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Corrective Ad Valorem and Unit Taxes: A Welfare Comparison

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Corrective Ad Valorem and Unit Taxes: 
A Welfare Comparison

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Abstract

The ad valorem versus unit taxes debate has traditionally emphasized tax yield. For this criterion, ad valorem taxes outperform unit taxes in terms of welfare for a wide range of imperfect competition settings, including Dixit-Stiglitz monopolistic competition. Yet, in a number of policy fields, such as environmental, health or trade economics, policy makers apply taxes to target the production/consumption volume in an industry, i.e. aim at a certain corrective effect rather than tax yield. This paper compares the two tax instruments with respect to equal corrective-effect in a Dixit-Stiglitz setting with love of variety, entry, exit, and redistribution of tax revenues. We find that unit taxes lead to more firms in the industry, less output per firm, less tax revenue, but higher welfare compared to ad valorem taxes.

Keywords: Externalities; Monopolistic competition; Taxes; Specific Taxes; Welfare

JEL classification: H23, L13

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1 Introduction

In a number of policy areas policy makers use tax instruments not merely in pursuit of tax revenue, but also as a means for influencing the total production/consumption in an industry. For example, in environmental policy there exist targets for reductions in pollution, and in order to achieve these targets corrective taxes are applied to polluting products (e.g. motor vehicle tax, eco-tax on fuel\(^1\)). Similarly, in health policy the proclaimed objective of taxes on, for example, tobacco or alcohol is to limit the consumption of these noxious products.\(^2\) Furthermore, trade policy can favor domestic producers with market power by limiting the import volume of foreign competitors through tariffs.

In all of these examples the aim of imposing taxes actually is to regulate some external effect (pollution, health risk, foreign competition) that rises proportionally with total industry output, imports or consumption of these outputs. The generated tax revenues may be welcome – and indeed as may be the case with taxes on tobacco or alcohol, few governments could do without them – but tax revenue is not necessarily the relevant measure, when comparing alternative taxes aiming at correcting such externalities. Yet, tax revenue is what a large part of existing public economics literature focuses on, namely, comparing the effectiveness and/or welfare effects of different taxes with respect to an equal-yield criterion. The comparison of ad valorem and unit (specific) taxes is no exception. Since the writings of Cournot and Wicksell, it is well-known that the equivalence of unit and ad valorem fiscal tools under perfect competition ceases in monopolistic settings. Suits and Musgrave (1953) and Delipalla and Keen (1992) provide the tools comparisons under a wide range of imperfect competitive settings, finding that ad valorem taxes outperform unit taxes for the equal tax-yield criterion.\(^3\)

The present paper reconsiders the welfare comparison of ad valorem

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\(^1\)See e.g. Haughton and Sarkar (1996) for estimates of the corrective-effect of Gasoline taxes in the US.

\(^2\)See e.g. the European Charter on Alcohol, 1995, of the World Health Organization. http://www.euro.who.int/AboutWHO/Policy. See also O'Donoghue and Rabin (2003) for a perspective on government policies that correct individual behavior, or optimal paternalism, in their terminology.

and unit taxes under monopolistic competition, yet we do not use the equal tax-yield criterion, but an equal corrective-effect criterion. What is crucial for this comparison, of course, is that the generated tax revenues are treated adequately, i.e. that they enter welfare. Furthermore, we assume that the size of an externality is proportional to total production of an industry or the total consumption of products from the industry. Therefore, the tax addresses the industry’s production/consumption. Accordingly, the analysis must consider both, number and size of individual firms. This comprises the entry and exit, and the scale decisions firms make after tax interventions. One modelling approach that can capture these various forces, and the one chosen here, is Dixit-Stiglitz-type monopolistic competition. The Dixit-Stiglitz (1977) approach towards monopolistic competition provides a consistent framework where consumers value product variety and where product differentiation, economies of scale and market exit and entry exist. Furthermore, since it is inherently a general equilibrium approach, the full consequences of redistributing the revenues that are generated by the two types of taxes – such that all generated tax-yield re-enters the consumers’ pockets – can be studied. Firms make zero profits in equilibrium and thus, welfare takes its starting point in the consumer utility function, which accounts, via the redistribution of revenues, for the superior yield performance of a tax.

The central finding of this model is that for an equal externality correction criterion, unit taxes dominate ad valorem taxes in terms of welfare. This is the reverse of the usual finding under the equal-yield criterion. In particular, we find that unit taxes induce more entry of firms to the industry, less output per firm, less tax revenue, but higher welfare compared to ad valorem taxes.

The present paper relates to the existing literature on corrective taxes. To the best of our knowledge there is no previous work that provides ad valorem versus unit tax welfare comparisons for corrective taxes under monopolistic competition. However, the crucial role of market structure for the assessment of corrective taxes has already been pointed out by Buchanan (1969). He demonstrates that the welfare impact of a corrective tax (a unit tax in his case), though positive in the case of a perfectly competitive industry, may turn out to be negative in case of taxing a monopoly. More recently Sheshinski (2004) compares corrective taxes when taxing an externality generating product and taxing households that are heterogenous in consumption characteristics and when income redistribution is desirable. Brett and Keen (2000) develop a po-
litical economy model with corrective taxes where the revenues generated by those taxes are earmarked. Earlier Dillén (1995) has examined self-financed corrective tax-subsidy schemes to correct for the distortions stemming from imperfect competition itself. Shrestha (2001) studies a situation where uncertainties as to the cost and benefit of an environmental policy intervention on some perfect competitive but polluting industry exist and are correlated. The paper derives tools comparisons for linear taxes, non linear-taxes (i.e. a tax that mirrors the expected schedule of the marginal benefit of the environmental policy) and quantitative restrictions, and identifies parameter ranges of the correlation for which the various tools dominate each other.

The regulation of an industry’s total production/consumption in order to tackle an externality may occur in many policy fields. The analysis of the present paper attempts to keep a broad perspective by disentangling the tools comparison from its respective setting in environmental, health or trade economics. Instead we model the externality problem in general functional form, such that the presence of the externality leads to a situation where the welfare maximizing choice is some generally defined ceiling on total industry volume. The ad valorem and unit tax instruments under consideration are then imposed in order to achieve this welfare maximizing level, based on any conceivable mix of the two tools. In our model, we find that an ad valorem tax will induce some firms to exit the industry. On the one hand, this exit reduces the aggregate production/consumption externality, but on the other hand, at the firm level, output remains unchanged. In contrast, the unit tax creates in fact entry into the industry, but per firm output (and hence the aggregate externality) decreases. With a unit tax, firms over-shift the tax to consumers and create more profits. Then, smaller per-firm production suffices to create the break-even point where firms recover fixed costs. If now welfare rankings are established, we find that unit taxation dominates. It is the consumers’ “love of variety” that drives this result. Entry of firms, contrary to exit under the ad valorem tax, increases variety and thus consumers’ welfare. This result is obtained even though the ad valorem tax yields the larger tax revenue and all tax revenues are redistributed.\footnote{The ability to trigger additional entry by unit taxes is also present in the equal-tax-yield comparison in for example Schröder (2004), but for the equal-yield comparison this effect does not suffice to compensate for the higher efficiency of ad valorem taxes in collecting revenue.}
These results carry some policy implications for policy areas where corrective taxes are applied to regulate the externality of monopolistic competitive industries. For example, concerning environmental policy, the findings of the paper indicate that environmental regulators should reconsider their reliance on ad valorem taxes in favor of unit taxes. E.g. if the registration tax for vehicles is supposed to limit the number of cars, it would be welfare superior to impose it as a unit tax rather than an ad valorem tax.\(^5\) In the case of taxes on tobacco and alcohol most European countries use in fact excise (i.e. unit) taxes (Martinez-Serrano and Patterson, 2003). Thus, this would be in line with the welfare optimal corrective tax found in the present paper. Finally in trade policy, specific (unit) tariffs even though avoided under the majority of WTO regulation, could be reconsidered on grounds of consumer welfare. In particular, when converting non-tariff barriers into tariffs (tariffication) the application of specific tariffs may be able to command a welfare gain compared to the application of ad valorem tariffs, while maintaining the same level of total import volume (for discussions of tariffication in WTO rounds see e.g. Ingco, 1996; and Nguyen et al., 1993).

The Dixit-Stiglitz type monopolistic competition setting is used by several other authors for comparisons of ad valorem and unit taxes. Closely related to the present paper is Schröder (2004) who compares ad valorem and unit taxes. Yet, Schröder (2004) uses the traditional equal-yield criterion and confirms the welfare superior performance of ad valorem taxes established for a wide range of imperfect competitive settings (Delipalla and Keen, 1992). Furthermore, and in contrast to the present paper, this does only consider either ad valorem or unit taxes and ignores the possibility of mixing the two tools. Dröge and Schröder (2005) apply a Dixit-Stiglitz monopolistic competition model in order to consider ad valorem and unit taxes in comparison with ad valorem and unit subsidies in an environmental economics setting. Their measure of comparison is the share of green produce in total output. They find that when policy makers want to achieve such a ratio target, that the ranking of instruments depends on the share of polluting output within an industry. For a small share, a unit tax is welfare superior to an ad valorem tax, but for high market shares of dirty products the ranking turns upside down. Other authors that use monopolistic competition settings to study environmental policy issues and instruments include Heijdra and

\(^5\)Currently, in 10 out of EU-15 countries the vehicle registration tax is levied as an ad valorem tax. See Kuhfeld and Kunert (2000).
van der Ploeg (1995) and Haupt (2000). These papers address optimality when inducing abatement.

Finally, the superior performance of the unit instrument can be linked to the results of several other papers that are situated in various imperfect competition settings. Pirttilä (2002) shows that a Delipalla and Keen (1992)-type model, once augmented with a second distortion (apart from imperfect competition), does generate rankings of unit and ad valorem taxes that depend on the relative size of the two distorting factors. Anderson et al. (2001) establish in a love of variety setting different from Dixit-Stiglitz, using a discrete choice framework for consumer demand, paired with Bertrand competition and differentiated products that the welfare dominance of ad valorem taxes can be challenged by a unit tax once there is free entry and sufficient love of variety. Vetter (2001) establishes that quotas (modelled as a unit tool) are superior to taxes (modelled as an ad valorem tax), although a first best solution can only be obtain using a mix of both tools. Apart from comparing quotas to taxes, and being placed in a Dixit-Stiglitz and Spence (1976a,b)-type framework, the work by Vetter (2001) differs from the present paper by not considering the redistribution of the revenues generated by the policy intervention. As discussed above this can be problematic, in particular since the inferior instrument in the present paper (and in Vetter, 2001), is the ad valorem tool, which however commands the larger revenue. Finally, Jørgensen and Schröder (2005) present an intra-industry trade model following Krugman (1980), based on Dixit-Stiglitz love of variety. Comparing either ad valorem or specific (unit) tariffs they establish that specific tariffs may welfare dominate ad valorem tariffs. The work by Jørgensen and Schröder (2005) differs from our paper by setting up a two country model, applying a different utility function and not considering policy mixes of the two instruments.

In the following section we introduce the model. Section 3 shows the equilibrium number of firms, total externality, tax revenues and welfare. Section 4 concludes and discusses policy implications.
2 The Model

Consider an industry which causes some kind of externality and whose market conditions are characterized by monopolistic competition. The industry has a large number of potential variants (firms), $N$, which enter symmetrically into demand. The number of variants actually produced is $n < N$ and is assumed to be large. The output of firm $i$ is denoted by $x_i$, and the externality caused by a firm is given by $d_i = \delta x_i$, where $\delta \geq 0$ and identical for all firms. Thus the externality grows proportional in firm output volume. All firms have the same cost structure $l_i(x_i) = (f + \beta x_i)$ where $l_i$ is labor – the only factor of production –, where $f$ is the fixed costs of production and $\beta$ are marginal costs. The economy wide wage rate is $w$. The consumer price is given by the inverse demand function $P(x_i)$. Internalization policy uses both, an ad valorem tax $\tau < 1$ and a unit tax $T$ expressed in real terms as $t = \frac{T}{w}$. As in this type of model, all output is actually consumed and all firms and all consumers behave identically, hence output/production taxation is equivalent to consumption taxation. Under these conditions, firm $i$’s profit is

$$\pi_i = (1 - \tau)P(x_i)x_i - (f + (\beta + t)x_i)w .$$

(1)

Consumers are assumed to be identical, denoting by $c_i$ consumption of good $i$ and by $L$ the labor force assumed to be equal to the number of consumers, goods market clearing implies $x_i = Lc_i$.

The model is completed by imposing a consumer utility function. Denote by $D = \sum d_i$ the economy wide aggregated externality and by $\bar{D}$ the externality level prevailing in the absence of any corrective policy. Denote by $u$ a sub-utilility function composed from the consumption of an un-taxed homogeneous good, $l$ (say leisure), and the consumption of differentiated goods, $c_i$, from the monopolistic competitive sector described above. In particular

$$U(u, D) \quad \text{where}$$

$$\frac{\partial U}{\partial u} > 0 , \frac{\partial U}{\partial D} < 0 , \frac{\partial u}{\partial x} > 0 , \frac{\partial l}{\partial D} |_{D \rightarrow 0} > 0 \quad \text{and} \quad \frac{\partial U}{\partial D} |_{D \rightarrow \bar{D}} < 0 .$$

(2)

The latter two total derivatives imply that the shape of the damage function and the way in which it enters utility is such that maximisation of (2) with respect to $D$ defines an optimal externality level

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6This section presents a straightforward application of Dixit and Stiglitz (1977) to the problem at hand.
$D^* \in ]0, \bar{D}[$. Notice that $D = \delta x_n$, and hence $\frac{D^*}{\delta}$ can define a target production/consumption volume for the industry, which can be achieved via ad valorem and unit taxes.

The sub-utility, $u$, identical for all individuals is assumed to be given by

$$u = \left( \frac{l}{L} \right)^{1-\alpha} \left( \sum_{i=1}^{n} \theta_i \right)^{\alpha},$$

where $0 < \theta < 1$ represents the taste for variety parameter, $0 < \alpha < 1$ defines the share of income consumers spend on the differentiated goods and $\frac{l}{L}$ is the per capita consumption of the homogeneous good at price $w$. Labour market clearing requires $\sum l_i + l = L$. In this specification the elasticity of substitution between goods from the two different sectors is 1 which is less than the elasticity of substitution between products from within the differentiated sector, $\frac{1}{1-\theta}$. This implies e.g. that different varieties of a harmful product, say, cigarettes are closer substitutes to each others than they are to consumption of leisure. Further, maximising (3), expenditure shares on homogeneous and differentiated products will be $(1-\alpha)$ and $\alpha$ respectively. Income-expenditure clearing requires $wL + R = wl + \sum p_ix_i$, where $R$ are redistributed tax receipts. Hence, the total expenditure on the homogeneous good, must fulfill $wl = (1-\alpha)(wL+R)$, i.e. consumption of leisure is affected by policies aimed at the polluting differentiated goods sector via redistributed revenue.

3 Results

The inverse demand function for the polluting industry is calculated by maximising utility from the consumption of differentiated goods

$$\max (\sum c_i^\theta)^\alpha - \lambda(p_1c_1 + \cdots + p_ic_i + \cdots + p_nc_n - \alpha(wL+R)).$$

Given the large number of firms assumption, firms do not realise the impact of their price on overall sales (and on tax revenue). The first order conditions are of the form

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7This sub-utility and the following solution steps are in fact the utility function used in Schröder (2004) who examines the tax yield and welfare consequences of ad valorem and unit taxes, but does ignore the issue of obtaining certain corrective production volumes nor does Schröder (2004) deal with the case were ad valorem and unit taxes are in place simultaneously.
\[ P(x_i) = \frac{\theta c_i^{\theta-1}}{\lambda \xi}, \quad (4) \]

where \( \xi = \alpha \left( \sum c_i^{\theta} \right)^{1-\alpha} \) assumed to be constant and where \( c_i = \frac{x_i}{L} \) (from the goods market clearing condition). Calculating the price elasticity of demand in absolute terms yields \( \epsilon = \frac{1}{1-\theta} \).

Equilibrium in the polluting industry is characterized by prices, per firm output and the number of firms. Due to symmetry we can restrict our analysis to one variant (firm), hence, omitting the subscript \( i \). After setting (4) into (1) firm profits read:

\[ \pi = (1 - \tau)^{\theta} \left( \frac{x}{\lambda \xi} \right)^{\theta-1} x - (f + (\beta + t)x)w. \]

Maximisation yields the profit-maximising price

\[ p = \frac{(\beta + t)w}{(1 - \tau)^{\theta}}. \quad (5) \]

Notice that from (5) it follows that the ad valorem tax is shifted entirely onto consumers, i.e. post tax firm prices are \( (1 - \tau)^{\theta} \left( \frac{x}{\lambda \xi} \right)^{\theta-1} x = \frac{(\beta + t)w}{(1 - \tau)^{\theta}} \), thus unaffected by the ad valorem tax. In contrast the unit tax \( t \) is overshifted onto consumers with factor \( 1/\theta \).

**Entry, Exit and Scale**

Equating the profit-maximising price with the price implied by zero profits, \( p_0 = \frac{(f + (\beta + t)x)w}{(1 - \tau)x} \), gives the scale decision of firms, i.e. per firm output under free entry and exit:

\[ x = \frac{\theta f}{(1 - \theta)(\beta + t)}. \quad (6) \]

Revenues to be redistributed are both the ad valorem tax revenues, \( \tau pxn \), and the revenues from unit taxes, \( Txn \). Total revenues can thus be written as \( R = \tau pxn + twxn \). With this expression we are now able to deduce the number of firms actually producing in equilibrium via market clearing using the \( x \) and \( p \) derived above. In particular, expenditure-income clearing, \( pxn = \alpha(wL + R) \), must hold. Substituting and solving for \( n \) gives:

\[ n = \frac{L \alpha (1 - \theta)}{f} \left( \frac{(\beta + t)(1 - \tau)}{(\beta + t - \alpha(t\theta + t\tau + \beta\tau - t\tau\theta))} \right). \quad (7) \]
Differentiating (6) and (7) with respect to $\tau$ and $t$ we arrive at:

$$\frac{dn}{d\tau} = \frac{L(\alpha - 1)\alpha(1 - \theta)(\beta + t)^2}{f(\beta(1 - \alpha\tau) + t(1 - \alpha(\theta + \tau - \theta\tau)))^2}, \tag{8}$$

$$\frac{dn}{dt} = \frac{L\alpha^2(1 - \theta)\beta(1 - \tau)^2}{f(\beta(1 - \alpha\tau) + t(1 - \alpha(\theta + \tau - \theta\tau)))^2}. \tag{9}$$

By inspection one finds that $\frac{dn}{d\tau} < 0$, $\frac{dn}{dt} > 0$. Furthermore, the derivatives $\frac{dx}{d\tau} = 0$, $\frac{dx}{dt} < 0$ follow by inspection of (6). Hence, one can state:

**Proposition 1.** An increase in the ad valorem tax, $\tau$ reduces the number of firms in the industry and leaves the per firm externality level unaffected. An increase in the unit tax $t$ increases the number of firms in the industry and reduces the per firm externality level.

Proposition 1 paints the following picture of the two tax tools. While the ad valorem tax reduces the number of firms, it has no effect on the per firm quantity produced, and hence on the per firm externality level. Accordingly, a reduction in the aggregate damage via the ad valorem tax is driven by exit from the industry. In contrast, the unit tax reduces per firm production (see equation (6)) and hence per firm externality, while at the same time the unit tax increases the number of firms in the industry. What motivates this entry into the industry? The fulcrum of this result is the operating surplus of firms. From (5) it followed that firms over-shift the unit tax with factor $1/\theta$, this over-shift in turn increases the operating surplus. Under monopolistic competition, firm entry into the industry occurs until all firms just break-even, i.e. are just able to cover their fixed production cost $f$. With a larger operating surplus smaller product runs suffice to achieve this break-even point, hence an increase in the unit tax may in fact generate entry into the industry, while at the same time reducing the output volume – and thus the externality – of every single firm.

**Aggregate Externality and Revenues**

It is now possible to calculate the total volume of externality, i.e. the aggregate damage level entering utility (2) and defined by $D = \sum d = n\delta x$. Using (6) and (7) we get:

$$D = \frac{\delta L\alpha\theta(1 - \tau)}{\beta(1 - \alpha\tau) + t(1 - \alpha(\theta + \tau - \theta\tau))}. \tag{10}$$
Differentiating (10) with respect to \( \tau \) and \( t \) gives:

\[
\frac{dD}{d\tau} = \frac{\delta \theta (\alpha - 1)(t + \beta)}{(\beta(1 - \alpha \tau) + t(1 - \alpha(\theta + \tau - \theta\tau)))^2},
\]

\[
(11)
\]

\[
\frac{dD}{dt} = \frac{\delta \alpha \theta (1 + \alpha(\theta(\tau - 1) - \tau))(\tau - 1)}{(\beta(1 - \alpha \tau) + t(1 - \alpha(\theta + \tau - \theta\tau)))^2}.
\]

\[
(12)
\]

By inspection \( \frac{dD}{d\tau} < 0 \), \( \frac{dD}{dt} < 0 \). Notice that even though the number of firms increases in \( t \) (Proposition 1) the accompanied reduction in per firm externality is so large that the unit tax is able to deliver reductions of total damage. This leads to the following:

**Proposition 2.** Increasing either the ad valorem tax \( \tau \) or the unit tax \( t \) reduces the total pollution level \( D \).

The damage level in the absence of any corrective tax policy \((\tau = t = 0)\) is \( \bar{D} = \frac{\delta \theta}{\beta} \). Then there exists some \( \gamma > 0 \) such that \( D = \gamma \bar{D} \), where \( \gamma \) is a simple measure for the damage level. Furthermore there must exist some \( \gamma^* \) such that \( \gamma^* \bar{D} = D^* \), i.e. denoting the level of damage that maximizes utility (2) and depends on the degree and way in which the externality (e.g. environmental damage, health risks, foreign competition) reduces utility. With this specification, after setting (10) equal to \( \gamma \bar{D} \) we can express \( t \) as a function of \( \tau \) to get a function of policy mixes for a fixed damage level:

\[
t(\tau) = \frac{\beta}{\gamma} \frac{1 - \tau - \gamma(1 - \alpha \tau)}{1 - \alpha \theta - \alpha(1 - \theta)\tau}.
\]

\[
(13)
\]

The function in (13) depicts combinations of \( \tau \) and \( t \) that result in the same damage level (denoted by \( \gamma \)). This is relevant also for the optimal damage level, \( \gamma^* \bar{D} = D^* \). Thus, as long as we apply (13) – substituting \( \tau \) for \( t \) – changes in \( \tau \) represent a change in the policy mix, in particular switching away from unit over to ad valorem taxes as we increase \( \tau \). Furthermore, with (13) we can also explore policy mixes with subsidies. On the contrary from (13) it follows that for a very tough ad valorem tax, \( \tau \to 1 \), the unit tax must turn into a subsidy \( t < 0 \), to maintain a certain level of damage, notice that once \( \gamma \) – and therewith the damage level \( D \) – becomes very small, the unit tax will still be positive also for \( \tau \) close to one.

With the iso-damage level relation given in (13) we can now calculate total revenue from the corrective policy and how revenues react to a
shift in the policy mix, while maintaining a certain aggregate externality level. Calculating $R = \tau pxn + twxn$ when using the $p$, $x$ and $n$ derived above and expressing $t$ in terms of $\tau$ from (13) we get:

$$\bar{R} = \frac{Lw\alpha(\tau + \theta(1 - \gamma - \tau))}{1 - \alpha(\theta - (1 - \theta)\tau)},$$

(14)

Differentiating (14) with respect to $\tau$ gives:

$$\frac{d\bar{R}}{d\tau} = \frac{Lw\alpha(1 - \theta)(1 - \alpha \gamma \theta)}{(1 - \alpha(\theta(1 - \tau) + \tau))^2},$$

(15)

which by inspection yields $\frac{d\bar{R}}{d\tau} > 0$. We arrive at the following result for the revenue efficiency of the two types of taxes:

**Proposition 3.** For any given level of corrective-effect – including the optimal level of aggregate externality $D^*$ – the substitution of ad valorem taxes, $\tau$, for unit taxes, $t$, in the policy mix, raises total revenues.

Proposition 3 says that the ad valorem tax is superior in raising revenues compared to the unit tax. This result is in fact a common finding in public economics where ad valorem tax tools are usually superior in extracting profits from imperfect competitive markets – when compared to specific tools such as the unit tax modelled here; e.g. Delipalla and Keen (1992), Schröder (2004). The ad valorem tax performs better with respect to revenues because it does not induce an over-shift and hence is also more efficient in addressing the externality problem compared to the unit tax which leads to an over-shift $1/\theta$, see (5).

**Welfare**

Proposition 1 and 3 comprise the fundamental conflict between the two types of taxes: ad valorem and unit. While the unit tax appears to be superior in terms of maintaining higher entry into the sector, which should increase consumer welfare, the ad valorem tax ensures a higher level of revenues which, since all revenues are redistributed to consumers, has a positive impact on welfare. Hence, we have two opposing effects.

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8It is obvious that when not keeping the externality level constant an increase in either policy tool leads to an increase of revenues and a reduction in the aggregate externality.
To obtain a comprehensive ranking of the two taxes, and thus an evaluation of the two opposing effects, we must calculate total welfare. Due to entry and exit, profits are zero and since all revenues are redistributed to the population, total consumer utility is a measure of welfare. Utility, U, (2) depends only on $D$ and sub-utility, $u$ and we only consider situations with constant damage $D$ – including the optimal damage level $D^*$. Therefore, a ranking of the sub-utility levels also implies a ranking of overall utility and hence a welfare ranking. Total consumer sub-utility, $W = Lu$, where $u$ is given in (3), can be written as:

$$W = \left(\left(1 - \alpha\right)\frac{wL + R}{w}\right)^{1-\alpha} (nx^\theta)^\alpha,$$

which after setting in the $x$ and $n$ from above and the $t$ and $R$ given in (13) and (14), respectively, becomes:

$$\bar{W} = \left(\frac{L (-1 + \alpha)}{-1 - \alpha \theta + \alpha (-1 + \theta \tau)}\right)^{1-\alpha} \left(\frac{\alpha \gamma \theta \left(- \left(\frac{f \gamma \theta (-1-\alpha \theta + \alpha (-1+\theta \tau))}{L \beta (-1+\theta)(-1+\alpha \gamma \theta (-1+\tau))}\right)^{-1+\theta}\right)^\alpha}{\beta}.\quad(16)$$

The expression denotes total consumer sub-utility for constant levels of damage, as depicted by $\gamma$, thus applies also to the optimal damage level. Differentiating (16) with respect to $\tau$, the following welfare ranking of the two tax tools is obtained.

**Proposition 4.** For any given level of a corrective-effect – including the optimal level of aggregate externality $D^*$ – the substitution of unit taxes, $t$, for ad valorem taxes, $\tau$, increases total welfare.

Proposition 4 establishes, that unit taxes are welfare superior compared to ad valorem taxes. The derivative of $\bar{W}$ is given in the appendix. This welfare ranking applies across the entire parameter range, for any mix of policy, and for any mix of industries, $\alpha$. Moreover, it is independent of the degree to which the externality enters utility – which in fact depends on the action taken by policy makers –, and the degree of product differentiation, $\theta$. This result is obtained even though the ad valorem tax raises more revenues then the unit tax, and it is derived for a fairly general specification of how damage influences utility. The driver for the welfare superior performance of the unit tax is the entry/exit behavior of firms in the industry. The unit tax directly enters firms marginal costs and thus reduces their individual output and externality volume. Yet, by increasing the firms’ operating surplus, entry into the industry is secured, and this obtains a given level of total externality with a larger
number of firms (each producing relatively less externalities compared to the ad valorem tax case). It is the larger number of firms that exists in the sector under the unit tax that results in the higher welfare.

4 Conclusions

In this paper we compare the performance of corrective ad valorem and unit taxes with respect to the efficiency in cutting an external effect under imperfect competition, in particular cutting externalities by taxing the associated industries output or the consumption of this output. In a Dixit-Stiglitz-type monopolistic competition setting, solved in general equilibrium with full redistribution of all tax revenues, we establish an analytical welfare ranking of the two tax instruments for an equal but generally formulated corrective effect, thus abandoning the traditional equal tax-yield criterion usually applied in tools comparisons.

The central findings are that unit taxation generates more entry into the industry than ad valorem taxation, i.e. more firms enter and, given the consumers taste for variety, this should increase welfare. However, each firm operates at a lower scale when subject to unit taxation as compared to a situation with ad valorem taxation. More importantly, ad valorem taxation leads to larger tax revenues, i.e. extracting more profits from the industry in question. Since tax revenues are redistributed, this has a positive impact on consumer welfare. These various forces lead to a welfare ranking in which unit taxes turn out to dominate ad valorem taxes. This is in disaccord with the traditional findings made in a wide range of imperfect competitive settings for an equal tax yield target. Thus, for the equal corrective effect criterion, the present paper shows that unit taxes might challenge the usual dominance of ad valorem taxes. The major force behind this finding is the entry or exit of firms associated with firms’ different shifting of the tax burden of the two tax instruments: A unit tax leads to a more than proportional increase in prices and this entails higher profits, which in turn are competed away by entry of new firms. The specification of monopolistic competition, in which consumers favor variety and where each new firm offers a new variant, then produces higher consumer utility. While this additional welfare from extra varieties is insufficient to reverse the ranking of the two tax instruments for the equal yield criterion, it suffices once we compare the tools along their ability to implement a corrective effect on industry volume, and thus
some proportionally associated externality.

Our findings are relevant for a number of policy areas such as environmental, health or trade policy. Whenever ad valorem tools are imposed as corrective instruments, and not with the predominant purpose of generating tax revenues, the choice of the tax tool should be reconsidered depending on the mode of competition in the industry in question. However, the analysis made represents only a first step towards a broader approach for tool comparisons. Every state intervention provokes firms and consumers to reconsider their technological equipment and consumption habits, respectively. This issue is widely discussed in the application to environmental economics. If, e.g. the externality that motivates a tax is air pollution, firms can either abate more (and thus improve the output-externality performance), pay, or exit. The present paper ignores the first option but addresses the other two. Moreover, the technological choice of firms may also be relevant for health or trade protection. However, health risks caused by alcohol or tobacco consumption are unlikely to be cured via technological improvement on the production side, thus in this sectors firms can only improve on general production costs, while on the consumption side a tax intervention may induce a switch to alternative and potentially more harmful substances. Furthermore, concerning trade policies aiming at the reduction of total import volume, our findings suggest that this is best achieved via specific tariffs. Yet, specific tariffs come at great administrative cost compared to ad valorem tariffs, which is another issue left for future research.
Appendix

Derivative of $\bar{W}$

Differentiating (16) with respect to $\tau$ gives:

\[
\frac{d\bar{W}}{d\tau} = \frac{\alpha (-1 + \theta) \tau \left( \frac{L (-1 + \alpha (1 + \alpha \gamma \theta))}{1 - \alpha \theta + \alpha (-1 + \theta) \tau} \right)^{2-\alpha} \left( \frac{\alpha \gamma \theta (-\frac{f_\theta (1 - \alpha \theta + \alpha (-1 + \theta) \tau)}{L \beta (-1 + \alpha \theta + \alpha (-1 + \theta) \tau))}^{1+\theta}\alpha}{L (-1 + \alpha \gamma \theta) (-1 + \tau)} \right)}{(17)}
\]

The denominator is always positive, while $(-1 + \theta)$ generates a negative sign in the numerator. The expression $(1 - \alpha \theta + \alpha (-1 + \theta) \tau)$ can be re-written as $(1 - \alpha(\theta + (1 - \theta) \tau))$ which is always positive, since $(\theta + (1 - \theta) \tau) < 1$. 
References


Heijdra, Ben; van der Ploeg, Frederick (1995), Fiscal and environmental


Sheshinski, Eytan (2004), On Atmosphere Externality and Corrective
References


