

3 Compensatory and Equivalent Variations: Two Types of Welfare Measurement

The next two questions will help us understand the difference between compensatory and equivalent variations: If relative price of two given commodities had to change, what would the change in income be if we needed to maintain the consumer at the same welfare level given this new situation? If relative price did not change, what would the equivalent change in income be that could produce the same impact as the relative price?

- a) **Compensatory and equivalent variations.** Let us adopt the Cobb-Douglas behavioural function and define a utility function as follows:

$$U = q_1^\gamma q_2^{1-\gamma}$$

U : level of direct utility;

q_1 : consumed quantity of good 1;

q_2 : consumed quantity of good 2;

γ : utility elasticity with respect to consumed good 1

$1-\gamma$: utility elasticity with respect to consumed good 2.

- b) **Indirect utility.** Functions q_1 and q_2 , which maximize utility, have the form:

$$q_1 = \gamma \frac{YM}{p_1}$$

$$q_2 = (1-\gamma) \frac{YM}{p_2}$$

where p_1 and p_2 are prices of goods and YM nominal income of consumers.

Replacing the above functions in the function of direct utility leads to the indirect utility function presented below:

$$UI = \frac{HYM}{p_1^\gamma p_2^{1-\gamma}}$$

with

$$H = \gamma^\gamma (1-\gamma)^{1-\gamma}$$

To facilitate the interpretation of indirect utility UI , let us recall that:

$$\frac{p_1^\gamma p_2^{1-\gamma}}{H}$$

is the composite price of a commodity basket when direct utility is equal to U .

In the case of perfect competition, elasticity γ corresponds to the fraction of bought commodity.

- c) **Compensatory and equivalent variations.** If we assign an index “0” to the reference situation and index “1” to the new situation, then one may compare two welfare states in two manners:

$$\text{a) } UI^0 = \frac{HYM^0}{(p^0_1)^\gamma (p^0_2)^{1-\gamma}}$$

where UI^0 means the level of indirect utility at the reference situation.

$$\text{b) } UI^1 = \frac{HYM^1}{(p^1_1)^\gamma (p^1_2)^{1-\gamma}}$$

in the case of new situation.

For further derivations, let us choose here, as a base of comparison, the new situation:

$$UI^0 - UI^1 = \frac{HYM^0}{(p^0_1)^\gamma (p^0_2)^{1-\gamma}} - \frac{HYM^1}{(p^1_1)^\gamma (p^1_2)^{1-\gamma}}$$

and, converting the above quantity into nominal values, we get compensatory variation CV:

$$CV = (UI^0 - UI^1) \frac{(p^1_1)^\gamma (p^1_2)^{1-\gamma}}{H} = \left(\frac{p^1_1}{p^0_1} \right)^\gamma \left(\frac{p^1_2}{p^0_2} \right)^{1-\gamma} (YM^0 - YM^1)$$

If $CV < 0$, then we have welfare improvement.

In simulation exercises through CGE models, the government first introduces a carbon tax to targeted polluting sectors. Simultaneously, it will reduce factor taxes in distorting production sectors. The next step is observing changes in price and on the household real income level. On this basis, one derives the compensatory variation CV presented above.

A Theoretical Example: CGE Model and Double-Dividend (DD)-Oriented Policies

In this section we will present a theoretical non-extensive cross-entropy (NCE) model to estimate parameters of an environmentally extended CGE model to assess impact of the DD hypothesis. The main issue to be underscored remains the rationale for applying the NCE approach in place of the traditional Kullback-Leibler cross-entropy model (Go *et al.*, 2015). The response lies in the statistical properties of power law-related NCE. Due to the estimated parameters of constant elasticity of production models, outputs from both techniques must diverge with a higher performance in the case of NCE estimator (see V.5.3 for details). Both solutions will be similar only when the modelled phenomena display Gaussian distribution.

As presented earlier in this part of the book, the negative externalities resulting from pollution is one of the economy-distorting factors that prevents reaching a general equilibrium and Pareto optimum. Pearce (1991) proposed a model to generate double positive impact by introducing a tax imposition on pollutant activities. Once again, the first positive impact results in reducing pollutants through the increase of their production cost. If we use income generated by the tax imposition on pollutant goods to reduce tax in other sectors, a DD may result. This section presents a theoretical CGE model in the context of DD hypothesis testing. The proposed model can enable assessing to what extent the carbon tax can be identified as an important factor affecting the size of the DD, identifying the existence of a strong DD in the economy, or highlighting the weight of certain factors in affecting the presence and size of the DD.

This CE formulation may be written as follows:

$$\begin{aligned}
 \text{Min} \xrightarrow{p,r} & \alpha_1 \frac{1}{q-1} \sum_k \sum_m p^E_{km} \left[(p^E_{km})^{q-1} - (q^E_{km})^{q-1} \right] - \\
 & - \sum_k \sum_m (p^E_{km} - q^E_{km}) (q^E_{km})^{q-1} + \\
 & + \alpha_2 \frac{1}{q-1} \sum_t \sum_n \sum_j r^E_{tnj} \left[(r^E_{tnj})^{q-1} - (s^E_{tnj})^{q-1} \right] - \sum_t \sum_n \sum_j (r^E_{tnj} - s^E_{tnj}) (s^E_{tnj})^{q-1}
 \end{aligned} \tag{5.11a}$$

Subject to:

$$F(X_t^E, Z_t^{0E}, Z_t^{uE}, B^E, \delta^E) = 0, \quad \forall t \in T$$

$$\delta^E = P(Z_t^E, B^E)$$

$$Y_t^E = G(X_t^E, Z_t^{0E}, Z_t^{uE}, B^E, \delta^E) + e_t \quad \forall t \in T$$

$$B_k^E = \sum_{m=1}^M p^E_{km} v_{km} \quad \forall k \in K$$

$$e_{tn}^E = \sum_{j=1}^J r^E_{tnj} w_{tnj} \quad \forall t \in T, n \in N$$

$$\sum_{m=1}^M p^E_{km} = 1, \quad \forall k \in K$$

$$\sum_{j=1}^J r^E_{tnj} = 1, \quad \forall t \in T, n \in N.$$

The above formulation (5.11a) is the same as the one presented in Part V (Equation 5.11), the only difference being the introduction in the model of an environmental sector. Consequently, this suggests the extension of the pre-existing model by adding

environmental relations, as shown in the above optimization system where variables and parameters bear the superscript “E”.

Depending on the target of the environmental model, the above CGE block $F(..) = 0$ will include additional endogenous variables (e.g., emitted CO_2 , the dirty commodity, and its prices), exogenous variables like the environmental tax, the behavioral parameters like the elasticity of the environmental input, etc.

For example, to show the impact of the DD policy, one can add in the above CGE system $F(..) = 0$ the next equations, see (Sasmaz, 2016)⁶¹ for counterfactual simulation purposes:

$$CO_{2t} = \alpha_{1t} + \beta_1 envtax_t + \beta_2 GDP_t + \beta_3 enecons_t + \zeta_{1t} \quad (6.1)$$

$$U nemp_t = \alpha_{2t} + \beta_4 envtax_t + \beta_5 GDP_t + \zeta_{2t} \quad (6.2)$$

Where CO_{2t} (Equation 6.1) is an endogenous variable which depends on the environmental tax rate $envtax_t$, the overall level of gross domestic production GDP_t , and the level of energy consumption $enecons_t$. Likewise, the endogenous variable $U nemp_t$ (Equation 6.2) explains the rate of unemployment which depends on the $envtax_t$ and the GDP_t . The index of time t is related to the targeted time series elements of the environmentally extended social accounting matrices Y_t^E . Parameters α_{1t} and α_{2t} are the constants. Parameters β_j (with $j = 5$) explain the long-run marginal change of the respective endogenous variable induced by a unit change of the explicative variable. Indeed, parameter interpretation in the model explains a long-run marginal change since we are dealing with an entropy model, the estimates of which will be generated by the maximum entropy principal rule under the CGE constraints. As noted in the previous section where the question was posed as to whether or not the maximum entropy solution is Pareto optimum, the obtained model solution may be different from the solution of the traditional CGE optimal computation, which is not based upon a probabilistic distribution.

Besides the proposed illustrative model in (6.1–6.2), there exists a large formulation of the DD policy—oriented through CGE models (e.g., (Fraser & Waschik, 2013) (Takeda, 2007), (Taheripour, Khanna i Nelson, 2008), (Bento & Jacobsen, 2007)). As an interesting case study, Fraser and co-author presented in (Fraser & Waschik, 2013) a CGE model to assess a DD hypothesis where three taxes were examined: tax on the production of energy goods, on the production of carbon, and on the usage of carbon. To show the existence of DD for each tax revenue raised, authors proposed to offset pre-existing distortions in the CGE model through an endogenous tax that adjusts to keep constant government revenue. Finally, the author’s outputs led to the existence of a strong DD associated with the existence of specific (immobile) factors

⁶¹ Note that author has checked the DD hypothesis through an econometric panel data model.

in the production of energy goods. Note that these outputs were generated through a traditional CGE model.

Concluding Remarks

In this section we have presented a theoretical NCE general formulation model in the case of general economic disequilibrium owing to, for example, environmental distortions. At the same time, the principal aspects of the empirical literature on the DD hypothesis has been presented. This represents a useful device for environmental economists who find in it a way to kill two birds with one stone, that is, reducing the carbon emission while creating better conditions for a balanced economic growth.

Future implementation of a PL-related NCE approach to estimate a CGE model, in general, and an environmentally extended CGE model, in particular, could reveal significant results. Indeed, the capacity of the PL-related NCE approach to handle non-linear inverse problem systems present in some sub-systems of the CGE model should produce positive outcomes.

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