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# A Real-Business-Cycle model with endogenous discounting and a government sector

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**Abstract**

We introduce an endogenous discount factor as in Uzawa (1968) and Schmitt-Grohe and Uribe (2003) into a real-business-cycle setup with Greenwood *et al.* (1988) preferences and augment the model with a detailed government sector. We calibrate the artificial economy to Bulgarian data for the period following the introduction of the currency board arrangement (1999-2016). We investigate the quantitative importance of endogenous discounting for the propagation cyclical fluctuations in Bulgaria. The presence of an endogenous discount factor improves the model performance against data, and in addition this extended setup dominates the standard RBC model framework with a constant discount factor, *e.g.*, Vasilev (2009).

**Keywords:** Business cycles; Uzawa preferences; Endogenous discounting; Bulgaria

*JEL Classification Codes:* E24, E32

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# 1 Introduction and Motivation

In the wide class of dynamic stochastic general-equilibrium models (also known as "real-business-cycle" models), a representative one-member household is faced with the dynamic choice of consumption, investment and hours worked sequences, which are optimal with respect to the individual's utility function, the budget constraint, and the rate of time preference, or the discount rate. More specifically, the discount factor, i.e., the inverse of the discount rate plus unity, in those models is assumed to be constant across time periods, and thus independent of the time profile of the utility stream associated with the set of optimal decision sequences. One shortcoming of the class of models with a constant discount factor, is that when calibrated to Bulgarian data, model predicted consumption volatility is too high relative to that observed in data, even when the consumption variable includes consumption of durables as well.

In order to address that shortcoming of these models, we follow Uzawa (1968), who analyzed the time preference structure of a representative household through the lens of neoclassical choice theory and derived a formulation specifying the rate by which an individual discounts future levels.[1] According to his study, the rate by which future utility is discounted, should importantly depend on present consumption. More specifically, the higher the level of real income today, the lower is the rate by which the individual discounts tomorrow's real income (or consumption). That is, it seems more realistic to think that the wealthier a person is, the more impatient the household should be, and thus the higher the agent's preference is for immediate consumption. We take those analytical results seriously and extend the standard model with an endogenous discount factor. The modelling choice follows the modification presented in Schmitt-Grohe and Uribe (2003) due to its computational simplicity, as compared to the original representation in Uzawa (1968).[2] Another important difference from Uzawa (1968) is that the endogenous discount factor will also depend positively on hours

[1] Alternative ways to descrease consumption volatility in the model is to allow for consumption habits (Vasilev 2018a), or Epstein-Zin preferences featuring a desire for "early resolution of uncertainty" on the household's side (Vasilev 2018b).

[2] Other papers using such preferences include Obstfeld (1990), Mendoza (1991), Schmitt-Grohe (1998), and Uribe (1997). Kim and Kose (2001) compare the business-cycle properties of this model to those of the standard model with a constant discount factor.

worked. In other words, a higher labor supplied is associated with a lower wealth, and hence the more patient the household is.

In addition to the endogenous discount factor, the utility function in the extended model uses the formulation utilized in Greenwood *et al.* (1988). Many authors, such as Mendoza (1991) and Correia *et al.* (1995) have demonstrated that these preferences improve the ability of these setups to capture business cycle facts.[3] More specifically, these preferences generate a labor supply response that is independent of the intertemporal consumption-saving decision, and the inter-temporal substitution (income) effect - which is a central mechanism in a large class of dynamic macroeconomic models - is thus eliminated.[4] This form of the utility function then allows us to study and emphasize different transmission mechanism in this paper.

The augmented model setup in this paper will be then used to quantitatively investigate the effect of endogenous discounting on the business cycle fluctuations in Bulgaria after the introduction of the currency board arrangement (1999-2016), which was a period of macroeconomic stability. Beside the second-moment matching exercise, the model will be validated using the methodology suggested in Canova (2007). In addition, the model is able to address the criticism of Nelson and Plosser (1992), Cogley and Nason (1995) and Rotemberg and Woodford (1996b), who argue that the RBC class of models do not have a strong internal propagation mechanism besides the strong persistence in the total factor productivity (TFP) process. We show those critiques are unfounded in the Bulgarian case, where the persistence of the TFP is much lower than that in the US, for example.

The rest of the paper is organized as follows: Section 2 describes the model framework and describes the decentralized competitive equilibrium system, Section 3 discusses the calibration procedure, and Section 4 presents the steady-state model solution. Sections 5 proceeds with the out-of-steady-state dynamics of model variables, and compared the simulated second moments and auto- and cross-correlation functions of theoretical variables against their empirical counterparts. Section 6 concludes the paper.

[3]Benhabib *et al.* (1991) show that such preferences can be interpreted as reduced-form ones for an economy with home production.

[4]In other words, the elasticity of intertemporal substitution associate with leisure is zero.

# 2 Model setup

There is a representative households which derives utility out of consumption and leisure. The time available to households can be spent in productive use or as leisure. The government taxes consumption spending and levies a common tax on all income, in order to finance non-productive purchases of government consumption goods, and government transfers. On the production side, there is a representative firm, which hires labor and capital to produce a homogenous final good, which could be used for consumption, investment, or government purchases.

## 2.1 Representative Household

There is a representative household, which maximizes its expected utility function, which features an instantaneous felicity function, as in Greenwood *et al.* (1988), and Schmitt-Grohe and Uribe (2003):

$$E_0 \sum_{t=0}^{\infty} \theta_t \frac{\left[c_t - \frac{h_t^\nu}{\nu}\right]^{1-\sigma}}{1-\sigma} \tag{2.1}$$

where

$$\theta_0 = 1 \tag{2.2}$$

$$\theta_{t+1} = \beta(c_t, h_t)\theta_t, \qquad t \geq 0, \tag{2.3}$$

$$\beta(c_t, h_t) = \left[1 + c_t - \frac{h_t^\nu}{\nu}\right]^{-\psi}. \tag{2.4}$$

Parameter $\sigma > 1$ captures the curvature of the utility function, $\nu > 0$ is the labor supply elasticity, and $\psi > 0$ is the curvature parameter of the discount factor function. The novelty in the setup is that the subjective discount factor $\theta_t$ is no longer a fixed scalar, but rather a function of current individual consumption and labor supply, and thus an endogenous variable. Furthermore, the discount factor is time-varying and assumed to be decreasing in consumption, and increasing in hours. In other words, as pointed out in Uzawa (1968), agents become more impatient the more they consume.[5] The household will internalize the

[5]This utility function is equivalent to a specification with a separable term containing government consumption, e.g. Baxter and King (1993). Since in this paper we focus on the exogenous (observed) policies,

effects of consumption and hours worked on the discount factor when choosing optimally how much to consume and how much labor to supply in each period.

Next, the household starts with an initial stock of physical capital $k_0 > 0$, and has to decide how much to add to it in the form of new investment. The law of motion for physical capital is

$$k_{t+1} = i_t + (1 - \delta)k_t \tag{2.5}$$

and $0 < \delta < 1$ is the depreciation rate. Next, the real interest rate is $r_t$, hence the before-tax capital income of the household in period $t$ equals $r_t k_t$. In addition to capital income, the household can generate labor income. Hours supplied to the representative firm are rewarded at the hourly wage rate of $w_t$, so pre-tax labor income equals $w_t h_t$. Lastly, the household owns the firm in the economy and has a legal claim on all the firm's profit, $\pi_t$.

Next, the household's problem can be now simplified to

$$\begin{aligned} E_0 \sum_{t=0}^{\infty} \theta_t \Bigg\{ & \frac{[c_t - \frac{h_t^\nu}{\nu}]^{1-\sigma}}{1-\sigma} \\ & + \lambda_t \Bigg[ -(1+\tau^c)c_t - k_{t+1} + (1-\delta)k_t + (1-\tau^y)[r_t k_t + w_t h_t] + g_t^t + \pi_t \Bigg] \Bigg\} \\ & + \eta_t \Bigg[ \theta_{t+1} - [1 + c - \frac{h_t^\nu}{\nu}]^{-\psi} \theta_t \Bigg], \end{aligned} \tag{2.6}$$

where $\lambda_t$ is the Lagrangean multiplier of the period-$t$ budget constraint, while $\eta_t$ is the Lagrangean multiplier associated with the evolution of the endogenous discount factor. Note also that $\lambda_t$ is discounted, while $\eta_t$ is not.

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and the household takes government spending as given, the presence of such a term is irrelevant. For the sake of brevity, we skip this term in the utility representation above.

The first-order optimality conditions (FOCs) are as follows:

$$c_t \quad : \quad [c_t - \frac{h_t^\nu}{\nu}]^{-\sigma} + \psi\eta_t[1 + c_t - \frac{h_t^\nu}{\nu}]^{-\psi-1} = \lambda_t(1+\tau^c) \tag{2.7}$$

$$h_t \quad : \quad h_t^{\nu-1}[c_t - \frac{h_t^\nu}{\nu}]^{-\sigma} = \lambda_t(1-\tau^y)w_t - \psi\eta_t h_t^{\nu-1}[1 + c_t - \frac{h_t^\nu}{\nu}]^{-\psi-1} \tag{2.8}$$

$$k_{t+1} \quad : \quad \lambda_t = [1 + c_t - \frac{h_t^\nu}{\nu}]^{-\psi} E_t\lambda_{t+1}[1 + (1-\tau^y)r_{t+1} - \delta] \tag{2.9}$$

$$\theta_{t+1} \quad : \quad \eta_t = -E_t\frac{[c_t - \frac{h_t^\nu}{\nu}]^{1-\sigma}}{1-\sigma} + E_t\eta_{t+1}[1 + c_{t+1} - \frac{h_{t+1}^\nu}{\nu}]^{-\psi} \tag{2.10}$$

$$TVC \quad : \quad \lim_{t\to\infty} \theta^t\lambda_t k_{t+1} = 0 \tag{2.11}$$

The interpretation of the first-order conditions above is standards: for each household, the marginal utility of consumption (taking into consideration the effect of of consumption on the discount factor) equals the marginal utility of wealth, corrected for the consumption tax rate. With endogenous discounting, the marginal utility of consumption contains an additional term, which reflects the fact that an increase in consumption this period lowers the discount factor. More specifically, a unit decline in the discount reduces t-period utility by the value of the Lagrange multiplier, $\eta_t$, which is now an additional state variable in the system. Alternatively, $\eta_t$ can be regarded as the present discounted value of utility from $t+1$ onward (or the "continuation value"). Substituting forward in (2.10) yields:

$$\eta_t = -E_t\sum_{j=1}^{\infty}\left(\frac{\theta_{t+j}}{\theta_{t+1}}\right)\frac{[c_t - \frac{h_t^\nu}{\nu}]^{1-\sigma}}{1-\sigma} \tag{2.12}$$

Similarly, the second equation states that when choosing labor supply optimally, at the margin, each hour spent by the household working for the firm should balance the benefit from doing so in terms of additional income generates, and the cost measured in terms of lower utility of leisure, plus the correction from the effect of work on the discount factor. The third equation is the so-called "Euler condition," which describes how the household chooses to allocate physical capital over time. The difference here is that it features a an endogenous discount factor. The last condition is called the "transversality condition" (TVC): it states that at the end of the horizon, the value of physical capital should be zero.[6]

[6]The evolution of the continuation value is not presented, as it was discussed above.

## 2.2 Firm problem

There is a representative firm in the economy, which produces a homogeneous product. The price of output is normalized to unity. The production technology is Cobb-Douglas and uses both physical capital, $k_t$, and labor hours, $h_t$, to maximize static profit

$$\Pi_t = A_t k_t^{\alpha} h_t^{1-\alpha} - r_t k_t - w_t h_t, \tag{2.13}$$

where $A_t$ denotes the level of technology in period $t$. Since the firm rents the capital from households, the problem of the firm is a sequence of static profit maximizing problems. In equilibrium, there are no profits, and each input is priced according to its marginal product, *i.e.*:

$$k_t \quad : \quad \alpha \frac{y_t}{k_t} = r_t, \tag{2.14}$$

$$h_t \quad : \quad (1-\alpha) \frac{y_t}{h_t} = w_t. \tag{2.15}$$

In equilibrium, given that the inputs of production are paid their marginal products, $\pi_t = 0$, $\forall t$.

## 2.3 Government

In the model setup, the government is levying taxes on labor and capital income, as well as consumption, in order to finance spending on wasteful government purchases, and government transfers. The government budget constraint is as follows:

$$g_t^c + g_t^t = \tau^c c_t + \tau^y [w_t h_t + r_t k_t] \tag{2.16}$$

Tax rates and government consumption-to-output ratio would be chosen to match the average share in data, and government transfers would be determined residually in each period so that the government budget is always balanced.

## 2.4 Dynamic Competitive Equilibrium (DCE)

For a given process followed by technology $\{A_t\}_{t=0}^{\infty}$ average tax rates $\{\tau^c, \tau^y\}$, initial capital stock $\{k_0\}$, the decentralized dynamic competitive equilibrium is a list of sequences $\{c_t, i_t, k_t, h_t, \theta_t\}_{t=0}^{\infty}$ for the household, a sequence of government purchases and transfers

$\{g_t^c, g_t^t\}_{t=0}^{\infty}$, and input prices $\{w_t, r_t\}_{t=0}^{\infty}$ such that (i) the household maximizes its utility function subject to its budget constraint; (ii) the representative firm maximizes profit; (iii) government budget is balanced in each period; (iv) all markets clear.

# 3 Data and Model Calibration

To characterize business cycle fluctuations with an endogenous depreciation rate in Bulgaria, we will focus on the period following the introduction of the currency board (1999-2016). Quarterly data on output, consumption and investment was collected from National Statistical Institute (2017), while the real interest rate is taken from Bulgarian National Bank Statistical Database (2017). The calibration strategy described in this section follows a long-established tradition in modern macroeconomics: first, as in Vasilev (2016), the steady-state value of the discount factor, $\beta(c,h) = 0.982$, is set to match the steady-state capital-to-output ratio in Bulgaria, $k/y = 13.964$, in the steady-state Euler equation. The labor share parameter, $1 - \alpha = 0.571$, is obtained as in Vasilev (2017d), and equals the average value of labor income in aggregate output over the period 1999-2016. This value is slightly higher as compared to other studies on developed economies, due to the overaccumulation of physical capital, which was part of the ideology of the totalitarian regime, which was in place until 1989. Next, the average income tax rate was set to $\tau^y = 0.1$. This is the average effective tax rate on income between 1999-2007, when Bulgaria used progressive income taxation, and equal to the proportional income tax rate introduced as of 2008. Similarly, the tax rate on consumption is set to its value over the period, $\tau^c = 0.2$.

Next, the curvature of the utility function is set to $\sigma = 2$, a standard value in the literature, while the curvature of the endogenous discount factor function, $\psi$, is calibrated to match the steady-state value of the discount factor. In turn, the labor supply elasticity, $\nu$, is calibrated to match that in steady-state consumers would supply one-third of their time endowment to working. This is in line with the estimates for Bulgaria (Vasilev 2017a) as well over the period studied. Next, the steady-state depreciation rate of physical capital in Bulgaria, $\delta = 0.013$, was taken from Vasilev (2016). It was estimated as the average quarterly depreciation rate over the period 1999-2014. Finally, the processes followed by total factor

productivity (TFP) is estimated from the detrended series by running an AR(1) regression and saving the residuals. Table 1 below summarizes the values of all model parameters used in the paper.

Table 1: Model Parameters

| Parameter | Value | Description | Method |
|---|---|---|---|
| $\beta$ | 0.982 | Discount factor | Calibrated |
| $\alpha$ | 0.429 | Capital Share | Data average |
| $1-\alpha$ | 0.571 | Labor Share | Calibrated |
| $\sigma$ | 2.000 | Curvature of the utility function | Set |
| $\nu$ | 1.400 | Labor supply elasticity | Calibrated |
| $\psi$ | 0.110 | Curvature of the discount factor function | Calibrated |
| $\delta$ | 0.013 | Depreciation rate on physical capital | Data average |
| $\tau^y$ | 0.100 | Average tax rate on income | Data average |
| $\tau^c$ | 0.200 | VAT/consumption tax rate | Data average |
| $\rho_a$ | 0.701 | AR(1) persistence coefficient, TFP process | Estimated |
| $\sigma_a$ | 0.044 | st. error, TFP process | Estimated |

# 4 Steady-State

Once the values of model parameters were obtained, the steady-state equilibrium system solved, the "big ratios" can be compared to their averages in Bulgarian data. The results are reported in Table 2 on the next page. The steady-state level of output was normalized to unity (hence the level of technology $A$ differs from one, which is usually the normalization done in other studies), which greatly simplified the computations. Next, the model matches consumption-to-output and government purchases ratios by construction; The investment ratios are also closely approximated, despite the closed-economy assumption and the absence of foreign trade sector. The shares of income are also identical to those in data, which is an artifact of the assumptions imposed on functional form of the aggregate production function. The after-tax return, where $\bar{r} = (1-\tau^y)r - \delta$ is also relatively well-captured by the model. Lastly, given the absence of debt, and the fact that transfers were chosen residually to balance

the government budget constraint, the result along this dimension is understandably not so close to the average ratio in data.

Table 2: Data Averages and Long-run Solution

| Variable | Description | Data | Model |
|---|---|---|---|
| $y$ | Steady-state output | N/A | 1.000 |
| $c/y$ | Consumption-to-output ratio | 0.648 | 0.674 |
| $i/y$ | Investment-to-output ratio | 0.201 | 0.175 |
| $k/y$ | Capital-to-output ratio | 13.96 | 13.96 |
| $g^c/y$ | Government consumption-to-output ratio | 0.151 | 0.151 |
| $wh/y$ | Labor income-to-output ratio | 0.571 | 0.571 |
| $rk/y$ | Capital income-to-output ratio | 0.429 | 0.429 |
| $h$ | Share of time spent working | 0.333 | 0.333 |
| $\bar{r}$ | After-tax net return on capital | 0.014 | 0.016 |

# 5 Out of steady-state model dynamics

Since the model does not have an analytical solution for the equilibrium behavior of variables outside their steady-state values, we need to solve the model numerically. This is done by log-linearizing the original equilibrium (non-linear) system of equations around the steady-state. This transformation produces a first-order system of stochastic difference equations. First, we study the dynamic behavior of model variables to an isolated shock to the total factor productivity process, and then we fully simulate the model to compare how the second moments of the model perform when compared against their empirical counterparts.

## 5.1 Impulse Response Analysis

This subsection documents the impulse responses of model variables to a 1% surprise innovation to technology. The impulse response functions (IRFs) are presented in Fig. 1 on the next page. As a result of the one-time unexpected positive shock to total factor productivity, output increases upon impact. This expands the availability of resources in the

economy, so used of output - consumption, investment and government consumption also increase contemporaneously. With an endogenous discount factor, the response in consumption is smoothed ("excess smoothness" in consumption), while the response in investment is increased. This "excess sensitivity" in investment behavior is due to the fact that the consumer internalizes the effect of consumption on the discount factor. As a result, consumption volatility drastically decreases. In turn, with smooth consumption, the adjustment happens with saving (i.e., physical capital accumulation in the model). In turn, physical capital becomes more volatile, and exhibits a hump-shaped behavior.

At the same time, the increase in productivity increases the after-tax return on the two factors of production, labor and capital. The representative households then respond to the incentives contained in prices and start accumulating capital, and supplies more hours worked. In turn, the increase in capital input feeds back in output through the production function and that further adds to the positive effect of the technology shock. In the labor market, the wage rate increases, and the household increases its hours worked. In turn, the increase in total hours further increases output, again indirectly.

Over time, as capital is being accumulated, its after-tax marginal product starts to decrease, which lowers the households' incentives to save. As a result, physical capital stock eventually returns to its steady-state, and exhibits a hump-shaped dynamics over its transition path. The rest of the model variables return to their old steady-states in a monotone fashion as the effect of the one-time surprise innovation in technology dies out.

## 5.2 Simulation and moment-matching

As in Vasilev (2017b), we will now simulate the model 10,000 times for the length of the data horizon. Both empirical and model simulated data is detrended using the Hodrick-Prescott (1980) filter. Table 3 on the next page summarizes the second moments of data (relative volatilities to output, and contemporaneous correlations with output) versus the same moments computed from the model-simulated data at quarterly frequency.[7] To minimize the

[7]The model-predicted 95 % confidence intervals are available upon request.

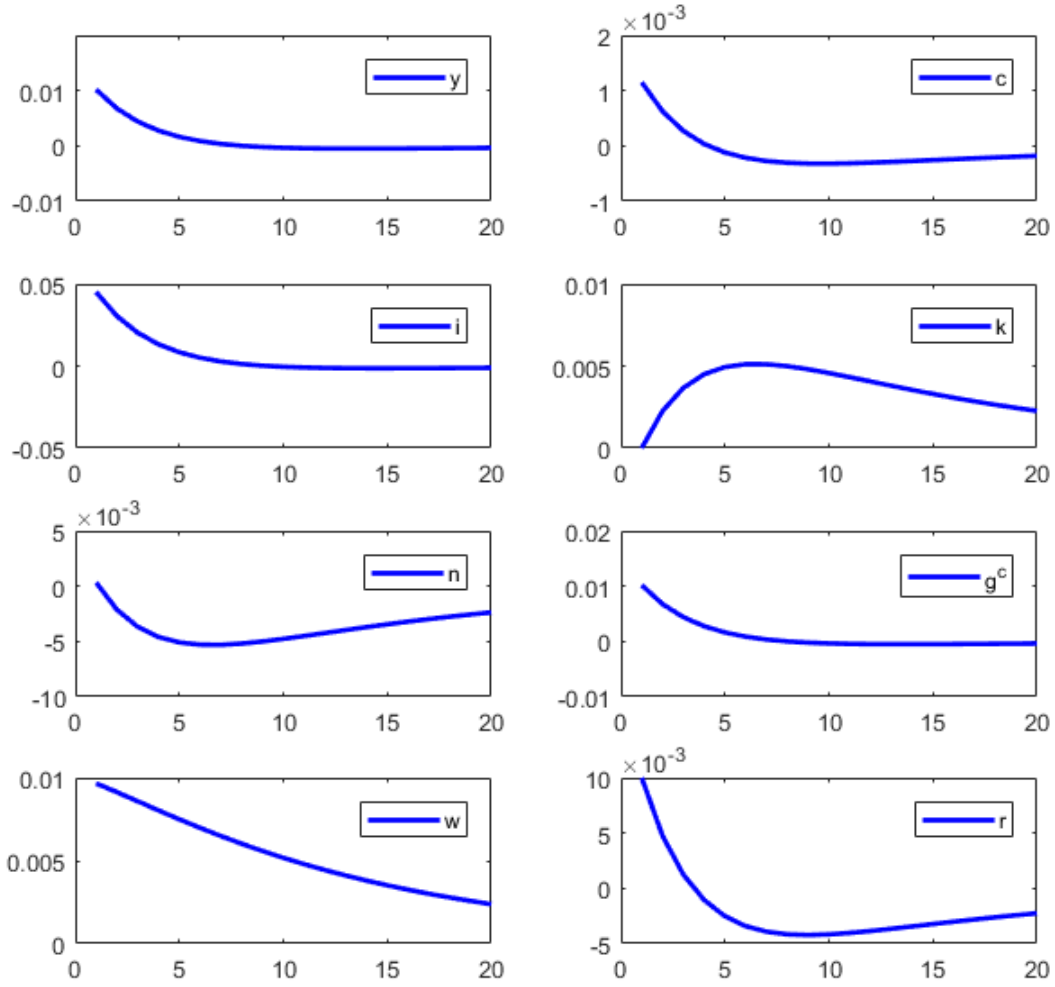


Figure 1: Impulse Responses to a 1% surprise innovation in technology

sample error, the simulated moments are averaged out over the computer-generated draws. As in Vasilev (2016, 2017b, 2017c), the model matches quite well the absolute volatility of output. By construction, government consumption in the model varies as much as output. The model with endogenous discount factor underestimates the variability in consumption, but predicted volatility of investment is too large.[8] Still, the model is qualitatively consistent with the stylized fact that consumption generally varies less than output, while investment is more volatile than output.

[8]One way to fix that is to add capital adjustment costs, which would decrease investment variability.

Table 3: Business Cycle Moments

| | Data | Model |
|---|---|---|
| $\sigma_y$ | 0.05 | 0.05 |
| $\sigma_c/\sigma_y$ | 0.55 | 0.37 |
| $\sigma_i/\sigma_y$ | 1.77 | 4.90 |
| $\sigma_g/\sigma_y$ | 1.21 | 1.00 |
| $\sigma_h/\sigma_y$ | 0.63 | 2.69 |
| $\sigma_w/\sigma_y$ | 0.83 | 3.52 |
| $\sigma_{y/h}/\sigma_y$ | 0.86 | 3.52 |
| $\sigma_u/\sigma_y$ | 3.22 | 2.80 |
| $corr(c,y)$ | 0.85 | 0.15 |
| $corr(i,y)$ | 0.61 | 0.95 |
| $corr(g,y)$ | 0.31 | 1.00 |
| $corr(h,y)$ | 0.49 | 0.14 |
| $corr(w,y)$ | -0.01 | 0.42 |
| $corr(u,y)$ | -0.47 | -0.14 |

With respect to the labor market variables, the variability of employment and wages predicted by the model is much higher than that in data, but the variability of unemployment in the model is not that far away from the observed volatility in data. This is yet another confirmation that the perfectly-competitive assumption, e.g. Vasilev (2009), as well as the benchmark calibration here, does not describe very well the dynamics of labor market variables. Next, in terms of contemporaneous correlations, the model systematically overpredicts the pro-cyclicality of the main aggregate variables - investment, and government consumption. This, however, is a common limitation of this class of models. The puzzle from the endogenous discount factor (containing consumption and hours) is that the predicted contemporaneous consumption correlation with output is too low. Also, along the labor market dimension, the contemporaneous correlation of employment with output, and unemployment with output, are a bit too weak. With respect to wages, the model predicts moderate cyclicality, while wages in data are acyclical. This shortcoming is well-known in

the literature and an artifact of the wage being equal to the labor productivity in the model.

In the next subsection, as in Vasilev (2016), we investigate the dynamic correlation between labor market variables at different leads and lags, thus evaluating how well the model matches the phase dynamics among variables. In addition, the autocorrelation functions (ACFs) of empirical data, obtained from an unrestricted VAR(1) are put under scrutiny and compared and contrasted to the simulated counterparts generated from the model.

## 5.3 Auto- and cross-correlation

This subsection discusses the auto-(ACFs) and cross-correlation functions (CCFs) of the major model variables. The coefficients empirical ACFs and CCFs at different leads and lags are presented in Table 4 below against the averaged simulated AFCs and CCFs. For the sake of brevity, we present only results for the model with consumption habits. Following Canova (2007), this is used as a goodness-of-fit measure.

Table 4: Autocorrelations for Bulgarian data and the model economy

| | | k | | | |
|---|---|---|---|---|---|
| Method | Statistic | 0 | 1 | 2 | 3 |
| Data | $corr(u_t, u_{t-k})$ | 1.000 | 0.765 | 0.552 | 0.553 |
| Model | $corr(u_t, u_{t-k})$ | 1.000 | 0.958 | 0.910 | 0.856 |
| | (s.e.) | (0.000) | (0.024) | (0.047) | (0.068) |
| Data | $corr(n_t, n_{t-k})$ | 1.000 | 0.484 | 0.009 | 0.352 |
| Model | $corr(n_t, n_{t-k})$ | 1.000 | 0.958 | 0.910 | 0.856 |
| | (s.e.) | (0.000) | (0.024) | (0.047) | (0.068) |
| Data | $corr(y_t, y_{t-k})$ | 1.000 | 0.810 | 0.663 | 0.479 |
| Model | $corr(y_t, y_{t-k})$ | 1.000 | 0.953 | 0.896 | 0.829 |
| | (s.e.) | (0.000) | (0.028) | (0.054) | (0.078) |
| Data | $corr(a_t, a_{t-k})$ | 1.000 | 0.702 | 0.449 | 0.277 |
| Model | $corr(a_t, a_{t-k})$ | 1.000 | 0.955 | 0.901 | 0.837 |
| | (s.e.) | (0.000) | (0.027) | (0.051) | (0.075) |
| Data | $corr(c_t, c_{t-k})$ | 1.000 | 0.971 | 0.952 | 0.913 |
| Model | $corr(c_t, c_{t-k})$ | 1.000 | 0.958 | 0.909 | 0.851 |
| | (s.e.) | (0.000) | (0.025) | (0.052) | (0.076) |
| Data | $corr(i_t, i_{t-k})$ | 1.000 | 0.810 | 0.722 | 0.594 |
| Model | $corr(i_t, i_{t-k})$ | 1.000 | 0.955 | 0.899 | 0.834 |
| | (s.e.) | (0.000) | (0.027) | (0.052) | (0.076) |
| Data | $corr(w_t, w_{t-k})$ | 1.000 | 0.760 | 0.783 | 0.554 |
| Model | $corr(w_t, w_{t-k})$ | 1.000 | 0.958 | 0.909 | 0.854 |
| | (s.e.) | (0.000) | (0.025) | (0.048) | (0.070) |

As seen from Table 4 above, the model compares relatively well vis-a-vis data. Empirical ACFs for output and investment are slightly outside the confidence band predicted by the model, while the ACFs for total factor productivity and household consumption are well-approximated by the model. The persistence of labor market variables are also relatively well-described by the model dynamics. Overall, the model with habits in consumption generates too much persistence in output and both employment and unemployment, and is subject

to the criticism in Nelson and Plosser (1992), Cogley and Nason (1995) and Rotemberg and Woodford (1996b), who argue that the RBC class of models do not have a strong internal propagation mechanism besides the strong persistence in the TFP process. In those models, e.g. Vasilev (2009), and in the current one, labor market is modelled in the Walrasian market-clearing spirit, and output and unemployment persistence is low.

Next, as seen from Table 5 below, over the business cycle, in data labor productivity leads employment. The model, however, cannot account for this fact. As in the standard RBC model a technology shock can be regarded as a factor shifting the labor demand curve, while holding the labor supply curve constant. Therefore, the effect between employment and labor productivity is only a contemporaneous one.

Table 5: Dynamic correlations for Bulgarian data and the model economy

| | | k | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Method | Statistic | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
| Data | $corr(n_t, (y/n)_{t-k})$ | -0.342 | -0.363 | -0.187 | -0.144 | 0.475 | 0.470 | 0.346 |
| Model | $corr(n_t, (y/n)_{t-k})$ | -0.091 | -0.135 | -0.198 | -0.947 | -0.514 | -0.425 | -0.348 |
| | (s.e.) | (0.346) | (0.304) | (0.259) | (0.047) | (0.272) | (0.312) | (0.347) |
| Data | $corr(n_t, w_{t-k})$ | 0.355 | 0.452 | 0.447 | 0.328 | -0.040 | -0.390 | -0.57 |
| Model | $corr(n_t, w_{t-k})$ | -0.091 | -0.135 | -0.198 | -0.947 | -0.514 | -0.425 | -0.348 |
| | (s.e.) | (0.346) | (0.304) | (0.259) | (0.047) | (0.272) | (0.312) | (0.347) |

# 6 Conclusions

We introduce an endogenous discount factor as in Uzawa (1968) and Schmitt-Grohe and Uribe (2003) into a real-business-cycle setup with Greenwood *et al.* (1988) preferences and augment the model with a detailed government sector. We calibrate the artificial economy to Bulgarian data for the period following the introduction of the currency board arrangement (1999-2016). We investigate the quantitative importance of endogenous discounting for the propagation cyclical fluctuations in Bulgaria. The presence of an endogenous discount factor

improves the model performance against data, and in addition this extended setup dominates the standard RBC model framework with a constant discount factor, *e.g.*, Vasilev (2009).

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