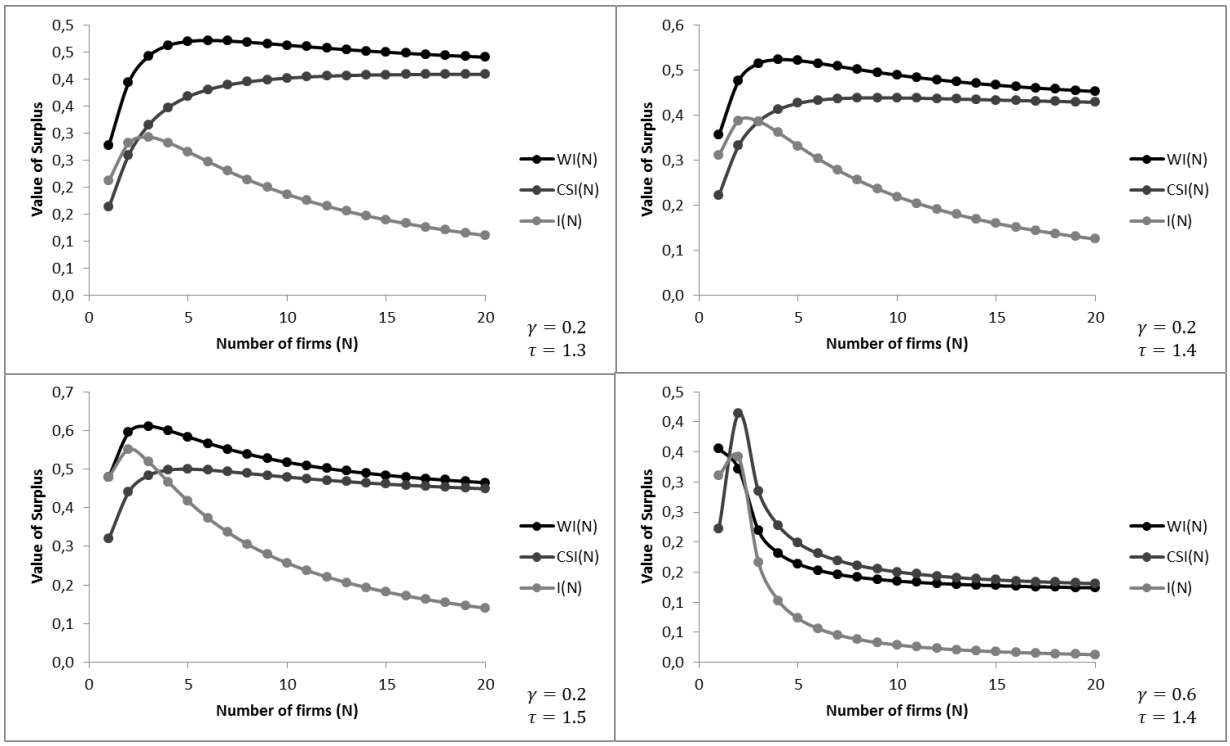
The variation of welfare according to the number of firms can be written:

$$\frac{\partial WI(N)}{\partial N} = W (1 + \varepsilon_W) = (\alpha - c_0) q (1 + \varepsilon_q) + F (1 + \varepsilon) - CS (1 + \varepsilon_{CS}) \tag{12}$$

with ε_q, the elasticity of output according to the number of firms $\varepsilon_q = \frac{N}{q} \frac{\partial q}{\partial N}$. Moreover, $\varepsilon = \frac{2N}{q} \frac{\partial q}{\partial N}$, this means that $\varepsilon_q = 2\varepsilon$ and $\varepsilon_{CS} = \varepsilon + \frac{\gamma N}{(1+\gamma(N-1))}$

We can verify that technical progress tends to decrease the number of firms maximizing welfare. Indeed, this number is lower than the one that maximizes consumer surplus and higher than the one that maximizes investment, (exepted for high τ and high γ for which the maximum welfare is achieved for monopoly. see in the annexes).

The graph below represents the evolution of Investment, Consumer surplus and welfare with technical progress at industry level:



Technical progress decreases the number of firms maximizing I, CSI and WI

The simulation is made with $(\alpha - c_0) = 0.4$. For $\gamma = 0.2$ and $\tau = 1.3$, maximum investment is achieved for 3 firms, maximum welfare for 6 firms and maximum consumer surplus for more than 20 firms. For $\gamma = 0.2$ and $\tau = 1.4$, maximum investment is achieved for 2 firms, maximum welfare for 4 firms and maximum consumer surplus for 9 firms. For $\gamma = 0.2$ and $\tau = 1.4$, maximum investment is achieved for 2 firms, maximum welfare for 3 firms and maximum consumer surplus for 5 firms. This shows that technical progress decreases the number of firm that maximizes Investment of the industry, consumer surplus and welfare. For $\gamma = 0.6$ and $\tau = 1.4$, maximum investment is achieved for 2 firms, maximum welfare for 1 firm and maximum consumer surplus for 2 firms. This last example illustrates the fact that, for sufficiently high γ and τ, it possible that maximum welfare is achieved for a monopoly while maximum investment is achieved for a duopoly.

5 Asymmetrical market

In this section, we allow firms to have different efficiency $\exists\ i, j;\ i \neq j$ such that $c_{0i} \neq c_{0j}$. We consider in this section the competition as the friction among firms regardless of the number of firms, like the degree of substitutability.

We have seen in section 3 that competition, like technical progress increases

the market share of the most efficient firms whose market share are above the average at the expense of the firms whose market share are under the average. This increases investment of the most efficient firms but decreases the investment of the least efficient ones. However, as function $F(x)$ is increasing and convexe, the increase in investment of the most efficient firms is higher than the decline in investment of the least efficient ones and as a result, investment of the industry increases.

Indeed, if $\sigma_1 > \sigma_2$, then $q_1 > q_2$, $F(q_1) > F(q_2)$ and $\frac{\partial F}{\partial q}(q_1) > \frac{\partial F}{\partial q}(q_2)$. As a result, $\frac{\partial F}{\partial q}(q_1) - \frac{\partial F}{\partial q}(q_2) > 0$. Competition, like technical progress increases asymmetry and asymmetry increases the investment of the industry, thus competition increases investment of the industry as long as firms have a positive market share. Indeed, there is a limit when the least efficient firm has no more output. In that case, an increase in competition beyond this limit implies the exit of the least efficient firm. This exits has a negative impact on competition and may reduce investment, but after the exit, the increase in competition still increase investment and entails another exit beyond a certain degree. Investment of the industry is higher for the second exit than for the first, because market is more asymmetric and output is more concentrated in the most efficient firms. The growth of competition thus eliminates in turn all the firms up to the most efficient one, which remains alone. When the most efficient firm is alone, it seeks to avoid the return of the last eliminated competitor. However, the fiercer the competition, the more easy it is because competition tends to increase the differences in efficiency between firms. As a result, the investment required to prevent the return of the competitor decreases with competition. This investment is the investment of the industry since the most efficient firm has a monopoly.

In summary, investment of the industry tends to increase with competition, even if it falls punctually at each exit, until the most efficient firm remains alone. Beyond this point, competition tends to decrease investment of the industry. There is thus an inverted U relationship between competition and investment. The degree of competition that maximizes investment of the industry is achieved at the point the second most efficient firm is about to exit.

An increase in technical progress, like competition amplifies the differences in efficiency among firms. As a result, exits occurs for a lower degree of competition under increased technical progress. The last exit also occurs for a lower degree of competition. This means that technical progress decreases the degree of competition that maximizes investment of the industry.

Impact on consumer surplus and Welfare:

The degree of substitutability that maximizes Consumer surplus and Welfare decreases with technical progress. As Investment, Consumer surplus and Welfare have a relative maximum for the degree of substitutability for which the less efficient firm is about to give up to. As a result, maximum Consumer Surplus and maximum Welfare are achieved for this degree of substitutability $\gamma = \gamma^*$ or for $\gamma = 0$. An increase in technical progress decrease this degree and thus the degree of substitutability that maximizes Consumer Surplus and

welfare. However, the level of technical progress for which the shift between γ^* and 0 occurs is not necessarily the same.

5.1 Impact of technical progress in the parametric model:

Investment of the industry is: $I = \sum_{i=1}^{N} \tau q_i^2/2$.

To illustrate the impact of technical progress when the competitive pressure increases with the degree of substitutability, γ, we use the parametric model defined above and, in this case, to simplify the analysis we choose $N = 2$. For simplification, we denote $E_i = (\alpha_i - c_{oi})$, the efficiency of firm i. In that case, for Schubik and Levitan demand function, (results for Singh & Vives demand function are in the annexes), equation (8) in which $N = 2$, thus $A = 1$ provides demand for firm i:

$$q_i = \frac{\left(2 - \gamma^2 - \tau\right) E_i - \gamma E_j}{\left(2 - \gamma^2 - \tau\right)^2 - \gamma^2} \tag{13}$$

This expression is available as soon as the output of the less efficient firm is positive. The coefficient of substitutability increases the competitive pressure and reduces the output of the less efficient firm. If firm i is the most efficient firm and firm j the less efficient one, the output of firm j is positive as soon as $\gamma \leq \gamma^*$with:

$$\gamma^* = \frac{\sqrt{E_i^2 + 4\left(2 - \tau\right) E_j^2} - E_i}{2E_j} \tag{14}$$

We can notice that an increase in τ decreases γ^*.

Denoting $\delta = \frac{q_i - q_j}{2}$ and $q = \frac{q_i + q_j}{2}$, the average output. Investment of the industry, for $\gamma \leq \gamma^*$ can be written: $I(\gamma) = \tau\left(q^2 + \delta^2\right)$. We can verify that when γ is close to γ^*, then investment of the industry is increasing: $\frac{\partial I}{\partial \gamma}\left(\gamma^{*^-}\right) > 0$ (see proof in the annexes)

For higher values of γ, $\gamma > \gamma^*$, firm j exits the market and firm i ,alone, exerts a monopoly which increases price and decreases total output and thus investment at the industry level. However, if we consider that the monopoly is contestable, when output of firm j equals zero, $q_j = 0$, $x_j = 0$, firm $j's$ price is marginal cost, and marginal cost is initial marginal cost as firm j does not invest any more: $p_j = c_j = c_{oj}$.

When the monopoly is contestable, the notion of substitutability still make sense. In this case, the output of firm i for the Schubik and Levitan demand function deduced from equation (6) is:

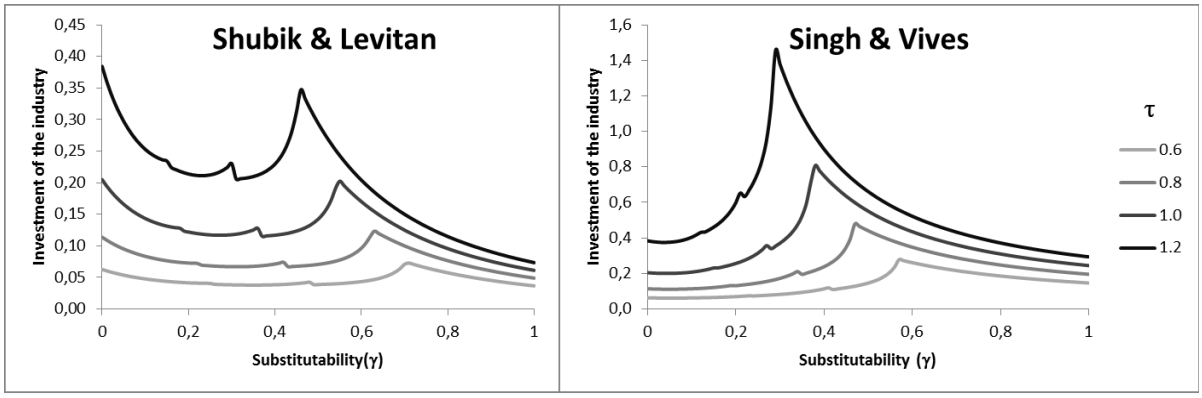
$$q_i = \frac{E_j}{\gamma}$$

Output of firm i decreases with the substitutability when $\gamma > \gamma^*$ then $\frac{\partial I}{\partial \gamma}\left(\gamma^{*^+}\right) < 0$

As a result, $\frac{\partial I}{\partial \gamma}\left(\gamma^{*^-}\right) > 0$ and $\frac{\partial I}{\partial \gamma}\left(\gamma^{*^+}\right) < 0$, There is a discontinuity in γ^* due to the exit of the least efficient firm and γ^* is a relative maximum of the investment of the industry.

γ^* is an absolute maximum if $I(\gamma^*) > I(0)$, 0 is an absolute maximum if $I(\gamma^*) < I(0)$ and 0 and γ^* are two equals relatives maximums if $I(\gamma^*) = I(0)$(proof in the annexes)

The graph below (5.1) represents the evolution of Investment at the industry level in function of substitutability γ for different level of technical progress τ for both demand functions when monopoly is contestable:



Technical progress decreases the degree of substitutability maximizing investment.

In this graph the efficiency of firms are: $(\alpha_1 - c_{01}) = 0.45$; $(\alpha_2 - c_{02}) = 0.35$; $(\alpha_3 - c_{03}) = 0.25$ and $(\alpha_4 - c_{04}) = 0.15$

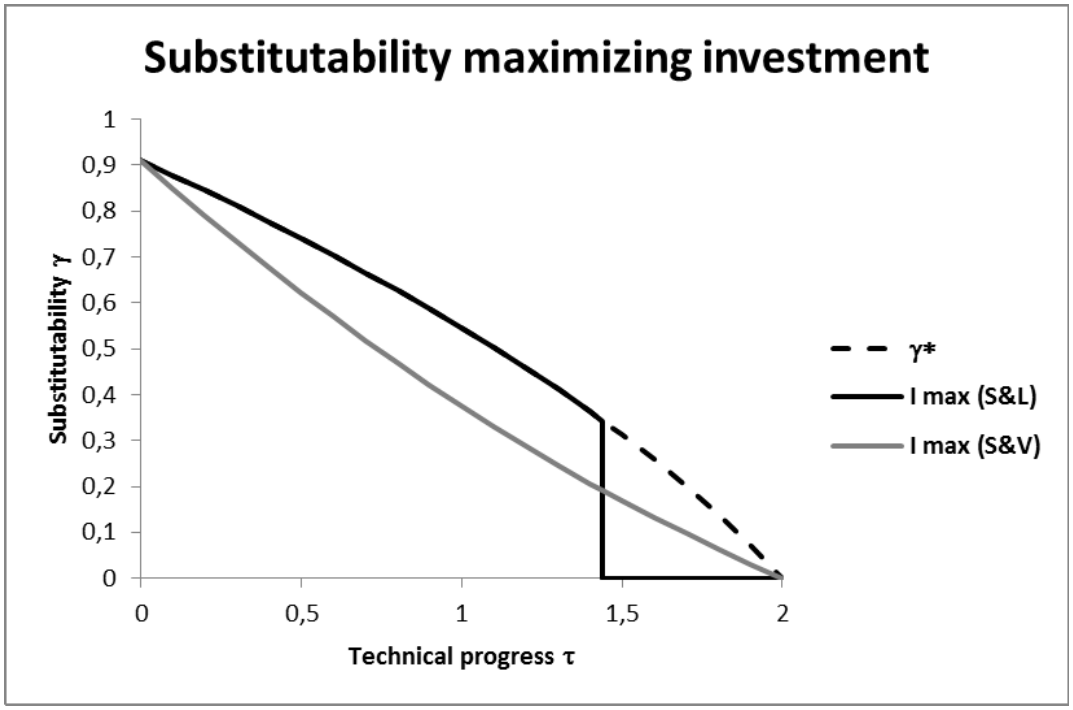
This graph shows that the degree of substitutability that maximizes investment of the industry decreases with technical progress. Each curve represents the evolution of the investment of the industry according to the degree of substitutability for a given level of technical progress. The darker the curves, the higher the technical progress, τ. In this example, the initial number of firms is 4, and we can see that less efficient firms exit as the degree of substitutability increases. Exits are illustrated by falls in investment for the degree of substitutability that cause the exit. The fall is all the more sharp as the number of firms decreases. In the left part, the demand function is Shubik & Levitan and in the right part, it is Singh and Vives demand function. We can observe that the degree of substitutability of the exits decrease with technical progress. It is also the case for the exit of the second most efficient firm which occurs for $\gamma = \gamma^*$. Indeed, we can notice, in equation (14) that γ^* is decreasing in τ. For the Singh & Vives demand function, the maximium investment of the industry

is achieved for $\gamma = \gamma^*$ which decreases with technical progress.For the Shubik & Levitan demand function, the maximum investment of the industry is achieved for $\gamma = \gamma^*$ or for $\gamma = 0$ when τ is high enough.

We denote $\tau^* = \frac{(\sqrt{E_i^2+E_j^2})\gamma^*+2E_j}{E_j}$, the value of technical progress such that $I(\gamma^*) = I(0)$. As long as $\tau \leq \tau^*$, $I(\gamma^*) \geq I(0)$ and if $\tau > \tau^*$ then $I(\gamma^*) < I(0)$. In both case, an increase in technical progress reduces the degree of substitutability that maximizes investment.

In the case we consider that monopoly is not contestable[2], the investment still decreases with the substitutability and, therefore, the maximum investment is still achieved for $\gamma = \gamma^*$.

The graph below (5.1), represents the degree of substitutability maximizing investment in function of technical progress for both demand functions.



Degree of substitutability maximizing Investment

The dotted black curve represents γ^* as a function of τ for Shubik & Levitan demand function and the full black curve represents the degree of substitutability maximizing investment as a function of τ still for Shubik & Levitan demand function. Both lines are mingled until τ^* and full line equals zero beyond. The

[2] in that case, output of the monopolist, firm i no longer depends on substitutability. $q_i = \frac{\alpha_i - c_{0i}}{2-\tau}$ for Shubik and Levitan demand function and $\frac{\alpha_i - c_{0i}}{2-\tau} < \frac{\alpha_j - c_{0j}}{\gamma^*}$

full gray curve represents the degree of substitutability maximizing investment as a function of τ for Singh & Vives demand function. In both cases technical progress decreases the degree of substitutability maximizing investment.

Consumer surplus and Welfare.

Consumer surplus can be calculated from equations (5) and (6) for $N = 2$:

$CS(\gamma) = \frac{q_i^2+q_j^2}{2} + \gamma q_i q_j$ which yields after some manipulations:

$$CS(\gamma) = \frac{(1+\gamma)(E_i+E_j)^2}{4(2-\tau-\gamma^2+\gamma)^2} + \frac{(1-\gamma)(E_i-E_j)^2}{4(2-\tau-\gamma^2-\gamma)^2} \tag{15}$$

which can be rewriten in a more compact form: $CS(\gamma) = (1+\gamma)q^2 + (1-\gamma)\delta^2$

For $\gamma \leq \gamma^*$, consumer surplus is increasing close to γ^*, $\frac{\partial CS}{\partial \gamma}\left(\gamma^{*^-}\right) > 0$ (proof in the annexes)

For $\gamma \geq \gamma^*$, $q_j = 0$, thus $CS(\gamma) = \frac{q_i^2}{2} = \frac{(\alpha_j - c_{0j})^2}{2\gamma^2}$. It is obvious that $CS(\gamma)$ is decreasing. $\frac{\partial CS}{\partial \gamma}\left(\gamma^{*^+}\right) < 0$. As a result, γ^* is a relative maximum of consumer surplus.

γ^* is an absolute maximum if $CS(\gamma^*) > CS(0)$, 0 is an absolute maximum if $CS(\gamma^*) < CS(0)$ and 0 and γ^* are two equals relatives maximums if $CS(\gamma^*) = CS(0)$. (proof in the annexes)

Moreover, we can notice that $CS(\gamma^*) = \frac{I(\gamma^*)}{\tau}$ and $CS(0) = \frac{I(0)}{\tau}$. This means that Investment of the industry and consumer surplus have the same absolute maximum. If technical progress $\tau < \tau^*$, γ^* is the absolute maximum and if $\tau > \tau^*$, 0 is the absolute maximum.

Welfare is given by: $W(\gamma) = \pi_1(\gamma) + \pi_2(\gamma) + CS(\gamma)$, which yields:

$W(\gamma) = (\alpha_i - c_{0i})q_i + (\alpha_j - c_{0j})q_j + \frac{(\tau-1)}{2}\left(q_i^2 + q_j^2\right) - \gamma q_i q_j$ or after some manipulations:

$$W(\gamma) = \frac{\left(3-\tau-2\gamma^2+\gamma\right)(E_i+E_j)^2}{4(2-\tau-\gamma^2+\gamma)^2} + \frac{\left(3-\tau-2\gamma^2-\gamma\right)(E_i-E_j)^2}{4(2-\tau-\gamma^2-\gamma)^2} \tag{16}$$

which can be rewriten in a more compact form: $W(\gamma) = \left(3-\tau-2\gamma^2+\gamma\right)q^2 + \left(3-\tau-2\gamma^2-\gamma\right)\delta^2$

For $\gamma \leq \gamma^*$ welfare is increasing close to γ^*, $\frac{\partial W}{\partial \gamma}\left(\gamma^{*^-}\right) > 0$ (proof in the annexes)

For $\gamma > \gamma^*$, $q_j = 0$, thus $W(\gamma) = E_i q_i + \frac{(\tau-1)}{2} q_i^2 = \frac{\left(3-\tau-2\gamma^2\right)E_j^2}{2\gamma^2}$. It is obvious that $W(\gamma)$ is decreasing. $\frac{\partial W}{\partial \gamma}\left(\gamma^{*^+}\right) < 0$. As a result, γ^* is a relative maximum of welfare.

γ^* is an absolute maximum if $W(\gamma^*) > W(0)$, 0 is an absolute maximum if $W(\gamma^*) < W(0)$ and 0 and γ^* are two equals relatives maximums if $W(\gamma^*) = W(0)$. (proof in the annexes)

Moreover, we can notice that $W(\gamma^*) = \frac{3-\tau-2\gamma^{*2}}{\tau} I(\gamma^*)$ and $W(0) = \frac{3-\tau}{\tau} I(0)$. In particular, if $I(\gamma^*) = I(0)$ then $W(\gamma^*) < W(0)$. This means that Investment of the industry and welfare do not have necessarily the same absolute maximum.

We denote τ^{**}, the level of technical progress such that if $\tau = \tau^{**}$ then $W(\gamma^*) = W(0)$. if $\tau < \tau^{**}$, γ^* is the absolute maximum and if $\tau > \tau^{**}$, 0 is the absolute maximum. We can show that $\tau^{**} \leq \tau^*$(proof in the annexes). This means that for $\tau < \tau^{**}$, γ^* is the absolute maximum for investment of the industry, consumer surplus and welfare. For $\tau^{**} < \tau < \tau^*$, γ^* is the absolute maximum for investment of the industry and consumer surplus and 0 is the maximum for welfare. For $\tau > \tau^*$, 0 is the absolute maximum for investment of the industry, consumer surplus and welfare.

6 Conclusion:

This paper shows that technical progress tends to reduce the degree of competition (measured as the number of firms or as the degree of substitutability) that maximizes investment at the industry level, Consumer Surplus and Welfare.

Technical progress, understood as technological opportunity, act as a catalyst of competition. It reduces the cost of investment aiming to improve efficiency. The highest the technological progress, the more firms are encouraged to improve efficiency and the more they invest. This race for investment is, in a way, a form of competition. As a result, when there is a level of competition that maximizes investment, an increase in the level of technical progress strenghtens competition and requires a reduction of the level of competition (number of competitors or substitutability) to keep the optimal level.

In the case of competition by the number of competitors, we have seen that investment of the industry occurs when the elasticity of investment per firm according to the number of firm is equal to -1. An increase in the level of technical progress makes investment per firm more elastic and therefore, requires a reduction of the number of firms to keep it at its optimal value of -1.

In the case of competition measured with the degree of substitutability in assymmetrical markets, competition increases the assymmetry between competitors. The degree of substitutability increases[3] investment of the industry until the less efficient competitor is forced out of the market. The maximum investment is achieved for this degree of substitutability. An increase in the

[3] at least starting from a certain degree. Indeed for Schubik and Levitan demand function, Investment of the industry starts to decrease with substitutability before increasing.

level of technical progress strenghtens competition and reduces the degree of substitutability for which the less efficient firm is forced out.

In both cases, the catalyst effect of technical progress on competition entails a reduction of the degree of competition that maximizes investment of the industry.

Investment in efficiency improvement[4] is a main driver of consumer surplus and welfare. As a result, technical progress also decreases the degree of competition that maximizes consumer surplus and welfare.

This property of technical progress has policy implications. In industries where the level of technical progress is high, like information technologies, telecommunications, biological or nanotehnological engineering, the degree of competition shoud be adjusted to a lower level than industries enjoying a lower level of technical progress in order to maximize investment, consumer surplus and welfare.

This paper highlights a theoretical point of view, a comparison of different industries enjoying different level of technological progress, would be usefull to empirically test this theory.

7 Annexes:

Proof of the output reallocation effect in parametric model:

From equation 8, we can write the market share can be written:

$\sigma_i = \frac{A\left(A^2+(1-\gamma)(A+\gamma)-A\tau\right)(\alpha_i-c_{0i})-\gamma A^2\sum_{j\neq i}(\alpha_j-c_{0j})}{A\sum_j(\alpha_j-c_{0j})[A^2+(1-\gamma)(A+\gamma)-A\tau-\gamma A(N-1)]}$

The derivation yields:

$\frac{\partial\sigma_i}{\partial\tau} = \frac{\gamma A^4\sum_j(\alpha_j-c_{0j})\left[(\alpha_i-c_{0i})(N-1)-\sum_{j\neq i}(\alpha_j-c_{0j})\right]}{\left(A\sum_j(\alpha_j-c_{0j})[A^2+(1-\gamma)(A+\gamma)-A\tau-\gamma A(N-1)]\right)^2}$

$sign\left(\frac{\partial\sigma_i}{\partial\tau}\right) = sign\left(\left(\alpha_i-c_{0i}\right)(N-1)-\sum_{j\neq i}\left(\alpha_j-c_{0j}\right)\right) = sign\left(\left(\alpha_i-c_{0i}\right)-\sum_j\left(\alpha_j-c_{0j}\right)/N\right)$

which means that $\frac{\partial\sigma_i}{\partial\tau}>0$ if firm i's efficiency is above average. Same manner, $\frac{\partial\sigma_i}{\partial\tau}<0$ if firm i's efficiency is below average, and $\frac{\partial\sigma_i}{\partial\tau}=0$ if firm i's efficiency is exactly average.

Verification that $sign(\varepsilon) = sign(\frac{\partial\varepsilon}{\partial\tau})$

In the parametric model, $F=\frac{\tau q^2}{2}$, $\frac{\partial F}{\partial N}=\tau q\frac{\partial q}{\partial N}$ and

$\varepsilon = \frac{2N}{q}\frac{\partial q}{\partial N} = \frac{2N\gamma\left((1-\gamma)\gamma-(\gamma(N-2)+1)^2\right)}{(\gamma(N-2)+1)((1-\gamma)(\gamma(N-1)+1)-(\gamma(N-2)+1)\tau+(\gamma(N-2)+1)(\gamma(N-1)+1))}$

and $\frac{\partial\varepsilon}{\partial\tau} = \frac{2N\gamma\left((1-\gamma)\gamma-(\gamma(N-2)+1)^2\right)}{((1-\gamma)(\gamma(N-1)+1)-(\gamma(N-2)+1)\tau+(\gamma(N-2)+1)(\gamma(N-1)+1))^2} = \frac{q}{(\alpha-c_0)}\varepsilon$

As $\frac{q}{(\alpha-c_0)}>0$, then $sign(\varepsilon)=sign(\frac{\partial\varepsilon}{\partial\tau})$

[4] other types of investment may not have the same consequences.

Verification that if $\varepsilon_{CS} < 0$ then $\frac{\partial \varepsilon_{CS}}{\partial \tau} < 0$

We know that $CS(N) = \frac{F(N)(1+\gamma(N-1))}{\tau}$, thus $\varepsilon_{cs}(N) = \frac{\partial F}{\partial N}\frac{N}{F} + \frac{\gamma N}{1+\gamma(N-1)} = \varepsilon + \frac{\gamma N}{1+\gamma(N-1)}$. As a result, $\varepsilon(N) \leq \varepsilon_{cs}(N)$ and if $\varepsilon_{CS}(N) < 0$ then $\varepsilon(N) < 0$. Moreover, $\frac{\partial \varepsilon_{CS}}{\partial \tau} = \frac{\partial \varepsilon}{\partial \tau}$ and since $sign\left(\frac{\partial \varepsilon}{\partial \tau}\right) = sign(\varepsilon)$, $\varepsilon(N) < 0$ means $\frac{\partial \varepsilon}{\partial \tau} < 0$ and $\frac{\partial \varepsilon_{CS}}{\partial \tau} < 0$.

If $0 > \varepsilon > \frac{-\gamma N}{1+\gamma(N-1)}$ then $\varepsilon_{cs}(N) > 0$, $\varepsilon(N) < 0$ and thus $\frac{\partial \varepsilon}{\partial \tau} < 0$ and $\frac{\partial \varepsilon_{CS}}{\partial \tau} < 0$.

Verification that if N^* maximizes investment of the industry and N^{**} maximizes consumer surplus, then $N^{**} \geq N^*$

The elasticity of consumer surplus writes: $\varepsilon_{cs}(N) = \varepsilon(N) + \frac{\gamma N}{1+\gamma(N-1)}$. Thus if N^* maximizes investment of the industry and N^{**} maximizes consumer surplus, then $\varepsilon_{CS}(N^{**}) \geq \varepsilon(N^{**})$. We know that $\varepsilon_{CS}(N^{**}) = -1$ thus $\varepsilon(N^{**}) \leq -1$. However, we know also that $\varepsilon(N^*) = -1$ and $\frac{\partial \varepsilon}{\partial N}(N^*) < 0$. Moreover, $\frac{\partial \varepsilon_{cs}}{\partial N}(N^{**}) = \frac{\partial \varepsilon}{\partial N}(N^{**}) + \frac{\gamma(1-\gamma)}{(1+\gamma(N-1))^2}$, therefore $0 > \frac{\partial \varepsilon_{cs}}{\partial N}(N^{**}) > \frac{\partial \varepsilon}{\partial N}(N^{**})$. This means that $N^{**} \geq N^*$.

Verification that maximum welfare is achieved for a lower number of firms than consumer surplus and a higher number of firms than investment (exepted for high τ and/or high γ where maximum welfare is achieved for monopoly)

remember $A = 1 + \gamma(N-2)$

If N_{CS} is the number of firms maximizing consumer surplus of the industry, then $\varepsilon_{CS} = -1$, $\varepsilon = -\left(\frac{1+\gamma(2N_{CS}-1)}{1+\gamma(N_{CS}-1)}\right)$ and $\varepsilon_q = -\left(\frac{1+\gamma(2N_{CS}-1)}{2(1+\gamma(N_{CS}-1))}\right)$, for simplification, we denote $A_{CS} = 1 + \gamma(N_{CS}-2)$. Replacing those expressions in equation (12) yields, after some manipulations: $\frac{\partial WI}{\partial N}(N_{CS}) = (\alpha - c_0)\frac{q}{2}\left(\frac{(1-\gamma)A_{CS} - A_{CS}\tau + (1-\gamma)^2}{(1-\gamma)(A_{CS}+\gamma) - A_{CS}\tau + A_{CS}(A_{CS}+\gamma)}\right)$

As the denominator is positive, this expression is negative if $\tau \geq (1-\gamma)\left(\frac{1-\gamma}{A_{CS}} + 1\right)$ and we can verify that it is the case for all values of τ and γ, because otherwise, if $\tau < (1-\gamma)\left(\frac{1-\gamma}{A_{CS}} + 1\right)$ then $A_{CS}\tau < (1-\gamma)(1-\gamma+A_{CS})$. In this case, $\varepsilon_{CS} = \frac{2N\gamma\left((1-\gamma)\gamma - A_{CS}^2\right)}{[(1-\gamma)(A_{CS}+\gamma) - A_{CS}\tau + A_{CS}(A_{CS}+\gamma)]A_{CS}} + \frac{\gamma N}{A_{CS}+\gamma} > -1$ which is not possible because by assumption $\varepsilon_{CS} = -1$ therefore, $\frac{\partial WI}{\partial N}(N_{CS}) < 0$ and denoting N_w, the number of firms maximizing welfare, $N_w < N_{CS}$. As N_{CS} decreases with technical progress, then N_w also decreases with technical progress.

If N_I is the number of firms maximizing investment of the industry, then $\varepsilon = -1$ and $\varepsilon_q = -1/2$. We denote $A_I = 1 + \gamma(N_I - 2)$ Replacing $\varepsilon = -1$ in equation(12) yields $\varepsilon_{CS} = \frac{\gamma N_I}{(A_I+\gamma)} - 1$, as a result, after some manipulations $\frac{\partial WI}{\partial N}(N_I) = (\alpha - c_0)\frac{q}{2} - \frac{q^2\gamma N_I}{2} = (\alpha - c_0)\frac{q}{2}\left[\frac{(1-\gamma)(2A_I+\gamma) - A_I\tau}{(1-\gamma)(A_I+\gamma) - A_I\tau + (A_I+\gamma)A_I}\right]$

As the denominator is positive, this expression is positive if $\tau < (1-\gamma)\left(\frac{(1+\gamma(N_I-1))}{(1+\gamma(N_I-2))} + 1\right)$. In this case, $\frac{\partial WI}{\partial N}(N_I) > 0$ and $N_w > N_I$. This is more likely to occur when

τ and γ are low. Otherwise, if τ and γ are sufficiently high such that $\tau > (1-\gamma)\left(\frac{(1+\gamma(N_I-1))}{(1+\gamma(N_I-2))}+1\right)$ then $N_w < N_I$. However, this can only occur when $N_w < 2$ which means that the maximum welfare is achieved for the monopoly.

Indeed, $\varepsilon = \frac{2\gamma N_I\left((1-\gamma)\gamma - A_I^2\right)}{[(1-\gamma)(A_I+\gamma)-A_I\tau+(A_I+\gamma)A_I]A_I} = -1$, if $\tau > (1-\gamma)\left(\frac{(1+\gamma(N_I-1))}{(1+\gamma(N_I-2))}+1\right)$ then $A_I\tau > (1-\gamma)(2A_I+\gamma)$ and $(1-\gamma)(A_I+\gamma) - A_I\tau + (A_I+\gamma)A_I < (1-\gamma)(A_I+\gamma) - (1-\gamma)(2A_I+\gamma) + (A_I+\gamma)A_I$

$(1-\gamma)(A_I+\gamma) - A_I\tau + (A_I+\gamma)A_I < \gamma N A_I$

replacing in $\varepsilon = -1$ yields $\frac{2\gamma N_I\left((1-\gamma)\gamma - A_I^2\right)}{\gamma N_I A_I^2} < -1$ or as $\left((1-\gamma)\gamma - A_I^2\right)$ is negative, $2\gamma N_I\left((1-\gamma)\gamma - A_I^2\right) > -\gamma N_I A_I^2$

which means $A_I^2 < 2(1-\gamma)\gamma > 0$ which is possible only if $N_I \leq 3/2$ and $\gamma \geq 1/3$.

Therefore, it is possible that $N_w < N_I$ with $N_I > 1$. As N_I is integer, it is possible that $N_{I\ \max} = 2$ and $N_{W\ \max} = 1$.

However, if $N_{I\ \max} > 2$, then $N_{W\ \max} \geq N_{I\ \max}$

As a result, exepted for $N_{W\ \max} = 1$, we have $N_{CS} > N_w > N_I$.

Proof that investment of the industry is increasing for $\gamma \leq \gamma^*$ when γ is close to γ^*

$\delta = \frac{q_i - q_j}{2}$ and $q = \frac{q_i+q_j}{2}$ means $q_i = q+\delta$ and $q_j = q-\delta$

Investment of the industry for $\gamma \leq \gamma^*$ writes $I(\gamma) = \frac{\tau}{2}\left[(q+\delta)^2 + (q-\delta)^2\right] = \tau\left(q^2+\delta^2\right)$

Thus $\frac{\partial I}{\partial \gamma}(\gamma) = 2\tau\left(q\frac{\partial q}{\partial \gamma}(\gamma) + \delta\frac{\partial \delta}{\partial \gamma}(\gamma)\right)$ and $\frac{\partial I}{\partial \gamma}\left(\gamma^{*-}\right) = 2\tau q\left(\frac{\partial q}{\partial \gamma}(\gamma^*) + \frac{\partial \delta}{\partial \gamma}(\gamma^*)\right)$ because $q_j(\gamma^*) = 0$ which yields $\delta(\gamma^*) = q(\gamma^*)$.

We know that γ reallocates output from j to i as i is more efficient, thus $q_i(\gamma)$ is increasing and thus

$\frac{\partial q_i}{\partial \gamma}(\gamma) = \frac{\partial q}{\partial \gamma}(\gamma) + \frac{\partial \delta}{\partial \gamma}(\gamma) > 0$. As a result $\frac{\partial I}{\partial \gamma}\left(\gamma^{*-}\right) > 0$

Proof that γ^* is an absolute maximum if $I(\gamma^*) > I(0)$ and 0 is an absolute maximum is $I(\gamma^*) < I(0)$ and 0 and γ^* are two equals relatives maximums if $I(\gamma^*) = I(0)$

$I(\gamma)$ is convex, indeed, $I(\gamma) = \tau\left(q^2+\delta^2\right)$, $\frac{\partial I}{\partial \gamma}(\gamma) = 2\tau\left(q\frac{\partial q}{\partial \gamma}(\gamma) + \delta\frac{\partial \delta}{\partial \gamma}(\gamma)\right)$

$\frac{\partial^2 I}{\partial \gamma^2}(\gamma) = 2\tau\left(\left(\frac{\partial q}{\partial \gamma}(\gamma)\right)^2 + q\frac{\partial^2 q}{\partial \gamma^2}(\gamma) + \left(\frac{\partial \delta}{\partial \gamma}(\gamma)\right)^2 + \delta\frac{\partial^2 \delta}{\partial \gamma^2}(\gamma)\right)$

q is the symmetric output for $N=2$, thus $q = \frac{E_i+E_j}{2(2-\gamma^2-\tau+\gamma)}$

$\frac{\partial q}{\partial \gamma}(\gamma) = \frac{-(E_i+E_j)(1-2\gamma)}{2(2-\gamma^2-\tau+\gamma)^2}$ and $\frac{\partial^2 q}{\partial \gamma^2}(\gamma) = \frac{(E_i+E_j)(2(3-\tau)-6\gamma(1-\gamma))}{2(2-\gamma^2-\tau+\gamma)^3}$. This expression is positive, thus $\frac{\partial^2 q}{\partial \gamma^2}(\gamma) \geq 0$.

$\delta(\gamma)$ is also convex , indeed $\delta(\gamma) = \frac{E_i - E_j}{2(2-\gamma^2-\tau-\gamma)}$; $\frac{\partial \delta}{\partial \gamma}(\gamma) = \frac{(E_i-E_j)(1+2\gamma)}{2(2-\gamma^2-\tau-\gamma)^2}$ and $\frac{\partial^2 \delta}{\partial \gamma^2}(\gamma) = \frac{((3-\tau)+3\gamma(1-\gamma))(E_i-E_j)}{(2-\tau-\gamma^2-\gamma)^3}$ This expression is positive, indeed $\left(2-\tau-\gamma^2-\gamma\right) > 0$ for $\gamma \leq \gamma^*$ because $\delta(\gamma) > 0$

Thus $\frac{\partial^2 \delta}{\partial \gamma^2}(\gamma) > 0$. As a result, $\frac{\partial^2 I}{\partial \gamma^2}(\gamma) > 0$, $I(\gamma)$ is convex for $\gamma \leq \gamma^*$. This convexity implies that if $I(\gamma^*) > I(0)$ then $\forall \gamma \in [0, \gamma^*]$, $I(\gamma^*) > I(\gamma)$ which means γ^* is an absolute maximum. Otherwise, if $I(\gamma^*) < I(0)$ then $\forall \gamma \in [0, \gamma^*]$, $I(0) > I(\gamma)$ $\gamma = 0$ is an absolute maximum. If $I(\gamma^*) = I(0)$, 0 and γ are two relatives maximums.

Proof that consumer surplus is increasing for $\gamma \leq \gamma^*$ when γ is close to γ^*

Using equation(15), $q_i = q + \delta$ and $q_j = q - \delta$, $q_i q_j = (q+\delta)(q-\delta) = q^2 - \delta^2$ and consumer surplus can be written $CS(\gamma) = (1+\gamma) q^2 + (1-\gamma)\delta^2$

For $\gamma \leq \gamma^*$, the evolution of Consumer surplus according to γ is given by: $\frac{\partial CS}{\partial \gamma}(\gamma) = 2(1+\gamma) q \frac{\partial q}{\partial \gamma}(\gamma) + 2(1-\gamma)\delta \frac{\partial \delta}{\partial \gamma}(\gamma) + q^2 - \delta^2$ this yields $\frac{\partial CS}{\partial \gamma}\left(\gamma^{*-}\right) = 2q\left[(1+\gamma^*)\frac{\partial q}{\partial \gamma}(\gamma^*) + (1-\gamma^*)\frac{\partial \delta}{\partial \gamma}(\gamma^*)\right]$ because $\delta(\gamma^*) = q(\gamma^*)$.

using the fact that $q_j(\gamma^*) = 0$, we can write $\left(2-\gamma^{*2}-\tau\right)(\alpha_j - c_{0j}) - \gamma(\alpha_i - c_i) = 0$ and this yields $(\alpha_i - c_{0i}) = \frac{\left(2-\gamma^{*2}-\tau\right)(\alpha_j - c_j)}{\gamma^*}$; this allows to write: $\frac{\partial q}{\partial \gamma}(\gamma^*) = \frac{(1-2\gamma^*)E_j}{2\gamma(2-\gamma^{*2}-\tau+\gamma^*)}$ and $\frac{\partial \delta}{\partial \gamma}(\gamma^*) = \frac{(1+2\gamma^*)E_j}{2\gamma(2-\gamma^{*2}-\tau-\gamma^*)}$ replacing in $\frac{\partial CS}{\partial \gamma}\left(\gamma^{*-}\right)$ leads to $\frac{\partial CS}{\partial \gamma}\left(\gamma^{*-}\right) = q\left[\frac{(2-\tau)\left(1-2\gamma^{*2}\right)+2\gamma^{*4}}{\gamma\left((2-\gamma^{*2}-\tau)^2-\gamma^{*2}\right)}\right] E_j > 0$

Proof that γ^* is an absolute maximum if $CS(\gamma^*) > CS(0)$, 0 is an absolute maximum if $CS(\gamma^*) < CS(0)$ and 0 and γ^* are two equals relatives maximums if $CS(\gamma^*) = CS(0)$.

Consumer surplus, as investment is convex for $\gamma \in [0, \gamma^*]$. $\frac{\partial^2 CS}{\partial \gamma^2}(\gamma) = 4\left[q\frac{\partial q}{\partial \gamma}(\gamma) - \delta\frac{\partial q}{\partial \gamma}(\gamma)\right] + 2\left[(1+\gamma)\left(\frac{\partial q}{\partial \gamma}(\gamma)\right)^2 + (1-\gamma)\left(\frac{\partial \delta}{\partial \gamma}(\gamma)\right)^2\right] + 2\left[(1+\gamma)q\frac{\partial^2 q}{\partial \gamma^2}(\gamma) + (1-\gamma)\delta\frac{\partial^2 \delta}{\partial \gamma^2}(\gamma)\right]$

$\frac{\partial^2 CS}{\partial \gamma^2}(\gamma) > 0$ thus consumer surplus is convex

The convexity of CS implies that if $CS(\gamma^*) > CS(0)$

then γ^* is an absolute maximum and if $CS(\gamma^*) < CS(0)$ then 0 is an absolute maximum and if $CS(\gamma^*) = CS(0)$ 0 and γ are two relatives maximums.

Proof that welfare is increasing for $\gamma \leq \gamma^*$ when γ is close to γ^*

$W(\gamma) = \left(3-\tau-2\gamma^2+\gamma\right)q^2 + \left(3-\tau-2\gamma^2-\gamma\right)\delta^2$

For $\gamma \leq \gamma^*$, the evolution of Consumer surplus according to γ is given by: $\frac{\partial W}{\partial \gamma}(\gamma) = 2\left(3-\tau-2\gamma^2+\gamma\right)q\frac{\partial q}{\partial \gamma}(\gamma) + 2\left(3-\tau-2\gamma^2-\gamma\right)\delta\frac{\partial \delta}{\partial \gamma}(\gamma) + (1-2\gamma)q^2 - (1+2\gamma)\delta^2$ this yields $\frac{\partial W}{\partial \gamma}\left(\gamma^{*-}\right) = 2q\left[\left(3-\tau-2\gamma^{*2}+\gamma^*\right)\frac{\partial q}{\partial \gamma}(\gamma^*) + \left(3-\tau-2\gamma^{*2}-\gamma^*\right)\frac{\partial \delta}{\partial \gamma}(\gamma^*)\right] - 4\gamma^* q^2$ because $\delta(\gamma^*) = q(\gamma^*)$

using the values of $\frac{\partial q}{\partial \gamma}(\gamma^*)$ and $\frac{\partial \delta}{\partial \gamma}(\gamma^*)$ calculated above yields:

$\frac{\partial W}{\partial \gamma}\left(\gamma^{*^-}\right) = 2q\left[\frac{\left(3-\tau-2\gamma^{*2}+\gamma^*\right)(1-2\gamma^*)}{2\gamma(2-\gamma^{*2}-\tau+\gamma^*)} + \frac{\left(3-\tau-2\gamma^{*2}-\gamma^*\right)(1+2\gamma^*)}{2\gamma(2-\gamma^{*2}-\tau-\gamma^*)} - 2\gamma^*\left(\frac{\left(3-\tau-2\gamma^{*2}+\gamma^*\right)}{2\gamma(2-\gamma^{*2}-\tau+\gamma^*)}\right)\right]E_j$

leads to $\frac{\partial W}{\partial \gamma}\left(\gamma^{*^-}\right) = q\left[\frac{2(2-\tau)\left(3-\tau+2\gamma^{*2}\right)(1-\gamma)+\gamma^{*2}(1+2\gamma^*)}{\gamma\left((2-\gamma^{*2}-\tau)^2-\gamma^{*2}\right)}\right]E_j > 0$

Proof that γ^* is an absolute maximum if $W(\gamma^*) > W(0)$, 0 is an absolute maximum if $W(\gamma^*) < W(0)$ and 0 and γ^* are two equals relatives maximums if $W(\gamma^*) = W(0)$.

$\frac{\partial^2 W}{\partial \gamma^2}(\gamma) > 0$, Welfare is convex

The convexity of W implies that if $W(\gamma^*) > W(0)$

then γ^* is an absolute maximum and if $W(\gamma^*) < W(0)$ then 0 is an absolute maximum and if $W(\gamma^*) = W(0)$, 0 and γ are two relatives maximums.

Proof that $\tau^{**} \leq \tau^*$

We know that $\tau^* = \frac{\left(\sqrt{E_i^2+E_j^2}\right)\gamma^*+2E_j}{E_j}$,thus $(2-\tau^*)^2 = \frac{\gamma^{*2}\left(E_i^2+E_j^2\right)}{E_j^2}$

$W(\gamma^*) = W(0)$ implies $(3-\tau^{**})\gamma^{*2}\left(E_i^2+E_j^2\right) = \left(3-\tau^{**}-2\gamma^{*2}\right)(2-\tau^{**})^2 E_j^2$

we replace $\gamma^{*2}\left((\alpha_i-c_{0i})^2+(\alpha_j-c_{0j})^2\right) = (\alpha_j-c_{0j})^2(2-\tau^*)^2$ in the previous equation and thus

$(3-\tau^{**})E_j^2(2-\tau^*)^2 = \left(3-\tau^{**}-2\gamma^{*2}\right)(2-\tau^{**})^2 E_j^2$ this yields:

$(2-\tau^{**})^2 = \frac{(3-\tau)}{(3-\tau-2\gamma^{*2})}(2-\tau^*)^2$. The term $\frac{(3-\tau)}{(3-\tau-2\gamma^{*2})} > 1$, as a result, $\tau^{**} < \tau^*$.

Robusteness check with the utility function from Singh and Vives (1984):

$$U(q_1,...,q_N) = \sum_{i=1}^{N}\alpha_i q_i - \frac{1}{2(1+\gamma)}\left(\sum_{i=1}^{N}q_i^2 + 2\gamma\sum_{j\neq i}q_i q_j\right) - \sum_{i=1}^{N}p_i q_i \quad (17)$$

$$p_i = \alpha_i - \frac{1}{1+\gamma}\left(q_i + \gamma\sum_{j\neq i}q_j\right) \quad (18)$$

$$q_i = \frac{(1+\gamma)\left[(\alpha_i-p_i)\left[\gamma(N-2)+1\right] - \gamma\sum_{j\neq i}(\alpha_j-p_j)\right]}{(1-\gamma)\left[\gamma(N-1)+1\right]} \quad (19)$$

We derive the general expression of q_i function of initial marginal costs using equations (2), (3) and (19):

$$q_i = \frac{(1+\gamma)B\left[\left(B^2+(1-\gamma)A-(1+\gamma)B\tau\right)(\alpha_i-c_{0i}) - \gamma B\sum_{j\neq i}(\alpha_j-c_{0j})\right]}{(1-\gamma)AB^2 + \left((1-\gamma)A-(1+\gamma)B\tau\right)\left((1-\gamma)A+B(1+B-(1+\gamma)\tau)\right)} \quad (20)$$

Symmetric Market:

Output:

$$q = \frac{(1+\gamma)(\gamma(N-2)+1)(\alpha-c_0)}{(1-\gamma)(\gamma(N-1)+1)-(1+\gamma)(\gamma(N-2)+1)\tau+(\gamma(N-2)+1)(\gamma(N-1)+1)}$$

Assymmetric duopoly:

Output:

$$q_i = \frac{(1+\gamma)\left[\left(2-\gamma^2-(1+\gamma)\tau\right)E_i-\gamma E_j\right]}{(2-\gamma^2-(1+\gamma)\tau)^2-\gamma^2}$$

Maximum substitutability:

$$\gamma^* = \frac{\sqrt{((\alpha_i-c_{0i})E_i+\tau E_j)^2+4(2-\tau)E_j^2}-E_i-\tau E_j}{2E_j}$$

and for the Singh and Vives demand function deduced from equation(19) is:

$$q_i = \frac{(1+\gamma)E_j}{\gamma}$$

References

[1] Arrow, K. (1962). Economic welfare and the allocation of resources for invention. In The rate and direction of inventive activity: Economic and social factors (pp. 609-626). Princeton University Press.

[2] Boone, J. (2008). A new way to measure competition. The Economic Journal, 118(531), 1245-1261.

[3] Aghion, P., Bloom, N., Blundell, R., Griffith, R., & Howitt, P. (2005). Competition and innovation: An inverted-U relationship. The Quarterly Journal of Economics, 120(2), 701-728.

[4] Aghion, P., Akcigit, U., & Howitt, P. (2014). What do we learn from Schumpeterian growth theory?. In Handbook of economic growth (Vol. 2, pp. 515-563). Elsevier.

[5] Dasgupta, P., & Stiglitz, J. (1980). Industrial structure and the nature of innovative activity. The Economic Journal, 90(358), 266-293.

[6] Fudenberg, D., & Tirole, J. (1985). Preemption and rent equalization in the adoption of new technology. The Review of Economic Studies, 52(3), 383-401.

[7] Gilbert, R. J., & Newbery, D. M. (1982). Preemptive patenting and the persistence of monopoly. The American Economic Review, 514-526.

[8] Kamien, M. I., & Schwartz, N. L. (1982). Market structure and innovation. Cambridge University Press.

[9] Houngbonon, G. V., & Jeanjean, F. (2016). What level of competition intensity maximises investment in the wireless industry?. Telecommunications Policy, 40(8), 774-790.

[10] Jeanjean, F. (2017). Technical Progress in the Relationship between Competition and Investment. ssrn papers.

[11] Jeanjean, F., & Houngbonon, G. V. (2017). Market structure and investment in the mobile industry. Information Economics and Policy, 38, 12-22.

[12] Motta, M., & Tarantino, E. (2017). The effect of horizontal mergers, when firms compete in prices and investments (No. 1579).

[13] Reinganum, J. F. (1981). On the diffusion of new technology: A game theoretic approach. The Review of Economic Studies, 48(3), 395-405.

[14] Scherer, F. M., & Ross, D. (1990). Economic Performance. Boston: Houghton-Mifflin.

[15] Schmutzler, A. (2013). Competition and investment—A unified approach. International Journal of Industrial Organization, 31(5), 477-487.

[16] Shubik, M., & Levitan, R. (1980). Market structure and behavior. Harvard Univ Pr.

[17] Singh, N., & Vives, X. (1984). Price and quantity competition in a differentiated duopoly. The RAND Journal of Economics, 546-554.

[18] Tandon, P. (1984). Innovation, market structure, and welfare. The American Economic Review, 74(3), 394-403.

[19] Vives, X. (2008). Innovation and competitive pressure. The Journal of Industrial Economics, 56(3), 419-469.