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## The Economics of Two-way Interconnection

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Ulrich Berger

# The Economics of Two-way Interconnection



Ulrich Berger

## **The Economics of Two-way Interconnection**

This book studies the economics of telecommunications networks characterized by two-way interconnection. Special emphasis is put on the role of access charges. Starting from the standard model used in the literature on network competition, the effect of departing from three of this model's less realistic assumptions is investigated. First, call externalities are integrated into the model. Secondly, competition between three or more networks is studied in a dynamic setting. Finally, a local interaction structure between agents is introduced to replace the unrealistic assumption of balanced calling patterns. In each of these cases, some of the conventional wisdom on the role of access charges is overturned by new results.

For his research on the topic of this book Ulrich Berger was awarded the Research Prize of the Vodafone Foundation and the WU Best Paper Award.

Ulrich Berger holds a doctoral degree in mathematics from the University of Vienna and a doctoral degree in economics from Vienna University of Economics. He is at the faculty of the latter's Department of Economics. He also worked on game theory and industrial organization at the IIASA in Laxenburg and the University of Munich.

## The Economics of Two-way Interconnection

# **Forschungsergebnisse der Wirtschaftsuniversität Wien**

Band 13



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Ulrich Berger

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# Chapter 1

## Introduction

### 1.1 Two-Way Interconnection

This work is concerned with the economics of networks characterized by two-way interconnection. The term *interconnection* is widely used in the literature on network industries to describe the linking of different networks. Specifically, for telecommunications networks, the European Community's interconnection directive 97/33/EC defines interconnection in the following way:

*'Interconnection' means the physical and logical linking of telecommunications networks used by the same or a different organization in order to allow the users of one organization to communicate with users of the same or another organization, or to access services provided by another organization.*

The problem with this formulation is that it subsumes two quite different meanings in one word. The usual understanding of interconnection is the one given in the first part of the definition, i.e. *the physical and logical linking of telecommunications networks used by the same or a different organization in order to allow the users of one organization to communicate with users of the same or another organization*. The second part of the definition also allows the term *interconnection* to refer to the linking of telecommunications networks in order to allow the users of one organization to *access services provided by another organization*. This, however, is something rather different. In order to avoid confusion, the type of interconnection addressed in the second part of the definition is usually called *access* to a network,

or, alternatively, *one-way interconnection*. The first part of the definition is then what is often referred to as *two-way interconnection*, and this is also the interpretation of the term interconnection we use in this work.

We do not intend to study the technical problems of interconnection, nor do we address the legal obligations which are partly responsible for the existence of interconnection of networks. We start with the observation that networks, especially telecommunications networks *are* interconnected, and then we ask what are the economic implications of this for the network industry, the single networks, and the end users of the networks. This is the meaning of the title of this book, ‘The economics of two-way interconnection’.

## 1.2 Telecommunications

Why is the market for telecommunications an interesting object of study? What makes it so different from the market for, say, apples? A short answer would be that telecommunications involves all the characteristics which are typical for so-called *network products*. This is not the whole truth, however, and the question deserves a more detailed answer. In the following paragraphs, we provide some keywords which serve to make clear some of the important special features of the market for telecommunications. We start with a simple situation: Imagine Anna wants to talk to Bob, who lives in another town. The simplest way to do this is to make a phone call to Bob. What are the characteristics of the telecommunications market Anna encounters by doing so?

- *Complementarities*: Anna could go and search for a public phone booth, but if she wants to call Bob regularly, she might find it useful to buy a telephone of her own. However, owning a telephone is not enough. Additionally, Anna’s telephone must be connected to an active telephone line which makes access to the public telephone network possible. Anna’s telephone is only useful in connection with the telecommunications services provided by the operator of the public network. Without a telephone, on the other hand, these services are themselves useless. The telephone and the services provided are *complements*.
- *Network externalities*: Even if Anna owns a telephone and is connected to a network, this will only enable her to talk to Bob, if Bob is also

connected and has a telephone. This is true for any person Anna might want to call. Hence the value of the phone and the service to Anna depends strongly on the number of other users which are connected. If Bob buys a phone and subscribes to the network, this increases Anna's willingness to pay for her own services, a positive externality which is called *network externality*.

- *Call externalities*: Anna calls Bob because she likes to talk to him. When Bob's telephone rings, he will most probably answer it, even if he does not know who the caller is. The reason he answers the phone is that his expected utility of doing so is positive. Bob might receive nuisance calls now and then, but usually, and on average, he benefits from being called. The utility he receives from Anna's call is a positive externality known as the *call externality*.
- *Economies of scale*: If Bob lives in the same country as Anna, and if both use a fixed-line telephone for their call, it is most likely that they are connected to the same network. The reason for this is that fixed-line network operators within a country are usually monopolists. Establishing a fixed-line network involves huge sunk costs, basically because it requires wiring the whole country with copper lines and physically connecting each household to the nearest switch of the network via the 'last mile'. Once the network is set up, however, delivering calls through it generates only marginal costs which are quite small. These significant *economies of scale* have led to the view that the telecommunications industry is a 'natural monopoly', and governments have usually licensed only a single company to provide telecommunications services.
- *Interconnection*: If Anna and Bob live in different countries, or if they use a mobile phone for their call, it is quite possible that they will be connected to different networks. The natural monopoly argument does not apply for mobile telecommunications, since it is much easier and less costly to establish a mobile network, where the 'last mile' is bridged via electromagnetic waves rather than via copper cables. However, if Anna and Bob are subscribed to different mobile telecommunications networks, they can only conduct a phone call if these networks are *inter-connected* (in the sense of two-way interconnection as explained above). Since different networks, even if they are interconnected, usually com-



pete with each other for customers, this raises additional problems on the supply side.

- *CPP vs. RPP*: A part of the costs of Anna's call to Bob is borne by the network Bob is connected to. These are particularly the costs of transmitting the call from the point of interconnection to the base station next to Bob's position, and the costs of terminating the call at Bob's mobile phone. There are basically two possibilities for Bob's network to be compensated for these costs. It could either charge Bob for receiving a call on his mobile phone, or it could charge Anna's network, which will then collect (part of) this charge from Anna via increased retail prices.<sup>1</sup> The first payment system is called the 'Receiving Party Pays' (RPP) system, and the second one the 'Calling Party Pays' (CPP) system. CPP is the de facto standard in Europe, while RPP is used in the USA, Canada, and Hong Kong. In this work we concentrate exclusively on the CPP system.
- *Termination-based price discrimination*: If Anna calls Bob on her mobile phone, the price she pays for a call-minute will most likely depend on whether Bob is connected to the same or a different network. In the former case, the call does not leave their network, it is called an *on-net call*. In the latter case, the call is transferred from Anna's to Bob's network at the point of interconnection, a switch where calls are routed into the network where they terminate. In this case we speak of an *off-net call*. If a network's price for on-net calls differs from the price for off-net calls, the network engages in *termination-based price discrimination*. Interestingly, this type of price discrimination introduces another kind of externalities:
- *Tariff-mediated network externalities*: Assume for the moment, that the price of on-net calls is lower than the price of off-net calls. Assume also, that the market is covered, i.e. each consumer is subscribed to some network. Under these circumstances the standard network externalities discussed above are exhausted, the sum of the networks' customers cannot grow any more. Nevertheless, Anna benefits if the market share of her network increases. The reason is that she can reach more of her

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<sup>1</sup>In principle Bob's network could also charge Anna directly, but this possibility is usually ruled out for technical and/or legal reasons, because it would require Bob's network to be able to bill virtually any person who might want to call one of its customers.

calling partners on-net, where it is cheaper to call. Hence Anna benefits (in expectation) whenever some customer of another network switches to her network. This constitutes a positive externality. However, since it does not work directly, but is mediated through the discriminating tariffs, it is called a *tariff-mediated network externality*. It should be mentioned that some economists refuse to call this effect an externality at all, since it relies on the price system to work. However, this is mainly a matter of how strict one defines the term ‘externality’.

- *Balanced calling patterns*: Note that the presence of tariff-mediated network externalities as explained above relies on the implicit assumption that there is a positive probability for Anna to call any other customer. To see this, imagine that Anna exclusively calls Bob, and nobody else. Then, as long as Bob is subscribed to the same network as Anna, she does not care at all about her network’s market share. The tariff-mediated network externalities exist only between Anna and her calling partners. If one assumes, as is regularly done in the literature, that Anna is equally likely to call any other customer, then we speak of *balanced calling patterns*. If calling patterns are balanced, the percentage of all calls terminating on some network is equal to the percentage of all calls originating on this network, and both are equal to this network’s market share. This makes calculations much easier in our models, but the question remains if the assumption of balanced calling patterns is really well-founded.
- *Nonlinear pricing*: Anna and Bob do not only pay for their phone calls, they are also likely to pay a fixed monthly fee for being connected. This means that the pricing scheme is *nonlinear*.<sup>2</sup> If a fixed fee is combined with a per-minute price for calls, we speak of a *two-part tariff*. A network using a two-part tariff has two distinct instruments to compete for market share and generate profit. Note that the fixed fee and the price for a call affect demand rather differently. While demand for subscription in general depends on both the fixed fee and the per-minute charge, a subscribed customer’s call volume is independent of the fixed fee, at least in the absence of wealth effects.

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<sup>2</sup>Total payment as a function of minutes called is still an *affine* linear function, but the term *linear pricing* solely denotes the case where this function is strictly linear, i.e. where there is no fixed fee.

- *Access charges*: If Anna and Bob are subscribed to different networks, then, as mentioned above, the call originating from Anna's mobile phone is switched to Bob's network at some point of interconnection. From there it is transmitted to the base station next to Bob, and the 'last mile' to Bob's mobile phone, where the call terminates, is bridged via electromagnetic waves. This means that parts of the total marginal costs of the phone call are borne by Bob's network. Under CPP, Bob's network will charge Anna's network a per-minute price for terminating Anna's call. This price is sometimes called *termination charge* or *interconnection fee*. Unfortunately, in the academic literature the somewhat misleading term *access charge* is the most commonly used one. While access is something different from interconnection, as discussed in Section 1.1, we will nevertheless, for the sake of continuity, also use the term 'access charge' for what should really be called 'termination charge'.

### 1.3 The Role of the Access Charge

Even if there is a large number of network operators in a telecommunications market, a single network is not subjected to any competition in one particular part of its services, namely in terminating calls to its customers. It is obvious, and seems technically unavoidable, that networks retain a monopoly position with respect to termination of such calls, and since networks can generate profits on incoming calls by raising their access charge above marginal cost, all the regulatory concerns usually associated with monopoly power also arise in these markets.

Should access charges be regulated, and if so, how? Before we address this question within a formal model, we will try to provide some intuition about the role these access charges play for prices, profits, and welfare.

The first thing to notice is that access charges are part of the *perceived marginal costs* of an off-net call. As mentioned earlier, an off-net call generates 'true', technical marginal costs for the originating network up to the point where the call is handled over to the rival network. The remaining part of the marginal costs as perceived by the network originating the call are the access charges paid to the network terminating the call. Since these access charges need not be equal to true marginal costs of termination, the

perceived total marginal costs of an off-net call will in general be different from the true total marginal costs.

When setting its prices, a profit maximizing network will equalize marginal revenue and marginal costs. However, it is the *perceived*, not the true marginal costs, it will take into account in this optimization procedure. Hence end-user prices of a network are (partly) determined by perceived marginal costs, and hence by its rivals' access charges. Particularly, a network will raise its price (for off-net calls, in the case of termination-based price discrimination), if one of the rivals increases its access charge.

Now let us consider what happens if two competing networks are allowed to set their access charges independently, i.e. noncooperatively.

Consider two symmetric networks competing in a covered market. By 'symmetric' we mean that the networks are basically identical, they have the same cost structure, pricing method, quality delivered, and so on. Under some additional assumptions, outlined in the following chapters, there will be a symmetric equilibrium, where these networks set the same access charges, offer the same prices, and share the market equally. With balanced calling patterns, the total number of calls originating from a network and terminating on the other network, i.e. the number of off-net calls, will be the same for both networks. Hence the total access charge payments a network makes to its rival is exactly equal to the total access charge payments it receives. These payments cancelling out, it seems as if profits are completely unaffected by the access charge.

This is not the case, however. Indeed, the belief that access charges play no role is widespread,<sup>3</sup> and has been termed the *bill-and-keep fallacy* by Laffont and Tirole (2000). As we have explained above, access charges have no direct, but an indirect effect on profits through their influence on end-user prices. If a network, let us call it network A, unilaterally increases its access charge, its rivals' average end-user price will go up, and as a consequence, indirect utility of the customers of A's rivals goes down. Since customers compare the net utilities they receive from the available networks in their subscription decision, they will tend to switch to network A, increasing A's market share and profit.

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<sup>3</sup>Armstrong (1998) mentions a paper by the New Zealand Ministry of Commerce supporting this wrong intuition.

The same is true, however, for the rival network. In the end, both networks will increase their access charges, and this may well raise call prices above the monopoly level. This is detrimental to both consumer and producer surplus, and hence calls for regulatory intervention. Note that this effect, which has been called the *raise-each-other's-cost effect*, is similar to the *double marginalization problem* arising in vertically structured industries and well known in the theory of industrial organization.

For these reasons, it is commonly agreed that networks should not be allowed to set their access charges noncooperatively. One way of alleviating the double marginalization problem is to impose *reciprocity* of access charges, i.e. to demand that both networks charge the same unit access fee. This can be achieved by a regulator setting an appropriate reciprocal access charge, or by letting the networks freely negotiate over the access charge, subject only to reciprocity. In many OECD countries, interconnection arrangements are indeed handled in the latter way, with regulatory intervention only if negotiations fail. Now, while collusion over retail prices is illegal in general, cooperative agreement on a reciprocal access charge is not only allowed, but often encouraged. This makes sense only if firms are not able to indirectly collude over retail prices by colluding over the access charge. Unfortunately this is by no means obvious.

## 1.4 Literature Overview

### 1.4.1 Linear Pricing Models

In the second half of the 1990s, serious concerns have been raised in the literature about firms' ability to use a cooperatively determined access charge as a collusion device (see e.g. Brennan (1997)). As noted already by Katz et al. (1995), networks have an incentive to agree on a high (above marginal cost) reciprocal access charge in order to achieve high end user prices. Together with the confirming results from the first explicit models (see below for details on this literature), this has led many researchers to adopt the view that collusion in the retail market is associated with *high* access charges. This view was only slightly clouded by subsequent opposite results arising from refinements of the basic models, which tried to eliminate some of the less realistic assumptions of these models.

The first to show the negative welfare effects of cooperatively determined access charges within an explicit model were Armstrong (1998), Laffont et al. (1998a) — henceforth LRTa — and Carter and Wright (1999). They employ models where two networks are horizontally differentiated in the Hotelling style and compete for customers in linear, nondiscriminating prices. The model of LRTa is by now widely accepted as the “standard model” of two-way interconnection, and most of the subsequent literature uses this model as a starting point. Basic assumptions of LRTa’s model include that consumers do not benefit from receiving calls and that calling patterns are balanced. All these authors conclude that the negotiated access charge may be used as a collusive device and will definitely exceed the marginal cost of access.

### 1.4.2 Nonlinear Pricing Models

If networks may compete in nonlinear prices, e.g. two-part tariffs, this result does no longer hold. As LRTa show, equilibrium profits are independent of the access charge, leaving networks indifferent about the price of interconnection. The intuition is that although usage fees still increase with the access charge, networks can counterbalance the negative impact on market share by lowering the fixed fee. Thus competition remains strong, and the access charge loses its collusive function.

Dessein (2003) studies a model where consumers differ in volume demand or subscription demand. He shows that introducing heterogeneity in volume demand leaves the neutrality of the access charge unaffected. This result is also supported by Hahn (2004). If demand for subscription is elastic, however, some consumers may choose not to subscribe in equilibrium. As Dessein (2003) and Schiff (2002) show, this leads networks to prefer an access charge *below* marginal cost. The reason for this is the emergence of positive network externalities in the absence of full participation.

### 1.4.3 Termination-Based Price Discrimination

The mentioned models do best describe local fixed-line telecommunications networks. With the rise of mobile telecommunications, however, the practice of termination-based price discrimination became apparent. In mobile networks it is commonly observed that different prices are charged for calls



terminating in different networks. Termination-based price discrimination was already studied by Economides et al. (1996). However, their results differ substantially from the results discussed below, since they assume that subscription decisions are made before prices are set, which renders market shares effectively exogenous.

A seminal paper introducing price discrimination into the models mentioned above is Laffont et al. (1998b), henceforth referred to as LRTb<sup>4</sup>. Among other results they show that with linear pricing, the collusive role of the access charge is reduced by the possibility of price discrimination. The reason is that similar to the case of two-part tariffs above, a higher access charge is reflected in a higher off-net price, but the building of market share is not necessarily linked to an increase in the access deficit, since customers can be attracted by lowering the on-net price. However, as opposed to the nondiscriminatory, nonlinear pricing case, the collusive role of a high access charge is not completely removed. Proposition 2 of LRTb states that the access charge still *locally* acts as a collusion device, which means that profits increase locally, if the access charge is increased above marginal cost.

As in the nondiscriminatory case, the corresponding result for nonlinear prices is quite different. Gans and King (2001) demonstrate that networks competing in two-part tariffs with discriminating call prices will negotiate a very *low* (below marginal cost) access charge in order to soften competition. They also conclude that the widespread bill-and-keep arrangements, corresponding to a zero access charge, may be undesirable from the consumers' perspective. As Cherdron (2001) notes, however, their result, predicting off-net prices below on-net prices, is somewhat at odds with what can be observed in existing mobile networks.

Also other authors have asked if bill-and-keep arrangements, which are usually argued to save transaction costs, are actually anticompetitive. A natural benchmark against which the welfare effects of such an agreement can be evaluated is cost-based access pricing, which sets access charges equal to marginal cost, corresponding to conventional regulatory wisdom. As mentioned above, Gans and King (2001) favor cost-based access charges, arguing that bill-and-keep arrangements may be used to soften competition. An opposing position is taken by Cambini and Valletti (2003), who demonstrate

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<sup>4</sup>A summary of the results of LRTa and LRTb is given in Laffont et al. (1997).

that bill-and-keep arrangements may be beneficial due to a positive impact on investments in quality prior to the competition stage.

Given the important characteristics of mobile telecommunications markets outlined above, it is surprising that the literature completely lacks a model of a caller-pays system incorporating nonlinear pricing and termination-based price discrimination as well as call externalities. Laffont et al. (1998b), Gans and King (2001), and Cambini and Valletti (2003) study the case of nonlinear discriminatory pricing, but without call externalities. Kim and Lim (2001), DeGraba (2003), and Jeon et al. (2004) take into account the call externality, but they concentrate on receiver-pays systems (where the importance of call externalities is more obvious). Hahn's (2003) model has nonlinear pricing and call externalities but studies a monopolistic network. Finally, Armstrong's (2002) extensive survey includes a small study of nonlinear pricing in the presence of call externalities, but without price discrimination.

## 1.5 Introducing Call Externalities

Summarizing the above, while under nonlinear pricing networks are either indifferent about the access charge or prefer an access charge below marginal cost, the work concerned with the linear pricing case unanimously suggests that networks will negotiate a high access charge to maximize joint profits.

Subsequently, we will show that actually the opposite might be the outcome of network competition in linear prices, and networks might well make use of a reciprocal access charge *below* marginal cost. This result may look similar to the one of Gans and King (2001), but there is an important difference. While their result has been criticized for being out of line with observed price structures, this does not apply to our findings, at least in the linear pricing case. Access might be sold at a discount, but off-net prices still exceed on-net prices in equilibrium. Moreover, there turns out to be little scope for regulatory intervention against bill-and-keep arrangements. These arrangements might result from collusion, but then they are also welfare improving compared with cost-based access pricing.

However, for competition in two-part tariffs, the Gans and King (2001) result is confirmed if receivers' utility is taken into account. The negotiated access



charge is always below marginal cost, and off-net calls are cheaper than on-net calls.

All of the papers discussed in the introduction share the basic assumption that a call generates utility only for the caller and not for the receiver. In this work we divert from this assumption by introducing call externalities. The obvious point that a call generates utility also for the receiver has been recognized<sup>5</sup>, but nonetheless widely neglected in the literature. Only recently, Kim and Lim (2001), and Jeon et al. (2004) have come up with similar models incorporating a call externality. However, they study a RPP system, where both the caller and the receiver of a call are charged. Note that the receiver of a phone call incurs the opportunity costs of the time the call takes. Hence he must get some strictly positive utility from a call, otherwise he would not answer the call. On the other hand it might be argued that at least on average the utility of the receiver will be smaller than the utility of the caller. Whatever the “real” average magnitude of receivers’ utility, neglecting it is likely to introduce a relevant distortion in the analysis of network competition.

First, however, it can be seen that under nondiscriminatory pricing the analysis of competition remains unchanged<sup>6</sup>. It is clear that volume demand is independent of any call externality. Obviously, nondiscriminating prices also make the subscription decision independent of receivers’ utility. Hence neither subscription nor volume demand or profits are influenced by the level of passive utility. This means that the results derived from the standard model of nondiscriminatory pricing discussed above carry over to the extension we study here. The only deviation from LRTa’s model arises in the judgement of welfare implications. Indeed, neglecting the call externality underestimates social welfare. To implement the social optimum, the price of a call would have to be below marginal cost.

Volume demand stays of course independent of the call externality also with termination-based price discrimination, but the subscription decision is influenced if on-net prices differ from off-net prices. This is because the utility from receiving calls contributes to the positive network externality under on-net prices (say) below off-net prices. An increase in a network’s market share

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<sup>5</sup>DeGraba (2003) suggests that the total utility generated by a call is shared equally between the calling parties. See also the discussion in Hahn (2003).

<sup>6</sup>See also the discussion in Schiff (2001).

raises the number of calls received by (and hence benefits the) subscribers of this network. In their subscription decision, consumers compare the net utilities they receive from joining either network. If a network raises its off-net price, this has two effects. First, the net utility of this network's customers decreases, and second, since these customers' demand for off-net calls falls, also the rival network's customers suffer, because they less frequently enjoy the benefit of being called. This second effect lowers customers' incentives to switch to the rival network. As the access charge, the call externality is reflected in equilibrium prices, which determine profits. Indeed, if the utility of receiving calls is sufficiently high, the second effect explained above becomes so strong that networks will prefer an access *discount* in order to keep the resulting off-net prices below the monopoly price.

This analysis rests on the assumption that profits are directly determined only by prices. Note, however, that in the case of two-part tariffs profits also depend on the fixed fee. As mentioned above, this has a deep impact on the nature of competition. The case of termination-based price discrimination with two-part tariffs is analyzed in chapter 5.2 of Jeon et al. (2004). Although their work is devoted to the RPP system, they include a short study of their model in the absence of reception charge, which of course coincides with a caller pays system. Interestingly, they show that if receivers' utility is high enough (equal to callers' utility), then for any given level of the access charge, the price for off-net calls in a symmetric equilibrium becomes infinite, resulting in *connectivity breakdown*. The intuition for this is the following. Any off-net call made generates utility for the caller and the receiver. However, since only the caller pays for the call, if receivers' utility is high, net surplus is higher for the receiver than for the caller. This means that while raising the off-net price may decrease the direct profit from off-net calls, at the same time it makes the own network more attractive, resulting in an increase in market share. The total effect on profit becomes positive, if receivers' utility is high. Furthermore, if receivers' utility is high enough, the total effect on profit is positive *regardless of the level of the off-net price*. This, of course, means that the only equilibrium has an infinite off-net price.

We conclude that the introduction of call externalities has a strong impact on the outcome of competition in the case of termination-based price discrimination. This is the case we study in Chapters 3 and 4 of this work.

## 1.6 Outline

In Chapter 2, as a starting point, we describe the standard model of LRTb used in the majority of the literature on network competition. In Chapter 3 we extend the standard model with linear pricing by call externalities. We show that some of the conventional wisdom on the collusive role of the access charge is overturned under this extension. The impact of call externalities on competition in the standard model with two-part tariffs is studied in Chapter 4. Here we argue that the traditional reasons put forward against the access pricing practice known as bill-and-keep turn out to be ill-founded once call externalities are taken into account. Chapter 5 departs from the duopoly assumption and studies the case of three or more competing networks. We show that the determination of market shares calls for a dynamic setting, and that this raises several serious problems like nonexistence or multiplicity of stable equilibria, or nonconvergence of market shares even for fixed prices. In Chapter 6 we introduce a local interaction structure between agents, thereby giving up the standard but unrealistic assumption of balanced calling patterns. For the usually observed case of on-net prices below off-net prices we show that this typically generates a multitude of consumer equilibria, which creates severe problems for the prediction of market shares.

# Chapter 2

## The Basic Model

In this section we introduce the model. It is based on the model of LRTb, but for simplicity we will neglect the fixed costs (which does not change the results qualitatively). On the other hand, we extend the model by adding the call externality.

### 2.1 Cost and Price Structure

Imagine a telecommunications market which is served by two interconnected networks labeled 1 and 2. Both networks have full coverage, i.e. every consumer can be reached by all other consumers, no matter which network they are subscribed to. The marginal cost of originating or terminating a call is  $c_0 > 0$ , and the total marginal cost of a call is

$$c = 2c_0 + c_1,$$

where  $c_1 \geq 0$  is the marginal cost of transmitting a call from the originating to the terminating end. The reciprocal unit access charge is  $a \geq -(c_0 + c_1)$ . Note that access charges may be negative. Setting a negative access charge corresponds to subsidising termination. The subsidy cannot be larger than the costs of originating and transmitting a call, however, since otherwise a network could make unlimited profits by installing a computer which permanently calls into the rival network.

Networks either compete in linear prices  $p_{ii}$  (for on-net calls within network  $i$ ) and  $p_{ij}$  (for off-net calls originating in network  $i$  and terminating in network  $j$ ), or, in the case of two-part tariffs, also in the fixed fee  $F_i$ .

## 2.2 Subscription Decision and Demand

On the demand side there is a large number of consumers, formally a continuum. A consumer can be member of at most one network. From the consumers' point of view the networks are horizontally differentiated, and this differentiation is of the Hotelling type. The networks are located at the extreme points of the unit interval  $[0, 1]$ , and each consumer is located at some address  $x \in [0, 1]$ . The total mass of consumers, normalized to 1, is distributed uniformly on this interval. The degree of horizontal differentiation is measured by a parameter  $t$  corresponding to the "transport costs". This means that a consumer located at address  $x$  faces a disutility of  $t|x - x_i|$  if he subscribes to network  $i$ , where  $x_1 = 0$  and  $x_2 = 1$  are the locations of the two networks.<sup>1</sup>

Consumers have homogeneous preferences for calls to other consumers. Calls to distinct other consumers constitute independent goods and total utility is assumed to be additively separable. The utility from an active call of length  $q$  is given by  $u(q)$ , where  $u' > 0$  and  $u'' < 0$ . For technical reasons we also assume that the Inada conditions  $\lim_{q \rightarrow 0} u'(q) = \infty$ ,  $\lim_{q \rightarrow \infty} u'(q) = 0$  are fulfilled, guaranteeing strictly positive and finite demand for all positive prices. It is helpful here to imagine that each consumer makes exactly one call to each other consumer, and only the length of a call is variable.

Consumers also derive utility from receiving calls. We denote the *passive* utility of receiving a call of length  $q$  by a strictly increasing and strictly concave function  $\bar{u}(q)$  with the same qualitative properties as  $u(q)$ .

A consumer with income  $y$ , subscribed to network  $i$  and located at  $x$ , making a call of length  $q_{out}$  to some other consumer and receiving a call of length  $q_{in}$  from some consumer, enjoys a total utility of

$$v_0 + y + u(q_{out}) + \bar{u}(q_{in}) - t|x - x_i|,$$

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<sup>1</sup>It is not necessary to take the disutility interpretation of the Hotelling transport costs literally,  $t$  may simply be interpreted as a parameter measuring the intensity of price competition.

where  $v_0$  is some fixed surplus from being connected, large enough to guarantee full participation, i.e. to prevent consumers from not subscribing in equilibrium.

The timing is as follows. First, networks cooperatively choose a reciprocal access charge, then they (noncooperatively) set on- and off-net prices, and the fixed fee in case of two-part tariffs. Consumers choose a network to subscribe to and then they choose the length of their on- and off-net calls.

Let

$$q(p) = \operatorname{argmax}_q \{u(q) - pq\}$$

be the consumer's demand, writing  $q_{ij}$  short for the demand for on- and off-net calls  $q(p_{ij})$ . Denoting by

$$v(p) = \max_q \{u(q) - pq\}$$

net surplus, under price discrimination with given market shares  $\alpha_1$  and  $\alpha_2$ , network  $i$  offers its subscribers a total net surplus of<sup>2</sup>

$$w_i = \alpha_i[v(p_{ii}) + \bar{u}(q_{ii})] + \alpha_j[v(p_{ij}) + \bar{u}(q_{ji})] - F_i. \quad (2.1)$$

Letting

$$h_{ij} = v(p_{ij}) + \bar{u}(q_{ji}),$$

we may write

$$w_i = \alpha_i h_{ii} + \alpha_j h_{ij} - F_i. \quad (2.2)$$

## 2.3 Existence and Stability of Equilibria

### 2.3.1 Existence of Consumer Equilibria

Imagine prices are fixed, and consumers have to decide which network to subscribe to. Due to the tariff-mediated network externalities and the call externality, a consumer's utility is influenced by all other consumers' subscription decisions. His decision problem is therefore not a simple optimization, but a strategic one. For fixed prices, consumers are actually playing a multi-person

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<sup>2</sup>Throughout this work, let  $\{i, j\} = \{1, 2\}$  if not indicated otherwise.

game. Hence their subscription decisions will depend on their expectations about future market shares, i.e. on the subscription decisions of all other consumers. In this situation, the natural solution concept is the Nash equilibrium of the corresponding game. To distinguish between this game among consumers and the extended two-stage extensive-form game where networks decide on their prices prior to consumers' subscription decisions, we call a Nash equilibrium of the game among consumers a *consumer equilibrium*. Thus, a consumer equilibrium is given if the market shares are such that no consumer has an incentive to unilaterally deviate from his subscription decision and switch to the other network.

A consumer equilibrium need not be unique. To see this, suppose the on-net prices as well as the off-net prices are the same for both networks, but the on-net price is below the off-net price. Suppose also, that the degree of differentiation between networks is small. If all consumers expect all others to subscribe to network 1, then it is optimal for them also to subscribe to network 1. The same is true for network 2, however. In this case the game turns out to be a coordination game, where it is an equilibrium for consumers to coordinate on one of the networks. If this happens, then we say that the market is *cornered* by one network.

If the market is cornered, i.e. if  $\alpha_i = 1$  for some  $i$ , then even the consumer with the weakest preferences for network  $i$  (the consumer located at  $x_j$ ) chooses to subscribe to this network. If the difference between on-net prices and off-net prices is not large enough to overcome the horizontal differentiation for this extreme consumer, the result of the subscription decisions cannot be a cornered market. Both networks will have a strictly positive market share in any consumer equilibrium then. Such an outcome is called a *shared market equilibrium*.

If there is a shared market equilibrium with  $0 < \alpha_i < 1$ , then the consumer located at  $x = \alpha_1$  is just indifferent between the networks. The market share  $\alpha_1 = \alpha$  (and  $\alpha_2 = 1 - \alpha$ ) in a shared market equilibrium can thus be calculated from the indifference condition

$$w_1 - t\alpha = w_2 - t(1 - \alpha),$$

and reads

$$\alpha = \frac{1}{2} + \sigma(w_1 - w_2). \quad (2.3)$$

Here,  $\sigma = 1/2t$  measures the degree of substitutability between the two networks. If  $t \rightarrow 0$ , networks become perfect substitutes, whereas for  $t \rightarrow \infty$  substitutability vanishes and the networks become local monopolies.

Note that (2.3) gives the market shares only implicitly, since these occur also in  $w_i$  on the right-hand side. To calculate the market shares explicitly, we insert from (2.2), setting  $\alpha = \alpha_1 = 1 - \alpha_2$ . Solving for  $\alpha$  then yields

$$\alpha = \frac{H_1}{H_1 + H_2}, \quad (2.4)$$

with

$$H_i = 1/2 + \sigma(h_{ij} - h_{jj} + F_j - F_i).$$

Obviously, for a shared market equilibrium to exist, i.e. for  $\alpha > 0$ ,  $H_1$  and  $H_2$  must have the same sign:  $H_1 H_2 > 0$ .

### 2.3.2 Stability of Consumer Equilibria

As already discussed above, there may be multiple consumer equilibria for given prices. However, some of these can usually be eliminated by pointing out that an economically meaningful equilibrium has to be *stable* with respect to an appropriate adjustment dynamic.

In the case where the consumers' game is a symmetric coordination game, there are two cornered market equilibria. However, there is also a third equilibrium, where each consumer subscribes to the network he is located closer to. Note that in this case the networks share the market equally, and net surplus offered to the consumers is the same for both networks. Hence each consumer's decision is optimal, minimizing his transport costs. However, this shared market equilibrium is highly unstable. To see this, consider a slight deviation in consumers' expectations about the market share. This will lead the marginal consumers to switch to the network with the higher market share. As a consequence, expectations are further biased in favor of this network, leading even more consumers to subscribe to it. The outcome of this positive feedback loop between expectations and subscription decisions ultimately leads to all consumers subscribing to one network, resulting in a cornered market. This phenomenon is called *market tipping* in network economics, and is commonly observed in the presence of positive network



externalities. We will analyze the dynamics of market shares in more detail in Chapter 5.

Since there are two pure strategies for each consumer, corresponding to subscription to one of the two networks, we obtain the general result (see also LRTb), that generically there are either three consumer equilibria, namely the two cornered market outcomes and an unstable shared market equilibrium, if both  $H_1$  and  $H_2$  are negative, or a unique, stable consumer equilibrium, which is a cornered market one if  $H_1 H_2 < 0$ , and a shared market equilibrium if both  $H_1$  and  $H_2$  are positive. That the number of equilibria is generically three or one is a consequence of the odd-number theorem for equilibria of finite games with generic payoffs. The stability of a unique equilibrium follows from the one-dimensionality of the state space. More on the properties of dynamics in such games can be found e.g. in Hofbauer and Sigmund (1998).

In the following we will assume that the consumer equilibrium is unique, i.e. a stable shared market equilibrium exists. We will see later on, that this assumption is justified if the degree of substitutability between networks is low enough. The attractive features of such an outcome are its uniqueness and its stability. This guarantees, that the extension of the consumers' game by a pricing stage prior to the subscription decision yields an extensive-form game with a unique subgame perfect equilibrium.

### 2.3.3 Network Equilibria

Imagine prices (including the fixed fee) are set and a corresponding stable consumer equilibrium has been realized. If in this situation neither network can gain by unilaterally changing its prices or fixed fee (taking into account the dependence of consumer equilibria on these values), then these values constitute what we call a *network equilibrium*. More generally, a network equilibrium is a subgame perfect equilibrium of the game between consumers and both networks, where networks simultaneously set prices in the first stage and consumers simultaneously choose a network to subscribe to in the second stage, having observed the prices set by networks. Since the networks are assumed to be identical in their cost structure, for the remainder of this work we restrict ourselves to *symmetric* network equilibria. These are network equilibria where the on-net and the off-net prices as well as the fixed fee

of the networks are the same, and the corresponding consumer equilibrium divides the market equally between the two networks, i.e.  $\alpha = 1/2$ .



# Chapter 3

## Linear Pricing

In this chapter we examine the case of linear pricing, which means that networks do not use a fixed fee, and therefore the total payment for a call is a linear function of minutes called. So in this chapter let  $F_1 = F_2 \equiv 0$ .

### 3.1 First Order Conditions

In order to analyze networks' pricing decisions, we first have to derive their objective function, i.e. the profit functions. For given prices and a corresponding stable consumer equilibrium  $\alpha$ , profit of network 1 is given by

$$\pi_1 = \alpha^2(p_{11} - c)q_{11} + \alpha(1 - \alpha)[(p_{12} - c)q_{12} + (a - c_0)(q_{21} - q_{12})],$$

and an analogous equation holds for  $\pi_2$ .

If we write

$$M_{ij} = [p_{ij} - c(1 + m)]q_{ij} + mcq_{ji}$$

for the *unit profit* of network  $i$  (the profit a single customer of network  $i$  generates with one active call to and one passive call from network  $j$ ), denoting by

$$m = (a - c_0)/c > -1$$

the (relative) markup on access, profit of network  $i$  can also be written in the form

$$\pi_1 = \alpha^2 M_{11} + \alpha(1 - \alpha)M_{12}.$$

Taking into account that  $M_{ii}$  depends only on  $p_{ii}$ , the first order conditions for a shared market equilibrium are given by

$$\begin{aligned}\frac{\partial \pi_1}{\partial p_{11}} &= 2\alpha \frac{\partial \alpha}{\partial p_{11}} M_{11} + \alpha^2 \frac{\partial M_{11}}{\partial p_{11}} + \frac{\partial \alpha}{\partial p_{11}} (1 - 2\alpha) M_{12} = 0, \\ \frac{\partial \pi_1}{\partial p_{12}} &= 2\alpha \frac{\partial \alpha}{\partial p_{12}} M_{11} + \alpha(1 - \alpha) \frac{\partial M_{12}}{\partial p_{12}} + \frac{\partial \alpha}{\partial p_{12}} (1 - 2\alpha) M_{12} = 0,\end{aligned}$$

and the respective equations for network 2.

At a symmetric shared market equilibrium, where  $p_{11} = p_{22}$ ,  $p_{12} = p_{21}$ , and  $\alpha = 1/2$ , the first order conditions for network  $i$  read

$$\frac{\partial \alpha_i}{\partial p_{ii}} M_{ii} + \frac{1}{4} \frac{\partial M_{ii}}{\partial p_{ii}} = 0, \quad \frac{\partial \alpha_i}{\partial p_{ij}} M_{ii} + \frac{1}{4} \frac{\partial M_{ij}}{\partial p_{ij}} = 0.$$

Inserting from (2.4), rearranging terms, and with a little abuse of notation treating  $\bar{u}(q_{ij})$  as an indirect utility function  $\bar{u}(q(p_{ij}))$  of  $p_{ij}$ , we get

$$\frac{\partial M_{ij}}{\partial p_{ij}} = \frac{\partial M_{ii}}{\partial p_{ii}} \frac{(v' - \bar{u}')(p_{ij})}{(v' + \bar{u}')(p_{ii})}, \quad (3.1)$$

$$\frac{\partial M_{ii}}{\partial p_{ii}} = -\sigma M_{ii} \frac{(v' + \bar{u}')(p_{ii})}{H_i}. \quad (3.2)$$

What can we infer from these equations about the prices in a stable shared market equilibrium? First, note that  $M_{ii}$ , the simple unit profit  $(p_{ii} - c)q_{ii}$ , is positive for  $p_{ii} > c$ , and upward sloping for  $p_{ii} < p^M$ , where  $p^M$  denotes the monopoly price (for marginal cost  $c$ )

$$p^M = \operatorname{argmax}_p \{(p - c)q(p)\}.$$

We also know that  $v' + \bar{u}' < 0$  and that  $H_i$  must be positive for the shared market equilibrium to be stable. From (3.2) then follows that the unit profit  $M_{ii}(p_{ii})$  has the same sign as its derivative. Hence, necessarily,  $c < p_{ii} < p^M$ . In this sense the equilibrium on-net price is “well-behaved”. This need not be the case for the off-net price. As equation (3.1) suggests, the sign of  $\partial M_{ij}/\partial p_{ij}$  depends on the sign of  $v' - \bar{u}'$ , which may well be positive if marginal passive utility is high.

## 3.2 Constant Elasticity of Demand

To be a bit more specific, we now invoke the explicit utility function

$$u(q) = \frac{q^{1-1/\eta}}{1-1/\eta},$$

with  $\eta > 1$ , which was also used by LRTb and yields the constant elasticity demand function

$$q(p) = p^{-\eta},$$

indirect utility

$$u(q(p)) = \frac{\eta}{\eta-1} p^{1-\eta},$$

net surplus

$$v(p) = \frac{1}{\eta-1} p^{1-\eta},$$

and a monopoly price of

$$p^M = \frac{\eta c}{\eta-1}.$$

Furthermore, for simplicity we assume that the utility from passive calls is a fixed fraction  $\beta \geq 0$  of the utility from active calls:

$$\bar{u}(q) = \beta u(q).$$

With these specifications, we can analyze our model in more detail. Inserting in (3.1) and (3.2) and rearranging terms, the first order conditions for network 1 can be expressed by the following system of equations.

$$p_{12}^{-1} = \frac{1}{1+m} \left( \frac{1}{p^M} \frac{2\beta\eta}{1+\beta\eta} + \frac{1-\beta\eta}{1+\beta\eta} p_{11}^{-1} \right), \quad (3.3)$$

$$p_{12}^{-1} = \left[ \frac{p^M}{\eta(p^M - p_{11})p_{11}^{\eta-1}} - \frac{\eta-1}{2\sigma(1+\beta\eta)} \right]^{\frac{1}{\eta-1}}. \quad (3.4)$$

We have intentionally written these equations so as to describe the reciprocal value of the off-net price as a function of the reciprocal value of the on-net

price. This allows us to draw the graphs of the two functions, and find all symmetric candidate equilibria as points of intersection of the corresponding curves.

The next proposition establishes the existence of a unique, stable, symmetric equilibrium for low substitutability. The proof relies on the quasi-concavity of the profit function in the limit as  $\sigma \rightarrow 0$ . It is similar to the proof of Proposition 1 in LRTb. Note, however, that the result is slightly different from LRTb's. Indeed, the call externality prevents the existence of equilibrium in the case of too high substitutability even for  $a = c_0$ , which is not the case in LRTb's model. For example, if  $a = c_0$ , i.e.  $m = 0$ , and  $t \rightarrow 0$ , i.e.  $\sigma \rightarrow \infty$ , the curve given by (3.4) admits its minimum at  $p_{11} = p_{12} = c$ . For any positive value of  $\beta$ , however, (3.3) yields  $p_{12} > c$  at  $p_{11} = c$ , and hence there is no point of intersection in the relevant region for large enough values of  $\sigma$ .

**Proposition 1** *For given access charge  $a$ , if  $\sigma$  is small enough, there exists a unique, stable, symmetric network equilibrium. Its price constellation is given by the intersection of (3.3) and the downward sloping part of (3.4).*

*Proof:* Consider the case  $\sigma \rightarrow 0$ . Then the networks are monopolies and the prices are at their respective monopoly levels. Graphically, (3.4) degenerates to a vertical line at  $p_{11} = p^M$ , intersecting (3.3) in  $p_{12} = (1 + m)p^M$ . This symmetric candidate equilibrium is thus unique and stable (since  $H_i = 1/2 > 0$ ). Moreover, the market shares become constant for  $\sigma \rightarrow 0$ . Hence, given the candidate equilibrium values of  $p_{22}$  and  $p_{21}$ , network 1's profit is

$$\pi_1(p_{11}, p_{12}) = \frac{1}{4}[(p_{11} - c)q_{11} + (p_{12} - c(1 + m))q_{12} + mcq_{21}].$$

This function is quasi-concave in  $(p_{11}, p_{12})$ , hence  $(p^M, (1 + m)p^M)$  is its unique maximum. For positive values of  $\sigma$  the slope of (3.4) becomes finite, which means that the candidate equilibrium on-net price falls below the monopoly price. The candidate equilibrium remains unique for small values of  $\sigma$ , and by continuity of  $H_i$  in  $\sigma$  it remains stable. Also, by continuity of the market share in prices and in  $\sigma$ , network 1's profit function remains quasi-concave. Hence the second order conditions are fulfilled for low substitutability. *QED*

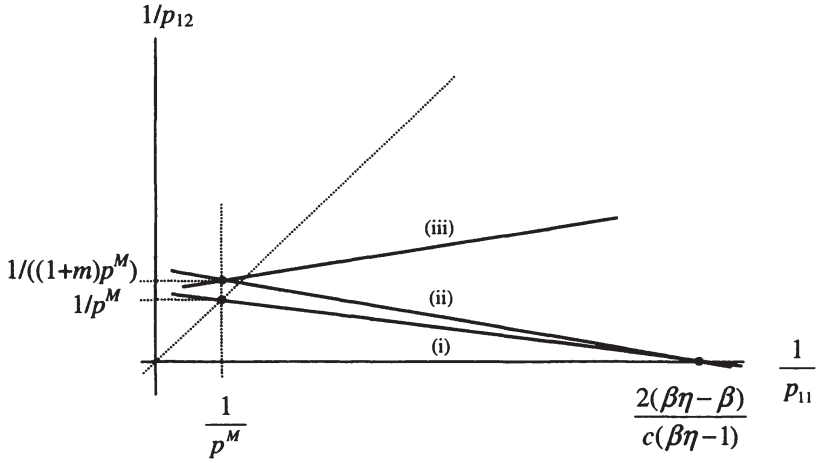


Figure 3.1: Three different positions of the line given by (3.3): (i)  $a = c_0$  and  $\beta\eta > 1$ , (ii)  $a < c_0$  and  $\beta\eta > 1$ , (iii)  $a < c_0$  and  $\beta\eta < 1$ .

In the following we will analyze the graphs of (3.3) and (3.4) more closely, allowing us to derive quickly and easily a variety of comparative statics results.

### 3.3 Graphical Analysis

Let us first have a closer look at (3.3). The right hand side of this equation is an affine linear function of  $p_{11}^{-1}$ , which depends on the parameters  $m$ ,  $\eta$ , and  $\beta$ , but not on  $\sigma$ . Its slope decreases with  $\beta$ , falling from  $(1+m)^{-1}$  for  $\beta = 0$  to zero for  $\beta = 1/\eta$  and approaching  $-(1+m)^{-1}$  for  $\beta \rightarrow \infty$ . At the monopoly price  $p_{11} = p^M$ , we have

$$p_{12}^{-1} = \frac{1}{(1+m)p^M},$$

which is independent of  $\beta$ . Graphically this means that by increasing the relative importance  $\beta$  of passive utility, the line in the  $p_{11}^{-1}$ - $p_{12}^{-1}$ -plane given by (3.3) is rotated clockwise around the point  $\left(\frac{1}{p^M}, \frac{1}{(1+m)p^M}\right)$ , see Figure 3.1.



Note that without the call externality, i.e. for  $\beta = 0$ , equation (3.3) reduces to

$$p_{12} = (1 + m)p_{11},$$

the *proportionality rule* from LRTb. For  $\beta\eta = 1$ , the line (3.3) is horizontal at  $p_{12} = (1 + m)p^M$ .

If we increase the access charge  $a$ , and hence the markup  $m$ , holding all other parameters constant, the line (3.3) rotates clockwise (if its slope is positive,  $\beta\eta < 1$ ) or counterclockwise (if its slope is negative,  $\beta\eta > 1$ ) around the point  $\left(\frac{2(\beta\eta - \beta)}{c(\beta\eta - 1)}, 0\right)$ , where it intersects the horizontal axis. In both cases the equilibrium moves downwards along the curve (3.4). It cannot reach the horizontal axis, however, since the point of intersection of (3.3) with this axis is always either on the negative side or to the right of  $2/c$ , i.e. outside the relevant region  $c < p_{11} < p^M$ . Hence, there is no scope for connectivity breakdown, meaning  $p_{12} \rightarrow \infty$ , contrary to Jeon et al.'s (2004) result for the nonlinear pricing case.

Turning to (3.4), we can see that this equation does not involve  $a$ , the access charge. Whenever  $1/(\eta - 1)$  is not an integer, the right hand side of (3.4) is defined only if the expression in square brackets is nonnegative. The second term of this expression is a negative constant, it does not depend on  $p_{11}$ . The first term is positive for  $p_{11} < p^M$  and — viewed as a function of  $p_{11}^{-1}$  — downward sloping from its vertical asymptote at  $p_{11} = p^M$  to its minimum at  $p_{11} = c$ , see Figure 3.2. For  $p_{11}^{-1} > c^{-1}$  the function given by (3.4) is strictly increasing and unbounded, its slope converging to  $\eta^{-1}/(\eta - 1)$  for  $p_{11}^{-1} \rightarrow \infty$ . Furthermore, this function is convex at least for values of  $p_{11}$  slightly below  $p^M$ . The second term in square brackets shifts the curve up (for  $\sigma \rightarrow \infty$ ) or down (for  $\sigma \rightarrow 0$ ).

Since (3.4) has a negative slope in the relevant region  $c < p_{11} < p^M$ , there exists at most one point of intersection with (3.3), if the slope of this line is nonnegative, i.e. if  $\beta\eta \leq 1$ . If  $\beta$  exceeds  $1/\eta$ , the slope of (3.3) is negative, and there exist two points of intersection. However, the second point is outside the relevant region if  $\sigma$  is small.

As announced above, we concentrate exclusively on the case where substitutability is low enough to guarantee existence of a unique stable equilibrium.

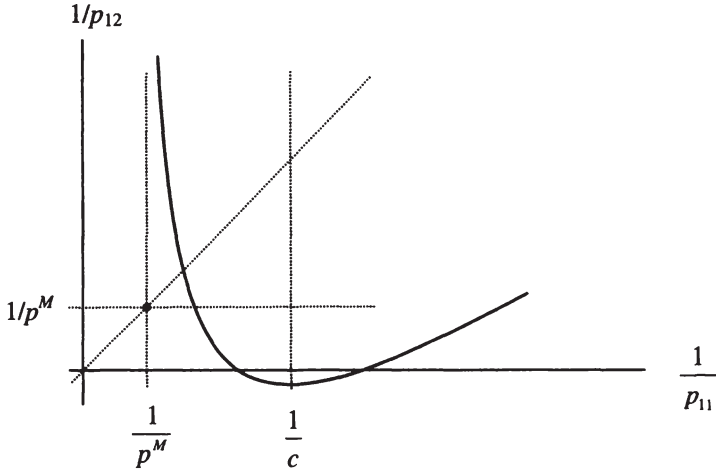


Figure 3.2: The curve given by (3.4).

We then ask, in which way the equilibrium depends on the parameters of the model. All the results are derived using the graphical analysis.

### 3.4 Comparative Statics

The next lemma shows that while the on-net price always decreases with the substitutability parameter  $\sigma$ , the direction of movement of the off-net price depends on the strength of the call externality and on the elasticity of demand. On the other hand, an increase in the access charge always lowers the on-net price and raises the off-net price.

**Lemma 1** (i) *The on-net price decreases with  $\sigma$  and the off-net price decreases with  $\sigma$  if  $\beta\eta < 1$ , increases with  $\sigma$  if  $\beta\eta > 1$ , and is constant at  $p_{12} = (1 + m)p^M$  if  $\beta\eta = 1$ .*

(ii) *The on-net price decreases in  $a$ , while the off-net price increases in  $a$ .*

*Proof:* An increase in  $\sigma$  shifts the graph of (3.4) upwards and does not influence the graph of (3.3). The point of intersection thus moves to the right, i.e.  $p_{11}^{-1}$  increases. The vertical direction of movement depends on the

slope of (3.3). If  $\beta\eta < 1$  (this includes the LRTb case  $\beta = 0$ ), the slope is positive, so also  $p_{12}^{-1}$  increases. If  $\beta\eta > 1$  the slope is negative and the intersection point moves down, and if  $\beta\eta = 1$  the line is horizontal at

$$p_{12}^{-1} = [(1 + m)p^M]^{-1}.$$

Increasing  $a$  or, equivalently,  $m$ , shifts the line (3.3) downwards. Since (3.4) slopes downward in the relevant region, the point of intersection moves down and to the right. This means  $p_{11}$  falls and  $p_{12}$  rises. *QED*

Part (ii) of the lemma appears to contradict a result of LRTb, since the case of no call externality is not excluded. On p. 48 they state that the off-net price may *decrease* in  $a$  if  $\sigma$  is not small enough, and give a numerical example for this. However, the values they provide ( $\eta = 2$  and  $\sigma = c = m = 1$ ) lead to the candidate equilibrium prices  $p_{11} = 1 = c$  and  $p_{12} = 2$ . A small increase in  $a$  then does indeed decrease the off-net price, but simultaneously the on-net price falls below marginal cost and in this region any candidate equilibrium is *unstable* and will therefore not be realized. In the region  $c < p_{11} < p^M$ , where the consumer equilibrium is stable, (3.4) is strictly decreasing and hence the off-net price inevitably rises with the access charge.

In contrast to the result in LRTb, more substitutability exerts upward pressure on the off-net price, if  $\beta$  is large enough. Intuitively, if the call externality induced negative effect of an increasing off-net price on the rival's customers is large, higher substitutability creates incentives for the networks to exploit this effect and raise the off-net price while lowering the on-net price to compensate their own customers.

### 3.5 The Collusive Role of the Access Charge

Part (ii) of Lemma 1 states that varying the access charge results in the equilibrium prices moving in opposite directions. We know that the equilibrium on-net price is always below the monopoly price. If this is also the case for the off-net price, the impact on profits of varying the access charge is ambiguous.<sup>1</sup> If, however, the off-net price is above the monopoly price, both

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<sup>1</sup>In a symmetric equilibrium, access charges payed and received cancel out. Thus, the relevant monopoly price for off-net calls is based on technical marginal costs  $c$ , not on

prices will move towards this monopoly price (and hence raise profits) if and only if the access charge is lowered.

Imagine now that  $\beta\eta > 1$ . This is not an unrealistic case, since  $\eta > 1$  and  $\beta$  may well be only slightly below 1. The slope of (3.3) is then negative, and for  $\sigma > 0$  we have  $p_{12} > (1+m)p^M$  in equilibrium. Now let the access charge equal marginal termination cost, so  $m = 0$ . Then the off-net price exceeds the monopoly price, and we have the situation described above. In order to maximize equilibrium profits, both networks will negotiate an access charge  $a$  below  $c_0$ .

If  $\beta\eta = 1$ , the equilibrium off-net price is  $(1+m)p^M$ , independently of  $\sigma$ . For  $a = c_0$  then  $p_{12}$  is at the monopoly level, while  $p_{11}$  is below  $p^M$ . Starting from these values, a small decrease in  $a$  raises  $p_{11}$  towards the monopoly price and thereby has a positive first-order effect on profits from on-net calls, but only a second-order (negative) effect on profits from off-net calls. In sum, profits rise. By continuity this continues to hold if  $\beta\eta$  is not too far below 1. This shows that networks may prefer an access discount even for  $\beta\eta < 1$ . For very low values of  $\beta$ , of course, this need not be the case.

Graphically, this can easily be seen if we keep in mind that since (3.4) is independent from the access charge, networks can only shift the line (3.3) up or down by varying the access charge. Thereby they can select any point on (3.4), subject to the restriction  $m > -1$ . Maximizing profits, they will choose the point where their isoprofit curve is tangent to (3.4), see Figure 3.3. The point of tangency is unique, at least if  $\sigma$  is not too large, since (3.4) is convex in the vicinity of  $p_{11} = p^M$  and the equilibrium profit function is quasi-concave in equilibrium prices (the upper-contour sets of the isoprofit curves are convex), peaking at the monopoly point  $(1/p^M, 1/p^M)$ . It follows immediately that the point of tangency will lie northeast from the monopoly point, as illustrated in Figure 3.4. This means that with the negotiated profit-maximizing access charge, both on- and off-net prices are smaller than the monopoly price. If the slope of (3.3) is negative or only slightly positive, of course, this implies that this line intersects  $\{p_{11} = p^M\}$  above the monopoly point. Hence

$$[(1+m)p^M]^{-1} > (p^M)^{-1},$$

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perceived marginal costs  $(1+m)c$ , and coincides with the monopoly price  $p^M$  for on-net calls.

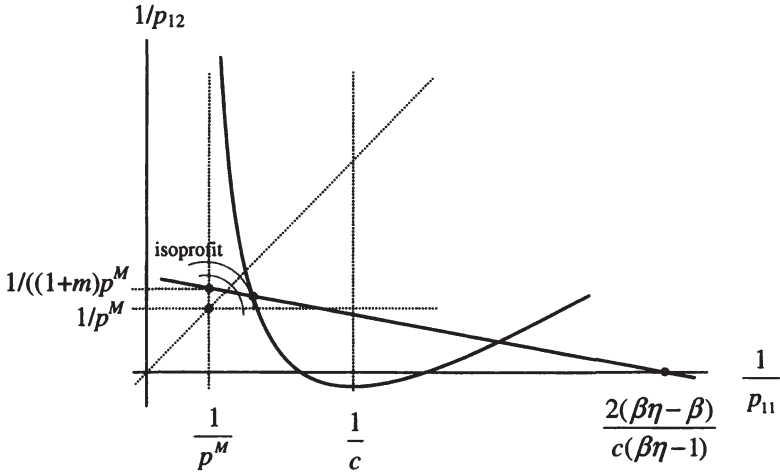


Figure 3.3: The collusive choice of  $a$ , illustrated for  $\beta\eta > 1$ .

or  $m < 0$ . This analysis proves the first part of the next proposition.

**Proposition 2** *Fix  $\sigma > 0$  small enough. There exists  $0 < k < 1$  such that if  $\beta\eta > k$ , networks will agree on an access discount, if  $\beta\eta < k$ , networks will negotiate an access markup, and if  $\beta\eta = k$ , networks will agree on  $a = c_0$ .*

The case  $\beta = 0$  is the case without passive utility, and we could in principle just refer to Proposition 2 of LRTb for the proof. In this proposition they state that for small  $\sigma > 0$  (and for  $\beta = 0$ ) the profit maximizing access charge exceeds  $c_0$ . While this statement turns out to be true, unfortunately their proof is flawed. In their proof, LRTb (p. 49) argue that for small  $\sigma > 0$  their Lemma 2 shows that both on-net and off-net prices increase with the access charge. From this they infer that starting from  $a = c_0$ , a small increase in the access charge raises both prices toward the monopoly level and therefore leads to higher profits. However, actually their Lemma 2 (correctly) states that for small  $\sigma > 0$  the on-net price *decreases* in  $a$ . Hence it is not obvious that an increase in  $a$  does indeed raise profits.

In the following we give the correct version of the proof.

*Proof:* It suffices to show that networks will negotiate a markup on access if  $\beta = 0$ . Given the analysis in the last paragraph, the second and third part

of Proposition 2 then follow immediately from continuity of the negotiated access charge in  $\beta\eta$  and from the intermediate value theorem, respectively.

Note, that for  $a = c_0$  and  $\beta = 0$ , the line (3.3) is the diagonal  $\{p_{12} = p_{11}\}$ . By symmetry of the equilibrium profit function in  $p_{11}$  and  $p_{12}$ , the slope of the isoprofit curves is equal to  $-1$  along the diagonal. The slope of (3.4) at the intersection with the diagonal, on the other hand, converges to  $-\infty$  as the point of intersection approaches the monopoly point, i.e. as  $\sigma \rightarrow 0$ . Thus, for small  $\sigma$  the point of tangency is below the diagonal (see Figure 3.4), where  $p_{11} < p_{12}$ , and by the proportionality rule,  $m > 0$ , i.e. a markup on access, is a necessary condition for this. *QED*

As noted, for small  $\sigma$  the profit maximizing point of tangency lies below the diagonal. Since networks will choose an access charge which lets this point become an equilibrium, we obtain the following corollary:

**Corollary 1** *If  $\sigma$  is positive but small and networks may cooperatively determine the access charge, then the resulting equilibrium prices will show a markup on off-net calls.*

### 3.6 The Socially Optimal Access Charge

From the social viewpoint, the optimal access charge is the access charge that maximizes welfare, the sum of profits and consumer surplus, in equilibrium:

$$W(p_{11}, p_{12}) = \frac{1}{2}[(1 + \beta)u(q_{11}) - cq_{11} + (1 + \beta)u(q_{12}) - cq_{12}]. \quad (3.5)$$

To maximize welfare, the caller would have to be induced to extend the length of his calls up to the point where marginal *total* utility created equals marginal cost. This means

$$(1 + \beta)u'(q_{ij}) = c$$

and is induced by a price of  $p_{ij} = (1 + \beta)^{-1}c$ . Of course these prices cannot be sustained in an equilibrium, since they are below marginal cost for  $\beta > 0$ .

Assume a benevolent regulator can set an arbitrary access charge subject to the technical constraint  $a > -c_0$ . By symmetry, the iso-welfare curves surrounding the unconstrained optimum have a slope of  $-1$  along the diagonal

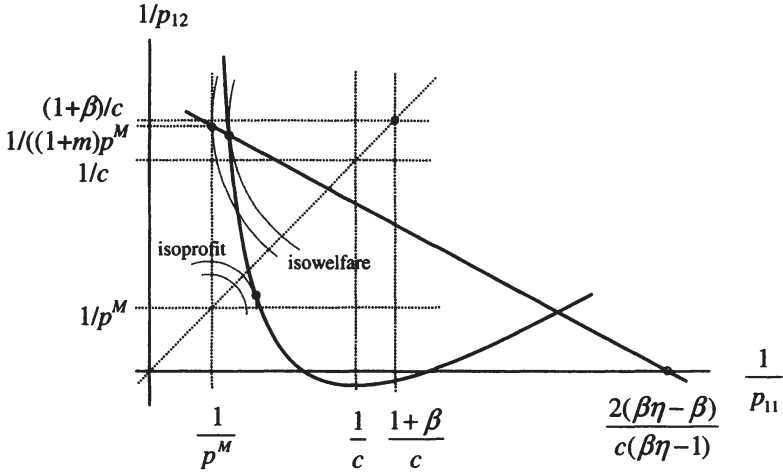


Figure 3.4: The socially optimal choice of  $a$ , illustrated for  $\beta\eta > 1$ .

$\{p_{11} = p_{12}\}$ . Since the slope of (3.4) at the intersection with the diagonal is smaller than  $-1$  for small  $\sigma$ , we can conclude that for small  $\sigma$  the point of tangency of (3.4) and the iso-welfare curves lies above the diagonal, and therefore also above the profit maximizing point on (3.4), as shown in Figure 3.4. This means that the welfare maximizing access charge is below marginal cost and also below the profit-maximizing access charge.

Moreover, we can show that the welfare maximizing access charge might actually fall below zero. It follows from the additively separable form of (3.5) that the iso-welfare curves have vertical tangents at  $p_{12} = c(1 + \beta)^{-1}$ . Since (3.4) becomes vertical at  $p_{11} = p^M$  for  $\sigma \rightarrow 0$ , the point of tangency approaches  $(1/p^M, (1 + \beta)/c)$ . Denoting the socially optimal access charge by  $a^w$ , this implies that  $(1 + \frac{a^w - c_0}{c})p^M$  converges to  $\frac{c}{1 + \beta}$ , or  $a^w \rightarrow c_0 \left(1 - 2\frac{1 + \beta\eta}{\eta + \beta\eta}\right)$ .

It can be seen that the sign of  $a^w$  depends on the relative size of  $\beta$  and  $\eta$ . Note that for  $\eta < \frac{2}{1 - \beta}$  the expression in brackets is negative, and so is  $a^w$ . The profit maximizing access charge  $a^\pi$ , on the other hand, is always positive for small  $\sigma > 0$ . We summarize this as follows.

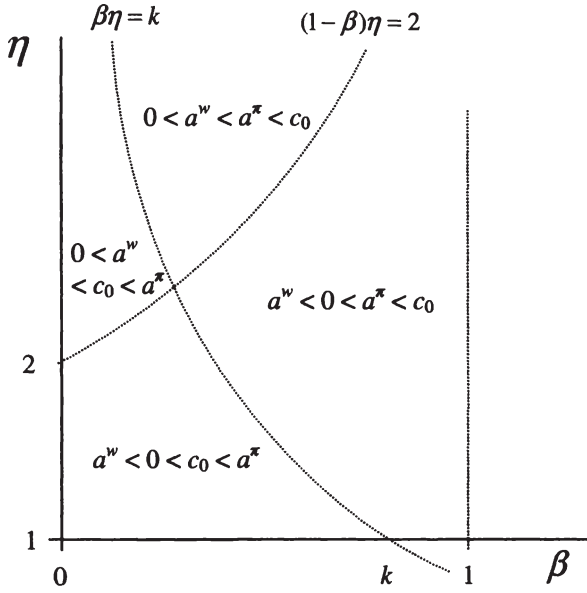


Figure 3.5: The four different regions in  $\beta$ - $\eta$ -space.

**Proposition 3** (i)  $a^w < c_0$  for small  $\sigma$ .

(ii) If  $\eta < \frac{2}{1-\beta}$ , then  $a^w < 0 < a^\pi$  for small  $\sigma$ .

(iii) If  $\eta > \frac{2}{1-\beta}$ , then  $0 < a^w < a^\pi$  for small  $\sigma$ .

The more relevant of the cases (ii) and (iii) of this proposition seems to be (ii), especially if we assume that  $\beta$  is close to 1. The four different regions in  $\beta$ - $\eta$ -space corresponding to cases  $\beta\eta > 1$  and  $\beta\eta < 1$  of Proposition 2 and cases (ii) and (iii) of Proposition 3 are shown in Figure 3.5. Note that in this case networks may actually agree on a bill-and-keep arrangement, setting  $a = 0$ . This might result from the consideration that in existing mobile phone networks, bill-and-keep helps to save transaction costs of interconnection, a point not included in our model. If transaction costs are substantial and were taken into account, bill-and-keep might indeed turn out to be profit maximizing. Note, however, that contrary to the view of Gans and King (2001), from Proposition 3(ii) it follows that bill-and-keep is also welfare improving compared with cost-based access pricing.



### 3.7 Discussion

In this chapter we introduced call externalities in LRTb's standard model of network competition with linear prices and termination-based price discrimination. We showed that this has a significant effect on the strategic incentives of network operators.

Corroborating the findings of Gans and King (2001), Dessein (2003), and others, our findings emphasize the point that collusion over the access charge will result in access sold at a discount. Nevertheless, we seem not to encounter this phenomenon in existing mobile phone networks, and regulators are usually struggling with bringing access charges down to cost.

The reason for this might be the nonexistence of a fixed-line network in our models. Indeed, if networks are not allowed to price discriminate in access, high access charges may well be the result of networks' incentives to boost profits from incoming calls originating on the fixed-line network. Alternatively, as Gans et al. (2005) suggest, even with price discrimination in access, networks may agree to keep mobile-to-mobile access charges at high levels in order to prevent customer arbitrage, i.e. consumers' substitution of mobile-to-mobile calls with fixed-to-mobile calls. A detailed study of these arguments is beyond the scope of this work.

Recent market research (Horvath and Maldoom, 2002) suggests that there is a strong tendency of mobile telephony to substitute for fixed-line telephony, and some business representatives in the field even believe that voice telephony over fixed-line networks will ultimately disappear completely. If this is true, then the policy implications which can be derived from the present model might indeed turn out to be of strong relevance.

# Chapter 4

## Nonlinear Pricing

In this chapter we study network competition and compare bill-and-keep with cost-based access pricing within the framework of a simple model where two symmetric networks compete in nonlinear and discriminatory prices in the presence of call externalities. In contrast to Gans and King's result, and corroborating the view of Cambini and Valletti, we argue in favor of bill-and-keep, showing that such an arrangement is indeed welfare improving compared to cost-based access pricing.

Again our analysis is based on the model of LRTb, which was also utilised by Gans and King and Cambini and Valletti. As in Chapter 2, we neglect fixed costs, but extend the model to include call externalities.

Networks now compete in two-part tariffs discriminating between on-net and off-net calls. A customer of network  $i$  with volumes  $q_{ii}$  of on-net and  $q_{ij}$  of off-net calls, respectively, is charged

$$T_i(q_{ii}, q_{ij}) = F_i + p_{ii}q_{ii} + p_{ij}q_{ij},$$

where  $F_i$  is a fixed fee and, as usual,  $p_{ij}$  is the price for a call from network  $i$  to network  $j$ .

### 4.1 Equilibrium Analysis

Under the assumption of a balanced calling pattern, profit of network  $i$  is

$$\pi_i = \alpha_i^2(p_{ii} - c)q_{ii} + \alpha_i\alpha_j[(p_{ij} - c)q_{ij} + (a - c_0)(q_{ji} - q_{ij})] + \alpha_i F_i. \quad (4.1)$$

Remember that the market share  $\alpha_i$  of network  $i$  is determined by the indifferent consumer and given by (2.3). Inserting from (2.1), we get

$$\begin{aligned}\alpha_i = \frac{1}{2} &+ \sigma\alpha_i[v(p_{ii}) - v(p_{ji}) + \bar{u}(q_{ii}) - \bar{u}(q_{ij})] - \\ &- \sigma\alpha_j[v(p_{jj}) - v(p_{ij}) + \bar{u}(q_{jj}) - \bar{u}(q_{ji})] + \\ &+ \sigma(F_j - F_i).\end{aligned}\tag{4.2}$$

Given the choices of the other network, we can derive the first order conditions in the following way: Imagine that first, for fixed market shares, a network maximizes profits holding its market share constant. It does this by choosing optimal prices  $p_{ii}$  and  $p_{ij}$ , while the fixed fee is used to offset deviations of the market share. In a second step, the network chooses its profit maximizing market share. If  $\alpha_i$  is held constant, differentiating (4.2) with respect to  $p_{ii}$  yields

$$0 = \alpha_i[v'(p_{ii}) + \bar{u}'(q_{ii})q'(p_{ii})] - \frac{\partial F_i}{\partial p_{ii}}.$$

This can be rewritten as

$$\frac{\partial F_i}{\partial p_{ii}} = \alpha_i[\bar{u}'(q_{ii})q'(p_{ii}) - q_{ii}].\tag{4.3}$$

On the other hand, maximizing profit (4.1) with respect to  $p_{ii}$  for constant market shares yields

$$0 = -\alpha_i[q_{ii} + (p_{ii} - c)q'(p_{ii})] - \frac{\partial F_i}{\partial p_{ii}},$$

or

$$\frac{\partial F_i}{\partial p_{ii}} = \alpha_i[(c - p_{ii})q'(p_{ii}) - q_{ii}].\tag{4.4}$$

Comparing (4.3) and (4.4), we derive the identity  $c - p_{ii} = \bar{u}'(q_{ii})$ , and since  $\bar{u} = \beta u$  and  $u'(q_{ii}) = p_{ii}$ ,

$$p_{ii} = \frac{c}{1 + \beta}.\tag{4.5}$$

Hence the profit maximizing on-net price is always at the social optimum — since  $\bar{u} = \beta u$ , this price maximizes  $u(q(p)) + \bar{u}(q(p)) - cq(p)$  — regardless of the market share.

Analogously, we can differentiate with respect to  $p_{ij}$  and compare the expressions for  $\frac{\partial F_i}{\partial p_{ij}}$ . This yields

$$p_{ij} = \frac{(1 - \alpha_i)(c + a - c_0)}{1 - \alpha_i(1 + \beta)} \quad (4.6)$$

for  $\alpha_i < 1/(1 + \beta)$ . For  $\alpha_i \rightarrow 1/(1 + \beta)$  from below, the optimal off-net price goes to  $+\infty$ , i.e. for  $\alpha_i \geq 1/(1 + \beta)$ , it is optimal for network  $i$  to deter any off-net call.

In a symmetric shared market equilibrium we have  $\alpha_i = \alpha_j = 1/2$  and hence

$$p_{ii}^* = \frac{c}{1 + \beta}, \quad p_{ij}^* = \frac{c + a - c_0}{1 - \beta}. \quad (4.7)$$

As usual when competing in two-part tariffs, networks set prices so as to maximize social welfare, and then extract consumer surplus via the fixed fee. For the on-net price, the call externality is internalized by the network's pricing decision, while this is not the case for the off-net price. If a network lowers its off-net price, also its rival's customers benefit through the call externality. In equilibrium this leads to prohibitively high off-net prices if  $\beta$  is large. Indeed, as already noted by Jeon et al. (2004), for  $\beta \rightarrow 1$  the off-net price goes to  $+\infty$ , resulting in *connectivity breakdown*. For the rest of the analysis we assume that  $\beta < 1$ .

## 4.2 Profit Maximizing Access Charge

In a symmetric equilibrium, where  $\alpha_i = 1/2$  and  $p_{ij} = p_{ji}$ , differentiating profit with respect to the fixed fee yields

$$\frac{\partial \pi_i}{\partial F_i} = \frac{\partial \alpha_i}{\partial F_i} [(p_{ii} - c)q_{ii} + F_i] + \frac{1}{2}. \quad (4.8)$$

From (4.2),  $\partial \alpha_i / \partial F_i$  can be replaced by

$$\frac{\partial \alpha_i}{\partial F_i} = \frac{\sigma}{2\sigma[v(p_{ii}) - v(p_{ij}) + \bar{u}(q_{ii}) - \bar{u}(q_{ij})] - 1}, \quad (4.9)$$

leading to

$$F_i = \frac{1}{2\sigma} - [v(p_{ii}) - v(p_{ij}) + \bar{u}(q_{ii}) - \bar{u}(q_{ij})] - (p_{ii} - c)q_{ii}. \quad (4.10)$$

In a symmetric equilibrium, network profit is given by

$$\pi_i = \frac{1}{4}(p_{ii} - c)q_{ii} + \frac{1}{4}(p_{ij} - c)q_{ij} + \frac{1}{2}F_i. \quad (4.11)$$

Differentiating with respect to  $a$  and inserting the equilibrium values of  $p_{ij}$  and  $F_i$  derived above, yields a profit maximizing access charge which is implicitly given by

$$a^\pi = c_0 + \frac{(1 - \beta)q(p_{ij}^*)}{(1 + 2\beta)q'(p_{ij}^*)} - \frac{3\beta c}{1 + 2\beta}. \quad (4.12)$$

Note that since  $q'$  is negative,  $a^\pi$  is smaller than  $c_0$ . Hence networks will invariably negotiate an access charge below marginal cost. Inserting (4.12) into (4.7) yields

$$p_{ii}^* - p_{ij}^* = \frac{1}{1 + 2\beta} \left( \frac{\beta c}{1 + \beta} - \frac{q(p_{ij}^*)}{q'(p_{ij}^*)} \right) > 0. \quad (4.13)$$

Hence the resulting off-net price is always below the on-net price, independently of  $\beta$ .

While the off-net price for any *given* access charge goes to  $+\infty$  for  $\beta \rightarrow 1$ , this is not the case for the off-net price resulting from the collusive choice of the access charge — both the nominator and the denominator go to zero at the same rate in the expression

$$p_{ij}^* = (c + a^\pi - c_0)/(1 - \beta).$$

Intuitively, connectivity breakdown cannot be optimal for networks that are maximizing joint profits.

## 4.3 Welfare Maximizing Access Charge

The socially optimal access charge  $a^w$  would be the one giving rise to equilibrium prices

$$p_{ii} = p_{ij} = c/(1 + \beta).$$

From (4.7), this is achieved by

$$a^w = c_0 - \frac{2\beta c}{1 + \beta}. \quad (4.14)$$

Clearly, the socially optimal access charge is below marginal cost. On the other hand, comparing (4.12) and (4.14) yields

$$a^\pi - a^w = \frac{1 - \beta}{(1 + \beta)(1 + 2\beta)} \left( (1 + \beta) \frac{q(p_{ij}^*)}{q'(p_{ij}^*)} - \beta c \right) < 0, \quad (4.15)$$

and this means that the profit maximizing access charge is even smaller than the socially optimal one. Summarizing,

$$-c_0 - c_1 < a^\pi < a^w < c_0.$$

This shows that with two-part tariffs and discriminatory prices, cost-based access pricing can never be optimal from the social viewpoint, if the call externality is taken into account.

If we agree that ‘realistic’ values of  $\beta$  exceed  $1/3$ , then from (4.14) we always have  $a^w < 0$ . It follows that from the social viewpoint, bill-and-keep, i.e.  $a = 0$ , is a strict improvement over cost-based access pricing. Indeed, bill-and-keep is exactly socially optimal if  $\beta$  happens to equal  $\frac{c_0}{3c_0 + 2c_1}$ .

Some authors, e.g. DeGraba (2003), have suggested that the caller and the receiver share the value of a call, i.e.  $\beta \approx 1$ . Since for  $\beta \rightarrow 1$ , both  $a^\pi$  and  $a^w$  decrease to  $-c_0 - c_1$ , this implies that networks’ and regulators’ incentives are almost perfectly aligned, eliminating the need for regulatory intervention altogether.

## 4.4 Discussion

In this chapter we studied network competition under a caller-pays system with two-part tariffs and termination-based price discrimination in the presence of call externalities. As in the linear pricing case, it turned out that both the profit maximizing and the welfare maximizing access charges are below marginal cost. Moreover, we made a point for the widespread bill-and-keep arrangements. While we agree with Gans and King that these arrangements

may be a result of tacit collusion, we showed that they are welfare improving compared with cost-based access pricing, corroborating the positive view of Cambini and Valletti. Finally, we demonstrated that if the value of a call is approximately shared between caller and receiver, the need for regulatory intervention tends to vanish altogether.

# Chapter 5

## The Dynamics of Market Shares

### 5.1 Existence, Multiplicity, and Stability of Equilibria

Economists traditionally assume that economic systems are in equilibrium. If a system were not, the argument goes, then some individuals do not optimize, or have wrong beliefs. In the long run, such a state cannot persist. Therefore, at least in the long run, a system must necessarily be in equilibrium.

But given that a system might initially be out of equilibrium, how does it get to equilibrium? Usually economists refer to some kind of evolutionary pressures or learning processes which drive the system to equilibrium. However, this answer raises new questions. First of all, it is well known that some systems do not even have an equilibrium. As an example, recall Proposition 1 above. It can be shown that for high substitutability, i.e. large values of  $\sigma$ , a network equilibrium (in pure strategies) does not exist. In such a case, economists have difficulties to say anything about the outcome of competition.

Another problem is the possible multiplicity of equilibria. It is quite common that economic systems admit multiple equilibria. As an example, we have already discussed the potential multiplicity of consumer equilibria in Section 2.3.1. If several equilibria exist, the question arises, which of the equilibria will be the long run state of the system. As we have argued previously, some



equilibria turn out to be dynamically unstable. Such equilibria, like the shared market consumer equilibrium in the three equilibria case cannot be long run outcomes. However, it is quite possible that there are multiple *stable* equilibria. Which equilibrium prevails in the long run is then often dependent on initial conditions. Under such circumstances we say the system exhibits *path dependence*.

The situation seems most predictable if there is a unique equilibrium, as is the case for the network equilibrium we analyzed in the previous chapters under certain conditions. Nevertheless, as soon as we leave the simple case of a one-dimensional state space, even a unique equilibrium need not suffice to predict long run behavior of a system. First, a unique equilibrium might well be unstable. Second, even asymptotic stability does not guarantee a predictable long run outcome. What we need is *global* asymptotic stability. By definition, global asymptotic stability guarantees that from every initial condition the system converges to the equilibrium in the long run. However, global asymptotic stability is a rather special case if the state space is two- or higher-dimensional.

## 5.2 Out-of-Equilibrium Dynamics

### 5.2.1 Dynamical Systems

Stability is not a property that equilibria do or do not possess by themselves. When speaking of stability of equilibria, we must refer to some underlying *dynamics*. Which dynamics to assume is always a matter of the special situation we are dealing with. Dynamical systems can be set up in discrete or continuous time, they can be stochastic or deterministic, they may be autonomous (time-independent) or nonautonomous, they can admit unique or multiple solutions, and so on.

In this work we consider only *deterministic* systems, i.e. dynamical systems which involve no stochastic elements. Such systems are usually easier to work with. If a deterministic system has unique solutions, its behavior is in principle completely predictable. Given any initial state, we can compute the state for any later point of time. However, for practical purposes this is often not sufficient. Such systems might well suffer from *sensitivity to initial conditions*, which means that arbitrarily close initial states diverge

after relatively short amounts of time. This leads to chaotic behavior, making these systems practically unpredictable for longer horizons.

When dealing with dynamical systems which describe the behavior of an economic system, we must distinguish between Nash equilibria of the economic system and dynamic equilibria of the dynamical system. The latter are usually called *stationary points* of the dynamical system, and we stick to this term in order to avoid confusion.

What are the forces determining the behavior of dynamical systems? When these systems are set up to model the behavior of a group of economic agents, it is usually the strategic incentives which drive the behavior of agents, and therefore the dynamical system. By definition, Nash equilibria are states in which no agent has an incentive to deviate from his strategy. Hence Nash equilibria typically are stationary states of the dynamics. However, in a non-strict equilibrium some agents have no incentive not to deviate, either. So, at least if there are multiple solutions, such equilibria need not be stationary states. On the other hand, stationary states need not be equilibria for some dynamics. To see this, imagine a dynamics based only on imitation of successful agents.<sup>1</sup> If we start with a homogeneous population, i.e. with all agents using the same strategy, then there is nothing else to imitate, and the system will necessarily remain in this state, even if it is not an equilibrium.

### 5.2.2 Levels of Rationality

Dynamical systems describing the behavior of a population of agents can roughly be classified along a line indicating the level of rationality they assume. At the zero-rationality end of this line are the pure *evolutionary* dynamics, the best-known of which is the *replicator dynamics* (see Maynard Smith, 1982). This dynamics assumes that agents are hard-wired to some strategy throughout their life, and that this strategy is inherited by their offspring. Payoffs are interpreted as biological fitness in this model. The more successful a strategy, i.e. the higher an individual's payoff compared to the average payoff in the population, the more offspring this individual has. Hence the frequency of relatively successful strategies increases through a purely evolutionary force. This dynamics stems from the biological sciences and its usefulness in economic contexts is at least questionable. Interestingly

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<sup>1</sup>For an overview of imitation learning see Schlag (1998).

however, it has been shown (Börger and Sarin, 1997) that this dynamics also arises from a model of agents adapting by reinforcement learning.

On the other end of the line there are the dynamics which assume full rationality of agents, and sometimes even more than that. For example, Matsui and Matsuyama's (1995) *perfect foresight dynamics* assumes that agents are 'omniscient', implying that they have perfect foresight and know from the outset which path the dynamical system will take. Given this knowledge, they optimize their selection of strategies along this path, and as a result, in aggregate the system follows exactly the path they have foreseen.

Between these two extremes, there is the large area of bounded rationality. The dynamics in this area have in common that they assume agents are boundedly rational, groping for optimality under the constraints of limited computing power, limited foresight, and/or limited knowledge and information about the behavior of other agents. This area includes a lot of rather different models of behavior which have been developed during the last decades, e.g. models of reinforcement learning, imitation learning, best response dynamics, fictitious play, etc. Some of these dynamics have been designed for small groups of players, others for large interacting populations.

Our topic of interest is the behavior of consumers and network operators in a telecommunications market. In order to point out the difficulties arising in a dynamic analysis of this market we concentrate on consumers' behavior for a given price structure of the networks. Hence we assume that networks' prices and fixed fees are constant, and consumers adapt their choice of network subscription to the changing market shares induced by these choices. What is the appropriate dynamical system to set up for such a model? To answer this question we make several simplifying assumptions, but try to stay close to what we consider consumers' real world behavior to be like.

### 5.2.3 Simplifying Assumptions

First of all, given that the number of individuals participating in mobile telephony is several millions even in smaller countries, we assume that we are dealing with an infinite population of consumers. We also assume that all these consumers are identical, they do not differ in their demand for phone calls. They also do not prefer any of the networks per se, so in contrast to

the previous chapters we assume here that networks are not differentiated from the consumers' perspective.

We further simplify the analysis by neglecting the obstacles which consumers usually encounter when trying to change their network subscription. The sum of these obstacles, be they of monetary nature or not, expressed in monetary units, is called *switching costs*. We simply assume that there are no switching costs. While this assumption might be called unrealistic, it does not play an important role in our analysis. Moreover, with number portability (the possibility to keep one's phone number when changing the network) likely to be implemented in many mobile telecommunications markets in the near future, actual switching costs will be greatly diminished.

As in the previous models, we stick to the assumptions of a covered market and of balanced calling patterns. As already discussed in the introduction, this implies that the percentage of calls a consumer makes into a certain network is exactly equal to the market share of this network.

Consumers cannot switch whenever they like. This reflects the fact that subscription contracts, which usually include a heavily subsidized handset, are binding for some period of time, most commonly 12 months. This induces *inertia* in the dynamics. Not all consumers get the possibility to switch at the same time. Instead, every time period only a small fraction of consumers is allowed to switch. For our purposes the most natural time period is one day. Given that subscription decisions are binding for 12 months, and assuming that contract initial dates are uniformly distributed over the days of the year, each day about one in 365 consumers is allowed to switch. However, for convenience we assume that each day the consumers getting a switching possibility are randomly selected from the population. While this implies that the same consumer may be allowed to switch on two consecutive days, it makes the analysis tractable by freeing us from the need to keep track of the contract length of each individual in the population. Furthermore, a consumer's *expected* waiting time till the next switching possibility is still 365 days, and we will see that this assumption does not change the direction of movement of the state space but only the velocity, implying that the orbits of the corresponding dynamical systems are the same.

Concerning consumers' beliefs, we assume that consumers have full information about current market shares. Thus, consumers' beliefs are correct, they know networks' current market shares at any point in time. This as-

sumption is a strong one, but it may be justified by consumers' possibility to inform themselves about current market shares from the media, or by consumers' ability to estimate these market shares from the distribution of network choice among their calling partners.

However, we assume that consumers are myopic, i.e. shortsighted. This means they act as if they believe that during the time period where their contract is binding the market share will not change. This assumption appears not too unrealistic, since it is often observed that consumers act short-sightedly in their daily decisions, particularly if the stakes are not very high. Furthermore, we assume that consumers are rational. Given their beliefs, consumers optimize, i.e. they make an optimal subscription decision given the current market shares.

In the absence of termination-based price discrimination market shares do not matter for a consumer's subscription decision. Indeed, in this case for generic combinations of price and fixed fee there is always a network corresponding to a strictly dominant strategy. For any given initial market shares, this dominant network's share will monotonically increase to 1.

The interesting case is the one with termination-based price discrimination. If on-net prices differ from off-net prices, it depends on the market shares, which network is currently optimal. While the state of the population moves to this 'corner' of the market, the optimal network may change, inducing the population state to change the direction of movement, and the question then is, what behavior this dynamical process will show in the long run.

Note that the system as we have set it up need not admit unique paths of the state. If for some distribution of market shares there are several optimal networks, we do not prescribe a certain choice to the consumers. In such a case, it is possible that some fraction of switching consumers chooses one of the optimal networks, while the remaining fraction chooses another one of those. Starting from such a population state, several different future paths are possible.

The system as described up to now is in discrete time. Given that the stepsize of the system, i.e. the distance between successive states, is rather small (at most  $\frac{1}{365}$  of the distance between two cornered market outcomes in the state space), we will approximate this discrete system by one in continuous

time. This facilitates the analysis and does not change the system's behavior qualitatively.

### 5.3 Best Response Dynamics

Let us now state the dynamic model of this chapter in a more formal way. There are  $n$  telecommunications networks. Each network  $i$  charges its customers a price of  $p_{ij}$  for a single call into network  $j = 1, \dots, n$ . We denote by  $p_{ii}$  network  $i$ 's on-net price and by  $p_{ij}$  for  $i \neq j$  network  $i$ 's off-net prices.

There is a continuum of agents, each of whom can choose among  $n$  networks to subscribe to. Each agent makes telephone calls to other agents. Demand for calls is completely inelastic, and we assume that each agent calls a fixed number, normalized to 1, of randomly selected other agents during a unit time period. Agents' total (active and passive) utility from a call is  $U$ . Utility is quasilinear in money, i.e. an agent's net surplus, given market shares  $\mathbf{x} = (x_i)_{i=1, \dots, n}$  with  $\sum_i x_i = 1$ , is

$$v_i(\mathbf{x}) = U - \sum_{j=1}^n p_{ij} x_j, \quad (5.1)$$

if the agent is subscribed to network  $i$ . Here we assume that  $U$  is large enough to prevent net surplus from becoming negative.

Time  $t \geq 0$  is continuous, and in each small time interval  $dt$ , a randomly selected fraction  $\delta dt$  of agents receives a switching opportunity. An agent receiving a switching opportunity chooses a myopic pure best response from the set  $BR(\mathbf{x}(t))$  of (pure or mixed) best responses to  $\mathbf{x}(t)$ , i.e. he subscribes to a network  $i$  maximizing his instant expected payoff  $v_i(\mathbf{x}(t))$ .

With these assumptions agents are playing a *population game*. Networks correspond to pure strategies, and the vector  $\mathbf{x}$  of market shares is the state of the population, corresponding to the population's mixed strategy. Each agent repeatedly plays a symmetric two-person  $n \times n$  game (a *matrix game*) against his calling partners. Since the latter are randomly drawn from the population, and since agents are (myopic) expected utility maximizers, we

can say each agent plays ‘against the population’. The payoff matrix of the game is given by the  $n \times n$  matrix

$$A = \begin{bmatrix} U - p_{11} & \cdots & U - p_{1n} \\ \vdots & U - p_{ij} & \vdots \\ U - p_{n1} & \cdots & U - p_{nn} \end{bmatrix}.$$

The motion of the population state  $\mathbf{x}(t) \in S_n$  is then described by the set of differential inclusions  $\dot{\mathbf{x}}(t) \in \delta[BR(\mathbf{x}(t)) - \mathbf{x}(t)]$ . Normalizing  $\delta = 1$ , we finally obtain

$$\dot{\mathbf{x}} \in BR(\mathbf{x}) - \mathbf{x}. \quad (5.2)$$

This dynamics is known as the *best response dynamics*. It was originally formulated by Gilboa and Matsui (1991) and Matsui (1992). Mathematically it is equivalent to Brown’s (1951) continuous-time *fictitious play* process with identical initial moves. These dynamics has thoroughly been studied by Hofbauer (1995), see also Berger (2001, 2005), and we will utilize their results for our analysis.

The right-hand side of (5.2) is set-valued, since best responses need not be unique. Therefore (5.2) is a system of *differential inclusions* rather than a system of differential equations. For details on differential inclusions see e.g. Aubin and Cellina (1984). The right-hand side of (5.2) is upper-semicontinuous with closed and convex values, guaranteeing existence of solutions through any initial value. However, in general these solutions need not be unique.

Obviously, if the pure strategy  $i$  is the *unique* best response to  $\mathbf{x}(t)$ , the best response path through  $\mathbf{x}(t)$  is a straight line, heading for  $i$ , as long as this strategy remains the unique best response. The sets of states  $\mathbf{x}$  with  $BR(\mathbf{x}) = \{i\}$  for different pure strategies  $i$  are disjoint, open and convex subsets<sup>2</sup> of  $S_n$ . If after some time the best response changes to  $i'$ , then the path suddenly heads for strategy  $i'$ . At the turning point  $\mathbf{x}$  the respective payoffs are equal:  $v_i(\mathbf{x}) = v_{i'}(\mathbf{x})$ .

If there is a constant solution through some point  $\mathbf{x}^* \in S_n$ , then we must have  $\mathbf{0} \in BR(\mathbf{x}^*) - \mathbf{x}^*$ , or  $\mathbf{x}^* \in BR(\mathbf{x}^*)$ , which means that  $\mathbf{x}^*$  is a (symmetric) Nash equilibrium of the game  $A$ .

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<sup>2</sup>Some of these sets might be empty. This is e.g. the case, if some pure strategy  $i$  is strictly dominated.



As already mentioned, revising agents choose some pure strategy which is a best response to the current state  $\mathbf{x}(t)$ . If there is more than one such pure best response, then the target of the best response path can be any convex combination of these pure best responses. In fact,  $\dot{\mathbf{x}}(t)$  is not uniquely determined for such a point in time. A consequence of this is that non-strict Nash equilibria need not be stable. While there is always a constant solution through them, there may be other solutions leaving such an equilibrium.

## 5.4 Two Networks

Let us briefly review the dynamics of market shares in the standard case of two competing networks. Generically there are 4 different cases:

- Network 1 dominates network 2.

In this case  $U - p_{11} > U - p_{21}$  and  $U - p_{12} > U - p_{22}$ . Put simply,  $p_{1j} < p_{2j}$  for  $j = 1, 2$ . Then network 1 is always the unique best response. No matter what the market shares, each consumer subscribes to network 1 whenever he gets a revision opportunity. The unit vector  $\mathbf{e}_1 = (1, 0)^t$ , corresponding to the state where all consumers are subscribed to network 1, is the unique Nash equilibrium of the respective matrix game and the unique globally asymptotically stable stationary state of (5.2). Interestingly, however, if the process starts out of equilibrium, the equilibrium is not reached in finite time. If  $x(t)$  denotes the market share of network 1, then (5.2) reads  $\dot{x}(t) = 1 - x(t)$ , with solution  $x(t) = 1 - (1 - x(0))e^{-t}$ . Thus, if  $x(0) < 1$ ,  $x(t) < 1$  for all  $t$ . The same is generally true whenever the best response path converges to a pure Nash equilibrium. However, this is an artefact of our assumptions of a continuum of consumers and of revision opportunities arriving randomly. It is not important for our main arguments.

- Network 2 dominates network 1.

This is just the mirror image of the first case.  $x = 0$  is the unique Nash equilibrium and is globally asymptotically stable.

- Both networks can corner the market.



For this case both  $x = 0$  and  $x = 1$  must constitute Nash equilibria. This is the case if  $p_{11} < p_{21}$  and  $p_{22} < p_{12}$ , i.e. each network's on-net price is below the other network's off-net price. Note, however, that in this case there is always a third equilibrium  $0 < \bar{x} < 1$  in mixed strategies, where a fraction  $\bar{x}$  of the population subscribes to network 1 and the remaining fraction  $1 - \bar{x}$  to network 2. In this equilibrium, all consumers must be indifferent between the networks, i.e.  $p_{11}\bar{x} + p_{12}(1 - \bar{x}) = p_{21}\bar{x} + p_{22}(1 - \bar{x})$ , from which network 1's market share can be calculated as

$$\bar{x} = \frac{p_{22} - p_{12}}{p_{11} - p_{21} + p_{22} - p_{12}}. \quad (5.3)$$

For market shares of network 1 below this value, network 2 is the best response, and if the market share of network 1 is higher than  $\bar{x}$ , consumers prefer network 1. As a consequence, any best response path starting at  $0 \leq x(0) < \bar{x}$  converges to 0 and any path starting at  $\bar{x} < x(0) \leq 1$  converges to 1. In the knife-edge case  $x(0) = \bar{x}$  there are infinitely many solutions: A path can remain at  $\bar{x}$  for an arbitrary amount of time and then head off to either of the two pure equilibria. From this analysis, both pure equilibria are locally asymptotically stable, while the mixed equilibrium is unstable.

- None of the networks can corner the market.

In this case there are no pure equilibria, i.e.  $p_{ii} > p_{ji}$  for  $j \neq i$ . Since the on-net price of a network is above the rival's off-net price, best response paths point inwards at both boundary points  $x = 0$  and  $x = 1$ . Again there exists a unique equilibrium in mixed strategies given by (5.3). However, this time the mixed equilibrium is globally asymptotically stable. This corresponds to the case of the stable shared market equilibrium we analyzed in the previous chapters. Any deviation of network 1's market share to a value below  $\bar{x}$  makes all consumers prefer to subscribe to network 1. Hence  $x(t)$  rises until  $x(t) = \bar{x}$  again. Analogously, if a deviation to a value above  $\bar{x}$  occurs,  $x(t)$  falls until equilibrium is re-established, and then remains there.

The four generic scenarios described above are valid not only for the best response dynamics, but for virtually any dynamics that respects the basic

assumption that the driving force is consumers' incentives. More precisely, any *payoff monotonic* dynamics, i.e. any dynamics with the property that  $\dot{x}(t) > 0$  if  $v_1(x(t)) > v_2(x(t))$  and vice versa, shows the same qualitative behavior as the best response dynamics in the case of two networks. The reason for this is that the state space is one-dimensional, and the direction of movement is completely determined by the payoff differences the consumers face. This is no longer true if we move to higher-dimensional state spaces. In the next section we treat the simplest of these non-trivial cases, the case of three networks.

## 5.5 Three Networks

If we have three different networks in the market, i.e.  $n = 3$ , then the state space is the two-dimensional probability simplex  $S_3 = \{\mathbf{x} = (x_1, x_2, x_3)^t : x_i \geq 0, \sum x_i = 1\}$ . If a best response path  $\mathbf{x}(t)$  changes direction, then at the turning point consumers must be indifferent between two of the three networks, and they must (weakly) prefer these two to the third one. Denote these two networks by  $i$  and  $j$ , and the third one by  $k$ , then the indifference condition is  $v_i(\mathbf{x}) = v_j(\mathbf{x}) \geq v_k(\mathbf{x})$ , or  $(A\mathbf{x})_i = (A\mathbf{x})_j \geq (A\mathbf{x})_k$ . The equality defines an affine linear subspace in  $S_3$ , which is generically a hyperplane, i.e. a line. The weak inequality then determines a (possibly empty) halfline in  $S_3$  which we denote by  $l_{ij}$ . If the three indifference halflines meet in a point  $\mathbf{x}^*$  in  $S_3$ , then this state is a Nash equilibrium (a completely mixed equilibrium, a partially mixed equilibrium, or a pure equilibrium, depending on whether it is in the interior of  $S_3$ , in the interior of one of the faces where  $x_i = 0$  for some  $i$ , or at a vertex). If the indifference lines do not intersect in the interior of  $S_3$ , then all Nash equilibria are on the boundary of the simplex.

We start our analysis by picking out three classes of price structures having particular symmetry properties. This allows us to focus on different important types of dynamic behavior illustrating our main points.

### 5.5.1 On-net Price below Off-net Price

Consider the class of games with the property that  $p_{ii} = p$  and  $p_{ij} = q$  for all  $i, j \in \{1, 2, 3\}$  and  $i \neq j$ . This means that all three networks charge the same

on-net price  $p$ , and the same off-net price  $q$  for calls to either of the other two networks. Assume furthermore that  $p < q$ , i.e. it is cheaper to call on-net. A consequence of this is that each network can corner the market. To see this, assume network  $i$ 's market share  $x_i$  is close enough to 1. Then every consumer prefers to subscribe to this network, since almost all his calls terminate in network  $i$ . The situation is qualitatively equivalent to the analogous case with only two networks. All three networks constitute strict Nash equilibria. As in the two-networks case, there is also a completely mixed equilibrium  $\bar{x}$ , which by symmetry of the networks lies in the barycenter of the simplex,  $\bar{x} = (1/3, 1/3, 1/3)^t$ . In addition to this completely mixed equilibrium there are three partially mixed equilibria on the three boundary faces of the simplex where one of the  $x_i$  vanishes. These three boundary faces just correspond to the one-dimensional simplex serving as the state space in the two-networks case above. By symmetry, the three partially mixed equilibria are

$$(1/2, 1/2, 0)^t, \quad (1/2, 0, 1/2)^t, \quad (0, 1/2, 1/2)^t. \quad (5.4)$$

To see that these points are indeed equilibria, consider as an example the state  $(1/2, 1/2, 0)^t$ . Networks 1 and 2 share the market, and all customers pay an average call price of  $P = (p + q)/2$ , since half of their calls are on-net and half of them are off-net. In this situation it does not pay for a customer to switch to network 3, since then all his calls would be off-net, with average price  $q > (p + q)/2 = P$ . Hence the state  $(1/2, 1/2, 0)^t$  constitutes a Nash equilibrium. However, it turns out that all four mixed equilibria are unstable. The instability of a partially mixed equilibrium follows from the analysis of the two-networks case. By the positive tariff-mediated network externality, a small deviation from equilibrium creates a positive feedback loop leading one of the networks to cover the market. The dynamics in this case are illustrated in Figure 5.1. The same argument works for the completely mixed equilibrium, and hence we obtain the following result.

**Theorem 1** *If the on-net price is below the off-net price, then in the long run a single network covers the market.*

## 5.5.2 On-net Price above Off-net Price

Next we analyze the reverse case. Again we assume that  $p_{ii} = p$  and  $p_{ij} = q$  for all  $i, j \in \{1, 2, 3\}$  and  $i \neq j$ . This time, however, suppose  $p > q$ , i.e. it

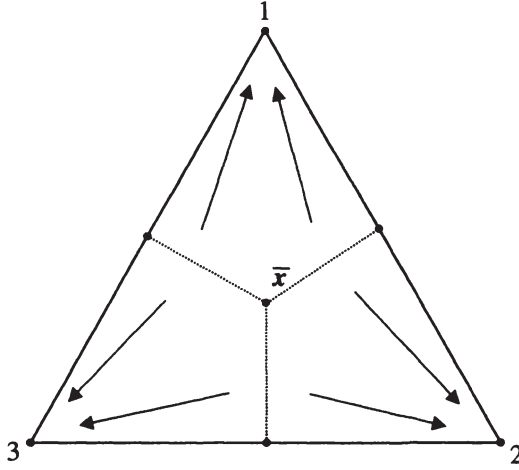


Figure 5.1: With positive tariff-mediated network externalities, any network can corner the market.

is cheaper to call off-net. With this price structure, it is clear that no single network can corner the market. If the market share of a network is close to one, almost all calls are on-net, and hence it pays to switch to another network, profiting from the low off-net price. Again we have the completely mixed symmetric equilibrium  $\bar{x}$  in the interior of the simplex. As opposed to the case above, this equilibrium is now unique. To see this, consider again the state  $(1/2, 1/2, 0)^t$ , where networks 1 and 2 share the market, and all customers pay an average call price of  $P = (p+q)/2$ . This time customers benefit from switching to network 3, since then all calls are off-net, with average price  $q < (p+q)/2 = P$ . Hence the state  $(1/2, 1/2, 0)^t$  is not a Nash equilibrium, and neither are the other two partially mixed symmetric states. The completely mixed equilibrium is globally asymptotically stable., see Figure 5.2. Analogous to the two-networks case, an on-net price above the off-net price creates negative tariff-mediated network externalities. The resulting negative feedback on market shares always makes the smallest network grow. This can be shown as follows. Consider any state  $x$ . A customer of network  $i$  pays an average price of  $P_i = px_i + q(x_j + x_k)$ , with  $\{i, j, k\} = \{1, 2, 3\}$ , which can also be written as  $P_i = px_i + q(1 - x_i) = q + (p - q)x_i$ . Since  $p > q$ ,  $P_i$  is increasing in  $x_i$ . This means that the lowest average price is offered by the

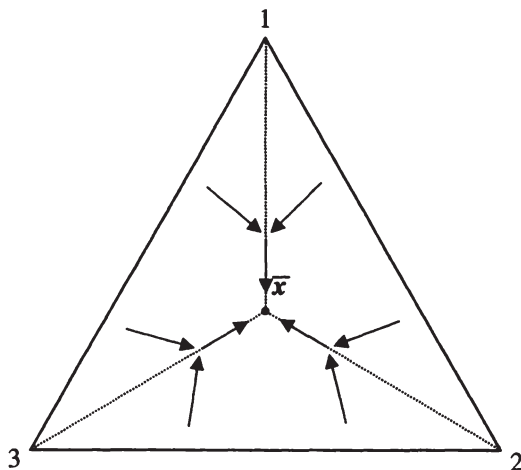


Figure 5.2: With negative tariff-mediated network externalities, no network can corner the market.

smallest network. With customers beginning to switch to this network, its market share grows until another network becomes the smallest. The process comes to a halt if and only if all three networks are of the same size, i.e. if  $x = \bar{x}$ . This proves our next result.

**Theorem 2** *If the on-net price is above the off-net price, then in the long run the three networks share the market equally.*

### 5.5.3 Cyclic Symmetry

The third case we analyze here is more difficult. We keep the symmetry assumption, but this time we assume cyclic symmetry. Consider the following pricing structure: Network  $i$  charges an on-net price of  $p$ , an off-net price of  $q$  for calls to network  $i + 1$  (where indices are counted modulo 3), and an off-net price of  $r$  for calls to network  $i + 2$ . We assume the ordering  $r < p < q$  of these prices. This means that for customers of network  $i$  it is cheaper to call on-net than to call to network  $i + 1$ , but on-net calls are more expensive than calls to network  $i - 1$ .

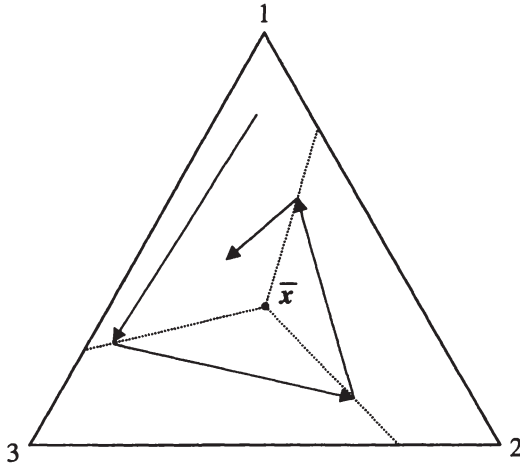


Figure 5.3: The vector of market shares cycles around the completely mixed equilibrium.

Note that this kind of pricing leads to a cyclic best response structure. The resulting matrix game is a variant of what is called the *Rock-Scissors-Paper game*, since its best response structure is the same as in the well-known children's game where one of the three symbols is shown by two players simultaneously, and rock beats scissors, which beats paper, which wins against rock.

It is straightforward in this case to show that the symmetric state  $\bar{x}$  is the unique equilibrium. No single network  $i$  can cover the market, since then all customers would strictly prefer to switch to network  $i + 1$ . Two networks cannot share the market. E.g. in state  $(x_1, x_2, 0)^t$ , customers of network 1 face an average price between  $p$  and  $q$ , while customers of network 2 pay an average between  $r$  and  $p$ . Since the latter is smaller, network 1 customers would always prefer network 2 over network 1. Note that this need not mean that they switch to network 2. If  $x_2$  is small enough, customers prefer network 3 over both 1 and 2. The most interesting question here concerns the stability of the completely mixed equilibrium. As we have seen above, the cyclic symmetry of the call prices induces all best response paths to surround  $\bar{x}$ , following the best response cycle  $1 \rightarrow 3 \rightarrow 2 \rightarrow 1$ , as shown in Figure 5.3. Since the state space is a plane, and since best response paths

do not intersect, a single path off the equilibrium can either spiral inwards or outwards, or form a closed cycle. If it forms a closed cycle, then this cycle consists of linear pieces, i.e. it is a polygon in the simplex, in our case a triangle. Such closed paths have first been found by Shapley (1964), and have therefore been termed *Shapley polygons* by Gaunersdorfer and Hofbauer (1995). In following their analysis, we can state the next result.

**Theorem 3** *With cyclically symmetric prices, networks share the market equally in the long run if and only if  $p - r \geq q - p$ . If this is not the case, the market shares converge to a unique Shapley triangle.*

Indeed, Gaunersdorfer and Hofbauer show that this theorem can be extended to more general price structures. Suppose network  $i$  charges on-net price  $p_i$  and off-net prices  $q_i$  to network  $i + 1$  and  $r_i$  to network  $i + 2$ , where  $r_{i+2} < p_{i+1} < q_i$  for all  $i$ . Then there exists a unique, completely mixed Nash equilibrium, and the following generalization of Theorem 3 holds.

**Theorem 4** *The completely mixed Nash equilibrium is globally asymptotically stable if and only if*

$$(p_1 - r_2)(p_2 - r_3)(p_3 - r_1) \geq (q_3 - p_1)(q_1 - p_2)(q_2 - p_3).$$

*In case of instability, the market shares converge to a unique Shapley triangle.*

The case of instability offers a new phenomenon, which is impossible to occur with only two networks. Irrespective of the initial state, in the long run the vector of market shares approaches a Shapley triangle in the interior of the state space, cycling along this triangle forever. Hence at any point in time all three networks are present in the market, and only their market shares change continually. The size of the Shapley triangle depends on the price differences. If  $p - r$  is only a bit lower than  $q - p$ , the Shapley triangle is very small, and market shares are always close to equilibrium in the long run. If, however,  $q - p$  is much larger than  $p - r$ , the Shapley triangle is close to the boundary triangle of the simplex, and each network is sometimes close to covering the market, before eventually the next network takes over. This phenomenon impressively demonstrates how careful one must be when interpreting equilibrium in a strategic context. In the static version of the game, traditional comparative static analysis would in any case treat the market as being in equilibrium.

### 5.5.4 The General Case

The last sections have been devoted to very special payoff structures. With more general payoff structures there are many different modes of market share behaviors. We can ask the question under what circumstances coexistence of all three networks is possible in the long run. One possibility is that an asymptotically stable shared market equilibrium exists. As we have seen, however, coexisting networks need not be in equilibrium, so we must take into account the possibility of an asymptotically stable Shapley polygon.

It can be shown that the existence of a Shapley polygon always implies the existence of a completely mixed equilibrium in the interior of the polygon. Indeed, let  $L_1$  be an asymptotically stable Shapley polygon in  $S_3$  and let  $x$  be a point in the interior of  $L_1$ . Consider the  $\alpha$ -limit of the best response path  $x(t)$  through  $x(0) = x$ , i.e. the set of limit points of  $x(t)$  for  $t \rightarrow -\infty$ . This limit is well defined since the area enclosed by the Shapley polygon is backwards invariant. Since the backwards best response path cannot cross itself, it must converge to another limit cycle, i.e. a Shapley polygon  $L_2$ . Continuing in this fashion we can construct a sequence of Shapley polygons  $L_1, L_2, L_3, \dots$ , where  $L_{k+1}$  lies in the interior of  $L_k$ . If the sequence is infinite, then it must converge to a single point, which is a completely mixed equilibrium. If the sequence is finite, it must end in a degenerate Shapley polygon with empty interior, i.e. in a point. Again this point is necessarily a completely mixed equilibrium.

From the convexity of the different best response regions it follows that all Shapley polygons are Shapley triangles. However, actually the procedure suggested above always ends in the completely mixed equilibrium after one step, because Shapley triangles turn out to be unique here. An elegant argument from projective geometry establishes this as follows (see Gaunersdorfer and Hofbauer, 1995). Suppose there are two Shapley polygons. Their vertices lie pairwise on the lines where  $(Ax)_i = (Ax)_j$ , and these three lines intersect in the completely mixed equilibrium  $\bar{x}$ . Hence the vertices of the triangles are perspective from  $\bar{x}$ . By Desargues Theorem, the extensions of their edges must pairwise intersect in three collinear points. However, this yields a contradiction, since the intersection points are the vertices of the simplex  $S_3$ , which are obviously not collinear.



The arguments from the last two paragraphs establish that coexistence of all three networks is possible if and only if there exists an asymptotically stable completely mixed Nash equilibrium or an asymptotically stable Shapley triangle. In the latter case there exists an unstable completely mixed equilibrium in the interior of the triangle. Hence existence of a completely mixed equilibrium  $\bar{x}$  is a necessary (but not sufficient) condition for coexistence. In this equilibrium, the payoffs are the same for all three pure strategies, i.e.  $(A\bar{x})_1 = (A\bar{x})_2 = (A\bar{x})_3$ , and the equilibrium is determined by the intersection of the three indifference halflines  $l_{12}$ ,  $l_{23}$ , and  $l_{31}$ . The best response sets are convex polytopes bounded by these lines. Consider now a small deviation from  $\bar{x}$  in the direction of network 1, i.e.  $e_1$ . The new state can be written as a convex combination

$$x_\varepsilon^1 = \varepsilon e_1 + (1 - \varepsilon)\bar{x} \quad (5.5)$$

for some small  $\varepsilon > 0$ . If  $e_1$  is a Nash equilibrium, i.e. if  $e_1 \in BR(e_1)$ , then by convexity of the best response sets and linearity of payoffs,  $e_1 \in BR(x_\varepsilon^1)$  for all  $1 \geq \varepsilon \geq 0$ . As a consequence, there is a best response path starting from  $\bar{x}$  and converging to  $e_1$ . In this case the completely mixed equilibrium is unstable and a Shapley triangle does not exist. This contradicts our assumption, hence we must have  $e_1 \notin BR(e_1)$ . Then either  $e_2$  or  $e_3$  are best responses to  $e_1$ .

Next consider a small deviation from  $\bar{x}$  in the direction of  $e_2$ . As before, the new state can be written as a convex combination  $x_\varepsilon^2 = \varepsilon e_2 + (1 - \varepsilon)\bar{x}$  for some small  $\varepsilon > 0$ . Again we conclude that  $e_2 \notin BR(e_2)$ , implying that either  $e_1$  or  $e_3$  are best responses to  $e_2$ . A completely analogous argument establishes that either  $e_1$  or  $e_2$  are best responses to  $e_3$ .

Assume  $x(s)$  for  $s \in [0, 1]$  is a path (not a best response path) on the boundary of  $S_3$ , starting at  $e_1$  and moving clockwise to  $e_2$ , to  $e_3$ , and back to  $e_1$ . Let  $b = (i, j, k)$  denote the sequence of networks which are best responses to  $x(s)$  along this path. Since all three networks must be present in the sequence (otherwise there would not be a completely mixed equilibrium), and since  $e_1$  cannot be a best response to  $x(0) = e_1$ , we have the following four possible configurations:

$$b \in \{(2, 1, 3), (2, 3, 1), (3, 1, 2), (3, 2, 1)\} \quad (5.6)$$

We can w.l.o.g. renumber the networks in such a way that  $\mathbf{e}_2 \in BR(\mathbf{e}_1)$ . This leaves us with the two possibilities

$$b = (2, 1, 3) \quad \text{or} \quad b = (2, 3, 1) \quad (5.7)$$

Consider the first case,  $b = (2, 1, 3)$ . Here the first switch of the best response to  $\mathbf{x}(s)$  is from  $\mathbf{e}_2$  to  $\mathbf{e}_1$ , and it occurs at the intersection of the boundary of the simplex with the indifference halfline  $l_{12}$ . Since  $\mathbf{e}_2$  is not a Nash equilibrium, the indifference halfline  $l_{12}$  must intersect the edge connecting  $\mathbf{e}_1$  and  $\mathbf{e}_2$ . Call the intersection point  $\mathbf{x}^{12}$ . In state  $\mathbf{x}^{12}$  agents are subscribed only to networks 1 and 2, they are indifferent between these two networks and strictly prefer them to network 3. Hence  $\mathbf{x}^{12}$  is a Nash equilibrium. Again it follows that there is a best response path leaving  $\bar{\mathbf{x}}$  and converging to  $\mathbf{x}^{12}$ , rendering  $\bar{\mathbf{x}}$  unstable and implying the nonexistence of a Shapley triangle. This is a contradiction, so  $b \neq (2, 1, 3)$ . It follows that  $b = (2, 3, 1)$ .

The last paragraph proved that  $b = (2, 3, 1)$  is a necessary condition for coexistence. We show now that together with the nonexistence of pure strategy equilibria it is also sufficient.

Note that  $b = (2, 3, 1)$  together with the nonexistence of a pure strategy equilibrium effectively means that for any two networks  $i$  and  $j$ , the indifference halfline  $l_{ij}$  does not intersect the edge connecting  $\mathbf{e}_i$  and  $\mathbf{e}_j$ . This is equivalent to saying that there are no Nash equilibria on the edges of the simplex. In this case the boundary of the simplex is repelling for best response path, and every such path eventually stays in the interior of the simplex. Hence, nonexistence of a boundary Nash equilibrium is a sufficient condition for coexistence. The same is of course true in the two networks case. We subsume this analysis in the following theorem.

**Theorem 5** *In the case of at most three networks there is coexistence in the long run if and only if there are no Nash equilibria involving an unused network.*

Note that the analysis relies on special geometric properties of the plane which do not extend to higher dimensions. For this reason we cannot derive the same result for the case of four or more networks.

## 5.6 Discussion

The emphasis in this chapter lies on the point that the usual static interpretation of consumer equilibrium has a serious weakness in models of network competition. We have seen that as soon as three or more networks are present, existence of a stable consumer equilibrium is no longer guaranteed. Moreover, if such an equilibrium happens to exist, then there are typically multiple ones. An exception is the case we called on-net prices above off-net prices, where a unique and globally asymptotically stable shared market equilibrium exists. Unfortunately the pricing patterns observed in real-world telecommunications networks are just the opposite of this case. This leads to an interesting puzzle, to which the next chapter is devoted.

# Chapter 6

## The Coexistence Puzzle

### 6.1 Introduction

In existing mobile telecommunications markets throughout Europe an observable common phenomenon is that on-net prices are considerably below off-net prices. This is due to access charges above marginal cost, the reason for which has been discussed at the end of Chapter 4. At the same time, telecommunications services appear to be very close substitutes. Indeed, voice telephony services offered by different network operators can hardly be argued to show any significant characteristics of horizontal product differentiation. It seems reasonable, therefore, to treat these services as homogeneous. A consequence of this is that the results of the standard models, including their extensions such as in Chapters 3 and 4, are no longer applicable, since a common feature of these models is the nonexistence of equilibrium under close substitutability.

A study of market share dynamics for fixed pricing structures in the absence of product differentiation has been provided in the last chapter. Recall Theorem 1 for the three-networks case, and the analogous analysis for the case of two networks. It is easy to generalize this theorem for an arbitrary number of networks and asymmetric prices (but keeping each network's on-net price below its off-net price).

**Theorem 6** *Assume  $n \geq 2$  symmetric networks compete in the market, behavior of market shares is given by the best response dynamics, and each*

*network  $i$  offers on-net price  $p_i$  and off-net price  $q_i$  with  $p_i < q_i$ . Then in the long run a single network covers the market.*

*Proof:* Assume the initial state is  $\mathbf{x} \in S_n$ . The average price  $P_i(\mathbf{x})$  paid by network  $i$ 's customers is  $P_i(\mathbf{x}) = p_i x_i + q_i(1 - x_i) = q_i - (q_i - p_i)x_i$ , which is decreasing in  $x_i$ . The network currently offering the lowest average price will grow, and this lowers its average price even further. All other networks lose market share and thereby their average price rises. In the long run then, a single network covers the market. *QED*

As we have argued previously, it is a straightforward consequence of on-net prices below off-net prices, that positive tariff-mediated network externalities drive the initially largest network to eventually corner the market. Given that network services are homogeneous, we should expect to see the respective markets to be dominated by a single network. Nevertheless, several networks seem to coexist in these markets for years.

One obvious objection to this “coexistence puzzle” is that real-world markets are not yet in equilibrium. However, anecdotal evidence suggests that not even the movement of market shares is in line with our predictions. In many national markets, large established networks loose market share to newcomers or younger and smaller competing networks. How can this happen? In this chapter we suggest a solution to the coexistence puzzle which relies on the rejection of the balanced calling patterns assumption.

## 6.2 Local Interaction Models

All the models we studied up to now were based on the assumption of balanced calling patterns. Recall that this assumption means that each customer in the market calls each other customer, or at least that he has the same probability of calling each other consumer. This assumption was necessary to allow us to treat the interaction as a population game. For each customer the mixed strategy to best respond to is the average strategy in the population, i.e. the population state. As a consequence, if it is optimal for a customer to choose network  $i$ , then this is optimal for all customers, since all are playing the same game. Consider an extreme opposite case, where the whole population is split up into pairs of consumers and each consumer

exclusively calls his partner. It is clear that with this kind of calling patterns, under the assumptions of Theorem 6, each pair of consumers will coordinate on one of the networks after at least one consumer in each pair has received a switching opportunity. The long run market share of a network depends on the initial state, but can be any fraction in  $[0, 1]$ .

In real populations, the interaction structure is somewhere between these two extreme cases. While calling patterns will definitely not be balanced in the strict sense, most people typically make phone calls to more than one partner. A quick and non-representative survey among the author's acquaintances suggests that people usually direct a large majority of their private monthly phone call minutes to only a handful of close friends or relatives, while a rather small minority of call minutes terminates in the rest of the population. Such a pattern of interaction is called *local*, as opposed to the *global* interaction structure leading to balanced calling patterns. Consequently, evolutionary or social learning models assuming that each agent interacts only with a finite set of other agents are called *local interaction models*. The game theoretic literature on local interaction models started with Ellisons's (1993) extension of the basic stochastic evolution models of Kandori et al. (1993) and Young (1993). Important results are due to Blume (1993, 1995), Goyal (1996), Berninghaus and Schwalbe (1996), Young (1998), and Ellison (2000). Closely related are recent studies of the *contagion* effect, see Morris (2000) or Lee and Valentinyi (2000). The economics literature on local interaction is surveyed by Brock and Durlauf (2001). In the following we give a general definition of a local interaction game in the context of telecommunications.

Let  $X$  be a countable population of agents, where each agent  $x$  represents a consumer in a telecommunications market. Let  $\sim$  be a binary relation on  $X$ . This relation is meant to describe who makes phone calls to whom. We assume that each agent only calls his "friends", where  $x'$  is a friend of  $x$  if  $x \sim x'$ . We assume that friendship is irreflexive,  $x \not\sim x$ , and symmetric,  $x \sim x' \implies x' \sim x$ . Thus no agent calls himself, and each agent is the friend of each of his friends. Let  $F(x)$  be the set of  $x$ 's friends. We assume that each agent has at least 1 and at most  $m$  friends,  $1 \leq |F(x)| \leq m$ . Each agent can choose between  $n$  networks in  $N = \{1, \dots, n\}$ , where network  $i$  offers prices  $p_{ij}$  for calls to networks  $j \in N$ . Time proceeds in discrete steps  $t = 0, 1, 2, \dots$ , and for simplicity we assume that each agent receives a switching opportunity

at every time step. The state  $S$  of the population is here not the vector of market shares but the exact distribution of networks on the population of agents, i.e. a function  $S : X \rightarrow N$ , where  $N = \{1, \dots, n\}$ , giving for each agent  $x$  his network choice  $S(x)$ . The system starts with an arbitrary initial state  $S_0$ . At each time  $t + 1$  each agent  $x \in X$  chooses a best response to  $S_t$ , i.e. a network  $i$  minimizing his total payment  $P_i(x, S_t)$ :

$$S_{t+1}(x) \in BR(x, S_t) \equiv \operatorname{argmin}_{i \in N} P_i(x, S_t),$$

where

$$P_i(x, S_t) = \sum_{x' \in F(x)} p_{i, S_t(x')}.$$

A state  $S$  is a Nash equilibrium if no agent has an incentive to deviate from his choice. Clearly,  $S$  is a Nash equilibrium if and only if  $S \in BR(\cdot, S)$ .

The setting is very general up to now. However, we concentrate on a particularly simple model in order to isolate the differences in long run behavior as compared to the last chapter. Thus we consider only the two-networks case  $n = 2$ . Moreover, we assume again a symmetric pricing structure  $p_{11} = p_{22} = p$  and  $p_{12} = p_{21} = q$  with on-net price below off-net price,  $p < q$ , as in Theorem 6. Finally, we assume that each agent has exactly three friends<sup>1</sup>, i.e.  $|F(x)| = 3$  for all  $x \in X$ . With these specifications, the long run behavior of the population state depends — apart from the initial state — only on the exact graph of the friendship relation on  $X$ . One particular such graph is exemplified in the next section.

## 6.3 A Simple Hexagonal Graph Structure

Let  $X = \mathbb{Z}^2$  and define the following binary relation on  $X$ :

$$\begin{aligned} (x_1, x_2) \sim (x'_1, x'_2) \quad \Leftrightarrow \quad & [|x'_1 - x_1| = 1 \wedge x'_2 = x_2] \\ & \vee [x_1 + x_2 \text{ is even} \wedge (x'_1, x'_2) = (x_1, x_2 + 1)] \\ & \vee [x_1 + x_2 \text{ is odd} \wedge (x'_1, x'_2) = (x_1, x_2 - 1)] \end{aligned} \tag{6.1}$$

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<sup>1</sup>An odd number of friends helps to avoid the knife-edge case of indifference between the networks.

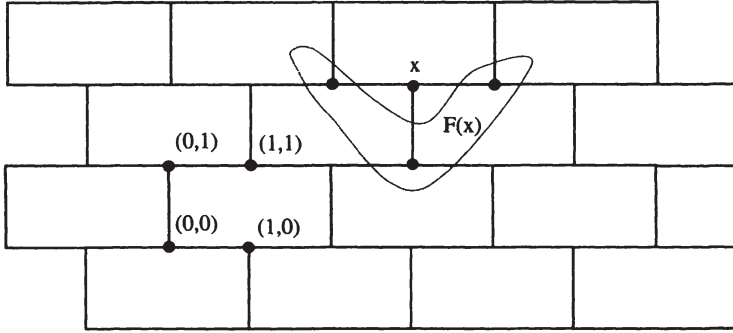


Figure 6.1: The graph of the friendship relation given in (6.1)

With these specifications each point in the plane with integer coordinates corresponds to an agent, and each agent has three friends: His immediate left neighbor, his immediate right neighbor, and alternatingly his immediate upper or lower neighbor, respectively. The corresponding graph is illustrated in Figure 6.1. Of course this graph is just one particular example. Indeed, even if  $X$  is taken to be finite, the size of the set of all graphs meeting the assumptions of the last paragraph grows exponentially in the number of elements of  $X$ . The graph we study here is particularly simple because of its obvious translation invariance. For each agent  $(x_1, x_2)$  with even sum  $x_1 + x_2$  we call the set of the six agents

$$\{(x_1, x_2), (x_1 + 1, x_2), (x_1 + 2, x_2), (x_1, x_2 + 1), (x_1 + 1, x_2 + 1), (x_1 + 2, x_2 + 1)\}$$

a *hexagon*. A hexagon thus consists of the six agents on the boundary of one of the rectangles visible in Figure 6.1.

It is now easy to see the following:

**Lemma 2** *Let  $H$  be a hexagon and assume  $S_T(x) = i$  for all  $x \in H$ . Then  $S_t(x) = i$  for all  $x \in H$  and all  $t \geq T$ .*

In other words, if all agents in a hexagon use the same network  $i$  at time  $t = T$ , then they will use the same network  $i$  at any time  $t \geq T$ . The reason for this is that each agent  $x$  in a hexagon has exactly two friends in the same hexagon. So if all agents in a hexagon use network  $i$ , then at least 2/3 of the



friends of  $x$  use network  $i$ . Therefore  $x$ 's total payment when subscribing to network  $i$  is strictly smaller than when subscribing to network  $j$ , as<sup>2</sup>

$$P_i(x, S_i) = 2p + p_{ik} \leq 2p + q < 2q + p \leq 2q + p_{jk} = P_j(x, S_i),$$

where  $k$  is the network used by the third friend of  $x$ .

This simple result has a deep impact on the nature of Nash equilibria of our local interaction game. The next theorem follows immediately from this lemma and the observation that the complement in  $X$  of the union of hexagons is again a union of hexagons.

**Theorem 7** *Let  $M$  be the union of arbitrary many hexagons in  $X$ . Define the state  $S^*$  by  $S^*(x) = i$  for all  $x \in M$  and  $S^*(x) = j$  for all  $x \in X - M$ . Then  $S^*$  is a Nash equilibrium.*

This result says that virtually *any* distribution of market shares on the two networks can be generated in equilibrium. Of course this plethora of equilibria is devastating for the predictability of market shares, even for given prices.

## 6.4 Discussion

As mentioned before, the graph we studied in this chapter is just one particularly simple example. As such, it does not show some of the characteristics of calling patterns found in real societies, e.g. the “small world” phenomenon (see Watts, 2000). Nevertheless, in its basic structure it comes close to what might be considered the important such characteristics. The decisive assumption here is that a person typically has only a handful of frequent calling partners, whereas only a minority of its call minutes are distributed more or less randomly over the rest of the population. If this person's friends happen to coordinate on a single network, then the incentive for it to subscribe to the same network is overwhelming, if only on-net prices are significantly below off-net prices. Since it is not unrealistic (albeit ruled out in our model) to assume that small groups of friends will tend to deliberately coordinate

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<sup>2</sup>It is straightforward to extend this argument to an arbitrary (but odd) number of friends under the symmetry assumptions made: Each agent will join the network the *majority* of his friends has joined.

in this way, there is considerable room for smaller networks to gain market share, despite the positive feedback loop which would inevitably favor large networks under balanced calling patterns.

The analysis of networks' strategic price setting behavior under a local interaction structure, however, is made extremely difficult by the multitude of existing consumer equilibria. While we have tried to answer some of the questions arising in the study of the economics of two-way interconnection, many more such questions became apparent. This research issue, therefore, is still far from being settled.



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Georgia Chioni

# The Rise and Fall of Satellite Personal Communication Systems

## Business and Legal Issues

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Satellite personal communications were expected to form a very successful market, reaching many millions of customers. The market functioned, however it has been, instead, a big business failure. The book examines the rise and fall of the Satellite Personal Communication Systems (SPCS) from an interdisciplinary approach. It describes the technology of a market formed on the basis of heavy investment, which traded international personal communication capacity through the use of mobile satellite networks. Moreover, it analyses the main actors and their business strategies that have lead pioneers to failure. In addition, it focuses on the regulatory efforts, which proved unsuccessful in saving the market from collapsing. The analysis, based on the interaction of various parameters (technology, law, actors and market) forms an interesting level-playing field.

*Contents:* Satellite Personal Communications · Satellite Technology · Global Economic Networks-Strategic Alliances · Telecommunications · Legal framework of Satellite Communications · Business Strategies in Satellite Personal Communications · Iridium

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