ECONSTOR
Make Your Publications Visible.

A Service of

ZBW Leibniz-Informationszentrum Wirtschaft
Leibniz Information Centre for Economics

Mumtaz, Haroon; Theodoridis, Konstantinos

Working Paper

The Federal Reserve's implicit inflation target and macroeconomic dynamics: A SVAR analysis

Working Paper, No. 820

Provided in Cooperation with:
School of Economics and Finance, Queen Mary University of London

Suggested Citation: Mumtaz, Haroon; Theodoridis, Konstantinos (2017) : The Federal Reserve's implicit inflation target and macroeconomic dynamics: A SVAR analysis, Working Paper, No. 820, Queen Mary University of London, School of Economics and Finance, London

This Version is available at:
https://hdl.handle.net/10419/184771

Standard-Nutzungsbedingungen:

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

Terms of use:

Documents in EconStor may be saved and copied for your personal and scholarly purposes.

You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.

WWW.ECONSTOR.EU

Mitglied der Leibniz-Gemeinschaft

Uif Gfefsbm Sftfswf't jnqmjdju jogmbujpo ubshfu boe Nbdspfdpopnjd ezobnjdt/ B TWBS bobmztjt/

Haroon Mumtaz boe Konstantinos Theodoridis

Working Paper No. 931 Bqsjm 3128 ISSN 1473-0278

School of Economics and Finance

Queen Mary University of London

The Federal Reserve's implicit inflation target and Macroeconomic dynamics. A SVAR analysis.*

Haroon Mumtaz†
Queen Mary University

Konstantinos Theodoridis‡
Bank of England
Lancaster University Management School

April 2017

Abstract

This paper identifies shocks to the Federal Reserve's inflation target as VAR innovations that make the largest contribution to future movements in long-horizon inflation expectations. The effectiveness of this scheme is documented via Monte-Carlo experiments. The estimated impulse responses indicate that a positive shock to the target is associated with a large increase in inflation, GDP growth and long-term interest rates. Target shocks are estimated to be a vital factor behind the increase in inflation during the pre-1980 period and are an important driver of the decline in long-term interest rates over the last two decades.

Key words: SVAR, DSGE model, inflation target.
JEL codes: C5, E1, E5, E6

1 Introduction

In the aftermath of the Great Recession, nominal interest rates in the US have remained persistently close to the zero lower bound (ZLB). This unique situation has highlighted the potential limitations of conventional monetary policy. While many central banks have pursued unconventional monetary policies such as quantitative easing, their effectiveness in stimulating the real economy is not undisputed (see Gambacorta *et al.* (2014)). In this scenario, the role of the Federal Reserve's (Fed) implicit inflation target has returned to the forefront. Some commentators have argued that a target higher than 2 percent should be considered in order to mitigate the negative effects of the ZLB on the effectiveness of monetary policy (see Ball (2014)). Potential changes in the Fed's implicit target are also of interest from a historical perspective. In a seminal paper, Ireland (2007) shows that this target level varied substantially and was one of the main factor behind the 'great inflation' of the 1970s. Gürkaynak *et al.* (2005) show that adding a time-varying inflation target is crucial in trying to understand the response of forward interest rates to macroeconomic developments.

In this paper, we propose a structural vector autoregression (SVAR) that can be used to identify shocks to the inflation target. The identification scheme exploits the idea that innovations to the inflation target are the main driver of long-horizon inflation expectations in the medium to the

*Any views expressed are solely those of the author(s) and so cannot be taken to represent those of the Bank of England or to state Bank of England policy. This paper should therefore not be reported as representing the views of the Bank of England or members of the Monetary Policy Committee, Financial Policy Committee or Prudential Regulation Authority Board.

†Email:h.mumtaz@qmul.ac.uk
‡konstantinos.theodoridis@bankofengland.co.uk

long-run. In other words, we apply the identification of Uhlig (2004a) to estimate the inflation target shock as the VAR innovation that makes the largest contribution to the forecast error variance (FEV) of a measure of long-horizon inflation expectations. Using an extensive Monte-Carlo experiment, we show that this simple strategy can successfully recover the inflation target shock and its impact on macroeconomic and financial variables.

Estimates of this SVAR using US data indicate that an inflation target shock that raises long-horizon inflation expectations by 1 percent has a large impact on the economy. In our benchmark model, this shock is associated with an impact on CPI inflation of about 1.2 percent, one year ahead, and a peak increase in short and long-term interest rates of about 3 percent. Real GDP growth is estimated to increase by about 0.8 percent in response to this shock after one year, albeit, with the estimated response displaying less persistence than the interest rate and inflation responses. The contribution of this shock to the FEV of GDP growth and inflation is 30 percent and 55 percent, respectively, at the 5 year horizon. The shock is especially important for long-term interest rates with a contribution to FEV of over 70 percent at this horizon. A historical decomposition indicates that in the absence of inflation target shocks, the 1970s would have seen low interest rates and inflation and the persistent decline of long term interest rates witnessed since the 1990s would have been substantially smaller.

The role of the Fed's inflation target has been considered in a number of recent studies. Kozicki and Tinsley (2005) present a generalised VAR model that includes a law of motion for the inflation target set by the monetary authority and the target perceived by the private sector, both of which are assumed to be unobserved by the econometrician. The estimates of these quantities suggest substantial time-variation with the target declining in the post-1980 period. Similar results are reported by papers such as Ireland (2007) and De Graeve *et al.* (2009). This strand of the literature pursues a more structural approach and incorporates time-varying inflation targets into DSGE and Macro-finance DSGE models, respectively.

Relative to this literature, the approach developed in the current paper is more data-driven. This offers two potential advantages. The first is simplicity and applicability – our SVAR model can be readily applied to economies that have a range of features. In contrast, approaches based on DSGE model and state-space models require additional assumptions that need to tailored to the economy in question. Second, our VAR based approach is likely to provide a better fit to the data when compared to models that impose a larger degree of cross-equation restrictions and thus provides an empirical benchmark.

The analysis in the paper has important policy implications. The results, again, highlight the important part played by the Fed's target in shaping the temporal evolution of inflation and long-term interest rates. The estimated dynamic responses to target shocks suggest that this aspect of the Fed's policy can be an effective tool in stimulating the economy and raising the level of inflation. This latter result is of potential interest to monetary authorities in several OECD countries that have faced persistently low inflation over the recent past.

The paper is organised as follows: Section 2 describes the SVAR used in this study and provides a description of the Monte-Carlo experiment used to test the method's performance. Empirical results are presented in section 3 while section 4 concludes.

2 Identifying shocks to the inflation target

Our empirical strategy is based on the following simple idea. From a theoretical point of view, long-horizon inflation expectations π^{LH} are driven by shocks to the central bank's inflation target $\left(\varepsilon_t^{\pi^*}\right)$ and range of additional disturbances $(\tilde{\varepsilon}_t)$ which include technology shocks and policy and

non-policy aggregate demand shocks:

$$\pi^{LH} = f\left(\varepsilon_t^{\pi^*}, \tilde{\varepsilon}_t\right) \tag{1}$$

However, over the medium to long-run horizons the role of $\varepsilon_t^{\pi^*}$ dominates in relative terms. While $\tilde{\varepsilon}_t$ can affect long-horizon inflation expectations, fluctuations in π^{LH} in the medium to long-run are largely driven by shocks to the inflation target. In other words, if on average across the sample, the monetary authority reacts systematically to changes in inflation and is, at least, perceived to be credible in the long-run, then long-horizon inflation expectations would coincide with the inflation target. As a consequence, any further changes in long-horizon inflation expectations reflect shocks to the inflation target.

We use a VAR model to approximate these economic disturbances. In particular, we estimate:

$$Y_t = \alpha + \sum_{j=1}^{P} \beta_{t-j} Y_{t-j} + A_0 \varepsilon_t \tag{2}$$

where the endogenous variables Y_t include a measure of long-horizon inflation expectations $\hat{\pi}^{LH}$ and a set of macroeconomic and financial variables X_t, and we order $\hat{\pi}^{LH}$ first for simplicity. The orthogonal shocks are denoted by ε_t and A_0 represents the contemporaneous impact matrix such that $A_0 A_0' = \Sigma$. It is well known that A_0 is not unique but the space spanned by these matrices can be written as $\tilde{A}_0 Q$ where Q is an orthonormal rotation matrix such that $Q'Q = I$

The shock to the inflation target is then identified by imposing the restriction that this shock makes the largest contribution the forecast error variance (FEV) of $\hat{\pi}^{LH}$. Consider the VAR in structural moving average form:

$$Y_t = B(L) A_0 \varepsilon_t$$

The k period ahead forecast error of the ith variable is given by:

$$Y_{it+k} - \hat{Y}_{it+k} = e_1 \left[\sum_{j=0}^{k-1} B_j \tilde{A}_0 Q \varepsilon_{t+k-j} \right]$$

where e_1 is a selection vector that picks out $\hat{\pi}^{LH}$ in the set of variables. Following Uhlig (2004a), the proposed identification scheme thus amounts to finding the column of Q that solves the following maximisation problem:

$$\arg\max_{Q_1} e_1' \left[\sum_{k=0}^{K} \sum_{j=0}^{k-1} B_j \tilde{A}_0 Q_1 Q_1' \tilde{A}_0' B_j' \right] e_1$$

such that $Q_1' Q_1 = 1$. Here Q_1 is the column of Q that corresponds to the shock that explains the largest proportion of the FEV of the first variable in the VAR, $\hat{\pi}^{LH}$. As shown by Uhlig (2004b), the maximisation can be re-written as eigenvalue eigenvector problem and a solution can be readily obtained.

The proposed identification differs from the method used in Michelis and Iacoviello (2016) to identify target shocks. These authors impose exact long-run restrictions on the behaviour of inflation and interest rates. In contrast, our approach is more agnostic and focuses on the medium run dynamics of variables rather than their behaviour in the infinite future which may be hard to pin down with a limited sample (see Erceg *et al.* (2005)). In a recent contribution, Arias *et al.* (2016) uses a mix of sign and zero restrictions to identify the systematic component of monetary

policy as embodied in the contemporaneous coefficients in a policy rule. In contrast, our approach focuses on the implicit inflation target and imposes no explicit restrictions on the interest rate equation in the SVAR model.

In order to evaluate the effectiveness of this identification scheme we consider an extensive Monte-Carlo experiment. As described in detail below, we use a DSGE model that allows for a time-varying inflation target as our data generating process (DGP). The aim is to check if the proposed identification scheme is able to recover the inflation target shock and provide a reliable estimate of its transmission.

2.1 DSGE Model

We use a DSGE model based on De Graeve and Theodoridis (2016). The technical appendix contains a detailed description of the model equations. Here we describe the key features of the model economy. In this economy, households consume, supply labour and accumulate capital – subject to an investment adjustment cost. Households have monopoly power over their wages. A fraction of them receive a random signal to set their wage optimally, with the remaining agents set wages based on a backward looking indexation rule. Households decide optimally about the degree of the capital utilisation, which is again subject to a cost, that determines the level of capital services. Intermediate good producers hire labour and capital services from households in order to produce. They have a monopoly over the price they charge, with price-setting subject to the same friction as wages. All the model features discussed so far are common with those in the literature (Smets and Wouters (2007), Christiano *et al.* (2005), Justiniano *et al.* (2010)).

In addition, in our model, agents have access to both short $\left(b_t^S\right)$ and long-term government debt $\left(\bar{b}_t^L\right)$. Following Woodford (2001) the latter asset is modelled as perpetuities that cost $p_{L,t}$ at time t and pay an exponentially decaying coupon κ^s at time $t+s+1$ where $0<\kappa\leq 1$. As explained in Woodford (2001) and Chen *et al.* (2012), the advantage of this formulation is that the price in period t of a bond issued s periods ago $p_{L-s,t}$ is a function of the current price $p_{L,t}$

$$p_{L-s,t}=\kappa^s p_{L,t} \tag{3}$$

This relation allows us to express the balance sheet equation and government budget constraint in a simple form (see the discussion in Chen *et al.* (2012)). Furthermore, in order to keep the analysis simple and tractable, we rule out the possibility of a secondary market for long-term bonds, implying that agents who buy long-term debt must hold it until maturity. In addition, long-term debt is a modelled as a less liquid asset than short-term debt. This is achieved by making the profit function of the j financial intermediary to be subject to an adjustment cost:

$$\xi_{j,t}=\underbrace{b_{j,t}^h+\frac{r_{t-1}^S}{\pi_t}b_{j,t-1}^S+\frac{p_{L,t}r_t^L}{\pi_t}b_{j,t-1}^L}_{\text{revenues}}-\underbrace{\frac{b_{j,t}^S}{\varepsilon_t^{b^S}}-\frac{p_{L,t}b_{j,t}^L}{\varepsilon_t^{\bar{b}^L}}-\frac{r_{t-1}^h}{\pi_t}b_{j,t-1}^h-\frac{x}{2}\left(\delta_{j,t-1}^B-\vartheta\delta_{t-2}^B-(1-\vartheta)\,\delta^B\right)^2\frac{\Gamma_{t-1}}{\pi_t}}_{\text{expenditures}}$$

where $\delta_{j,t}^B=\frac{b_{j,t}^S}{\bar{b}_{j,t}^L}$, r_t^S is the policy rate, r_t^L denotes the long-term interest rate, $\varepsilon_t^{b^S}$ and $\varepsilon_t^{\bar{b}^L}$ are Smets and Wouters (2007) 'haircut' asset-specific shocks that affect the balance sheet of the financial intermediary:

$$b_{\kappa,t}^h=\frac{b_{\kappa,t}^S}{\varepsilon_t^{b^S}}+\frac{\bar{b}_{\kappa,t}^L}{\varepsilon_t^{b^L}} \tag{4}$$

$\frac{x}{2}\left(\delta_{j,t-1}^B-\vartheta\delta_{t-2}^B-(1-\vartheta)\,\delta^B\right)^2$ is the liquidity friction and its size is captured by the parameter

x. Finally, the parameter ϑ indicates the degree of liquidity smoothing. Our set-up gives rise to an endogenous term-premium and this drives a wedge between expectations about the policy rate and the long-term interest rate.

The Government's budget constraint is given by

$$b_t^S + \bar{b}_t^L + T_t = \frac{r_{t-1}^S}{\pi_t} b_{t-1}^S + \frac{r_t^L}{\pi_t} \frac{p_{L,t}}{p_{L,t-1}} \bar{b}_{t-1}^L + g_t y_t$$

where the left hand side is the total (short plus long-term) debt issued by the government at time t plus lump-sum taxes (T_t) used to pay interest. Taxes are adjusted according to the following rule:

$$T_t = \Phi \left(\frac{b_{t-1}^S + \bar{b}_{t-1}^L}{b^S + \bar{b}^L} \right)^{\theta} \tag{5}$$

As in De Graeve *et al.* (2009) monetary authorities set policy based on the following rule

$$\frac{\bar{\pi} r_t}{r \bar{\pi}_t} = \left(\frac{\bar{\pi} r_{t-1}}{r \bar{\pi}_{t-1}} \right)^{\phi_R} \left(\frac{\pi_t}{\bar{\pi}_t} \right)^{(1-\phi_R)\phi_\pi} \left(\frac{y_t}{y} \right)^{(1-\phi_R)\phi_y} e^{\sigma_R \omega_{R,t}} \tag{6}$$

The inflation target shock evolves according to

$$\bar{\pi}_t - 0.999 \bar{\pi}_{t-1} = \rho_{\bar{\pi}} \left(\bar{\pi}_{t-1} - 0.999 \bar{\pi}_{t-2} \right) + \sigma_{\bar{\pi}} \omega_{\bar{\pi},t} \tag{7}$$

2.1.1 Perceived versus Actual Inflation Target Shocks

Agents in the model discussed above observe both the inflation target and monetary policy shocks. Following Erceg and Levin (2003), Gürkaynak *et al.* (2005) and Del Negro and Eusepi (2011) we also consider a version of the model where this assumption is relaxed. In this version, agents do not observe the source of departure of the (log linearly approximated) policy rate from its rule, namely

$$\hat{r}_t = \phi_R \hat{r}_{t-1} + (1 - \phi_R) \left(\phi_\pi \hat{\pi}_t + \phi_y \hat{y}_t \right) + \hat{o}_t \tag{8}$$

where

$$\hat{o}_t = \left[1 - (1 - \phi_R)\phi_\pi \right] \hat{\bar{\pi}}_t - \phi_R \hat{\bar{\pi}}_{t-1} + \hat{m}_t \tag{9}$$

In other words agents in the economy observe $\hat{o}_t$ but not its individual components, however, they can obtain an estimate about the inflation target $\left(\hat{\bar{\pi}}_t^{KF} \right)$ and monetary policy shock $\left(\hat{m}_t^{KF} \right)$ shocks by solving a Kalman Filter extraction problem. In this economy it is the Kalman Filter estimate of the inflation target shock $\left(\hat{\bar{\pi}}_t^{KF} \right)$ that enters in the price and wage Phillips curve equations and not the actual inflation target process which is assumed to be unobserved.

2.1.2 Active versus Passive Monetary Policy

The benchmark model does not incorporate the possibility that over some periods, monetary policy may have been passive with $\phi_\pi < 1$. Under these circumstances, inflation expectations are de-anchored and respond persistently to structural shocks. In order to take this into account, we consider a Markov switching version of the benchmark model that allows for regime switches in the policy rule between an active ($\phi_\pi > 1$) and passive ($\phi_\pi < 1$) state:

$$\hat{r}_t - \hat{\bar{\pi}}_t = \phi_R \left(\hat{r}_{t-1} - \hat{\bar{\pi}}_{t-1} \right) + (1 - \phi_R) \left(\phi_\pi \left(s_t \right) \left(\hat{\pi}_t - \hat{\bar{\pi}}_t \right) + \phi_y \hat{y}_t \right) + \hat{m}_t \tag{10}$$

where s_t follows a first order Markov chain with transition probabilities $P = \begin{bmatrix} p_{11} & 1-p_{22} \\ 1-p_{11} & p_{22} \end{bmatrix}$.

2.2 Monte Carlo Simulations

The benchmark model is estimated using Bayesian likelihood techniques discussed in An and Schorfheide (2007)[1] We use the data set of Smets and Wouters (2007) with the sample truncated at $2007Q4$ to avoid issues with the zero lower bound. Information about the prior and the posterior distribution of the structural parameter vector can be found in the technical Appendix. Briefly, the prior moments of the structural parameters coincide with those in Smets and Wouters (2007) study, while the posterior ones are very similar to those in the existing literature.

The Monte-Carlo experiment is based on 1000 simulated data sets of 200 observations.[2]At each replication we simulate series for output, inflation, the short-term and the long-term interest rate and 10 year ahead inflation expectations. This set of endogenous variables is used to estimate a $VAR(P)$ model where the lag length is determined via the Schwarz criterion. The identification scheme discussed above is applied to identify the target shock and the resulting impulse responses are stored.

Figure 1 shows the results from the experiment when the structural parameter vector for the benchmark DGP is set equal to the estimated posterior mode. The black line and the shaded area correspond to the point wise 50, 5 and 95 percentiles of the simulated distribution using the proposed identification scheme. The blue circle line represents the underlying response to the inflation target shock in the DSGE model. The figure suggests that the VAR estimates of the response to this shock provide a close approximation of true responses in the DGP.

Next, we check if the performance of the identification scheme is robust to different DSGE parameter values. To this end, we draw 1000 parameter vectors from the estimated DSGE posterior distribution and use them to simulate the 1000 data sets for the experiment.

Figure 2 illustrates again that the success of the VAR identification scheme does not depend on a particular DSGE parameter vector.

Next, we investigate whether the econometrician can identify the inflation target shock successfully when the agents in the economy cannot observe them. We repeat the simulation steps mentioned in the first experiment but we use the model where inflation target and monetary policy shocks unobserved. Figure 3 suggests that even in this case the identification scheme performs remarkably well. This is because agents learn about the true shock and the inference error does not survive long enough to have an effect on the *long-horizon* inflation expectations.

Our final exercise examines the ability of the scheme to recover the true shocks when monetary authorities' preferences towards inflation switch stochastically between "dovish" and "hawkish" regimes. This simulation is carried out by setting all parameter values except ϕ_π to the posterior mode of the benchmark model. We assume that in the hawkish regime, $\phi_\pi(s_t = 1) = 1.83$, while in the dovish regime this parameter is set less than one ($\phi_\pi(s_t = 2) = 0.83$). The estimates of Bianchi and Melosi (2016) are used to calibrate the transition probabilities at $p_{11} = 0.95$ and $p_{22} = 0.7$ and the model is solved using the RISE toolbox (see (Maih, 2015)). As before, we simulate data form this model and estimate the SVAR and the response to target shocks.

Figure 4 suggests that the VAR identification continues to perform remarkably well even in this environment. This is because when the sample is considered as a whole, the impact of dovish

[1]The estimation and simulations are implemented using Dynare 4.4.3. All the files can be downloaded from authors' personal web page.

[2]Actually 10200 observations are simulated each time and the first 10000 pseudo data points are dropped to eliminate the effects caused by the initial conditions.

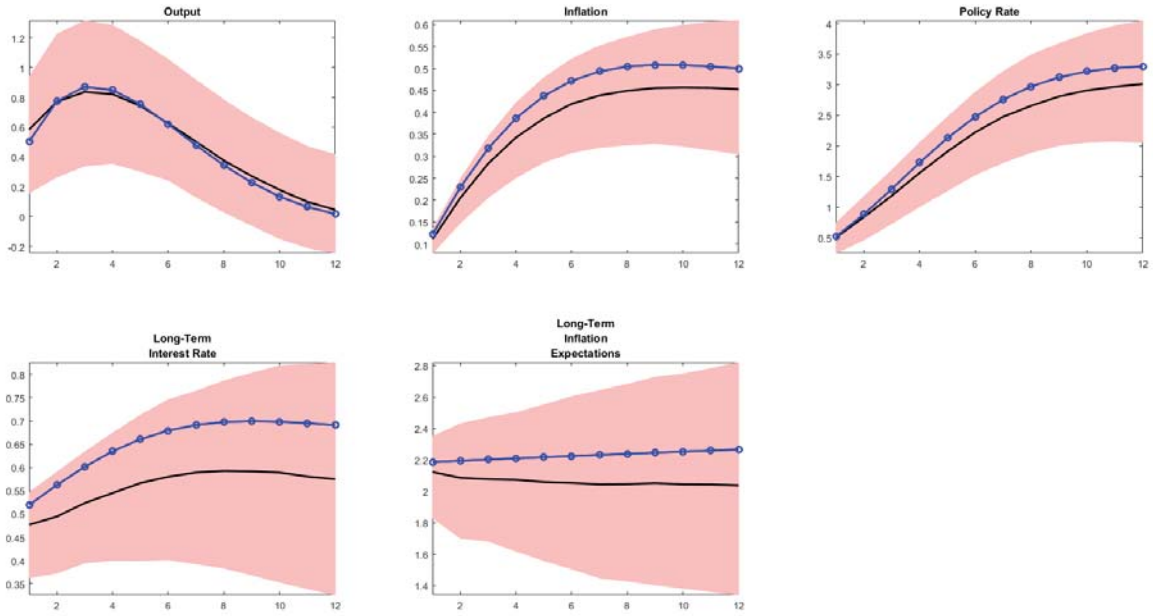

Figure 1: Using posterior mode estimates. The black line and the shadow pink area correspond to the pointwise 50, 5 and 95 percentiles of the simulated distribution. The blue circle line represents the DSGE true response to an inflation target shock.

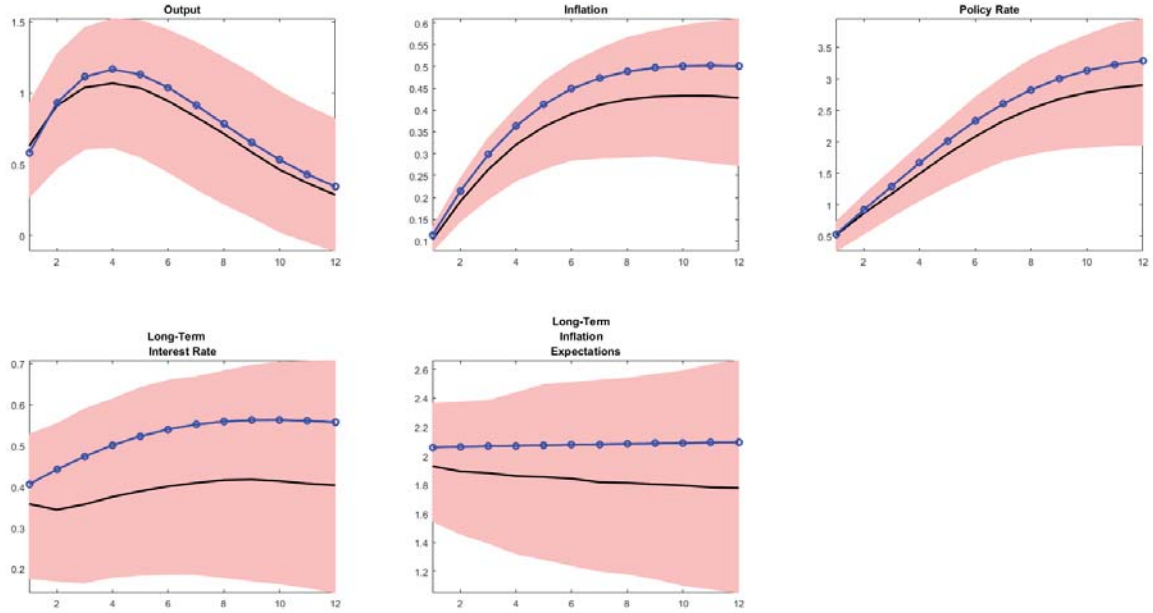

Figure 2: Using draws from the estimated DSGE posterior. See notes to figure 1.

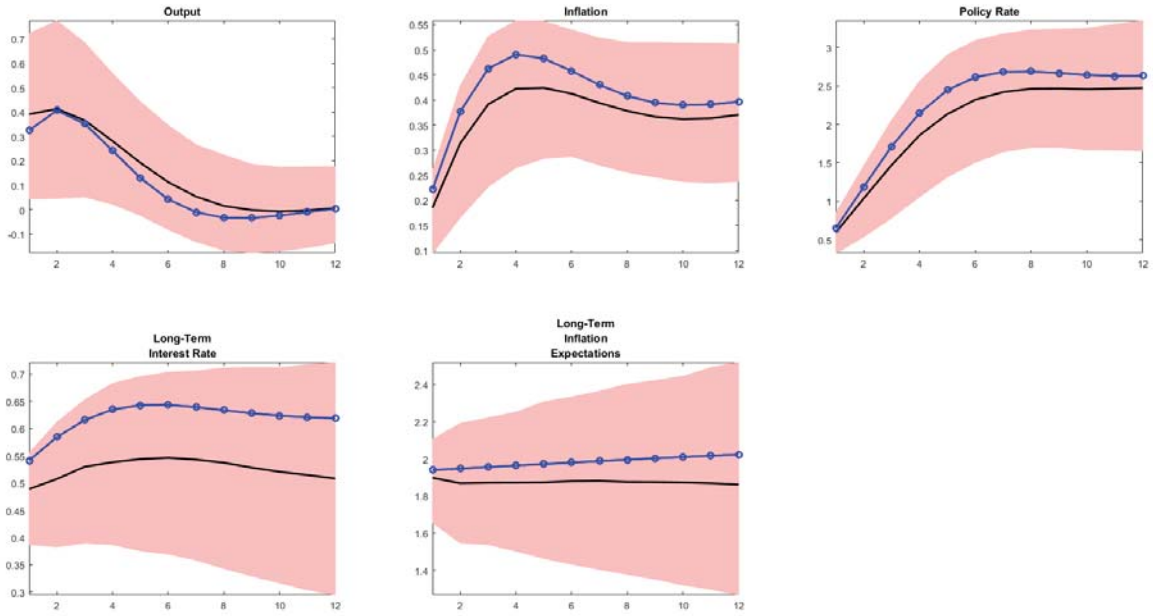

Figure 3: Using the version of the model with unobserved target and policy shock. See notes to figure 1.

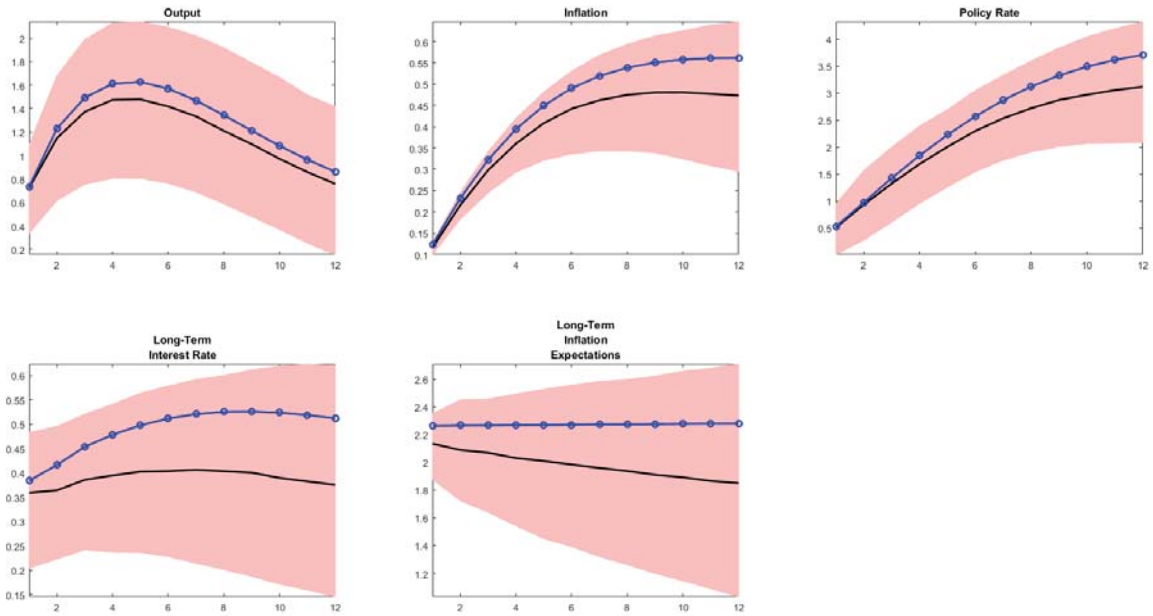

Figure 4: Using the version of the model with regime switching. The DSGE response is an average across the two regimes. See notes to figure 1.

regimes is mitigated by the presence of regimes during which the monetary authority responds actively to inflation. As a consequence, inflation target shocks continue to exert a major influence on long-horizon inflation expectations in this economy.

3 Empirical Analysis

3.1 Data and model specification

Implementation of the SVAR model described above requires a series for long-horizon inflation expectations. The existing literature typically uses either survey based measures or those derived from financial market prices. For our purpose, an important concern relates to the span of the data available. In other words, it may be the case that shocks to the inflation target are not observed regularly and therefore the proposed identification strategy is more effective in a reasonably long sample. With this in mind, our benchmark proxy for π^{LH} is a spliced survey based measure of long horizon PCE inflation expectations used in the Federal Reserve board model. This measure (with mnemonic PTR) is available on a quarterly basis from 1968Q1 to 2016Q3. In the earlier part of the sample PTR uses estimates of inflation expectations from Kozicki and Tinsley (2001). Data for the 1980s is obtained from the discontinued Decision Makers poll (DMP). Published by Richard Hoey, this survey aimed to capture the 5 to 10 year ahead expectations of participants in the financial markets. From 1991Q4 onwards, the series is based on 1 to 10 year ahead inflation expectations taken from the Survey of Professional Forecasters.

We check the robustness of the results by using an alternative measure based on blue chip economic indicators and the Livingstone survey. 10 year ahead expectations regarding CPI inflation are available from the latter survey since 1991 and are published in June and December of each year. This survey represents the views of academic and non-academic economists. Prior to 1991, this series is based on long horizon forecasts of CPI inflation included in blue chip economic indicators. These forecasts are published in March and October and are available from October 1979.[3] As this data is bi-annual, we use a mixed frequency version of the VAR model (see Schorfheide and Song (2015)) described below. As this series is available at a lower frequency and for a smaller period, PTR remains our preferred proxy for π^{LH}.

The benchmark VAR model is given by:

$$Y_t = \alpha + \sum_{j=1}^{P} \beta_{t-j} Y_{t-j} + v_t \tag{11}$$

where α is a vector of intercepts, Y_t includes $\hat{\pi}^{LH}$, real GDP growth (y_t), CPI inflation (π_t), the 10 year government bond yield (I_t) and the three month treasury bill rate (R_t). The data for the 10 year yield is obtained from Global Financial data. All remaining variables are taken from the Federal Reserve bank of St. Louis FRED database. Appendix A describes the data sources in detail. When the blue chip and Livingstone series is used for $\hat{\pi}^{LH}$, the model is augmented by an observation equation that implements the assumption that the observed expectations data is an average of missing observations in the previous and current quarter. These missing observations are treated as latent variables with the transition equation given by 11 in companion form.

[3] From 1979 to 1983, the blue chip forecasts are for the GNP deflator. A combined series is made available by the Federal Reserve Bank of Philadelphia.

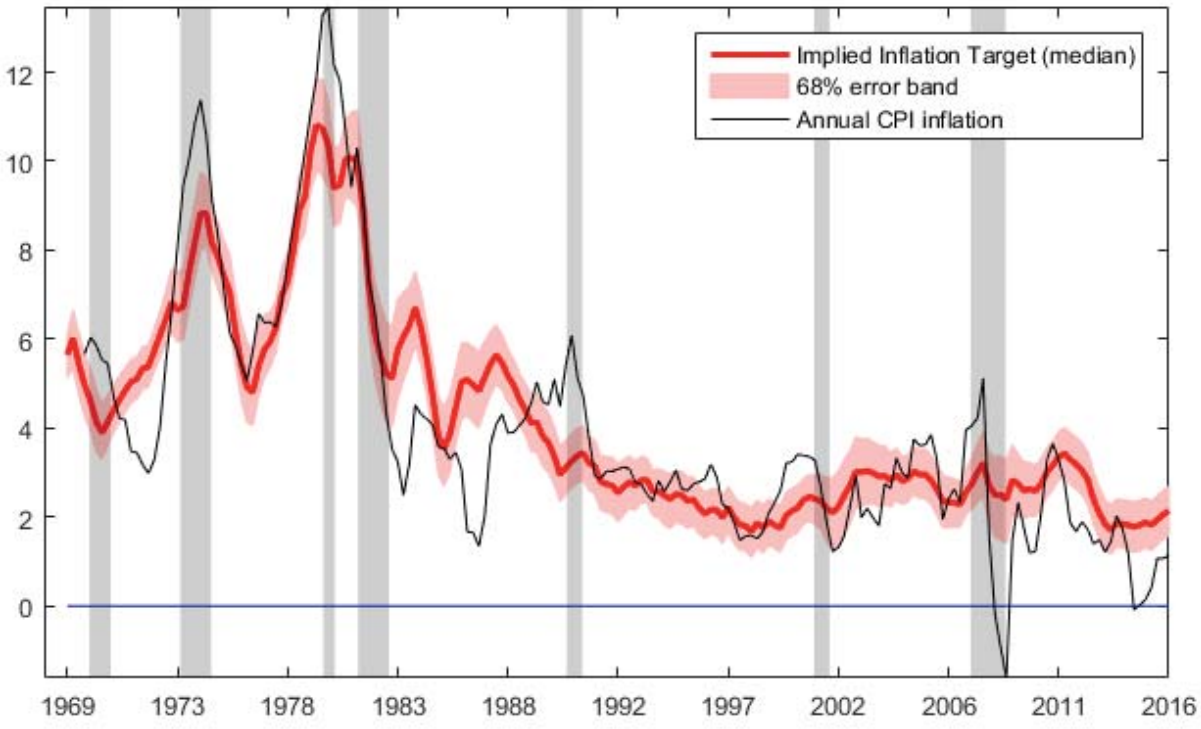

Figure 5: The Federal Reserve's inflation target and actual inflation.

The lag length P for the benchmark model is set to 2.[4] We adopt a Bayesian approach to model estimation and use a Gibbs sampling algorithm to approximate the posterior distribution of the parameters. As described in the technical appendix, we use a natural conjugate prior with tightness set to values commonly used in the literature for US data. As discussed below, the results are not sensitive to the tightness of the prior. The appendix provides details of the conditional posterior distributions and the steps of the Gibbs sampler. We also provide a description of the extended algorithm used to estimate the mixed frequency VAR model when the inflation expectations based on the blue chip and Livingstone surveys is employed in the estimation.

As discussed above, the shock to the implicit inflation target is identified via the restriction that this shock makes the largest contribution to the FEV of $\hat{\pi}^{LH}$ at horizon K. The benchmark model uses $K = 40$, but the results are robust to using a longer horizon.

3.2 Empirical Results

Figure 5 presents the inflation target implied by the historical decomposition from the benchmark SVAR model that uses PTR as the measure of long-horizon inflation expectations. This is calculated as the portion of (annual) inflation driven by the identified inflation target shocks, with the remaining innovations set to zero. The estimates of the target level show a number of similarities to estimates of this object presented in Kozicki and Tinsley (2005), Coibion and Gorodnichenko (2011)

[4]Both the Schwarz criterion and a marginal likelihood comparison suggests a lag length of 1. We employ an extra lag to capture any remaining dynamics. The results are robust to using $P = 1$ or $P = 4$.

and Ireland (2007). In particular, the mid and the late 1970s saw the target rise substantially with peaks of 8 to 10 percent. The rise in the estimated target in these periods lagged the increase in actual inflation suggesting that the Fed was accommodating the impact of inflationary shocks (see Kozicki and Tinsley (2005)). After the appointment of Paul Volcker to the chairmanship of the Fed in 1979Q4, both target and actual inflation declined. After the mid-1980s, the target level has hovered close to the 2 percent mark.

3.2.1 Impulse responses

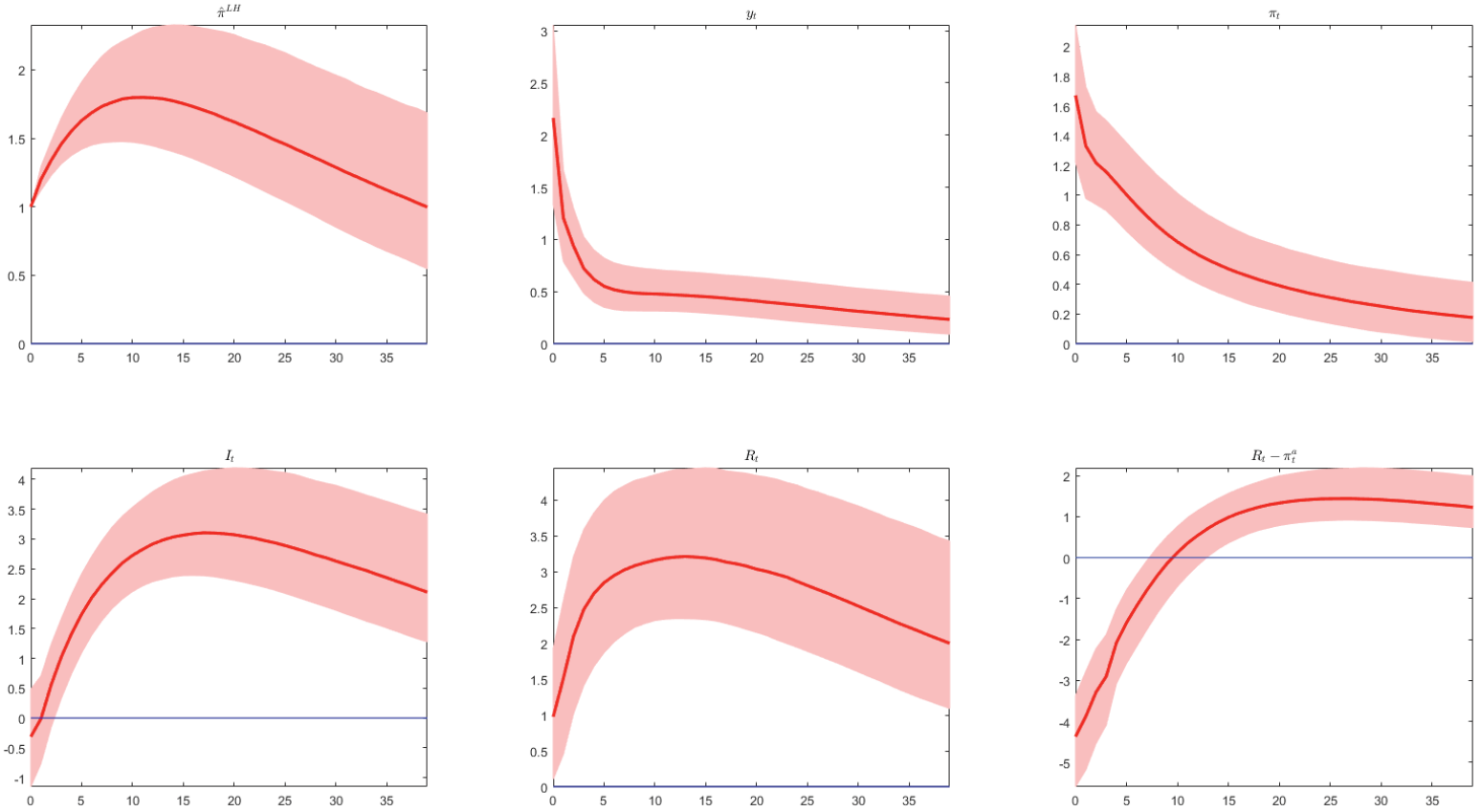

Figure 6: Impulse response to an inflation target shock. The responses are shown for PTR $\left(\hat{\pi}^{LH}\right)$, real GDP growth (y_t), CPI inflation (π_t), the 10 year goverment bond yield (I_t), the three month treasury bill rate (R_t) and the real interest rate where $\pi_t^a = \sum_{i=0}^{3} \pi_{t-i}$. The solid lines are posterior medians while the shaded area is the 68% error band.

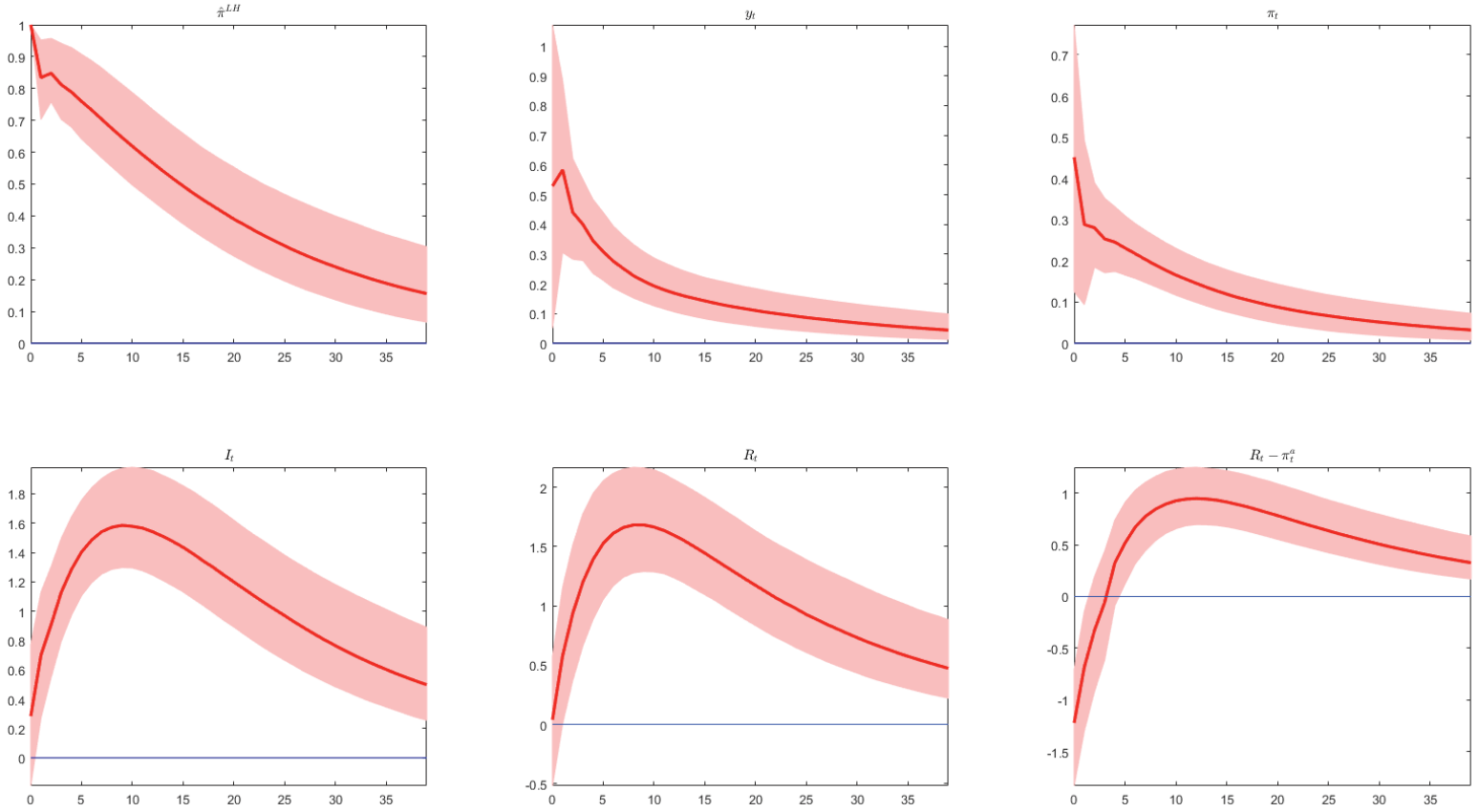

Figure 7: Impulse response to an inflation target shock. The responses are shown for Blue Chip/Livingstone survey $\left(\hat{\pi}^{LH}\right)$, real GDP growth (y_t), CPI inflation (π_t), the 10 year goverment bond yield (I_t), the three month treasury bill rate (R_t) and the real interest rate where $\pi_t^a = \sum_{i=0}^{3} \pi_{t-i}$. The solid lines are posterior medians while the shaded area is the 68% error band.

Figure 6 presents the impulse responses to inflation target shocks. The figure presents the response to a shock that raises $\hat{\pi}^{LH}$ by 1 percent on impact. The impact effect on GDP growth and inflation is estimated to be large but dissipates quickly. At the one year horizon the response of GDP growth is 0.8 percent, while inflation rises by 1.2 percent, with the response estimated to be persistent. The last panel in the figure shows that the implied increase in annual inflation π_t^a is larger compared to the rise in the short-rate over the first three years of the horizon. As a consequence, the ex-post real interest rate declines substantially, before increasing by about 1.5 percent over the second half of the horizon. While the response of the long rate I_t is more sluggish than that of the short-rate, the magnitude of the two responses is very similar at long horizons.

In order to check the robustness of these results to the measure of long-horizon inflation expectations, we re-estimate the VAR model using the series based on the blue chip forecasts and the Livingstone survey. As discussed above, this VAR model entails the estimation of missing inflation expectations data, as well as the VAR parameters. Note also that the estimation sample excludes the 1970s due to data unavailability. The impulse responses from this alternative model are shown in figure 7. The response of GDP growth and inflation to this shock is highly persistent and the null hypothesis of a zero response can be rejected over most of the sample period. The magnitude of the two responses, however, is estimated to be smaller than the benchmark model. As in the benchmark model, the real rate declines, with the response reversing after one year. Both the short and the long-term interest rate responses are large and persistent, albeit the peak response is estimated to be smaller than the benchmark case. In summary, the estimated responses from this model are qualitatively similar to the benchmark case. The estimated magnitude is smaller than benchmark. This may reflect the fact that the mixed frequency VAR treats some of the observations on $\hat{\pi}^{LH}$ as unobserved and thus contains less information than the benchmark model for the task of identifying the inflation target shock. An alternative explanation can be based on the fact that the post-1980 sample is dominated by the Great Moderation and a decline in the impact of structural shocks over this period is well documented in the literature (see for example Galí and Gambetti (2009)). The latter explanation would suggest that these estimates may represent the lower end of the range of the impact of this shock. Nevertheless, the effects of the inflation target shock are still estimated to be sizeable and are clearly different from zero from a statistical perspective.

The results of the benchmark model are robust to other changes in model specification. Details of these additional robustness checks are given in the on-line technical appendix. First, we expand the benchmark VAR model by adding the first three principal components extracted from a large panel of macroeconomic and financial data for the US (FRED-QD database provided by the St. Louis Fed). This allows us to incorporate a large amount of information into the model and to account for potential information insufficiency (see Forni and Gambetti (2014)). The impulse responses from this model are shown in Figure 2 of the technical appendix. The results are very close to the benchmark estimates indicating robustness with respect to this issue. Similarly, when the benchmark model is estimated on pre-2007 data, the results are largely unaffected (see Figure 3 in the technical appendix). As mentioned above, using a lag length of 1 or 4 does not alter the key conclusions (See Figures 4 and Figure 5 in the technical appendix). Figure 6 in the technical appendix shows that if the FEV restrictions are imposed using a horizon of 80 quarters, the key results survive. Finally, Figure 7 shows that the use of a flat prior leads to conclusions that are similar to the benchmark case.

While the Monte-Carlo experiment in section 2.1 provides strong evidence on the effectiveness of the proposed identification scheme, it is interesting to further scrutinise the properties of the identified shock. For this purpose, we follow Ben Zeev and Pappa (2017) and consider the cross-correlation between the estimated target shock and shocks identified in the literature to be important for business cycle fluctuations. As shown in Figure 8, we consider monetary policy

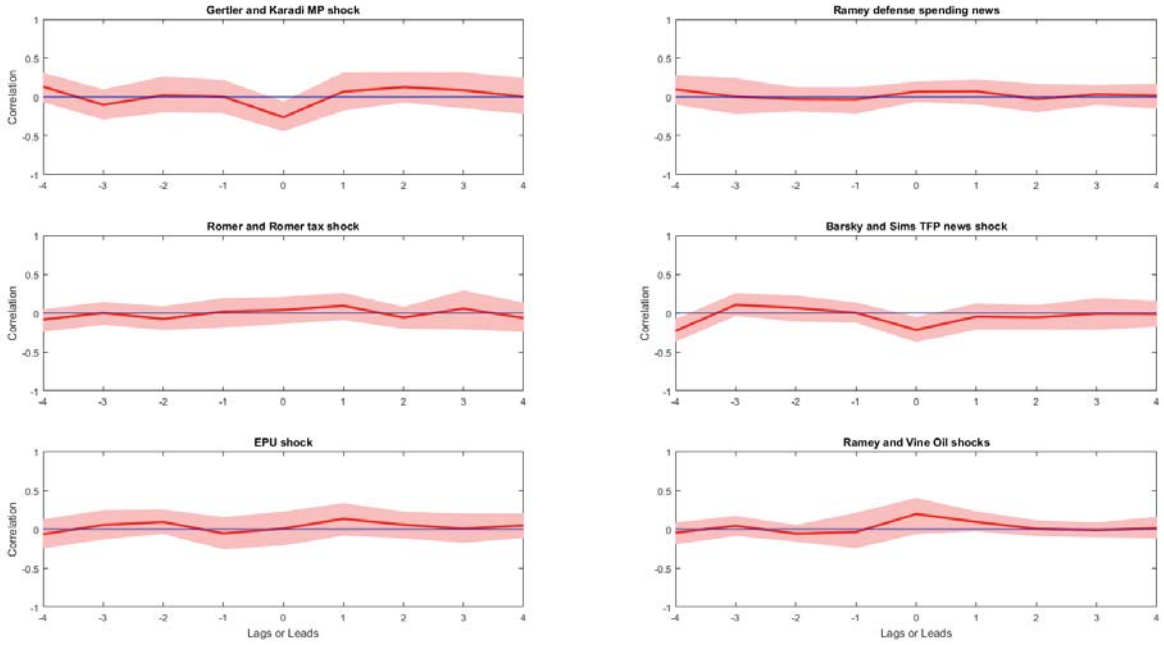

Figure 8: Correlation between the target shock and other structural shocks. The sold line is the median correlation, while the error band represents the 95 percent confidence interval obtained via bootstrap.

shocks identified by Gertler and Karadi (2015) and fiscal shocks taken from Ramey (2011) and Romer and Romer (2010). Policy uncertainty (EPU) shocks are proxied as residuals to an AR(4) model using the economic policy uncertainty index of Baker *et al.* (2017). TFP news shocks are taken from Barsky and Sims (2011) while we use the measure of oil shocks constructed by Ramey and Vine (2011). Figure 8 shows that the estimated correlations are insignificant at most lags or leads. Moreover, the magnitude of the estimated correlations is small in all cases. This provides evidence that the shock identified in our SVAR model is distinct from other policy and non-policy disturbances.

In summary, the benchmark results and the sensitivity analysis suggests the following conclusion: a unit inflation target shock is estimated to have an impact on US GDP growth, CPI inflation, the short and long term interest rate that is sizeable from an economic and statistical perspective. We now turn to evaluating the contribution of inflation target shocks.

3.2.2 Variance and historical decomposition

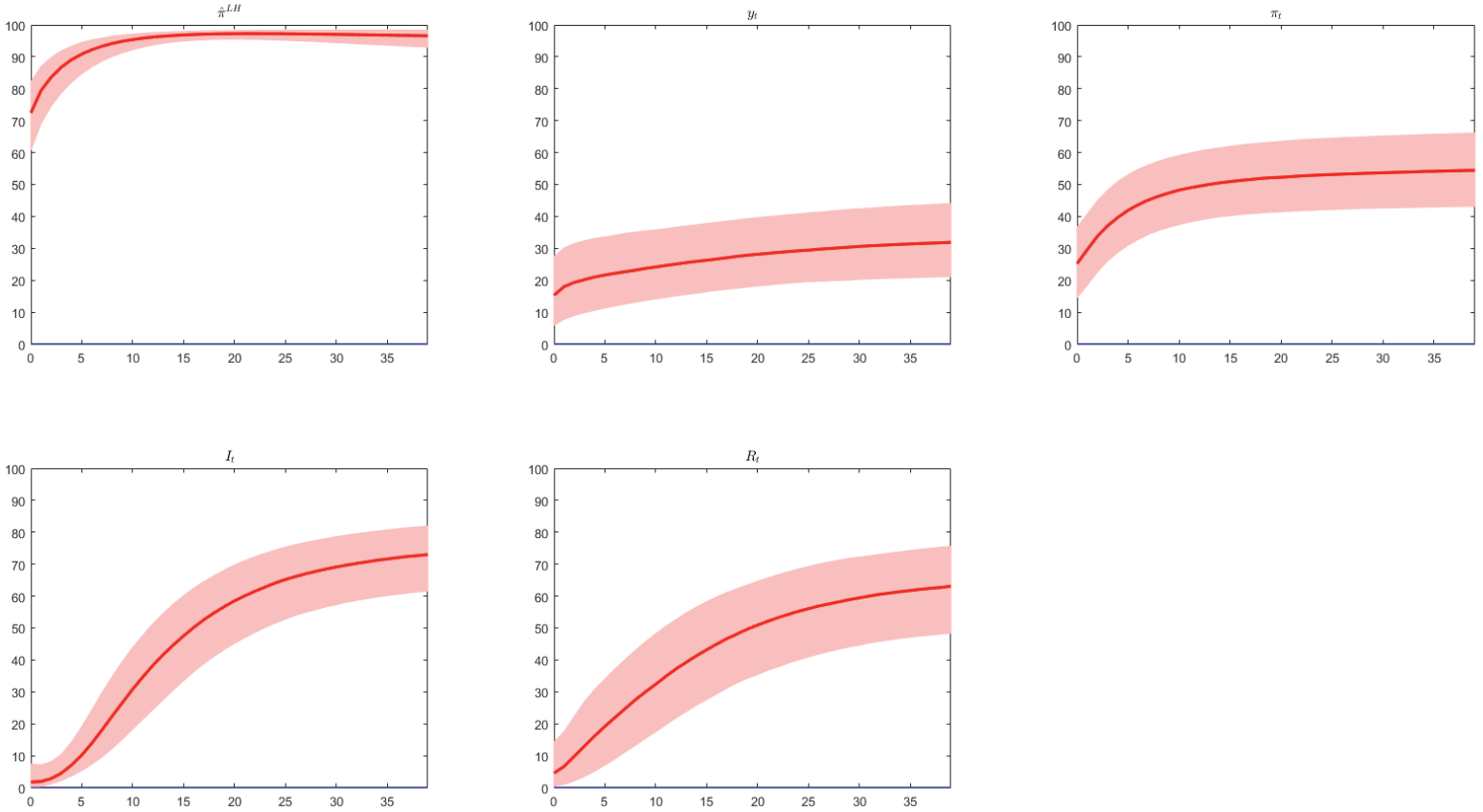

Figure 9: Contribution of the inflation target shock to the FEV. The contributions are shown for PTR $\left(\hat{\pi}^{LH}\right)$, real GDP growth (y_t), CPI inflation (π_t), the 10 year goverment bond yield (I_t), the three month treasury bill rate (R_t). The solid lines are posterior medians while the shaded area is the 68% error band.

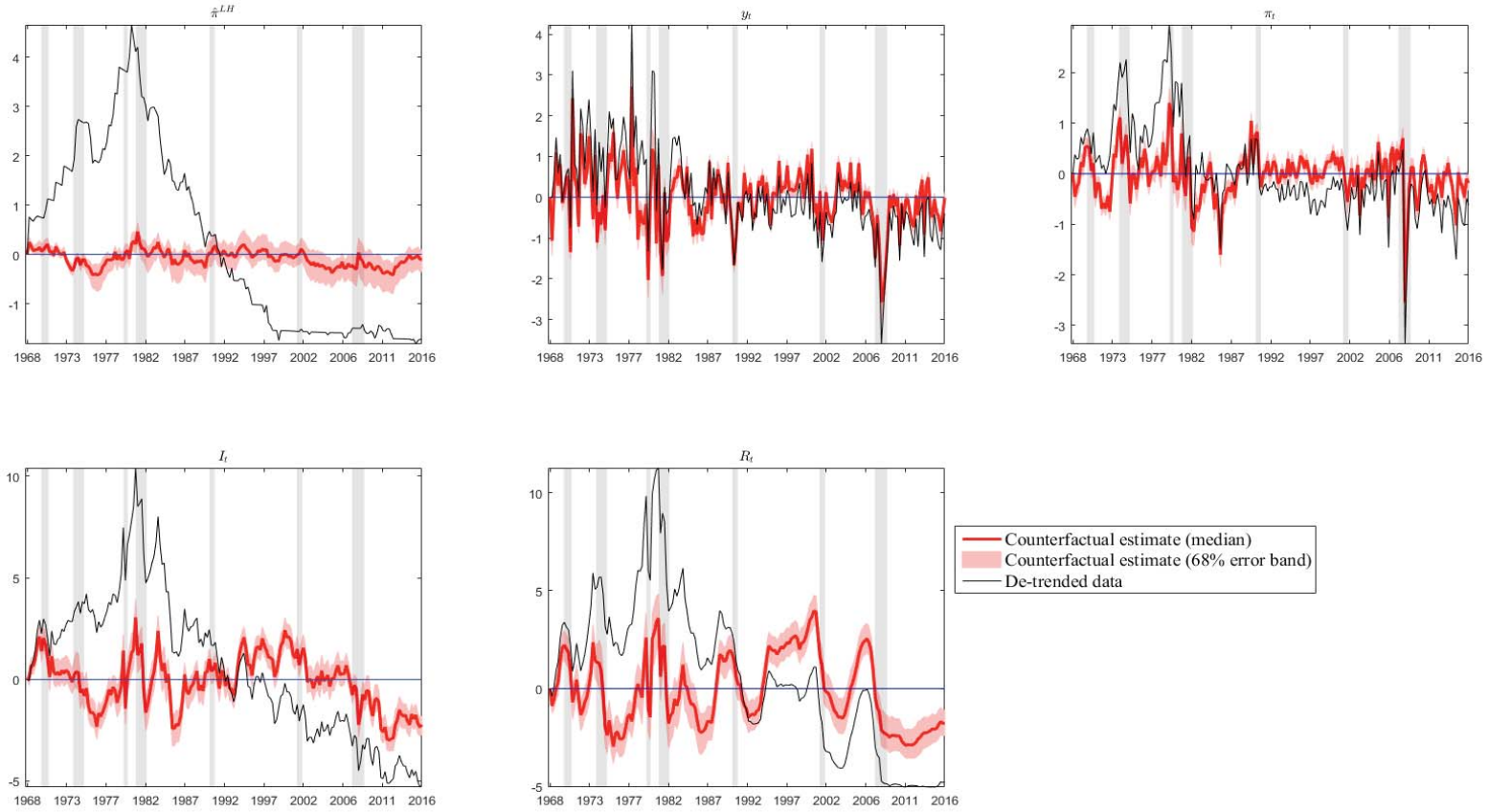

Figure 10: Contribution of shocks other than inflation target shocks to the de-trended data. The contributions are shown for PTR $\left(\hat{\pi}^{LH}\right)$, real GDP growth (y_t), CPI inflation (π_t), the 10 year goverment bond yield (I_t), the three month treasury bill rate (R_t). The solid lines are posterior medians while the shaded area is the 68% error band. The shaded vertical bands show NBER recession dates.

Figure 9 shows the contribution of the inflation target shock to the forecast error variance (FEV) estimated using the benchmark model. By construction, the shock explains the bulk of the FEV of $\hat{\pi}^{LH}$. At the two year horizon, the shock contributes about 23% to the FEV of GDP growth. The contribution to FEV of CPI inflation at this horizon is estimated to be about 48%. It is interesting to note that these contributions are larger than those typically reported for the monetary policy shock (see for example Bernanke *et al.* (2005)). The bottom panels of the figure show that this shock makes the most important contribution to the FEV of I_t and R_t explaining the bulk of the FEV at long horizons.

A similar conclusion is reached when examining the contribution of this shock to the historical fluctuations in these interest rates. Figure 10 plots (de-trended) data for the endogenous variables along with the counter-factual estimates from the VAR assuming that the inflation target shock equals zero at each point in time. The second row of the figure shows that after the early 1990s the counter-factual estimate of the interest rates is above the observed data, implying that fluctuations in the implicit target of the Fed helped to keep these rates at depressed levels over the last two decades. The inflation target shock made a strong positive contribution to inflation during the mid and late-1970s. The great inflation is largely absent in the counter-factual scenario and the impact of inflationary shocks in the early and the late 1970s is muted. The counter-factual estimate of inflation largely remains above the actual data after the mid-1980s providing some support for the hypothesis that systematic policy contributed to low inflation seen over the Great Moderation. It is interesting to note that in the absence of these shocks, GDP growth is estimated to have been higher over the last two decades.

Decomposition results based on the VAR that uses the blue chip\Livingstone inflation expectation series suggests similar conclusions. The inflation target shock is estimated to explain about 50 percent of the FEV of the short and long term interest rate. As in the benchmark case, the shock makes a modest contribution to GDP growth. We estimate that the contribution to the inflation FEV is about 20 percent at the two year horizon. While still sizeable, this estimate is smaller than the benchmark case. As discussed above, this may reflect the decline in the impact of shocks over the Great Moderation period or the fact that the blue chip\Livingstone data is available at a lower frequency and for a smaller time span.[5]

4 Conclusions

We propose to identify shocks to the Fed's implicit inflation target as innovations in an SVAR that explain the bulk of the FEV of long-horizon inflation expectations. When this scheme is applied to data simulated from a DSGE model that features a time-varying inflation target, we are able to recover the target shock and estimate its transmission with precision. This result remains robust when the target is assumed to be unobserved by the agents in the model.

Application of this SVAR to US data suggests that the impact of a 1 percent positive inflation target shock is large with the peak effect on GDP growth and inflation, one year ahead, estimated to be 0.8 percent and 1.2 percent, respectively. The shock has its largest impact on both the short and the long-term interest rate. Decompositions from the SVAR indicate that this shock was the major driving force behind the great inflation of the 1970s and contributed substantially to the persistent decline observed in long-term interest rates.

In future work, it would be interesting to investigate the transmission of this shock in other OECD countries. It may also be useful to estimate the spillover effects of US inflation target shocks to inflation rates and long term yields in industrialised countries and emerging markets.

[5] These additional results are presented in the technical appendix.

A Appendix A: Data sources

FRED is Federal Reserve Economic data (http://research.stlouisfed.org/fred2/) and GFD refers to Global Financial Data (http://www.globalfinancialdata.com/).

Data on inflation expectations

- PTR. Downloaded from FRB/US model webpage.
- Blue Chip/Livingstone survey. Available from the Philadelphia Fed.

Macroeconomic/Financial data for the United States

- Real GDP: Real GDP (FRED series id GDPC96).
- CPI (FRED series id CPIAUCSL). We calculate inflation as the quarterly growth in CPI.
- 3 month Treasury Bill rate (FRED series id TB3MS).
- 10 Year government bond yield (GFD code IGUSA10D)

References

An, Sungbae and Frank Schorfheide, 2007, Bayesian Analysis of DSGE Models, *Econometric Reviews* **26**(2-4), 113–172.

Arias, Jonas E., Dario Caldara and Juan F. Rubio-Ramirez, 2016, The Systematic Component of Monetary Policy in SVARs: An Agnostic Identification Procedure, *FRB Atlanta Working Paper 2016-15*, Federal Reserve Bank of Atlanta.

Baker, Scott R, Nicholas Bloom and Steven J Davis, 2017, Measuring Economic Policy Uncertainty, *The Quarterly Journal of Economics* **Forthcoming**.

Ball, Laurence M., 2014, The Case for a Long-Run Inflation Target of Four Percent, *IMF Working Papers 14/92*, International Monetary Fund.

Barsky, Robert B. and Eric R. Sims, 2011, News shocks and business cycles, *Journal of Monetary Economics* **58**(3), 273–289.

Ben Zeev, Nadav and Evi Pappa, 2017, Chronicle of a War Foretold: The Macroeconomic Effects of Anticipated Defence Spending Shocks, *The Economic Journal* pp. n/a–n/a.

Bernanke, Ben S., Jean Boivin and Piotr Eliasz, 2005, Measuring the Effects of Monetary Policy: A Factor-Augmented Vector Autoregressive (FAVAR) Approach*, *The Quarterly Journal of Economics* **120**(1), 387.

Bianchi, Francesco and Leonardo Melosi, 2016, Constrained Discretion and Central Bank Transparency, *The Review of Economics and Statistics* p. forthcoming.

Chen, Han, Vasco Curdia and Andrea Ferrero, 2012, The Macroeconomic Effects of Large-scale Asset Purchase Programmes, *Economic Journal* **122**(564), F289–F315.

Christiano, Lawrence, Martin Eichenbaum and Charles Evans, 2005, Nominal Rigidities and the Dynamic Effects of a shock to Monetary Policy, *Journal of Political Economy* **113**, 1–45.

Coibion, Olivier and Yuriy Gorodnichenko, 2011, Monetary Policy, Trend Inflation, and the Great Moderation: An Alternative Interpretation, *American Economic Review* **101**(1), 341–70.

De Graeve, Ferre and Konstantinos Theodoridis, 2016, Forward Guidance, Quantitative Easing, or both?, *Working Paper Research 305*, National Bank of Belgium.

De Graeve, Ferre, Marina Emiris and Raf Wouters, 2009, A structural decomposition of the US yield curve, *Journal of Monetary Economics* **56**(4), 545–559.

Del Negro, Marco and Stefano Eusepi, 2011, Fitting observed inflation expectations, *Journal of Economic Dynamics and Control* **35**(12), 2105–2131.

Erceg, Christopher J. and Andrew T. Levin, 2003, Imperfect credibility and inflation persistence, *Journal of Monetary Economics* **50**(4), 915–944.

Erceg, Christopher J., Christopher Gust and Luca Guerrieri, 2005, Can Long-Run Restrictions Identify Technology Shocks?, *Journal of the European Economic Association* **3**(6), 1237–1278.

Forni, Mario and Luca Gambetti, 2014, Sufficient information in structural {VARs}, *Journal of Monetary Economics* **66**, 124 – 136.

Galí, Jordi and Luca Gambetti, 2009, On the Sources of the Great Moderation, *American Economic Journal: Macroeconomics* **1**(1), 26–57.

Gambacorta, Leonardo, Boris Hofmann and Gert Peersman, 2014, The Effectiveness of Unconventional Monetary Policy at the Zero Lower Bound: A Cross-Country Analysis, *Journal of Money, Credit and Banking* **46**(4), 615–642.

Gertler, Mark and Peter Karadi, 2015, Monetary Policy Surprises, Credit Costs, and Economic Activity, *American Economic Journal: Macroeconomics* **7**(1), 44–76.

Gürkaynak, Refet S., Brian Sack and Eric Swanson, 2005, The Sensitivity of Long-Term Interest Rates to Economic News: Evidence and Implications for Macroeconomic Models, *American Economic Review* **95**(1), 425–436.

Ireland, Peter N., 2007, Changes in the Federal Reserve's Inflation Target: Causes and Consequences, *Journal of Money, Credit and Banking* **39**(8), 1851–1882.

Justiniano, Alejandro, Giorgio Primiceri and Andrea Tambalotti, 2010, Investment shocks and business cycles, *Journal of Monetary Economics* **57**(2), 132–45.

Kozicki, Sharon and P. A. Tinsley, 2001, Shifting endpoints in the term structure of interest rates, *Journal of Monetary Economics* **47**(3), 613–652.

Kozicki, Sharon and P.A. Tinsley, 2005, Permanent and transitory policy shocks in an empirical macro model with asymmetric information, *Journal of Economic Dynamics and Control* **29**(11), 1985 – 2015. Expectations, learning, and monetary policyExpectations, learning, and monetary policy.

Maih, Junior, 2015, Efficient perturbation methods for solving regime-switching DSGE models, *Working Paper 2015/01*, Norges Bank.

Michelis, Andrea De and Matteo Iacoviello, 2016, Raising an inflation target: The Japanese experience with Abenomics, *European Economic Review* **88**, 67 – 87. SI: The Post-Crisis Slump.

Ramey, Valerie A., 2011, Identifying Government Spending Shocks: It's all in the Timing, *The Quarterly Journal of Economics* **126**(1), 1–50.

Ramey, Valerie A. and Daniel J. Vine, 2011, Oil, Automobiles, and the U.S. Economy: How Much Have Things Really Changed?, *NBER Macroeconomics Annual 2010, Volume 25*, NBER Chapters, National Bureau of Economic Research, Inc, pp. 333–367.

Romer, Christina D. and David H. Romer, 2010, The Macroeconomic Effects of Tax Changes: Estimates Based on a New Measure of Fiscal Shocks, *American Economic Review* **100**(3), 763–801.

Schorfheide, Frank and Dongho Song, 2015, Real-Time Forecasting With a Mixed-Frequency VAR, *Journal of Business & Economic Statistics* **33**(3), 366–380.

Smets, Frank and Rafael Wouters, 2007, Shocks and Frictions in US Business Cycles: a Bayesian DSGE Approach, *American Economic Review* **97**, 586–606.

Uhlig, Harald, 2004a, DO TECHNOLOGY SHOCKS LEAD TO A FALL IN TOTAL HOURS WORKED?, *Journal of the European Economic Association* **2**(2-3), 361–371.

Uhlig, Harald, 2004b, What moves GNP?, *Econometric Society 2004 North American Winter Meetings 636*, Econometric Society.

Woodford, Michael, 2001, Fiscal Requirements for Price Stability, *Journal of Money, Credit and Banking* **33**(3), 669–728.

The Federal Reserve's implicit inflation target and Macroeconomic dynamics. (Technical Appendix: SVAR results and DSGE model)*

Haroon Mumtaz†
Queen Mary University

Konstantinos Theodoridis‡
Bank of England
Lancaster University Management School

April 2017

Abstract

Key words:
JEL codes: C5, E1, E5, E6

1 Estimation of the Bayesian VAR model

Consider the reduced form VAR

$$Y_t = \alpha\Gamma_t + \sum_{j=1}^{P} \beta_{t-j} Y_{t-j} + v_t, var\left(v_t\right) = \Omega$$

where Γ_t is $1 \times M$ vector of exogenous regressors, which in our application includes an intercept. We adopt a Bayesian approach to estimation of the reduced form VAR model. We introduce a natural conjugate prior for the VAR parameters (see Banbura *et al.* (2010)):

$$Y_{D,1} = \begin{pmatrix} \frac{diag(\gamma_1\sigma_1 ... \gamma_N\sigma_N)}{\tau} \\ 0_{N\times(P-1)\times N} \\ ..... \\ diag\left(\sigma_1 ... \sigma_N\right) \\ ..... \\ 0_{M\times N} \end{pmatrix}, and\ X_{D,1} = \begin{pmatrix} \frac{J_P \otimes diag(\sigma_1 ... \sigma_N)}{\tau} & 0_{NP\times M} \\ 0_{N\times NP+M} & \\ ..... & \\ 0_{M\times NP} & I_M \times c \end{pmatrix} \quad (1)$$

where γ_1 to γ_N denotes the prior mean for the coefficients on the first lag, τ is the tightness of the prior on the VAR coefficients and c is the tightness of the prior on the constant terms. In our application, the prior means are chosen as the OLS estimates of the coefficients of an AR(1) regression estimated for each endogenous variable using a training sample. We set $\tau = 0.1$. The scaling factors σ_i are set using the standard deviation of the error terms from these preliminary

*Any views expressed are solely those of the author(s) and so cannot be taken to represent those of the Bank of England or to state Bank of England policy. This paper should therefore not be reported as representing the views of the Bank of England or members of the Monetary Policy Committee, Financial Policy Committee or Prudential Regulation Authority Board.

†Email:h.mumtaz@qmul.ac.uk

‡konstantinos.theodoridis@bankofengland.co.uk

AR(1) regressions. Finally we set $c = 1/1000$ in our implementation indicating a flat prior on the constant. We also introduce a prior on the sum of the lagged dependent variables by adding the following dummy observations:

$$Y_{D,2} = \frac{diag\left(\gamma_1\mu_1...\gamma_N\mu_N\right)}{\lambda},\ X_{D,2} = \left(\begin{array}{cc} \frac{(1_{1\times P})\otimes diag(\gamma_1\mu_1...\gamma_N\mu_N)}{\lambda} & 0_{N\times M} \end{array}\right) \tag{2}$$

where μ_i denotes the sample means of the endogenous variables calculated using the training sample. As in Banbura *et al.* (2010), the tightness of this sum of coefficients prior is set as $\lambda = 10\tau$. Given the natural conjugate prior, the conditional posterior distributions of the VAR parameters $B = vec(\left[\alpha, \beta_1; \beta_2..; \beta_j\right]\ )$ and Ω take a simple form and are defined as:

$$G\left(B\left|\Omega\right.\right) \sim N(B^*, \Omega \otimes \left(X^{*\prime}X^*\right)^{-1}) \tag{3}$$

$$G\left(\Omega\left|B\right.\right) \sim IW(S^*, T^*). \tag{4}$$

where X denotes the right hand side variables $\Gamma_t, Y_{t-1}, ..Y_{t-P}$. The posterior means are given by $B^*=(X^{*\prime}X^*)^{-1}\left(X^{*\prime}Y^*\right)$ and $S^*=(Y^*-X^*)\tilde{B}'(Y^*-X^*\tilde{B})$, where $Y^*=[Y; Y_{D,1}; Y_{D,2}]$, $X^*=[X; X_{D,1}; X_{D,2}]$ and $\tilde{B}$ is the draw of the VAR coefficients B reshaped to be conformable with X^*. T^* denotes the number of rows of Y^*. A Gibbs sampler offers a convenient method to simulate the posterior distribution of B and Ω by drawing successively from these conditional posteriors. We employ 25,000 iterations using the last 5000 for inference.

Once the iterations are past the burn-in stage, we calculate the contemporaneous impact matrix. As described in the main text, the the VAR in structural moving average form is:

$$Y_t = B\left(L\right)A_0\varepsilon_t$$

The k period ahead forecast error of the ith variable is given by:

$$Y_{it+k} - \hat{Y}_{it+k} = e_1\left[\sum_{j=0}^{k-1} B_j\tilde{A}_0 Q\varepsilon_{t+k-j}\right]$$

where e_1 is a selection vector that picks out $\hat{\pi}^{LH}$ in the set of variables. Following Uhlig (2004a), the proposed identification scheme thus amounts to finding the column of Q that solves the following maximisation problem:

$$\arg\max_{Q_1} e_1'\left[\sum_{k=0}^{K}\sum_{j=0}^{k-1} B_j\tilde{A}_0 Q_1 Q_1'\tilde{A}_0'B_j'\right]e_1$$

such that $Q_1'Q_1 = 1$. Here Q_1 is the column of Q that corresponds to the shock that explains the largest proportion of the FEV of the first variable in the VAR, $\hat{\pi}^{LH}$. This objective function can be written as

$$\arg\max_{Q_1} Q_1'SQ_1$$

where $S = \sum_{k=0}^{K}\sum_{j=0}^{k-1}\tilde{A}_0'B_j'\left(e_1e_1'\right)B_j\tilde{A}_0$. The maximisation problem can be expressed as a Lagrangian

$$L = Q_1'SQ_1 - \lambda\left(Q_1'Q_1 - 1\right)$$

The first order condition is $SQ_1 = \lambda Q_1$. As shown by Uhlig (2004b), this is an eigenvalue-eigenvector problem. The column Q_1 that maximises the variance is the eigenvector associated

with the largest eigenvalue λ. We use the matlab code written by Kurmann and Otrok (2013) to implement this solution.

2 Estimation of the Bayesian Mixed Frequency VAR model

The observation equation of the model is defined as

$$y_t = H_t \beta_t + V_t$$

where $y_t = \left[\begin{array}{cc} \hat{\pi}_t^{LH} & z_t \end{array} \right]$ where z_t denotes the remaining $N-1$ endogenous variables in the VAR. As discussed in the text, the Blue Chip\Livingstone survey proxy for $\hat{\pi}^{LH}$ is unavailable every quarter. β_t denotes the $ns \times 1$ vector of state variables $\left(\tilde{\pi}_t^{LH}, z_t, \tilde{\pi}_{t-1}^{LH}, z_{t-1}\right)$ where $\tilde{\pi}_t^{LH}$ denotes the unknown higher frequency data on inflation expectations. At time t, if $\hat{\pi}_t^{LH}$ is missing, the matrix H_t is defined as

$$H_t = \left[\begin{array}{ccc} & 0_{1\times ns} & \\ 0_{N-1\times 1} & I_{N-1} & 0_{N-1\times ns-N} \end{array} \right]$$

and $var\left(V_t\right) = 1e10$.

At time t, if $\hat{\pi}_t^{LH}$ is available and $\hat{\pi}_{t-1}^{LH}$ is missing, we assume that $\hat{\pi}_t^{LH}$ is an average of the current and the previous quarters (unobserved) expectations data. Thus:

$$H_t = \left[\begin{array}{cccc} \frac{1}{2} & 0_{1\times N-1} & \frac{1}{2} & 0_{1\times N-1} \\ 0_{N-1\times 1} & I_{N-1} & 0_{N-1\times ns-N} & \end{array} \right]$$

and $var\left(V_t\right) = 0$.

For some periods both $\hat{\pi}_t^{LH}$ and $\hat{\pi}_{t-1}^{LH}$ are available. Then we assume that for that period inflation expectations are observed at the quarterly frequency:

$$H_t = \left[\begin{array}{ccc} & 1 \quad 0_{1\times ns-1} & \\ 0_{N-1\times 1} & I_{N-1} & 0_{N-1\times ns-N} \end{array} \right]$$

and $var\left(V_t\right) = 0$.

The transition equation is a $VAR(2)$ written in companion form

$$\beta_t = \mu + F\beta_{t-1} + e_t$$

where $var\left(e_t\right) = \Omega$.

As in the standard VAR model, we assume a natural conjugate prior for the VAR parameters. Given β_t, the conditional posterior of the VAR coefficients μ, F and the error covariance Ω is as defined for the BVAR above and these parameters can be easily draw. Conditional on a draw for $\Xi = \mu, F, \Omega$, the Carter and Kohn (1994) algorithm is used to draw β_t. This conditonal posterior is defined as

$$H\left(\tilde{\beta}_T \backslash \Xi\right) = H\left(\beta_T \backslash \tilde{z}_T\right) \prod_{t=1}^{T-1} H\left(\beta_t \backslash \beta_{t+1}, \tilde{z}_t\right)$$

where $\tilde{\beta}_T = \left[\beta_1, \beta_2, ..\beta_T\right], \tilde{z}_T = \left[z_1, z_2, ..z_T\right]$. The mean and variance of the normal density $H\left(\beta_T \backslash \tilde{z}_T\right)$ can be obtained via the Kalman filter. Carter and Kohn (1994) provide the updating equations to obtain the mean and variance of the normal densities $H\left(\beta_t \backslash \beta_{t+1}, \tilde{z}_t\right)$. We use 25000 iterations of the Gibbs Sampler and retain the last 5000 iterations for inference.

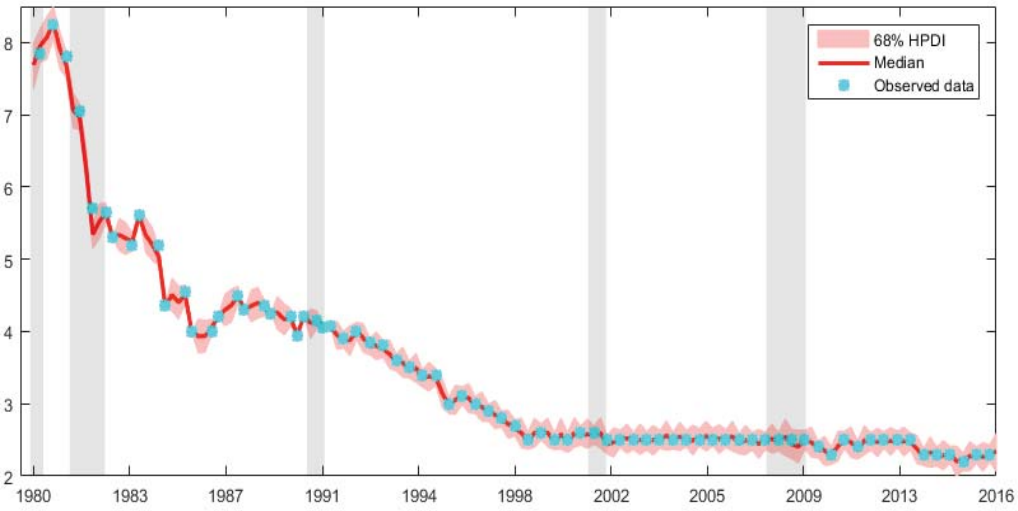

Figure 1: Estimated quarterly data for long-horizon blue chip\Livingstone survey inflation expectations.

Figure 1 presents the estimated posterior distribution of $\tilde{\pi}_t^{LH}$ and compares it with the observed low frequency data.

3 Sensitivity Analysis

In this section, we present a number of robustness checks regarding the benchmark VAR model. First, we expand the benchmark VAR model by adding the first three principal components extracted from a large panel of macroeconomic and financial data for the US (FRED-QD database provided by the St Louis Fed). This allows us to incorporate large amount information into the model and to potentially account for potential information insufficiency (see Forni and Gambetti (2014)). The impulse responses and FEV decomposition from this model are very similar to the benchmark case (see figure 2). When the model is estimated truncated the sample to 2007, the main conclusions are largely unaffected (see figure 3). Figures 4 and 5 show that altering the lag length to 1 and 4 respectively does not affect the benchmark results greatly. Figure 6 shows that the results are similar to benchmark when a horizon of 80 quarters is used for shock identification. Figure 7 shows that the results are not materially different if a flat prior for the VAR parameters is used. Figure 8 shows the variance decomposition using the mixed frequency VAR model.

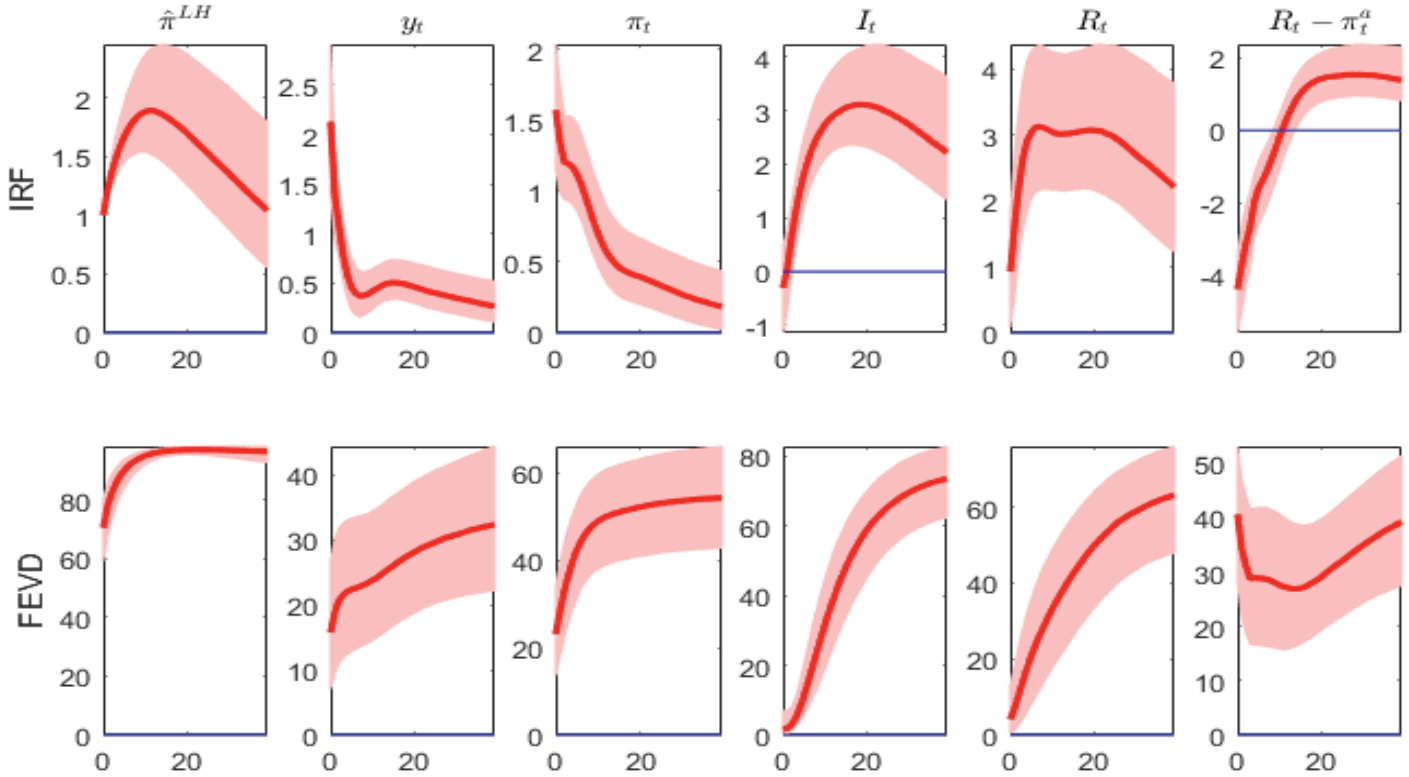

Figure 2: Response from a version of the benchmark model that includes principal components from a large data set.

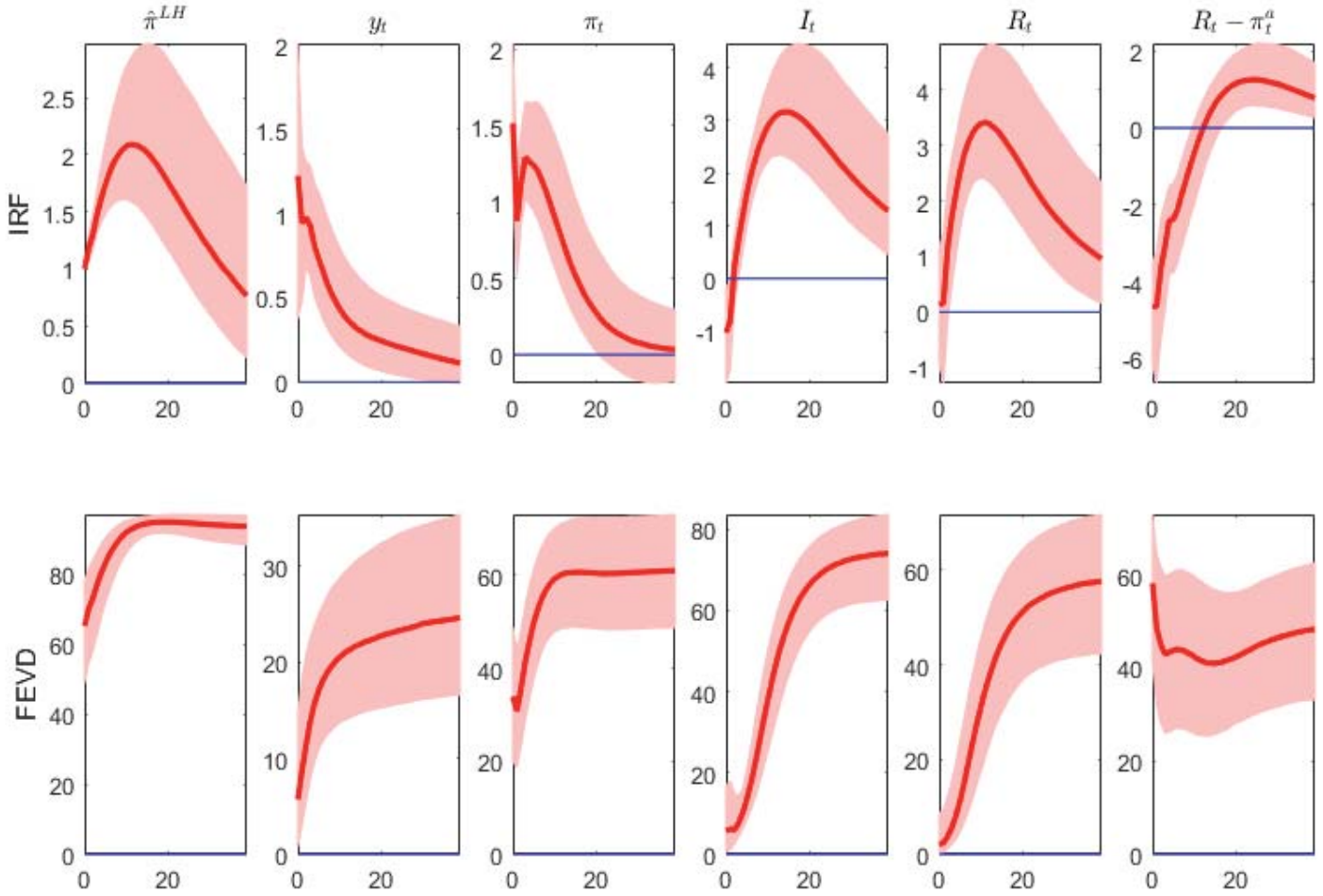

Figure 3: Response from a version of the benchmark model estimated using data up to 2006Q4.

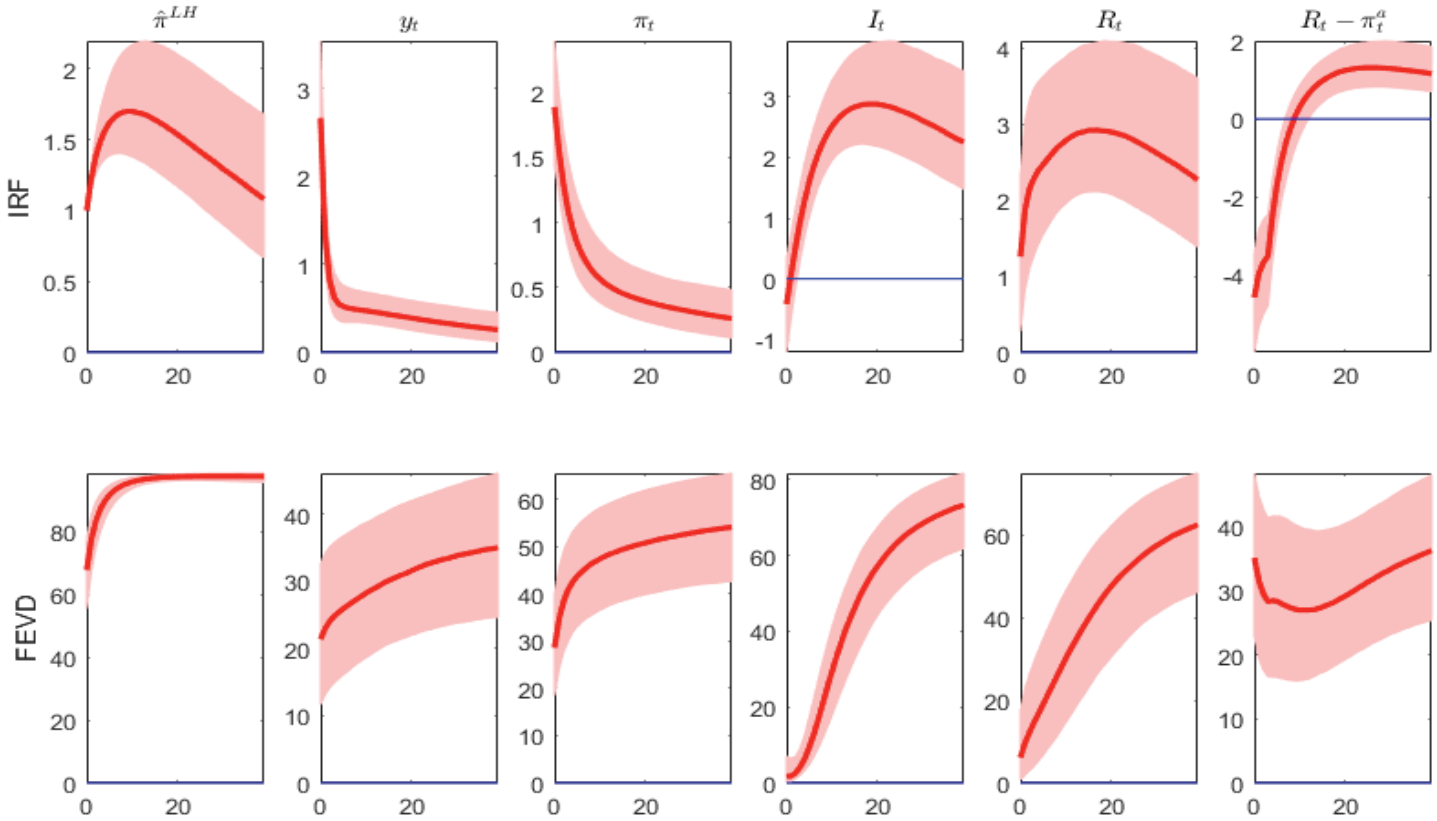

Figure 4: Response from a version of the benchmark model estimated using a lag length of 1.

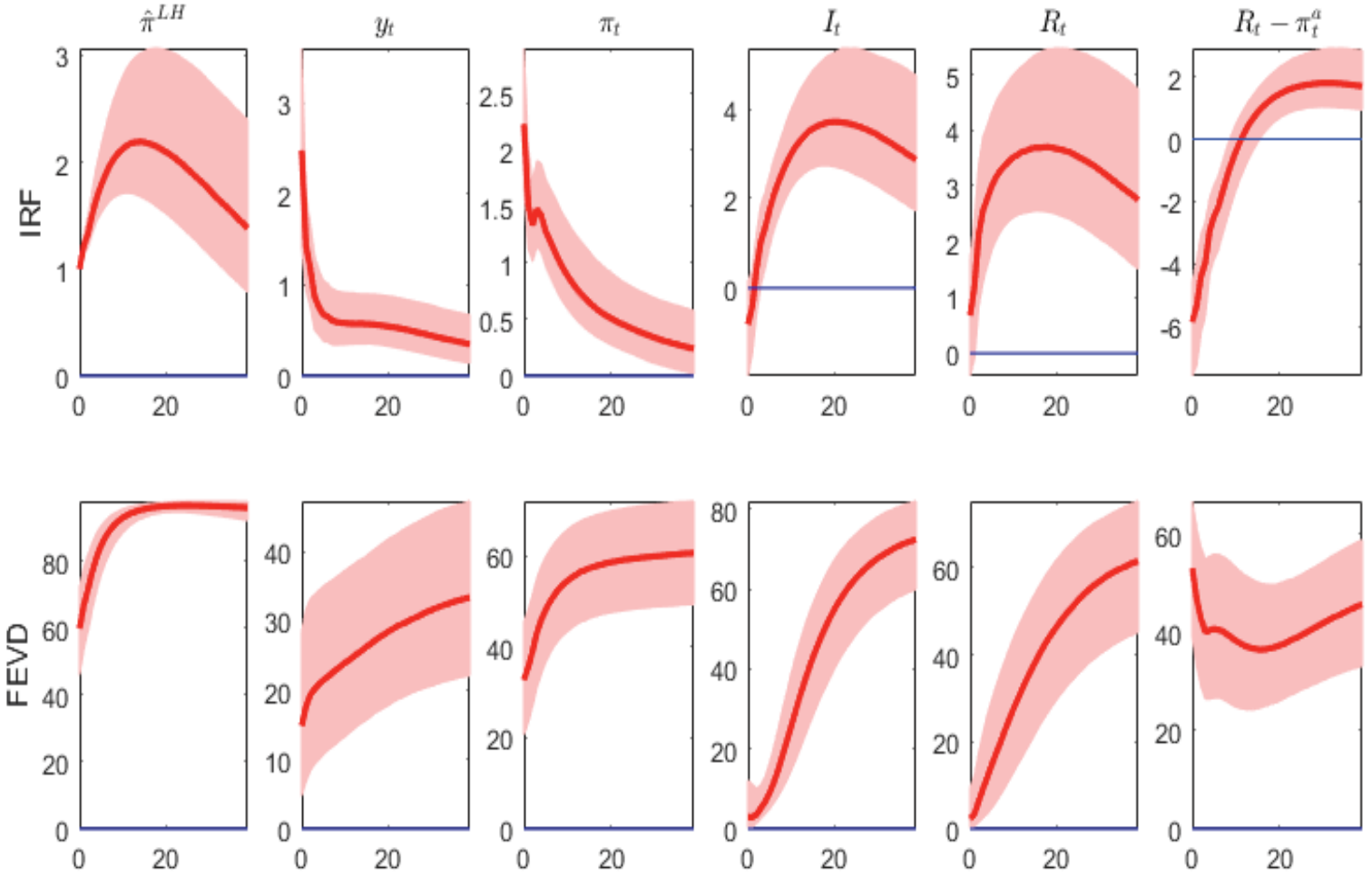

Figure 5: Response from a version of the benchmark model estimated using a lag length of 4.

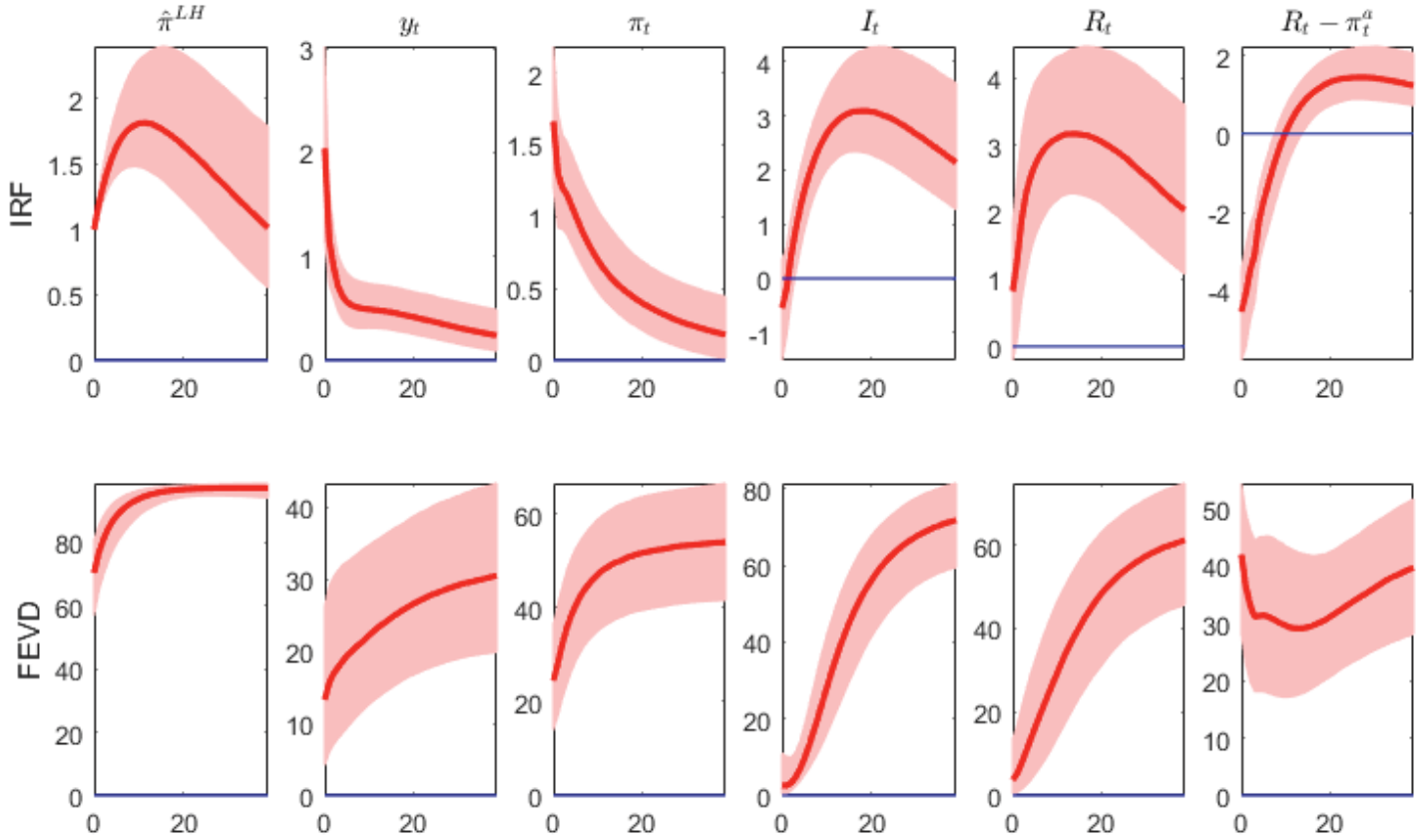

Figure 6: Response from a version of the benchmark model estimated using a horizon of 80 quarters to identify the target shock.

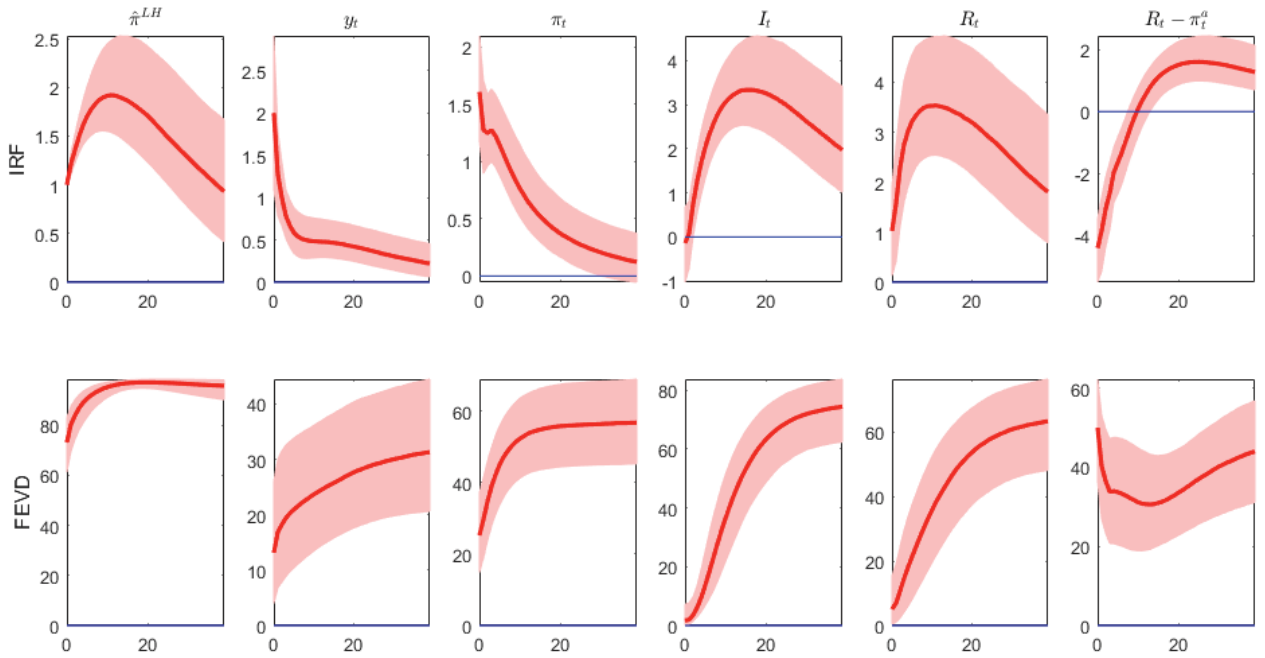

Figure 7: Response from a version of the benchmark model estimated using a flat prior.

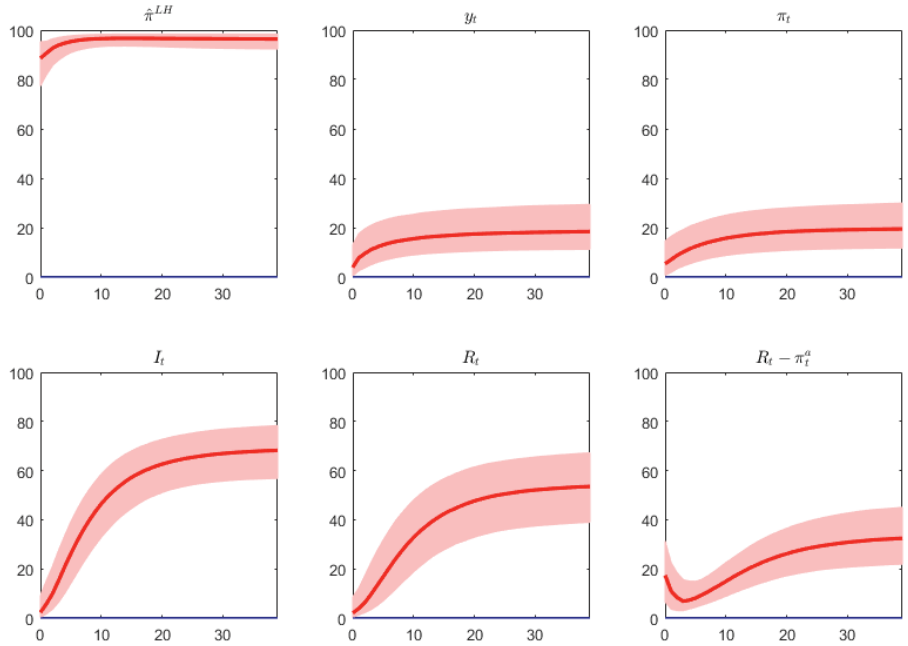

Figure 8: Contribution to FEV from the mixedfrequency VAR using Blue Chip/Livingstone survey expectations.

4 Theoretical Model

4.1 Firms

Three types of firms are operated in the domestic economy. The intermediate monopolistically competitive domestic firms use labour supplied by households to produce a differentiated good that is sold to a final good producer who employs a continuum of these differentiated goods in her constant elasticity of substitution – CES – production to deliver the final good. The monopolistically competitive importing firms use a costless technology and turn a homogenous good – bought in the world market – into a differentiated good, which is then sold to the domestic consumers. The exporting monopolistically competitive firms use similar 'brand naming' technology and transform the domestic final good into a differentiated product that is sold to foreign households.

This sector consists of three firms, the 'labour packer' who hires labour from households and transforms it into a homogenous input good – h_t^d, a continuum of monopolistically competitive firms that buys h_t^d and produces an intermediate $y_{i,t}$ and the final good producer who combines all these intermediate products into a single good consumed by households. The final good producer's CES production function is given by

$$y_t^d = \left[\int_0^1 y_{i,t}^{\frac{1}{\lambda_{y,t}}} di\right]^{\lambda_{y,t}} \tag{5}$$

where

$$\lambda_{y,t} = \left(1-\rho_{\lambda_y}\right)\lambda_y + \rho_{\lambda_y}\lambda_{y,t-1} + \sigma_{\lambda_y}\omega_{\lambda_y,t} \tag{6}$$

denotes the time-varying mark-up in the domestic good market. The final good producer's demand curve for $y_{i,t}$ arises from the profit minimisation problem – $\max_{y_{i,t}} \left\{ p_t \left[\int_0^1 y_{i,t}^{\frac{1}{\lambda_{y,t}}} di\right]^{\lambda_{y,t}} - \int_0^1 p_{i,t} y_{i,t} \right\}$

$$y_{i,t} = \left(\frac{p_{i,t}}{p_t}\right)^{-\frac{\lambda_{y,t}}{\lambda_{y,t-1}}} y_t^d \tag{7}$$

The final good price index is obtained by combining 5 and 7

$$p_t = \left[\int_0^1 p_{i,t}^{\frac{1}{1-\lambda_{y,t}}} di\right]^{1-\lambda_{y,t}} \tag{8}$$

Intermediate good producers use the following production function

$$y_{i,t} = z_t \left(\Gamma_t h_{i,t}^d\right)^{\alpha} k_{i,t}^{1-\alpha} \tag{9}$$

where

$$z_t = (1-\rho_z)\, z + \rho_z z_{t-1} + \sigma_z \omega_{z,t} \tag{10}$$

$$\gamma_t = \ln\left(\frac{\Gamma_t}{\Gamma_{t-1}}\right) = \left(1-\rho_\gamma\right)\gamma + \rho_\gamma \gamma_{t-1} + \sigma_\gamma \omega_{\gamma,t} \tag{11}$$

is a stationary exogenous technological process and $h_{i,t}^d$ is the amount of homogeneous labour rented

by the firm i^{ih}. The intermediate firm select $h_{i,t}^d$ in order to minimise its production cost

$$\min_{h_{i,t}^d} p_t w_t h_{i,t}^d + p_t r_t^k k_t + mc_{i,t} p_t \left[y_{i,t} - z_t \left(\Gamma_t h_{i,t}^d \right)^{\alpha} k_{i,t}^{1-\alpha} \right] \tag{12}$$

The real marginal cost for the intermediate firms is given by the first order conditions of (12) with respect to $h_{i,t}^d$ and $k_{i,t}$

$$mc_{i,t} = \frac{w_t}{\alpha \frac{y_{i,t}}{h_{i,t}^d}} \tag{13}$$

$$mc_{i,t} = \frac{r_t^k}{(1-\alpha)\frac{y_{i,t}}{k_{i,t}}}$$

A fraction – $\left(1-\xi_y\right)$ – of intermediate firms receive a random signal and they are allowed to optimally reset their prices – $p_{i,t}^{new}$. The proportion – ξ_y – of firms that cannot reoptimise prices will set p_t based on backward-looking rule

$$p_t = \pi_{t-1}^{\kappa_y} \bar{\pi}_t^{1-\kappa_y} p_{t-1} \tag{14}$$

where $\pi_t = \frac{p_t}{p_{t-1}}$ is the gross inflation and κ_y is the indexation parameter. The pricing problem of firm i is then

$$\max_{p_{i,t}^{new}} E_t \sum_{j=0}^{\infty} \left(\beta \xi_y\right)^j \frac{\lambda_{t+j}}{\lambda_t} \left\{ \left(\prod_{s=1}^{j} \pi_{t+s-1}^{\kappa_y} \bar{\pi}_{t+s}^{1-\kappa_y} \frac{p_{i,t}^{new}}{p_{t+j}} - mc_{t+j} \right) y_{i,t+j} \right\} \tag{15}$$

subject to

$$y_{i,t+j} = \left(\prod_{s=1}^{j} \pi_{t+s-1}^{\kappa_y} \bar{\pi}_{t+s}^{1-\kappa_y} \frac{p_{i,t}^{new}}{p_{t+j}} \right)^{-\frac{\lambda_{y,t}}{\lambda_{y,t-1}}} y_{t+j}^d \tag{16}$$

The first-order condition is expressed as system of difference equations

$$f_{1,t} = \lambda_t mc_t y_t^d + \beta \xi_y E_t \left(\frac{\pi_t^{\kappa_y} \bar{\pi}_{t+1}^{1-\kappa_y}}{\pi_{t+1}} \right)^{-\frac{\lambda_{y,t}}{\lambda_{y,t-1}}} f_{1,t+1} \tag{17}$$

$$f_{2,t} = \lambda_t \ddot{\pi}_t y_t^d + \beta \xi_d E_t \left(\frac{\pi_t^{\kappa_y} \bar{\pi}_{t+1}^{1-\kappa_y}}{\pi_{t+1}} \right)^{-\frac{1}{\lambda_{y,t-1}}} \left(\frac{\ddot{\pi}_t}{\ddot{\pi}_{t+1}} \right) f_{2,t+1} \tag{18}$$

$$0 = \lambda_{y,t} f_{1,t} - f_{2,t} \tag{19}$$

$$1 = \xi_y \left(\frac{\pi_{t-1}^{\kappa_y} \bar{\pi}_t^{1-\kappa_y}}{\pi_t} \right)^{-\frac{1}{\lambda_{y,t-1}}} + \left(1-\xi_y\right) \ddot{\pi}_t^{-\frac{1}{\lambda_{y,t-1}}} \tag{20}$$

where $\ddot{\pi}_t \equiv \frac{p_t^{new}}{p_t}$.

Market clearing condition in the domestic sector

$$y_t = \int_0^1 y_{i,t} di = \int_0^1 \left(\frac{p_{i,t}}{p_t} \right)^{-\frac{\lambda_{y,t}}{\lambda_{y,t-1}}} di y_t^d = \upsilon_t^p y_t^d \tag{21}$$

where $\upsilon_t^p = \int_0^1 \left(\frac{p_{i,t}}{p_t}\right)^{-\frac{\lambda_{d,t}}{\lambda_{d,t}-1}} d_i$ is the price dispersion term and it is given by

$$\upsilon_t^p = \xi_y \left(\frac{\pi_{t-1}^{\kappa_y}\pi^{1-\kappa_y}}{\pi_t}\right)^{-\frac{\lambda_{y,t}}{\lambda_{y,t}-1}} \upsilon_{t-1}^p + \left(1-\xi_y\right)\bar{\pi}_t^{-\frac{\lambda_{y,t}}{\lambda_{y,t}-1}} \tag{22}$$

4.2 Households

The domestic economy is populated by a continuum of households that attain utility from consumption – $c_{\kappa,t+j}$ – and leisure – $h_{\kappa,t+j}$. Household's preferences are separable

$$E_t \sum_{j=0}^{\infty} \beta^j \left\{ d_{t+j} \frac{\left(c_{\kappa,t+j} - b c_{\kappa,t+j-1}\right)^{1-\sigma_c}}{1-\sigma_c} - \psi_{t+j}\Gamma_{t+j}^{1-\sigma_c} \frac{h_{\kappa,t+j}^{1+\varphi}}{1+\varphi} d\kappa \right\} \tag{23}$$

where d_t is a discount factor shock

$$d_t = \left(1-\rho_d\right) d + \rho_d d_{t-1} + \sigma_d \omega_d \tag{24}$$

β is the discount factor, φ the inverse of the Frisch elasticity, σ_c the inverse of intertemporal elasticity of substitution and b the habit formation parameter. They also invest on fiscal capital that is rented to firms, furthermore capiltal accumulation is subject to investment adjustment cost

$$\bar{k}_t = \left(1-\delta\right)\bar{k}_{t-1} + \phi_t \left(1 - \frac{\psi_I}{2}\left(\frac{i_t}{\gamma i_{t-1}} - 1\right)\right) i_t \tag{25}$$

$$k_t = \upsilon_t \bar{k}_{t-1}$$

Household's real budget constraint is given by

$$D_{\kappa,t}^h + c_{\kappa,t} + i_{\kappa,t} + \Upsilon_{\kappa,t} = \frac{r_{t-1}^h}{\pi_t} D_{\kappa,t-1}^h + w_{\kappa,t} h_{\kappa,t} + r_t^k \upsilon_{\kappa,t} \bar{k}_{\kappa,t-1} - u\left(\upsilon_{\kappa,t}\right)\bar{k}_{\kappa,t-1} + F_{\kappa,t} - T_{\kappa,t} \tag{26}$$

The household κ uses its labour income – $w_{\kappa,t}h_{\kappa,t}$, net return on capital services – $r_t^k \upsilon_{\kappa,t}\bar{k}_{\kappa,t-1} - u\left(\upsilon_{\kappa,t}\right)\bar{k}_{\kappa,t-1}$, gross interest rate financial intermediary deposits – $\frac{r_{t-1}}{\pi_t^c} D_{\kappa,t-1}^h$, government transfers – T_t – and profits – F_t – to finance consumption, investment and new purchases of financial assets – $c_{\kappa,t} + i_{\kappa,t} + D_{\kappa,t} + \Upsilon_{\kappa,t}$. The household maximises (23) with respect to $c_{\kappa,t}$, $i_{\kappa,t}$, $\bar{k}_{\kappa,t-1}$ and $D_{\kappa,t}^h$ subject to (26) and (25)

$$\frac{d_t}{\left(c_{\kappa,t} - b c_{\kappa,t-1}\right)^{\sigma_c}} - E_t \frac{\beta b d_{t+1}}{\left(c_{\kappa,t+1} - h c_{\kappa,t}\right)^{\sigma_c}} = \lambda_{\kappa,t} \tag{27}$$

$$\lambda_{\kappa,t} = \beta E_t \left\{ \lambda_{\kappa,t+1} \frac{r_t^h}{\pi_{t+1}} \right\} \tag{28}$$

$$r_{\kappa,t}^k = u'\left(\upsilon_{\kappa,t}\right) \tag{29}$$

$$q_{\kappa,t} = \beta \frac{\lambda_{\kappa,t+1}}{\lambda_{\kappa,t}} \left[r_{t+1}^k \upsilon_{\kappa,t+1} - u\left(\upsilon_{\kappa,t+1}\right) + \left(1-\delta\right) q_{\kappa,t+1} \right] \tag{30}$$

$$1 = q_{\kappa,t} \left\{ 1 - \frac{\psi_I}{2}\left(\frac{i_t}{i_{t-1}} - \gamma\right)^2 - \psi_I \left(\frac{i_t}{i_{t-1}} - \gamma\right)\frac{i_t}{i_{t-1}} \right\} + \beta \frac{\lambda_{\kappa,t+1}}{\lambda_{\kappa,t}} q_{\kappa,t} \psi_I \left(\frac{i_{t+1}}{i_t} - \gamma\right)\left(\frac{i_{t+1}}{i_t}\right)^2 \tag{31}$$

4.3 Financial intermediary

The financial intermediary firm issues deposits to households paying a gross interest rate r_t^h. The firm then purchases a portfolio of short and long term government issued bonds paying interest r_t^S and r_t^L.

Similar to Andres *et al.* (2004), Chen *et al.* (2012), Harrison (2012) and Liu *et al.* (2014) we follow the formulation in Woodford (2001) and long-term bonds are perpetuities that cost $p_{L,t}$ at time t and pay an exponentially decaying coupon κ^s at time $t+s+1$ where $0<\kappa\leq 1$. As it is explained in Woodford (2001) and Chen *et al.* (2012) the advantage of this formulation is that the price in period t of a bond issued s periods ago $p_{L-s,t}$ is a function of the coupon the current price $p_{L,t}$

$$p_{L-s,t}=\kappa^s p_{L,t} \tag{32}$$

This relation allows to express the balance sheet equation and government budget constraint (below) in a familiar form that it is easy to work with it (see the discussion in Chen *et al.* (2012)). Furthermore, in order to keep things simple, we rule out the possibility of a secondary market for long-term bonds, meaning that agents who buy long-term debt must hold it until maturity.[1] Finally, for simplicity we assume that all government bonds issued are purchased by this firm.

The intermediary's balance sheet is given

$$b_{\kappa,t}^h=\frac{b_{\kappa,t}^S}{\varepsilon_t^{b^S}}+\frac{p_{L,t}b_{\kappa,t}^L}{\varepsilon_t^{b^L}}$$

or

$$b_{\kappa,t}^h=\frac{b_{\kappa,t}^S}{\varepsilon_t^{b^S}}+\frac{\bar{b}_{\kappa,t}^L}{\varepsilon_t^{b^L}} \tag{33}$$

Motivated by the work of Smets and Wouters (2007) we assume the balance sheet equation is subject to two 'financial' shocks: a short and a long-term risk premium shocks dented by $\varepsilon_t^{b^S}$, $\varepsilon_t^{\bar{b}^L}$ and $\varepsilon_t^{\bar{b}^{L,*}}$, respectively.

Intermediary's profit function is then given by

$$\xi_t=\underbrace{b_{\kappa,t}^h+\frac{r_{t-1}^S}{\pi_t}b_{\kappa,t-1}^S+\frac{p_{L,t}r_t^L}{\pi_t}b_{\kappa,t-1}^L}_{\text{revenues}}-\underbrace{\frac{b_{\kappa,t}^S}{\varepsilon_t^{b^S}}-\frac{p_{L,t}b_{\kappa,t}^L}{\varepsilon_t^{\bar{b}^L}}-\frac{r_{t-1}^h}{\pi_t}b_{\kappa,t-1}^h-\frac{x}{2}\left(\delta_{\kappa,t-1}^B-\vartheta\delta_{t-2}^B-(1-\vartheta)\,\delta^B\right)^2\frac{\Gamma_{t-1}}{\pi_t}}_{\text{expenditures}}$$

where

$$\delta_{\kappa,t}^B=\frac{b_{\kappa,t}^S}{\bar{b}_{\kappa,t}^L}$$

Intermediary's profits are subject to two adjustment costs. The fist one captures the idea that altering the foreign debt held by domestic intermediaries to GDP ratio is costly.[2] The second term reflects the situation where although intermediaries prefer to hold more long than short-term debt that decreases 'liquidity'.

[1] See the discussion in Andres *et al.* (2004) for the advantages of that assumption.

[2] In our model this term is not required to make the net foreign asset position of the model stationary (see the discussion in Schmitt-Grohe and Uribe (2003)) as we properly model long-term debt in the foreign economy.

Using the balance sheet equation the profit function becomes

$$\begin{aligned} E_t\xi_{t+1} &= \frac{r_t^S}{\pi_{t+1}}b_{\kappa,t}^S + E_t\left\{\frac{r_{t+1}^L}{\pi_{t+1}}\frac{p_{L,t+1}}{p_{L,t}}\right\}\bar{b}_{\kappa,t}^L - E_t\left\{\frac{r_t^h}{\pi_{t+1}}\right\}b_{\kappa,t}^h \\ &\quad -\frac{x}{2}\left[\delta_{\kappa,t}^B - \vartheta\delta_{t-1}^B - (1-\vartheta)\,\delta^B\right]^2\frac{\Gamma_t}{E_t\pi_{t+1}} \end{aligned} \tag{34}$$

Profit maximisation with respect to short, domestic and foreign long-term debt and subject to the balance sheet condition delivers an expression for the effective rate faced by the household, long-term interest rate and exchange rate

Short-term debt

$$\begin{aligned} 0 &= \frac{r_t^S}{E_t\pi_{t+1}} - \frac{r_t^h}{\varepsilon_t^{b^S}E_t\pi_{t+1}} - x\left[\delta_{\kappa,t}^B - \vartheta\delta_{t-1}^B - (1-\vartheta)\,\delta^B\right]\frac{\Gamma_t}{\bar{b}_{\kappa,t}^L}\frac{1}{E_t\pi_{t+1}} \\ \frac{r_t^h}{\varepsilon_t^{b^S}} &= r_t^S - x\left[\delta_{\kappa,t}^B - \vartheta\delta_{t-1}^B - (1-\vartheta)\,\delta^B\right]\frac{\Gamma_t}{\bar{b}_{\kappa,t}^L} \end{aligned} \tag{35}$$

Long-term domestic debt

$$0 = E_t\left\{\frac{r_{t+1}^L}{\pi_{t+1}}\frac{p_{L,t+1}}{p_{L,t}}\right\} - \frac{r_t^h}{\varepsilon_t^{\bar{b}^L}E_t\pi_{t+1}} + x\left[\upsilon_{\kappa,t} - \vartheta\upsilon_{t-1} - (1-\vartheta)\,\upsilon\right]\frac{\Gamma_t b_{\kappa,t}^S}{\left(\bar{b}_{\kappa,t}^L\right)^2}\frac{1}{E_t\pi_{t+1}} \tag{36}$$

$$E_t\left\{r_{t+1}^L\frac{p_{L,t+1}}{p_{L,t}}\right\} = \frac{r_t^h}{\varepsilon_t^{\bar{b}^L}} - x\left[\delta_{\kappa,t}^B - \vartheta\delta_{t-1}^B - (1-\vartheta)\,\delta^B\right]\frac{\Gamma_t b_{\kappa,t}^S}{\left(\bar{b}_{\kappa,t}^L\right)^2} \tag{37}$$

4.4 Wages

We follow Erceg *et al.* (2000) and assume that each monopolistically competitive household supplies a differentiated labour service to the production section. They set their nominal wage and supply any amount of labour demanded by the firms at that wage rate. For convenience, we assume that there exist a representative firm that combines households' labour inputs into a homogenous input hood - h_t^d - using a CES production function

$$h_t^d = \left[\int_0^1 h_{\kappa,t}^{\frac{1}{\lambda_w}}\,d\kappa\right]^{\lambda_w} \tag{38}$$

where λ_w is the wage mark-up. Taking w_t and $w_{\kappa,t}$ as given the aggregator's demand for the labour hours of household κ results its profit maximisation $\max_{h_{\kappa,t}}\left\{w_t\left[\int_0^1 h_{\kappa,t}^{\frac{1}{\lambda_w}}\,d\kappa\right]^{\lambda_w} - \int_0^1 w_{\kappa,t}h_{\kappa,t}\right\}$

$$h_{\kappa,t} = \left(\frac{w_{\kappa,t}}{w_t}\right)^{-\frac{\lambda_w}{\lambda_w-1}}h_t^d \tag{39}$$

The aggregate wage arise from the profit condition and the demand curve

$$w_t = \left[\int_0^1 w_{\kappa,t}^{\frac{1}{1-\lambda_w}}\,d\kappa\right]^{1-\lambda_w} \tag{40}$$

In each period, a function – $1-\xi_w$ – of households receive a random signal and they are allowed to reset wages optimally – w_t^{new}. All other households can only partially index their wages by past inflation. The problem of setting wages can be described as follows

$$\max_{w_t^{new}} E_t \sum_{j=0}^{\infty} (\beta\xi_w)^j \left\{ -\psi_{t+j}\Gamma_{t+j}^{1-\sigma_c} \frac{h_{\kappa,t+j}^{1+\varphi}}{1+\varphi} + \lambda_{t+j} \prod_{s=1}^{j} \frac{\pi_{t+s-1}^{\kappa_w} \bar{\pi}_t^{1-\kappa_w} \gamma}{\pi_{t+s}} w_{\kappa,t} h_{\kappa,t+j} \right\} \tag{41}$$

subject to

$$h_{\kappa,t+j} = \left(\prod_{s=1}^{j} \frac{\pi_{t+s-1}^{\kappa_w} \bar{\pi}_{t+s}^{1-\kappa_w} \gamma}{\pi_{t+s}} \frac{w_{\kappa,t}}{w_{t+j}} \right)^{-\frac{\lambda_w}{\lambda_w - 1}} h_{t+j}^d \tag{42}$$

Substituting labour demand into the former equation

$$\max_{w\kappa,t} E_t \sum_{j=0}^{\infty} (\beta\xi_w)^j \left\{ \begin{array}{c} -\frac{\psi_{t+j}\Gamma_{t+j}^{1-\sigma_c}}{1+\varphi} \left(\prod_{s=1}^{j} \frac{\pi_{t+s-1}^{\kappa_w} \bar{\pi}_{t+s}^{1-\kappa_w} \gamma}{\pi_{t+s}} \frac{w_{\kappa,t}}{w_{t+j}} \right)^{-\frac{\lambda_w(1+\varphi)}{\lambda_w - 1}} \left(h_{t+j}^d \right)^{1+\varphi} \\ +\lambda_{t+j} \prod_{s=1}^{j} \frac{\pi_{t+s-1}^{\kappa_w} \bar{\pi}_{t+s}^{1-\kappa_w} \gamma}{\pi_{t+s}} w_{\kappa,t} \left(\prod_{s=1}^{j} \frac{\pi_{t+s-1}^{\kappa_w} \bar{\pi}_{t+s}^{1-\kappa_w} \gamma}{\pi_{t+s}} \frac{w_{\kappa,t}}{w_{t+j}} \right)^{-\frac{\lambda_w}{\lambda_w - 1}} h_{t+j}^d \end{array} \right\}$$

which can be simplified further

$$\max_{w\kappa,t} E_t \sum_{j=0}^{\infty} (\beta\xi_w)^j \left\{ \begin{array}{c} -\frac{\psi_{t+j}\Gamma_{t+j}^{1-\sigma_c}}{1+\varphi} \left(\prod_{s=1}^{j} \frac{\pi_{t+s-1}^{\kappa_w} \bar{\pi}_{t+s}^{1-\kappa_w} \gamma}{\pi_{t+s}} \frac{w_{\kappa,t}}{w_{t+j}} \right)^{-\frac{\lambda_w(1+\varphi)}{\lambda_w - 1}} \left(h_{t+j}^d \right)^{1+\varphi} \\ +\lambda_{t+j} \left(\prod_{s=1}^{j} \frac{\pi_{t+s-1}^{\kappa_w} \bar{\pi}_{t+s}^{1-\kappa_w} \gamma}{\pi_{t+s}} \frac{w_{\kappa,t}}{w_{t+j}} \right)^{-\frac{1}{\lambda_w - 1}} w_{t+j} h_{t+j}^d \end{array} \right\}$$

The first order is summarised by the following recursive equations

$$v_{1,t} = \frac{1}{\lambda_w} \left(w_t^{new}\right)^{\frac{1}{1-\lambda_w}} \lambda_t w_t^{\frac{\lambda_w}{\lambda_w - 1}} h_t^d + \beta\xi_w E_t \left(\frac{\pi_t^{\kappa_w} \bar{\pi}_{t+1}^{1-\kappa_w} \gamma}{\pi_{t+1}} \right)^{\frac{1}{1-\lambda_w}} \left(\frac{w_{t+1}^{new}}{w_t^{new}} \right)^{\frac{1}{\lambda_w - 1}} v_{1,t+1} \tag{43}$$

$$v_{1,t} = \psi_t \gamma_t^{1-\sigma_c} \left(\frac{w_t}{w_t^{new}} \right)^{\frac{(1+\varphi)\lambda_w}{\lambda_w - 1}} \left(h_t^d \right)^{1+\varphi} + \beta\xi_w E_t \left(\frac{\pi_t^{\kappa_w} \bar{\pi}_{t+1}^{1-\kappa_w} \gamma}{\pi_{t+1}} \right)^{\frac{(1+\varphi)\lambda_w}{1-\lambda_w}} \left(\frac{w_{t+1}^{new}}{w_t^{new}} \right)^{\frac{(1+\varphi)\lambda_w}{\lambda_w - 1}} v_{1,t+1} \tag{44}$$

$$w_t^{\frac{1}{1-\lambda_w}} = \xi_w \left(\frac{\pi_{t-1}^{\kappa_w} \bar{\pi}_t^{1-\kappa_w} \gamma}{\pi_t} \right)^{\frac{1}{1-\lambda_w}} w_{t-1}^{\frac{1}{1-\lambda_w}} + (1-\xi_w) \left(w_t^{new}\right)^{\frac{1}{1-\lambda_w}} \tag{45}$$

The market clearing condition in the labour market is

$$h_t = \int_0^1 h_{\kappa,t} d\kappa = v_t^w h_t^d \tag{46}$$

where $\upsilon_t^w = \int_0^1 \left(\frac{w_{i,t}}{w_t}\right)^{-\frac{\lambda_w}{\lambda_w - 1}} d_i$ is the wage dispersion term and its evolution is described

$$\upsilon_t^w = \xi_w \left(\frac{\pi_{t-1}^{\kappa_w} \bar{\pi}_t^{1-\kappa_w} \gamma}{\pi_t}\right)^{\frac{\lambda_w}{1-\lambda_w}} \left(\frac{w_{t-1}}{w_t}\right)^{\frac{\lambda_w}{1-\lambda_w}} \upsilon_{t-1}^w + (1-\xi_w) \left(\frac{w_t^{new}}{w_t}\right)^{\frac{\lambda_w}{1-\lambda_w}} \tag{47}$$

4.5 Government

Government's budget constraint adjusted for long-term debt is given by

$$b_t^S + \bar{b}_t^L + T_t = \frac{r_{t-1}^S}{\pi_t} b_{t-1}^S + \frac{r_t^L}{\pi_t} \frac{p_{L,t}}{p_{L,t-1}} \bar{b}_{t-1}^L + G_t$$

where the left hand side is the total (short plus long-term) debt issued by the government at time t. Finally, transfers are adjusted according to the following rule

$$T_t = \Phi \left(\frac{b_{t-1}^S + \bar{b}_{t-1}^L}{b^S + \bar{b}^L}\right)^{\theta} \tag{48}$$

$$G_t = g_t y_t \tag{49}$$

4.6 Monetary policy

The monetary authority sets its instrument short-term interest rate according to a Taylor rule

$$\frac{r_t}{r} = \left(\frac{r_{t-1}}{r}\right)^{\phi_R} \left(\frac{\pi_t}{\bar{\pi}_t}\right)^{(1-\phi_R)\phi_\pi} \left(\frac{y_t}{y}\right)^{(1-\phi_R)\phi_y} m_t \tag{50}$$

where $m_t = m_{t-1}^{\rho_m} e^{\sigma_R \omega_{R,t}}$. In other words, the policymaker adjusts the nominal interest rate in response to its lag value, to inflation deviations from the target

$$\bar{\pi}_t - 0.999\bar{\pi}_{t-1} = \rho_{\bar{\pi}} \left(\bar{\pi}_{t-1} - 0.999\bar{\pi}_{t-2}\right) + \sigma_{\bar{\pi}} \omega_{\bar{\pi},t} \tag{51}$$

(De Graeve *et al.* (2009)) and to trend output gap deviations from its long-run equilibrium – y.

4.7 Market clearing conditions

$$y_t^d = c_t + G_t + i_t \tag{52}$$

4.8 Stationary Equations

$$\frac{y_t}{\gamma_t} = \tilde{y}_t = z_t \left(h_t^d\right)^{\alpha} \tilde{k}_t^{1-\alpha} \tag{53}$$

$$\bar{k}_t = (1-\delta) \frac{\bar{k}_{t-1}}{\gamma_t} + \phi_t \left(1 - \frac{\psi_I}{2} \left(\frac{\tilde{\imath}_t \gamma_t}{\tilde{\imath}_{t-1}} - \gamma\right)^2\right) \tilde{\imath}_t \tag{54}$$

$$r_t^k = u'\left(\upsilon_t\right) \tag{55}$$

$$q_t = \beta \frac{\tilde{\lambda}_{t+1}}{\tilde{\lambda}_t \gamma_{t+1}} \left[r_{t+1}^k \upsilon_{t+1} - u\left(\upsilon_{t+1}\right) + (1-\delta) q_{t+1}\right] \tag{56}$$

$$1 = q_t \left\{ 1 - \frac{\phi_I}{2} \left(\frac{\tilde{\imath}_t \gamma_t}{\tilde{\imath}_{t-1}} - \gamma \right)^2 - \phi_I \left(\frac{\tilde{\imath}_t \gamma_t}{\tilde{\imath}_{t-1}} - \gamma \right) \frac{\tilde{\imath}_t \gamma_t}{\tilde{\imath}_{t-1}} \right\} + \beta \frac{\tilde{\lambda}_{t+1}}{\tilde{\lambda}_t \gamma_{t+1}} q_{t+1} \phi_I \left(\frac{\tilde{\imath}_{t+1} \gamma_{t+1}}{\tilde{\imath}_t} - \gamma \right) \left(\frac{\tilde{\imath}_{t+1} \gamma_{t+1}}{\tilde{\imath}_t} \right)^2 \tag{57}$$

$$mc_t = \frac{\tilde{w}_t}{\alpha \frac{\tilde{y}_t}{h_t}} = \frac{\tilde{w}_t h_t}{\alpha \tilde{y}_t} \tag{58}$$

$$mc_t = \frac{r_t^k \upsilon_t}{\alpha \frac{\tilde{y}_t \gamma_t}{\tilde{k}_{t-1}}} \tag{59}$$

$$\frac{d_t}{\left(\tilde{c}_t - b \frac{\tilde{c}_{t-1}}{\tilde{\gamma}_t} \right)^{\sigma_c}} - E_t \frac{\beta b d_{t+1}}{\left(\tilde{c}_{t+1} \tilde{\gamma}_{t+1} - h \tilde{c}_t \right)^{\sigma_c}} = \tilde{\lambda}_t$$

$$\tilde{\lambda}_t = \beta E_t \left\{ \frac{\tilde{\lambda}_{t+1}}{\tilde{\gamma}_{t+1}^{\sigma_C}} \frac{r_t^h}{\pi_{t+1}^c} \right\} \tag{60}$$

$$\tilde{v}_{1,t} = \frac{1}{\lambda_w} \left(\tilde{w}_t^{new} \right)^{-\frac{1}{\lambda_w - 1}} \tilde{\lambda}_t \tilde{w}_t^{\frac{\lambda_w}{\lambda_w - 1}} h_t^d + \beta \xi_w E_t \left(\frac{\pi_t^{\kappa_w} \bar{\pi}_{t+1}^{1-\kappa_w} \gamma}{\pi_{t+1}} \right)^{\frac{1}{1-\lambda_w}} \left(\frac{\tilde{w}_{t+1}^{new} \tilde{\gamma}_{t+1}}{\tilde{w}_t^{new}} \right)^{\frac{1}{\lambda_w - 1}} \tilde{v}_{1,t+1} \tilde{\gamma}_{t+1}^{1-\sigma_C} \tag{61}$$

$$\tilde{v}_{1,t} = \psi_t \left(\frac{\tilde{w}_t}{\tilde{w}_t^{new}} \right)^{\frac{(1+\varphi)\lambda_w}{\lambda_w - 1}} \left(h_t^d \right)^{1+\varphi} + \beta \xi_w E_t \left(\frac{\pi_t^{\kappa_w} \bar{\pi}_{t+1}^{1-\kappa_w} \gamma}{\pi_{t+1}} \right)^{\frac{(1+\varphi)\lambda_w}{1-\lambda_w}} \left(\frac{\tilde{w}_{t+1}^{new} \tilde{\gamma}_{t+1}}{\tilde{w}_t^{new}} \right)^{\frac{(1+\varphi)\lambda_w}{\lambda_w - 1}} \tilde{v}_{1,t+1} \tilde{\gamma}_{t+1}^{1-\sigma_C} \tag{62}$$

$$\tilde{w}_t^{\frac{1}{1-\lambda_w}} = \xi_w \left(\frac{\pi_{t-1}^{\kappa_w} \bar{\pi}_t^{1-\kappa_w} \gamma}{\pi_t} \right)^{\frac{1}{1-\lambda_w}} \left(\frac{\tilde{w}_{t-1}}{\tilde{\gamma}_t} \right)^{\frac{1}{1-\lambda_w}} + (1 - \xi_w) \left(\tilde{w}_t^{new} \right)^{\frac{1}{1-\lambda_w}} \tag{63}$$

$$\tilde{f}_{1,t} = \tilde{\lambda}_t mc_t \tilde{y}_t^d + \beta \xi_y E_t \left(\frac{\pi_t^{\kappa_w} \bar{\pi}_{t+1}^{1-\kappa_w}}{\pi_{t+1}} \right)^{-\frac{\lambda_y}{\lambda_y - 1}} \tilde{f}_{1,t+1} \tilde{\gamma}_{t+1}^{1-\sigma_C} \tag{64}$$

$$\tilde{f}_{2,t} = \tilde{\lambda}_t \ddot{\pi}_t \tilde{y}_t^d + \beta \xi_d E_t \left(\frac{\pi_t^{\kappa_w} \bar{\pi}_{t+1}^{1-\kappa_w}}{\pi_{t+1}} \right)^{-\frac{1}{\lambda_y - 1}} \left(\frac{\ddot{\pi}_t}{\ddot{\pi}_{t+1}} \right) \tilde{f}_{2,t+1} \tilde{\gamma}_{t+1}^{1-\sigma_C} \tag{65}$$

$$0 = \lambda_y \tilde{f}_{1,t} - \tilde{f}_{2,t} \tag{66}$$

$$\frac{\tilde{b}_t^S}{\varepsilon_t^{b^S}} + \frac{\bar{b}_t^L}{\varepsilon_t^{\bar{b}^L}} + \tilde{T}_t = \frac{r_{t-1}^S}{\pi_t^c} \frac{\tilde{b}_{t-1}^S}{\tilde{\gamma}_t} + \frac{r_t^L}{\pi_t^c} \frac{p_{L,t}}{p_{L,t-1}} \frac{\bar{b}_{t-1}^L}{\tilde{\gamma}_t} - \tilde{G}_t \tag{67}$$

$$\tilde{y}_t^d = \tilde{c}_t + \tilde{G}_t + \tilde{\imath}_t \tag{68}$$

$$\frac{r_t^h}{\varepsilon_t^{b^S}} = r_t^S - x \left[\delta_t^B - \vartheta \delta_{t-1}^B - (1 - \vartheta) \delta^B \right] \frac{1}{\bar{b}_{\kappa,t}^L} \tag{69}$$

$$E_t \left\{ r_{t+1}^L \frac{p_{L,t+1}}{p_{L,t}} \right\} = \frac{r_t^h}{\varepsilon_t^{\bar{b}^L}} - x \left[\delta_t^B - \vartheta \delta_{t-1}^B - (1 - \vartheta) \delta^B \right] \frac{\tilde{b}_{\kappa,t}^S}{\left(\bar{b}_{\kappa,t}^L \right)^2} \tag{70}$$

4.9 Steady states

Household's Euler equation implies

$$r^h = \frac{\tilde{\gamma}^{\sigma_C}\pi^c}{\beta} \tag{71}$$

Household's effective interest rate spread is given by financial intermediary first order condition

$$\frac{r^h}{r^S} = r^S = r^L$$

Set $y = 1$ and $l = 1/3$

$$k = \left(\frac{\tilde{y}}{zh^{\alpha}}\right)^{\frac{1}{1-\alpha}} \tag{72}$$

$$i = \delta k \tag{73}$$

$$c = (1-g)\,y - i \tag{74}$$

$$\tilde{\lambda} = \frac{d\left(\tilde{\gamma}^{\sigma_c} - \beta b\right)}{\left(\tilde{c}\tilde{\gamma} - b\tilde{c}\right)^{\sigma_c}} \tag{75}$$

$$\begin{aligned} mc &= \frac{wh}{\alpha y} = \frac{1}{\lambda_p} \\ w &= \frac{\alpha y}{h\lambda_p} \end{aligned} \tag{76}$$

$$\tilde{w}^{new} = w \tag{77}$$

$$\tilde{v}_1 = \frac{\frac{1}{\lambda_w}\tilde{\lambda}\tilde{w}h}{1 - \beta\xi_w\tilde{\gamma}^{1-\sigma_C}} \tag{78}$$

$$\psi = \frac{\left(1 - \beta\xi_w\tilde{\gamma}^{1-\sigma_C}\right)}{h^{1+\varphi}} \tag{79}$$

$$\tilde{f}_1 = \frac{\tilde{\lambda}mc\tilde{y}}{1 - \beta\xi_y\tilde{\gamma}^{1-\sigma_C}} \tag{80}$$

$$\tilde{f}_2 = \frac{\tilde{\lambda}\bar{\pi}\tilde{y}}{1 - \beta\xi_y\tilde{\gamma}^{1-\sigma_C}} \tag{81}$$

4.10 Linearised Equations

$$\hat{r}_t^h = \hat{r}_t^S + \hat{\varepsilon}_t^{b^S} - \tilde{x}\left(\hat{\delta}_t^B - \vartheta\hat{\delta}_{t-1}^B\right) \tag{82}$$

$$\hat{r}_{t+1}^L + \hat{p}_{L,t+1} - \hat{p}_{L,t} = \hat{r}_t^h - \hat{\varepsilon}_t^{\bar{b}^L} - \frac{\tilde{x}}{\delta}\left(\hat{\delta}_t^B - \vartheta\hat{\delta}_{t-1}^B\right) \tag{83}$$

$$\hat{\delta}_t^B = \hat{b}_t^S - \hat{\bar{b}}_t^L$$

$$\hat{b}_t^S + \frac{\hat{\bar{b}}_t^L}{\delta^B} = \frac{1}{\beta}\left(\hat{b}_{t-1}^S + \frac{\hat{\bar{b}}_{t-1}^L}{\delta^B} - \left(1+\frac{1}{\delta^B}\right)\hat{\gamma}_t\right) - \frac{T}{b^S}\hat{T}_t + \frac{1}{\beta}\left\{\hat{r}_{t-1}^S - \hat{\pi}_t^c + \frac{1}{\delta^B}\left(\hat{r}_t^L - \hat{\pi}_t^c + \hat{p}_{L,t} - \hat{p}_{L,t-1}\right)\right\} - \frac{gy}{b^S}\left(\hat{g}_t + \hat{y}_t\right) \tag{84}$$

$$\frac{T}{b^S}\hat{T}_t = \theta\left(\hat{b}_{t-1}^S + \frac{\hat{\bar{b}}_{t-1}^L}{\delta^B} - (1+\delta)\hat{\gamma}_t\right) \tag{85}$$

$$mc_t = \hat{w}_t + \hat{h}_t - \hat{y}_t \tag{86}$$

$$mc_t = \hat{w}_t + \hat{h}_t - \hat{y}_t$$

$$\hat{\lambda}_t = \hat{\lambda}_{t+1} - \sigma_C\hat{\gamma}_{t+1} + r_t^h - \hat{\pi}_{t+1} \tag{87}$$

$$\frac{\gamma^{\sigma_C} - \beta b}{\gamma^{\sigma_C}}\hat{\lambda}_t = \left[\hat{d}_t - \frac{\beta\gamma}{\gamma^{\sigma_C}}\hat{d}_{t+1}\right] - \frac{\sigma_C}{1-\frac{b}{\gamma}}\left\{\begin{array}{c}\left(1+\frac{\beta b^2}{\gamma^{1+\sigma_C}}\right)\hat{c}_t - \frac{b}{\gamma}\left(\hat{c}_{t-1} - \hat{\gamma}_t\right) \\ -\frac{\beta b}{\gamma^{\sigma_C}}\left(\hat{c}_{t+1} + \hat{\gamma}_{t+1}\right)\end{array}\right\} \tag{88}$$

$$\begin{aligned}\left[\Delta\hat{w}_t + \hat{\gamma}_t + \hat{\pi}_t - \kappa_w\hat{\pi}_{t-1} + (1-\kappa_w)\bar{\pi}_t\right] &= \beta\gamma^{1-\sigma_c}\left[\Delta\hat{w}_{t+1} + \hat{\gamma}_{t+1} + \hat{\pi}_{t+1} - \kappa_w\hat{\pi}_t - (1-\kappa_w)\bar{\pi}_{+1}\right] \\ &+ \frac{\left(1-\beta\xi_w\gamma^{1-\sigma_c}\right)(1-\xi_w)\left[\lambda_w - 1\right]}{\left[(\varphi+1)\lambda_w - 1\right]\xi_w}\left\{\hat{\psi}_t + \varphi\hat{h}_t - \hat{\lambda}_t - \hat{w}_t\right\} \\ &+ \hat{\bar{\lambda}}_{w,t}\end{aligned}$$

$$\hat{y}_t = \hat{z}_t + \alpha\hat{h}_t + (1-\alpha)\hat{k}_t \tag{89}$$

$$\hat{k}_t = \hat{v}_t + \bar{k}_{t-1} - \hat{\gamma}_t \tag{90}$$

$$\hat{v}_t = \frac{1}{\psi_v}\hat{r}_t^k \tag{91}$$

$$\bar{k}_t = \left(1-\frac{i}{\bar{k}}\right)\left(\bar{k}_{t-1} - \hat{\gamma}_t\right) + \frac{i}{\bar{k}}\hat{\imath}_t + \left(1-\frac{1-\delta}{\gamma}\right)\left(1+\beta\gamma^{1-\sigma_C}\right)\gamma^2\psi_I\hat{\phi}_t$$

$$\hat{p}_{L,t} = -\frac{r}{r-\kappa}\hat{r}_t^L \tag{92}$$

$$\bar{b}_t^L = \hat{p}_{L,t} + \hat{b}_t^L \tag{93}$$

$$\hat{y}_t = \frac{c}{(1-g)\,y}\hat{c}_t + \frac{g}{1-g}g_t + \frac{i}{(1-g)\,y}\hat{\imath}_t + \frac{r^k k}{(1-g)\,y}\hat{v}_t \tag{94}$$

$$\begin{aligned}\hat{\pi}_t - \hat{\bar{\pi}}_t &= \frac{\beta}{1+\beta\kappa_y}E_t\left(\hat{\pi}_{t+1} - \hat{\bar{\pi}}_{t+1}\right) + \frac{\kappa_y}{1+\beta\kappa_y}\left(\hat{\pi}_{t-1} - \hat{\bar{\pi}}_t\right) - \frac{\beta\kappa_y}{1+\beta\kappa_y}\left(\hat{\bar{\pi}}_t^c - \hat{\bar{\pi}}_{t+1}\right) \\ &+ \frac{\left(1-\xi_y\right)\left(1-\beta\xi_y\right)}{\xi_y\left(1+\beta\kappa_y\right)}\widehat{mc}_t + \lambda_{y,t}\end{aligned} \tag{95}$$

4.11 Estimation

A number of parameters (Table 1) is decided prior to the estimation of the model. The time discount factor ($\beta = 0.99$), the steady state value of the productivity growth ($\gamma = 1$) and the inflation target

($\bar{\pi} = 1$) imply that the steady state value of the interest rate is 4%. We follow the literature and assume log consumption preferences ($\sigma_C = 1$,Justiniano *et al.* (2010)). Similar to Smets and Wouters (2007) steady-state price and wage markup are set equal to 20% and 10% respectively. The share of capital in the production (α) and its depreciation rate (δ) have been calibrated to 0.36 and 0.025, numbers typically used in the literature (Christiano *et al.* (2005), Trabandt and Uhlig (2011) and Jermann and Quadrini (2012)). δ_B has been set equal to 1 to match the steady state short- to long-term debt ratio in the data (see De Graeve and Theodoridis (2016)). As in Smets and Wouters (2007),Leeper *et al.* (2010), and Traum and Yang (2011) the steady-state government spending to GDP ratio (g), short-term to GDP ration $\left(\frac{b^S}{y}\right)$ and the lump-sum tax response coefficient to debt (θ) equal 0.18, 0.65 and 0.025, respectively.

Table 1: Calibrated Parameters

Mnemonic	Description	Value
β	Time Discount Factor	0.990
γ	Steady State Productivity Growth	1.000
$\bar{\pi}$	Steady State Inflation Target	1.000
σ_C	Inverse Intertemporal Substitution Elasticity	1.000
λ_y	Steady State Price Markup	1.200
λ_w	Steady State Wage Markup	1.100
θ	Tax Response to Debt	0.025
α	Production Capital Share	0.360
δ	Capital Depreciation Rate	0.025
$\frac{1}{\delta^B}$	Steady State Long to Short Term Debt Ratio	1.000
h	Steady State Hours	0.333
g	Steady State Government Spending to GDP Ratio	0.180
$\frac{b^S}{y}$	Steady State Short Term Debt to GDP Ratio	0.650

Table 2 summarises the prior moments and density functions of the structural parameter vector estimated. The same prior moments are employed for the estimation of both full and limited information models. These moments are those employed by Smets and Wouters (2007).

Tables 3 and 4 report the posterior moments of the Full and Limited Information models respectively. Both set of estimates are very similar with each other but also with those in the literature.

References

Andres, Javier, J. David Lopez-Salido and Edward Nelson, 2004, Tobin's imperfect asset substitution in optimizing general equilibrium, *Journal of Money, Credit, and Banking* **36**(4), 665–90.

Banbura, Marta, Domenico Giannone and Lucrezia Reichlin, 2010, Large Bayesian vector auto regressions, *Journal of Applied Econometrics* **25**(1), 71–92.

Carter, C. K. and R. Kohn, 1994, On Gibbs Sampling for State Space Models, *Biometrika* **81**(3), pp. 541–553.

Chen, Han, Vasco Curdia and Andrea Ferrero, 2012, The Macroeconomic Effects of Large-scale Asset Purchase Programmes, *Economic Journal* **122**(564), F289–F315.

Christiano, Lawrence, Martin Eichenbaum and Charles Evans, 2005, Nominal Rigidities and the Dynamic Effects of a shock to Monetary Policy, *Journal of Political Economy* **113**, 1–45.

De Graeve, Ferre and Konstantinos Theodoridis, 2016, Forward Guidance, Quantitative Easing, or both?, *Working Paper Research 305*, National Bank of Belgium.

De Graeve, Ferre, Marina Emiris and Raf Wouters, 2009, A structural decomposition of the US yield curve, *Journal of Monetary Economics* **56**(4), 545–559.

Erceg, Christopher J., Dale W. Henderson and Andrew T. Levin, 2000, Optimal monetary policy with staggered wage and price contracts, *Journal of Monetary Economics* **46**(2), 281–313.

Forni, Mario and Luca Gambetti, 2014, Sufficient information in structural {VARs}, *Journal of Monetary Economics* **66**, 124 – 136.

Harrison, Richard, 2012, Asset purchase policy at the effective lower bound for interest rates, *Bank of England working papers 444*, Bank of England.

Jermann, Urban and Vincenzo Quadrini, 2012, Macroeconomic Effects of Financial Shocks, *American Economic Review* **102**(1), 238–71.

Justiniano, Alejandro, Giorgio Primiceri and Andrea Tambalotti, 2010, Investment shocks and business cycles, *Journal of Monetary Economics* **57**(2), 132–45.

Kurmann, Andr? and Christopher Otrok, 2013, News Shocks and the Slope of the Term Structure of Interest Rates, *American Economic Review* **103**(6), 2612–2632.

Leeper, Eric M., Michael Plante and Nora Traum, 2010, Dynamics of fiscal financing in the United States, *Journal of Econometrics* **156**(2), 304–321.

Liu, Philip, Haroon Mumtaz, Konstantinos Theodoridis and Francesco Zanetti, 2014, Changing Macroeconomic Dynamics at the Zero Lower Bound, *mimeo*.

Schmitt-Grohe, Stephanie and Martin Uribe, 2003, Closing small open economy models, *Journal of International Economics* **61**(1), 163–185.

Smets, Frank and Rafael Wouters, 2007, Shocks and Frictions in US Business Cycles: a Bayesian DSGE Approach, *American Economic Review* **97**, 586–606.

Trabandt, Mathias and Harald Uhlig, 2011, The Laffer curve revisited, *Journal of Monetary Economics* **58**(4), 305 – 327.

Traum, Nora and Shu-Chun S. Yang, 2011, Monetary and fiscal policy interactions in the post-war U.S, *European Economic Review* **55**(1), 140–164.

Uhlig, Harald, 2004a, DO TECHNOLOGY SHOCKS LEAD TO A FALL IN TOTAL HOURS WORKED?, *Journal of the European Economic Association* **2**(2-3), 361–371.

Uhlig, Harald, 2004b, What moves GNP?, *Econometric Society 2004 North American Winter Meetings 636*, Econometric Society.

Woodford, Michael, 2001, Fiscal Requirements for Price Stability, *Journal of Money, Credit and Banking* **33**(3), 669–728.

Table 2: Estimated Parameters: Prior Moments

Mnemonic	Description	Density	Mean	STD
σ_L	Inverse Labour Supply Elasticity	Normal	1.50	0.25
χ	Liquidity Adjustment Cost	Normal	5.00	0.25
b	Habit Smoothing	Beta	0.75	0.05
κ_y	Price Indexation	Beta	0.50	0.15
ξ_y	Calvo Price Reset Probability	Beta	0.50	0.10
κ_w	Wage Indexation	Beta	0.50	0.15
ξ_w	Calvo Wage Reset Probability	Beta	0.50	0.10
ϕ_R	Policy Smoothing	Beta	0.75	0.10
ϕ_π	Inflation Policy Response	Normal	1.50	0.10
ϕ_y	Output Policy Response	Normal	0.12	0.05
ψ_ν	Capital Utilisation Cost	Normal	5.00	0.50
ϕ_I	Investment Adjustment Cost	Normal	4.00	0.50
ρ_γ	AR Persistence Non Stationary Productivity Process	Beta	0.50	0.20
ρ_{b^S}	AR Persistence of Short Term Debt Supply Process	Beta	0.50	0.20
ρ_{λ_y}	AR Persistence Price Markup Process	Beta	0.50	0.20
σ_{λ_y}	STD Price Markup Process	Beta	0.50	0.20
ρ_{λ_W}	AR Persistence Wage Markup Process	Beta	0.50	0.20
μ_{λ_W}	MA Persistence Wage Markup Process	Beta	0.50	0.20
$\rho_{\bar{\pi}}$	AR Persistence Inflation Target Process	Beta	0.50	0.20
ρ_m	AR Persistence Monetary Policy Process	Beta	0.50	0.20
ρ_g	AR Persistence Government Spending Process	Beta	0.50	0.20
ρ_ϕ	AR Persistence Investment Specific Process	Beta	0.50	0.20
σ_γ	STD Non Stationary Productivity Process	Inv-Gamma	0.10	2.00
σ_{b^S}	STD of Short Term Debt Supply Process	Inv-Gamma	0.10	2.00
μ_{λ_y}	MA Persistence Price Markup Process	Inv-Gamma	0.10	2.00
σ_{λ_W}	STD Persistence Wage Markup Process	Inv-Gamma	0.10	2.00
$\sigma_{\bar{\pi}}$	STD Inflation Target Process	Inv-Gamma	0.10	2.00
σ_{r^S}	STD Monetary Policy	Inv-Gamma	0.10	2.00
σ_g	STD Government Spending Process	Inv-Gamma	0.10	2.00
σ_ϕ	STD Investment Specific Process	Inv-Gamma	0.10	2.00

Notes: STD denotes the standard deviation and Inv-Gamma the inverse gamma distribution.

Table 3: Full Information Model Estimated Parameters: Posterior Moments

Mnemonic	Description	Mode	5th	95th
σ_L	Inverse Labour Supply Elasticity	1.53	0.83	1.81
χ	Liquidity Adjustment Cost	4.05	4.12	5.40
b	Habit Smoothing	0.71	0.71	0.76
κ_y	Price Indexation	0.92	0.78	0.92
ξ_y	Calvo Price Reset Probability	0.89	0.89	0.92
κ_w	Wage Indexation	0.75	0.32	0.84
ξ_w	Calvo Wage Reset Probability	0.46	0.46	0.91
ϕ_R	Policy Smoothing	0.74	0.76	0.87
ϕ_π	Inflation Policy Response	1.90	1.87	2.11
ϕ_y	Output Policy Response	0.14	0.13	0.31
ψ_ν	Capital Utilisation Cost	5.17	4.21	5.84
ϕ_I	Investment Adjustment Cost	5.45	4.63	5.99
ρ_γ	AR Persistence Non Stationary Productivity Process	0.29	0.14	0.39
ρ_{b^S}	AR Persistence of Short Term Debt Supply Process	0.86	0.33	0.85
ρ_{λ_y}	AR Persistence Price Markup Process	0.89	0.84	0.94
σ_{λ_y}	STD Price Markup Process	1.00	0.99	1.00
ρ_{λ_W}	AR Persistence Wage Markup Process	0.95	0.71	0.97
μ_{λ_W}	MA Persistence Wage Markup Process	0.83	0.60	0.87
$\rho_{\bar{\pi}}$	AR Persistence Inflation Target Process	0.17	0.15	0.57
ρ_m	AR Persistence Monetary Policy Process	0.41	0.12	0.41
ρ_g	AR Persistence Government Spending Process	0.95	0.95	1.00
ρ_ϕ	AR Persistence Investment Specific Process	0.96	0.94	0.97
σ_γ	STD Non Stationary Productivity Process	1.03	0.94	1.13
σ_{b^S}	STD of Short Term Debt Supply Process	0.99	1.12	4.00
μ_{λ_y}	MA Persistence Price Markup Process	0.17	0.14	0.18
σ_{λ_W}	STD Persistence Wage Markup Process	1.08	0.90	1.15
$\sigma_{\bar{\pi}}$	STD Inflation Target Process	0.10	0.10	0.12
σ_{r^S}	STD Monetary Policy	0.23	0.20	0.24
σ_g	STD Government Spending Process	0.59	0.55	0.65
σ_ϕ	STD Investment Specific Process	0.31	0.29	0.39

Notes: The columns 5th and 95th refer to the 5th and 95th percentiles of the posterior distribution of the structural parameter vector.

Table 4: Limited Information Model Estimated Parameters: Posterior Moments

Mnemonic	Description	Mode	5th	95th
σ_L	Inverse Labour Supply Elasticity	1.57	1.19	1.99
χ	Liquidity Adjustment Cost	4.32	3.92	4.76
b	Habit Smoothing	0.65	0.62	0.68
κ_y	Price Indexation	0.89	0.81	0.92
ξ_y	Calvo Price Reset Probability	0.70	0.66	0.74
κ_w	Wage Indexation	0.79	0.61	0.91
ξ_w	Calvo Wage Reset Probability	0.29	0.23	0.36
ϕ_R	Policy Smoothing	0.75	0.69	0.80
ϕ_π	Inflation Policy Response	1.91	1.88	1.96
ϕ_y	Output Policy Response	0.09	0.05	0.12
ψ_ν	Capital Utilisation Cost	5.34	4.53	6.03
ϕ_I	Investment Adjustment Cost	4.52	3.89	5.28
ρ_γ	AR Persistence Non Stationary Productivity Process	0.22	0.12	0.34
ρ_{b^S}	AR Persistence of Short Term Debt Supply Process	0.86	0.82	0.88
ρ_{λ_y}	AR Persistence Price Markup Process	0.98	0.95	0.99
σ_{λ_y}	STD Price Markup Process	0.64	0.48	0.73
ρ_{λ_W}	AR Persistence Wage Markup Process	0.96	0.92	0.99
μ_{λ_W}	MA Persistence Wage Markup Process	0.73	0.59	0.83
$\rho_{\bar{\pi}}$	AR Persistence Inflation Target Process	0.11	0.04	0.23
ρ_m	AR Persistence Monetary Policy Process	0.39	0.27	0.54
ρ_g	AR Persistence Government Spending Process	0.95	0.92	0.97
ρ_ϕ	AR Persistence Investment Specific Process	0.95	0.92	0.97
σ_γ	STD Non Stationary Productivity Process	1.02	0.94	1.12
σ_{b^S}	STD of Short Term Debt Supply Process	0.92	0.79	1.10
μ_{λ_y}	MA Persistence Price Markup Process	0.16	0.14	0.18
σ_{λ_W}	STD Persistence Wage Markup Process	1.22	1.06	1.36
$\sigma_{\bar{\pi}}$	STD Inflation Target Process	0.10	0.10	0.11
σ_{r^S}	STD Monetary Policy	0.24	0.23	0.27
σ_g	STD Government Spending Process	0.59	0.54	0.65
σ_ϕ	STD Investment Specific Process	0.31	0.28	0.35

Notes: The columns 5th and 95th refer to the 5th and 95th percentiles of the posterior distribution of the structural parameter vector.

School of Economics and Finance

Queen Mary
University of London

This working paper has been produced by
the School of Economics and Finance at
Queen Mary University of London

Copyright © 2017 Haroon Mumtaz and
Konstantinos Theodoridis. All rights reserved

School of Economics and Finance
Queen Mary University of London
Mile End Road
London E1 4NS
Tel: +44 (0)20 7882 7356
Fax: +44 (0)20 8983 3580
Web: www.econ.qmul.ac.uk/research/workingpapers/