

4 From a SAM to a CGE Model: a Cobb-Douglas Economy

4.1 CGE Building Steps

The CGE we want to present below belongs to the class of models described by Dervis *et al.* (1982) and remains similar to those reported by Robinson *et al.* (1990) and Devarajan *et al.* (1983). Thus the model is SAM based, which means that the SAM serves to identify the agents in the economy and provides a database with which the model is calibrated. As such, the modelling approach follows the influential Pyatt's *SAM Approach to Modelling* (Pyatt and Graham, 1988). In particular, since the model contains the important assumption of the law of one price, prices are common across the rows of the SAM. The SAM also provides an important organizational role since the groups of agents identified by the SAM structure are also used to define sub-matrices of the SAM for which behavioural relationships need to be defined. The model concerns a real economy without monetary sector.

As displayed by Figure 2, generating a CGE model proceeds in twelve steps (e.g., Decaluwé *et al.* 2001). However, since some of these steps are concerned mainly with data collection and processing, we describe below the six key steps directly related to model construction.

The first stage is the identification of the behavioural relationships. Since in the previous steps collected statistical data have been checked for their internal consistency with respect to macroeconomic theory⁴⁹, these are defined by reference to the sub-matrices of the SAM within which the associated transactions are recorded.

The second stage is formal and involves a further definition of the components of the transactions recorded in the SAM between respective sellers (rows) and buyers (columns). Thus, this step gives mathematical substance to behavioural relationships.

In the third stage, an algebraic statement of the general equilibrium model is provided. The equations of the model are succinctly presented in Figure 2, where standard accounts of the model's equations and variables are presented, too.

In the fourth stage, there is a discussion on macroeconomic closure rules and the choice of the most appropriate ones in the case of a modelled economy. This stage recognizes the fact that in CGE models, the number of variables is always higher than the number of equations, and exceeding variables must be rendered exogenous via appropriate macroeconomic theory.

⁴⁹ However, as will be demonstrated by the extended input-output table or the social accounting matrix for taking into account environmental aspects in the economy a SAM displays new internal consistencies, associating economy with environment.

1. Issue of the problem
2. Collecting statistical information
3. Checking internal consistency of collected data vs economic theory, e.g., a SAM must balance
4. Definition of behavioural relationships
5. Definition of transaction relationships
6. Algebraic statement of the model
7. Model closure rules
8. Choice of numerical values for the parameters of the model, i.e., calibration
9. Reproduction of the referential situation
10. Preparation of the simulation plan
11. Simulation and outputs of the new situation
12. Interpretation of outputs or comparison between referential and new situation results

Figure 2: The twelve steps for building a standard computable general equilibrium model⁵¹

The fifth stage consists of determining the numerical parameters of the model through the calibration technique. Then, some parameters not directly available must be derived from equations from the second stage so that the whole system of equations becomes determined. At this point, the application of the non-extensive entropy econometrics approach will reveal advantages over the traditional calibration technique. As shown in the preceding chapters, that new approach is conceptually better to solve inverse problems, nonlinear (e.g., the CES models) or not (e.g., the Cobb Douglas models).

The last stage consists of replication (reproduction) of the initial situation through the already built model. When the model exactly replicates initial variable values contained within the SAM⁵⁰, the modeller can then provide various simulations to assess the macroeconomic impact of different shocks.

Behavioural relationships. The SAM accounts reflect the agents' behaviour related to different transactions that took place during a given period of time. A behavioural model is defined to describe the agent achieving each transaction posted in the SAM accounts. These behavioural relationships are defined by a set of linear and nonlinear functions reflecting the way agents involved in the model will respond to exogenous changes in the parameters and/or variables of the model. Next, the 'law of one price' should apply for each row of the SAM (McDonald, 2007), but this is not true in the present case.

⁵⁰ This means that we have retained here the hypothesis that the SAM constitutes a basis for processing data.

⁵¹ This figure is an adaptation of Decalue *et al.* (2001) and Scot Mc Donald *et al.* (2007).

Households. Households maximize utility by choosing the bundles of consumption commodities through the Cobb-Douglas utility function type. Consumption commodities represent a set of composite goods and services from domestically produced and imported commodities. To reflect the hypothesis that home-produced and imported goods and services are imperfect substitutes, the above composite commodities are formally expressed as a Constant Elasticity of Substitution (CES) function. The optimal ratios of imported and domestic goods and services are set up through their relative prices. On the basis of the existence of product differentiation, Armington (1969) proposed the CES relation under the assumption of imperfect substitution (see Devarajan *et al.* (1983). According to this assumption, the same generic multi-regional commodity is not perceived in the same way by different consumers. For instance, for a Polish consumer, Arabica coffee from Kenya may present a different taste from Arabica coffee from Brazil for various reasons. One assumes in this model the country to be a price taker for all imported goods and services.

Domestic production. A two-stage nested technology is used in the domestic production process. At the first level aggregate, one combines intermediate inputs with aggregate primary inputs, then generates the outputs of activities at the basic prices. Depending on the production technology chosen, the proportion of aggregate intermediates and aggregate primary inputs varies with the (composite) prices in the case of a CES specification while in the case of a Leontief model aggregate, intermediates and primary inputs are in fixed proportions, then reflecting a homogenous relation of degree one. The second level aggregate uses Leontief technology to generate intermediate input demands in fixed proportions with respect to aggregate intermediate inputs of each sector activity. The intermediate input commodity is a composite product which mixes domestic and imported raw materials. Also at next second level, primary inputs combine to generate aggregate value added through the Cobb-Douglas or CES technology. Once again, the relative factor prices determine optimal ratios of primary inputs. Each activity produces one aggregated commodity. Hence, each vector of goods and services demanded corresponds to one vector of activity outputs.

The vector of home commodities demanded is determined by the home demand (including self-consumption by households) and export, in both cases for home-produced commodities. Assuming imperfect transformation between home demand and export demand, formally explained by a Constant Elasticity of Transformation (CET) function, the relative prices on the alternative markets determine the optimal distribution of home-produced goods and services between home and export markets. In accordance with the country's position on the world export market, the model can be specified for a small country—and thus a price taker—or for a country with a dominant position—and thus a price-maker country—on all export markets. In fact, for a price-giver country, selected export commodities can be deemed to influence the world price.

The remaining behavioural relationships constructed on the basis of SAM accounts remain generally linear. Those concern principally inter-institution exchanges. Thus,

we have tax rates, i.e., transfers from households or enterprises to government; social allocations, i.e., secondary distribution of income; and reciprocal transfers between enterprises and households or between local institutions and the rest of the world. To this list, which is not exhaustive, one can also add fixed rate of savings or investment.

4.2 The Standard Model of Transaction Relationships

In the standard model description below, we follow—for clearer presentation—symbol indexes found in McDonald and Thierfelder (2007) unless the context requires some changes. The transaction relationships are presented in Fig 3. The prices of domestically consumed (composite) commodities are defined as $PQDc$. This price is weighted prices of domestic and imported price at *CIF* (cost insurances and freight) value to which are added indirect taxes, e.g., value added tax (*VAT*). The quantities of commodities demanded domestically are, as expected, divided between intermediate demand, $QINTDc$, and final demand, with final demand further subdivided between transactions by households, $QCDc$, enterprises, $QENTDc$, government, $QGDc$, investment, $QINVDc$, and stock changes, $dstocconstc$. When no analytical need is explained, these last two categories of final demand are grouped together under investment. The value of total domestic demand, at purchaser prices, is therefore $PQDc * QQc$. As already explained, export demand, QEc , as an entry in the commodity row is not taken into account here since the domestic prices of exported commodities, $PEc = PWEc * ER$, suggest a separate row for export commodity according to the law of *one row one price*. Here $PWEc$ means world prices and ER exchange rate. Abbreviations are merely a way to save space in the table. Price of exported commodities is additionally affected by export duties, TEc , which are entered into the commodity columns. Commodity supplies come from domestic producers who receive the prices, $PXCc$, for each commodity c . with the total sectorial domestic production of commodities being denoted $QXCc$. Commodity imports, QMc , are valued carriage insurance and freight (*CIF*) paid, in a way that the domestic price of imports, PMc , is defined as a world price, $PWMc$, times the exchange rate, ER , plus an ad valorem adjustment for import TMc .

All domestically consumed commodities are subject to a variety of indirect taxes, such as sales taxes, TSc , and excise taxes, $TECc$. Other taxes can be readily added.

We assume that each activity produces a unique output QXa . Domestic production activities receive average prices for their output, PXA , being an aggregated price of intermediary input price and primary factors price. This means that in addition to intermediate inputs, activities also purchase primary inputs, $FDfa$, for which they pay average prices, WFf . The model allows the prices of each factor to differ among activities. Some or all activities pay production taxes, at the rates TXa , proportionately to the value of activity outputs.

	Enterprises	Government	Capital	RoW	Total
Commodities	$(PQD_e * QENTD_e)$	$(PQD_e * QGD_e)$	$(PQD_e * QINVD_e)$ $(PQD_e * dstocconst_e)$	$(PWE_e * QE_e * ER)$	$(PQD_e * QQ_e)$
Activities	0	0	0	0	$(PX_a * QX_a)$
Factors	0	0	0	$(factwor_f * ER)$	YF_f
Households	$hoentconst_h$	$(hogovconst_h * HGADJ)$	0	$(howor_h * ER)$	YH_h
Enterprises	0	$(entgovconst * EGADJ)$	0	$(entwor * ER)$	$EENT$
Government	$(TYE * YE)$	0	0	$(govwor * ER)$	EG
Capital	$(YE - EENT)$	$(YG - EG)$	0	$(CAPWOR * ER)$	$TOTSAV$
Rest of World	0	0	0	0	Total 'Expenditure' Abroad
Total	YE	YG	$INVEST$	Total 'Income' from Abroad	

Figure 3: The standard model transaction relationships.

Source: PROVIDE project, 2009 *Enterprises*

The model allows for the domestic use of both domestic and foreign owned factors of production and for payments by foreign activities for the use of domestically owned factors. Factor incomes, therefore, accrue from payments by domestic activities and foreign activities, *factwor_f*, where payments by foreign activities are assumed exogenously determined and are denominated in foreign currencies.

After allowing for depreciation, *deprec_f*, and the payment of factor taxes, *TF_f*, the residual factor incomes, *YFDIST_f*, are divided between domestic institutions (households, enterprises, and government) and the rest of the world in fixed proportions. Households receive incomes from factor rentals and/or sales, inter-household transfers, *hohoconst_{hh}*, transfers from enterprises, *hoentconst_h*, and government, *hogovconst_h*, and remittances from the rest of the world, *howor_h*, where remittances are defined in terms of the foreign currency.

Household expenditures consist of payments of direct/income taxes, *TY_h*. The household income after tax deduction is then posted to inter-household transfers and consumption expenditures, with the pattern of consumption expenditures determined by the household utility functions. The residual household income is posted to savings, where the saving rate, *SHH_h*, is an exogenously fixed coefficient.

The enterprise account receives income primarily from capital returns in the form of retained profits. Depending on the scale and field of activity, the enterprise may receive subsidies from government, *entgovconst*, or transfers from the rest of the world, *entwor*.

Expenditures then consist of the payment of direct/income taxes to government, *TYE*, transfers of dividends, and other remittances to households, *hoentconst_h*. Enterprises may transfer to the rest of the world a fraction of their profits or pay for foreign

factor. Residuals which are the difference between income YE and expenditures $EENT$ constitute the source of savings for enterprises.

Government. Government income comes primarily from direct and indirect taxation of factors, commodities, and from various penalties related to public administration. Thus, incomes accrue from the various tax instruments (import and export duties; sales, production, and factor taxes; and direct taxes), that can all vary due to changes in the values of production, trade, and consumption. The government may receive transfers (in foreign currency) from the rest of the world, $govwor$, as a consequence of various international conventions.

Expenditures consist of transfers to enterprises for subsidies of some productive activities. Other transfers concern the secondary distribution of income by government and are oriented toward households. Transfers may concern the rest of the world, particularly in the form of salary packages to nationals residing out of the country. In these models, all transfers to different domestic institutions are fixed in real terms but may vary in nominal terms with consumer prices. There is an analogous treatment of government savings as in the case of the enterprise, i.e., the internal balance, which is defined as the difference (residual) between government income, YG , and committed government expenditure, EG . The quantities of commodities consumed by the government are fixed in real terms, and hence government consumption expenditure will vary with commodity prices. In some circumstances, it can be realistic to endogenize government consumption behaviour for the determination of government consumption expenditure.

Domestic investment. Domestic investment demand consists of fixed capital formation, $QINVDc$, and stock business changes, $dstocconstc$. The departure point is that in an open economy model (an economy with foreign trade and capital flows), private investment plus governmental borrowing must equal private savings plus foreign investment. The value of fixed capital formation will vary with commodity prices while the volume of fixed capital formation can vary both as a consequence of the volume of savings changing or changes in exogenously determined parameters. In this basic version of the model, domestic savings is made up of savings by households, enterprises, the government (internal balance), and foreign savings, i.e., the balance on the capital account or external balance, $CAPWOR$.

Savings and capital accounts. The various closure rules available within the model allow for alternative assumptions about the determination of domestic savings. One could make assumptions about, for instance, flexible vs. fixed savings rates for households or flexible vs. fixed exchange rates for the value of foreign savings. Foreign savings come out as a discrepancy between foreign incomes and expenditures. The incomes to the rest of the world account, i.e., expenditures by the domestic economy towards the rest of the world, consist of the values of imported commodities and factor services. On the other hand, expenditures by the rest of the world account, i.e., incomes to the domestic economy from the rest of the world, consist of the values of exported commodities and net transfers by institutional accounts. The difference

between these incomes and expenditures represent the capital account, the value of which depends on a variable exchange rate.

Prices. As in any standard model, the supply prices of the composite commodities $PQSc$ are defined as the weighted averages of the domestically produced and consumed commodities PDc and the domestic prices of imported commodities PMc . Domestic prices of imported commodities are defined as the products of world prices of commodities $PWMc$ and the exchange rate ER increased by ad valorem import duties TMc and by a consumption tax applied to domestic commodities, such as sales taxes TSc and excise taxes $TEXc$. These weights are updated in the model through first order conditions for optima. The above average prices will give the composite consumer price $PQDc$.

The producer prices of commodities $PXCc$ are defined as the weighted averages of the prices received for domestically produced and sold commodities $PQSc$ and export PEc markets. These weights are updated in the model through first order conditions for optima. Given that the country is by hypothesis small, the prices received on the export market are defined as the world price of exports $PWEc$ and the exchange rate ER less any export duties due, which are defined by ad valorem export duty rates TEc .

The average price per unit of output received by each activity PXa is defined as the weighted average of the domestic producer prices. After paying indirect/production/output taxes TXa , this is divided between payments to aggregate value added $PVAa$, i.e., the amount available to pay primary inputs and aggregate intermediate inputs $PINTa$. Total payments for intermediate inputs per unit of aggregate intermediate input are defined as the weighted sums of the prices of the inputs $PQDc$. Let us recall that input prices are explained in composite price since imported commodities are demanded as intermediate inputs.

General Equilibrium conditions. As already mentioned several times, total demand for the composite commodities, QQc , consists of demand for intermediate inputs, $QINTDc$, consumption by households, $QCDc$, enterprises, $QENTDc$, and government, $QGDc$, gross fixed capital formation, $QINVDc$, and stock changes, $dstocconstc$.

Total supply, that is, supplies from domestic producers, QDc , plus imports, QMc , meets these demands; equilibrium conditions ensure that total supply and demand for all composite commodities must balance. Commodities are delivered to both the domestic and export, QEc , markets subject to equilibrium conditions that require all domestic commodity production, $QXCc$, to be either domestically consumed or exported.

As far as equilibrium of factors market WF is concerned, supply must balance with demand. In the case of the hypothesis of an inter-sector non-mobility of factors, equilibrium will have a sector character. The next balance concerns the current-account balance for the rest of the world in foreign currency. It implies that import spending plus factors to the rest of the world must balance with export income, plus revenues, institutional transfers from the rest of the world, and foreign savings. Next,

equilibrium concerns the government balance, which means that government income YG must balance with government expenditures EG plus government savings $GSAV$. Finally, the savings-investment relation must balance, which means that non-government savings, plus government savings and foreign savings, must balance with fixed investment plus stock change.

Standard Mathematical formulation. Standard mathematical formulation of a CGE model is presented in almost each presentation related to this class of model, and interested readers can refer to Lofgren *et al.* (2000) or Herault (2006) amongst others. However, in the present model, we add some changes concerning the estimation of behavioural parameters to react to one of the main criticisms of CGE models. Production relationships by activities are defined by a series of nested Constant Elasticity of Substitution (*CES*) production functions. In the standard model, there is, however, additional limits imposed by economic meaningfulness and the availability of empirical data that allow for the inclusion of information (elasticity of substitution) about the possibilities for substitution between and within sub-groups of factors. This point is one of the repeated criticisms of CGE models since, in this particular case, different modellers should find different outputs, as the selection of elasticity parameters is arbitrary. In the present work, we bypass this problem thanks to a new maximum entropy approach introduced in the first parts of this document. The same procedure will be carried out for the constant elasticity of transformation between domestic and export product or Armington elasticity of substitution between domestic product and imports. Coming back to activity output, it is worth reminding that it is a *CES* aggregate of the quantities of aggregate intermediate inputs $QINT$ and value added QVA while aggregate intermediate inputs are a Leontief aggregate of the (individual) intermediate inputs. Furthermore, aggregate added value is a *CES* aggregate of the quantities of ‘primary’ inputs demanded by each activity, where the primary inputs can be natural factors—types of labour, capital, and land. In the traditional model, optimal combinations of each natural and/or aggregate in each *CES* aggregate are determined by first order conditions based on relative prices.