

Baake, Pio; Mitusch, Kay

**Working Paper**

## Mobile Phone Termination Charges with Asymmetric Regulation

DIW Discussion Papers, No. 500

**Provided in Cooperation with:**

German Institute for Economic Research (DIW Berlin)

*Suggested Citation:* Baake, Pio; Mitusch, Kay (2005) : Mobile Phone Termination Charges with Asymmetric Regulation, DIW Discussion Papers, No. 500, Deutsches Institut für Wirtschaftsforschung (DIW), Berlin

This Version is available at:

<http://hdl.handle.net/10419/18351>

**Standard-Nutzungsbedingungen:**

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

**Terms of use:**

*Documents in EconStor may be saved and copied for your personal and scholarly purposes.*

*You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.*

*If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.*

Discussion Papers

# 500

Pio Baake\*  
Kay Mitusch\*\*

Mobile Phone Termination Charges  
with Asymmetric Regulation

Berlin, Juli 2005



**DIW** Berlin

German Institute  
for Economic Research

\* DIW Berlin, Department Information Society and Competition, pbaake@diw  
\*\* TU Berlin, Workgroup for Infrastructure Policy (WIP), km@wip.tu-berlin

## IMPRESSUM

© DIW Berlin, 2005

DIW Berlin  
Deutsches Institut für Wirtschaftsforschung  
Königin-Luise-Str. 5  
14195 Berlin  
Tel. +49 (30) 897 89-0  
Fax +49 (30) 897 89-200  
[www.diw.de](http://www.diw.de)

ISSN print edition 1433-0210  
ISSN electronic edition 1619-4535

All rights reserved.  
Reproduction and distribution  
in any form, also in parts,  
requires the express written  
permission of DIW Berlin.

# Mobile Phone Termination Charges with Asymmetric Regulation

Pio Baake\*      Kay Mitusch†  
*DIW Berlin*      *TU Berlin*

June 21, 2005

## Abstract

We model competition between two unregulated mobile phone companies with price-elastic demand and less than full market coverage. We also assume that there is a regulated full-coverage fixed network. In order to induce stronger competition, mobile companies could have an incentive to raise their reciprocal mobile-to-mobile access charges above the marginal costs of termination. Stronger competition leads to an increase of the mobiles' market shares, with the advantage that (genuine) network effects are strengthened. Therefore, 'collusion' may well be in line with social welfare.

*JEL-classification numbers:* L41, L96.

*Keywords:* telecommunication, mobile phones, mobile-to-mobile access charges, network effects

---

\*DIW Berlin, Königin-Luise-Str. 5, 14191 Berlin, Germany, Email: pbaake@diw.de.

†TU Berlin, Sekr. H 33, Straße des 17. Juni 135, 10623 Berlin, Germany, E-mail: mitusch@wip.tu-berlin.de

# 1 Introduction

The regulation of termination charges on the fixed network is widely considered to be a cornerstone for the promotion of competition in liberalized telecommunication markets. In Germany, as in Europe in general, the desire to foster competition led to an asymmetric treatment of the dominating fixed network provider (Deutsche Telekom AG which owns and controls more than ninety percent of the local loop) on the one hand and the mobile phone providers (two larger and two smaller firms, with Deutsche Telekom being the largest) on the other hand. Under the German regime only the fixed network's termination charges are regulated, not those of the mobile networks. Moreover, the retail prices of mobile companies are also unregulated whereas those of the fixed network are subjected to a price cap. The fixed network provider is, however, allowed to pass the fixed-to-mobile termination charges on to its consumers. That is, the price of a call originating on the fixed network and terminating on a mobile network equals the (regulated) fixed network retail price plus the (unregulated) termination charge set by the respective mobile company.

This paper investigates the economic consequences of such an asymmetric regulation on the remaining, unregulated access prices and tariffs. We will analyze competition between two independent mobile phone companies under the assumption that total demand for mobile connections is price elastic, i.e., that the market is only partially covered by mobile providers. This assumption is in contrast to the standard model of competition between networks, see for example Laffont, Rey, and Tirole (1998, section 5), Gans and King (2001). We will additionally assume that every customer also connects on to the single, regulated fixed network provider. As the coexistence of fixed and mobile phones opens up the possibility to call a person in different ways (from fixed to fixed, from fixed to mobile, from mobile to fixed, and from mobile to mobile), we will assume that different *kinds* of calls are imperfect substitutes.

We allow for two-part tariffs with price differentiation for on-net and off-net calls. Mobile companies will set the variable prices for different kinds of calls by taking into account the substitutabilities between the different kinds of calls as well as the perceived marginal costs, which may include termination charges. The fixed components of the tariffs, i.e., the connection prices, are used to compete for customers and, of course, to extract rents.

It turns out that mobile companies set their termination charges for calls originating from the fixed network above the marginal interconnection costs. The negotiated (reciprocal)

termination charge for mobile-to-mobile calls is used as a strategic instrument to either *soften* or *strengthen* competition for customers. In the latter case, a termination charge *above* marginal costs leads to tariff-induced network effects which induce tougher competition for customers. As a consequence, the number of mobile phone users and hence the firms' market shares increase. This is desirable for the companies for two reasons. First, since mobile connections generate a genuine positive network externality (by an improvement of reachability) mobiles become the more attractive the more consumers already have a mobile. Second, the more customers are connected to a mobile network the higher the number of calls either originating from or terminating on the mobile networks. Both effects raise the profits of mobile companies, so that mobile companies have, in principle, an incentive to use high termination charges as a commitment device for strong competition. Note that this is also perfectly in line with social welfare maximization. The networks effects imply that 'collusion' for stronger competition increases social welfare.

The extent to which mobile companies will strengthen competition by choosing a high termination charge for mobile-to-mobile calls depends in a complex way on the degree of substitutability (which we assume to be the same between all different kinds of calls) and the implied market shares. Our model shows that if there were no substitutes at all, i.e. all kinds of calls are independent goods (a very unrealistic case), the additional utility from using a mobile is rather high which in turn leads to relatively high market shares. The marginal importance of market coverage is then rather small, which also implies that the mobile companies will try to *soften* competition by setting the termination charge below marginal cost. At the other extreme, if different kinds of calls were perfect substitutes, there would be no room for mobiles in the first place (given that their costs are no less than those of the cost-price regulated fixed network). However, for an intermediate degree of substitutability we obtain the above mentioned result, namely, that the negotiated termination charge is set *above* marginal costs of termination. The degree of substitutability is therefore an important variable, as it affects the relative strength of the different effects and the viability of mobiles in the first place.

The model yields a structure of access and retail prices that seems to be in line with casual empiricism. Mobile-to-mobile termination charges being above marginal costs of termination resemble observed pricing behavior of firms. Moreover, when this relationship holds it turns out that the fixed-to-mobile termination charges will exceed both the regulated mobile-to-fixed and the mobile-to-mobile charges. The implied retail prices for fixed-to-

mobile calls exceed those for on-net mobile calls. Furthermore, off-net mobile-to-mobile calls are more expensive than those on-net.

This structure of access and retail prices is in contrast to the results obtained in the standard model with full market coverage, see Gans and King (2001, Proposition 2), Cambini and Valletti (2003) and De Bijl and Peitz (2002, section 6.4). Full market coverage implies that reciprocal access charges are set below marginal costs, which leads to other implausible price relationships, namely, prices for off-net mobile-to-mobile calls being lower than those for on-net mobile calls. While Schiff (2002) considers partial market coverage, he also assumes that the firms' tariffs affect the consumers' ex ante participation decisions which leads to fiercer competition for customers. Neglecting price discrimination between off-net and on-net calls and concentrating on two networks, Schiff shows that access charges below marginal cost are again optimal.

Optimal access charges above marginal costs are obtained by Poletti and Wright (2004) and Valletti and Cambini (2005). Focusing on different consumer groups Poletti and Wright (2004) show that access charges above marginal costs can increase the firms' profits by increasing the marginal tariffs for *light* users and thus relaxing the incentive constraints for *heavy* users. Valletti and Cambini (2005) analyze quality improving investments. Since access charges above marginal costs impose an access deficit on a network with a larger market share, the firms' incentives to attract more customers by enhancing the quality of their networks are negatively correlated with the access charges. Thus, access charges above marginal costs serve as a mechanism to reduce costly investments in quality.

We know of no model that addresses the issue of mobile-to-mobile access charges in a situation with only partial market coverage by mobiles and an alternative fixed network in the background. The existing literature on mobile phone telecommunication focuses on fixed-to-mobile access charges. To that aim, Houpis and Valletti (2004) assume that there are no mobile-to-mobile calls. Gans and King (2000) use the same assumption in one part of their paper; in the other part they assume that mobile-to-mobile access charges are exogenously set at the marginal costs of termination. These papers find that fixed-to-mobile access charges are set too high (from a welfare point of view) and thus should be regulated, as is the case in the UK and Australia. In our model, mobile companies will also set the fixed-to-mobile access charges above marginal costs of termination. However, here we take the German regulatory regime as given and focus on its implications on mobile-to-mobile access charges.

The next section introduces the model. Section 3 analyzes demand. The equilibrium determination of market shares is quite complex and allows for no closed solution of the demand expressions, even if a simple underlying utility function is used. We therefore have to resort to a numerical example for the final analysis of price competition and negotiated access charges in section 4. Section 5 discusses variations in the degree of substitutability between different kinds of calls. Section 6 summarizes.

## 2 The Model

### 2.1 Firms

We consider competition between three different telecommunication firms,  $A$ ,  $B$ , and  $F$ . Firm  $F$  is the single (incumbent) fixed network provider, while firms  $A$  and  $B$  are two mobile phone companies. We have to keep track where telephone calls originate and where they terminate. For  $i, j \in \{A, B, F\}$ , we denote by  $X_j^i$  the number of calls originating at network  $i$  and terminating at network  $j$ . Figure 1 lists all possible kinds of calls.

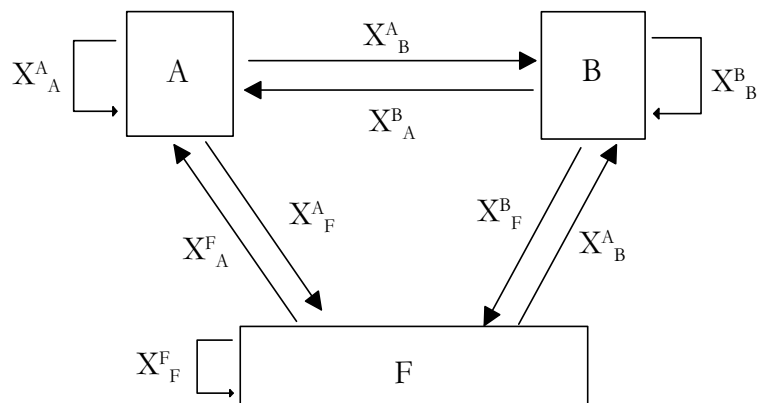


Figure 1: Inter- and Intranetwork calls

A firm faces three cost components. First, there is a fixed cost  $k^i$  per customer subscribing to its network, where we assume  $k^A = k^B \geq k^F$ , since mobiles usually incur higher costs. Next, there are variable costs per call originating or terminating on a firm's network. For the fixed network,  $F$ , the variable costs are normalized to zero. Again, in order to capture the



commonly observed cost differential of mobile networks, we allow for variable costs  $c \geq 0$  of firms  $A$  and  $B$ . Note that operating an on-net call within a mobile network costs  $2c$ . The third cost component are the access charges for interconnecting calls. For  $i \neq j$ , we denote by  $a_j^i$  the termination charge firm  $i$  has to pay to firm  $j$  for a call originating at network  $i$  and terminating at network  $j$ .

Firms' proceeds consist of the termination charges they receive from other firms and the tariffs paid by their own customers. We will consider tariffs consisting of a subscription fee and prices per calls. The fixum charged by firm  $i$  to each subscriber is denoted by  $s^i$ . The price charged by firm  $i$  to a customer who calls someone at network  $j$  from its network is denoted by  $p_j^i$  (including the case  $i = j$ ).

We assume that the total mass of consumers is 1 and that *all* consumers connect to the fixed network. A consumer may in addition connect to either one of the two mobile networks (it is assumed that no-one wants to carry around two mobiles). Letting  $n^A$  resp.  $n^B$  denote the numbers of customers subscribing to firm  $A$  resp.  $B$ , we arrive at the following profit expressions:

$$\Pi^F = (s^F - k^F) + p_F^F X_F^F + (p_A^F - a_A^F) X_A^F + (p_B^F - a_B^F) X_B^F + a_F^A X_F^A + a_F^B X_F^B \quad (1)$$

$$\begin{aligned} \Pi^A = & (s^A - k^A) n^A + (p_A^A - 2c) X_A^A + (p_F^A - c - a_F^A) X_F^A \\ & + (p_B^A - c - a_B^A) X_B^A + (a_A^F - c) X_A^F + (a_A^B - c) X_B^F \end{aligned} \quad (2)$$

$$\begin{aligned} \Pi^B = & (s^B - k^B) n^B + (p_B^B - 2c) X_B^B + (p_F^B - c - a_F^B) X_F^B \\ & + (p_A^B - c - a_A^B) X_A^B + (a_B^F - c) X_B^F + (a_B^A - c) X_A^A \end{aligned} \quad (3)$$

Asymmetric regulation is captured by the following assumptions. As termination charges of the fixed network are regulated, we assume that  $a_F^A = a_F^B = 0$ , since firm  $F$ 's variable costs are zero. In contrast, termination charges of mobile networks are not regulated. We assume that mobile companies enter bargaining over a reciprocal bilateral access charge  $a := a_B^A = a_A^B$ . Afterwards each mobile company unilaterally sets the termination charge that has to be paid by the fixed network,  $a_A^F$  resp.  $a_B^F$ , as well as its own tariff; these are unregulated. In contrast, tariffs of the fixed network are regulated. We assume that they just cover costs, so that  $s^F = k^F$ ,  $p_F^F = 0$  and  $p_i^F = a_i^F$  for  $i = A, B$ , since the fixed network provider is allowed to pass termination charges of mobile companies on to consumers.<sup>1</sup> Note

---

<sup>1</sup>Recall that variable costs of the fixed network are assumed to be zero. If they were positive, say  $c^F > 0$ , then  $a_F^A = a_F^B = c^F$ ,  $p_F^F = 2c^F$ , and  $p_i^F = c^F + a_i^F$ .

that the fixed network provider is not an active player in this game.

We are now able to summarize the timing of the model.

1. Mobile companies bargain over their reciprocal access charge  $a$ .
2. Mobile companies simultaneously choose fixed network access charges,  $a_A^F$  resp.  $a_B^F$ , and their own tariffs,  $s^i$  and  $p_j^i$  for  $i = A, B$  and  $j = A, B, F$ .
3. Consumers subscribe to the fixed network and decide about subscription to a mobile company.
4. Consumers decide about the amounts of calls they want to make.

In order to solve the model by backward induction we have to describe consumer choice.

## 2.2 Consumers

Consumers are ex ante differentiated with respect to their valuation of having a mobile, as well as to their preferences among the two mobiles. After the connecting decisions have been made, they are additionally differentiated according to their ‘subscriber types’. These are denoted by  $i \in \{A, B, F\}$  as follows: If a consumer subscribes to a mobile company (and the fixed network) he becomes an ‘ $A$ -subscriber’ resp. ‘ $B$ -subscriber’; if he does not subscribe to any mobile, but only to the fixed network, he becomes an ‘ $F$ -subscriber’. Letting  $CS^i$  denote the surplus an  $i$ -subscriber derives from calls (to be described below) and introducing two consumer-type variables  $\theta$  and  $\mu$  (to be explained immediately) we arrive at the following total utility expression.

$$\text{Utility of Consumer } (\mu, \theta) = \begin{cases} CS^A - s^A - s^F + \theta - \mu & \text{if he subscribes to } A \\ CS^B - s^B - s^F + \theta - (1 - \mu) & \text{if he subscribes to } B \\ CS^F - s^F & \text{if he does not subscribe} \end{cases}$$

The consumer types  $\theta$  and  $\mu$  introduce vertical as well as horizontal product differentiation from an ex ante point of view. The larger a consumer’s  $\theta$  the more he desires a mobile. On the other hand, the larger his  $\mu$  the more he is enticed by the image and design of mobile company  $B$ , as compared to  $A$ ; while consumer  $\mu = 1/2$  is intrinsically indifferent between

the two. We assume that consumer preferences are such that  $\theta \in [0, 1]$  and  $\mu \in [0, 1]$  are uniformly and independently distributed.

Calls enter utility as follows. We assume that only outgoing calls are valued (i.e. the calls someone makes, not the ones he receives). We also assume a uniform calling behavior, i.e. an  $i$ -subscriber makes to every  $j$ -subscriber the same amounts of calls. However, there are different kinds of calls to the same person. We assume that using a mobile or a conventional telephone are imperfect substitutes. For example, the conventional phone offers a higher quality of connection and is more convenient to use, if it is in easy reach. But if one is away from it, a mobile is more convenient. Similarly, if one wants to call someone who is at home or in his office one will call him on his fixed telephone. But if he is likely to be somewhere else, one might try to call him on the mobile first (if he has one). Thus, there are potentially four different modes of calling the same person. In order to express agents' utilities, we introduce the following notation. The kind of phone used will be indicated by either  $f$ , fixed, or  $m$ , mobile. Then, for  $i, j \in \{A, B, F\}$  and  $k, l \in \{f, m\}$ , denote

$x_{jl}^{ik}$  : The amount of calls by an  $i$ -subscriber using his  $k$ -kind of phone to each  $j$ -subscriber on his  $l$ -kind of phone.

Note, superscripts describe origination (Who calls, using which kind of phone?) and subscripts termination (Who is being called, which kind of phone is ringing?). Examples are:

$x_{Bf}^{Am}$  : The amount of calls by an  $A$ -subscriber using his mobile to each  $B$ -subscriber on his fixed phone.  
 $x_{Bm}^{Ff}$  : The amount of calls by a non-subscriber (using his fixed phone, of course) to each  $B$ -subscriber on his mobile.

Clearly,  $x_{jl}^{Fk} = 0$  if  $k \neq f$  and  $x_{Fl}^{ik} = 0$  if  $l \neq f$ , since non-subscribers have no mobiles. All other  $x_{jl}^{ik}$  can be positive and in the following we will presume that they are.

Since  $x_{jl}^{ik}$  are defined as numbers of calls to *each*  $j$ -subscriber it follows that, for example, an  $A$ -subscriber will make a total of  $n^B x_{Bm}^{Am}$  mobile-to-mobile calls to all the  $B$ -subscribers. An  $A$ -subscriber's utility from calling  $B$ -subscribers in the four different modes is assumed to be homogenous in the number of calling partners:

$$n^B U(x_{Bf}^{Af}, x_{Bm}^{Af}, x_{Bf}^{Am}, x_{Bm}^{Am})$$

where  $U(\cdot)$  is a standard concave utility function with imperfect substitutes. Denoting the number of non-subscribers by  $n^F := 1 - n^A - n^B$ , a non-subscriber's utility from calling other non-subscribers is similarly given by  $n^F U(x_{Ff}^{Ff}, x_{Fm}^{Ff}, x_{Ff}^{Fm}, x_{Fm}^{Fm}) = n^F U(x_{Ff}^{Ff}, 0, 0, 0)$ . And so on. An  $i$ -subscriber's consumer surplus (before deducing fixed elements) is therefore:

$$CS^i = \sum_{j=A,B,F} n^j [U(x_{jf}^{if}, x_{jm}^{if}, x_{jf}^{im}, x_{jm}^{im}) - p_F^i x_{jf}^{if} - p_j^i x_{jm}^{if} - p_F^i x_{jf}^{im} - p_j^i x_{jm}^{im}] \quad (4)$$

Note, implicit in this formulation is a positive externality from an increase of  $n^A$  or  $n^B$  (at the expense of  $n^F$ ) on every consumer. The reason is that only non-subscribers are subject to the constraints  $x_{jl}^{Fk} = 0$  if  $k \neq f$  and  $x_{Fl}^{ik} = 0$  if  $l \neq f$ , while everyone can be called on the fixed network. Hence, if all the other  $x_{jl}^{ik}$  are strictly positive at the consumption optimum, the maximized terms in the square brackets of (4) must be higher for  $j = A$  and  $j = B$  than for  $j = F$ .

As a particular example of the  $U$ -function we will use a generalized Dixit-function:

$$U(x_{jf}^{if}, x_{jm}^{if}, x_{jf}^{im}, x_{jm}^{im}) = \sum_{k=f,m} \sum_{l=f,m} \left( x_{jl}^{ik} - \frac{1}{2} x_{jl}^{ik} \left( (1 - \gamma) x_{jl}^{ik} + \gamma \sum_{\kappa=f,m} \sum_{\lambda=f,m} x_{j\lambda}^{i\kappa} \right) \right) \quad (5)$$

where  $\gamma \in [0, 1)$  measures the degree of substitutability of the goods, assumed to be identical for all kinds of calls.

Before analyzing the model, we will state the relations between individual and aggregate amounts of calls. Recall that an  $A$ -subscriber makes a total of  $n^B x_{Bm}^{Am}$  mobile-to-mobile calls to the  $B$ -subscribers. This implies that the total number of calls originating at network  $A$  and terminating at network  $B$  equals  $X_B^A = n^A n^B x_{Bm}^{Am}$ . Similarly, we arrive at the following relationships between the different patterns of use ( $x_{jl}^{ik}$ ) and the total amount of connections ( $X_j^i$ , see Figure 1 above). For  $i, j \in \{A, B\}$ :

$$X_j^i = n^i n^j x_{jm}^{im} \quad (6)$$

$$X_F^i = n^i (n^F x_{Ff}^{im} + n^A x_{Af}^{im} + n^B x_{Bf}^{im}) \quad (7)$$

$$X_i^F = n^i (n^F x_{im}^{Ff} + n^A x_{im}^{Af} + n^B x_{im}^{Bf}) \quad (8)$$

$$X_F^F = n^F (n^F x_{Ff}^{Ff} + n^A x_{Af}^{Ff} + n^B x_{Bf}^{Ff}) + n^A (n^F x_{Ff}^{Af} + n^A x_{Af}^{Af} + n^B x_{Bf}^{Af}) \quad (9)$$

$$+ n^B (n^F x_{Ff}^{Bf} + n^A x_{Af}^{Bf} + n^B x_{Bf}^{Bf})$$

### 3 Consumer Choices

#### 3.1 Demand for calls

At the final stage 4 of the game,  $i$ -subscribers, for  $i \in \{A, B, F\}$ , will decide about the numbers of calls they make:

$$\begin{aligned} \max_{x_{jl}^{ik}} CS^i \quad \text{subject to } x_{jl}^{ik} = 0 \quad & \begin{cases} \text{if } i = F \text{ and } k \neq f \\ \text{or } j = F \text{ and } l \neq f \end{cases} \\ j \in \{A, B, F\} \\ k, l \in \{f, m\} \end{aligned}$$

In the following we denote the vector of variable prices by  $p = (p_j^i)$ , for all  $i, j \in \{A, B, F\}$ , the vector of market shares by  $n = (n^A, n^B, n^F)$ , and the solutions to the above program by  $x_{jl}^{ik}(p, n)$ . For the Dixit utility function given in (5) the Appendix shows that these functions are linear in prices, decreasing in the respective own prices (i.e., the direct costs as given in (4)) and increasing in the other prices. Inserting the solution back into  $CS^i$  gives, as a preliminary result, the maximized consumer surplus for *given* market shares. These values will be denoted by  $v^i(p, n)$  and their vector by  $v(p, n) = (v^A(p, n), v^B(p, n), v^F(p, n))$ .

#### 3.2 Connection decisions

At stage 3 consumers make the subscription decisions. Assuming that all subscriber groups are strictly positive, we can identify the respective indifferent consumers. In the following we omit the arguments of functions where this does not lead to confusion. Among the subscribers to a mobile company, those who are indifferent between companies  $A$  and  $B$  are given by

$$v^A - s^A - \mu + \theta = v^B - s^B - (1 - \mu) + \theta \Rightarrow \mu^*(v(p, n), s) := \frac{1}{2}(1 + v^A - s^A - (v^B - s^B))$$

where  $s := (s^A, s^B)$ . Hence, consumers with  $\mu > \mu^*$  will subscribe to company  $B$ , if at all. The consumers who are indifferent between non-subscription to a mobile (i.e. ‘ $F$ -subscription’) and subscription to mobile company  $A$  resp.  $B$  are given by the conditions

$$\begin{aligned} v^F = v_A - s^A - \mu + \theta & \Rightarrow \theta^A(\mu, v(p, n), s^A) := v^F - v^A + s^A + \mu \\ v^F = v^B - s^B - (1 - \mu) + \theta & \Rightarrow \theta^B(\mu, v(p, n), s^B) := v^F - v^B + s^B + (1 - \mu) \end{aligned}$$

Hence, consumers with  $\mu < \mu^*$  and  $\theta > \theta^A$  will subscribe to mobile company  $A$  and to  $F$ . Consumers with  $\mu > \mu^*$  and  $\theta > \theta^B$  will subscribe to mobile company  $B$  and to  $F$ . The remainder will only subscribe to  $F$ .

We can now calculate the equilibrium amounts of subscribers. Defining  $\mu^A(v(p, n), s) := \min\{\mu^*, 1 - (v^F - (v^A - s^A))\}$  and  $\mu^B(v(p, n), s) := \max\{\mu^*, 1 - (v^F - (v^B - s^B))\}$ , one obtains

$$n^A = \int_0^{\mu^A(v(p, n), s)} (1 - \max\{\theta^A, 0\}) d\mu \quad (10)$$

$$n^B = \int_{\mu^B(v(p, n), s)}^1 (1 - \max\{\theta^B, 0\}) d\mu \quad (11)$$

and  $n^F = 1 - n^A - n^B$ . After inserting  $\mu^*$  and  $v_i(p, n)$ , the system of equations can be solved for  $(n^A, n^B, n^F)$ . Note that this solution gives us the market shares as well as the consumer surpluses as functions of the tariffs  $(p, s)$  only, i.e. a vector  $n(p, s)$  and a vector  $v(p, s) := v(p, n(p, s))$ . Figure 2 illustrates a possible market segmentation that could be the result of firms' pricing decisions (in the example shown it holds that  $\mu^A = \mu^B = \mu^*$  and  $\theta^A > 0, \theta^B > 0$  for all  $\mu \in [0, 1]$ ).

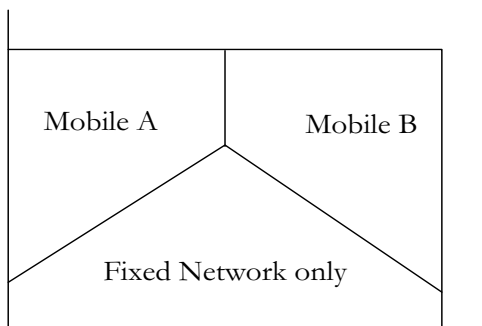


Figure 2: Possible market segmentation

Finally, total demands for connections,  $X_j^i$ , which enter the firms' profit functions, can be calculated using (6) to (9). Note that the demand expressions are highly non-linear in prices even if they are based on a Dixit utility function. Although the individual choices of calls,  $x_{jl}^{ik}(p, n)$ , are simple linear functions in the variable prices, the equilibrium pattern of market segmentation yields complicated expressions for the  $n^i(p, s)$ , which will then have

to be multiplied to get the  $X_j^i$ . However, the Dixit utility function yields unique demand expressions with interior solutions for appropriate price ranges and with comparative statics having the expected signs.

## 4 Choices of mobile phone companies

### 4.1 Tariffs and fixed-to-mobile access charges

At stage 2, mobile companies individually and simultaneously set the fixed-to-mobile access charges as well as their own tariffs (taking their reciprocal access charge  $a$  as given). That is, firm  $i = A, B$  sets  $a_i^F$ ,  $s^i$  and  $p_j^i$  for  $j = A, B, F$ . Recalling regulation of the fixed network,  $s^F = k^F$ ,  $p_F^F = 0$ ,  $a_F^i = 0$  and  $p_i^F = a_i^F$  (since the fixed network is allowed to pass access charges for off-net calls on to consumers), we now look at reaction functions and Nash equilibrium for stage 2.

Defining  $c_i^i := 2c$  and  $c_j^i := c + a_j^i$  for  $j \neq i$  the profit expression of mobile phone company  $A$ , given by (2), can be restated as:

$$\Pi^A = \sum_{j=A,B,F} (p_j^A - c_j^A) X_j^A + \sum_{j=B,F} (a_A^j - c) X_A^j + n^A (s^A - k^A)$$

For firm  $B$  analogously. In order to characterize reaction functions by first-order conditions, we have to take account of the fact that consumers' connection decision equilibrium  $(n^A, n^B)$  depends on the tariffs,  $s^i$  and  $p_j^i$  for  $i, j = A, B, F$ .

The first-order condition for  $s^A$  is

$$\sum_{j=A,B,F} \left[ (p_j^A - c_j^A) \frac{dX_j^A}{ds^A} \right] + \sum_{j=B,F} \left[ (a_A^j - c) \frac{dX_A^j}{ds^A} \right] + \frac{\partial n^A}{\partial s^A} (s^A - k^A) + n^A = 0 \quad (12)$$

where the effects on firm  $A$ 's demand components are, for  $j = A, B, F$

$$\frac{dX_j^A}{ds^A} = \sum_{i=A,B} \frac{\partial X_j^A}{\partial n^i} \frac{\partial n^i}{\partial s^A} \quad \text{and} \quad \frac{dX_A^j}{ds^A} = \sum_{i=A,B} \frac{\partial X_A^j}{\partial n^i} \frac{\partial n^i}{\partial s^A}$$

Turning to the variable price components, the first-order conditions for  $p_h^A$ , for  $h = A, B, F$ , are initially given by

$$\sum_{j=A,B,F} \left[ (p_j^A - c_j^A) \frac{dX_j^A}{dp_h^A} \right] + \sum_{j=B,F} \left[ (a_A^j - c) \frac{dX_A^j}{dp_h^A} \right] + X_h^A + \frac{\partial n^A}{\partial p_h^A} (s^A - k^A) = 0 \quad (13)$$

with, for  $h, j = A, B, F$

$$\frac{dX_j^A}{dp_h^A} = \frac{\partial X_j^A}{\partial p_h^A} + \sum_{i=A,B} \frac{\partial X_j^A}{\partial n^i} \frac{\partial n^i}{\partial p_h^A} \quad \text{and} \quad \frac{dX_A^j}{dp_h^A} = \frac{\partial X_A^j}{\partial p_h^A} + \sum_{i=A,B} \frac{\partial X_A^j}{\partial n^i} \frac{\partial n^i}{\partial p_h^A}$$

where we denote by  $\partial X_j^A / \partial p_h^A$  resp.  $\partial X_A^j / \partial p_h^A$  the effects of price changes on demand flows, holding market segmentation constant.<sup>2</sup> The first-order condition (13) can be simplified considerably using the following facts. First, Roy's identity implies  $\partial v_A / \partial p_h^A = -X_h^A / n^A$  (note, since  $X_h^A$  is total demand for  $A$ - $h$ -connections, an  $A$ -subscriber's individual demand is  $X_h^A / n^A$ ). Second, one checks that conditions (10) and (11) imply  $\partial n^i / \partial v_j = -\partial n^i / \partial s^j$  for  $i, j = A, B$ . It follows, for  $i = A, B$  and  $j = A, B, F$

$$\frac{\partial n^A}{\partial p_j^i} = -\frac{\partial n^A}{\partial s^i} \frac{\partial v_A}{\partial p_j^i} \quad \text{and} \quad \frac{\partial n^B}{\partial p_j^i} = -\frac{\partial n^B}{\partial s^i} \frac{\partial v_B}{\partial p_j^i}$$

Inserting these expressions, substituting  $X_h^A$  by Roy's identity, and using (12), the first-order conditions (13) for the variable prices  $p_h^A$  ( $h = A, B, F$ ) reduce to

$$\sum_{j=A,B,F} \left[ (p_j^A - c_j^A) \frac{\partial X_j^A}{\partial p_h^A} \right] + \sum_{j=B,F} \left[ (a_A^j - c) \frac{\partial X_A^j}{\partial p_h^A} \right] = 0 \quad (14)$$

Thus, in the initial first-order condition (13), the marginal effects of prices on market segmentation cancel out with the direct price effects, if the first-order condition for  $s^A$  is satisfied. This is also intuitive, and a standard property. Firms use the fixed price component to attract subscribers, and the variable price components to affect calling behavior in the most profitable way.

Finally, turn to the first-order condition for the fixed-to-mobile access charge,  $a_A^F = p_A^F$ . For this price, a similar simplification as for the other variable prices is not possible (since mobile companies do not control the respective fixed price component,  $s^F$ ). We therefore state the first-order condition in its general form

$$\sum_{j=A,B,F} \left[ (p_j^A - c_j^A) \frac{\partial X_j^A}{\partial p_A^F} \right] + \sum_{j=B,F} \left[ (a_A^j - c) \frac{\partial X_A^j}{\partial p_A^F} \right] + X_A^F + \frac{\partial n^A}{\partial p_A^F} (s^A - k^A) = 0 \quad (15)$$

with, for  $j = A, B, F$

$$\frac{dX_j^A}{dp_A^F} = \frac{\partial X_j^A}{\partial p_A^F} + \sum_{i=A,B} \frac{\partial X_j^A}{\partial n^i} \frac{\partial n^i}{\partial p_A^F} \quad \text{and} \quad \frac{dX_A^j}{dp_A^F} = \frac{\partial X_A^j}{\partial p_A^F} + \sum_{i=A,B} \frac{\partial X_A^j}{\partial n^i} \frac{\partial n^i}{\partial p_A^F}$$

---

<sup>2</sup>That is,  $\partial X_j^A / \partial p_h^A$  is the sum of  $(\partial X_j^A / \partial x_{gl}^{ik}) (\partial x_{gl}^{ik} / \partial p_h^A)$  over all  $i, g \in \{A, B, F\}$  and all  $k, l \in \{f, m\}$ .



Reaction functions of firm  $B$  are given analogously.

While it is impossible to find a closed form solution for all relevant reaction functions, using the Dixit utility function allows us to solve (14) for the equilibrium prices  $p_h^A$  (see the Appendix). To solve (12) and (15) together with the equations determining  $n^A$  and  $n^B$  (see (10) and (11)) we have to rely on numerical specifications. Focusing on symmetric Nash equilibria, i.e., equilibria with  $s^A = s^B$ ,  $p_F^A = p_F^B$ , and employing the Dixit function reveals that there exists a unique symmetric Nash equilibrium. Using  $c = 0$ ,  $k^A = k^B = 0.2$  and  $\gamma = 0.5$  we get the equilibrium prices and the firms' market shares  $n^A = n^B$  as functions of the reciprocal termination charge  $a$ , shown in Figure 3.<sup>3</sup>

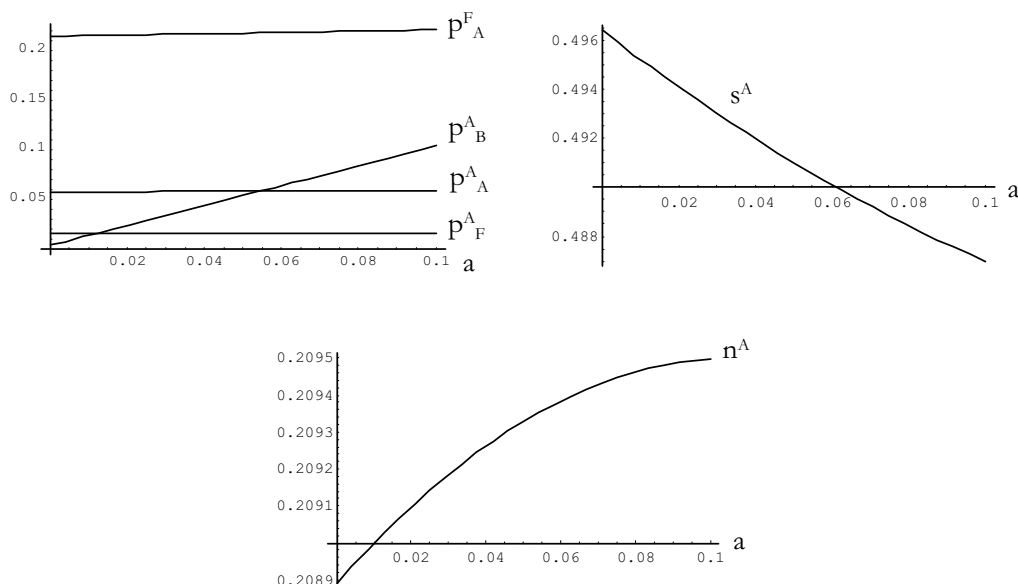


Figure 3: Prices and market shares as functions of  $a$

While the price for on-net mobile calls,  $p_A^A$ , is almost unaffected by  $a$  (slightly increasing), the price for off-net mobile calls,  $p_B^A$ , is markedly increasing, so as to pass the increased access charge  $a$  on to consumers. However, the fixed subscription price  $s^A$  is decreasing in  $a$ , since firms have an incentive to attract more customers to their own network if interconnection gets more costly (tariff-induced network effects). The combined effect of these tariff changes on the joint market shares of mobile phone companies,  $2n^A$ , is positive. That is, the positive effect of the decrease in  $s^A$  basically outweighs the negative effect of the increase in  $p_B^A$ .

<sup>3</sup>Note that  $k^F$  has no effect on model results.

## 4.2 Mobile-to-mobile access charges

Turning to the first stage of the game, i.e., the determination of the termination charge  $a$ , we assume that  $a$  is determined in bilateral bargaining between the mobile companies. Again, focusing on symmetric equilibria,  $a$  will be chosen such that the mobile companies' profits are maximized.

Inspection of Figure 3 clearly indicates that the mobil firms have to balance two main effects when they decide on  $a$ . While there is negative correlation between  $a$  and the consumers' subscription price,  $s^A$ , the correlation between  $a$  and the firms' market shares  $n^A$  is positive. Taking into account that higher market shares have a positive network effect on all consumers, and particularly on the mobile phone subscribers (genuine network effects), it might be attractive for the mobile companies to raise  $a$  above the marginal costs of termination. Employing numerical values  $c = 0$ ,  $k^A = k^B = 0.2$  and  $\gamma = 0.5$ , the left graph in Figure 4 shows that, indeed, profits achieve their maximum at a positive bilateral termination charge  $a$  (which is the solution to stage 1 of the game), although marginal costs are  $c = 0$  in this example.

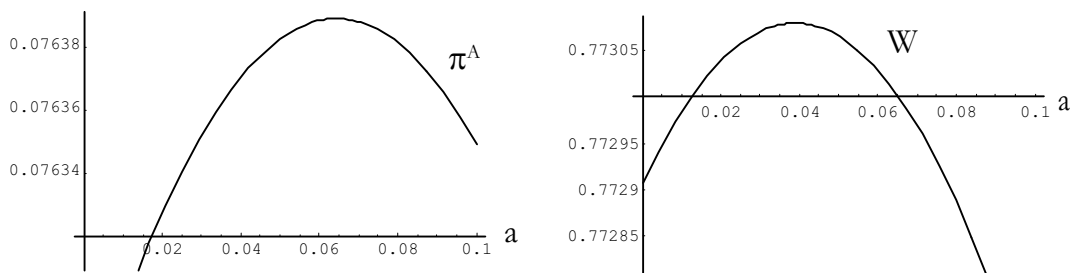


Figure 4: Profits and welfare in  $a$

This result contrasts a standard result in the literature on fixed network competition, e.g. Gans and King (2001), Cambini and Valletti (2003), and De Bijl and Peitz (2002, section 6.4). In these models the two firms would agree to set the reciprocal access charge even below the marginal cost of termination (in our notation  $a < c = 0$  if possible), in order to soften ensuing competition and induce higher subscription prices. In our model, with less than full market coverage by mobile companies, there is an incentive to raise  $a$  in order to increase market shares and realize network effects.

From the point of view of antitrust one might call an access charge  $a > c$  ‘too high’ and

an incident of ‘collusion’. Of course, in a technical sense there is collusion, since firms set  $a$  cooperatively. However, while conventional wisdom does anticipate that firms want to induce an ensuing equilibrium with higher variable prices (here  $p_B^A$  and  $p_A^B$ ), it seems to overlook that firms also want to induce lower subscription prices ( $s^A$  and  $s^B$ ). We might, in fact, interpret ‘collusion’ as a commitment to more aggressive competitive behavior, in order to enhance market coverage, which is quite in contrast to the conventional antitrust view.

To make the point more explicit we also calculated social welfare (consumers’ willingness’ to pay minus their payments, plus firms’ profits). The right graph in Figure 5 shows that the socially optimal level of  $a$  is positive as well, and it is closer to the profit maximizing level of  $a$  than to  $c = 0$ . Thus, even an agent in charge of welfare maximization would raise  $a$  above marginal costs in order to induce more intense competition among the mobile companies which leads to increased positive (genuine) network effects.

Furthermore, comparing Figures 3 and 4, our example leads to realistic price relationships. With  $a^* = 0.065$  as the optimal termination charge (see Figure 4), the retail prices satisfy  $p_B^A > p_A^A$  (see Figure 3a), that is, on-net mobile calls are cheaper than off-net calls.<sup>4</sup> Moreover, our example exhibits  $p_A^F > p_A^A > p_F^A$ , which seems to be realistic as well. Finally, mobiles’ subscription prices are  $s^A = 0.49$ , above the cost of connection ( $k^A = 0.2$ ) but not as extraordinarily high as would be predicted by the standard model of fixed network competition. The relationship of access charges,  $p_F^A = a_A^F > a > a_F^A = 0$ , also corresponds to reality, while the standard model with full market coverage would predict  $a < a_F^A$ .

## 5 Comparative statics in the degree of substitutability

Table 1 shows the dependence of results on the degree of substitutability,  $\gamma$ , for selected values. All prices and the market share are calculated for the corresponding optimal negotiated access charge  $a^*$ .

---

<sup>4</sup>The exact numbers are given in Table 1 below, row  $\gamma = 0.5$ .

$\gamma$	$a^*$	$s^A$	$p_F^A$	$p_A^A$	$p_B^A$	$p_A^F$	$n^A$	$a^W$
0.05	-0.03	0.621	0.001	0.021	-0.029	0.448	0.44	0.1
0.25	0.048	0.501	0.02	0.058	0.051	0.33	0.293	0.075
0.5	0.065	0.49	0.016	0.059	0.069	0.209	0.209	0.04
0.75	0.04	0.48	0.009	0.036	0.043	0.109	0.167	0.015
0.95	0.01	0.473	0.002	0.008	0.011	0.022	0.144	0.005

Table 1: Impact of  $\gamma$  on the optimal access charges and equilibrium prices

The negotiated termination charge  $a^*$  is not monotone in  $\gamma$ , illustrating the complicated interactions in this model. For  $\gamma = 0.05$  we get the ‘traditional’ result that firms set the termination charge below marginal costs *and* that the market shares of mobiles are relatively high.<sup>5</sup> In fact, the low degree of substitutability leads to market segmentation as shown in Figure 5.

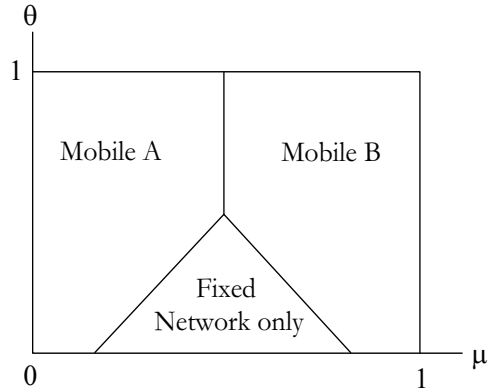


Figure 5: Equilibrium market shares with  $\gamma = 0.05$

In this case the number of consumers who can be additionally attracted by a decrease of the subscription price is relatively low. The positive effects from fiercer competition for customers are therefore small, with the consequence that firms have an incentive to soften competition by setting negotiated termination charges below marginal costs.

<sup>5</sup>Recall that we normalized variable costs of the fixed network to zero. Hence, negative values of prices must not actually be negative. Otherwise, since negative values are implausible, the respective prices would have to be set at the boundary of zero.

Increasing  $\gamma$  (starting from  $\gamma = 0.05$ ) implies higher substitutability and therefore lower additional utilities from connecting to a mobile network. *Ceteris paribus* this reduces the number of mobile customers, which also implies that the positive effects from increasing their market shares become more important for the mobile companies (compare Figures 2 and 5). Consequently, the optimal  $a^*$  is higher than marginal costs.<sup>6</sup>

However, an increase of  $\gamma$  from 0.5 to 0.75 and to 0.95 leads to a decrease of  $a^*$ . Here, the high substitutability makes it hard to get any market share for mobiles at all (since everyone already has a phone). Additionally, the genuine network effects of mobile phones are weak if substitutability is high, since the added utility is marginal.

The welfare maximizing access charge  $a^W$  is monotone decreasing in  $\gamma$ . For low substitutability one would like to induce stronger competition between mobile companies in order to promote propagation of mobiles, which leads to higher realized network effects. For high substitutability, mobile phones and network effects are of less importance and the socially optimal access charge comes close to the marginal costs of termination. The difference between socially optimal and negotiated access charges,  $a^W - a^*$ , is positive for low  $\gamma$ , since the firms have an incentive to soften competition. For high  $\gamma$ , the difference becomes negative which indicates that the firms' motive to strengthen competition leads to market shares which are inefficiently high.

## 6 Summary

We analyze competition between two unregulated mobile phone companies with only partial market coverage when there is already a regulated fixed network with full market coverage in the background. Calls with a mobile and calls with the fixed telephone, to any given person, are imperfect substitutes.

Mobile companies set the fixed-to-mobile access charges above the marginal cost of termination, in order to profit from the calls originating at the fixed network and terminating on their own network. The variable price components of tariffs are set above the perceived marginal costs of termination. The fixed price components are used to gear the tradeoff between attracting customers in competition (i.e. increase the mobile firms' market share) and extracting rents.

---

<sup>6</sup>A more detailed analysis reveals that  $a^*$  exceeds marginal costs ( $c = 0$ ) for  $\gamma \geq 0.09$ .

The most interesting result concerns the choice of the mobile-to-mobile access charges. We assume that these charges are reciprocal and negotiated in bilateral bargaining. In a symmetric equilibrium, the reciprocal access charge has no direct effect on firms' profits. However, firms can use it as a strategic variable to affect their own behavior in the competition stage. From the standard model of competition between full-coverage fixed networks with full price discrimination, it is well known that a higher access charge induces stronger competition, due to the tariff-induced network effects. The same is true in our model.

However, in the standard model the implication is that firms will set access charges below marginal costs of termination, in order to soften competition. In our model the opposite is true, in some numerical examples. Mobile companies have an incentive to induce stronger competition, by setting the access charge above marginal costs of termination, with the aim of increasing joint market coverage and thereby realizing network effects. The network effects are due to the fact that the utility of a phone, including a mobile phone, is increasing with the number of mobile owners. Thus, by raising the access charge, mobile companies use the tariff-induced network effects instrumentally with the aim of realizing genuine network effects.

This strategy is well in line with the aim of social welfare maximization and can hardly be called 'collusion'. In fact, the welfare maximizing level of access charges is also above marginal costs of termination and may be higher or lower than the negotiated access charge.

We showed that the degree of substitutability has an important but non-monotonic effect on the negotiated access charge. Starting from a low degree of substitutability, the negotiated access charge is at first increasing in the degree of substitutability, probably because the issue of gaining market coverage (and thus realized network effects) initially gets more important when fixed and mobile phones become more substitutable. When substitutability is already high, however, a further increase can lead to a decrease of the negotiated access charge. This may be due to the fact that market coverage is then not an issue anyway and, particularly, that the network effects are vanishing (mobiles become useless in the case of perfect substitutability, since everyone already has a phone). Thus the effect described above is particularly strong for the realistic case of an intermediate degree of substitutability.

In that case the model also generates a realistic structure of prices and access charges. Concerning access charges, fixed-to-mobile exceed mobile-to-mobile, which in turn exceed mobile-to-fixed access charges. Concerning retail prices, fixed-to-mobile calls are more

expensive than on-net mobile calls, which in turn are more expensive than mobile-to-fixed calls. Finally, off-net mobile-to-mobile calls are more expensive than those on-net.

The model demonstrates that the standard full-coverage model of fixed network competition is ill-applied for analyzing mobile network competition. For the latter it is important to acknowledge partial market coverage, the existence of a full-coverage fixed network in the background, imperfect substitutability between fixed and mobile calls, and the genuine network effects that arise in such a setting.

## Appendix

Starting with consumers' demand and using the Dixit utility function (5) we get the following demand functions of the consumers who are only connected to the fixed network (using  $p_F^F = 0$ ):

$$x_{Ff}^{Ff} = 1; \quad x_{jf}^{Ff} = \frac{1}{1-\gamma^2}(1-\gamma(1-p_j^F)); \quad x_{jm}^{Ff} = \frac{1}{1-\gamma^2}(1-\gamma-p_j^F) \quad (16)$$

The demand functions of the consumers connected to both the fixed network and mobile network  $i$  are as follows, for  $i, j = A, B$

$$x_{Ff}^{if} = \frac{1}{(1-\gamma^2)}(1-\gamma(1-p_F^i)); \quad x_{Ff}^{im} = \frac{1}{(1-\gamma^2)}(1-p_F^i-\gamma) \quad (17)$$

$$x_{jf}^{if} = \frac{1}{(1-\gamma)(1+3\gamma)}(1-\gamma(1-p_F^i-p_j^F-p_j^i)) \quad (18)$$

$$x_{jf}^{im} = \frac{1}{(1-\gamma)(1+3\gamma)}(1-(1+2\gamma)p_F^i-\gamma(1-p_j^F-p_j^i)) \quad (19)$$

$$x_{jm}^{if} = \frac{1}{(1-\gamma)(1+3\gamma)}(1-(1+2\gamma)p_j^F-\gamma(1-p_F^i-p_j^i)) \quad (20)$$

$$x_{jm}^{im} = \frac{1}{(1-\gamma)(1+3\gamma)}(1-(1+2\gamma)p_j^i-\gamma(1-p_j^F-p_F^i)) \quad (21)$$

Using (16)–(21) and analyzing the consumers' indirect utility functions  $v_A, v_B$  and  $v_F$  it is easy to show that

$$\frac{\partial}{\partial n^i}(v^i - v^F) < 1 \quad \text{and} \quad \left( \frac{\partial v^i}{\partial n^i} - \frac{\partial v^i}{\partial n^j} \right) - \left( \frac{\partial v^j}{\partial n^i} - \frac{\partial v^j}{\partial n^j} \right) < 1$$

This implies that the stability conditions for (10) and (11) are satisfied. Substituting (16)–(21) in the firms' first order conditions for the optimal prices  $p_i^i, p_j^i, p_F^i$ , equation (14), leads

to (with  $i, j = A, B, i \neq j$ )

$$p_i^i = \frac{1}{(1+2\gamma)(1+\gamma(2+\gamma(n^i+n^j)))} [(1+2\gamma)c(2+\gamma(3+\gamma n^i)) + \gamma^2(2+3\gamma)cn^j + \gamma(1+\gamma(2+n^i+2\gamma n^i+\gamma n^j))p_i^F] \quad (22)$$

$$p_j^i = \frac{1}{(1+2\gamma)(1+\gamma(2+\gamma(n^i+n^j)))} [c + a_j^i(1+2\gamma)(1+\gamma(2+\gamma(n^i+n^j))) + \gamma(c(4+\gamma(4+n^j+\gamma(n^i+2n^j))) + \gamma(1+\gamma)n^i p_i^F)] \quad (23)$$

$$p_F^i = \frac{1}{1+\gamma(2+\gamma(n^i+n^j))} [c(1+\gamma(2-n^i+\gamma n^j)) + \gamma(1+\gamma)n^i p_i^F] \quad (24)$$

Turning to the optimal choices of  $s^i$  and  $p_i^F$  note first that the respective derivatives can be written as

$$\frac{d\Pi^i}{ds^i} = \frac{\partial\Pi^i}{\partial s^i} + \frac{\partial\Pi^i}{\partial n^i} \frac{\partial n^i}{\partial s^i} + \frac{\partial\Pi^i}{\partial n^j} \frac{\partial n^j}{\partial s^i} \quad (25)$$

$$\frac{d\Pi^i}{dp_i^F} = \frac{\partial\Pi^i}{\partial p_i^F} + \frac{\partial\Pi^i}{\partial n^i} \frac{\partial n^i}{\partial p_i^F} + \frac{\partial\Pi^i}{\partial n^j} \frac{\partial n^j}{\partial p_i^F} \quad (26)$$

with  $\partial\Pi^i/\partial s^i = n^i$  and (using  $a_F^i = 0$ )

$$\begin{aligned} \frac{\partial\Pi^i}{\partial n^i} &= s^i - k^i + 2n^i(p_i^i - 2c)x_{im}^{im} - n^j(a+c-p_j^i)x_{jm}^{im} - n^i(c-p_F^i)(x_{if}^{im} - x_{Ff}^{im}) \\ &\quad + (p_F^i - c)(n^i x_{if}^{im} + n^j x_{jf}^{im} + x_{Ff}^{im} - (n^i + n^j)x_{Ff}^{im}) \\ &\quad + (a-c)n^j x_{im}^{jm} + n^i(p_i^F - c)(x_{im}^{if} - x_{im}^{Ff}) \\ &\quad + (p_i^F - c)(n^i x_{im}^{if} + n^j x_{im}^{jf} + (1-n^i-n^j)x_{im}^{Ff}) \end{aligned} \quad (27)$$

$$\begin{aligned} \frac{\partial\Pi^i}{\partial n^j} &= n^i(-(a+c-p_j^i)x_{jm}^{im} - (p_F^i - c)(x_{jm}^{if} - x_{if}^{if})) \\ &\quad + (a-c)x_{im}^{jm} + (p_i^F - c)(x_{im}^{jf} - x_{im}^{Ff}) \end{aligned} \quad (28)$$

$$\begin{aligned} \frac{\partial\Pi^i}{\partial p_i^F} &= \frac{1}{(-1+c)(1+c)(1+3c)} [n^i(-1-c+2p_i^F + c(-2+c(-3+3n^i+n^j)) \\ &\quad + n^i(-2(-1+p_i^i+p_F^i)) - n^j(-2+a+p_i^j+p_F^j) \\ &\quad + c(3+n^i(c-2(1+p_i^i+p_F^i-2p_i^F)) \\ &\quad + n^j(2+a+c+p_i^j+p_F^j-4p_i^F)) + 6p_i^F)] \end{aligned} \quad (29)$$

In order to derive  $\partial n^j/\partial s^i$  and  $\partial n^j/\partial p_i^F$  for  $i, j = A, B$  we can use (10) and (11) and the implicit function theorem which leads to the following two equations, for  $\phi \in \{s^i, p_i^F\}$

$$\frac{\partial n^A}{\partial \phi} = \frac{\partial G^A}{\partial \phi} + \frac{\partial G^A}{\partial n^A} \frac{\partial n^A}{\partial \phi} + \frac{\partial G^A}{\partial n^B} \frac{\partial n^B}{\partial \phi} \quad (30)$$

$$\frac{\partial n^A}{\partial \phi} = \frac{\partial G^B}{\partial \phi} + \frac{\partial G^B}{\partial n^A} \frac{\partial n^A}{\partial \phi} + \frac{\partial G^B}{\partial n^B} \frac{\partial n^B}{\partial \phi} \quad (31)$$



Taking into account the relevant border conditions, i.e.,  $\mu^i \geq \mu^*$  and  $\theta^i \geq 0$  with  $i = A, B$ , and solving (30) and (31) we obtain  $\partial n^j / \partial s^i$  and  $\partial n^j / \partial p_i^F$ . Finally, substituting (16)–(24), (27)–(29) as well as  $\partial n^j / \partial s^i$  and  $\partial n^j / \partial p_i^F$  in (25) and (26) the optimal  $s^i$  and  $p_i^F$  are determined by

$$\frac{d\Pi^i}{ds^i} = 0 \quad \text{and} \quad \frac{d\Pi^i}{dp_i^F} = 0 \quad (32)$$

together with (10) and (11). Analyzing this system of equations shows that (10), (11) and (32) are highly non-linear in  $s^i$ ,  $p_i^F$  and  $n^i$ . In order to derive the solutions we first used (10) and (11) to eliminate  $s^i$ . Relying on numerical calculations and assuming symmetry we were then able to show that the two equations in (32) have a unique solution in  $n^A = n^B$  and  $p_A^F = p_B^F$  which also leads to a maximum in the firms' profits.

## References

Cambini, C., and Valletti, T. (2003): Network competition with price discrimination: 'bill and keep' is not so bad after all. *Economics Letters*, 81, 205–213.

De Bijl, P., and Peitz, M. (2002): *Regulation and Entry into Telecommunications Markets*. Cambridge University Press, Cambridge.

Gans, J.S., and King, S.P. (2000): Mobile network competition, customer ignorance, and fixed-to-mobile call prices. *Information Economics and Policy*, 12, 301–327.

Gans, J., and King, S. (2001): Using 'Bill and Keep' Interconnect Arrangements to Soften Network Competition. *Economics Letters*, 71, 413–420.

Houpis, G., and Valletti, T.M. (2004): *Mobile termination: what is the "right charge?"*. Mimeo.

Laffont, J.-J., Rey, P., and Tirole, J. (1998): Network competition: II. Price discrimination. *The RAND Journal of Economics*, 29, 38–56.

Poletti, S. and Wright, J. (2004): Network Interconnection with Participation Constraints. *Information Economics and Policy*, 16, 347–373.

Schiff, A. (2002): Two-Way Interconnection with Partial Consumer Participation, *Networks and Spatial Economics*, 2, 295-315.

Valletti, T.M. and Cambini, C. (2005): Investments and Network Competition. *RAND Journal of Economics*, forthcoming.