

Vasilev, Aleksandar

Working Paper

Insurance-markets Equilibrium with a Non-convex Labor Supply decision, Unobservable Effort, and Efficiency Wages of the "No-shirking" Type

Suggested Citation: Vasilev, Aleksandar (2018) : Insurance-markets Equilibrium with a Non-convex Labor Supply decision, Unobservable Effort, and Efficiency Wages of the "No-shirking" Type, ZBW – Leibniz Information Centre for Economics, Kiel, Hamburg

This Version is available at:

<https://hdl.handle.net/10419/184667>

Standard-Nutzungsbedingungen:

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

Terms of use:

Documents in EconStor may be saved and copied for your personal and scholarly purposes.

You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.

Insurance-markets Equilibrium with a Non-convex Labor Supply decision, Unobservable Effort, and Efficiency Wages of the "No-shirking" Type

Aleksandar Vasilev*

November 2, 2018

Abstract

The purpose of this paper is to describe the lottery- and insurance-market equilibrium in an economy with non-convex labor supply decision, unobservable effort, and efficiency wages of the no-shirking type a la Shapiro and Stiglitz (1984). The presence of indivisible labor creates a market incompleteness, which requires that an insurance market for (un)employment be put in operation to "complete" the market.

Keywords: Indivisible labor, Lotteries, Unobservable effort, No-shirking Efficiency wages, Insurance.

JEL Classification Codes: E1, J22, J41

*Lecturer, Lincoln International Business School, UK. E-mail for correspondence: AVasilev@lincoln.ac.uk.

1 Introduction and Motivation

The purpose of this paper is to describe the lottery- and insurance-market equilibrium in an economy with non-convex labor supply decision, unobservable effort, and efficiency wages of the no-shirking type a la Shapiro and Stiglitz (1984). We show how lotteries as in Rogerson (1988) can be used to convexify consumption sets. With a discrete labor supply decisions, the markets are incomplete. The particular focus in this paper is on the lottery- and insurance-market equilibrium in an economy with indivisible labor supply, unobservable effort and efficiency wages. The presence of non-convexity requires that an insurance market for employment be put in operation to achieve market completeness.

2 Model setup

The theoretical setup follows to a great extent Vasilev (2017). There is a unit mass of households, indexed by i and distributed uniformly on the $[0, 1]$ interval, as well as a representative firm. In the exposition below, we will use small case letters to denote individual variables and suppress the index i to save on notation. To simplify the analysis, the model economy is static, without physical capital, and agents will face a non-convex labor supply decision. The firm produces output using labor and capital, but cannot observe the effort exerted by workers. Given that effort is not directly contractible (due to its unobservability on the firm's side), the firm sets a reservation wage to induce an optimal level of effort.

2.1 Description of the model

Each household maximizes the following utility function:

$$U = \ln c + \eta \ln(1 - eh - \xi), \quad (2.1)$$

where $\eta > 0$ is the weight attached to leisure, and as in Burnside *at al.* (1993, 1996), $\xi > 0$ is some fixed cost of working.¹ Variable c denotes household i 's consumption, h denotes hours worked, and e is the amount of effort exerted. The time available to each worker is

¹Parameter ξ is to be interpreted as some kind of organizational or planning cost, e.g the time spent on planning how to spent the day productively. Note that if the household decides to supply zero hours of labor, then $\xi = 0$.

normalized to unity. In addition, we assume that that worker's effort will be imperfectly observable by firms.

All households have equal share in the firm's profit. Total profit is pooled together (within the "family" of households), and then distributed equally among all households. In this way, households can partially insure one another against unfavorable outcomes in the labor market, e.g. not being selected for work. The common consumption can be represented as

$$c^h = \pi, \tag{2.2}$$

$\pi = \Pi$, or the sum of individual profit income equals firm's total profit. The other type of income is the labor income, and households would differ in each period depending on their employment status.

From the perspective of firms, all individuals are identical, so employment outcome could be viewed as random, *i.e.* the firm will choose a certain share of households for work, and leave the rest unemployed. Since the level of effort is not directly observable by firms, some of the employed workers will work and exert the required effort level, e , stipulated in the contract, while others may decide to shirk. If caught, which happens with probability d due to the imperfect technology of detection, the individual is fired and receives a fraction $0 < s < 1$ of the wage.²

The labor contract that the firms then needs to offer provide is to be one that induces workers not to cheat in equilibrium. The contract would specify a wage rate, an effort level, and an implementable rule that a worker caught cheating on the job will be fired and paid only a fraction s of the wage, $0 < s < 1$. All workers know this in advance, and take the terms of the contract and the labor demand as given. In general, the supply of labor will exceed labor demand, so in equilibrium there is going to be involuntary unemployment.

In addition, each employed transfers/contributes T units of income to the unemployment pool, where the proceeds are used to payout to the unemployed. The level of transfers is

²As in Burnside *et al.* (2000), the household does not observe whether the others shirked, or were fired, only the initial employment status.

such that individuals who are not selected for work by the firm are at least as well off as employed workers who are caught shirking. The consumption of an employed worker who does not decide to shirk then equals:

$$c = c^h + wh - T, \quad (2.3)$$

where w is the hourly wage rate. Note that an employed worker who decided to shirk, but is not caught, obtains the same consumption as the conscientious worker, but a higher utility of leisure due to the zero effort exerted and thus no fixed cost of work is incurred.

In contrast, a worker who is employed, decides to cheat, and is caught, receives

$$c^s = c^h + swh - T. \quad (2.4)$$

Alternatively, as proposed in Alexopoulos (2004), this is identical to a case where the firm pays swh upfront, and $(1 - s)wh$ at the end of the period, which is retained in case the worker is caught cheating.

Note that not everyone will be employed, thus the employment rate $\lambda < 1$, and $0 < 1 - \lambda < 1$ would denote the mass of unemployed.³ The consumption of unemployed individuals, c^u , is then

$$c^u = c^h + \frac{\lambda}{1 - \lambda}T, \quad (2.5)$$

where the transfer received by each unemployed equals $\frac{\lambda}{1 - \lambda}T$.⁴ Note that if a household is selected for work and rejects the job offer, there will be no unemployment insurance, or it would receive just the common consumption c^h . Therefore, no household selected for work would have an incentive to reject, so the participation constraint will be trivially satisfied.

Depending on whether a household is selected for work or not, the corresponding instantaneous utility levels are:

$$u(c^u, e^u = 0, h^u = 0) = \ln c^u + \eta \ln 1 = \ln c^u, \quad (2.6)$$

³This result is established in Vasilev (2018).

⁴It is straightforward to reformulate the model so that a self-financing unemployment insurance program is provided by the government rather than the household. Therefore, this setup is very close to the one using unemployment lotteries in Rogerson (1988) and Hansen (1985)

if unemployed,

$$u(c, e, h) = \ln c + \eta \ln(1 - eh - \xi), \quad (2.7)$$

if employed and the worker does not shirk,

$$u(c, e, h) = \ln c + \eta \ln(1) = \ln c, \quad (2.8)$$

if the person shirks, but is not caught, and

$$u(c^s, e^s = 0, h^s = 0) = \ln c^s + \eta \ln(1) = \ln c^s, \quad (2.9)$$

if the person shirks, and is caught.

Let λ^s be the proportion of shirkers and given a detection probability d of a shirker being caught, this implies $d\lambda^s$ would be the proportion of shirkers being caught, and $(1 - d)\lambda^s$ are the shirkers not being caught. In turn, $\lambda - \lambda^s$ are the employed individuals who decide not to shirk.

Finally, note that the leisure (in efficiency units) of shirkers that are caught, and leisure enjoyed by unemployed individuals is the same. Thus, the lump-sum transfer should be chosen so that the consumption levels of the two groups is equalized, or

$$c^s = c^u \quad (2.10)$$

$$c^h + sw h - T = c^h + \frac{\lambda}{1 - \lambda} T. \quad (2.11)$$

or

$$T = (1 - \lambda)sw h. \quad (2.12)$$

In this setup the aggregate household takes as given the effort level and wage rate $\{e, w\}$, which are specified in the contracts that the firm offers. This means that the household takes firm's labor demand as given, which would produce involuntary unemployment. Thus, the household chooses $\{c^h\}$ to maximize (where we have already used the fact that $c^u = c^s$)

$$(\lambda - \lambda^s)[\ln c + \eta \ln(1 - \xi - eh)] + \lambda^s[(1 - d) \ln c + d \ln c^s] + (1 - \lambda) \ln c^s \Big\} \quad (2.13)$$

s.t

$$\begin{aligned} (\lambda - d\lambda^s)c + (d\lambda^s + 1 - \lambda)c^s = \\ (\lambda - d\lambda^s)wh + d\lambda^s sw h. \end{aligned} \quad (2.14)$$

The first-order condition (FOCs) is as follows:

$$c^h : \frac{(\lambda - d\lambda^s)}{c} + \frac{(d\lambda^s + 1 - \lambda)}{c^s} = \mu, \quad (2.15)$$

where μ is the Lagrange multiplier attached to the budget constraint.

2.2 Firm

There is a perfectly competitive representative firm that produces output via the following Cobb-Douglas production function ($H = nh$)

$$y_t = (He)^{1-\alpha}. \quad (2.16)$$

The firm chooses the employment rate, capital input, wage rate (and thus effort level) to maximize

$$(He)^{1-\alpha} - wnh \quad (2.17)$$

s.t. "no shirking condition" (the ICC):

$$\ln c + \eta \ln(1 - \xi - h) \geq (1 - d) \ln c + d \ln c^s \quad (2.18)$$

or

$$d \ln c + \eta \ln(1 - \xi - h) \geq d \ln c^s \quad (2.19)$$

In equilibrium, the firm chooses the optimal employment. In addition the firm offers an efficiency wage rate w to induce a certain optimal effort level, i.e. $e = e(w)$.⁵

$$n : wh = (1 - \alpha) \frac{y}{n}. \quad (2.20)$$

$$w : H = (1 - \alpha) \frac{y}{e} e'(w) \quad (2.21)$$

⁵As in Solow (1979), we assume that the wage rate is a function of effort.

Dividing the FOC for employment and wages, we obtain the standard Solow (1979) condition

$$\frac{we'(w)}{e} = 1 \quad (2.22)$$

or

$$\frac{w}{e(w)} = (1 - \alpha) \frac{y}{H}. \quad (2.23)$$

In other words, this is an equation that characterizes firm's labor demand. Note that the firm minimizes cost per efficiency unit here.⁶ Firms want to hire labor as cheaply as possible, and $w/e(w)$ is the cost per unit of effective labor.

Next, for a given wage rate, the "no-shirking" condition indicated a maximum effort level the firm can obtain from each worker. Rearranging further the constraint, we obtain

$$e \leq e(w) = \frac{1 - \xi}{h} - \frac{1}{h} (c^s/c)^{d/\eta}. \quad (2.24)$$

The firm takes T as given, so the right-hand side is only a function of w , since

$$\frac{c}{c^s} = \frac{c^h + wh - T}{c^h + swh - T} \quad (2.25)$$

Also

$$e'(w) = -\frac{d}{\eta} \frac{1}{h} \left(\frac{c^s}{c}\right)^{d/\eta-1} \left(\frac{c^s - sc}{(c^s)^2}\right) h. \quad (2.26)$$

and

$$w = \frac{c - c^s}{(1 - s)h} \quad (2.27)$$

since the ratio of consumptions is a function of the wage rate, a result that follows from the Solow condition, the effort equation and the wage expression above. Combining the Solow condition, the effort equation, and the wage expression above, it follows that there is only one value for $\frac{c^s}{c}$ that solves this equation and produces a positive level of effort in

⁶If the firm pays higher efficiency wages to induce more effort, that decreases labor demand (because of the wage premium incorporated in the efficiency wage) and produces involuntary unemployment. Also note that the firm adjusts the extensive margin (employment rates), while hours per person are not changing.

equilibrium. Thus the ratio of consumptions is a constant (denoted by χ), and a function of model parameters, *i.e.*

$$\frac{c}{c^s} = \frac{c^h + wh - T}{c^h + sw h - T} = \chi > 1 \quad (2.28)$$

In general, the optimal level of employment, will not coincide with the proportion of workers wishing to accept the contract $(w, e(w))$. As long as firm's demand for labor is less than the labor supply, the "no-shirking" constraint will be binding (hold with equality), and there will be involuntary unemployment in equilibrium.

3 Insurance Market: Stand-in Insurance Company

An alternative way to represent the labor selection arrangement is to regard workers as participants in a lottery with the proportion employed equal to the probability of being selected for work. Therefore, we can introduce insurance markets, and allow households to buy insurance, which would allow them to equalize the actual income received independent of the employment status. More specifically, the structure of the insurance industry is as follows: there is one representative insurance company, which services all households and maximizes profit. It receives revenue if a household is working in the market sector and makes payment if it is not. At the beginning of the period, the households decide if and how much insurance to buy against the probability of being chosen for work. Insurance costs q per unit, and provides one unit of income if the household is not employed. Thus, household will also choose the quantity of insurance to purchase b ; we can think of insurance as bonds that pay out only in case the household is not chosen for work.

The amount of insurance sold by the insurance company is a solution to the following problem: Taking $q(i)$ as given, $b(i)$ solves

$$\max_{b(i)} \lambda(i)q(i)b(i) - [1 - \lambda(i)]b(i). \quad (3.1)$$

With free entry profits are zero, hence

$$\lambda(i)q(i)b(i) - [1 - \lambda(i)]b(i) = 0, \quad (3.2)$$

hence the insurance market for each household clears.

4 Decentralized Competitive Equilibrium (DCE) with lotteries

4.1 Definition of the DCE with lotteries

A competitive Equilibrium with Lotteries for this economy is a list

$$(c(i)^w, c(i)^s, e(i)^w, \lambda(i), w, \pi) \quad (4.1)$$

such that the following conditions are fulfilled.

1. Consumers maximization condition. Taking prices w, π as given, for each i , the sequence

$$\sigma = (c(i)^w, c(i)^s, e(i)^w, \lambda(i)) \quad (4.2)$$

solves the maximization problem

$$\begin{aligned} \max_{\sigma \in \Sigma} & (\lambda(i) - \lambda(i)^s) [\ln c(i)^w + \eta \ln(1 - \xi - eh)] + \lambda(i)^s [(1 - d) \ln c(i)^w + d \ln c(i)^s] \\ & + (1 - \lambda(i)) \ln c(i)^s \end{aligned} \quad (4.3)$$

s.t

$$\begin{aligned} & [\lambda(i) - d\lambda(i)^s]c(i)^w + [d\lambda(i)^s + (1 - \lambda(i))]c(i)^s = \\ & (\lambda(i) - d\lambda(i)^s)wh + d\lambda(i)^s sw h. \end{aligned} \quad (4.4)$$

with

$$c(i)^w \geq 0, c(i)^s \geq 0, 0 < \lambda(i) < 1, \lambda(s)^s < \lambda(i) \forall i, \quad (4.5)$$

where Σ is the constraint defined by relations (4.4)-(4.5).

2. Firm maximization condition. Taking prices w, π as given,

$$\max_H (He)^{1-\alpha} - wnh \quad (4.6)$$

s.t.

$$d \ln c(i)^w + \eta \ln(1 - \xi - h) \geq d \ln c(i)^s \quad (4.7)$$

3. Market-clearing condition. We have

$$h \int_i \lambda(i) di = H, \quad (4.8)$$

$$\int_i \{[\lambda(i) - d\lambda(i)^s]c(i)^w + (d\lambda(i)^s + (1 - \lambda(i))c(i)^s)\} di = (He)^{1-\alpha}, \quad (4.9)$$

where the first equation describes the clearing in the labor market, while the second equation captures the goods-market clearing.

4.2 Characterizing the DCE

The household's problem is as follows:

$$\begin{aligned} \mathcal{L} = \max_{\sigma \in \Sigma} & (\lambda(i) - \lambda(i)^s)[\ln c(i)^w + \eta \ln(1 - \xi - eh)] + \lambda(i)^s[(1 - d) \ln c(i)^w + d \ln c(i)^s] \\ & + (1 - \lambda(i)) \ln c(i)^s \\ & - \mu \left[[\lambda(i) - d\lambda(i)^s]c(i)^w + [d\lambda(i)^s + (1 - \lambda(i))]c(i)^s - (\lambda(i) - d\lambda(i)^s)wh - d\lambda^s sw h \right], \end{aligned} \quad (4.10)$$

where μ is the Lagrangian multiplier in front of the households' budget constraint. The first-order optimality conditions are as follows:

$$c(i)^w : \frac{1}{c(i)^w} = \mu, \forall i, \quad (4.11)$$

$$c(i)^s : \frac{\lambda^s d}{c(i)^s} = \mu[d\lambda^s + (1 - \lambda)], \forall i. \quad (4.12)$$

it follows that

$$\frac{c^w}{c^s} = 1 + \frac{1 - \lambda}{\lambda^s d} \neq \chi \quad (4.13)$$

Note that since it cannot be that $c(i)^s = 0$, it follows that $\lambda(i)^s = 0$. That is, in equilibrium nobody will be shirking.⁷

We simplify the Lagrangian by suppressing all consumption superscripts and i notation in the derivations to follow

$$\lambda(i) : \ln(c^w/c^s)(1 - \xi - eh)^\eta = \mu[c^w - c^s - wh] \quad (4.14)$$

⁷And thus taking first-order conditions with respect to λ^s makes no sense.

This condition states that the marginal rate of substitution between effort in the market sector and consumption equals the wage rate. This implicitly characterizes optimal market sector participation rate λ . Note that it is optimal from the benevolent planner/government point of view to choose randomly λ and to introduce uncertainty. With randomization, choice sets are convexified, and thus market completeness is achieved. Now we extend the commodity space to include insurance markets explicitly.

5 Decentralized Competitive Equilibrium (DCE) with insurance markets

6 Definition of the DCE with insurance markets

A competitive Equilibrium with Lotteries and insurance markets for this economy is a list

$$(c(i)^w, c(i)^s, e(i)^w, \lambda(i), b(i), q(i), p, w, \pi) \quad (6.1)$$

such that the following conditions are fulfilled.

1. Consumers maximization condition. Taking prices p, w, π as given, for each i , the sequence

$$\sigma = (c(i)^w, c(i)^s, e(i)^w, \lambda(i), b(i), q(i)) \quad (6.2)$$

solves the maximization problem

$$\begin{aligned} \max_{\sigma \in \Sigma} & (\lambda(i) - \lambda(i)^s) [\ln c(i)^w + \eta \ln(1 - \xi - eh)] + \lambda(i)^s [(1 - d) \ln c(i)^w + d \ln c(i)^s] \\ & + (1 - \lambda(i)) \ln c(i)^s \end{aligned} \quad (6.3)$$

s.t.

$$pc(i)^w + b(i)q(i) = wh + \pi \quad (6.4)$$

$$pc(i)^s = b(i) + \pi \quad (6.5)$$

$$c(i)^w \geq 0, c(i)^s \geq 0, 0 < \lambda(i) < 1, \forall i \quad (6.6)$$

or

$$pc(i)^w + pq(i)c(i)^s = wh + (1 + \pi)q(i), \quad (6.7)$$

where Σ is the constraint defined by relations (6.4)-(6.6).

2. Firm maximization condition. Taking prices w, π as given,

$$\max_H (He)^{1-\alpha} - wnh \quad (6.8)$$

s.t.

$$d \ln c(i)^w + \eta \ln(1 - \xi - h) \geq d \ln c(i)^s \quad (6.9)$$

3. Insurance-company condition. Taking $q(i)$ as given, $b(i)$ solves

$$\max_{b(i)} \lambda(i)q(i)b(i) - [1 - \lambda(i)]b(i). \quad (6.10)$$

With free entry profits are zero, hence

$$\lambda(i)q(i)b(i) - [1 - \lambda(i)]b(i) = 0, \quad (6.11)$$

hence the insurance market for each household clears.

4. Market-clearing condition. We have

$$h \int_i \lambda(i) di = H, \quad (6.12)$$

$$\int_i \{[\lambda(i) - d\lambda(i)^s]c(i)^w + (d\lambda(i)^s + (1 - \lambda(i))c(i)^s)\} di = (He)^{1-\alpha}, \quad (6.13)$$

where the first equation describes the clearing in the labor market, while the second equation captures the goods-market clearing.

6.1 Characterization of the DCE with insurance markets

$$\begin{aligned} \mathcal{L} = & \max_{\sigma \in \Sigma} (\lambda(i) - \lambda(i)^s) [\ln c(i)^w + \eta \ln(1 - \xi - eh)] + \lambda(i)^s [(1 - d) \ln c(i)^w + d \ln c(i)^s] \\ & + (1 - \lambda(i)) \ln c(i)^s - \mu [pc(i)^w + pq(i)c(i)^s - wh - (1 + \pi)q(i)] \end{aligned} \quad (6.14)$$

Without loss of generality, normalize $p = 1$. We also obtained that $\lambda(i)^s = 0$, *forall* i . The resulting first-order conditions are as follows:

$$c(i)^w : \frac{\lambda(i)}{c(i)^w} = p\mu, \forall i, \quad (6.15)$$

$$c(i)^s : \frac{1 - \lambda(i)}{c(i)^s} = pq(i)\mu, \forall i. \quad (6.16)$$

Optimal λ ($\lambda(i) = \lambda, \forall i$) is implicitly characterized by the zero-profit condition from the insurance company:

$$\frac{\lambda}{1 - \lambda} = \frac{1}{q}, \quad (6.17)$$

which implies that the price of the insurance equals the ratio of probabilities of the two events (“the odds ratio”). Combining this with the other optimality condition, we obtain that conditional on an efficiency wage schedule that discourages shirking, households buy full insurance to equalize consumption,

$$c^w = c^s, \forall i. \quad (6.18)$$

That is, in the presence of uncertainty, we need an insurance company to achieve market completeness.

7 Conclusions

This paper describes the lottery- and insurance-market equilibrium in an economy with non-convex labor supply decision, unobservable effort, and efficiency wages of the no-shirking type a la Shapiro and Stiglitz (1984). The presence of indivisible labor creates a market incompleteness, which requires that an insurance market for (un)employment be put in operation to “complete” the market.

References

Alexopoulos, M. (2004) “Unemployment and the business cycle,” *Journal of Monetary Economics* 51: 277-298.

Burnside, C., Eichenbaum, M. and Rebelo, S. (1993) "Labor Hoarding and the Business Cycle," *Journal of Political Economy*, vol. 101(2), pp. 245-73.

Burnside, C. and M. Eichenbaum (1996) "Factor Hoarding and the Propagation of Business Cycle Shocks," *American Economic Review* 86, 1154-74.

Burnside, C., Eichenbaum, M., and J. Fisher (2000) "Fiscal Shocks in an Efficiency Wage Model," *NBER Working Paper* No. 7515.

Hansen, G. (1985) "Indivisible labor and the business cycle," *Journal of Monetary Economics*, 16(3), 309-328.

Rogerson, R. (1988) "Indivisible labor, lotteries and equilibrium," *Journal of Monetary Economics*, vol. 21(1), pp. 3-16.

Shapiro, C. and Stiglitz, J.E. (1984) "Equilibrium unemployment as a worker discipline device," *American Economic Review* 36, pp. 1291-1303.

Vasilev, A.Z. (2018) "Aggregation with non-convex labor supply, unobservable effort, and efficiency wages of the no-shirking type," *Journal of Mathematical Economics and Finance*, Vol. IV, Issue 1(6), pp. 37-51.

Vasilev, A.Z. (2017) "A Real-Business-Cycle model with efficiency wages and fiscal policy: the case of Bulgaria," *Bulgarian Economic papers* 01-2017, *Center for Economic Theories and Policies*, Sofia University St. Kliment Ohridski, Sofia, Bulgaria.