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IFN Working Paper No. 1236, 2018

Credit Ratings and Structured Finance

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Credit Ratings and Structured Finance*

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Abstract

The poor performance of credit ratings of structured finance products in the financial crisis has prompted investigation into the role of credit rating agencies (CRAs) in designing and marketing these products. We analyze a two-period reputation model in which a CRA both designs and rates securities that are sold both to investors who require highly rated securities and investors who are unconstrained. Assets are pooled and senior and junior tranches are issued with a waterfall structure. When rating standards are low, the CRA will include only risky assets in the securitization pool, serving both types of investors without any rating inflation. Rating inflation may be non-monotonic in rating standards, with no rating inflation when rating standards are low and high.

Keywords: Credit rating agencies, reputation, structured finance
JEL Codes: G24, L14

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1 Introduction

The recent financial crisis has prompted much investigation into the role of credit-rating agencies (CRAs). With the dramatic increase in the use of structured finance products, the agencies quickly expanded their business and earned outsized profits (Moody's, for example, tripled its profits between 2002 and 2006). Ratings quality seems to have suffered, as structured finance products were increasingly given top ratings shortly before the financial markets collapsed. In this paper, we ask how CRAs influence the structure of such products, and how the products' structure changes with market incentives.

The structuring process is marked by close collaboration between issuers and rating agencies. Issuers depend on rating agencies to certify quality and to be able to sell to regulated investors. Beyond directly paying CRAs for ratings (the "issuer pays" system), Griffin and Tang (2012) write that "the CRA and underwriter may engage in discussion and iteration over assumptions made in the valuation process." Agencies also provide their models to issuers even before the negotiations take place (Benmelech and Dlugosz, 2009). These products are characterized by careful selection of the underlying asset pool and private information about asset quality.

We present a reputation-based two-period model of rating structured products. The model incorporates the endogenous structuring and rating of securities. Each period, an issuer has a set of safe and risky assets that it can put into a pool and issue securities against. A monopoly CRA assists in the structuring of these securities and rates them. The prospect of earning future profits can give the CRA reputational incentives to provide accurate ratings.

We model reputation by positing that the CRA is long-lived and can be one of two types: truthful or opportunistic. Securities are sold to rational investors who cannot observe the type of the CRA or the quality of the securities, but who make inferences from the payoffs realizations, ratings and the size of the asset pool. The type of the CRA is revealed between periods with an endogenous probability that depends directly on the amount of rating inflation.

The issuer and CRA design securities that have a waterfall/priority structure, i.e. with junior securities that face the first losses and senior securities that pay out their promised amount until the value of the junior securities has gone to zero. This is a typical structure for structured finance products (Coval, Jurek, and Stafford, 2009).

A principal motivation for securitization is to appeal to investor groups with heterogeneous preferences. The *senior* securities are designed to appeal

to constrained investors, who can only purchase investment grade securities, i.e. securities with a rating above a certain level. Constrained investors may be regulated, such as banks, pension funds, and insurance companies. The securities are designed using a rating standard, a probability that the senior securities will not be paid in full. This probability is exogenous in the model, but may be determined by the standards of the CRA, the regulator, and/or the investors. The *junior* securities are designed to appeal to unconstrained investors, such as hedge funds, who can purchase any type of security.

The CRA includes safe assets in the pool to be able to sell to constrained investors. This creates scope for rating inflation, which relies on passing off risky assets as safe ones. The trade-off for the opportunistic CRA is that this allows it to extract more rents from the issuer, who gets to retain the safe assets, but makes it more likely that the CRA will be identified as opportunistic and thereby decreases its expected future profits.

Our first result is that, for some parameters, the equilibrium has both types of CRA only including risky assets in the asset pool. This implies there is no rating inflation, as there is no scope for substituting risky for safe assets. Nevertheless, through the structuring process, there will still be senior securities to sell. And this achieves the first best allocation, as there are larger gains from trade for risky assets. This equilibrium will be more likely to occur when there is less demand for safe securities, more demand for risky ones, and the rating standard is low.

When CRAs include safe assets in the asset pool, rating inflation is possible. Rating inflation is important to examine as it is directly related to surplus. Rating inflation (a) measures the amount of adverse selection/misallocation there is, and (b) leads to excess risk being taken on by investors (which, outside of the model, may be surplus reducing due to systemic effects).

We examine how rating inflation depends on the rating standard. For parameters such that rating inflation is possible, it is locally decreasing in the rating standard. On the one hand, increased rating standards lead to lower future payoffs as it is harder to securitize large pools of assets, which incentivizes more rating inflation. On the other hand, increased standards lead to less risky assets in today's pool, which make it easier for investors to detect rating inflation. The second effect dominates in the model.

Nevertheless, we also look at how rating inflation changes with the rating standard globally, and find that this effect may be non-monotonic. For low rating standards, only risky assets are included, which, as discussed above, leads to no rating inflation. For high rating standards, there may not be any rating inflation due to a higher likelihood of being caught. Intermediate rating standards involve positive amounts of rating inflation. Interestingly, this indicates that new markets that don't have many constrained investors may

not need to be heavily scrutinized by regulators. More advanced markets, consequently, deserve strict scrutiny.

We also show that rating inflation is increasing in the wealth of unconstrained investors, as it allows more risky assets into the pool, decreasing the likelihood that inflation is discovered. Rating inflation is also increasing in the value of retaining safe assets, as this makes it more desirable to substitute risky for safe assets.

Finally, we provide two new motivations for the pooling of assets: (a) a mechanical reason of tailoring products for constrained investors; and (b) a novel explanation of the CRA balancing the informational advantage over investors with the need to maintain its reputation by choosing the right mix of safe and risky assets to include.

The paper is organized as follows. In the next subsection, we review the literature. In Section 2, we set up the model. In Section 3, we analyze the equilibrium of the second period and in Section 4 the equilibrium of the first period. In Section 5, we look at the determinants of rating inflation in the model. Section 6 concludes. The Appendix contains all mathematical proofs not in the text.

1.1 Related Literature

There is substantial evidence of asymmetric information and strategic asset pool selection for structured finance products.¹ There is also much empirical support for our finding that rating inflation is an important element of structured finance.²

In the theory literature, Mathis, McAndrews, and Rochet (2009), Fulghieri, Strobl, and Xia (2014), Bar-Isaac and Shapiro (2013), and Strausz (2005) examine dynamic models of certification agencies with reputation concerns.³ Our model of reputation is similar, but we allow for multiple risky assets, which permits the CRA to tranche securities as well as rate them.

Daley, Green, and Vanasco (2017a, 2017b) examine the interaction between retention, security design, ratings, and origination. Ratings in their main model are similar to public information. We focus on a CRA's strategic incentives to undertake security design and ratings.

¹See, e.g., Downing, Jaffee, and Wallace (2009), An, Deng, and Gabriel (2011), Elul (2011), and Ashcraft, Goldsmith-Pinkham, and Vickery (2011).

²See, e.g., Gorton and Metrick (2013), Cornaggia, Cornaggia, and Hund (2017), Ashcraft, Goldsmith-Pinkham, and Vickery (2011), Vickery (2012), Griffin and Tang (2012), He, Qian, and Strahan (2012), and Stanton and Wallace (2012).

³There is a large recent theoretical literature on CRAs, including Sangiorgi and Spatt (2017), Cohn, Rajan, and Strobl (2016), and Bolton, Freixas, and Shapiro (2012).

Opp, Opp, and Harris (2013) examine how ratings-contingent regulation affects the informativeness of ratings in a setting where a CRA rates risky projects with binary outcomes. The constrained investors in our model rely on a rating that states it respects a rating standard. The rating standard may depend on regulation. However, the standard may be violated in our model by the CRA. Moreover, we allow for security design.

2 Model

There are three types of agents in the model: issuers, investors, and a CRA. All agents are risk-neutral. We begin by focusing on issuers.

2.1 Issuers

The issuer has two types of assets that it would like to sell to a set of investors: risky assets and safe assets. Risky assets pay off \tilde{X} per unit, a random variable distributed uniformly over the unit interval. For simplicity, the payoffs of different risky assets are assumed to be perfectly correlated with each other. Risky assets are worth $r \in (0, 1/2)$ to the issuer. Safe assets pay off 1 per unit with probability one. They are worth $s \in (2r, 1)$ to the issuer.

The issuer's valuations of the assets are lower than the investors' values for the assets. This can occur for several reasons: the issuer may have valuable alternative investment opportunities, regulatory capital requirements for holding the assets, and/or the need to transfer risk off of its balance sheet. The assumption that $s > 2r$ implies that under full information the net profits to the issuer from selling a dollar of risky assets is greater than the net profits from selling a dollar of safe assets.

The issuer's supply of safe assets is $\sigma > 0$ and its supply of risky assets is large (for simplicity, we assume it is infinite).⁴ We denote the measure of safe assets the issuer includes in the securitization pool by σ^I and the measure of risky assets by ρ^I .

2.2 Investors

There are two types of investors: unconstrained, U , with aggregate wealth $w_U > 0$ and constrained, C , with aggregate wealth $w_C > 0$.⁵ Constrained

⁴Note that assuming that the issuer has a large supply of risky assets guarantees that an issuer can create a pool of any size that contains only risky assets. This will be important for the lemons problem we analyze.

⁵We assume that investors are credit-constrained, which might arise from borrowing frictions (see, for example, Boot and Thakor (1993)).

investors can only purchase securities that receive an investment grade rating. We define investment grade ratings below. Constrained investors may be constrained by regulations (for example, banks, pension funds, and insurance companies are often restricted in the types of assets they may hold), internal by-law restrictions, or their portfolio hedging requirements. The unconstrained investors are willing to purchase any security. These investors may be hedge funds or other institutional investors. We assume that both types of investors are rational, in the sense that they update given available information and maximize their expected payoffs.⁶ Investors' reservation utility is normalized to zero.

2.3 Securitization

With the help of a credit rating agency (CRA), the issuer can issue securities backed by a portfolios of safe and risky assets for investors through *securitization*. We define securitization as creating a senior and junior tranche with a waterfall/priority structure. This means that if the payoff of the underlying pool of assets is sufficiently large, both tranches receive a payoff; the senior tranche receives its promised repayment and the junior tranche receives the residual. When the payoff of the pool is sufficiently low, the junior tranche receives nothing, and the senior tranche receives the whole payoff (which may be below the payoff promised to them). This model is a stylized version of how securitization works (see Coval, Jurek, and Stafford (2009) for a detailed description of the process).⁷

Formally, suppose that the CRA in conjunction with the issuer decides on the assets to include in the pool and how to structure it. To simplify this process for the model, we assume the CRA's fee gives it a fraction f of the surplus from the issuance. The issuer and the CRA decide jointly on the number of safe and risky assets to include in the securitization and the specification of the structure of the senior and junior tranches. For simplicity of exposition, we will refer to all decisions as being made by the CRA.

More precisely, suppose the CRA can market the securities of a junior tranche and a senior tranche. The junior tranche securities are intended for unconstrained investors, while the senior tranche securities are intended for

⁶There has been much discussion about the naïveté of investors in the RMBS market; e.g. Bolton, Freixas, and Shapiro (2012). However, not all structured finance markets are necessarily characterized in such a way, as Stanton and Wallace (2012) point out: "All agents in the CMBS market can reasonably be viewed as sophisticated, informed investors."

⁷In a previous version of this paper, we modelled pass-through securities, where investors get pro-rata shares of cash flows from the underlying mortgages.

constrained investors. Let $z^I \in [0, \sigma^I + \rho^I]$ be the face value of the senior tranche – i.e. the total payoff of the senior securities if they are paid in full. The senior tranche securities are paid in full if and only if the payoff of all assets in the pool is sufficient to make this payment:

$$\sigma^I + \rho^I \tilde{X} \geq z^I.$$

Define x^I as the the realization of \tilde{X} such that the inequality binds:

$$\sigma^I + \rho^I x^I = z^I.$$

For realizations x of \tilde{X} below x^I , the senior tranche receives the total payoff of the pool and the junior tranche receives nothing. Notice that having the CRA choose the face value of the senior tranche, z^I , is equivalent to having the CRA choose the cutoff $x^I \in [0, 1]$. As it will be notationally simpler to use the cutoff x^I , we do so in most of the paper.

The realized payoff of the senior tranche is thus:

$$\sigma^I + \rho^I \min \{x, x^I\},$$

and its expected payoff:

$$\begin{aligned} & \sigma^I + \rho^I (1 - x^I) x^I + \rho^I x^I (x^I/2) \\ &= \sigma^I + \rho^I x^I (1 - x^I/2). \end{aligned} \tag{1}$$

The realized payoff of the junior tranche is the residual value of the pool:

$$\max \{ \rho^I (x - x^I), 0 \},$$

and its expected value:

$$\begin{aligned} & \rho^I (1 - x^I) ((1 - x^I) + 0) / 2 \\ &= \rho^I (1 - x^I)^2 / 2. \end{aligned} \tag{2}$$

From the above, it follows that the realized payoff of the entire pool equals:

$$\sigma^I + \rho^I x$$

and its expected payoff:

$$\sigma^I + \rho^I / 2.$$

2.4 The CRA

We assumed that constrained investors are constrained in the sense that they may only purchase investment grade securities. We thus need to define the criteria a CRA uses for ratings.

2.4.1 Ratings

The simplest approach to define the rating criteria is to quantify the probability of default. We define an investment grade rating as signifying that the probability the senior tranche is not paid in full is less than or equal to an exogenous probability $P \in [0, 1]$.⁸ This gives us the following ratings constraint:

$$\Pr\left(\sigma^I + \rho^I \tilde{X} \leq \sigma^I + \rho^I x^I\right) \leq P,$$

which, given the uniform distribution assumption, is equivalent to stating that:

$$x^I \leq P.$$

It is natural to think of the probability of default of the senior tranche, P , as exogenous to the given securitization problem. This probability may represent the historical default rate for highly rated securities that the CRA wishes to maintain, a more lax standard that the CRA applies to securitizations, or a more conservative standard that the CRA applies due to pressure from regulators or investors. We will examine how changes to this standard affect ratings.

To simplify the problem, we will use a transformed version of the rating constraint:

$$(1 - x^I)^2 - R \geq 0,$$

where $(1 - P)^2 \equiv R$. A higher R thus means a more demanding rating constraint.

Lastly, we assume that an issuer can't sell rated securities on its own. It may sell securities, but without ratings it will not be able to access constrained investors. This is the first role of ratings in our model; a regulatory (or institutionalized) license to access certain clientele. Given this, the issuer may still sell to unconstrained investors. As the issuer is short lived, it faces a lemons problem, and can only include risky assets - if investors believed it

⁸We could make this definition more lax by stating that the probability that the realized loss is larger than a pre-specified amount is smaller than the exogenous probability P . The results are qualitatively similar in this case; the proofs are available upon request.

included safe assets it would switch them for risky assets. This is the second role of ratings in our model; reputation allows the CRA to overcome the lemons problem and sell safe assets. If the issuer decides not to deal with the CRA, it then receives an outside option where it sells as many risky assets/securities as possible to unconstrained investors, $\rho^I = 2w_U$ for a payoff of:

$$w_U(1 - 2r). \tag{3}$$

This outside option partially determines the fee paid to the CRA (which is a share of the surplus).

2.4.2 Reputation

We focus on a monopoly rating agency. The CRA reduces the lemons problem through the reputation it acquires over time. There are two types of rating agencies: truthful (T) and opportunistic (O).⁹ The opportunistic CRA's announcement and its choice of tranche structure depend on its incentives. The truthful CRA is behavioral in the sense that it (a) is restricted to truthful announcements and (b) structures the securities assuming that all investors believe it is truthful.¹⁰ The literature generally uses the behavioral player as a device to create reputational incentives for the opportunistic player. In our model, this limits the amount of rating inflation that the opportunistic CRA chooses in the first period.

Our model has two periods. The CRA is the same for both periods, but there is a different issuer in each period. The probability of facing a truthful CRA at the beginning of the period is given by θ_t , where $t \in \{1, 2\}$. In period 1, the probability is a prior given by nature, and in period 2, the probability is a posterior. We assume the prior, the structure of the game, and payoffs, are common knowledge.

The CRA perfectly observes the quality of the issuer's assets. As part of its services, the CRA structures and rates the securities offered by the issuer for a fee equal to a fraction $f > 0$ of the surplus generated. We assume the fraction f is exogenous and it is the same in both periods and for both types

⁹This follows the approach of Fulghieri, Strobl, and Xia (2014) and Mathis, McAndrews, and Rochet (2009) (who, in turn, follow the classic approach of modeling reputation of Kreps and Wilson (1984) and Milgrom and Roberts (1984)).

¹⁰We follow the reputation literature to assume a behavioral player. Nevertheless, point (b) rarely arises in the literature. Hartman-Glaser (2016) is the only example we know in which the truthful issuer can decide how much to retain of a security. In another version of this paper, we allow the truthful CRA to be strategic in structuring the securities.

of CRAs.¹¹ The actual fee is paid when the payoffs are realized.

While in practice the issuer will initially structure the securities and get feedback from the rating agencies about modifications necessary to achieve certain ratings,¹² we incorporate this back and forth into one step for simplicity. We defined the issuer's outside option in equation (3). The CRA's outside option is zero.

The issuer knows the type of the CRA. Due to the surplus sharing rule, the interests of the CRA and the issuer are perfectly aligned, and the CRA can easily signal its type to the issuer through the asset composition.

Denote a message that is sent to investors by a CRA of type $d \in \{T, O\}$ by $\bar{m}^d = (\bar{\sigma}^d, \bar{\rho}^d)$, where $\bar{\sigma}^d$ ($\bar{\rho}^d$) is the reported measure of safe (risky) assets in the pool. Denote the true measures of assets by σ^d and ρ^d . We assume that the total quantity of assets $\sigma^d + \rho^d$ is observable. The term z^d represents the face value of the senior tranche, which is also observable.

In each period, an action by a CRA of type d is defined by $s^d = (\bar{m}^d, \sigma^d, \rho^d, z^d)$. Since we assume that the true quantities are observable to investors, any message \bar{m}^d must fulfill $\bar{\sigma}^d + \bar{\rho}^d = \sigma^d + \rho^d$. If the CRA is truthful, then the strategy space is further restricted such that $(\bar{\sigma}^T, \bar{\rho}^T) \equiv (\sigma^T, \rho^T)$.

In the model, the announcement of \bar{m}^d and the observation of $\sigma^d + \rho^d$ and z^d supplies investors with enough information to directly calculate the expected values and probabilities of given realizations of the senior and junior tranches. The opportunistic CRA may substitute risky assets for safe assets and thus worsen the actual payoffs and inflate ratings. We say that *rating inflation* occurs when the opportunistic CRA includes a higher fraction of risky assets than reported.

To summarize, the timing of the first period game with is as follows:

0. Nature draws the type of the CRA. With probability θ_1 it's truthful and with probability $1 - \theta_1$ it's opportunistic.
1. The CRA specifies the measures of safe and risky assets to be included, the face value of the senior tranche z^d , and that ratings will be produced.

¹¹Fulghieri, Strobl, and Xia (2014) also use a surplus splitting rule.

¹²See details in Griffin and Tang (2012). Rating agencies also provide their basic model to issuers to communicate further. For example, Benmelech and Dlugosz (2009) write, "The CDO Evaluator software [from S&P, publicly available] enabled issuers to structure their CDOs to achieve the highest possible credit rating at the lowest possible cost. . . the model provided a sensitivity analysis feature that made it easy for issuers to target the highest possible credit rating at the lowest cost."

2. The CRA reports measures of safe and risky assets (ratings) to investors.¹³
3. Investors observe the size of the pool and of the senior tranche, and the announcement of the CRA, and buy securities at their conditional expected value.
4. Payoffs are realized and the CRA's fee paid out.

We suppose that the steps 1 to 4 are repeated in a second period and that the issuer is different in each period. Based on the message reported and payoff realization in the first period, the second-period investors update their prior about the type of the CRA accordingly. If the messages of the two types of CRAs are different, investors can deduce the type of the CRA with probability one. If the messages are identical, investors update their priors by observing the realization of security payoffs at the end of the first period. We describe this updating process in detail in our analysis of the first period.

In what follows, we work our way backward in solving the model, beginning with the second period. We use Perfect Bayesian Equilibrium as our equilibrium concept.

3 The Second Period

In this section, we will analyze the second period of the game. Since this is the last period, the opportunistic CRA has no reputation concerns. We begin by describing the behavior of the truthful CRA, as it does not act strategically (i.e., it doesn't take into account the fact that investors perceive it to be opportunistic with positive probability). Note that the behavior of the truthful CRA will be the same in both periods, as it doesn't recognize the need for reputation management and the issuers in each period have the same characteristics. Thereafter we derive the equilibrium behavior of the opportunistic CRA.

3.1 The truthful CRA

The truthful CRA maximizes its revenues disregarding the behavior of the opportunistic CRA. It solves the following program:

¹³The actual ratings to be reported could also be included in the contract, without altering the conclusions in the paper. However, such an inclusion could expose the opportunistic CRA (and the issuer) to lawsuits should the ratings not match the proposed asset quality. As the type of the CRA is known to the issuer (see earlier discussion), the ratings will be known by the issuer without contracting on them.

$$\begin{aligned}
& \max_{\check{\sigma}^T, \check{\rho}^T, \check{x}^T \geq 0} \{ \check{\sigma}^T (1 - s) + \check{\rho}^T (1/2 - r) - w_U (1 - 2r) \} \\
& \quad s.t. \quad \sigma - \check{\sigma}^T \geq 0 \\
& \quad \quad (1 - \check{x}^T)^2 - R \geq 0 \\
& \quad \quad w_C - \check{\sigma}^T - \check{\rho}^T \check{x}^T (1 - \check{x}^T/2) \geq 0 \\
& \quad \quad w_U - \check{\rho}^T (1 - \check{x}^T)^2 / 2 \geq 0
\end{aligned}$$

The objective function is the expected surplus generated by the CRA from selling securities to investors. The expected fee is a fraction $f > 0$ of this expression. The first constraint is the resource constraint for safe assets. The second constraint is the rating constraint, which implies that the probability that the senior tranche will pay out less than its full amount is P or less. The third constraint is the budget constraint of constrained investors, which makes use of the expected payoff of the senior tranche in equation (1). The fourth constraint is the budget constraint of unconstrained investors, which makes use of the expected payoff of the junior tranche in equation (2).

In the Appendix, we prove that the budget constraint of unconstrained investors binds in any solution (σ^T, ρ^T, x^T) to the truthful CRA's optimization program, and moreover that any solution has $\rho^T > 0$. This allows us to simplify the program considerably and to prove the following proposition, where we make use of the definition $W \equiv w_C + w_U - w_U/R$, which is explained below.

Proposition 1 *The solution to the truthful CRA's problem has the following properties:*

- i. *If $W \leq 0$, the budget constraint of the constrained investors and the non-negativity constraint for safe assets bind, implying:*

$$\begin{aligned}
\sigma^T &= 0, \\
\rho^T &= 2(w_C + w_U), \\
x^T &= 1 - \sqrt{w_U / (w_C + w_U)}.
\end{aligned}$$

- ii. *If $0 < W \leq \sigma$, the rating constraint and the budget constraint of the constrained investors bind, implying:*

$$\begin{aligned}
\sigma^T &= w_C + w_U (1 - 1/R), \\
\rho^T &= 2w_U/R, \\
x^T &= P.
\end{aligned}$$

iii. If $W > \sigma$, the rating constraint and the resource constraint bind, implying:

$$\begin{aligned}\sigma^T &= \sigma, \\ \rho^T &= 2w_U/R, \\ x^T &= P.\end{aligned}$$

The variable W represents the value of safe assets that the truthful CRA prefers to allocate to the pool when all it has to worry about is the rating standard and the budgets of investors. When constrained investors have more wealth, the truthful CRA finds it more profitable to include more safe assets and appeal to them. The opposite effect holds for unconstrained investors. When the rating standard is higher, the truthful CRA includes more safe assets to satisfy it. Since the measures of included assets can't be negative, the truthful CRA will not include any safe assets when W is negative. Finally, if W is larger than the supply of safe assets, σ , the truthful CRA will include a measure of safe assets equal to this supply.

We will start by solving for an equilibrium when $W > 0$, and revisit the case when $W \leq 0$ in the next section. Note that for $W > 0$, $\rho^T = 2w_U/R$, whereas $\sigma^T = \min\{w_C + w_U(1 - 1/R), \sigma\}$.

3.2 The opportunistic CRA

In this subsection, we examine the strategic choices of the opportunistic CRA.

Lemma 1 *In equilibrium, any securitization rated by the opportunistic CRA in the second period will only include risky assets in the pool of assets.*

The lemons problem here has two elements. First, risky assets have a higher margin than safe ones for the CRA. Second, investors do not observe the actual composition of assets. Therefore if the opportunistic CRA includes some safe assets and investor beliefs are fixed such that they anticipate these safe assets will be included, the opportunistic CRA has the incentive to replace the safe assets with risky assets. This problem arises directly from the existence of constrained investors - it is profitable to include safe assets in the pool (when $W > 0$) to be able to sell to constrained investors, which then allows for rating inflation (replacing safe assets with risky ones) to take place.¹⁴

¹⁴Note that this differs from rating inflation in much of the theoretical literature on CRAs. There, the issuer has one good (NPV positive) or bad (NPV negative) asset to

We say that the two types of CRAs *pool* at an information set where they are both called upon to act, if they report the same messages, include the same quantity of assets in the asset pools, and choose the same sizes of their senior tranches in equilibrium. If the two types of CRAs do not pool at such an information set, we say that they *separate*.

Lemma 2 *For $W > 0$, if the type of the opportunistic CRA is not fully revealed in the first period, the opportunistic CRA pools with the truthful CRA in the second period.*

If there were an equilibrium where the two types separated in the second period in spite of a positive posterior, the opportunistic CRA would be recognized, and would therefore only be able to sell securities backed by risky assets. For $W > 0$, this gives strictly lower revenues than the truthful CRA's pool that included safe assets, and therefore there is a profitable deviation. It is easy to see that a pooling equilibrium exists for off-the-equilibrium path beliefs such that any deviation is believed to be the opportunistic CRA for sure.

For a given posterior $\theta_2 \in (0, 1)$, the corresponding second-period surplus created by the opportunistic CRA is given by:

$$\begin{aligned} \pi_2(\theta_2) = & \theta_2 (\sigma^T + \rho^T/2) + (1 - \theta_2) (\sigma^T + \rho^T) / 2 \\ & - (\sigma^T + \rho^T) r - w_U (1 - 2r). \end{aligned}$$

The revenues received depends on investors' beliefs about the type of the CRA. With probability θ_2 , the CRA is truthful, and includes σ^T safe assets worth 1 per unit and ρ^T risky assets worth on average 1/2 per unit. With complementary probability, the CRA is opportunistic and includes only risky assets, but ensures that the number of assets is equal to the total number of assets the truthful CRA includes ($\sigma^T + \rho^T$). As these assets are all risky, they are worth on average 1/2. The opportunity cost of selling off those assets for the issuer is the quantity of assets multiplied by their payoff if retained r . Lastly the issuer can earn $w_U (1 - 2r)$ by securitizing without the CRA's help.

We can simplify this expression:

sell/issue a bond against. Investors don't want to invest in the bad asset, and rely on the CRA to screen the asset. However, the CRA may earn more by rating a bad asset good. For examples of this type of model, see Fulghieri et al (2014) and Piccolo and Shapiro (2018). Here, there are no bad assets, as risky assets are also NPV positive and investors are risk neutral.

$$\pi_2(\theta_2) = \theta_2 \sigma^T / 2 + (\sigma^T + \rho^T - 2w_U) (1 - 2r) / 2.$$

If the type of the opportunistic CRA is revealed to investors in period one ($\theta_2 = 0$), then it will include only risky assets. We derive the optimal tranching by the opportunistic CRA for this case in the following lemma (see Appendix B for a proof):

Lemma 3 *If investors know the type of the opportunistic CRA ($\theta_2 = 0$) in period 2, it will include a measure $\rho_2^O = \rho^T = 2w_U/R$ of risky assets and no safe assets in the pool. The corresponding surplus is given by:*

$$\begin{aligned} \pi_2(0) &= (1 - 2r) (\rho^T - 2w_U) / 2 \\ &= (1 - 2r) w_U (1/R - 1). \end{aligned}$$

Notice that even though only risky assets are included in the result of the lemma, the opportunistic CRA can still sell securities to constrained investors, as they are made safer through tranching.

Lastly, we derive the difference between the second period surplus when the CRA is pooling and when it is separating:

$$\pi_2(\theta_2) - \pi_2(0) = \sigma^T (\theta_2 + 1 - 2r) / 2 > 0. \quad (4)$$

4 The First Period

In this section, we analyze the strategic choice of the opportunistic CRA in the first period. The following lemma demonstrates that there are no separating equilibria in the first period.

Lemma 4 *There is no equilibrium where the opportunistic CRA separates in the first period.*

If the opportunistic CRA separated in the first period, it would have a strictly lower payoff in the second period than the payoff from pooling (see equation (4)). The truthful CRA makes the same choices in the first period as in the second period. Therefore, the opportunistic CRA can also guarantee a higher payoff by pooling with the truthful CRA in the first period, using the logic of Lemma 2 and the knowledge that it will get a higher payoff than separating in the second period. We can thus restrict ourselves to looking only at pooling equilibria in the first period.

In any pooling equilibrium, the opportunistic CRA chooses the same size of the pool as the truthful CRA ($\sigma^T + \rho^T$) and the same face value of the senior tranche (z^T), but may include a larger measure of risky assets, ρ_1^O .

A property of the uniform distribution is used here to simplify the problem. The likelihood ratio between the density function of the aggregate payoff of the assets pooled by the truthful CRA and the aggregate payoff of the assets pooled by the opportunistic CRA is constant for aggregate payoff realizations above σ^T . Hence, if investors know the aggregate payoff realization is above this level, then no additional information can be learned about the type of the CRA from knowing the exact aggregate payoff realization.

Moreover, if the aggregate payoff could have come from either type of CRA, then no inference can be made about the type of the CRA by observing the different payoff realizations of the two tranches. The reason is that, identical aggregate payoff realizations are split in the same fashion between the junior and senior tranches created by truthful and opportunistic CRAs.

In conclusion, no inference can be made about the type of the CRA, unless the aggregate payoff realization is below σ^T , the minimum payoff of the truthful CRA's pool, as this could not have been generated by a truthful CRA. Given the distribution of the risky asset's payoffs, the opportunistic type is therefore discovered ($\theta_2 = 0$) with probability:

$$\Pr((\sigma^T + \rho^T - \rho_1^O) + \rho_1^O \tilde{X} < \sigma^T) = 1 - \rho^T / \rho_1^O.$$

With the complementary probability, ρ^T / ρ_1^O , the type of the opportunistic CRA will not be revealed by the payoff realization and instead investors will increase their posterior probability that the CRA is truthful in the second period to:

$$\theta_2 = \theta_1 / (\theta_1 + (1 - \theta_1)(\rho^T / \rho_1^O)).$$

In any pooling equilibrium, the opportunistic CRA's choice of the measure of risky assets to include in the pool, ρ_1^O , must be optimal given the first-period choice of the truthful CRA, (σ^T, ρ^T, z^T) . Furthermore, the beliefs of investors are held fixed when the opportunistic CRA chooses the amount of risky assets to include, meaning that the choice does not affect the revenues received. We denote the amount of risky assets that investors expect to be included in the pool by an opportunistic CRA by ρ_1^{Oe} . More specifically, ρ_1^O must be a solution to the following maximization problem:

$$\max_{\check{\rho}_1^O \in [\rho^T, \sigma^T + \rho^T]} \left\{ \begin{array}{l} \theta_1 (\sigma^T + \rho^T / 2) + (1 - \theta_1) (\sigma^T + \rho^T - \rho_1^{Oe} + \rho_1^{Oe} / 2) \\ - (\sigma^T + \rho^T - \check{\rho}_1^O) s - \check{\rho}_1^O r - w_U (1 - 2r) + \\ \delta \pi_2 (\theta_1 / (\theta_1 + (1 - \theta_1)(\rho^T / \rho_1^{Oe}))) \rho^T / \check{\rho}_1^O + \\ + \delta \pi_2 (0) (1 - \rho^T / \check{\rho}_1^O) \end{array} \right\}$$

The first line represents the revenues in the first period. As the price depends on the equilibrium beliefs of investors, and the quantity is observable and identical for both types of CRAs, revenues are held fixed in the decision problem for the opportunistic CRA. The second line represents the issuer's opportunity cost of not holding on to the assets and the payoff it could obtain without the CRA. The third and fourth lines represent the expected second-period surplus, which depends on the probability that the CRA is discovered or not. Note that the probability depends on the opportunistic CRA's choice, as more distortion away from the reported value make it more likely to be discovered. The equilibrium second-period surplus does not depend on this choice, as the beliefs of investors about the updated type of the CRA are held fixed. Thus, the trade-off for the opportunistic CRA is to increase its payoff by retaining more safe assets and placing more risky assets in the pool versus having a higher probability of enjoying future rents. Finally, as the CRA receives a fraction of the surplus f in both periods, we leave it out of the maximization problem.

The above program is convex in $\check{\rho}_1^O$ since the second derivative equals:

$$2\delta(\pi_2(\theta_2) - \pi_2(0))\rho^T/(\check{\rho}_1^O)^3 > 0.$$

Hence, we will have a corner solution. The two possible solutions are (a) maximal rating inflation, in which the opportunistic CRA includes only risky assets ($\rho_1^O = \sigma^T + \rho^T$) or (b) zero rating inflation, where the opportunistic CRA includes the same measure of risky assets as the truthful CRA ($\rho_1^O = \rho^T$). Under maximal rating inflation, the posterior belief about the probability that the CRA is truthful is:

$$\theta'_2 = \frac{\theta_1}{\theta_1 + (1 - \theta_1)\rho^T/(\sigma^T + \rho^T)}.$$

An equilibrium with maximal inflation can be sustained if the expected surplus with the truthful CRA's amount of risky assets $\rho_1^O = \rho^T$ is smaller than the expected surplus with the maximal amount $\rho_1^O = \sigma^T + \rho^T$ under the beliefs $\theta_2 = \theta'_2$:

$$\delta\pi_2(\theta'_2) - \sigma^T s - \rho^T r \leq \frac{\rho^T}{\sigma^T + \rho^T} \delta\pi_2(\theta'_2) + \frac{\sigma^T}{\sigma^T + \rho^T} \delta\pi_2(0) - (\sigma^T + \rho^T)r.$$

Notice that here, the first period revenues are the same in both scenarios and are not included, as this looks at a deviation where beliefs are held fixed. The first period opportunity cost of including assets differ, as the assets included are different, and thus appear on both sides.

For zero rating inflation, the posterior belief is equal to the prior, θ_1 . Therefore, an equilibrium with zero rating inflation can be sustained if the expected surplus with the same amount of risky assets as the truthful CRA, $\rho_1^O = \rho^T$, is larger than the expected surplus with the maximal amount $\rho_1^O = \sigma^T + \rho^T$ under the beliefs $\theta_2 = \theta_1$:

$$\delta\pi_2(\theta_1) - \sigma^T s - \rho^T r \geq \frac{\rho^T}{\sigma^T + \rho^T} \delta\pi_2(\theta_1) + \frac{\sigma^T}{\sigma^T + \rho^T} \delta\pi_2(0) - (\sigma^T + \rho^T)r.$$

The above implies the following equilibrium actions:

$$\rho_1^O = \begin{cases} \sigma^T + \rho^T & \text{for } \delta(\pi_2(\theta'_2) - \pi_2(0)) \leq (\sigma^T + \rho^T)(s - r) \\ \rho^T & \text{for } \delta(\pi_2(\theta_1) - \pi_2(0)) \geq (\sigma^T + \rho^T)(s - r) \end{cases} \quad (5)$$

The intuition behind these expressions is simple. If the current gain from inflating ratings (substituting risky assets for safe assets) is higher than the present value of the future surplus from being more truthful, then the opportunistic CRA prefers to inflate ratings as much as possible. If the present value of the future surplus from truthful ratings is higher than the current gain from inflating ratings, then the opportunistic CRA prefers not to inflate ratings at all.

These expressions define discount factors for which maximal rating inflation and truth-telling are equilibria. Maximal rating inflation is an equilibrium when the discount factor is below a cutoff, which we define as $\underline{\delta}$. Plugging in second-period payoffs, this cutoff is defined as:

$$\underline{\delta} \equiv \frac{2(s - r)(\sigma^T + \rho^T)}{\sigma^T(\theta'_2 + 1 - 2r)} \quad (6)$$

No rating inflation is an equilibrium when the discount factor is above a cutoff, which we define as $\bar{\delta}$. Plugging in second-period payoffs, this cutoff is defined as:

$$\bar{\delta} \equiv \frac{2(s - r)(\sigma^T + \rho^T)}{\sigma^T(\theta_1 + 1 - 2r)} \quad (7)$$

Note also that since second-period surplus is increasing in the posterior beliefs, $\theta_1 < \theta'_2$ and thus $\bar{\delta} > \underline{\delta}$, for $W > 0$. This means that there is a range of δ such that neither maximal nor minimal inflation is part of an equilibrium. For such δ , we conjecture that there are equilibria in mixed strategies where the opportunistic CRA chooses maximal rating inflation with probability $p \in (0, 1)$ and zero rating inflation with probability $1 - p$ in period one.¹⁵

¹⁵Due to the convexity of the objective function, we can rule out mixed equilibria where asset allocations with intermediate inflation are played with positive probability.

The posterior in period two conditional on the risky assets defaulting will be:

$$\theta_2^{mix}(p) \equiv \frac{\theta_1}{\theta_1 + (1 - \theta_1)(1 - p\sigma^T / (\sigma^T + \rho^T))}$$

Note that $\theta_2^{mix}(p)$ is an increasing function of p such that $\theta_2^{mix}(0) = \theta_1$ and $\theta_2^{mix}(1) = \theta_2$.

The opportunistic CRA will be indifferent between the two extremes whenever:

$$\delta(\pi_2(\theta_2^{mix}(p)) - \pi_2(0)) = (s - r)(\sigma^T + \rho^T). \quad (8)$$

Hence, for every δ such that

$$\frac{(s - r)(\sigma^T + \rho^T)}{\pi_2(\theta_2) - \pi_2(0)} < \delta < \frac{(s - r)(\sigma^T + \rho^T)}{\pi_2(\theta_1) - \pi_2(0)},$$

there is a unique mixed equilibrium p^* . We can solve for this equilibrium using the indifference condition (equation (8)) and plugging in the profits from equation (4):

$$\delta\sigma^T(\theta_2^{mix}(p^*) + 1 - 2r) = 2(s - r)(\sigma^T + \rho^T),$$

giving:

$$p^* = \frac{1}{1 - \theta_1} \left(1 + \rho^T / \sigma^T - \frac{\theta_1}{2(s - r) / \delta - (1 - 2r)\sigma^T / (\sigma^T + \rho^T)} \right). \quad (9)$$

We have thus established the following results.

Proposition 2 *If $W > 0$, in the first period, the opportunistic CRA's equilibrium choice is:*

- (a) *inflating maximally if $\delta \leq \underline{\delta}$,*
- (b) *reporting truthfully if $\delta \geq \bar{\delta}$, and*
- (c) *inflating maximally with probability $p^* \in (0, 1)$ and reporting truthfully with probability $1 - p^*$ if $\delta \in (\underline{\delta}, \bar{\delta})$.*

Recall that for any $W > 0$, the measure of risky assets included by the truthful CRA is given by the formula $\rho^T = 2w_U/R$ and the measure of safe assets $\sigma^T = \min\{w_C + w_U(1 - 1/R), \sigma\}$. We make these substitutions in the expressions for $\underline{\delta}$, $\bar{\delta}$ and p^* in Appendix C.

4.1 Only risky assets

If $W \leq 0$, the truthful CRA includes only risky assets. This implies that the opportunistic CRA will not substitute safe assets for risky in equilibrium and include only risky assets as well. The opportunistic CRA will issue the same type of securities as the truthful CRA in both periods and there will be *no rating inflation*. Even though there are only risky assets, due to tranching, there will still be a safer senior tranche (with $x^T = 1 - \sqrt{w_U / (w_C + w_U)}$) and a riskier junior tranche. Rewriting the inequality $W < 0$ as

$$w_C < w_U (1/R - 1)$$

reveals when it is preferable to include only risky assets and not inflate ratings. First, the wealth of constrained investors must be sufficiently low, since the size of the senior tranche that can be made from only risky assets is limited. Second, the wealth of unconstrained investors must be sufficiently high, to be able to include enough risky assets to create the senior tranche. Lastly, the rating constraint must be sufficiently lax (low R) to enable the creation of the the senior tranche. In addition to the absence of rating inflation, this equilibrium configuration has the benefit of maximizing the expected surplus given this set of parameters, since the budget of the constrained and unconstrained investors is exhausted, the profit margin is higher for risky than for safe assets, and the opportunistic CRA will be hired in period two with probability one.

5 Rating Inflation

Rating inflation is a direct measure of surplus in our model. It measures:

- The size of the lemons problem: The first best allocation is given by the solution of the truthful CRA; when $W > 0$, it is optimal to sell safe assets as well as risky assets. Distortions away from this allocation reduce surplus. The opportunistic CRA can't resist substituting risky assets for safe ones, but it is to its own detriment, as investors take into account this behavior. The extreme form of this behavior is when it fully inflates ratings, and includes only risky assets. Reputation allows the opportunistic CRA to partially circumvent this problem and include some safe assets.
- How much risk investors are taking on: Regulators often outsource the assessment of how much risk certain financial institutions (banks, insurance companies, pensions) take on to the ratings industry. We don't

model directly why the investment grade threshold is important, but its use in monitoring for regulatory purposes demonstrates that there are negative consequences from circumventing the threshold. Becker and Ivashina (2015) and Efung (2018) document institutions' (insurance companies and banks, respectively) efforts to arbitrage ratings by reaching for yield. Rating inflation in our model is a measure of by how much the opportunistic CRA is violating the investment grade threshold - causing a buildup of risk and facilitating the reach for yield.

In this section, we will first analyze the effect of small changes of some of the parameters on rating inflation and, thereafter, large changes.

5.1 Local comparative statics

We use three metrics to measure increases in rating inflation for $W > 0$:

1. There is no rating inflation when $\delta \geq \bar{\delta}$. Therefore if $\bar{\delta}$ increases, the range of discount factors for which there is no rating inflation shrinks.
2. There is maximal rating inflation when $\delta \leq \underline{\delta}$. Therefore if $\underline{\delta}$ increases, the range of discount factors for which there is maximal rating inflation increases.
3. In the mixed equilibrium ($\underline{\delta} < \delta < \bar{\delta}$), p^* is the probability with which maximal rating inflation is chosen. Thus, we interpret an increase in p^* as an increase in rating inflation.

We examine how the parameters of the model affect rating inflation in the following proposition:

Proposition 3 *For $W > 0$, rating inflation is:*

1. *Decreasing in the tightness of the rating constraint and the prior that the CRA is truthful. Formally, p^* , $\bar{\delta}$ and $\underline{\delta}$ are decreasing in R and θ_1 .*
2. *Increasing in the wealth of the unconstrained investors and the retention value of the safe assets. Formally, p^* , $\bar{\delta}$ and $\underline{\delta}$ are increasing in w_U and s .*
3. *Decreasing in the wealth of constrained investors if $W \leq \sigma$. Formally, p^* , $\bar{\delta}$ and $\underline{\delta}$ are decreasing in w_C .*
4. *Decreasing in the supply of safe assets if $W > \sigma$. Formally, p^* , $\bar{\delta}$ and $\underline{\delta}$ are decreasing in σ .*

The above results follow immediately from the expressions for p^* , $\bar{\delta}$ and $\underline{\delta}$ (see Appendix C), but we include a proof of the comparative statics with respect to the prior θ_1 in Appendix D, since it is slightly more involved.

We highlight three intriguing results from the proposition and summarize the rest. First, rating inflation is decreasing in the tightness of the rating constraint. There are two effects. One is that a tighter rating constraint makes it harder to sell in period 2, reducing the surplus, which increases rating inflation. Another is that a tighter rating constraint makes it harder to include risky assets, which increases the likelihood that the opportunistic CRA in period 1 is caught cheating investors. Proposition 3 demonstrates that the second effect is stronger in our model. The increase in the tightness of the rating constraint may come from investor scrutiny or regulation, and is effective.

Second, rating inflation is increasing in the wealth of unconstrained investors. Increasing demand from such investors makes the truthful CRA include more risky assets in the pool, decreasing the likelihood that an inflating opportunistic CRA is discovered. In addition, it reduces the difference between the second-period surplus when the opportunistic CRA is discovered and when it is not. Thus an inflow of money/investment (perhaps due to easy lending) by unconstrained investors such as hedge funds can foster an environment of rating inflation.

Third, rating inflation increases in the value of retaining safe assets, as the opportunistic CRA will have a higher desire to substitute risky for safe assets. The value of retaining safe assets may depend on the demand for safe assets, which have extra value due to their money-like features and use for collateral (Diamond, 2017).

We briefly summarize the other effects. Rating inflation decreases in the prior that the CRA is truthful, due to the increase in the period 2 payoff. In the intermediate case ($0 < W \leq \sigma$), when the constrained investors' budget constraint binds, rating inflation decreases in the wealth of constrained investors. The reason is that their wealth improves the second-period surplus, while at the same time increasing the probability that the opportunistic CRA is discovered after the first period. A similar result holds for the amount of safe assets when the resource constraint for safe assets binds ($W > \sigma$).

5.2 Global comparative statics

In this subsection, we analyze the effect of large changes in R on rating inflation.

Define $\underline{\delta}(R)$ ($\bar{\delta}(R)$) as the lower (upper) discount threshold evaluated at R . Assume $\delta > \bar{\delta}(R = 1)$, and let \underline{R} be defined implicitly by $\delta = \underline{\delta}(R = \underline{R})$ and

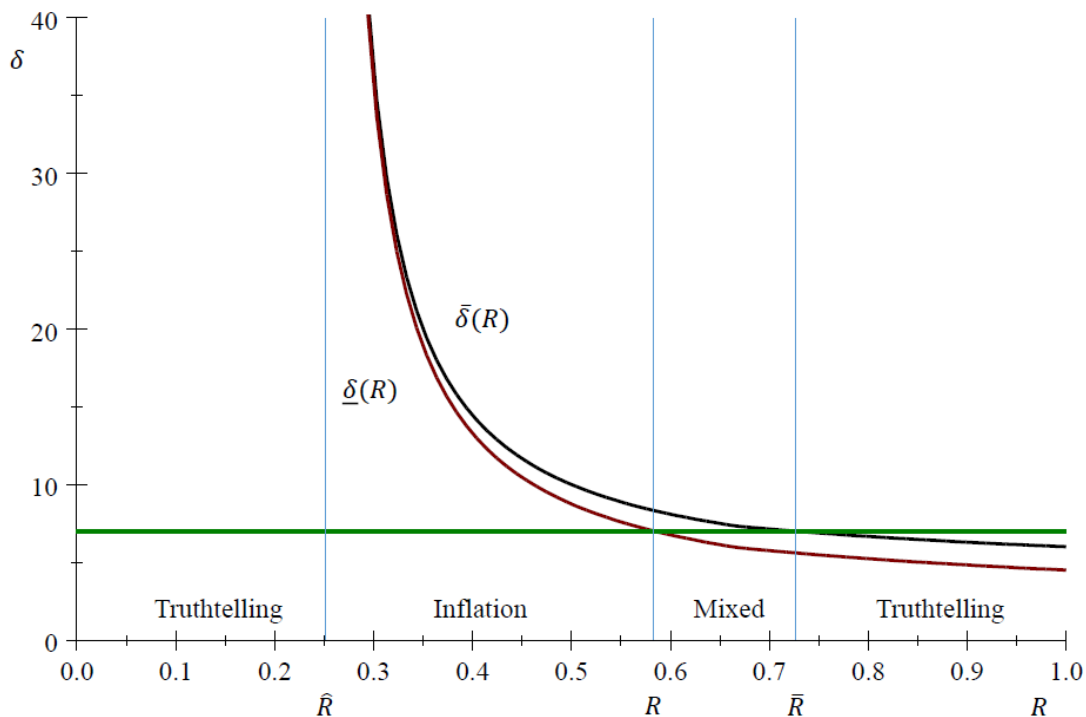


Figure 1: The cutoffs for the discount factor are depicted for the parameter values $w_C = 3$, $w_U = 1$, $\sigma = 2.5$, $\theta = 0.1$, $r = 0.4$, and $s = 0.9$. Assuming a discount factor of $\delta = 7$, we obtain $\hat{R} = 0.25$, $\underline{R} \approx 0.584$, and $\bar{R} \approx 0.727$.

\bar{R} by $\delta = \bar{\delta}(R = \bar{R})$ (in Appendix E we show that both exist and are unique under this assumption). Figure 1 depicts the discount factor thresholds in (δ, R) space.

Notice that these functions aren't defined for all R . This is because when $W \leq 0$, the truthful CRA does not include any safe assets and hence there is no possibility of rating inflation. Define $\hat{R} \equiv w_U / (w_U + w_C)$, the value of R such that $W = 0$. It follows immediately that $\hat{R} \in (0, 1)$. In Figure 1, $\hat{R} = 0.25$, the point where the discount factor curves asymptotically approach infinity from the right.

Using Proposition 3, it is straightforward to derive the following proposition, where we write $p^*(R)$ to denote the mixed strategy evaluated at $R \in (\underline{R}, \bar{R})$.

Proposition 4 *For any finite $\delta > \bar{\delta}(1)$, in the first period the opportunistic CRA:*

- (a) reports truthfully for $R \leq \hat{R}$
- (b) inflates maximally for $R \in (\hat{R}, \underline{R}]$,

- (c) *inflates maximally with probability $p^*(R)$ and reports truthfully with probability $1 - p^*(R)$ for $R \in (\underline{R}, \bar{R})$,*
 (d) *reports truthfully for $R \in [\underline{R}, 1]$.*

This proposition demonstrates that rating inflation may be non-monotonic in the rating standard R .¹⁶ For low R , it is more profitable for the truthful CRA to include only risky assets. Hence, there is no room for the opportunistic CRA to inflate. For intermediate R , it is more profitable for the truthful CRA to include safe assets. Hence, the opportunistic CRA will inflate ratings, substituting risky assets for safe assets. For high R , part (d) of the Proposition shows that ratings may return to being truthful. Here, the truthful CRA will include more safe assets which makes it easier to discover a cheating opportunistic CRA.

One might imagine that low rating standards prevail when financial products are new and have little track record or established models to estimate their risk. In this case the CRA will only include risky assets, but is still able to create some safe securities from them. As these products become more well understood, rating standards increase, but that opens up the possibility of rating inflation. Finally, if rating inflation becomes too pervasive and disruptive, regulatory and investor pressure may increase standards further, which could reduce rating inflation.

Nevertheless, within the model (i.e. abstracting away from the externalities of rating inflation and risk buildup), rating standards are a constraint imposed on the solution. Therefore, the solution with lower rating standards with no inflation has higher surplus than the solution with higher rating standards with no inflation.

6 Conclusion

In this paper, we derive the equilibrium of a simple model of security design where a CRA with reputation concerns both designs and rates securities. We show that the observed equilibrium outcome and rating inflation are sensitive to the investment grade standard as well as the relative demand by constrained and unconstrained investors. Intriguingly, for some parameters, the most efficient outcome (which has no rating inflation), is observed for

¹⁶Note that if we change the assumption in the proposition to $\delta \leq \bar{\delta}(1)$, the opportunistic CRA will still inflate maximally for intermediate values of R , but it will no longer report truthfully with probability one for high enough R . More precisely, if $\delta \leq \underline{\delta}(1)$, it will inflate maximally for all $R \in (\hat{R}, 1]$; and if $\bar{\delta}(1) < \delta \leq \underline{\delta}(1)$, it will inflate maximally for $R \in (\hat{R}, \underline{R}]$ and play the mixed equilibrium for $R \in (\underline{R}, 1]$.

low rating standard. The non-monotonicity of rating inflation with respect to rating standards may be an important concern to regulators. It would be of interest to study further possible systemic effects of rating inflation.

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Appendix

A Proof of Proposition 1 (The truthful CRA's solution)

In order to simplify the optimization program, the following lemma is useful.

Lemma 5 *The budget constraint for the unconstrained investors must bind in any solution.*

Proof: Suppose this is not the case. There are two cases to consider: $x^T = 0$ and $x^T > 0$. In the first case, by continuity, ρ^T can be increased without violating the the unconstrained investors' budget constraint. Neither will any of the other constraints be violated since none of them depends on $\check{\rho}^T$ for $x^T = 0$. However, this implies that the suggested solution can be improved upon since the objective function is strictly increasing in $\check{\rho}^T$ – a contradiction.

In the second case, by continuity, it is possible to reduce \check{x}^T slightly without violating the unconstrained investors' budget constraint. This relaxes all of the other constraints except the first, which remains unaffected (if

$\check{\sigma}^T = w_C$ and $\check{\rho}^T = 0$, the constrained investors' budget constraint cannot be relaxed, but this cannot be a solution since a strictly higher payoff can be achieved by setting $\check{x}^T = 0$, $\check{\sigma}^T = w_C$, and $\check{\rho}^T = 2w_U$). Hence, after the reduction in \check{x}^T it is possible to increase $\check{\rho}^T$ slightly without violating any constraint. Once again, the suggested solution can be improved upon since the objective function is strictly increasing in $\check{\rho}^T$ – a contradiction. ■

Corollary 1 $\rho^T > 0$ in any solution.

Proof: This follows immediately from Lemma 5. ■

The above allows us to solve for x^T from the binding budget constraint:

$$x^T = 1 - \sqrt{2w_U/\check{\rho}^T}.$$

We can now simplify the optimization program by substituting for x^T and rewriting the rating constraint:

$$\begin{aligned} \max_{\check{\sigma}^T, \check{\rho}^T \geq 0} & \{ \check{\sigma}^T (1 - s) + \check{\rho}^T (1/2 - r) - w_U (1 - 2r) \} \\ \text{s.t. } & \sigma - \check{\sigma}^T \geq 0 \\ & w_U - R\check{\rho}^T/2 \geq 0 \\ & w_C + w_U - \check{\sigma}^T - \check{\rho}^T/2 \geq 0 \end{aligned}$$

By the Karush–Kuhn–Tucker theorem, in any solution there are multipliers $\lambda_1, \lambda_2, \lambda_3 \geq 0$, one for each constraint, such that the following conditions holds, where the first condition holds with equality if $\check{\sigma}^T > 0$ and the second condition holds with equality due to Corollary 1.

$$\begin{aligned} \frac{\partial L}{\partial \check{\sigma}^T} &= 1 - s - \lambda_1 - \lambda_3 \leq 0, \\ \frac{\partial L}{\partial \check{\rho}^T} &= 1/2 - r - \lambda_2 R/2 - \lambda_3/2 = 0. \end{aligned}$$

We can immediately rule out certain solutions by observing the following:

1. If $\lambda_2 = 0$, then $\lambda_3 = 1 - 2r$, implying that the budget constraint of constrained investors binds. Substituting in the first first-order condition reveals that this must be negative and thus $\sigma^T = 0$.

2. If $\lambda_1 = 0$ and $\sigma^T > 0$, then $\lambda_3 = 1 - s$ and $\lambda_2 R = 1 - 2r - (1 - s) > 0$. Hence, the rating and the budget constraints of constrained investors bind.
3. If $\lambda_3 = 0$, then $\lambda_1 > 0$ and $\lambda_2 > 0$. Hence, the rating and the resource constraints bind.

This means that we have the following possible solutions:

1. The budget constraints of the constrained investors binds and $\sigma^T = 0$.
2. The budget constraints of the constrained investors and the rating constraint bind.
3. The resource and rating constraints bind.
4. $x^T = 0$.

A.1 Only risky assets

If the budget constraints of the constrained binds and there are no safe assets, we have $\sigma^T = 0$,

$$\begin{aligned}\rho^T &= 2(w_C + w_U), \\ x^T &= 1 - \sqrt{w_U / (w_C + w_U)},\end{aligned}$$

and the expected surplus is

$$(1 - 2r)w_C.$$

A necessary condition for this solution is that the rating condition is not violated. This holds if and only if:

$$w_C + w_U(1 - 1/R) \leq 0.$$

A.2 Budget and rating constraints binding

If the budget constraints of the constrained and the rating constraint bind, we have:

$$\begin{aligned}\sigma^T &= w_C + w_U(1 - 1/R), \\ \rho^T &= 2w_U/R, \\ x &= P.\end{aligned}$$

The associated surplus is given by:

$$(1 - s)(w_C + w_U(1 - 1/R)) + (1 - 2r)w_U(1/R - 1).$$

A necessary condition for this solution is that the resource constraint is not violated, which is true if and only if:

$$w_C + w_U(1 - 1/R) \leq \sigma.$$

In addition, the non-negativity constraint of σ^T must not be violated:

$$w_C + w_U(1 - 1/R) \geq 0.$$

Note that if the last inequality holds with equality, this solution is identical to the one with only risky assets.

A.3 Resource and rating constraints binding

If the resource and rating constraints bind, we have:

$$\begin{aligned}\sigma^T &= \sigma, \\ \rho^T &= 2w_U/R, \\ x^T &= P,\end{aligned}$$

and a surplus of

$$(1 - s)\sigma + (1 - 2r)w_U(1/R - 1).$$

A necessary condition is that the budget constraint of the constrained investors is not violated. This holds if and only if:

$$w_C + w_U(1 - 1/R) \geq \sigma.$$

Note that if the last inequality holds with equality, this solution is identical to the one where the budget and rating constraints bind.

A.4 Safe senior tranche

If the non-negativity constraint for x binds we obtain the following solution: $x^T = 0$, $\sigma^T = \min\{\sigma, w_C\}$, $\rho^T = 2w_U$, with corresponding surplus:

$$(1 - s)\min\{\sigma, w_C\}.$$

A necessary condition for this solution is that the rating constraint is not violated, but this is always true for $x^T = 0$. It is also easy to see that the budget constraint of the constrained investors is not violated. However, it is straightforward to demonstrate that the payoff in this case is strictly less than the payoff in all of the previous cases. Hence, this cannot be a solution to the optimization program.

A.5 Comparing the cases

1. The budget constraints of the constrained investors binds and $\sigma^T = 0$:
 $w_C + w_U (1 - 1/R) \leq 0$.
2. The rating constraint and the budget constraint of the constrained investors bind:
 $0 < w_C + w_U (1 - 1/R) \leq \sigma$.
3. The resource and rating constraints bind:
 $w_C + w_U (1 - 1/R) > \sigma$.

These are the cases listed in Proposition 1. ■

B Proof of Lemma 3 (Opportunistic CRA revealed in period two)

Suppose that the opportunistic CRA's type is revealed in period two. What securities would it sell if it were hired? From Lemma 1, it follows that the opportunistic CRA would not include any safe assets. From analogous arguments to those in Lemma 5, it follows that the budget constraint of the unconstrained investors will bind, meaning that $x_2^O = 1 - \sqrt{2w_U/\check{\rho}_2^O}$ and giving the following optimization program.

$$\begin{aligned} \max_{\check{\rho}_2^O \geq 0} & \{ \check{\rho}_2^O (1/2 - r) - w_U (1 - 2r) \} \\ \text{s.t. } & w_U - R\check{\rho}_2^O/2 \geq 0 \\ & w_C + w_U - \check{\rho}_2^O/2 \geq 0. \end{aligned}$$

It is easy to see that this program has two solutions:

1. $\rho_2^O = 2(w_C + w_U)$ if $W \leq 0$.
2. $\rho_2^O = 2w_U/R$ if $W > 0$. ■

C Discount-factor cutoffs and mixed equilibrium

C.1 Intermediate W

If $W \in (0, \sigma]$, $\rho^T = 2w_U/R$ and $\sigma^T = w_C + w_U - w_U/R$. Substituting in the formulas for the discount-factor cutoffs and the mixed-strategy equilibrium gives:

$$\begin{aligned}\underline{\delta} &= \frac{2(s-r)}{\frac{\theta_1}{\theta_1 + 2w_U/(R(w_C + w_U) - w_U)} + (1-2r)\frac{w_C + w_U - w_U/R}{w_C + w_U + w_U/R}}, \\ \bar{\delta} &= \frac{2(s-r)}{\frac{\theta_1}{1 + 2w_U/(R(w_C + w_U) - w_U)} + (1-2r)\frac{w_C + w_U - w_U/R}{w_C + w_U + w_U/R}}, \\ p^* &= \frac{1}{1-\theta_1} \left(\frac{1 + 2w_U/(R(w_C + w_U) - w_U)}{-\frac{\theta_1}{2(s-r)/\bar{\delta} - (1-2r)(w_C + w_U - w_U/R)/(w_C + w_U + w_U/R)}} \right).\end{aligned}$$

Note that p^* equals zero for $\delta = \bar{\delta}$ and one for $\delta = \underline{\delta}$.

C.2 High W

If $W > \sigma$, $\rho^T = 2w_U/R$ and $\sigma^T = \sigma$. Substituting in the formulas for the discount-factor cutoffs and the mixed-strategy equilibrium gives:

$$\begin{aligned}\underline{\delta} &= \frac{2(s-r)}{\theta_1/(\theta_1 + 2w_U/(\sigma R)) + (1-2r)\sigma/(\sigma + 2w_U/R)}, \\ \bar{\delta} &= \frac{2(s-r)}{\theta_1/(1 + 2w_U/(\sigma R)) + (1-2r)\sigma/(\sigma + 2w_U/R)}, \\ p^* &= \frac{1}{1-\theta_1} \left(\frac{1 + 2w_U/(\sigma R)}{-\frac{\theta_1}{2(s-r)/\bar{\delta} - (1-2r)\sigma/(\sigma + 2w_U/R)}} \right).\end{aligned}$$

Analogously to the previous case, p^* equals zero for $\delta = \bar{\delta}$ and one for $\delta = \underline{\delta}$.

D Proof of Proposition 3 (Local comparative statics with respect to θ_1)

The only comparative static which is non-trivial to demonstrate is the change in p^* with respect to θ_1 . We prove that p^* is decreasing in θ_1 . Define:

$$A \equiv 1 + \rho^T / \sigma^T,$$

$$C(\delta) \equiv \frac{1}{2(s-r)/\delta - (1-2r)\sigma^T / (\sigma^T + \rho^T)},$$

and note that $A > 1$ since $\rho^T > 0$. Using this notation, the mixed strategy can be written:

$$p^* = \frac{A - \theta_1 C(\delta)}{1 - \theta_1}.$$

This expression equals one for $\delta = \underline{\delta}$. Hence, we can solve for $C(\underline{\delta})$.

$$C(\underline{\delta}) = \frac{A - 1 + \theta_1}{\theta_1}.$$

Differentiating p^* with respect to θ_1 gives:

$$\frac{\partial p^*}{\partial \theta_1} = \frac{A - C(\delta)}{(1 - \theta_1)^2}.$$

We will show that this expression is negative. The mixed strategy is defined for $\delta \in [\underline{\delta}, \bar{\delta}]$ (at the boundaries it is degenerate), but since $C(\delta)$ is increasing in δ , it is enough to show the sign for $\delta = \underline{\delta}$.

$$\begin{aligned} \frac{\partial p^*}{\partial \theta_1}(\delta = \underline{\delta}) &= \frac{A - C(\underline{\delta})}{(1 - \theta_1)^2} \\ &= \frac{1 - A}{\theta_1(1 - \theta_1)} < 0. \end{aligned}$$

■

E Proof of Proposition 4 (Global comparative statics with respect to \mathbf{R})

Lemma 6 *For any finite $\delta > \bar{\delta}(R = 1)$, there are unique values \underline{R} and \bar{R} such that $\underline{\delta}(\underline{R}) = \delta$ and $\bar{\delta}(\bar{R}) = \delta$.*

Proof: For $R \in (\hat{R}, 1]$, the functions $\underline{\delta}(R)$ and $\bar{\delta}(R)$ are continuous in R since they are both continuous in σ^T for $\sigma^T > 0$ and since $\sigma^T = \min\{w_U + w_C - w_U/R, \sigma\}$ is positive and continuous in R over the same interval. It is easy to show that $\underline{\delta}(R) < \bar{\delta}(R)$ for $R > \hat{R}$ and moreover that:

$$\lim_{R \downarrow \hat{R}} \underline{\delta}(R) = \lim_{R \downarrow \hat{R}} \bar{\delta}(R) = \infty.$$

Hence, there exists $a \in (\hat{R}, 1]$ such that $\delta < \underline{\delta}(a) < \bar{\delta}(a)$. By assumption $\delta > \bar{\delta}(R = 1)$. The intermediate value theorem can thus be applied to the functions over the closed and bounded interval $[a, 1]$, proving existence of \underline{R} and \bar{R} . Uniqueness follows since the functions $\underline{\delta}(R)$ and $\bar{\delta}(R)$ are decreasing in R for $R \in (\hat{R}, 1]$ by Proposition 3. ■