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Inter-temporal Calculative Trust Design to Reduce Collateral Need for Business Credits

Silu Muduli, Shridhar Kumar Dash*

Credit rationing arising out of informational asymmetry and lack of collateral is a well-recognised economic constraint in the credit market. These constraints get magnified for small businesses. This paper attempts to capture the dimension of trustworthiness (calculative trust) by designing a multi-period, incentivised payment structure that will induce economic agents to reveal the existence of private information about any projects or true intentions of paying up the credit that is going to fund the project. The model dynamically estimates the collateral needed by taking into account the truthfulness of the borrower. The proposed design is compared with the benchmark model - credit scoring-based model. Randomised simulations are carried out for the ex ante solution for the borrower. We find that the proposed design outperforms from the perspective of lenders when the probability of default of any project is less than 80 per cent. Our simulation result also finds that building trust helps small business owner to significantly reduce the need for collateral.

JEL Classification : M21, R51, G21
Keywords : Calculative trust, collateral

Introduction

Creation of jobs is one of the most important political/economic issues facing developing economies. The progress of Small and Medium Enterprises (SMEs) is imperative for employment growth because, across developing economies, SMEs are a key employment generation sector (Chu, Benzing and McGee, 2007; Lee, 1998; Lin, 1998). This sector grows when entrepreneurs invest in new projects. Retained earnings may not be sufficient to meet the

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capital needed for growth and therefore additional capital may need to be raised in the form of debt. However, small businesses encounter financial constraints while raising capital to fund their growth (Panda and Dash, 2014; Thampy, 2010).

Constraints faced by small businesses are more binding due to the perceived credit risks associated with them. Credit risk can arise from two sources. First, risk arising from the inability of the project to pay back the loan, and, second, risk arising from unwillingness of the business owner to pay back the loan. There is a well developed risk mitigation strategy for a project’s inability to pay back the loan, driven by intermediaries such as credit rating agencies, credit bureaus, and internally-developed scorecards. On the other hand, the risk mitigation strategy with respect to unwillingness of a business owner to pay back the credit is not well developed. This is the grey area in the underwriting processes followed by lending institutions.

Willingness to pay back can be traced to trustworthiness of the business owner. This kind of trustworthiness can be visualised as calculative trust as discussed in Lewicki et al., (2006). One can argue that any economic relationship will start with a principal (lending institution) calculating the trustworthiness of the agent (borrower) on the basis of the perceived credit risk associated with dealing with an agent. As the principal deals with the agent over time, set of information with the principal expands, and the trustworthiness of the agent in the eyes of the principal changes. These inter-temporal changes in calculative trust have not been adequately captured in the available literature. Besides, they have not been modelled in a way to understand the risk arising out of an unwillingness to pay back the loan.

The inter-temporal dimensions of the perceived credit risk can also be looked at from a perspective of private information available with a business owner. This asymmetry of information between borrower and lender will lead to adverse selection (ex ante), and the creation of a moral hazard (ex post) problem. To deal with the asymmetric information problem, lending institutions will demand collateral from all types of borrowers (including the good ones). Non-availability of sufficient collateral may in turn result in non-availability of credit to the potentially good quality borrower. This is the classical credit rationing problem put forward by Stiglitz and Weiss (1981). The asymmetry of information problem is particularly binding in the case of SMEs, as they also face lack of collateral. Therefore, one can argue that small business owners are likely to face acute credit rationing.
This paper develops an incentivised metrics, which induces the small business owner to reveal certain amount of private information so that the lender can assess the risk associated with the borrower’s unwillingness to pay back the loan. The paper does not address the credit risk arising out of genuine risk of failure of the project due to external socio-economic and political circumstances. The paper assumes that these kinds of genuine risks can be taken care of by credit default scores, and pricing of credit. The paper is organised into five sections. Section II provides a review of the literature. Section III discusses the model, the \textit{ex ante} solution, and simulation results. Section IV provides the \textit{ex post} solution for the borrower and Section V concludes the paper.

\textbf{Section II}

\textbf{Literature Survey}

Non-availability of financial resources is a major constraint faced by start-ups and SMEs in developing countries (Cook, 2001; Gray, Cooley and Lutabingwa, 1997; Levy, 1993; Peel and Wilson, 1996). It is perceived that SMEs belong to a high risk credit category and hence this sector finds it difficult to raise debt capital to fund its business growth (Thampy, 2010). Stiglitz and Weiss (1981) showed that credit rationing will happen due to the existence of information asymmetry. The principal and agents have different sets of information about the project for which credit is needed. Though both are rational, they might not have sufficient incentives to work towards a common goal (Akerlof, 1970; Ross, 1973). However, the lender solves the problem of information asymmetry through contract and monitoring (Sahlman, 1990).

The purpose of monitoring is to make sure that there is an incentive for compatibility between the borrower and the lender so that the borrower does not take undue risk. The success of the contractual mechanism to reduce the agency risk depends on the completeness of the contract, and its enforcement. However, enforcement of the contract is a challenge, and is costly (Lawton, 2002).

In addition to an appropriate contract, and monitoring, the lender can ask for collateral to reduce the risk due to the failure of a funded project. The need for collateral as a means to reduce credit risk depends on the structure of the credit market (Besanko and Thakor, 1987; Bester, 1985). Chen (2006) showed that the riskier borrowers pledge higher collateral than safe borrowers, and
Jimenez and Saurina (2004) found evidence that highly collateralised loans have higher probability of default. The literature on the relationship between *ex ante* demand for collateral and *ex post* risk associated with the loan is inconclusive. Therefore, taking high amount of collateral may not reduce the credit default *ex post*. Logically, one can argue that the willingness to pay back plays an important role in assessing the risk associated with any borrower. A borrower who has low pledgeable collateral and is willing to pay back the loan is likely to be a safer borrower in comparison to a borrower with highly pledgeable collateral but with an unwillingness to pay back the loan.

An alternative way to addresses the asymmetry of information problem has been looked from the perspective of trust. Das and Teng (1998), Shepherd and Zacharakis (2001), and Vosselman and Van der Meer-Kooistra (2009) argue that the agency risk can be reduced by employing both contractual mechanisms and trust building in a dyadic relationship. Literature on uses of contract and trust can be visualised from three perspectives. First, the presence of a higher trust level will drive lower levels of control and a less stringent contract (Dyer and Singh, 1998). Second, contractual control increases the trust level, hence contractual control and trust are complementary to each other (Leifer and Mills, 1996; Poppo and Zenger, 2002). Third, trust itself is a type of control mechanism, hence both are substitutes (Bradach and Eccles, 1989).

Most of the literature on trust, with respect to the asymmetry of information problem, is discussed in the context of the relationship between venture capitalists and entrepreneurship where there is a greater degree of uncertainty. Panda and Dash (2016) studied different stages of entrepreneurial ventures and found evidence for trust-based control by Indian venture capitalists in the early stage of a firm. They also found a combination of trust and control-based risk mitigation methods adopted by the venture capitalists in the late stages of a firm. The role of trust in building mutually beneficial cooperation has been explored in the case of bank-entrepreneur relationships (Saparito *et al.*, 2004). At present, there is a lack of literature looking at trust in the context of both lender’s and borrower’s perspectives. This paper attempts to bring trust, which builds over time, to calculate the need for collateral. This model takes the concept of calculative trust, as discussed in Lewicki *et al.*, (2006), to facilitate trust building between a borrower and a lender. The proposed model is an attempt to mitigate the need for collateral by incentivising the borrower to be trustworthy over time. This model has
the objective of reducing credit constraints for individuals (non-wilful defaulters) without the availability of hard information. Since many of the credit institutions demand collateral from borrowers, the trustworthy borrowers face credit constraints. The model suggests a methodology to reduce this credit constraint. The model is discussed in the following section.

Section III
The Model

A design in the world of information asymmetry is a procedure to nudge the agent to behave in a pre-specified way, without cohesion. The importance of design can be seen in the Vickrey–Clarke–Groves (VCG) mechanism (Vickrey, 1961; Clarke, 1971; Groves, 1973), where the design helps us to address the following twin objectives: (a) the efficient allocation of public goods among agents which (b) forces them to reveal the true value of these public goods. This is possible by appropriately incentivising the agent to reveal the truth. Our model is inspired by the VCG mechanism to incentivise the borrower to reveal private information about the project for which the loan is sought.

At the time when a borrower seeks a loan for a project, it pays for the borrower to communicate in a way that will positively influence the lender to decide on giving the credit. For example, the borrower may overestimate the project cash flows to show a favourable picture in support of the project. The lender will suffer when a project with inflated cash flows is provided credit, and ultimately the project fails and the lender suffers losses. The proposed model creates a design where the borrower is disincentivised to inflate the cash flow numbers, and will make an attempt to tell the truth about the cash flow of the project (according to the information available with the borrower). Furthermore, our design incentivises the borrower not to deviate from the ex ante promises.

Parameters of the Model

The model assumes a rational borrower who has \( n \) projects. Each project has a probability of default, which is represented by \( \theta_i \in (0,1] \) for \( i \in \{1,2,3,...,n\} \). The \( \theta_i \) is exogenous to our model, and is known to the lender. The borrower approaches the lender. The lender does not have any creditworthiness information about the borrower, and finances one project at a
time. Funding of the project is done if a previous loan is paid in full, including interest as per the due date. The borrower’s tuple is \((B_i, T_i, W_i)\) for the project \(i \in \{1, 2, 3, \ldots, n\}\), where \(W_i\) is the borrower’s pledgeable asset, \(B_i\) is the amount of loan for project \(i\) for a time period \(T_i\). Similarly, the lender’s tuple is \((\overline{B_i}, a_i, r_i)\), where \(\overline{B_i}\) represents the upper bound of the loan sanctioned for the project \(i\), \(a_i\) represents fractions of the \(B_i(1 + r_i)T_i\) needed as collateral, and \(r_i\) is the interest rate of the loan. The interest rate for the project \(i\) depends upon the riskiness \(\theta_i\) of the project \(i\). Keeping in mind the credit risk, the sanctioned loan amount for each project is constrained by the expected cash flows from the project and the \(W_i\) available at that period.

The Design

Project – \(i\)

Loan for project \(i\) is sanctioned if all loans availed previously are repaid in full. The borrower has pledgeable asset \(W_i\). The project cash flow is random and distributed uniformly in \([\underline{y}_i, \overline{y}_i]\). The probability of default \(\theta_i\) for the project \(i\) is determined by the lender from its past experience of similar projects. For this project the lender charges interest rate \(r_i\) such that:

\[
r_i = r_f + r_p(\theta_i), \quad r_p(\theta_i) \geq 0, \quad \frac{dr_p(\theta_i)}{d\theta_i} > 0
\]

where \(r_f\) is risk-free rate and \(r_p(\theta_i)\) is risk premium which depends upon the probability of the default of the project \(i\). The upper bound for loan amount \(\overline{B_i}\) is decided by the lender for time period \(T_i\) by

\[
\overline{B_i} = \min \left\{ \frac{1}{\alpha_i} \left( \frac{\overline{y}_i + \underline{y}_i}{2} - \overline{C} \right), \sum_{i=1}^{r_i} \frac{1}{(1 + r_f)^i} \frac{1}{\alpha_i (1 + r_i)^{\theta_i}} \right\} = \overline{B_i}
\]

\(\alpha_i \in (0, 1]\) is percentage of \(B_i(1 + r_i)T_i\) that the lender needs to keep as collateral for the project \(i\); \(\overline{C}\) is subsistence consumption of the borrower. For project 1, it is assumed to be equal to 1, because the lender wants to mitigate all the risk by demanding 100 per cent of the loan amount, including interest, as collateral. Besides, a binding collateral requirement exists because the lender does not have any prior information of trustworthiness of the borrower. This construct is contrary to Bester (1985), which finds that higher collateral requirements will attract high-risk borrowers. Bester’s finding is static, while
our model takes an inter-temporal approach. Therefore, in the second period (when the borrower seeks funding for project 2), the collateral needed will be dynamically determined, and will come down if the borrower honours the reported payment schedule for project 1.

- The borrower pays the collateral \( \alpha_i B_i (1 + r_i)^t \) for the project \( i \).
- The borrower is asked to report a payment schedule:

\[
P_i = \{(P_{i1}, P_{i2}, \cdots, P_{iT_i-1}, P_{iT_i}) \in \mathbb{R}_+^{T_i}; \sum_{t=1}^{T_i} \frac{P_{it}}{(1 + r_i)^t} = B_i \}
\]

for the project \( i \). Here \( P_{it} \) represents reported payment for the time period \( t \) for the project \( i \), \( \mathbb{R}_+ \) is the set of non-negative real numbers.

- The borrower pays \( P'_{i1} \geq 0 \), and this payment is independent of \( P_{i1} \) in time 1, pays \( P'_{i2} \geq 0 \) in time 2, and so on.

- If

\[
\sum_{t=1}^{T_i} \frac{P'_{it}}{(1 + r_i)^t} < B_i
\]

the lender liquidates the collateral and retains an amount \( (M_i) \) from the proceeds from liquidation of the collateral such that

\[
\sum_{t=1}^{T_i} \frac{P'_{it}}{(1 + r_i)^t} + \frac{M_i}{(1 + r_i)^{T_i}} = B_i
\]

- Let \( T'_i \) be the time period in which the loan amount is fully paid. So \( T'_i \) can be less than or equal to \( T_i \).

Taking these into account, we define a term \( c_{it} \) that captures the trust-building process between the borrower and the lender in time \( t \) for the project \( i \),

\[
c_{it} = \begin{cases} 
0 & : P'_{it} = P_{it} = 0 \\
\frac{P'_{it}(P'_{it} - P_{it})}{P'_{it} + P_{it}} & : \text{Otherwise}
\end{cases}
\]

\( \forall \ t \in \{1,2,3,\cdots, T'_i\} \). As \( c_{it} \) increases over the period, the trustworthiness increases. We use this to incentivise or disincentivise the borrower depending on the behaviour of the borrower over time. The construct of the incentivising scheme is provided below.

Let \( n_{it} \) be the cardinality of the set \( S = \{c_{ij} : j = 1,2,3,\cdots, t - 1 & c_{ij} \geq 0\} \). This, \( n_{it} \), is the number of periods in which the borrower had kept its promise before time for the project.
Let’s define the following *reward–penalty function*:

\[
\beta_{it} = \begin{cases} 
    c_{it}^{2(t-n_{it})-1} : & c_{it} < 0, n_{it} \geq 0 \\
    \frac{1}{c_{it}^{1+n_{it}}} : & c_{it} > 0 
\end{cases}
\]  

(1)

For \( t = 2, 3, 4,...,T_i \) and \( \beta_i = c_i \).

The variable \( n_{it} \) plays a very important role in *reward–penalty function* \( \beta_{it} \). It controls the magnitude of penalty or reward depending on the promised payment schedule and the actual payment schedule. The complexity arises when \( n_{it} = 0 \), because \( n_{it} \) is 0 when the borrower pays everything in the first payment or when the borrower has not kept any promises. The former is trustworthy behaviour which is rewarded in our design and the latter is not trustworthy behaviour and is penalised in our design.

**Explanation for \( \beta_{it} \)**

Value of \( \beta_{it} \) is directly related to the trustworthiness of a borrower. Calculation of \( \beta_{it} \) depends on the behaviour of the borrower with respect to the promise that is made *ex ante*. While calculating \( \beta_{it} \), we have separated three types of borrowers for rewarding and penalising.

\( c_{it} < 0, n_{it} = 0 \): These are types of borrowers who historically have not kept any promises, and are not keeping the promise in time period \( t \) as well. Borrowers of these kinds are not trusted, and are asked to provide 100 per cent of the loan amount as collateral.

\( c_{it} < 0, n_{it} > 0 \): These are types of borrower who have kept their past promises at least once, but are not keeping the promise in time period \( t \). Borrowers of these kinds are not trusted fully and are penalised by asking for higher amount of collateral. The degree of penalty depends on the number of times promises are kept.

\( c_{it} \geq 0 \): This is a type of borrower who is keeping promises at time period \( t \). In this case \( n_{it} \) negatively affects the magnitude of reward, and the design incentivises the borrower to pay back the loan as early as possible.

If \( c_{it} = 0 \), for all \( t = 1,2,3,\cdots,T_i \), the borrower kept the promise and, hence, has to be rewarded. Then the lender asks for collateral \( \hat{\alpha} \sim U(0,1) \), *i.e.*, distributed uniformly over \((0,1)\), as a percentage of the amount \( B_{i+1}(1 + r_{i+1})^{i+1} \) for project \( i + 1 \). However, our objective is to reduce the collateral ratio by incentivising
the borrower to prepay the credit by deviating from the promised payment positively. Therefore, we have taken $E(\alpha) = 0.5$ as the collateral requirement ratio. In case $c_t \neq 0$ for some $t$, we define

$$\alpha_{t+1} = \frac{1}{\max\{1, 1 + \sum_{t=1}^{\tau_i} \beta_{it}\}}$$

For the next project $i + 1$, lender asks for $\alpha_{i+1}B_{i+1}(1 + r_{i+1})^{T_{i+1}}$, as value of collateral.

**Implications of the Design**

**Proposition 1:** It is optimal strategy for the borrower to report payment scheme $0, 0, 0, \ldots, 0, B_i(1 + r_i)^{T_i}$ and pay within time period $T_i < T_r$.

**Proof.** Let

$$\Delta_{it} = P'_it - P_{it}$$

If the borrower pays the loan amount (no default case), then

$$\sum_{t=1}^{T_i} \frac{\Delta_{it}}{(1 + r_i)^t} = 0 \tag{2}$$

and,

$$c_{it} = \frac{P'_it\Delta_{it}}{P'_it + P_{it}}, \forall t \in \{1, 2, 3, \ldots, T_i\}$$

One of the trivial solutions that satisfy Eq. (2), $\Delta_{it} = 0$ for all $t$. If $\Delta_{jt} < 0$ for some time period $j \in \{1, 2, 3, \ldots, T_i\}$, then it will increase $\alpha_{i+1}$. To have each $\Delta_{it} \geq 0$ and to minimise $\alpha_{i+1}$, the borrower should report to pay 0 till $T_i - 1$. Since $\Delta_{it} \geq 0$ for all $t < T_i - 1$ then $\Delta_{iT_i} < 0$. This will reduce the value of $\alpha_{i+1}$. Now the borrower thinks of paying the entire credit before $T_i$, so that the bank will not take $\Delta_{iT_i} < 0$ into consideration for calculation of $\alpha_{i+1}$. Therefore, the optimal strategy will be to report $0, 0, 0, \ldots, 0, B_i(1 + r_i)^{T_i}$ and pay within time period $T_i < T_r$.

Taking optimal reporting from Proposition-1 into account $P_{it} = 0, \forall t < T_i - 1$, therefore

$$c_{it} = \frac{P'_i(P'_it)}{P'_it + P_{it}} = P'_it$$

Hence,

$$\beta_{it} = c_{it}^T = \frac{1}{\beta_{it}}$$
**Ex ante Solution for the Borrower**

The Proposition-1 helps us to know the *ex ante* payment schedule of a rational borrower. However, the same rational borrower will minimise *ex post* $\alpha_{p,t}$. The *ex ante* optimisation problem for the borrower will be

\[
\min_{c_{1t}, c_{2t}, \ldots, c_{T_t-1}} \frac{1}{\max\{1, 1 + \sum_{t=1}^{T_t-1} c_{it}^\frac{1}{t}\}}
\]

Such that,

\[
\sum_{t=1}^{T_t-1} \frac{c_{it}}{(1 + r_t)^t} = B_t
\]

\[
c_{it} \geq 0
\]

Equivalently,

\[
\max_{c_{1t}, c_{2t}, \ldots, c_{T_t-1}} \sum_{t=1}^{T_t-1} \frac{c_{it}^\frac{1}{t}}{}
\]

Such that,

\[
\sum_{t=1}^{T_t-1} \frac{c_{it}}{(1 + r_t)^t} = B_t
\]

\[
c_{it} \geq 0
\]

Clearly, the function $\sum_{t=1}^{T_t-1} \frac{1}{t} c_{it}$ is a strictly concave function, hence the first order conditions solution will be sufficient for the optimal solution and it will be unique.

Using the Lagrangian multiplier method we can solve the above problem.

Lagrangian of the optimisation problem is given by:

\[
L_1 = \sum_{t=1}^{T_t-1} \frac{c_{it}^\frac{1}{t}}{} - \lambda_1 \left( \sum_{t=1}^{T_t-1} \frac{c_{it}}{(1 + r_t)^t} - B_t \right)
\]

Here ‘$\lambda_1$’ is the Lagrange multiplier. From the first order conditions, the solution of the above problem will be

\[
\lambda_1 = 1 + r_t
\]
The optimal payment scheme for borrower $P_{it}'$ is given by:

$$p_{it}' = c_{it}' = \frac{(1+r_{it})^t}{t!}, \forall t = 2,3,4,\ldots, T_i - 1.$$ (4)

$$p_{it}' = c_{it}' = B_i - \left( \sum_{t=2}^{\tau_i-1} \frac{c_{it}^t}{(1+r_{it})^t} \right)(1 + \tau_i)$$ (5)

The above optimal solution from our design incentivises the individual to create a payment schedule that decreases over time.

**Comparison with the Benchmark Model**

To showcase the value of the proposed design, we have compared the results of our model with a benchmark model. The benchmark case is where a lender provides loans to projects having probability of default less than $k$. The probability of default is predetermined exogenously. The lender takes collateral before providing the loan. Let the collateral amount be $\phi B_i (1 + r_i)^{T_i}$ for project $i$, where $\phi \in [0,1]$ which is a policy parameter that decides collateral amount. In case of default, the lender recovers some portion of the loan amount by liquidating the collateral. The liquidating factor is $\delta_i$, which lies in $[0,1]$.

Then expected profit $\pi$ for the risk-neutral lender under the benchmark model can be written as:

$$E(\pi_{\text{Benchmark Model}}) = \sum_{i \in [\mathcal{J} \cap \mathcal{J}_k]} [(1 - \theta_i)B_i(1 + r_i)^{T_i} + \theta_i \delta_i \phi B_i (1 + r_i)^{T_i}]$$

Similarly, in the proposed model, we keep asset as collateral, which has a liquidation factor $\gamma_i \in [0,1]$ for the project $i$. The expected profit for the risk-neutral lender when the proposed model is used can be written as:

$$E(\pi_{\text{Model}}) = \sum_{i = 1}^{n} [(1 - \theta_i)B_i(1 + r_i)^{T_i} + \theta_i \gamma_i \alpha_i B_i (1 + r_i)^{T_i}]$$

Taking both expressions into consideration:

$$E(\pi_{\text{Model}}) - E(\pi_{\text{Benchmark Model}})$$

$$= \sum_{i \in [\mathcal{J} \cap \mathcal{J}_k]} [(1 - \theta_i)B_i(1 + r_i)^{T_i} + \sum_{t=1}^{T_i} \theta_i \gamma_i \alpha_i B_i (1 + r_i)^{T_i} - \sum_{i \in [\mathcal{J} \cap \mathcal{J}_k]} \theta_i \phi \delta_i B_i (1 + r_i)^{T_i}]$$

$$= \sum_{i \in [\mathcal{J} \cap \mathcal{J}_k]} [(1 - \theta_i)B_i(1 + r_i)^{T_i} + \theta_i \gamma_i \alpha_i B_i (1 + r_i)^{T_i} - \sum_{i \in [\mathcal{J} \cap \mathcal{J}_k]} \theta_i \phi \delta_i B_i (1 + r_i)^{T_i}]$$

By using these equations, we can put down the following proposition.

**Proposition 2:** The welfare of a lender (expected profit) will always be higher in the designed model when $\gamma_i \geq \frac{\phi \delta_i}{\alpha_i}$. 
Simulation Result

The objective of the simulation is to compare the performance of the proposed model \textit{vis-à-vis} the benchmark model. The performances are measured in terms of two outcomes. First, profit generated by the lender; and second, total amount of collateral needed as a percentage of total loans given. The simulation is done by coding the model in R software, and has been done for 1000 \((n = 1000)\) projects. Each project has a probability of default, which has been generated randomly from a uniform distribution \(\theta_i \sim U[0,1]\). The proposed model is applied to generate designed collateral metrics for different projects. The result of the same has been plotted in Chart 1 and Chart 2.

**Chart 1: Distribution of \(\alpha\)**

![Chart 1](image)

\(N = 1000\) \hspace{1cm} Bandwidth = 0.004211

**Chart 2: Value of \(\alpha\) for Different Projects**

![Chart 2](image)
The amount of collateral required is \( \theta_iB_i \) and depends on \( \phi \), which is a policy parameter.

Notes: *p - value < 0.1; **p - value < 0.05; ***p - value < 0.01.

In the case of the benchmark model we have taken two cases. In the first case, the collateral requirement depends upon the values of \( \theta \), \( s \), and the amount of collateral required is \( \theta B_i (1 + r)^t \) where the amount borrowed is \( B_i \) (Table 1). In second case, collateral needed is independent of the default risk of the project (Table 2) and depends on \( \phi \), which is a policy parameter.

Table 1: Simulation Result

<table>
<thead>
<tr>
<th>Loan Amount</th>
<th>( k = 0.05 )</th>
<th>( k = 0.30 )</th>
<th>( k = 0.5 )</th>
<th>( k = 0.8 )</th>
<th>( k = 0.9 )</th>
<th>( k = 0.99 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0–25 lakhs</td>
<td>12493.85***</td>
<td>11348.91***</td>
<td>9155.42***</td>
<td>2528.79***</td>
<td>948.59***</td>
<td>-1820.46***</td>
</tr>
<tr>
<td>0–50 lakhs</td>
<td>24982.67***</td>
<td>22667.89***</td>
<td>18299.05***</td>
<td>5220.70***</td>
<td>1963.96***</td>
<td>-3713.25***</td>
</tr>
<tr>
<td>0–100 lakhs</td>
<td>49872.13***</td>
<td>45475.22***</td>
<td>36589.55***</td>
<td>10450.63***</td>
<td>3784.37***</td>
<td>-734.23***</td>
</tr>
</tbody>
</table>

Table 2: Simulation Result with Policy Parameter \( \phi \)

<table>
<thead>
<tr>
<th>Policy Parameter</th>
<th>( k = 0.05 )</th>
<th>( k = 0.30 )</th>
<th>( k = 0.5 )</th>
<th>( k = 0.8 )</th>
<th>( k = 0.9 )</th>
<th>( k = 0.99 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 0.01 )</td>
<td>49748.63***</td>
<td>45647.11***</td>
<td>37804.07***</td>
<td>18457.90***</td>
<td>9851.07***</td>
<td>365.75***</td>
</tr>
<tr>
<td>( \phi = 0.02 )</td>
<td>50159.46***</td>
<td>45590.56***</td>
<td>37611.09***</td>
<td>18605.34***</td>
<td>9689.72***</td>
<td>256.72***</td>
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<tr>
<td>( \phi = 0.03 )</td>
<td>49787.31***</td>
<td>45720.78***</td>
<td>37709.13***</td>
<td>18214.72***</td>
<td>9706.17***</td>
<td>134.76***</td>
</tr>
<tr>
<td>( \phi = 0.04 )</td>
<td>49885.81***</td>
<td>45812.82***</td>
<td>37538.95***</td>
<td>18109.82***</td>
<td>9434.03***</td>
<td>-15.30***</td>
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<tr>
<td>( \phi = 0.05 )</td>
<td>50231.55***</td>
<td>45566.65***</td>
<td>37424.77***</td>
<td>17930.45***</td>
<td>9331.92***</td>
<td>-152.74***</td>
</tr>
<tr>
<td>( \phi = 0.06 )</td>
<td>50102.85***</td>
<td>45813.46***</td>
<td>37370.11***</td>
<td>17703.19***</td>
<td>9204.74***</td>
<td>-234.56***</td>
</tr>
<tr>
<td>( \phi = 0.07 )</td>
<td>50000.21***</td>
<td>45335.13***</td>
<td>37255.71***</td>
<td>17635.43***</td>
<td>9143.52***</td>
<td>-380.43***</td>
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<tr>
<td>( \phi = 0.08 )</td>
<td>49958.29***</td>
<td>45517.59***</td>
<td>37343.81***</td>
<td>17412.35***</td>
<td>9040.47***</td>
<td>-500.24***</td>
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<tr>
<td>( \phi = 0.09 )</td>
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<td>45271.22***</td>
<td>37458.35***</td>
<td>17120.41***</td>
<td>8971.95***</td>
<td>-651.28***</td>
</tr>
<tr>
<td>( \phi = 0.1 )</td>
<td>50124.56***</td>
<td>45750.08***</td>
<td>37210.02***</td>
<td>15675.07***</td>
<td>8998.34***</td>
<td>-754.20***</td>
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<td>( \phi = 0.2 )</td>
<td>50278.28***</td>
<td>45785.05***</td>
<td>36751.19***</td>
<td>14185.70***</td>
<td>6507.99***</td>
<td>-39672.21***</td>
</tr>
<tr>
<td>( \phi = 0.3 )</td>
<td>50114.05***</td>
<td>45476.90***</td>
<td>36716.65***</td>
<td>12611.07***</td>
<td>4624.84***</td>
<td>-6356.89***</td>
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<tr>
<td>( \phi = 0.4 )</td>
<td>50672.44***</td>
<td>44952.48***</td>
<td>35959.41***</td>
<td>10986.27***</td>
<td>2470.28***</td>
<td>-9035.63***</td>
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<tr>
<td>( \phi = 0.5 )</td>
<td>50057.41***</td>
<td>44952.46***</td>
<td>35220.93***</td>
<td>9489.68***</td>
<td>497.98***</td>
<td>-10476.30***</td>
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<tr>
<td>( \phi = 0.6 )</td>
<td>50454.53***</td>
<td>44770.08***</td>
<td>34486.60***</td>
<td>7495.72***</td>
<td>-1731.09***</td>
<td>-13562.17***</td>
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<td>( \phi = 0.7 )</td>
<td>50409.29***</td>
<td>44928.74***</td>
<td>33902.88***</td>
<td>6289.84***</td>
<td>-3598.10***</td>
<td>-16547.25***</td>
</tr>
<tr>
<td>( \phi = 0.8 )</td>
<td>50129.72***</td>
<td>44042.05***</td>
<td>33995.35***</td>
<td>4643.14***</td>
<td>-5497.62***</td>
<td>-19042.21***</td>
</tr>
<tr>
<td>( \phi = 0.9 )</td>
<td>50112.10***</td>
<td>44062.76***</td>
<td>32348.97***</td>
<td>3976.87***</td>
<td>-7861.64***</td>
<td>-21473.02***</td>
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<tr>
<td>( \phi = 1 )</td>
<td>50529.24***</td>
<td>43792.57***</td>
<td>31993.65***</td>
<td>3071.99***</td>
<td>-9744.46***</td>
<td>-23782.32***</td>
</tr>
</tbody>
</table>

Notes: *p - value < 0.1; **p - value < 0.05; ***p - value < 0.01.
as defined before in the model. Liquidating factor $\delta_i$ for collateral of the $i$th project is generated randomly from a uniform distribution over support $[0,1]$. For the designed model, we have assumed a random liquidating factor $\gamma_i$ which is distributed uniformly over $[0,1]$.

The difference $E(\pi_{\text{Model}}) - E(\pi_{\text{Benchmark Model}})$ has been calculated and the difference between expected profits is tested (one tailed t-test).

$$H_0: E(\pi_{\text{Model}}) - E(\pi_{\text{Benchmark Model}}) = 0$$

$$H_1: E(\pi_{\text{Model}}) - E(\pi_{\text{Benchmark Model}}) > 0$$

Table 1 shows that the proposed model does better when a project having a probability of default less than 0.9 is accepted by the benchmark model, and the benchmark model will do better for projects with the probability of default greater than 0.9 (which is an unlikely event). Table 2 shows that the proposed model provides better result than the benchmark model for all values of $\phi$ where $k < 0.8$. The benchmark model does better when the lender provides loans to very high risk projects, and demands a very high percentage of loan as collateral. The previous two simulations shown in Tables 1 and 2 find the dominance of the proposed model for all projects with a probability of default at 0.8. Table 3 shows the calculated values of collateral needed as a percentage of the total loan amount for the proposed model for different cut-off values of $k$. We can see that there exists very little difference across the different value of $k$, which means that the proposed design is independent of the default probability of projects. However, the lender can decide on a cut-off level of $k$, and after that decide on the collateral need using the proposed model.

<table>
<thead>
<tr>
<th>Loan Amount Limits (in ₹ lakh)</th>
<th>Collateral Ratio = Collateral Amount of Borrowing</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$k = 0.05$</td>
</tr>
<tr>
<td>0 - 25</td>
<td>8.5</td>
</tr>
<tr>
<td>0 - 50</td>
<td>7.5</td>
</tr>
<tr>
<td>0 - 100</td>
<td>8.1</td>
</tr>
</tbody>
</table>

Table 3: Collateral Need as a Percentage of Total Borrowing for the Proposed Model
Section IV

Ex post Solution for the Borrower

Our _ex post_ solution assumes Proposition-1 to hold. Suppose in the first period, the borrower’s cash flow is $\psi$. In case $\psi \geq \bar{C} + P_{i1}'$, the borrower achieves the first best solution as discussed in the _ex ante_ solution for the borrower. If not (i.e., $\psi < \bar{C} + P_{i1}'$), then the borrower pays $P_{i1}' = C_{i1}' = \psi - \bar{C}$ and consumes the subsistence amount. In this case the optimisation problem becomes,

$$\max_{c_{i2}, c_{i3}, \ldots, c_{i(T-1)}} \sum_{t=2}^{T_i-1} \frac{1}{r} c_{it}$$

Such that

$$\sum_{t=2}^{T_i-1} \frac{c_{it}}{(1 + r_i)^t} = B_i - \frac{P_{i1}'}{1 + r_i}$$

$$c_t \geq 0$$

The Lagrangian of the above problem becomes

$$L_2 = \sum_{t=2}^{T_i-1} \frac{1}{r} c_{it} - \lambda_2 \left( \sum_{t=2}^{T_i-1} \frac{c_{it}}{(1 + r_i)^t} - B_i + \frac{P_{i1}'}{1 + r_i} \right)$$

Solving the first order condition, we have

$$P_{i1}' = c_{i1}' = \frac{(1+r_i)^t}{\lambda_2 t^{t-1}}$$

Putting it in the constraint, we have

$$Z(\lambda_2) = \sum_{t=2}^{T_i-1} \frac{1}{(\lambda_2 t)^{t-1}} - B_i + \frac{P_{i1}'}{1 + r_i} = 0$$

The function $Z(.)$ has the following characteristics

$$\frac{\partial Z}{\partial \lambda_2} < 0, \lim_{\lambda_2 \to 0} Z(\lambda_2) = \infty > 0, \lim_{\lambda_2 \to \infty} Z(\lambda_2) = -B_i + \frac{P_{i1}'}{1 + r_i} < 0$$

Thus there exists a unique positive solution for $\lambda_2$ of Eq.7.

Since Eq.7 is a non-linear equation, we need computational capabilities to find the optimal payment schedule. We have designed a programme in R to solve for the optimal schedule of the borrower, and the result is shown in Chart 3.

As we can see from Chart 3, in case the borrower is unable to pay the first best optimal payment, then the borrower can find the optimal solution for the remaining periods. The optimal schedule shows the borrower has the incentive to pay more in the next period in case the payment was missed in
the previous period, which will depend upon the cash flow from the project in the previous period. Thus, the proposed model incentivises the borrower not to default willingly, and pay back the loan as early as possible.

**Section VI**

**Conclusion**

The paper attempts to link mainstream literature on the need for collateral and credit rationing with management literature on trust building over time. The model compares the *ex ante* payment promise with *ex post* payment structure for trust building. This is done by creating a design, which incentivises the behaviour of honouring a commitment to a proposed *ex ante* payment schedule. In the proposed design, a small business owner can improve the creditworthiness over time, and can avail higher amounts of credit with a smaller amount of collateral.

The simulation results show that the lending institutions will be able to increase their profit by using the proposed model *vis-à-vis* the benchmark model. The proposed model will always outperform the benchmark model when the probability of default of a project is less than 80 per cent. Besides, with the help of trust building, a small business owner can bring down collateral requirements to as low as 10 per cent of the total borrowing.

The model can be improved by bringing the probability of default into the design, and simulation can be done on real life data of a lending institution.
References


