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Fiscal Competition, Capital-Skill Complementarity, and the Composition of Public Spending

Berlin, July 2005

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Abstract

Following Keen and Marchand (1997), the paper analyses the effect of fiscal competition on the composition of public spending in a model where capital and skilled workers are mobile while low skilled workers are immobile. Taxes are levied on capital and labour. Each group of workers benefits from a different kind of public good. Mobility of skilled workers provides an incentive for jurisdictions to spend ‘too much’ on public goods benefitting the skilled and ‘too little’ on those benefitting low skilled workers. In the case of capital-skill complementarity, this incentive is strengthened. The analysis is then extended to allow for mobility of unskilled labour.

JEL classification: H77, J24, J61.

Keywords: Tax competition, capital skill complementarity, public spending.

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1. Introduction

The literature on fiscal competition, starting with Oates (1972) and the classic pieces by Zodrow and Mieszkowski (1986) and Wilson (1986), has largely focussed on the effects of capital mobility on the level of public spending. While some authors have shown that tax competition can be efficiency enhancing, most of the literature has tended to argue that tax and spending levels will be inefficiently low due to the fiscal externalities of taxing mobile tax bases (see Wilson, 1999, for a survey).

The problem of the composition of public spending, however, has until recently been neglected. Keen and Marchand (1997) address this omission by analysing how fiscal competition affects the composition of public spending between consumption goods and public inputs. They find that the spending mix will be tilted towards spending on public inputs. The reason is that this kind of expenditure attracts mobile factors (namely, capital) while spending on consumption goods does not (since workers are assumed to be immobile). Hence, in equilibrium, all jurisdictions could be better off if they coordinated on spending more on consumption and less on public inputs.¹

This paper extends the Keen-Marchand model by introducing skill heterogeneity: There are high skilled and low skilled workers, each benefitting from different publicly provided goods. For instance, the high skilled may want to visit public theatres and opera houses, while low skilled workers may benefit more from public housing projects and other progressive social programs. I assume that capital and high skilled labour are complementary and, in addition, I initially assume that high skilled workers are interregionally mobile while low skilled workers are not (see, e.g., Mauro and Spilimbergo, 1999, for evidence).

These extensions are relevant for two reasons. First, on the empirical side, neglecting the interaction between different spending categories may blur the mechanisms by which communities try to attract mobile factors and by which they interact with neighbouring communities. Second, as far as the modelling side and its policy implications are concerned, it should be stressed that capital skill complementarity and the greater mobility of skilled than unskilled workers seem to be well documented.² Hence, jurisdictions may find that

¹Matsumoto (2000) shows that overprovision of public inputs does not necessarily hold when labour is mobile as well.
²See e.g. Griliches (1969), Bergström and Panas (1992), and Krusell et al. (2000) for evidence on capital skill complementarity. For evidence on the mobility of skilled versus unskilled workers, see Mauro
to attract capital, they also need to attract skilled workers, and to do so they may also use public goods which differentially benefit this group of workers. As a concrete example, when Boeing moved its headquarters from Seattle to Chicago, tax incentives played a major role. But in order to win the bidding war against competing cities, Chicago also “played up its cultural institutions and Lake Michigan location—much was made of the fact that Boeing CEO Phil Condit is a sailor and an opera fan” (Garcia-Mila and McGuire, 2002, 112). Likewise, Denver played its scenic beauty as an incentive. In fact, Denver “City Councilman Ed Thomas stressed the high quality of life in Denver saying, ‘I don’t know if we even need to compete on financial incentives’ ” (Garcia-Mila and McGuire, 2002, 112). In the end, Chicago may have won because of the best tax incentive package offered to Boeing, but the cited passages reflect the fact that the cities also stressed the role of amenities and public goods which benefit the company’s workers.

The analysis in this paper is similar to Matsumoto (2000), with the distinction that in his model, there is no skill heterogeneity and all workers are assumed to be mobile.\(^3\) In the present model, given that skilled workers are mobile, regions have an incentive to overprovide opera houses relative to public housing, since the former attract mobile workers while the latter does not. Further, if there is capital-skill complementarity, this incentive is strengthened since spending on operas attracts skilled workers as well as capital, both of which enlarge the tax base.\(^4\) The paper is also related to Huber (1999), who analyses factor taxation in an optimum tax model with skilled and unskilled labour. However, in his model the focus is on taxes rather than spending and labour is immobile.

The paper proceeds as follows. The next section introduces the model. In section 3, the analysis is extended to mobility of low skilled labour. The last section concludes.

## 2. Baseline model

### 2.1. Uncoordinated equilibrium

The model is based on Keen and Marchand (1997), who elaborated on Zodrow and Mieszkowski (1986). There are \(N\) jurisdictions, called regions, each with independent

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\(^3\)See also Matsumoto (2004), who analyses the provision of two different public inputs.

\(^4\)There is a large body of literature on CSC originating with Griliches (1969).
taxing and spending power. There are three factors of production: capital $k$, skilled labour (or human capital), $h$, and unskilled labour, $l$.\(^5\) Capital and skilled labour are mobile while unskilled labour is immobile. Furthermore, the mass of unskilled workers in each jurisdiction is normalised to one, as is the mass of initial skilled workers (before migration) in each jurisdiction. All jurisdictions are identical and possess one unit of land, which is used for production. Jurisdictions are small and treat the return to capital as well as the utility achieved by mobile skilled labour as given. Output is produced with a strictly concave and linear homogeneous production function, which in intensive form is written $y = f(k, h)$.

Firms maximise profits under perfect competition. The price of output is normalised to one. Denoting partial derivatives by subscripts, the production function satisfies $f_k, f_h > 0; f_{kk}, f_{hh} < 0$. The net return to capital will be denoted by $r$ and the after-tax skilled wage rate by $w^H$. The after-tax unskilled wage rate is $w^L \equiv f(k, h) - kf_k - hf_h$.

Throughout, the assumption of capital skill complementarity (CSC) will be maintained:\(^6\)

**Assumption 1** Capital and skilled labour are (gross) complements in the sense that $f_{kh} > 0$. Further, at the uncoordinated equilibrium, the high skilled wage exceeds the low skilled wage: $w^H > w^L$.

Furthermore, as in Keen and Marchand (1997), assume:

**Assumption 2** The capital to skilled labour ratio, $k/h$, is non-increasing in $r$, which is equivalent to $hf_{kh} + kf_{kk} \leq 0$, and non-decreasing in $w^H$, which is equivalent to $hf_{kh} + kf_{kk} \leq 0$.

Jurisdictions are assumed to choose fiscal policy non-cooperatively. There are two tax instruments available: a unit tax on capital at rate $t$, and a tax on labour at rate $\tau$. If skilled and unskilled labour could be taxed at different rates, efficiency could be achieved by taxing only unskilled labour (since labour supply is assumed inelastic). Hence, in line with the tax competition literature, I assume that both types of labour have to be taxed.

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\(^5\)In section 3, land will be introduced as another factor of production.

\(^6\)Note that the CSC hypothesis strictly speaking holds that the elasticity of substitution between capital and unskilled labour is higher than that between capital and skilled labour. In the present analysis, however, only the absolute complementarity between capital and skilled labour matters.
at the same rate.\textsuperscript{7} Note also that if it were possible to finance opera houses with fees, this would always lead to a first best allocation.

Taxes are used to finance two public goods, one benefitting skilled labour, $g_H$, and one benefitting unskilled labour, $g_L$. As mentioned in the Introduction, one might think of theatres or opera houses which primarily benefit the upper classes versus public housing or social assistance which benefit the poor.\textsuperscript{8}

Each individual is assumed to inelastically supply one unit of labour. An individual in jurisdiction $i$ receives income from wages and an identical share of the jurisdiction’s capital endowment ($\bar{k}_i$). Therefore, the budget constraints of a skilled and unskilled individual can be written:

\begin{align*}
  x^H_i &= w^H_i + r\bar{k}_i \\
  x^L_i &= w^L_i + r\bar{k}_i,
\end{align*}

where $x^K_i$ is private good consumption of skill type $K = H, L$ in jurisdiction $i$.

The government budget constraint is:

\[ g^H_i + g^L_i = t_i k_i + \tau_i(l_i + h_i). \]

An individual with skill level $K = H, L$ living in jurisdiction $i$ has a quasiconcave utility function $u(x^K_i, g^K_i)$. Since jurisdictions are small, factor mobility implies that each jurisdiction treats the net return to capital, $r$, and the utility of mobile skilled workers, $\bar{u}$,
as given. Together with profit maximisation by firms, this implies:

\[
\begin{align*}
    f_k(k_i, h_i) - t_i &= r \quad \text{for all } i \\
    f_h(k_i, h_i) - \tau_i &= w_i^H \quad \text{for all } i \\
    u(x_i^H, g_i^H) &= \bar{u} \quad \text{for all } i.
\end{align*}
\]

Equation (4) is the location equilibrium condition for capital, and (6) the corresponding condition for skilled labour. Note that since skilled labour receives utility from public goods, this condition implies that the net return to labour is equalised across jurisdictions only when public goods levels are identical.

Throughout, I will consider a symmetric equilibrium where \( k_i = h_i = l_i = 1 \) for all \( i \) (hence, jurisdiction specific subscripts will be dropped). Differentiation of (4)–(6) gives the reaction of the endogenous variables \( k, h, w_i^H, w_i^L \) to a change of the policy variables:

\[
\begin{align*}
    k_t &= \frac{f_{kh}}{D} < 0, \quad h_t = k_r = -\frac{f_{kh}}{D} < 0, \quad h_r = \frac{f_{kk}}{D} < 0, \quad w_i^H = w_r^H = 0 \\
    k_g &= \frac{u_y^H f_{kh}}{u_x^H D} > 0, \quad h_g = -\frac{u_y^H f_{kk}}{u_x^H D} > 0, \quad w_i^H = -\frac{u_y^H}{u_x^H} < 0, \\
    w_i^L &= -k, \quad w_r^L = -h, \quad w_i^L = -hw_i^H > 0,
\end{align*}
\]

where \( D \equiv f_{kk} f_{kh} - f_{kh}^2 > 0 \), and \( u_y^H \equiv \partial u(x_H, g_H)/\partial g_H, k_g \equiv \partial k/\partial g_H, w_i^H \equiv \partial w_i^H/\partial g_H, \) and \( w_i^L \equiv \partial w_i^L/\partial g_H. \)

With CSC, capital taxes lead to an outflow of capital and skilled labour, and the same is true of labour taxes. Spending on \( g_H \) causes an inflow of skilled labour and (with CSC) capital. Unskilled wages fall with the capital and labour tax rate and rise with \( g_H. \)

Government is assumed to maximise an additive social welfare function:

\[ W = u(g_L, x_L) + u(g_H, x_H). \]

Since skilled workers’ utility is given (by (6)), the problem can be reformulated to maximise the welfare of unskilled workers. Using (2) and (3), the problem can be written as follows:

\[
\max_{t, \tau, g_H} u(t k + \tau (l + h) - g_H, w^L + \tau \bar{k}) \quad \text{subject to (4), (5), and (6)}.
\]

The first order conditions for interior solutions are:

\[
\begin{align*}
    u_y^L(k + tk_t + \tau h_t) - u_x^L k &= 0 \\
    u_y^L(tk_r + l + h + \tau h_r) - u_x^L (h + l) &= 0 \\
    u_y^L(tk_g + \tau h_g - 1) + u_x^L w_y^L &= 0.
\end{align*}
\]
Rewriting (10), (11) and (12), using (8) and (9), gives:

\[
\frac{u^L_g}{u^L_{x}} = \frac{k}{k + tk_t + \tau h_t} > 1, \quad (13)
\]

\[
\frac{u^L_g}{u^L_{x}} = \frac{l + h}{l + h + \tau h_x + tk_x} > 1, \quad (14)
\]

\[
\frac{u^H_g}{u^H_{x}} / \frac{u^L_g}{u^L_{x}} = (1 - tk_g - \tau h_g) < 1. \quad (15)
\]

Equations (13) and (14) are the usual Samuelson conditions for the supply of public goods with distortionary taxes (Atkinson and Stern, 1974): the marginal benefit of the public good is equated to the marginal cost of funds, and the MCF is equated for the two distortionary tax sources. With CSC, (13) shows the standard result that public goods are ‘underprovided’ at the margin, in the rough sense that the marginal rate of substitution between public and private consumption exceeds the marginal cost of public goods.

Equation (15) shows the effect of tax competition on the composition of public goods in the spirit of Keen and Marchand (1997). In the first best optimum, public good provision should equate the marginal rates of substitution between public and private goods for skilled and unskilled workers. In the non-cooperative equilibrium, mobility of skilled labour implies that \( h_g > 0 \), which leads to ‘overprovision’ of opera houses relative to social assistance (in the usual loose sense). Intuitively, spending on opera houses attracts mobile workers, which increases the tax base. Hence, the marginal cost of opera houses to the jurisdiction is lower than the marginal cost of social assistance. Second, CSC implies \( k_g > 0 \), which further exacerbates this effect: spending on opera houses now attracts mobile workers and physical capital. Third, the unskilled wage rises with \( g_H \). As long as \( u^L_x > u^H_x \) (which holds, e.g., under additive separability of the utility function), the distributional effect of this will increase social welfare. Hence, there will be ‘too many’ opera houses and ‘too little’ social spending (in the usual rough sense).

2.2. Coordinated policy change

Consider a coordinated change of the spending mix for given tax rates. The following Proposition shows the effect on total welfare in a jurisdiction.
Proposition 1 Suppose utility is additively separable. With CSC, starting from an uncoordinated symmetric equilibrium, welfare in each jurisdiction would rise with a coordinated rebalancing of expenditures from public goods benefitting skilled labour to those benefitting unskilled labour.

Proof. Note that the allocation of capital and skilled labour and therefore also factor prices will not change, i.e., \( dk_i = dh_i = 0 \) and \( dr = dw_H^i = 0 \) for all \( i \). This implies \( dx_K = 0 \) for \( K = H, L \). Starting from the symmetric equilibrium, and using the government budget constraint (\( dg_L = -dg_H \)), the effect on welfare is then given by

\[
dW = u_L^g dg_L + u_H^g dg_H = (u_L^g - u_H^g)dg_L
\]

\[
= u_L^g \left( 1 - \frac{u_L^H}{u_L^K}(1 - tk_g - \tau h_g) \right) dg_L,
\]

use having been made of (15) and the fact that \( h = 1 \) at the symmetric equilibrium. Using (8) and the fact that with separability \( w^H > w^L \) \( \leftrightarrow \) \( u_L^x > u_H^x \), Assumption 1 then implies that \( dW/dg_L \) is positive.

This result can be understood as a fiscal externality of the kind analysed in the fiscal competition literature (e.g., Wildasin, 1989). Providing opera houses leads to an inflow of mobile workers and (because of CSC) also capital into a jurisdiction.\(^9\) Each jurisdiction perceives this factor inflow as a benefit, but neglects the negative effect of the factor outflow on the tax bases of other jurisdictions. Spending on goods which benefit immobile factors produces no such externality, and, hence, at the uncoordinated equilibrium, each jurisdiction would benefit from rebalancing its spending towards the latter.

3. Mobility of unskilled labour

One obvious criticism of the basic model is that it assumes the unskilled to be immobile. This assumption might be justified by appealing to empirical regularities, but it is legitimate to ask how low skilled mobility affects the model. In fact, mobility of low skilled

\(^9\)This argument assumes that workers consume public goods in the jurisdiction where they reside. There is a literature which studies the ‘exploitation’ of central cities which provide public goods that are used by residents of the suburbs (Neenan, 1970).
is a central issue in the alleged ‘race to the bottom’ in welfare policies (e.g., Brueckner, 2000). How then does Proposition 1 stand up when unskilled labour is mobile as well? This section will analyse that question.

Since from the individual jurisdiction’s viewpoint, mobility of labour means that workers’ utility levels are exogenously given, instead of welfare maximisation I now assume that jurisdictions maximise land rents (see, e.g., Wilson, 1995; Matsumoto, 2000).\(^\text{10}\) In order to do this, land is now introduced into the production function. The production function is now written \(f(k, h, l)\) with land in the background. The assumption of linear homogeneity of the production function is now dropped, so that firm profits can be viewed as land rents, \(R = f(k, h, l) - (r + \tau)k - (w_H + \tau)h - (w_L + \tau)l\). As in Matsumoto (2000), I assume that workers are equally endowed with land in all jurisdictions. Let \(y\) be the land rent income of a typical worker. It will also be assumed that since jurisdictions are small, they treat workers’ land rent income (which comes from land in all the nation’s jurisdictions) as exogenous.

Mobility of skilled and unskilled labour implies:

\[
\begin{align*}
  u(g_H, x_H) & \geq \bar{u}_H \\
  u(g_L, x_L) & \geq \bar{u}_L,
\end{align*}
\]

where net wages of the skilled and unskilled workers are given by

\[
\begin{align*}
  f_h - \tau &= w_H \\
  f_l - \tau &= w_L.
\end{align*}
\]

Equations (16), (17), (18) and (19) determine the skilled and unskilled wage as functions \(w_H(g_H), w_L(g_L)\), with

\[
\begin{align*}
  u'_{gH} &= -\frac{u_g^H}{u_x^H}, u'_{gL} = -\frac{u_g^L}{u_x^L}.
\end{align*}
\]

Together with (4), this system of equations determines \(k, h, l\) as functions of \(t, \tau, g_H,\)

\(^{10}\)It can be shown, however, that one can analyse the dual problem of maximising the welfare of mobile workers subject to a constraint on land rents, see Wilson (1995).
and $g_L$. Differentiating (18), (19), and (4), using (20), gives

\[ k_{gh} = \frac{1}{S}[f_{hl}f_{kl} - f_{kh}f_{hl}]w^H_{gh}, k_{gL} = \frac{1}{S}[f_{hl}f_{kh} - f_{hk}f_{kl}]w^L_{gL} \]  
\[ (21) \]

\[ h_{gh} = \frac{1}{S}[f_{kk}f_{kl} - f_{kh}f_{kl}]w^H_{gh}, h_{gL} = \frac{1}{S}[f_{kh}f_{kl} - f_{kh}f_{hl}]w^L_{gL} \]  
\[ (22) \]

\[ l_{gh} = \frac{1}{S}[f_{kh}f_{kl} - f_{kh}f_{hl}]w^H_{gh}, l_{gL} = \frac{1}{S}[f_{kh}f_{kk} - f_{kh}f_{hl}]w^L_{gL} \]  
\[ (23) \]

where concavity of $f(k, h, l)$ implies

\[ S = \begin{vmatrix} f_{kk} & f_{kh} & f_{kl} \\ f_{kh} & f_{hh} & f_{hl} \\ f_{kl} & f_{hl} & f_{ll} \end{vmatrix} < 0. \]

I now introduce a different version of CSC:

**Assumption 3** (i) The production function displays capital-skill complementarity in the sense that $f_{kh} > f_{kl}$ for $h = l$. (ii) The unskilled to skilled labour ratio, $l/h$, is non-increasing in $w_L$, which is equivalent to $hf_{hh} + lf_{hl} \leq 0$, and non-decreasing in $w_H$, which is equivalent to $hf_{kk} + lf_{kl} \leq 0$.

The typical jurisdiction’s maximisation problem is:

\[
\max_{t, \tau, g_H, g_L} f(k, h, l) - (r + t)k - (w_H + \tau)h - (w_L + \tau)l \\
\text{s.t.} \quad tk + \tau(h + l) = g_H + g_L.
\]

Letting $\lambda$ be the Lagrangean multiplier, the first order conditions can be written

\[
-k + \lambda(k + tk_l + \tau(h_l + l_l)) = 0 \tag{24}
\]
\[
-l - h + \lambda(tk_r + I + h + \tau(l_r + h_r)) = 0 \tag{25}
\]
\[
-hw^H_{gg} + \lambda(tk_{gg} + \tau(h_{gg} + l_{gh} - 1)) = 0 \tag{26}
\]
\[
-lw^L_{gl} + \lambda(tk_{gl} + \tau(h_{gl} + l_{gl} - 1)) = 0. \tag{27}
\]

Rewriting (26) and (27), using (20), gives

\[
\frac{u^H_g}{u^H_x} = \frac{1 - tk_{gg} - \tau(h_{gg} + l_{gh})}{1 - tk_{gl} - \tau(h_{gl} + l_{gl})}. \tag{28}
\]

Equations (26) and (27) show that the provision of public spending is capitalised in the mobile workers’ wages, where the marginal effect on the wage corresponds to the worker’s
marginal rate of substitution between public and private goods. Moreover, the cost of providing one unit of a public good is one minus the revenue effect of the respective good. As long as revenue is more responsive to $g_H$ than $g_L$, one would expect there to be relative overprovision of $g_H$ in equilibrium.

To see whether this is indeed the case, consider now the effect of a coordinated rebalancing of expenditure from $g_H$ to $g_L$. Since this has no effect on factor allocation, wages and rents are unaffected. Hence, the change of welfare can simply be traced to the change of workers’ utility due to the change of public good supply.

**Proposition 2** Suppose that utility is additively separable and $f_{hh} \geq f_{ll}$ for $h = l$. Then, starting from the symmetric non-cooperative equilibrium, a coordinated rebalancing of expenditure from $g_H$ to $g_L$ raises welfare in every jurisdiction.

**Proof.** See the Appendix. ■

The assumption that $f_{hh} \leq f_{ll}$ at the equilibrium ensures that the revenue effect of $g_H$ is larger than that of $g_L$, given that CSC holds (this can be seen by inspecting (21)–(23)). If this is true, overprovision of public goods benefitting skilled labour holds even when unskilled labour is mobile. The intuition is simply that with CSC, the fiscal externality implied by spending on $g_H$ is larger, at the margin, than the externality of spending on $g_L$. Therefore, shifting expenditure from $g_H$ to $g_L$ can improve welfare.

To sum up, jurisdictions in fiscal competition will tend to spend ‘too much’ on opera houses and other goods benefitting high skilled workers and too little on public housing or other goods benefitting low skilled under CSC even when the low skilled are mobile as well. Hence, the hypothesis of the paper is that CSC and differential mobility of the high skilled should lead to an overprovision of opera houses.\(^\text{11}\)

4. Conclusion

The paper has examined the composition of public spending in a model of fiscal competition with heterogeneous labour. In particular, when households are mobile and capital and skilled labour complementary, jurisdictions would benefit by coordinating on spending

\(^{11}\text{This is not to infer in any way that the author is biased against opera houses...}\)
more on public goods benefiting the low skilled and less on public goods benefiting the high skilled. This distortion of spending can also be shown to exist when both types of labour are mobile, as long as capital skill complementarity holds.

There is some circumstantial evidence that jurisdictions compete by offering tax incentives to firms, infrastructure, but also public goods benefiting mobile skilled labour. One question would be whether one can also find hard empirical evidence for the distortion of spending under fiscal competition. Another empirical application would be to simulate welfare effects of policy coordination based on estimates of substitution elasticities between factors of production. This would allow to gauge the magnitudes of the welfare effects analysed in the paper.

Appendix

Proof of Proposition 2. Starting from the symmetric equilibrium, the effect on welfare is given by

\[ u_g^H dg_H + u_g^L dg_L = (u_g^L - u_g^H) dg_L. \]  

(29)

Using (21)–(23) in (28) and solving implies:

\[
\frac{u_g^L}{u_g^H} - 1 = -\frac{1}{S} \frac{u_g^L}{u_g^H} (tA + \tau B),
\]

where

\[
A \equiv f_{hl}(f_{kl} - f_{kh}) + f_{kl}f_{hh} - f_{kh}f_{ll},
\]

\[B \equiv f_{kh}^2 - f_{kl}^2 + f_{kl}(f_{ll} - f_{kh}).\]

Since \( S < 0 \), a sufficient condition for \( \frac{u_g^L}{u_g^H} > 1 \) is \( A, B > 0 \). Now, note that as long as \( f_{kh} > 0 \), both \( A \) and \( B \) are increasing in \( f_{kh} \). Solving gives

\[ A > 0 \iff f_{kh} > \frac{f_{kl}(f_{hh} + f_{hl})}{f_{ll} + f_{hl}}, \]

(31)

\[ B > 0 \iff f_{kh} > \sqrt{f_{kl}^2 + f_{kl}(f_{hh} - f_{ll})}. \]

(32)

Assumption 3 implies the inequality in (31) is fulfilled as long as \( f_{hh} \geq f_{ll} \). Furthermore, with \( f_{hh} \geq f_{ll}, \) Assumption 3 also implies \( B > 0 \). Since separability together with \( w_H > w_L \) implies \( u_g^L > u_g^H \), it follows that \( u_g^L > u_g^H \).
References


