

Equality of *When*?

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Abstract

This paper analyses the temporal unit of egalitarian concern. In the intertemporal context, the differences between egalitarian views can be appreciated not only in inequality analysis but also as regards the ideal egalitarian distribution to be established. In this paper, three intergenerational egalitarian principles (*Complete Lives Egalitarianism*, *Corresponding Segments Egalitarianism* and *Simultaneous Segments Egalitarianism*) are analysed and *CSE* is argued to be the appropriate egalitarian benchmark. The relations between the three principles and other moral ideals, namely maximin and utilitarianism, are also analysed. It is proved that *CLE* and *CSE* are compatible with a concern for the worst off and, partially, with a utilitarian concern, while the adoption of *SSE* implies a worse trade-off between egalitarianism and the other moral ideals.

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1 Introduction

The publication of John Rawls' *A Theory of Justice* [22] has marked a renewal of interest in egalitarian theories, and has sparked a large literature. One of the central issues of contention has been the choice of the appropriate equalisandum: income, wealth, utility, primary goods or, more recently, capabilities, functionings, and opportunities. As Amartya Sen [27] forcefully put it, one of the central questions for egalitarians raised by Rawls' theory, and by his forceful criticism of utility as the relevant equalisandum, is *equality of what?*

The analysis of the appropriate unit of egalitarian concern - and indeed other debates on the foundations of egalitarian theory following Rawls' seminal contribution - has been carried out in the main within the confines of an essentially static environment. The choice of the appropriate equalisandum, for example, has been explored abstracting from the time structure of individual lives and focusing on a single generation (e.g., Sen [27]; Dworkin [9, 10]; Arneson [1]; Cohen [6]; Roemer [23]). Similarly, the analysis of different measures of inequality has typically focused on the distribution of the relevant variable in a single period (e.g., Sen [26, 28]; Temkin [31]).

To be sure, time and dynamics have played a significant role in egalitarian debates. Many empirical studies have analysed the dynamics of inequality over time or across generations, and the intergenerational and intertemporal impact of different distributive policies. Theoretically, a large literature has emerged on intergenerational justice and the evaluation of infinite utility streams,¹ and the issues that a dynamic approach poses to egalitarianism, including the trade-offs between distribution and growth, have been highlighted almost immediately after the publication of *A Theory of Justice*.²

Yet, even when distributive dynamics, growth and time have been considered, the complex economic and philosophical implications of the fact that agents' lives develop over time have often been overlooked by focusing either on the analysis of a sequence of time slices (e.g., in the analysis of the evolution of income distribution over time) or by implicitly assuming that the relevant unit of distributive concern can be reduced to a single variable measuring the lifetime attainment of the relevant equalisandum. As Milton Friedman [11, p.38] famously put it, "the identification of low measured income with 'poor' and high measured income with 'rich' is justified only if measured income can be regarded as an estimate

¹For a discussion see Mariotti and Veneziani [16] and Lombardi et al. [15].

²See, for example, the classic papers by Arrow [2] and Dasgupta [8]. For a more recent discussion see Roemer and Veneziani [24, 25].

of expected income over a lifetime or a large fraction thereof”. A similar view is held by most political and moral philosophers: “the subject of an egalitarian principle is not the distribution of particular rewards to individuals at some time, but the prospective quality of their lives as a whole, from birth to death.” (Nagel [21, p.69]).

In a seminal article, Dennis McKerlie [17] questioned the standard view - defined *complete lives egalitarianism (CLE)* - according to which agents’ lives, taken as a whole, are the proper unit of egalitarian concern. For, McKerlie argued, *CLE* is insensitive to inequalities occurring within particular segments of agents’ lives, no matter how severe such inequalities are, provided they cancel out over the course of agents’ lives. He suggested that egalitarians focus instead on inequalities occurring between contemporaries at a given time - a view called *simultaneous segments egalitarianism (SSE)*. More generally, McKerlie [17] argued that because agents’ lives extend over time, a sound egalitarian analysis requires the definition of the proper unit of egalitarian concern, i.e. whole lives or selected parts of them. Egalitarian principles based on different temporal units incorporate different moral concerns and have different policy implications.

The issues raised by McKerlie [17] are deep, go beyond the boundaries of liberal egalitarian approaches, and have sparked a growing literature across the disciplinary borders of philosophy and economics.³ Veneziani [32, 33] has shown, for example, that the temporal structure of individual lives has relevant implications in exploitation theory: a capitalist economy in which individuals switch roles over time so that everyone is exploited for an equal amount of time is not necessarily a just economy. Moreover, according to various authors, the fact that individual lives are structured into normatively relevant segments raises difficult, if not intractable issues for egalitarianism.

In a series of contributions, for example, Temkin [30, 31] has analysed the three main intertemporal egalitarian principles, namely *CLE*, *SSE* and also *corresponding segments egalitarianism (CSE)*, which holds that inequalities must be measured between corresponding stages of agents’ lives - e.g., childhood, middle age, old age, etc. According to him, in the analysis of inequalities no principle is entirely satisfactory: “several views are possible, each of which seems plausible in some cases and implausible in others” (Temkin [31, p.291]). Indeed, the absence of a single approach properly capturing egalitarian views suggests that we should reconsider our intuitions and acknowledge that equality is inevitably

³See, for example, Temkin [30, 31]; Kappel [13]; McKerlie [18, 19, 20]; Huemer [12]; Carter [5]; Attas [3]; and Bidanure [4].

a complex notion. So much so that it may sometimes be difficult to formulate clear and consistent egalitarian considered judgements.⁴

Kappel [13] has taken Temkin's [30, 31] conclusion further, suggesting that we should actually abandon egalitarianism. In his view, none of the egalitarian views is completely satisfactory because it is normatively irrelevant and even misleading to compare the relative attainments of the relevant variable across individuals. Once we consider the temporal structure of individual lives, it becomes clear that "what matters in our egalitarian judgement is giving priority to the worse off" (Kappel [13, p.204]).

More radically, Huemer [12] has argued that the conflicting intuitions arising in the intertemporal context reveal a deep conceptual problem for egalitarianism. According to him, once we acknowledge that individual lives are divided into temporal stages, egalitarianism cannot be coherently defined: intuitions concerning complete lives and about temporal segments inevitably conflict leading to inconsistencies.

This paper analyses the issues that time and the temporal structure of individual lives pose in normative economics, and in particular for egalitarianism. To be specific, we reconsider the three main intertemporal egalitarian principles proposed in the literature in order to identify the appropriate *temporal unit of egalitarian concern* - that is, to answer the question posed in the title: equality of *when*?⁵ We argue that, although critics have highlighted some important and often neglected issues, their arguments do not pose any intractable problems for egalitarians.⁶ It may be true that no principle fully captures our egalitarian intuitions when evaluating inequalities, but this does not mean that egalitarianism is incoherent or that no satisfactory egalitarian benchmark can be defined.

Indeed, we argue that an important distinction has been overlooked in the literature, which is a peculiar feature of the intertemporal context. Unlike in the static setting, apart from differing in the analysis of *unequal* distributions, intertemporal egalitarian principles also define different *egalitarian states* to reach.

⁴A similar point is made by Attas [3], who argues that the temporal structure of agents' lives creates some fundamental indeterminacies for egalitarian principles in the Rawlsian tradition which makes them virtually empty.

⁵To be sure, *CLE*, *CSE* and *SSE* do not exhaust the set of logically conceivable intertemporal egalitarian views. Yet they do capture the most common egalitarian considered judgements. Lippert-Rasmussen [14], for example, considers also *ranking-order segments egalitarianism* according to which egalitarian evaluations should be based on the comparison of equivalent rank-order segments of individual lives - that is, the best segment, the second-best segment, and so on - in terms of the attainment of the relevant variable. However, the ethical foundations of this approach are rather unclear.

⁶We focus in particular on the arguments advanced by Temkin [30, 31] and Kappel [13]. For a thorough critical analysis of Huemer's [12] argument, see Carter [5].

The two issues are connected but they should be kept conceptually distinct in the choice of the appropriate principle. This is even more evident for policy purposes, - e.g., from the viewpoint of a government concerned with equality, - since the definition of the ideal “steady-state” egalitarian distribution and the design of the transition process to that state raise different problems. In order to implement an egalitarian strategy, in addition to a correct analysis of the status quo, it is necessary to define the appropriate egalitarian benchmark.

This paper focuses on the latter issue. Section 2 briefly reviews the main results of the existing literature on the properties of the three egalitarian views in the evaluation of unequal distributions. Then, the distinction between the evaluation of existing inequalities and the definition of the appropriate egalitarian distribution is introduced, and it is argued that, as regards the distribution to establish, *CSE* defines the appropriate intertemporal egalitarian benchmark.

Because the evaluation of a distribution, e.g., for policy purposes, is influenced by more than one normative concern, in section 3, a formal analysis of the trade-offs between the different egalitarian principles and other normative views is presented, which aims to provide a formal basis for *all things considered judgements* (Temkin [31]). The relations of *CLE*, *CSE*, and *SSE* with two non-primarily-egalitarian normative concerns, Rawls’s difference principle and utility,⁷ are analysed. A stylised model is set up, which generalises Arrow [2] and Dasgupta [8] by considering overlapping generations (OLG), so that at each date there are two types of individual, young and old, rather than a representative agent. This allows us to analyse *intertemporal* as well as *intra*temporal equality. It is proved that the maximin solution yields *CSE* and *CLE*, but not *SSE*, and if the assumptions of the model are relaxed, *CLE* remains the egalitarian principle that can best accommodate Rawlsian or utilitarian concerns, and it is easier to reconcile these concerns with *CSE* than with *SSE*.

Finally, it is worth noting that our formal analysis yields some interesting insights on a vexed issue in normative economics, namely the well-known trade-off between Rawlsian distributive justice and growth. For we show that, once the temporal structure of individual lives is explicitly modelled, then contrary to Arrow’s [2] and Dasgupta’s [8] classic papers, the dynamics of the economy is *not* completely stationary, even though the application of the maximin principle precludes permanent growth. We briefly elaborate on this point in the concluding section. The proofs of all formal results are relegated to the Appendix.

⁷For a discussion of the relation between the maximin and egalitarianism, see Temkin [31].

2 Three Egalitarian Principles Compared

Let x be the relevant *equalisandum*, which shall be called ‘welfare’ (x could be income, utility, opportunities, primary goods, etc.). Assume that agents’ lives can be divided into an equal number T of well-defined periods of equal length. Let $X_{(T)} = \{\mathbf{x}_i^t = (x_{i1}^t, x_{i2}^t, \dots, x_{iT}^t), x_{ij}^t \in \mathbb{R}\}$ be the set of vectors describing the attainment of x_{ij}^t by agent i at date t in period j , $1 \leq j \leq T$ of her life. For the sake of simplicity, assume x to be interpersonally and intertemporally comparable, and additive along agents’ lives, so that $x_i = \sum_{j=1}^T x_{ij}^{t+j-1}$ is the lifetime attainment of x by agent i . These assumptions make the analysis comparable with Mc Kerlie [17] and Temkin [31] and they are quite natural if x is a variable such as income or an index of primary goods. On the other hand, if a subjective variable like utility is considered, these assumptions give the opportunity to compare the egalitarian principles *in vitro*, as a first step towards a more satisfactory and realistic analysis.

The three egalitarian principles can be interpreted as different ways of evaluating distributions of the \mathbf{x}_i^t vectors. Let D_1 , D_2 and D_3 denote inequality measures associated with *CLE*, *CSE*, and *SSE*, respectively. Formally, $D_y : X_{(T)} \times \dots \times X_{(T)} \rightarrow \mathbb{R}_+$ for $y = 1, 2, 3$. If $D_y = 0$, then a distribution is egalitarian according to the relevant principle $y = 1, 2, 3$. Given the definitions in section 1, $D_1 = 0$ if and only if $x_i = x_h$ for all agents i, h ; $D_2 = 0$ if and only if $x_{ij}^t = x_{hj}^\tau$ for all agents i, h , dates t, τ and corresponding life stages j and $D_3 = 0$ if and only if $x_{ij}^t = x_{hz}^t$ for all agents i, h , life stages j, z and simultaneous dates t .

In order to focus on the implications of the three egalitarian principles, - rather than on the features of specific measures, - no further restrictions are imposed on the D_y ’s. As in the static setting, where the problems of inequality measurement are reflected into the existence of several measures capturing different aspects of inequality (e.g., Gini index, Atkinson’s measure, etc.; see Temkin [31]) in principle there are many possible ways of measuring inequalities *according to each criterion*, that is, there are various specifications of every D_y . Actually, in the intertemporal context the choice of the appropriate inequality measure associated to each criterion is more complex, since the D_y ’s should rank distributions of *vectors* rather than distributions of real numbers.

As convincingly argued by Temkin [31], however, one of the specific features of intertemporal analysis is that, unlike in the atemporal context, even assuming a unique possible D_y associated to each principle, the issue of inequality measurement would not be solved: different egalitarian principles highlight different

kinds of inequalities and no principle, *CLE*, *CSE*, or *SSE*, seems completely satisfactory in the analysis of *unequal* distributions.

The main problem of *CLE* is that it leads to “changing places egalitarianism” (McKerlie [17]). If whole lives are the unit of egalitarian concern, in a “situation involving differential treatment of equally deserving people - no matter how significant, and even perverse those differing treatments are - there can be no egalitarian objection as long as the roles of the equally deserving people are interchanged so that each receives an equivalent share of the treatments meted out” (Temkin [31, p.236]). According to *CLE*, for example, an allocation where $\mathbf{x}_i^t = (5, 5, 5, 5)$, for each agent i , is equivalent to the following distribution:

Example 1.

$$\mathbf{x}_i^t = (10, 10, 0, 0), \text{ for } i = 1, 3, 5, \dots$$

$$\mathbf{x}_j^t = (0, 0, 10, 10), \text{ for } j = 2, 4, 6, \dots$$

CSE and *SSE* rule out changing places egalitarianism insofar as they do not allow present inequalities to compensate for past ones: in Example 1, both D_2 and D_3 would be different from zero. Yet, they do not represent entirely satisfactory alternatives in the evaluation of unequal distributions. By focusing only on inequalities in selected portions of agents’ lives, both principles can lead to the paradoxical conclusion that a distribution exhibiting changing places egalitarianism is as objectionable as one in which the agents’ roles are not interchanged and one agent is worse off in *every* relevant segment, as the next example shows:⁸

Example 2.

$$\mathbf{x}_i^t = (10, 10, 10, 10), \text{ for } i = 1, 3, 5, \dots$$

$$\mathbf{x}_j^t = (0, 0, 0, 0), \text{ for } j = 2, 4, 6, \dots$$

According to D_2 and D_3 , the latter distribution is as unequal as that in Example 1. Hence, Temkin [31, p.291] concludes that in the evaluation of existing inequalities, “several views are possible, each of which seems plausible in some cases and implausible in others”, and it may be opportune to use the information conveyed by *all* principles rather than adopting only one of them.

To be sure, it may be objected that the distribution in Example 1 *is* egalitarian and we find it intuitively objectionable because it fails on other criteria, e.g. fraternity (equality between contemporaries promotes feelings of solidarity),

⁸Consistent with the rest of the literature, we are assuming D_2 and D_3 to be additive. The basic arguments do not change if this assumption is relaxed.

or welfare (because of diminishing marginal utility).⁹ It is difficult to address this objection without more information on the nature of x and on the meaning of the welfare levels in Example 1. But, granting that information on the temporal structure of the attainment of x is at least *prima facie* relevant, then one may argue that the distribution in Example 1 is not equivalent to one in which $\mathbf{x}_i^t = (5, 5, 5, 5)$, for all i , and agents in Example 1 are not equally treated.

We need not adjudicate this issue here. For, even granting the relevance of changing places egalitarianism, we argue that this does not raise intractable problems for egalitarians. The key point to note is that Temkin’s [31] (and indeed Kappel’s [13]) arguments do not extend to the choice of the appropriate intertemporal *egalitarian benchmark*, which is quite a different issue from the analysis of past and present *inequalities*. In the static context, while the measurement of inequalities can be controversial, the definition of egalitarian states is uncontroversial: different inequality measures give the same answer if the distribution is egalitarian - that is, when all agents attain the same level of the relevant variable x .¹⁰ In contrast, in the intertemporal context, it is misleading to say that different views can “be regarded as built around ways of measuring the inequality between lives” (McKerlie [17, p.487]). The three principles stress different aspects of existing inequalities, but they also define different egalitarian states to reach, as shown by the fact that, unlike in the static context, $D_y = 0$ does not necessarily imply $D_{y'} = 0$, $y \neq y'$. The two issues are connected, but should be kept conceptually distinct in the choice of the appropriate egalitarian principle.¹¹

The difference between the two perspectives is particularly evident for policy purposes, since the definition of the ideal “steady-state” egalitarian distribution and the design of the transition process to that state raise different issues. In order to implement an egalitarian strategy, in addition to a correct analysis of the *status quo* (involving the evaluation of existing inequalities and claims for compensation of past ones), the proper intertemporal egalitarian benchmark must be defined.

This distinction has been largely overlooked in the literature, and so the scope of the conclusions reached is sometimes unclear. For example, McKerlie [17] discusses the choice of the egalitarian benchmark, but his arguments are based mainly on the analysis of the claims for compensation of past inequalities

⁹We are grateful to Robert Sugden for this suggestion.

¹⁰See, e.g., the measures discussed in Sen [26, 28] and in Temkin [31, chapter 5].

¹¹The distinction is also ignored in Lippert-Rasmussen’s [14] otherwise extremely insightful analysis.

implied by the different views. Similarly, Temkin’s [31] criticises *CLE* because it entails changing places egalitarianism but he finds *SSE* and *CSE* faulty based on the analysis of *unequal* distributions, which provides little information as to the features of the egalitarian distributions associated with them. In the rest of this section, we show that, once the distinction is rigorously drawn, it clearly emerges that even though no principle may fully capture our egalitarian intuitions when evaluating unequal distributions, this does not mean that egalitarianism is incoherent or that no satisfactory egalitarian benchmark can be defined.

Consider the three principles from the point of view of the distributions with $D_y = 0$, $y = 1, 2, 3$. As noted above, changing places egalitarianism may raise doubts on *CLE* as the intertemporal egalitarian benchmark. *SSE* is not a satisfactory alternative.

A first puzzling feature of *SSE* is reflected in the time dependence of D_3 and in particular in its sensitivity to small changes in the agents’ date of birth. In principle, *for given allocations* of x , it is sufficient a “slight” shift in the date of birth of an agent to change dramatically the value of D_3 and the egalitarian judgement. However, it is hard to see why if an agent is born, say, ten years later, or earlier, the judgement about an otherwise identical (and possibly *CL*- and *CS*-egalitarian) distribution should change. This is more evident the shorter the stages in which agents’ lives are divided.¹²

Second, according to *SSE*, *only* inequalities between contemporaries are ethically relevant, and therefore $D_3 = 0$ whenever agents’ lives do not overlap. However, let $T = 4$ and consider the following example.

Example 3.

$$\mathbf{x}_i^t = (1, 2, 3, 4), \forall i, t$$

In Example 3, $D_1 = D_2 = 0$, while D_3 would definitely be positive. Suppose next that the only available action to reach $D_3 = 0$ is the construction of a nuclear plant that will explode in $t = 10$ yielding the following distribution.

Example 4.

$$\mathbf{x}_i^t = (t + 100, t + 101, t + 102, t + 103), \text{ for } t \leq 6 \text{ and } \forall i,$$

$$\mathbf{x}_i^7 = (107, 108, 109, 0), \forall i,$$

$$\mathbf{x}_i^8 = (108, 109, 0, 0), \forall i,$$

¹²However, in the determination of the appropriate length of the stages, a trade-off arises between the robustness of the results (which tends to increase with the length of periods) and their relevance (since in the limit only whole lives matter).

$$\mathbf{x}_i^9 = (109, 0, 0, 0), \forall i, \text{ etc.}$$

According to *SSE*, if future generations' welfare is uniformly affected in each t , no other egalitarian consideration is necessary to evaluate a policy: the distribution in Example 4 is strictly preferable to that in Example 3 and it raises no egalitarian objection. Therefore the nuclear plant should be built. This conclusion would be rejected by most egalitarians and it raises serious doubts on *SSE* as the appropriate egalitarian benchmark.¹³

Examples 3 and 4 also show a more general point: the requirement of *CL*-equality cannot be abandoned without generating unappealing results (from an egalitarian perspective). This suggests that the analysis of intertemporal egalitarian benchmarks should focus on the choice of the most appropriate restriction on *CLE*. Indeed, only in the context of inequality analysis the "views are independent of each other, in the sense that each of their judgments may be in agreement or disagreement depending on the particular case in question" (Temkin [31, p.242]). If egalitarian distributions are analysed, then it is misleading to ask whether "the whole lives view [should] be rejected entirely, and replaced by some combination of the simultaneous and corresponding segments views" (Temkin [31, p.238]). Neither *CSE* nor a *simultaneous segments* restriction on *CLE* (discussed below) replaces the latter. Actually, in order to avoid changing places egalitarianism, any restriction on *CLE* should require all agents belonging to the same generation to have identical patterns of x during their lives. Hence, for a given x_i equal for all i , alternative restrictions will differ only in the admissible patterns of x for agents belonging to different generations.

One possibility, suggested by McKerlie [17, p. 484] is to impose *SS*-equality in addition to *CL*-equality. We find this proposal unconvincing. This version of *SSE* (hereafter *SSE*₂) is subject to the same time-dependency problem faced by the unconstrained *SSE* (hereafter, *SSE*₁). Moreover, the emphasis on simultaneity as the relevant egalitarian restriction on the allocation of x along agents' lives is not entirely compelling. *SSE*₂ removes changing places egalitarianism between agents belonging to the same generation, but the requirement of equality in the overlapping segments of the lives of agents belonging to different generations seems less convincing. According to *SSE*₂, the distribution in Example 3 - in which agents are treated identically regardless of the generation they belong to - is definitely non-egalitarian, while the following distribution is *SSE*₂-egalitarian.

¹³It is worth noting that Example 4 does not represent a variant of the so-called 'levelling down objection': what is objectionable is not that *SSE* leads to a lower welfare level in Example 4 than in Example 3, but rather that according to *SSE*, Example 4 must be considered better than Example 3 *from an egalitarian viewpoint*.

Example 5.

$$\mathbf{x}_i^t = (1, 2, 3, 4), \quad \forall i \text{ and } t = 4d, \quad d = 0, 1, 2, \dots$$

$$\mathbf{x}_i^t = (2, 3, 4, 1), \quad \forall i \text{ and } t = 1 + 4d, \quad d = 0, 1, 2, \dots$$

$$\mathbf{x}_i^t = (3, 4, 1, 2), \quad \forall i \text{ and } t = 2 + 4d, \quad d = 0, 1, 2, \dots$$

$$\mathbf{x}_i^t = (4, 1, 2, 3), \quad \forall i \text{ and } t = 3 + 4d, \quad d = 0, 1, 2, \dots$$

In Example 5, only agents born every four periods have the same pattern of attainment of x during their lives. However, unless agents are assumed to be myopic and to care only about the inequalities that they can actually observe in every t , it is hard to see why a distribution exhibiting such a cyclical pattern should be desirable from an egalitarian perspective, and indeed why it should be strictly preferable to that in Example 3. Notice that the egalitarian intuition behind SSE_2 is not the same as that behind SSE_1 : in the latter case, the idea is that inequalities between contemporaries are worse than inequalities between removed generations - e.g., between the present generation and people living in the middle age. Instead, given the same total level of x , the only role played by simultaneity in SSE_2 is to constrain the allocation of x during agents' lives.

Another possibility is to adopt CSE : since the distributions with $D_2 = 0$ are a strict subset of those with $D_1 = 0$, CSE can be naturally interpreted as a restriction on CLE .¹⁴ Moreover, unlike CLE and SSE_2 , CSE fully incorporates the egalitarian intuition that identical agents should be treated exactly in the same way, since in CS -egalitarian distributions they have an identical welfare allocation along their lives. Formally, unlike D_1 and D_3 , $D_2 = 0$ if and only if $\mathbf{x}_i = \mathbf{x}_h$, for all i, h ; that is, $D_2 = 0$ if and only if the vectors describing the pattern of the egalitarian variable along agents' lives are identical. Thus, all distributions in the class with $D_2 = 0$ can be simply described as follows.

Example 6.

$$\mathbf{x}_i = (p, q, r, s), \quad \forall i$$

regardless of agent i 's date of birth.

Unlike the distributions with $D_1 = 0$ or $D_3 = 0$, - as changing places egalitarianism and Example 4 respectively show, - those belonging to the class with $D_2 = 0$ are equivalent *from an egalitarian point of view*, as the comparison of any pair of CS -egalitarian distributions shows.

¹⁴If the duration of agents' lives is uncertain, neither CSE nor SSE_2 necessarily implies CLE *ex-post*, but the above arguments still hold *ex-ante*, if applied to expected welfare.

It is worth taking stock to summarise our argument here. Consider different welfare distributions from under a veil of ignorance, removing all normatively irrelevant information about personal identities. Equality requires that any agent randomly drawn from the economy be indistinguishable from any other in the normatively relevant (welfare) dimensions. In the static framework, every agent is simply defined by her level of x (a scalar). In an egalitarian distribution, every agent picked up at random from the economy would indeed be indistinguishable from every other agent as they all reach the same level of x .

In the intertemporal framework, agents are defined by a vector describing the attainment of x in each period of their lives. In a *CL*- or *SS*-egalitarian distribution, an agent picked up randomly from the economy would not necessarily be identical to any other agent. Assuming that all of the information contained in the vector \mathbf{x}_i^t is normatively relevant in principle (as it is natural to assume in the intertemporal context), under either *CLE* or *SSE* equally deserving agents would not necessarily be treated identically. In contrast, at a *CS*-egalitarian distribution any agent randomly drawn from the economy would have exactly the same vector \mathbf{x}_i^t . Discarding the (morally irrelevant) information about birth dates, a *CS*-egalitarian distribution indeed implies identical agents.

To be sure, it is not necessarily true that $p = q = r = s$, and thus *CSE* allows potentially great inequalities between people living in the same period and belonging to different age cohorts. For instance, a *CS*-egalitarian distribution could imply that in every period there are happy young people, while the elderly live in despair. However undesirable such a distribution may be, if $D_2 = 0$ - that is, if, when they were young, the elderly were treated as the current young - there should be no egalitarian objection to it, since identical people have an identical pattern of x during their lives.¹⁵

There may, of course, be non-egalitarian objections to the latter allocation and in general, distributions with $D_2 = 0$ are not equivalent *all things considered*. For instance, distributions with a higher overall welfare or without unbalanced welfare allocations along agents' lives may be preferred. As shown by Example 6, if *CSE* is adopted, egalitarian and non-egalitarian concerns can be clearly distinguished in the evaluation of a distribution. The former reduce to the requirement $\mathbf{x}_i = \mathbf{x}$, all i , while the latter are related to the features of \mathbf{x} , that is, the desirable pattern of the egalitarian variable along agents' lives. *All things*

¹⁵If identity changed during an agent's life, there might be an egalitarian objection to the distribution. However a similar critique can be moved to any intertemporal egalitarian principle, since it amounts to saying that the principle is analysed in the wrong context. Once the agents' identity is correctly specified, all the arguments in this paper remain valid.

considered a distribution with, say, $p > q = r = s$ may be rejected because of the unbalanced welfare allocation. However, this is an argument regarding the welfare pattern *along an agent's life* and not how she fares *relative to others* and therefore it is not an egalitarian reason to reject the distribution. A smoother welfare profile would probably be preferable but this would be the outcome, e.g., of the adoption of some kind of maximin principle, or a prioritarian view applied to portions of an agent's life.

3 Egalitarianism, Utility and the Maximin

In the static context, given the relevant equalisandum, different egalitarian views can “be regarded as built around ways of measuring the inequality between lives” (McKerlie [17, p.487]), but the egalitarian state to reach is unambiguously defined. As a result, the differences between the various views in relation with other normative principles can be shown in unequal distributions, but not if one evaluates the desirability of reaching the common egalitarian state in relation, say, to utilitarian concerns. This is not true in the intertemporal context: different principles yield different trade-offs between egalitarian and non-egalitarian concerns also in *egalitarian* distributions. Since the evaluation of a distribution is influenced by more than one ethical concern, it is important to analyse these trade-offs in a systematic way.

In this section, *CLE*, *CSE*, and *SSE* are analysed in relation to two non-primarily-egalitarian normative principles, namely Rawls's [22] difference principle and utilitarianism. If, as argued in section 2, it is appropriate to impose a restriction on *CLE*, then it is important to analyse whether this implies a welfare loss, whether different restrictions have different effects on welfare, and what are the consequences for the worst-off generation.

The problem is modelled in a stark way. We generalise Arrow [2] and Dasgupta [8], in which the maximin criterion is examined in a dynamic framework. Society exists for an infinite number of generations. Population is stationary. There is no technical progress and only one good that can be consumed or invested. Utility is the relevant egalitarian variable, and thus $x_j^t = u_j^t = u(c_j^t)$, where the subscript i denoting different agents belonging to the same age cohort is dropped in order to focus on *intergenerational* inequalities. Assume that $T = 2$ and $j = 1, 2$ (youth and old age), and agents have identical additively-separable utility functions:

$$W(c_1^t, c_2^{t+1}) = u(c_1^t) + \beta u(c_2^{t+1})$$

where c_1^t is consumption of the young in t , c_2^{t+1} is the consumption of the elderly in $t + 1$, $0 < \beta \leq 1$ is the subjective discount factor and u satisfies $u(0) = 0$, $u'(c_j^t) \equiv du/dc_j^t > 0$, $\lim_{c \rightarrow 0} u'(c) = \infty$, and $u''(c_j^t) \equiv d^2(u)/d(c_j^t)^2 < 0$.

Production possibilities can be represented by a production function $F(K, L)$ where K is the stock of capital and L is labour supply. F is continuous and homogeneous of degree one. L^t is proportional to population and normalised to one. Thus, if $k^t \equiv K^t/L^t$, then $F(K^t, L^t) = f(K^t/L^t, 1) = f(k^t)$. The function satisfies $f(0) = 0$, $f' > 0$, $f'' < 0$, and Inada conditions.

For any variable z , let $\{z^t\}_{t=0,1,\dots}$ denote an infinite sequence of values of z . The maximin programme (MP) can be written as follows.

$$\max_{\{c_1^t, c_2^{t+1}\}_{t=0,1,\dots}} \min_t W(c_1^t, c_2^{t+1}),$$

subject to $k^{t+1} - k^t + c_1^t + c_2^t = f(k^t)$, all $t \geq 0$, given k^0 and c_2^0 .

For any variable z , let $dz = z' - z$ denote a change in z . Propositions 1 and 2 provide necessary conditions for a maximin solution.

Proposition 1. *At the solution to (MP), $W(c_1^t, c_2^{t+1}) = W(c_1^{t+1}, c_2^{t+2})$ for all t .*

In other words, a welfare distribution must satisfy *CLE* in order to be the maximin solution. In this sense the maximin criterion poses an efficiency restriction on *CLE*: the maximin solution is the *CL*-egalitarian distribution with the highest level of equal welfare.

Proposition 2. *At the solution to (MP), $u'(c_1^t)/u'(c_2^{t+1}) = \beta(1 + f'(k^{t+1}))$, all t .*

By Proposition 2, agents born in t attain the highest $W(c_1^t, c_2^{t+1})$ given k^{j+1} , c_1^j and c_2^{j+1} , $j \neq t$. This suggests that the condition in Proposition 2 can be derived as the first order condition of a constrained optimisation problem, defining the optimal consumption allocation along an agent's life. Given the assumptions on u and f , Proposition 2 implies that the maximin solution is unique, while none of the egalitarian criteria identifies *per se* a particular welfare distribution. However, the main implication of Proposition 2 in the analysis of the egalitarian views is that in general $u(c_1^t) \neq \beta u(c_2^{t+1})$ so that the maximin allocation will not be *SS*-egalitarian.

Let $c = c_1^t + c_2^t$, all t , denote a generic constant (total) consumption programme and let $c^m \equiv f(k^0)$, all t .

Lemma 1. c^m is the maximum sustainable aggregate constant consumption.

Lemma 1 provides a natural benchmark for the maximin path. Let $c_2^m = c_2^0$ and $c_1^m = c^m - c_2^m$; for any given c_2^0 , no distribution in which $W(c_1^t, c_2^{t+1}) < W^m = W(c_1^m, c_2^m)$, some t , can solve (MP).¹⁶ Hence, let $R^t = f(k^t) + k^t - c_2^t$ denote the resources available to the generation born in t : if $R^t = R^0$ then all generations from t onwards can reach at least W^m . Consider the following sequence of maximisation programmes (P_t).

$$\max_{c_1^t, c_2^{t+1}} u(c_1^t) + \beta u(c_2^{t+1}),$$

subject to

$$k^{t+1} + c_1^t \leq R^t,$$

$$f(k^{t+1}) + k^{t+1} - c_2^{t+1} \geq R^{t+1}$$

$$\text{given } R^t, R^{t+1}.$$

Let (c_1^*, c_2^*, k^*) be the solution of P_t with $R^t = R^{t+1} = R^0$, where in general $k^* \neq k^0$. Let $V(R^t, R^{t+1})$ denote the maximum function associated with P_t . Let $W^* \equiv W(c_1^*, c_2^*) = V(R^0, R^0)$. The main theorem can now be proved.

Theorem 1. Let c_2^0 be given. The maximin solution corresponds to the vector (c_1^*, c_2^*, k^*) for each generation.

Theorem 1 states that although the maximin principle and *CSE* represent different restrictions on *CLE*, they coincide in the economy described, since at the solution to MP, agents have the same consumption - and welfare - allocation during their lives.¹⁷ Thus, if the egalitarian social planner also adopts an

¹⁶Alternatively, the benchmark path could be the solution to the following problem:

$$\max_{c_1, c_2} u(c_1) + \beta u(c_2), \text{ subject to } c_1 + c_2 = f(k^0)$$

In this case, the assumption of a given c_2^0 would be dropped and the constraint $c_2^0 \geq c_2$ would be necessary necessary to guarantee equal treatment of the generation born in $t = -1$. This choice would include generation $t = -1$ in the definition of the just path, allowing for an explicit treatment of the transition to justice, instead of taking its past consumption choices as given. However, the main results of this paper would not change.

¹⁷It can be proved that with a finite horizon this is not true. However, the adoption of the

intergenerational maximin criterion, Theorem 1 proves that the two objectives are not in contradiction if *CLE* or *CSE* are adopted, while if *SSE*₁ (or *SSE*₂) is chosen, a trade-off between the two concerns arises.¹⁸

Moreover, since the maximin solution coincides with the allocation that maximises agents' utility under a *CLE* constraint, the model allows us to introduce some utilitarian concern in the analysis. Consider, for instance, classical (average or total) utilitarianism. By Proposition 2, it is more difficult to reconcile a utilitarian concern with *SSE*₂ than with *CLE* or *CSE*, since *SSE*₂ does not allow for a constrained welfare-maximising allocation along agents' lives. Instead, if *SSE*₁ is adopted, in principle it is possible for infinitely many generations to reach a higher welfare level than at the maximin, with only a finite number of generations falling below it in order to start capital accumulation. Thus, due to the infinite gain in utility, a utilitarian would prefer the latter distribution to the maximin/*CSE* solution. In general, such a distribution might be appealing (as opposed to *CSE* or *CLE* distributions) not because it is *SS*-egalitarian but because some *CS*, or even *CL* inequalities can be outweighed by an infinite gain in utility, *all things considered*. In this sense, *SSE*₁ is the only intertemporal egalitarian principle compatible with sustained welfare growth and thus the principle that can best accommodate utilitarian concerns (although *SS*-equality could still imply some welfare loss with respect to unconstrained utility maximisation). However, this result derives from the exclusive focus of *SSE*₁ on *intratemporal* inequalities, and thus it should not be seen as a solution to the equality/growth dilemma, but rather as a way of bypassing it.

The model presented is highly stylised and some caution is necessary in interpreting the results. While the analysis of *SSE*₁ does not depend on any particular assumptions, in more general settings, *CLE* and *CSE* will not be equivalent as concerns their relations with other normative principles and the maximin solution will be neither *CS*- nor *CL*-egalitarian.¹⁹ However, despite its simplified structure, the model does capture in vitro some inherent features of the egalitarian views. As concerns utilitarianism, since *CSE* and *SSE*₂ distributions are

infinite horizon hypothesis is implied by the very nature of the problem, as there is no reason to restrict the analysis of a normative principle to an arbitrary, finite number of generations.

¹⁸It is worth noting that our results are consistent with Norman Daniels' [7] account of intertemporal justice, according to which the best way of synchronically distributing resources among people of different ages should be identified by thinking prudentially about a diachronic distribution across the different temporal stages of a single complete lifetime. In our model, however, this is not an assumption: it is a *result* of the application of the maximin principle.

¹⁹However, while Theorem 1 is more sensitive to changes in the assumptions, heterogeneous, non additive or non concave preferences, technical progress or more general production functions would leave Proposition 1 basically unchanged.

strict subsets of those with $D_1 = 0$, the *CLE* lifetime welfare level will always be at least as high as the *SSE*₁ and *CSE* levels. Moreover, from Proposition 2, it is legitimate to infer that even in more general settings *CLE* lifetime welfare would be at least as high as the *SSE*₂ welfare, since *SSE*₂ does not allow agents to allocate consumption optimally along their lives. Similarly, as regards Rawlsian concerns, the above results suggest that in a more general setting, if the maximin solution was not egalitarian, the *CLE* level would be at least as close to it as the *CSE* level, and the latter in turn would be at least as close to the maximin as the *SSE*₂ level.

4 Conclusion

In this paper three egalitarian views are analysed in the intertemporal context. Once the static setting is abandoned, egalitarian principles - apart from differing in the analysis of existing inequalities, - also define different ideal egalitarian distributions. While it may be important to use the different information conveyed by every criterion in the analysis of existing inequalities, when the egalitarian distributions associated with them are analysed, *CLE* and *SSE* have undesirable features while *CSE* represents the appropriate egalitarian benchmark.

The relations between the three egalitarian principles and other moral ideals, namely maximin and utilitarianism, are also analysed. As regards the maximin principle, Propositions 1-2 and Theorem 1 show that, unlike with *CLE* and *CSE*, the adoption of *SSE* implies a trade off between egalitarianism and a concern for the worst off. As regards utility, the same conclusion holds if one interprets *SSE* as a restriction on *CLE*, since it yields a lower egalitarian welfare level. This is not true if *SSE* is analysed *per se*, but this is just because in this case the *SSE* is a strictly *intratemporal* principle.

In closing this paper, it is worth noting that our formal analysis yields some interesting insights on a vexed issue in normative economics, namely the well-known trade-off between equality and growth. Arrow [2] and Dasgupta [8] proved that if agents are selfish, live for one period, and their lives do not overlap, then Rawls's maximin principle implies a stationary path of consumption, capital and welfare. Our model yields a more nuanced conclusion and suggests some interesting directions for further research. In our OLG framework, agents have an incentive to choose the optimal path of capital in order to maximise lifetime welfare and, under quite general assumptions, this implies growth in at least one period. In other words, although the application of the maximin principle

precludes permanent growth in the economy, as in Arrow [2] and Dasgupta [8], the dynamics of the economy is *not* completely stationary. Our conjecture is that an explicit and more realistic analysis of the temporal structure of agents' lives (which span over many periods), and the overlaps across generations, together with the introduction of uncertainty and irreversibility of investments may alter the justice/growth trade-off.²⁰ This indicates a promising line for further research on intertemporal and intergenerational justice.

²⁰A similar point is made by Silvestre [29], albeit in a rather different formal setting.

Appendix

Proof of Proposition 1.

Let W^* be the value of MP and suppose that, contrary to the statement, $W(c_1^0, c_2^1) > W^*$. By continuity, there is a sufficiently small $dc_1^0 < 0$, such that $W(c_1^0, c_2^1) > W^*$, $-dk^1 = dc_1^0$ and the amount of resources available in $t = 1$ increases by $[1 + f'(k^1)]dk^1$. Let $dc_1^1 = f'(k^1)dk^1 > 0$ and $dk^2 = dk^1$ and repeat the procedure for all $t \geq 2$ so that $dc_1^t = f'(k^t)dk^t > 0$, $dk^{t+1} = dk^t > 0$, and $W(c_1^t, c_2^{t+1}) > W^*$, all t , a contradiction. The proof of the case with $W(c_1^t, c_2^{t+1}) > W^*$, some $t > 0$, is similar. □

Proof of Proposition 2.

Suppose not. Then there is dc_1^t, dc_2^{t+1} such that $dc_2^{t+1} = -[1 + f'(k^{t+1})]dc_1^t$ and $u'(c_1^t)dc_1^t + \beta u'(c_2^{t+1})dc_2^{t+1} > 0$. By the concavity of W , this implies $W(c_1^t, c_2^{t+1}) > W(c_1^t, c_2^t)$ leaving unmodified c_1^j , all $j \neq t$ and k^j, c_2^j , all $j \neq t + 1$, violating Proposition 1. □

Proof of Lemma 1.

Consider $c > c^m$. At $t = 0$, $k^1 < k^0$ and thus $k^2 - k^1 = k^1 - k^0 + f(k^1) - f(k^0) < 0$ and $k^2 - k^1 < k^1 - k^0$, i.e. $|k^2 - k^1|/|k^1 - k^0| > 1$, and, by induction $|k^{t+1} - k^t|/|k^t - k^{t-1}| > 1$. Therefore $k^t = 0$ for t finite, and c is not sustainable. □

Proof of Theorem 1.

1. The existence and uniqueness of (c_1^*, c_2^*, k^*) is guaranteed by the assumptions on u and f . Note also that (c_1^*, c_2^*, k^*) satisfies the condition in Propositions 1 and 2.
2. Suppose it is possible to raise the welfare of all generations above W^* . Consider P_0 : by construction the first generation's welfare can increase over W^* if and only if $R^1 < R^0$. Consider now generation 2: clearly $V(R^1, R^0) < W^*$. Moreover $V(R^t, R^{t+1})$ is concave and its iso-welfare contours have slope $[1 + f'(k(R^t, R^{t+1}))]$, where $k(R^t, R^{t+1})$ is the optimum value of k^{t+1} from P_t . Hence, $W(c_1^1, c_2^2) > W^*$ implies $R^2 < R^0$, with $|R^2 - R^0| > [1 + f'(k(R^0, R^0))] |R^1 - R^0|$. Iterating the argument, $W(c_1^t, c_2^{t+1}) > W^*$ implies $|R^{t+1} - R^0|/|R^t - R^0| > [1 + f'(k(R^0, R^0))]$, all t , and the path violates the non-negativity of R^t at some finite t . □

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