Firm Dynamics, Misallocation and Targeted Policies

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Abstract

Access to external finance is a major obstacle for small and young firms; thus, providing subsidized credit to small and young firms is a widely-used policy option across countries. We study the impact of such targeted policies on aggregate output and productivity and highlight indirect general equilibrium effects. To do so, we build a model of heterogeneous firms with endogenous entry and exit, wherein each firm may be subject to forward-looking collateral constraints for their external borrowing. Subsidized credit alleviates credit constraints small and young firms face, which helps them to achieve the efficient and larger scale of production. This direct effect is, however, either reinforced or offset by indirect general equilibrium effects. Factor prices increase as subsidized firm demand more capital and labor. As a result, higher production costs induce more unproductive incumbents to exit, while replacing them selectively with productive entrants. This cleansing effect reinforces the direct effect by enhancing the aggregate productivity. However, the number of firms in operation decreases in equilibrium, and this, in turn, depresses the aggregate productivity.

Keywords: Firm dynamics; misallocation; financial frictions; firm size and age
JEL Classifications: E22, G32, O16

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1 Introduction

The recent literature has documented the large contribution of start-up firms to job creation in the U.S. (Haltiwanger, Jarmin, and Miranda (2016)). Young firms grow fast; however, they also tend to fail at greater rates, and they are especially vulnerable to negative shocks such as recessions and natural disasters.[1] In particular, credit constraints compound such adverse shocks, which leads to the high exit rates among young firms (Collier et al. (2016)).

Government policies that attempt to alleviate credit constraints faced by small and young businesses are widely adopted across countries. Providing subsidized credit to small and young firms is one of the prime examples. The targeted group of firms directly benefits from the subsidy. However, this type of targeted policies may accompany unintended impacts on the other firms in a general equilibrium environment. To evaluate the disproportionate impacts of such policies on aggregate output and productivity, we construct a general equilibrium model with heterogeneous firms that can endogenously enter and exit but face collateral constraints. Calibrating the model against both aggregate and microeconomic data, we examine the overall effect of targeted policies on an aggregate economy, and decompose it into two components; a direct effect from credit subsidies and indirect general equilibrium effects.

Our model has three key ingredients that allow us to capture the realistic cross sectional distribution of firms and their dynamics as observed in data. First, we employ a bounded Pareto distribution for firm-level productivity process. It is well-known that the empirical distribution of firm size in employment is highly skewed. That is, small firms dominate the business population, whilst large firms account for the largest fraction of aggregate employment.[2] Coupled with the decreasing-returns-to-scale production technology, the above specification of productivity process successfully replicates the empirical size distribution in our model economy.[3] Second, we allow the endogenous entry and exit behaviors by firms.[4] It is essential to introduce this setting to reproduce the empirical

[1]Since January 2011, the Obama Administration has been promoting a set of entrepreneur-focused policies, including Jumpstart Our Business Startups (JOBS) Act. The White House (2012) claim that these policies allow *"Main Street small businesses and high-growth enterprises to raise capital from investors more efficiently, allowing small and young firms across the country to grow and hire faster."*

[2]In 2012, the share of employment at large firms is 51.6 percent while the employment share of small firms is 16.7 percent in the US (Statistics of U.S. Businesses, 2012).

[3]Jo (2015) builds a business cycle model of such, and shows the important role of the empirical firm size distribution in driving business cycle fluctuation following financial shocks. Guner, Ventura, and Xu (2008) and Gourio and Roys (2014) study the aggregate implication of policies that distort the size distribution of firms.

[4]See Hopenhayn (1992) and Jovanovic (1982) for the workhorse models of firm selections and dynamics.

patterns of firm dynamics, including substantially lower survival rates among young firms. This model element also allows us to examine the importance of extensive margins which contribute to aggregate productivity.[5] Third, forward-looking collateral constraints are added, in the spirit of Kiyotaki and Moore (1997), and this, together with the second ingredient of our model, leads to the empirical age distribution of firms.[6] The presence of this collateral constraint hinders the immediate firm growth upon entry in the model. This helps us not only to have a substantial number of small firms in the economy as in the data, but also to hit the right pattern of firm lifecycle dynamics; newborn firms gradually build up capital stock by relying on debt issuance.

We use this model to study the aggregate implications of targeted credit subsidy policies in a general equilibrium environment. Credit subsidies in our model are lumpsum cash transfers from households to targeted firms. Therefore, none of the policy effects that we study in this paper arises from direct policy distortions as follows. We show that there are four effects, one direct and three indirect, of a credit subsidy policy in our model. The first one is a direct effect that emerges from the fact that small and young firms can achieve the efficient scale of production by receiving a subsidy. The second one is an indirect general equilibrium effect. Increased demand for capital and labor - from the recipients of the subsidy - will raise factor prices. Higher factor prices depress the scale of production of untargeted firms. The third effect is also indirect. Due to the increased factor prices, more unproductive incumbents exit which are replaced with less unproductive potential entrants (i.e., *cleansing effects*). The last effect is that there are fewer firms in operation because of higher costs of production, which in turn aggravates the aggregate productivity.[7]

Furthermore, we study the implications of policies that differ in their respective targets; one for small firms (*size-dependent policy*) and one for young firms (*age-dependent policy*). In each policy experiment, we analyze the associated impacts on firm size and age distributions, life cycle patterns of firms, and the relationship between firm dynamics and aggregate productivity. Our main finding is that, targeting young firms will be the most efficient productivity-enhancing policy amongst the considered. Targeting all small firms,

[5]Amongst others see Foster, Haltiwanger, and Syverson (2008), Osotimehin (2016) and Bartelsman, Haltiwanger, and Scarpetta (2013) for a growing body of works on the study of resource allocation.

[6]The importance of firm age has been studied by the following seminal papers: Davis, Haltiwanger, and Schuh (1996), Dunne, Roberts, and Samuelson (1989), and Evans (1987). More recently, Fort et al. (2013) emphasize the age dimension of firm heterogeneity during the Great Recession in the U.S. See Hsieh and Klenow (2014) for the lifecycle of manufacturing plants in Mexico and India, and the associated resource misallocatoin.

[7]Under the decreasing-returns-to-scale, the number of production units in a model economy is positively associated with the level of aggregate measured total factor productivity.

instead, is relatively less efficient, as they obtain little productivity gains in aggregate, even though the mass of targeted firms is large. Intuitively, this result is driven by the fact that capital misallocation due to financial frictions is more pervasive among young-small firms in our model, rather than among all small firms. Particularly, the following features in the model environment deliver our main quantitative results. First, most startups in our economy are born with a relatively high leverage ratio compared to that of incumbents. Second, the firm-level productivity process is persistent, implying that a young financially-constrained startup needs to maintain its high leverage ratio for a while. The firm then accumulates capital stock until it reaches the efficient scale which is consistent with prices and its own productivity. Third, and related to the second feature, if firms survive long enough and become grown-up, they are less likely financially constrained, regardless of firm size. At this stage, firm size is largely determined by productivity rather than by credit limit. Therefore, among small firms in our model economy, some are indeed financially constrained, and thus positive productivity gains are expected when undoing capital misallocation by relaxing their collateral constraints. However, there also exist firms that are small just because their productivity is low, not because they experience difficulties in borrowing. If the fraction of such low productive firms is large among small firms, then targeting all small firms may not deliver the desired outcome of enhancing productivity. Instead, since young firms tend to be financially constrained and, more importantly, some of them are actually highly productive but without sufficient capital, targeting young firms leads to a more efficient result in improving aggregate productivity level.

Related Literature In policy debates about small firms and their macroeconomic performances, there are various factors that are to blame for preventing small businesses from growing and hiring fast.[8] This paper focuses on the role of financial frictions for small and young firm dynamics. The main contribution is to build a general equilibrium model with rich and realistic firm dynamics, and to offer a quantitative analysis of different policy alternatives that are often argued to be useful. The result of this paper sheds lights on the importance of firm age dimension when it comes to considering policy options for enhancing the access to finance for small and young businesses. In particular, we show that *age-dependent policy* is more productivity-enhancing than *size-dependent policy*. This

[8]For example, targets of the Obama administration's entrepreneur-focused policy initiatives include finance, education, red tapes, innovation, and market opportunities for entrepreneurs. (White House, 2012) The policy recommendations by Lord Young for Prime Minister David Cameron include helping small UK businesses to secure loans, to get public procurements and education and support. (Young, 2015)

result is in line with what is argued in Fort et al. (2013) by showing the importance of controlling for firm age to determine firm dynamics. They also show that young and small firms are more sensitive to shocks than old and large firms are. This high sensitivity of small and young firms does not necessarily imply that a policy geared to them is the best recommendation for an aggregate economy. Our results, however, confirm that a targeted policy that unlocks access to capital for business startups indeed is the best possible option.

To the best of our knowledge, this paper is the first to conduct a quantitative policy simulation exercise that looks at the policy alternatives both for size-dependent and age-dependent options in a comprehensive way. Regarding size-dependent policies, Guner, Ventura, and Xu (2008) study the macroeconomic implications of restricting large firms, and find sizable effects on aggregate productivity.[9] Gourio and Roys (2014) also look at the quantitative impact of the French policy on size restriction, and show an important role of such policy distortion in explaining aggregate productivity. Nonetheless, age-dependent policy is only studied in our paper.

Behind our results from policy experiments, there are two main mechanisms at work on the allocative efficiency of resources: one across incumbents, and one across entrants and exiting firms. The role of financial frictions in generating resource misallocation and its aggregate implications have been studied by Khan and Thomas (2013), Buera and Moll (2015), Buera, Kaboski, and Shin (2011), and Midrigan and Xu (2014), among others.

Our model builds on a family of the standard theory of firm dynamics such as Hopenhayn (1992), and extends it into a general equilibrium environment with collateral frictions.[10] In studying firm dynamics with financial frictions, Cooley and Quadrini (2002) is the closest to ours. The simplified collateral constrains in our paper are well-suited for large panel simulations, and the general equilibrium setting allows us to examine the disproportionate impacts on small versus large, and on young versus old firms across alternative policies.[11] More recently, there are studies on the time-varying firm age distribution over business cycles, which emphasize the cohort effect on firm dynamics.[12] Our

[9]We differ from their approach by considering the implications financial frictions on small and young firms. Later, we discuss this in detail.

[10]Amongst others see Clementi and Palazzo (2015) and Siemer (2016).

[11]Cooley and Quadrini (2001) study the non-contingent debt contracts between firms and lenders, and show that their model can explain firm dynamics conditional on firm age and size simultaneously. Khan, Senga, and Thomas (2016) build an equilibrium business cycle model with non-contingent debt contracts and study how default risks shape firm dynamics and aggregate fluctuations following real and financial shocks.

[12]Both Moreira (2106) and Sedlacek and Sterk (2016) highlight the importance of aggregate conditions at firm birth in determining the initial employment size, by using the customer-base accumulation. Sed-

paper is distinct from these works by investigating the mechanism of financial frictions in shaping firm distributions, and also by quantifying the relevant policy implications.

2 Model

There is a large number of heterogeneous firms producing a homogeneous good. Each firm accumulates physical capital over time, and its external borrowing is limited by a collateral constraint. Individual decisions of employment, investment, and borrowing generate substantial differences across firms, along with persistent shocks to individual productivity. In each period, firms are allowed to endogenously exit, and potential entrants decide their entry to the production sector. This characterizes the realistic firm dynamics together with the rich firm-level heterogeneity in our model economy. We close the model by describing the household problem, and then define a recursive competitive equilibrium. We summarize the firm-level state of capital and borrowing by using a single state variable defined as cash-on-hand, which allows us to derive individual firm's decisions in a tractable way. Our model environment is based on Jo (2015), but we depart from it by introducing the endogenous entry and exit by firms.

2.1 Firms

2.1.1 Production and Financial Friction

The economy is consist of a continuum of firms. Each firm owns its predetermined capital stock, k, and hires labor, n, in a competitive labor market. The production technology is described by $y = z\epsilon F(k, n)$, where $F(\cdot)$ exhibits decreasing-returns-to-scale (DRS) property. A firm's productivity is determined by two components, z and ϵ. z is the aggregate level of productivity common across firms, and ϵ represents firm-specific idiosyncratic productivity. We assume that z follows a Markov chain with $\pi^z_{fg} \equiv Pr(z' = z_g|z = z_f) \geq 0$ and $\sum_{g=1}^{N_z} \pi^z_{fg} = 1$ for each $f = 1, 2, \ldots, N_z$. Similarly, $\epsilon \in \mathbf{E} \equiv \{\epsilon_1, \epsilon_2, \ldots, \epsilon_{N_\epsilon}\}$ independently follows a Markov chain with $\pi^\epsilon_{ij} \equiv Pr(\epsilon' = \epsilon_j|\epsilon = \epsilon_i) \geq 0$ and $\sum_{j=1}^{N_\epsilon} \pi^\epsilon_{ij} = 1$ for each $i = 1, 2, \ldots, N_\epsilon$.

In this model, not all firms are able to finance their desired investment due to financial frictions. Each firm faces an individual borrowing limit for obtaining one-period discount debt at the price of q. This borrowing constraint restricts the amount of newly determined

lacek (2016) focuses on the unemployment dynamics and the age distribution of firms during the Great Recession debt level today, b', not to exceed a firm's collateral. This is based on the idea of limited enforceability of financial contracts, and we assume that the firm's future period capital, k', serves as collateral for current borrowing. Therefore, for a firm choosing k' in the current period, the collateral constraint is given by $b' \leq \theta k'$, where the financial parameter, θ, captures the economy-wide level of financial frictions. Notice that θ is assumed to be common across firms, but the borrowing decision depends endogenously on each firm's state.[13] When θ is close to the real interest rate, $\frac{1}{q}$, the financial market allows firms to invest at their desired level. Note that our specification of collateral constraint is forward-looking in the spirit of Kiyotaki and Moore (1997). Because we do not restrict the sign of b, a firm can also accumulate its financial assets in terms of a negative value of b, once the firm achieves its optimal scale at a given productivity level, $z\epsilon$. In follows that the individual borrowing constraint is occasionally binding in our model economy.[14]

2.1.2 Entry and Exit

To model endogenous entry and exit by firms, we follow the standard approach in the literature.[15] One distinctive feature of our model from the existing works, however, is the presence of financial frictions at firm-level. Therefore, individual states of productivity, capital, and borrowing jointly affect the exit decisions made by firms. Together with endogenous entry decisions by potential entrants, this leads to a realistic firm dynamics in our model economy.

For simplicity, we assume that firms have to pay a fixed operation cost in order to remain in the production sector. At the beginning of each period, firms need to pay $\xi_o > 0$ in units of output before conducting production.[16] This fixed cost of operation creates a binary decision of endogenous exit. If a firm does not pay this cost, it has to exit the industry with zero value. Thus, only firms with enough profitability continue to produce within the period.

In addition to the endogenous exit margin, we assume that incumbent firms are also exposed to exogenous risk of exit in each period. This assumption reflects other possible

[13] We set the value of θ in our calibrated benchmark economy to be consistent with aggregate moments in data. Then, we conduct policy counterfactuals later in this paper by varying the value of θ across firms in a general equilibrium environment.

[14] In case of always binding borrowing constraint, θ directly maps into a firm's leverage ratio.

[15] Hopenhayn (1992) is the seminal work in this literature with industry dynamics by firms' endogenous entry and exit.

[16] The payment takes place after both individual and aggregate productivity shocks are realized, but before a firm's exogenous exit status is known. Our approach only differs in this timing of cost from the model in Hopenhayn (1992). Below, we discuss the specific timing of individual decisions made in a given period.

channels of firm exit behavior in an economy, while preventing immediate firm growth over time combined with financial frictions. Specifically, firms in our model are subject to a fixed probability of exogenous exit, π_d, in each period. We assume that the exogenous exit information arrives after firms decide to remain by paying the fixed cost ξ_o. Unlike the endogenous exit decision, however, firms are supposed to exit at the end of a period after production.[17] These exiting firms liquidate all their remaining earnings and assets, and only the continuing firms to the next period make intertemporal decisions of investment and borrowing.

We further assume that there is a fixed measure, M^e, of potential entrants in this economy. Before deciding its entry, a potential entrant randomly draws its initial productivity, ϵ_0, and capital, k_0, respectively from a time-invariant distribution, while the initial debt level is a fraction of k_0 which reflects the average leverage ratio of entrants.[18] When a potential entrant decides to enter, it needs to pay a fixed entry cost, $\xi_e > 0$, in units of output. In this way, we are able to setup a simple binary problem of endogenous entry by potential entrants, and this allows only firms with sufficient future profitability to actually enter the production sector by paying the fixed cost of entry. Firm entry takes place at the end of each period, and the actual entrants start operating in the next period.

2.1.3 Timing and Firm Distribution

We illustrate the timing of exogenous shocks and decisions made by firms in a given period of the model. At the beginning of a period, a firm is identified by its individual state vector, (k, b, ϵ); the predetermined capital, $k \in \mathbf{K} \subset \mathbb{R}_+$, the amount of debt carried from the previous period, $b \in \mathbf{B} \subset \mathbb{R}$, and the current period idiosyncratic productivity level, $\epsilon \in \mathbf{E}$. We summarize the distribution of firms in the model by a probability measure, $\mu(k, b, \epsilon)$, which is defined on a Borel algebra $\mathcal{S} \equiv \mathbf{K} \times \mathbf{B} \times \mathbf{E}$. For notational simplicity, we use $s \equiv (z, \theta)$ to denote the aggregate exogenous state with its transition probability, $\pi^s_{lm} \geq 0$. In this way, the aggregate state of the economy is described by (s, μ), and the evolution of μ follows a mapping Γ such that $\mu' = \Gamma(s, \mu)$.

After the exogenous shocks are realized on s and ϵ at the beginning of a period, a firm decides whether to remain in the economy by paying the fixed operation cost, ξ_o. If it

[17]Notice that the actual exogenous exit happens at the end of a period, while the endogenous exit is at the beginning of the period. We can also interpret our exit timing that firms exogenously exit first and then endogenous exit happens among the remaining firms across two adjacent periods.

[18]In our calibration, we assume that k_0 is drawn from a uniform distribution, $U(0, \bar{k}_0)$, where $\bar{k}_0$ is a fraction, χ_k, of the optimal level of incumbents' capital stock at the average productivity level. We choose a small value of χ_k to reflect the average size of entrants in relative to incumbents.

decides to exit, the firm immediately disappears. If remaining, on the other hand, the firm is then informed of its exogenous exit which takes place at the end of the period. Given the aggregate and individual states, $(k, b, \epsilon; s, \mu)$, the firm maximizes the sum of its current and future expected discounted dividends. After production, the firm pays its wage bill and clears the existing debt b. Conditional on its survival to the next period, the firm conducts the intertemporal decisions of investment, i, and borrowing, b', while deciding the current period dividend payments, D, to its shareholders. The capital accumulation of each firm is standard, $i = k' - (1 - \delta)k$, with $\delta \in (0, 1)$. In the meantime, potential entrants draw their initial capital and productivity, and decide whether to enter by paying the entry cost ξ_e. Entering firms are identified by (k_0, b_0, ϵ_0), and make their decisions starting from the next period. Markets are perfectly competitive, so firms take the wage rate, $w(s, \mu)$, and the discount debt price, $q(s, \mu)$, as given.

2.1.4 Firm's Problem

We make distinctions on a firm's value within a period, depending on the timing of entry and exit as described above. This is because of our explicit assumption on the timing of firm entry and exit in the model economy. Let $v^0(k, b, \epsilon; s, \mu)$ be the value of a firm with (k, b, ϵ) at the beginning of the current period, before making its decision of paying the operation cost ξ_o. Accordingly, define $v^1(k, b, \epsilon; s, \mu)$ as the value of a remaining firm after the endogenous exit decision within the period, but before the exogenous exit status is known. Finally, if a firm survives to the next period, its within-the-period value is given by $v(k, b, \epsilon; s, \mu)$. Then the firm's optimization problem can be recursively defined by using v^0, v^1, and v.

$$v^0(k, b, \epsilon_i; s_l, \mu) = \max\left\{0, v^1(k, b, \epsilon_i; s_l, \mu)\right\} \tag{1}$$

$$\begin{aligned} v^1(k, b, \epsilon_i; s_l, \mu) = \pi_d \cdot \max_n & \left[z_l \epsilon_i F(k, n) - w(s_l, \mu)n + (1-\delta)k - b - \xi_o\right] \\ & + (1 - \pi_d) \cdot v(k, b, \epsilon_i; s_l, \mu) \end{aligned} \tag{2}$$

With the current realization of (s_l, ϵ_i), the firm makes a binary decision of exit over the values of zero, *(endogenous exit)*, and $v^1(k, b, \epsilon_i; s_l, \mu)$, *(stay)*, in equation (1). After paying ξ_o, the firm takes a binary expectation over the possibility of exogenous exit given by π_d in equation (2). In case of exogenous exit, the firm still maximizes its liquidation value at the end of the period, without considering its dynamics decisions. The value of surviving firms to the next period is then given by $v(k, b, \epsilon; s, \mu)$ as below.

$$v(k, b, \epsilon_i; s_l, \mu) = \max_{n,k',b',D} \Big[D + \sum_{m=1}^{N_s} \pi^s_{lm} d_m(s_l, \mu) \sum_{j=1}^{N_\epsilon} \pi^\epsilon_{ij} v^0(k', b', \epsilon_j; s_m, \mu')\Big] \tag{3}$$

subject to

$$0 \leq D = z_l \epsilon_i F(k, n) - w(s_l, \mu)n + (1 - \delta)k - b - \xi_o - k' + q(s_l, \mu)b'$$
$$b' \leq \theta k'$$
$$\mu' = \Gamma(s_l, \mu)$$

In (3), the firm optimally chooses its labor demand, n, furture capital, k', and new debt level, b', to maximize the sum of the firm's current dividends, D, and the beginning-of-the-period value, $v^0(k', b', \epsilon_j; s_m, \mu')$, at the next period. The current period dividends are the residual in the firm's budget constraint, and we restrict the dividends to be non-negative. The firm's future value is discounted by using the stochastic discount factor, $d_m(s_l, \mu)$. The financial friction is introduced as a collateral constraint for the firm's new borrowing in the above maximization problem.

$$v^e = \max \Big\{0, -\xi_e + \sum_{m=1}^{N_s} \pi^s_{lm} d_m(s_l, \mu) v^0(k_0, b_0, \epsilon_0; s_m, \mu')\Big\} \tag{4}$$

Lastly, we define the value of a potential entrant with (k_0, b_0, ϵ_0) as v^e in (4) above. The potential entrant makes a binary decision of entry by whether paying the entry cost ξ_e. Once it enters, the firm starts operation in the next period given its initial capital and productivity.

2.2 Households and Equilibrium

2.2.1 Representative Household

We assume that there is a unit measure of identical households in the economy. In each period, the households earn their labor income by supplying a fraction of the time endowment. The period utility is given by $U(C, 1 - N)$, and the households discount the future utility by a subjective discount factor, $\beta \in (0, 1)$. The representative household holds a comprehensive portfolio of assets; one-period firm shares with measure λ and non-contingent discount bonds ϕ. The household maximizes the lifetime expected discounted utility by choosing the quantities of aggregate consumption demand, C^h, and labor supply,

N^h, while adjusting the asset portfolio in each period. The household value, V^h, is defined as below.

$$V^h(\lambda, \phi; s_l, \mu) = \max_{C^h, N^h, \lambda', \phi'} \left[U(C^h, 1 - N^h) + \beta \sum_{m=1}^{N_s} \pi^s_{lm} V^h(\lambda', \phi'; s_m, \mu') \right] \quad (5)$$

subject to

$$\begin{aligned} & C^h + q(s_l, \mu)\phi' + \int_{\mathcal{S}} \rho_1(k', b', \epsilon'; s_l, \mu)\lambda'(d[k \times b \times \epsilon]) \\ & \quad \leq w(s_l, \mu)N^h + \phi + \int_{\mathcal{S}} \rho_0(k, b, \epsilon; s_l, \mu)\lambda(d[k \times b \times \epsilon]) \\ & \mu' = \Gamma(s_l, \mu) \end{aligned}$$

We apply the conventional notation for the stock prices. In (5), $\rho_1(k', b', \epsilon'; s_l, \mu)$ denotes the ex-dividend prices of firm shares in the current period, and $\rho_0(k, b, \epsilon; s_l, \mu)$ is the dividend-inclusive values from the current shareholding, λ. Let $\Phi^h(\lambda, \phi; s, \mu)$ be the household's decision for bond holding, and $\Lambda^h(k', b', \epsilon', \lambda, \phi; s, \mu)$ be the choice of firm shareholding corresponding to future state (k', b', ϵ').

2.2.2 Recursive Competitive Equilibrium and Prices

We define a recursive competitive equilibrium (RCE) of the economy. For convenience, we denote the distribution of producing firms as μ^p, while the measure of actual entrants as μ^e and the measure of endogenously exiting firms as μ^{ex}. Whenever possible, we suppress the arguments of functions in the definition below for simplicity.

> A RCE is a set of functions; prices $\left(w, q, (d_m)_{m=1}^{N_s}, \rho_0, \rho_1\right)$, quantities $\left(N, K, B, D, C^h, N^h, \Phi^h, \Lambda^h\right)$, and values $\left(v^0, v^1, v, v^e, V^h\right)$ that solve the optimization problem and clear markets, and that the associated policy functions are consistent with the aggregate law of motion as in the following conditions.
>
> 1. v^0, v^1, and v solve equations (1)–(3), and (N, K, B, D) are the associated policy functions for firms.
> 2. V^h solves (5), and $(C^h, N^h, \Phi^h, \Lambda^h)$ are the associated policy functions for households.
> 3. The labor market clears, $N^h = \int_{\mathcal{S}} N(k, \epsilon; s, \mu) \cdot \mu^p(d[k \times b \times \epsilon])$.

4. The goods market clears.

$$C^h = \left(\int_{\mathcal{S}} \Big[z\epsilon F(k,N) - (1-\pi_d)\big(K(k,b,\epsilon;s,\mu) - (1-\delta)k\big) + \pi_d(1-\delta)k \Big] \right.$$
$$\left. \cdot\, \mu^p(d[k \times b \times \epsilon]) \right) + \int_{U(0,\bar{k}_0)} (k_0 - \xi_e) \cdot \mu^e(d[k_0 \times b_0]) - \int_{\mathcal{S}} k \cdot \mu^{ex}(d[k \times b \times \epsilon])$$

5. The law of motion for the firm distribution, μ, is consistent with the value functions, (v^0, v^1, v, v^e), and the measures, (μ^p, μ^e, μ^{ex}), and with the associated policy functions, where Γ defines the mapping form μ to μ' given π_d, $K(k,b,\epsilon;s,\mu)$, and $B(k,b,\epsilon;s,\mu)$.

As noted by Khan and Thomas (2013), it is convenient to modify the firm's value functions, (v^0, v^1, v, v^e), by using the equilibrium prices at the market clearing conditions in the above. Let C and N be the market clearing quantities for aggregate consumption and labor supply from the perspective of households. Then the equilibrium value of output goods can be expressed by using the marginal utility of the market clearing consumption, $D_1U(C, 1-N)$. Likewise, the real wage, $w(s,\mu)$, is equal to the marginal rate of substitution between leisure and consumption. Next, the inverse of the discount bond price, q^{-1}, equals the expected gross real interest rate. Lastly, the stochastic discount factor, $d_m(s_l,\mu)$, is equal to the household's intertemporal marginal rate of substitution across states. The following equations summarize the equilibrium prices in terms of marginal utilities.

$$w(s,\mu) = \frac{D_2U(C, 1-N)}{D_1U(C, 1-N)}$$
$$q(s,\mu) = \frac{\beta \sum_{m=1}^{N_s} \pi^s_{lm} D_1U(C'_m, 1-N'_m)}{D_1U(C, 1-N)}$$
$$d_m(s,\mu) = \frac{\beta D_1U(C'_m, 1-N'_m)}{D_1U(C, 1-N)}$$

By using the above price expressions, we solve the equilibrium allocations directly from the firm's problem in a consistent manner with the household's optimal decisions. Let $p(s,\mu) \equiv D_1U(C, 1-N)$ denote the marginal utility of consumption at C and N. Then firms value their current output and dividends by $p(s,\mu)$, and the value functions, (v^0, v^1, v, v^e), can be re-written in terms of $p(s,\mu)$ without carrying the stochastic discount factor. In particular, we define new value functions, (V^0, V^1, V, V^e), by respectively multiplying $p(s,\mu)$ to the original values. Then the following equations (6)–(9) newly summarize the firm's problem.

$$V^0(k,b,\epsilon_i;s_l,\mu) = \max\left\{0, V^1(k,b,\epsilon_i;s_l,\mu)\right\} \tag{6}$$

$$\begin{aligned} V^1(k,b,\epsilon_i;s_l,\mu) = {} & \pi_d \cdot \max_n p(s_l,\mu)\left[z_l\epsilon_i F(k,n) - w(s_l,\mu)n + (1-\delta)k - b - \xi_o\right] \\ & + (1-\pi_d)\cdot V(k,b,\epsilon_i;s_l,\mu) \end{aligned} \tag{7}$$

$$V(k,b,\epsilon_i;s_l,\mu) = \max_{n,k',b',D}\left[p(s_l,\mu)D + \beta\sum_{m=1}^{N_s}\sum_{j=1}^{N_\epsilon}\pi^s_{lm}\pi^\epsilon_{ij}V^0(k',b',\epsilon_j;s_m,\mu')\right] \tag{8}$$

subject to

$$\begin{aligned} & 0 \le D = z_l\epsilon_i F(k,n) - w(s_l,\mu)n + (1-\delta)k - b - \xi_o - k' + q(s_l,\mu)b' \\ & b' \le \theta k' \\ & \mu' = \Gamma(s_l,\mu) \end{aligned}$$

$$V^e = \max\left\{0, -p(s_l,\mu)\xi_e + \beta\sum_{m=1}^{N_s}\pi^s_{lm}V^0(k_0,b_0,\epsilon_0;s_m,\mu')\right\} \tag{9}$$

For the reminder of this paper, we occasionally suppress the aggregate state (s,μ) in the price functions and the decision rules for simplicity.

2.3 Characterizing Firm-level Decisions

2.3.1 Firm Types

To characterize the firm-level decision rules in the model, it is convenient to first define a subset of firms in the distribution whose decisions are not affected by the collateral constraints in any possible individual state. In particular, we follow the approach in Khan and Thomas (2013) to distinguish firm types in our model economy. Note that this distinction is solely for deriving all the intertemporal decisions made by firms, and that a firm may alter its type depending on the individual state over time.

We define a firm as *unconstrained* when it has already accumulated sufficient wealth such that it never worries about facing a binding collateral constraint in any possible future state. In this case, the unconstrained firm's all Lagrangian multipliers on its borrowing constraints become zero, and the firm is indifferent between paying dividends and internal saving. On the other hand, the rest of firms in the distribution is defined as *constrained*. Constrained firms may or may not experience binding borrowing constraints in the current period, and they put non-zero probability of facing a binding constraint in the future. These constrained firms then choose to pay zero dividends in the current period, because the shadow value of retained earnings is greater than that of paying dividends.

We begin by deriving the decision rules for the unconstrained firms. We then systematically characterize the entire decision rules in the next subsection, after reducing firms' individual state-space. First, firms are allowed to optimally hire their desired labor in each period, because we do not assume any explicit adjustment friction in labor demand. Thus, all firms with the same (k, ϵ) choose $n = N^w(k, \epsilon; s, \mu)$ which solves the static labor condition, $z\epsilon D_2 F(k, n) = w(s, \mu)$.

Next, we consider the choice of future capital, k', by the unconstrained firms. The collateral constraint is irrelevant for this type of firms, so we can easily derive their optimal level of $k' = K^w(\epsilon; s, \mu)$ as follows. Let $\Pi^w(k, \epsilon; s, \mu) \equiv z\epsilon F(k, N^w) - wN^w$ be the current earnings of a firm with its optimal labor hiring N^w. Given the Markov property for the assumed stochastic processes and the lack of any capital adjustment cost, K^w is the solution to the following problem.

$$\max_{k'} \Big[- p(s_l, \mu)k' + \beta \sum_{m=1}^{N_s} \pi^s_{lm} p(s_m, \mu') \sum_{j=1}^{N_\epsilon} \pi^\epsilon_{ij} \big(\Pi^w(k', \epsilon_j; s_m, \mu') + (1-\delta)k'\big)\Big]$$

With the policy functions N^w and K^w, we make use of the definition of an unconstrained firm to derive the optimal borrowing policy for b'. As in Khan and Thomas (2013), the *minimum savings policy*, $b' = B^w(\epsilon; s, \mu)$, is recursively defined by the following two equations.

$$B^w(\epsilon_i; s_l, \mu) = \min_{(s_m, \epsilon_j)_{j,m}} \Big(\tilde{B}\big(K^w(\epsilon_i, s_l, \mu), \epsilon_j; s_m, \mu'\big)\Big) \tag{10}$$

$$\begin{aligned} \tilde{B}(k, \epsilon_i; s_l, \mu) = {} & z_l \epsilon_i F(k, N^w) - wN^w - \xi_o + (1-\delta)k - K^w(\epsilon_i) \\ & + q \min\big\{B^w(\epsilon_i; s_l, \mu), \theta K^w(\epsilon_i)\big\} \end{aligned} \tag{11}$$

$\tilde{B}(K^w, \epsilon_j; s_m, \mu')$ in (10) denotes the maximum level of debt (or, the minimum level of saving) that an unconstrained firm can hold at the beginning of the next period in which the exogenous state (s_m, ϵ_j) is realized. Having chosen the unconstrained choice of capital, K^w, at the current period, the firm will remain unconstrained in the subsequent periods by definition. The minimum savings policy, B^w, ensures that the firm's debt never exceeds this threshold level, $\tilde{B}$, given all possible realizations of (s, ϵ). Moreover, the threshold function can be retrieved by using B^w and K^w at the current period state with $(k, \epsilon_i; s_l, \mu)$, as in (11). Notice that the minimum operator in (11) reflects the collateral constraint in the firm's problem, so that B^w truly is the optimal borrowing policy for this type of firms. In (10), B^w again is determined to have the unconstrained firms unaffected by the constraint cover any future path of (s, ϵ).

2.3.2 Cash-on-hand and Decision Rules

The incumbent firm's problem in (8) is a challenging object because of the occasionally binding constraints for D and b'. In addition, notice that a firm's individual state vector includes two endogenous variables, (k, b), with each having a continuous support. Following the approach in Jo (2015), we collapse these two continuous individual state variables into a newly defined variable called *cash-on-hand*. This method does not alter the equilibrium allocation of the model, because the new individual state variable is a sufficient statistic for the information contained in k and b. By reducing the state-space in this way, the firm-level decision rules can be fully identified as functions of the cash-on-hand.

Define the cash-on-hand, $m(k, b, \epsilon)$, of a firm with (k, b, ϵ) by using the optimal labor demand N^w.

$$m(k, b, \epsilon; s, \mu) \equiv z\epsilon F(k, N^w) - wN^w + (1 - \delta)k - b$$

The cash-on-hand, $m(k, b, \epsilon)$, itself is a function of the individual state, (k, b, ϵ), and it represents the firm's net worth after production.[19] Notice that the decisions of k' and b' made by continuing incumbents immediately determine the level of cash-on-hand in the future period, $m(k', b', \epsilon'; s', \mu')$, along with the realization of (ϵ', s'). By using this new state variable, we transform the incumbent firm's problem in (6)–(8) into the corresponding values W^0, W^1, and W.

$$W^0(m, \epsilon_i; s_l, \mu) = \max \left\{0, W^1(m, \epsilon_i; s_l, \mu)\right\} \tag{12}$$

$$W^1(m, \epsilon_i; s_l, \mu) = \pi_d \cdot p(s_l, \mu) \cdot (m - \xi_o) + (1 - \pi_d) \cdot W(m, \epsilon_i; s_l, \mu) \tag{13}$$

[19] The per-period operation cost is not included in our definition of m. It does not make any difference in deriving the optimal decision rules, even when we include the cost in m.

$$W(m, \epsilon_i; s_l, \mu) = \max_{m',k',b',D} \left[p(s_l, \mu)D + \beta \sum_{m=1}^{N_s} \sum_{j=1}^{N_\epsilon} \pi^s_{lm} \pi^\epsilon_{ij} W^0(m'_{jm}, \epsilon_j; s_m; \mu') \right] \tag{14}$$

subject to

$$\begin{aligned}
& 0 \leq D = m - \xi_o - k' + q(s_l, \mu)b' \\
& b' \leq \theta k' \\
& m'_{jm} \equiv m(k', b', \epsilon_j; s_m, \mu') \\
& \quad = z_m \epsilon_j F(k', N^w(k', \epsilon_j)) - w(s_m, \mu')N^w(k', \epsilon_j) + (1 - \delta)k' - b' \\
& \mu' = \Gamma(s_l, \mu)
\end{aligned}$$

By using the cash-on-hand, we not only reduce the state space for an individual firm's problem, but also derive its decisions as functions of m. Given the unconstrained policies K^w and B^w, we define a threshold level of m that distinguishes the unconstrained firms out of the entire firm distribution μ. From the firm's budget constraint in (14), an unconstrained firm pays the current dividends, $D^w = m - \xi_o - K^w + qB^w \geq 0$. It follows that this firm's cash-on-hand is greater or equal to a certain threshold level, $\tilde{m}(\epsilon; s, \mu) \equiv \xi_o + K^w(\epsilon; s, \mu) - q(s, \mu)B^w(\epsilon; s, \mu)$. Therefore, any firm with $m(k, b, \epsilon; s, \mu) < \tilde{m}(\epsilon; s, \mu)$ can be identified as constrained.

Recall that, in any given period, some constrained firms experience currently binding borrowing constraints while the others do not. We call the latter constrained firms as *Type-1*, and the firms with currently binding constraints as *Type-2*. Notice that Type-1 firms can still adopt the unconstrained capital policy, K^w, but not the minimum savings policy, B^w. The debt policy of Type-1 firms can be easily determined by the zero-dividend policy after substituting $k' = K^w$ in the budget constraint. On the other hand, Type-2 firms can only invest to the extent their borrowing limits allow. This constrained choice of capital is derived from the level of cash-on-hand, m, held by Type-2 firms. Specifically, $D = 0$ implies that a Type-2 firm's decision of k' and b' must be feasible, given the firm's available cash-on-hand after paying the fixed operation cost. Given m, we define the upper bound of a Type-2 firm's capital choice as $\bar{K}(m) \equiv \frac{m - \xi_o}{1 - q\theta}$, from the binding collateral constraint. Firms with more cash-on-hand, therefore, can relax this upper bound until they can choose the unconstrained capital policy, K^w. Notice also that the upper bound, $\bar{K}(m)$, approaches to infinity as the financial parameter θ becomes closer to q^{-1}, which illustrates the case of perfect credit markets. Below, we summarize the decision rules of k' and b' by firm type given (m, ϵ); unconstrained, Type-1, and Type-2.

- Firms with $m \geq \tilde{m}(\epsilon; s, \mu)$ are unconstrained, and therefore adopt $K^w(\epsilon)$ and $B^w(\epsilon)$.
- For constrained firms with $m < \tilde{m}(\epsilon; s, \mu)$, the upper bound for k' is $\bar{K}(m) \equiv \frac{m - \xi_o}{1 - q\theta}$.

– Firms with $K^w(\epsilon) \leq \bar{K}(m)$ are Type-1, and adopt $k' = K^w(\epsilon)$ and $b' = \frac{1}{q}\big(K^w(\epsilon) + \xi_o - m\big)$.

– Firms with $K^w(\epsilon) > \bar{K}(m)$ are Type-2, and adopt $k' = \bar{K}(m)$ and $b' = \frac{1}{q}\big(\bar{K}(m) + \xi_o - m\big)$.

3 Model Parameters

We parameterize the model, and quantitatively investigate the implications of macroeconomic policies aimed at specific groups of firms in an economy. The model is annual, and we set the model parameter values of our benchmark economy, to jointly match the key macroeconomic moments and the rich firm-level heterogeneity observed in the U.S. data. Table 1 summarizes the calibrated parameter values.

The period utility function of the representative household is standard as in the literature. We specifically assume the functional form of indivisible labor following Rogerson (1988), $U(C, 1-N) = \log C + \psi(1-N)$. The decreasing-returns-to-scale (DRS) production function is in Cobb-Douglas form, $F(k, n) = k^\alpha n^\nu$ with $\alpha, \nu > 0$ and $\alpha + \nu < 1$.

The value of the household subjective discount factor, β, is set to imply the long-run real interest rate of 4 percent. The preference parameter of labor disutility, ψ, is set to match the average hours of worked at 0.33. The annual depreciation rate, δ, is chosen to match the average aggregate investment to capital ratio of the postwar U.S. economy. The curvature of capital input in the production function, α, is set to imply the average capital to output ratio of 2.3. The other production parameter, ν, is set to have the average share of labor income of 0.6, following Cooley and Prescott (1995). For the time series of aggregate investment, output, and private capital stock, we use the Fixed Asset Tables and National Income and Product Accounts (NIPA) from the Bureau of Economic Analysis (BEA).

We jointly determine the rest of model parameter values, $(\theta, \xi_o, \xi_e, \pi_d, \epsilon_m, \epsilon_M, \gamma_\epsilon, \rho_\epsilon)$, to replicate the aggregate debt to asset ratio, the average exit rate of firms, and the size and age distributions of firms.[20] In addition, we control the distribution of producing firms to have a unit measure at the steady state of the model. The financial parameter in the collateral constraint, θ, is set to be 0.9, to be consistent with the average debt to asset ratio of the non-farm, non-financial businesses in the Flow of Funds from 1954 to 2007. The period operation cost, ξ_o, and the fixed entry cost, ξ_e, are set to imply about

[20]To our best knowledge of the literature, this paper is one of the first attempts to match both firm size and age distributions in data, endogenously from a model with both financial frictions and endogenous entry and exit.

10 percent of total exit rate by firms, along with the exogenous exit rate, π_d.[21] We use the data tables from the Business Dynamics Statistics (BDS) to calculate the average total exit rate by the private firms in the U.S.[22] The BDS also reports the emprical firm size and age distributions in detailed size and age groups, with which we match in 6 employment size bins and 11 age bins.

We assume that the idiosyncratic productivity, ϵ, is drawn from a time-invariant distribution, $G(\epsilon; \epsilon_m, \epsilon_M, \gamma_\epsilon)$ which is a bounded Pareto distribution.[23] In each period, a firm in our model economy retains its previous level of individual productivity with a fixed probability of ρ_ϵ. We set $\rho_\epsilon = 0.75$ to be consistent with the evidence on the average persistence of firm-level productivity in the data.[24] The bounds of ϵ support, (ϵ_m, ϵ_M), and the shape parameter, γ_ϵ, of the Pareto distribution are chosen to have both the employment share and the population share at each firm size bin aligned with the corresponding average values reported in BDS from 1977 to 2007. We discretize ϵ by 11 values in our numerical applications.

Table 1 : Calibration

Model Parameters				
	Value	Description and Target Value		Model
β	0.96	Annual gross real interest rate	1.04	$1/q = 1.04$
ν	0.60	Labor share of income	0.60	
ψ	2.408	Average hours worked	0.33	$N = 0.3337$
δ	0.069	Investmet to capital ratio (BEA)	0.069	$I/K = 0.069$
α	0.272	Capital to output ratio (BEA)	2.30	$K/Y = 2.2443$
Calibrated to jointly match the following data moments, firm size and age distributions				
θ	0.90	Debt to Asset ratio (Flow of Funds)	0.36	$D/A = 0.3401$
π_d	0.0874	Average firm exit rate (BDS)	0.11-0.13	Exit rate = 0.1005
ξ_o	0.01382	Steady state measure of firms	1.0	$\int \mu \cdot d\mu = 1.0069$
ξ_e	0.1522			

[21]Notice that these parameters also jointly shape the firm age distribution in the economy.

[22]The BDS database is based on the U.S. Census, and covers more than 90 percent of the U.S. private employment starting from 1977.

[23]Jo (2015) shows that the conventional log AR(1) identification of ϵ is not enough to generate the highly-skewed firm size distribution observed in data. The bounded Pareto distribution requires a minimal set of parameters to replicate the entire firm size distribution in the model economy.

[24]There is a vast literature of measuring firm-level productivity processes. Our choice of the persistence value is consistent with that estimated in Foster, Haltiwanger, and Syverson (2008).

4 Results

4.1 Steady State: Firm Heterogeneity and Decisions

We begin with describing the stationary distribution of firms in the model. Figure 1 shows the entire firm distribution over capital and debt levels, (k, b), at the steady state. It displays all three types of firms that we considered in the previous section; unconstrained, Type-1, Type-2 firms. The unconstrained firms are located at each endpoint-mass of negative debt levels of the figure. These negative debt levels represent the minimum savings policy, $B^w(\epsilon)$, taken by this type of firms. The constrained firms are distributed across capital and debt, and the majority is relatively small with positive leverages.

The above firm types are more easily distinguishable when we look at the snapshot of decision rules on capital, k', debt, b', and dividends, D, as functions of a firm's cash-on-hand level, m. Figure 2 shows these firm-level decision rules at the median value of ϵ. In the figure, the level of cash-on-hand, $m(k, b, \epsilon)$, is on the horizontal axis, and we add the two vertical lines to distinguish the firm types. The line near $m = 4$ represents the threshold of being unconstrained, $\tilde{m}(\epsilon)$, and the one near 0.3 is the threshold for being Type-1. Starting from the right-hand side of the figure, when a firm has survived and accumulated sufficient wealth over time such that $m \geq \tilde{m}(\epsilon)$, it is considered as unconstrained. The firm then adopts the unconstrained choices of capital, $K^w(\epsilon)$, and debt, $B^w(\epsilon)$, and starts paying positive dividends. Constrained firms with m less than the threshold value, on the other hand, follow the zero-dividend policy to accumulate their internal savings toward being unconstrained. Type-1 firms in the middle part between the two thresholds, can still adopt the optimal level of capital, $K^w(\epsilon)$, while gradually reducing debt as their m increase. Lastly, Type-2 firms with small m are only able to invest until their borrowing limits, so their choice of capital is constrained with positive borrowings.

As discussed earlier, the level of cash-on-hand is critical in determining a firm's optimal decisions of investment and borrowing. Upon entry, young firms in our model start relatively small and then gradually accumulate their cash-on-hand over time. These life-cycle dynamics of firms are illustrated in Figure 3. At age 0, an average firm in our model economy is financially constrained because they are short of collateral for external financing. Thus, the firm keeps raising its external debt level until age 4, and then gradually de-leverages once it can finance the optimal level of investment for $K^w(\epsilon)$ around age 6. In addition, firms still accumulate financial savings even after age 18, so that they become unconstrained and pay positive dividends. This is represented by the hump-shaped leverage curve in the lower panel of the figure.[25] Any policy targeting firms of specific age group, hence, will shift the average lifecycle dynamics in Figure 3, which eventually re-shapes the entire firm distribution in the model economy. Further, such policy will also affect the entry and exit margins of firm decision by affecting the equilibrium prices.

Next, we compare the model-generated firm size and age distributions with those observed in BDS database. As evident in Figure 4, our model economy exhibits substantial firm-level heterogeneity that are endogenously determined by the combination of persistent shocks, financial frictions, and firm dynamics. In the upper panel of the figure, we distinguish firm employment size by 6 bins, and report the average population share in each bin from 1977 to 2007 in BDS. As well known in the literature, the empirical firm size distribution is highly skewed, where more than 88 percent of firms hire less than 20 employees in a given year. We almost perfectly replicate this lower tail of firm size distribution in our benchmark economy, by using the Pareto-distributed ϵ process.[26] By the nature of our collateral constraint, small firms in the size distribution are more likely to be financially constrained, when their productivity is expected to rise while the accumulated cash-on-hand is insufficient to cover their investment. In the lower panel of Figure 4, we report the age distribution of firms generated from the model.[27] Since our focus is on small and young firms, we directly use the detailed firm age bins as distinguished by the BDS database. As observed in data, the firm age distribution in our model preserves the decreasing population shares as firms become mature.[28] These realistic firm size and age distributions in the benchmark economy serve as the foundation for our policy exercises aimed at small and young firms. Therefore, replicating the observed firm heterogeneity is a necessary step, before we quantitatively examine the aggregate consequences of such policies.

Lastly, we look at the endogenous exit decision by firms at the steady state of the model. Figure 5 shows the exit choice (=1) over capital and debt at the ergodic distribution of ϵ. Consistent with the conventional knowledge, a firm in the model decides to

[25]In Figure 3, we do not intend to completely replicate the firm-level lifecycle dynamics as observed in data. In particular, the growth of young firms is quite fast in our model. However, we instead target the entire firm size and age distributions in our benchmark economy with a minimal set of assumptions governing firm dynamics.

[26]The table for the entire firm size distribution is available upon request. The BDS employment shares across firm size bins are targeted in our model economy, and then we count the number of firms in each bin to compute the population shares.

[27]We repeatedly simulate 50,000 firms to get the average firm age distribution in the model at the steady state.

[28]Due to the censored firms in the BDS database, we calculate the average population shares between 1993 and 2007.

exit when it accumulates relatively larger debt than its existing capital stock. This is because the future expected value of such over-leveraged firm falls below 0, so that the firm finds it better to exit before paying the fixed operation cost at the beginning of a period. Young firms with both low productivity and small wealth are easily tempted to this outside option of exit, so we are also able to capture the disproportionately high exit rates among those firms.[29] Figure 5 also illustrates that the exit decision is nonlinear at the margin, and therefore, the equilibrium effect of a policy will be ambiguous for exactly identifying the resulting change of the exit threshold.

4.2 Policy Experiments

4.2.1 Overview of Counterfactual Exercises

We conduct policy counterfactuals in the benchmark economy at the steady state. In particular, we consider two different policies that differ by their targeted firms; young firms and small firms. Each policy aims at relaxing the financial frictions for its targeted firms in need of more external borrowing. That is, we now vary the financial parameter in the model, θ, by firm characteristics of age and size.[30] This approach of adjusting θ is analogous to an investment subsidy by a government which helps only firms with binding borrowing constraints conditional on size and age, when we assume that the government effectively identifies such firms.

For the *age-dependent policy* that aims at young firms with age between 0 and 4, we consider a counterfactual exercise specifically with $\theta = q^{-1} > \theta_{ss}$.[31] This implies that the government subsidizes the additionally required investment for the targeted firms such that they can obtain the efficient scale of production at a given level of productivity. Likewise, we apply the same value of θ for the *size-dependent policy* for small-medium sized firms (SME) hiring less than 499 employees.[32] The rest of firms not affected by the

[29] Note that, in our model, the exogenous exit applies symmetrically to all incumbent firms regardless of firm characteristics. The state-dependent decision of exit, on the other hand, helps replicating this micro-level evidence of firm exit behavior, once the skewed firm size distribution is considered.

[30] This is one of the main differences in our approach from the work by Guner, Ventura, and Xu (2008). They investigate the quantitative effects of distortionary taxes on factor input uses with heterogeneous firms. Thus, once a firm falls into the targeted group in their model, its size is always restricted by such policies. In our approach, however, only firms with binding borrowing constraints can be subsidized for their desired investment. Moreover, we also consider the age dimension of firm heterogeneity and the associated policy in this paper.

[31] Different values of θ do not alter the aggregate implications of the policies. The results are available upon request.

[32] Our definitions of small and young firms are consistent with the empirical literature. In particular, we follow Fort, Haltiwanger, Jarmin, and Miranda (2013). Moreover, firms eligible to business loan programs

above policies still maintain the value of θ at the benchmark economy. Notice that the above policies are intended to reduce the financial frictions experienced by the constrained firms, conditional on firm size or age. Therefore, the actual number of firms affected by each policy is a smaller subset of the targeted groups.

For the additionally required resources to relax firms' borrowing constraints (or, the aggregate subsidy), we assume that it is financed by a lumpsum transfer from the households. In short, the amount of resource transfer is determined by the distribution of firms and the level of unconstrained capital, $K^w(\epsilon)$, at a given price. To exactly measure the required resources under a certain policy, we isolate the firms with binding borrowing constraints. We then compute those firms' efficient investment, i, and the constrained level of investment without the policy, i^B. The resource transfer (rt) is computed by aggregating the gap between i and i^B,

$$rt(p) = \int_{\mu^B} (i - i^B) d\mu^B$$

, where μ^B denotes the measure of firms with binding borrowing constraints who are affected by the policy. We account this resource transfer in the market clearing condition of the equilibrium under each policy regime.[33]

We make comparisons of the aggregate consequences between the competing policies, in relative to the benchmark. To do so, we calculate the aggregate improvements in terms of efficiency under a certain policy, by considering its associated resource transfers. Specifically, let ΔTFP denote the relative gain in measured total factor productivity when a policy is implemented. Then the ratio, $\frac{\Delta TFP}{rt}$, serves as our measure of efficiency gain from each policy experiment.[34] In this way, we are able to compare the improvements in aggregate resource allocation across firms between size- and age-dependent policies, without arbitrarily controlling the required resource transfers.

from the U.S. Small Business Administration typically hire more than 500 employees in most industries. We also report the results from the size-dependent policy only for small firms with less than 20 employees in the next subsection.

[33]Note that the market clearing prices in turn affect the firm distribution in our model economy when θ is adjusted. In particular, changes in $K^w(\epsilon)$ and $\bar{K}(m)$ alter the composition of firm types in equilibrium; unconstrained, Type-1, and Type-2 firms.

[34]This efficiency measure is analogous to a net multiplier. By definition, the value is set to 0 in our benchmark economy.

4.2.2 Aggregate Results

We begin with comparing the aggregate results between *size-dependent* and *age-dependent* policies of relaxing collateral constraints faced by their respective targeted firms. Table 2 reports the policy counterfactual results, along with those from the benchmark economy. In each exercise, we maintain the model parameter values at the benchmark economy except θ, and then find new equilibrium allocations.

Table 2 : Aggregate Results from Policy Experiments

Policy Counterfactual: Aggregates				
	Benchmark	Age-dependent (age 0 to 4)	Size-dependent (SMEs)	Size-dependent (small only)
C	0.2103	0.2147 (2.11)	0.2157 (2.54)	0.2150 (2.23)
K	0.6322	0.6675 (5.58)	0.6817 (7.83)	0.6688 (5.79)
Y	0.2817	0.2887 (2.49)	0.2911 (3.34)	0.2890 (2.59)
N	0.3337	0.3350 (0.39)	0.3363 (0.78)	0.3349 (0.36)
D/A	0.3401	0.3664	0.3692	0.3658
firms (μ)	1.006848	0.978711	0.982922	0.981148
endo. exit rate	0.013172	0.020803	0.024077	0.020207
rt	–	0.136042	0.175993	0.129888
μ subsidized	–	0.041864	0.089157	0.073906
$\Delta TFP(\%)$	(0.616511)	0.746149	0.769246	0.815537
$\Delta TFP/rt$	–	0.033814	0.026947	0.038709
avg. rt	–	3.249618	1.973967	1.757476
$\Delta TFP_{\mu}(\%)$	–	1.112317	1.079934	1.149754

Note: We use (α, ν) values at the benchmark to compute the measured TFP under each policy. Values in parenthesis are percentage changes from their benchmark values.

As evident from reducing financial frictions by firm characteristics of size and age, all policies considered in Table 2 improve macroeconomic performance of the model economy. Specifically, the age-dependent policy of targeting young firms with age 0 to 4 raises aggregate output by 2.5 percent, in relative to the benchmark case. The aggregate output under the size-dependent policy for small-medium sized firms (SMEs), on the other hand, further increases by 3.3 percent. We can see this significant improvement mainly comes from helping the smallest firms in the economy, by looking at the last column of Table 2. In that, the aggregate output still increases by 2.6 percent when the size-dependent policy is adopted only for small firms hiring less than 20 employees. More importantly, the consumption under each policy still improves more than 2 percent in relative to the benchmark, which is after deducting the lumpsum resource transfers for the aggregate investment subsidy. Hence, those policies in our experiments not only boost aggregate outputs, but also enhance household welfare measured by consumption in each period.

These aggregate improvements are largely driven by the reduced resource misallocation across firms by the above policies. In our model, the collateral constraint prevents small but productive firms from undertaking their desired levels of investment. Thus, investment goods are not efficiently allocated across firms in the presence of financial frictions. This misallocation emerges as a loss in measured total factor productivity (TFP) at aggregate level, and it follows that the size- or age-dependent policy will lead to a higher aggregate TFP than that of the benchmark. In Table 2, we report these relative gains in measured TFP (ΔTFP). In overall, the gains in TFP are relatively small, ranging from 0.75 to 0.82 percent. However, this result is from subsidizing only firms with binding borrowing constraints which account for about 4 to 9 percent in the entire population.[35] In addition, the increases in TFP become more pronounced when we calculate the gains in average productivity per firm, ΔTFP_{μ}. The productivity gain per firm is 1.1 percent under the age-dependent policy, which is similar to the gain when only smallest firms are targeted. As mentioned earlier, we also consider how effective each policy is in terms of improving the measured TFP compared to the required resource transfers from households ($\Delta TFP/rt$). According to Table 2, the age-dependent policy is more effective in improving TFP per each unit of resource transfer than the size-dependent policy for SMEs. However, the effectiveness of the size-dependent policy becomes larger when its target becomes smaller as shown in the last column of the table. Therefore, these differences in policy effectiveness are critical in designing and implementing actual policies in practice.[36] In terms of the average resource transfer per firm, the age-dependent policy requires relatively larger subsidies for each firm than the other policies. This result indicates that it is young firms that are most severely affected by the borrowing constraints in our model economy.

We also highlight that the endogenous entry and exit by firms are important in quantifying the aggregate results of policies for small and young firms. In every policy counterfactual experiment that we report in Table 2, the total number of firms decreases by 1.7 to 2.1 percent, in relative to the benchmark economy. The relatively fewer firms in operation under the subsidizing policies represent the equilibrium price effects on firm

[35] Clearly, our counterfactuals involve much less number of financially constrained firms, when compared to its empirical counterpart. In this context, our aggregate results from the model economy can be viewed as conservative.

[36] As an example, the case of information asymmetry between firms and the government may distort the allocation of investment subsidies. In this paper, we do not consider this possibility, but the age-dependent policy in the above can be a possible solution, though not complete. This is because firm age information is clear in data, relative to that of firm size or productivity, and the number of total young firms is much smaller than that of total small firms.

level decisions of entry and exit. In labor markets, due to the boosted productivity in aggregate, higher labor input usages by firms lead to an increase in equilibrium wage rate. The rise in factor price puts more pressure on firms with low profitability to exit. This mechanism is represented by the higher endogenous exit rates under policies in Table 2, ranging from 2 to 2.4 percent in each period. Moreover, the higher wage rate also affects the entry decision by potential firms, which enhances the selection among them. The potential entrants observe higher profitability under each policy regime which relaxes borrowing constraints, but the increased cost of production prevents firms with low productivity from entering. As will be discussed in the next section on firm dynamics, the selection effect on the entry decision dominates, and hence the average productivity of actual entrants becomes higher in our policy experiments. The persistence of idiosyncratic productivity implies these entrants will still maintain a higher productivity level during their early stages of lifecycle, which improves the aggregate TFP further. Therefore, our policies targeting only incumbents adjust firm entry and exit margins actively through the equilibrium price channel, which accelerates the cleansing effect from replacing less productive incumbents with young firms.[37] This yields additional gains from such policies in our model economy.

4.2.3 Firm Dynamics

We now look at how firm dynamics are affected by the investment subsidizing policies considered in Table 2, in relative to the benchmark economy. In particular, we simulate 50,000 firms repeatedly at the steady state of the model economy under each policy, and then report the average size and productivity of firms at each age upon entry.

Figure 6 compares the average firm dynamics of entrants under each policy. The upper-left panel of the figure shows the typical growth pattern of entrants in our benchmark economy (*blue line*), similarly to what is observed in data. First, without any policy intervention, on average, firms start small in terms of capital size. That is, the relative size of entrants to that of mature firms at age 20 is about one-fourth. Over time, these entrants become gradually larger as they accumulate more cash-on-hand until age 9.[38]

[37] This result is related with the literature of creative destruction along with business cycles, in Caballero and Hammour (1994). They emphasize the cleansing effect of recessions from replacing low productive-outdated production units with innovative ones. In our work, however, we rather focus on the steady state comparisons of different policies for incumbents.

[38] We acknowledge that the duration of firm growth upon entry in our benchmark is relatively short, when compared to that in data. This is because the only ingredient that hinders immediate firm growth in our model is the borrowing constraints. Thus, our results can be viewed as conservative in the absence of other real or financial frictions for young firms.

As a cohort of firms gets older, the selection process among entrants forces unproductive and less-profitable firms out of economy, via their endogenous exit decisions. Hence, as shown in the upper-right panel of Figure 6, the average productivity of entrants increases over time as they become mature. Specifically, the productivity of age-0 firms in our benchmark economy is about 10 percent lower than that of firms at age 20.

Under the age-dependent policy for young firms, the relative size of entrants is almost the same as that of mature firms in Figure 6 (*red line*). Clearly, this is because the subsidizing policy for firms with age 0 to 4 removes any binding borrowing constraints for investment. Until age 5, the entrants keep growing even larger than the mature firms that face the financial frictions on average, while investments among them are efficiently allocated. Thus, the young firms in this case initially have the largest borrowing upon entry, and then gradually de-leverage over time, conditional on survival (lower-left panel). In addition, from the upper-right panel of Figure 6, the productivity of age-0 firms under the age-dependent policy is larger than its counterpart in the benchmark case. It follows that our policy intended for incumbents induces a strong selection among the potential entrants, so that relatively more productive firms actually enter and start operating, which in turn lifts the aggregate productivity. This is mainly through the equilibrium price feedback effect on firms' entry decision, and we will further discuss this in the next subsection. Once firms become older than 5, the average size reverts back to the case with borrowing constraints. However, since those firms were able to accumulate more cash-on-hand under this policy, they tend to start saving faster in terms of financial assets after age 5, as illustrated by the steeper slope of de-leveraging over time.[39] This implies that, at age 5, firms are better prepared to idiosyncratic risk due to their lower leverage ratio which is a relatively better financial position. Hence, we confirm that our age-dependent policy further supports the young-productive firms by allowing them to survive longer, in a similar way of *selectively protecting infant firms*.[40]

In the case of subsidizing SMEs (*green line* in Figure 6), on the other hand, we observe the typical firm dynamics patterns as in the benchmark. Indeed, the size-dependent policy for SMEs in our counterfactual experiment has only level effects on the average size and

[39]Recall that firms in our model economy operate with DRS production technology. Once a firm reaches its optimal scale, any profits can be used to accumulate financial wealth, when there is no change in its productivity.

[40]This result shares the similar view with Moll (2014), when firm-level productivity shocks are persistent. Moll (2014) distinguishes the transition and the steady state losses from misallocation due to financial frictions. When productivity shocks are sufficiently persistent, he shows that the loss at the steady state is relatively small, but the economy stays longer in transition. Our policy exercises consider the possibility of expediting the transition process by allowing firms' self-financing.

productivity by firm age.[41] In particular, the policy helps entrants start relatively large and then allows them to mature faster. This is mainly driven by the higher on average productivity in the early stage of a firm's lifecycle. These level effects are mainly due to the nature of our size-dependent policy which only affects financially constrained SMEs at each point of time in the model. That is, whenever a productive firm falls short of its collateral, regardless of age, the policy effectively finances the firm's optimal investment matching its productivity. The equilibrium price effects on firm entry and exit decisions still exist as in the age-dependent policy, but the selection among potential entrants are relatively small, which is represented by the initial average productivity of age-0 firms.

Next, we look at the implications of firm dynamics under a size- or age-dependent policy on the entire firm distributions. In Figure 7, we compare firm size and age distributions under each policy regime. From our results of firm dynamics in the above, it is evident that both type of subsidizing policies raise the population share of the smallest firm size group (upper panel) by a similar magnitude. On the other hand, the lower tail of a firm age distribution (lower panel) is largely affected by the type of policy. When the age-dependent policy is implemented, due to the relatively strong selection effect on entry, the number of age-0 firms is slightly lower than the benchmark. In contrast, the share of all young firms increases relatively more under the size-dependent policy.

4.2.4 General Equilibrium vs. Partial Equilibrium

So far, we conducted the previous policy counterfactuals in a general equilibrium (GE) environment. This involves endogenous changes in the equilibrium price of the model which is the real wage rate (w).[42] In fact, the equilibrium wage under each policy is higher than the benchmark wage. In the following, we illustrate the importance of the GE price effect on firm entry and exit margin by comparing the firm dynamics under GE against them under Partial Equilibrium (PE). In particular, we only report the results from the age-dependent policy because the main intuition still holds under different subsidizing policies.

Figure 8 compares firm dynamics with and without the equilibrium price adjustments under the age-dependent policy. Clearly from the figure, the dynamics of average firm size and productivity under PE are not much different from those in GE environment. One

[41]We plot the average capital and productivity in Figure 6 as their relative values at each age compared to the mature firms at age 20. In absolute values, the results from the size-dependent policy (*green line*) are almost parallel shifts from their counterparts in the benchmark (*blue line*).

[42]Since we only compare the outcomes from different stationary equilibria, the risk-free rate is the same across the policy experiments.

noticeable difference, however, is the absolute capital size at each age is relatively larger in PE. This indirectly but intuitively shows the effect of higher wage rate in GE, because both the optimal capital choice, $K^w(\epsilon)$, and the constrained capital choice, $\bar{K}(m)$, are inversely related with the wage at the steady state of the model. Note that the initial productivity level of an entrant is drawn from the same ergodic distribution of ϵ, which implies that the relative share of firms with each ϵ value is the same regardless of the total number of entrants. Thus, given the same average level of productivity in PE and GE, the average size of firms becomes smaller with an increase in w.

More importantly, the change in equilibrium wage adjusts the margins of exit and entry decisions. Other things being equal, a firm will be more inclined to go out of business, when the wage rate increases which hurts the profitability of the firm. This effect will be more pronounced when the firm is highly leveraged, so that it will find it better to exit instead of paying the fixed operation cost. The relatively higher endogenous exit rate under each subsidizing policy in Table 2, when compared to the benchmark value, reflects this active exit behavior by firms in response to a rise in w.

In addition, the rise in equilibrium wage strengthens the selection among the potential entrants. Without any change in wage, recall that both the age- and size-dependent policies raise the value of entry, V^1, in Equation (6). This implies that more firms are willing to pay the fixed entry cost, given a fixed mass of potential entrants, M^e. At the same time, the rise in wage accompanied by the above policies also lowers the value of entry by reducing the lifetime expected profits. This selects only productive-enough firms that can afford the higher cost of production enter the economy by paying the entry cost. Since the previous two effects of a policy work in opposite directions, the overall effect with the equilibrium price adjustment on firm entry is ambiguous. However, our quantitative experiments in GE show that the selection effect on entry is stronger, and therefore, the average productivity of entrants is higher under each policy, as shown in Figure 6.[43]

From the increase endogenous exit and the selection on firm entry, the equilibrium measure of firms, μ, decreases under each policy in Table 2.

Without these adjustments in exit and entry, under PE, the total number of firms rises by about 8 percent (Figure 8), which pumps-up the aggregate gains from the size-

[43]This result is somewhat different from what Buera, Moll, and Shin (2013) predict in a similar model environment. They show that credit subsidies improve aggregates in the short-run, while distorting the entry into entrepreneurship in the long-run due to the fixed subsidy scheme over time. However, our policies considered in this paper are dependent on firm size or age, and our results on firm entry shows a positive gain from the selection effect.

and age-dependent policies in a misleading way. In sum, our point here is that the GE environment is crucial in more precisely quantifying the macroeconomic impact of a policy for firms' external financing.

5 Concluding Remarks

Are policies targeted to small firms the answer for accelerating the aggregate productivity level? To address these questions, we construct a general equilibrium model with heterogeneous firms that are allowed to endogenously enter and exit but face collateral constraints. Then we calibrate the model to conduct policy counterfactual exercises to ask which policies could potentially nourish the firm dynamics that result in enhancing aggregate productivity. Our results critically highlight the role of indirect general equilibrium effects of the targeted policies in affecting micro-level firm dynamics with a strong selection process. This also reshapes firm size and age distributions.

Enhancing productivity in aggregate is an immediate policy relevance for many countries, especially for Japan and many European countries. A variety of industrial targeted policies have been implemented in these countries. The prime example includes the Act Concerning Temporary Actions to Facilitate Finance of Small and Medium Sized Enterprises, enacted in Japan on December 4, 2009. By this, banks are asked to make efforts to accept amendments to the terms and conditions of loans requested by small and medium enterprises. While our approach is general, we believe that it may be useful to make quantitative investigations for this type of a *Moratorium* policy. Lastly, since our policy analyses in this paper are silent on the aggregate dynamics, studies on the transitional dynamics with subsidized credits during a recession are relevant and promising. This is left for future research.

References

Atkeson, A. and P. J. Kehoe. 2005. "Modeling and Measuring Organization Capital." *Journal of Political Economy* 113 (5):1026–1053.

Bartelsman, E., J. Haltiwanger, and S. Scarpetta. 2013. "Cross-Country Differences in Productivity: The Role of Allocation and Selection." *American Economic Review* 103 (1):305–334.

Buera, F. J., J. P. Kaboski, and Y. Shin. 2011. "Finance and Development: A Tale of Two Sectors." *American Economic Review* 101 (5):1964–2002.

Buera, F. J. and B. Moll. 2015. "Aggregate Implications of a Credit Crunch: The Importance of Heterogeneity." *American Economic Journal: Macroeconomics* 7 (3):1–42.

Buera, F. J., B. Moll, and Y. Shin. 2013. "Well-intended policies." *Review of Economic Dynamics* 16 (1):216–230.

Buera, F. J. and Y. Shin. 2013. "Financial Frictions and the Persistence of History: A Quantitative Exploration." *Journal of Political Economy* 121 (2):221–272.

Caballero, R. J. and M. L. Hammour. 1994. "The Cleansing Effect of Recessions." *American Economic Review* 84 (5):1350–1368.

Clementi, G. L. and B. Palazzo. 2015. "Entry, Exit, Firm Dynamics, and Aggregate Fluctuations." *American Economic Journal: Macroeconomics* forthcoming.

Collier, B. L., A. F. Haughwout, H. C. Kunreuther, E. O. Michel-Kerjan, and M. A. Stewart. 2016. "Firm Age and Size and the Financial Management of Infrequent Shocks." NBER working paper 22612.

Cooley, T. F. and E. C. Prescott. 1995. "Economic Growth and Business Cycles." *Frontiers of Business Cycle Research* Princeton University Press.

Cooley, T. F. and V. Quadrini. 2001. "Financial Markets and Firm Dynamics." *American Economic Review* 91 (5):1286–1310.

Davis, S. J., J. Haltiwanger, and S. Schuh. 1996. "Job creation and job destruction." MIT Press.

Dunne, T., M. J. Roberts, and L. Samuelson. 1989. "The Growth and Failure of U. S. Manufacturing Plants." *Quarterly Journal of Economics* 104 (4):671–698.

Evans, D. S. 1987. "The Relationship Between Firm Growth, Size, and Age: Estimates for 100 Manufacturing Industries." *Journal of Industrial Economics* 35 (4):567–581.

Fort, T. C., J. C. Haltiwanger, R. S. Jarmin, and J. Miranda. 2013. "How Firms Respond to Business Cycles: The Role of Firm Age and Firm Size." *IMF Economic Review* .

Foster, L., J. C. Haltiwanger, and C. Syverson. 2008. "Reallocation, Firm Turnover, and Efficiency: Selection on Productivity or Profitability?" *American Economic Review* 98 (1):294–425.

Gourio, F. and N. Roys. 2014. "Size-dependent regulations, firm size distribution, and reallocation." *Quantitative Economics* 5 (2):377–416.

Guner, N., G. Ventura, and Y. Xu. 2008. "Macroeconomic implications of size-dependent policies." *Review of Economic Dynamics* 11 (4):721–744.

Haltiwanger, J. C., R. S. Jarmin, and J. Miranda. 2016. "Who Creates Jobs? Small versus Large versus Young." *Review of Economics and Statistics* 95 (2):347–361.

Hopenhayn, H. A. 1992. "Entry, exit, and firm dynamics in long run equilibrium." *Econometrica* 60 (2):1127–1150.

Hsieh, C. and P. J. Klenow. 2014. "The Life Cycle of Plants in India and Mexico." *Quarterly Journal of Economics* 129 (3):1035–1084.

Jo, I. 2015. "Firm Size and Business Cycles with Credit Shocks." Working paper.

Khan, A., T. Senga, and J. K. Thomas. 2016. "Default risk and aggregate fluctuations in an economy with production heterogeneity." Working paper.

Khan, A. and J. K. Thomas. 2013. "Credit Shocks and Aggregate Fluctuations in an Economy with Production Heterogeneity." *Journal of Political Economy* 121 (6):1055–1107.

Kiyotaki, N. and J. Moore. 1997. "Credit Cycles." *Journal of Political Economy* 105 (2):211–248.

Midrigan, V. and D. Y. Xu. 2014. "Finance and Misallocation: Evidence from Plant-Level Data." *American Economic Review* 104 (2):422–458.

Moll, B. 2014. "Productivity Losses from Financial Frictions: Can Self-Financing Undo Capital Misallocation?" *American Economic Review* 104 (10):3186–3221.

Moreira, S. 2016. “Firm Dynamics, Persistent Effects of Entry Conditions, and Business Cycles.” Working paper.

Osotimehin, S. 2016. “Aggregate productivity and the allocation of resources over the business cycle.” Working paper.

Rogerson, R. 1988. “Indivisible labor, lotteries and equilibrium.” *Journal of Monetary Economics* 21 (1):3–16.

Sedlacek, P. 2015. “Lost Generations of Firms and Aggregate Labor Market Dynamics.” Working paper.

Sedlacek, P. and V. Sterk. 2016. “The Growth Potential of Startups Over the Business Cycle.” Working paper.

Siemer, M. 2016. “Firm entry and Employment Dynamics in the Great Recession.” Working paper.

Figure 1

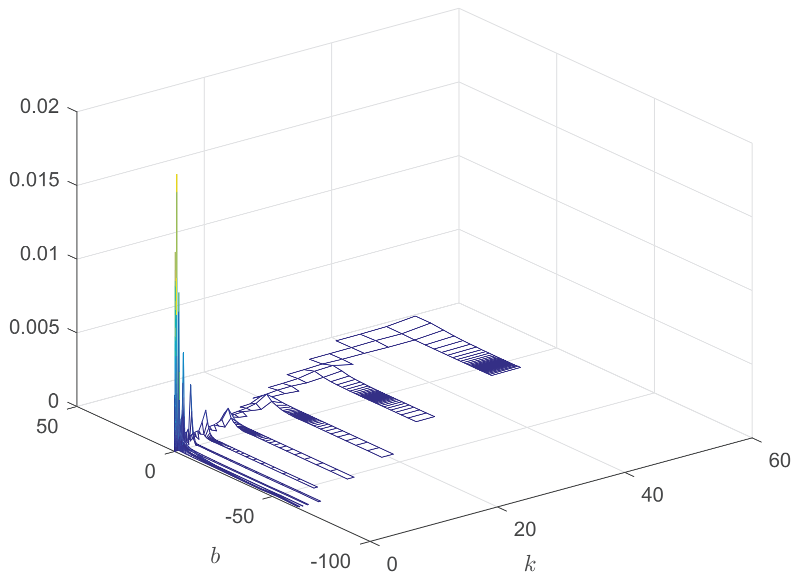


Figure 2

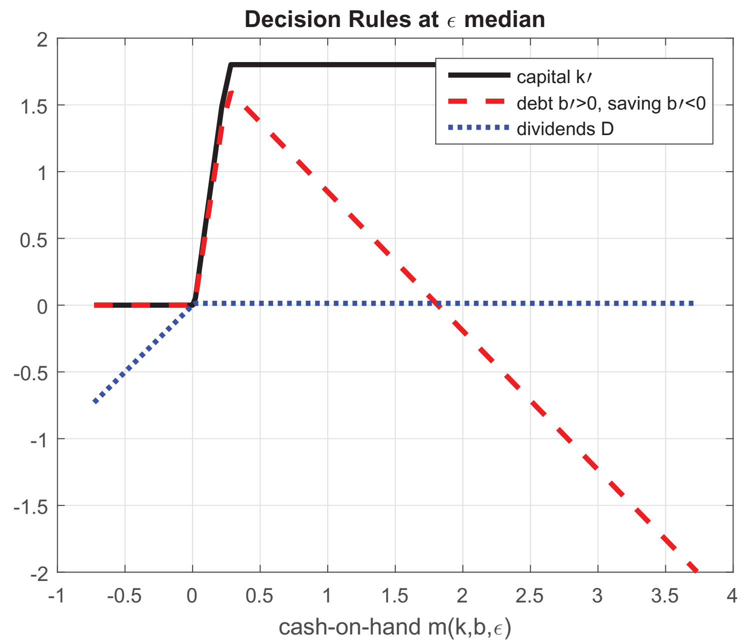


Figure 3

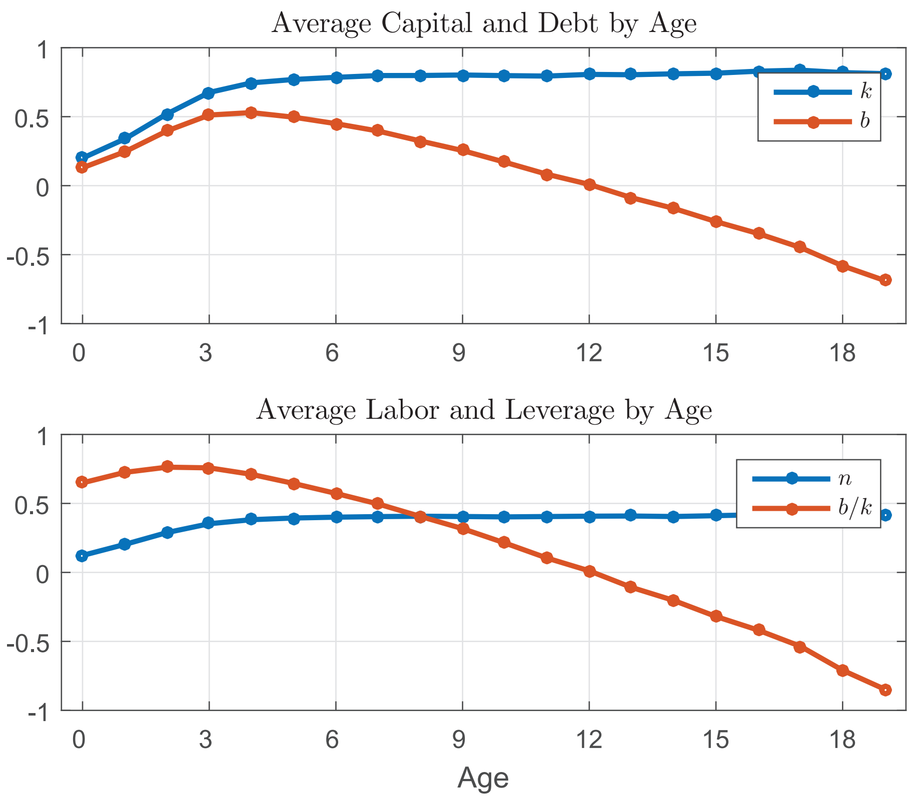
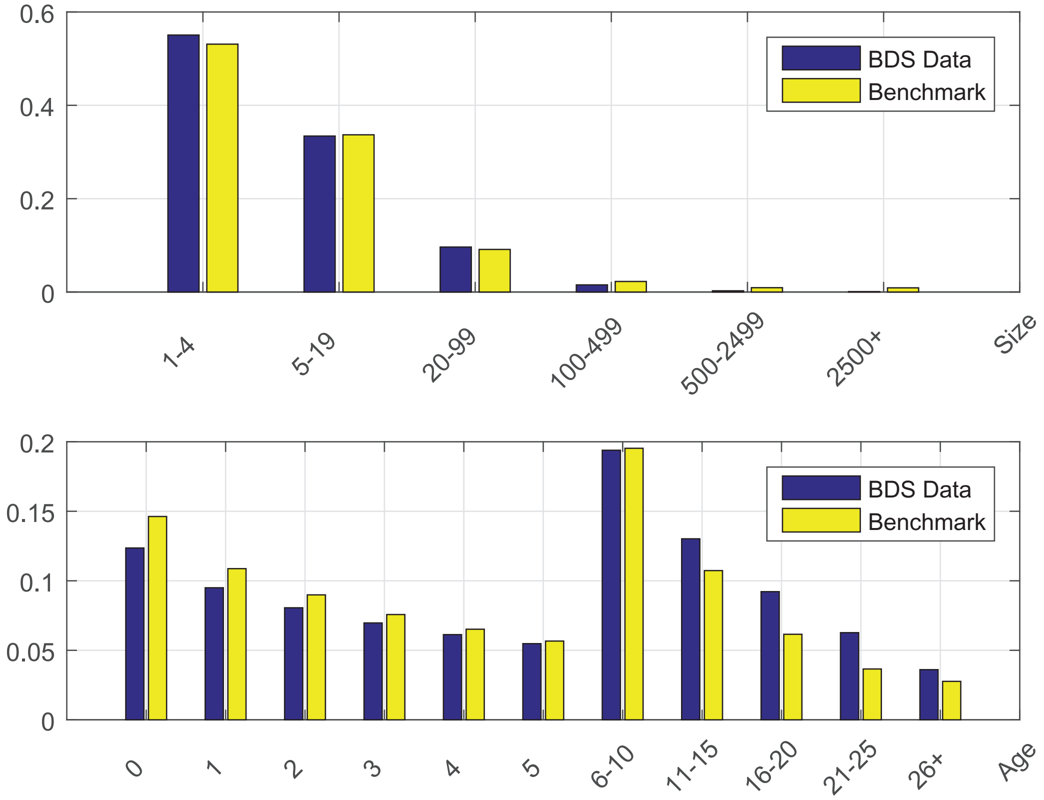

Average Capital and Debt by Age
k
b
Average Labor and Leverage by Age
n
b/k
Age

Figure 4



0.6
0.4
0.2
0
BDS Data
Benchmark
1-4
5-19
20-99
100-499
500-2499
2500+
Size
0.2
0.15
0.1
0.05
0
BDS Data
Benchmark
0
1
2
3
4
5
6-10
11-15
16-20
21-25
26+
Age

Figure 5

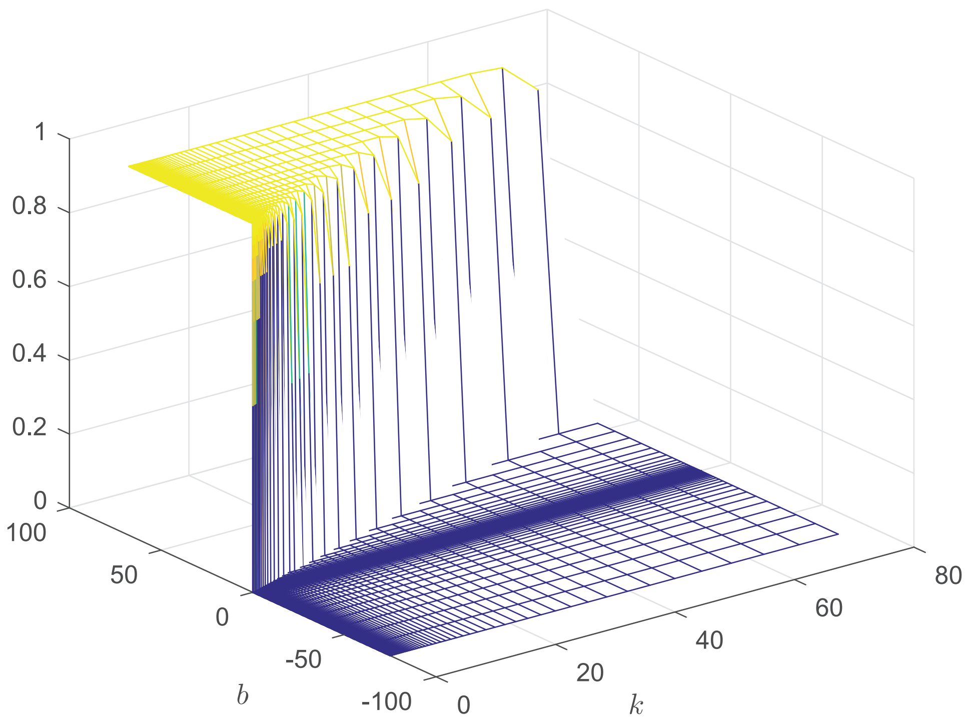
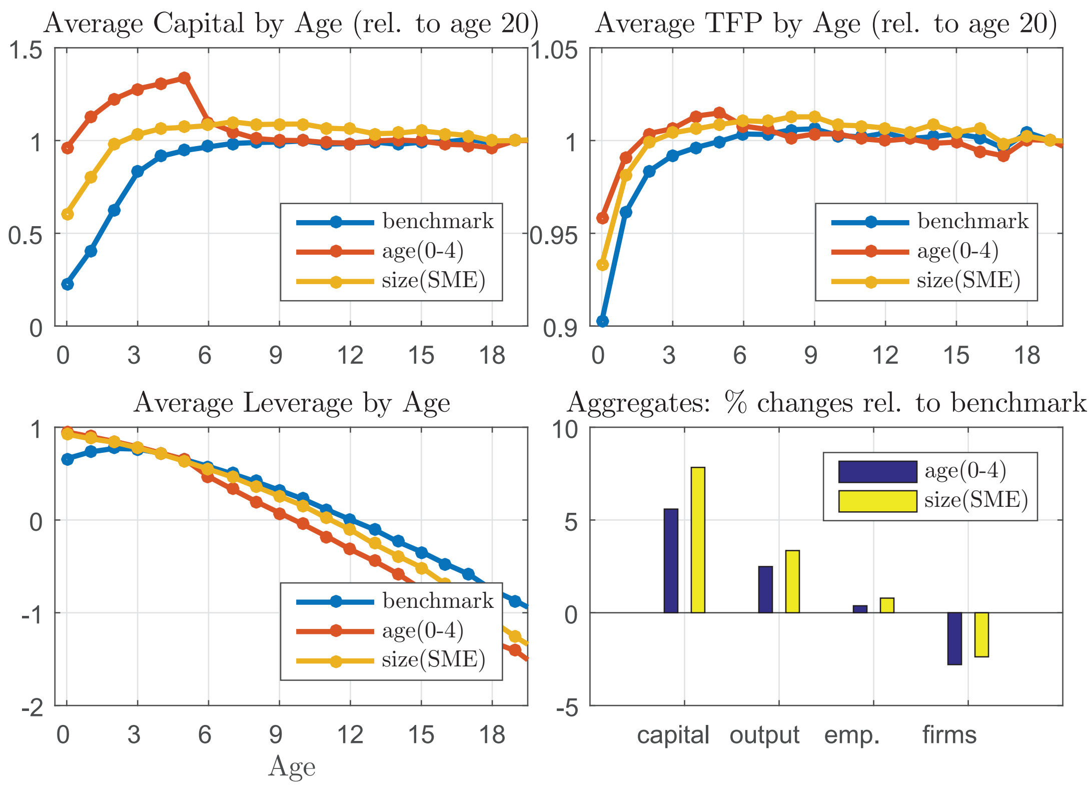


Figure 6



Average Capital by Age (rel. to age 20)
Average TFP by Age (rel. to age 20)
Average Leverage by Age
Aggregates: % changes rel. to benchmark
benchmark
age(0-4)
size(SME)
Age
capital
output
emp.
firms

Figure 7

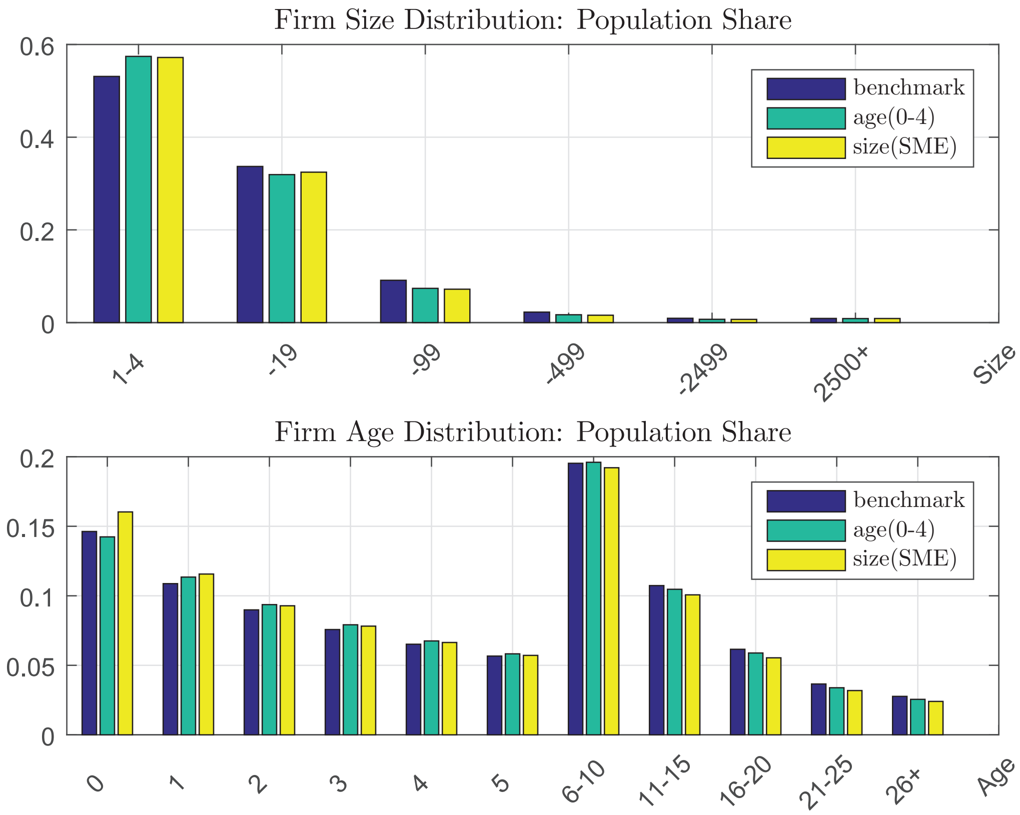
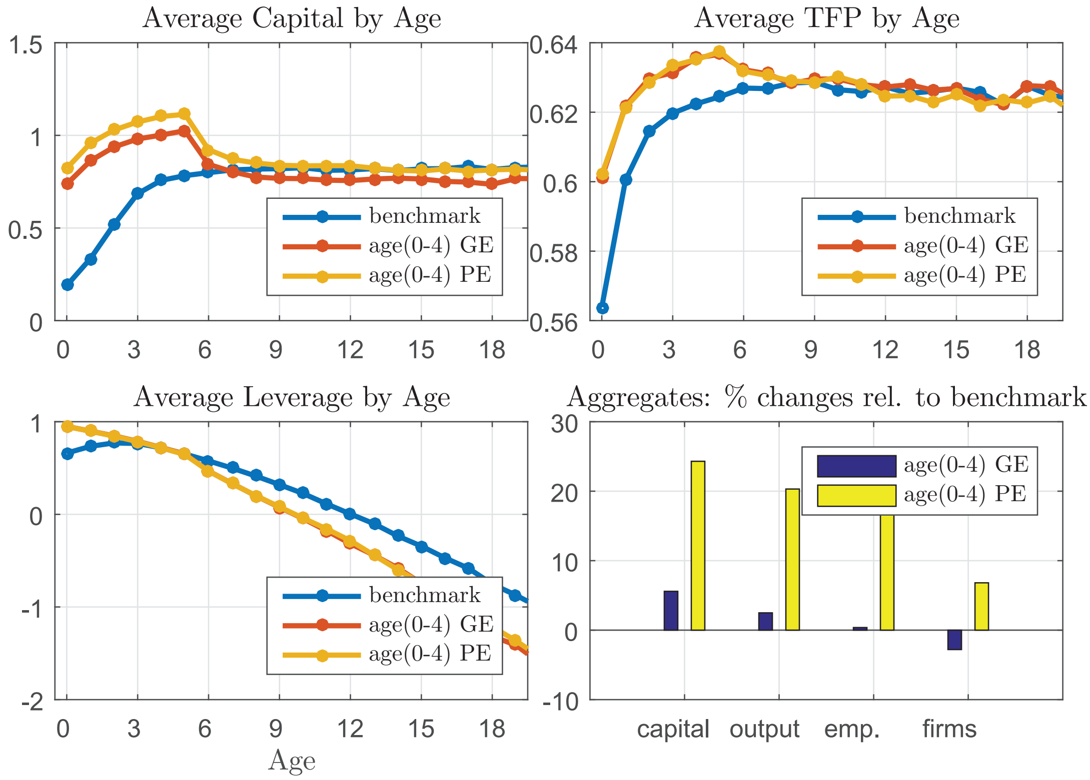

Firm Size Distribution: Population Share
0.6
0.4
0.2
0
benchmark
age(0-4)
size(SME)
1-4
-19
-99
-499
-2499
2500+
Size
Firm Age Distribution: Population Share
0.2
0.15
0.1
0.05
0
benchmark
age(0-4)
size(SME)
0
1
2
3
4
5
6-10
11-15
16-20
21-25
26+
Age

Figure 8



Average Capital by Age
1.5
1
0.5
0
0
3
6
9
12
15
18
benchmark
age(0-4) GE
age(0-4) PE
Average TFP by Age
0.64
0.62
0.6
0.58
0.56
0
3
6
9
12
15
18
benchmark
age(0-4) GE
age(0-4) PE
Average Leverage by Age
1
0
-1
-2
0
3
6
9
12
15
18
Age
benchmark
age(0-4) GE
age(0-4) PE
Aggregates: % changes rel. to benchmark
30
20
10
0
-10
capital
output
emp.
firms
age(0-4) GE
age(0-4) PE

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