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Preferential Treatment of Government Bonds in Liquidity Regulation – Implications for Bank Behaviour and Financial Stability

Ulrike Neyer, André Sterzel

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Abstract

This paper analyses the impact of different treatments of government bonds in bank liquidity regulation on financial stability. Using a theoretical model, we show that a sudden increase in sovereign default risk may lead to liquidity issues in the banking sector, implying the insolvency of a significant number of banks. Liquidity requirements do not contribute to a more resilient banking sector in the case of sovereign distress. However, the central bank acting as a lender of last resort can prevent illiquid banks from going bankrupt. Then, introducing liquidity requirements in general and repealing the preferential treatment of government bonds in liquidity regulation in particular actually undermines financial stability. The driving force is a regulation-induced change in bank investment behaviour.

JEL classification: G28, G21, G01.

Keywords: Bank liquidity regulation, government bonds, sovereign risk, financial contagion, lender of last resort.

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1 Introduction

The financial crisis of 2007/2008 was characterised by severe liquidity issues in many markets and illustrated the importance of liquidity with respect to a proper functioning of the financial system. The European Central Bank (ECB) provided massive liquidity to banks aiming to avoid the breakdown of the financial sector and to ensure financial stability. As a response to the crisis, the Basel Committee on Banking Supervision (BCBS) published global minimum liquidity standards for banks within the Basel III regulation framework. However, within this liquidity regulation framework, government bonds receive a preferential treatment. In particular, they are regarded as highly liquid assets, which means that banks can use government bonds to meet their liquidity requirements without applying any haircuts or quantitative limits, i.e. in liquidity regulation government bonds are treated as liquidity risk-free. However, this is actually not the case. In the European sovereign debt crisis, for example, the credit risk applied to some EU member states increased substantially and the sovereign bonds of these countries could not be easily and quickly liquidated without leading to substantial losses for banks (liquidity risk). Accordingly, the crisis has highlighted severe contagion effects from sovereigns to banks. Against this background, there is an ongoing debate addressing the abolishment of the favourable treatment of sovereign bonds in EU banking regulation. This paper adds to this debate by investigating in a theoretical way whether the contagion channel from sovereigns to banks can be weakened through the abolishment of the preferential treatment of government bonds in liquidity regulation.

In our model, there are three agents: depositors, banks and investors. The objective of banks is to maximise their depositors’ expected utility. The depositors have the usual Diamond-Dybvig preferences. In the banking sector, there is no aggregate liquidity risk, though banks face idiosyncratic liquidity risks. Banks can invest in three assets: a risk-free short-term asset, which does not earn any return, and in two risky long-term assets (government bonds and loans) with an expected positive return. Whereas loans are totally illiquid, government bonds are liquid as there exists an interbank market for this

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1 In the Basel III framework, sovereign bonds are also given privileged treatment with respect to capital requirements and to large exposure regimes.

2 Except for the bank regulation part, the model setup corresponds to the setup presented in Neyer and Sterzel (2017).
asset. Thus, investing in government bonds allows banks to hedge their idiosyncratic liq-
uidity risks.\(^3\) Besides deposits, banks can raise equity capital from risk-neutral investors
to finance their investments. Raising costly equity capital allows banks to transfer liq-
uidity risks associated with highly profitable but totally illiquid loans from risk-averse
depositors to risk-neutral investors which implies an increase in their depositors’ expected
utility. Banks may be subject to liquidity regulation, requiring them to hold more liquid
assets (short-term assets and government bonds) than they would choose to hold without
regulation.

Within this model framework, in a first step we analyse the banks’ investment and
financing behaviour under different liquidity regulations. As a starting point, we deter-
mine the bank optimal behaviour when there is no regulation. Then, we consider two
different possible liquidity regulation scenarios with respect to the regulatory treatment
of government bonds. In the first scenario, there is a preferential treatment of govern-
ment bonds, in the sense that government bonds and the short-term asset are classified
as equally liquid although there exists a market liquidity risk for government bonds. In
response to the introduction of this liquidity regulation, banks increase their liquid asset
holdings at the expense of a disproportionately high decrease of their loan investment and
a reduction in their equity capital. In the second scenario, the preferential treatment of
government bonds in liquidity regulation is repealed, by classifying government bonds as
less liquid than the short-term asset. This implies that the observed bank behaviour in
the first scenario is reinforced. Banks further increase their holdings of the short-term
asset as well as of government bonds and decrease their loan investment and reduce their
equity capital.

In a second step, we first investigate the banks’ shock-absorbing capacity in the absence
of liquidity regulation and then in the two different liquidity regulation scenarios with
respect to government bond treatment. We consider a shock in the form of an increase
in the default probability of sovereign bonds (government bond shock). These increasing
doubts about sovereign solvency may lead to a sovereign bond price drop and hence to
liquidity issues of a significant number of banks, implying illiquid but per se solvent banks

\(^{3}\)As pointed out by [BCBS (2017)](#), for example, banks hold government bonds for a variety of reasons. So
government bonds do play an important role in managing a bank’s daily activities. In our model, banks
hold government bonds to manage their liquidity.
going bankrupt (systemic crisis). We show that liquidity requirements do not increase the government-bond-shock-absorbing capacity of the banking sector. In this sense they do not increase financial stability. The shock-absorbing capacity will increase if a central bank as a lender of last resort (LOLR) exists, which provides additional liquidity against adequate collateral. In our model, loans serve as adequate collateral. However, then the introduction of liquidity requirements in general and repealing the preferential treatment of government bonds in liquidity regulation in particular actually reduce the government-bond-shock-absorbing capacity. The driving force is the regulation-induced change in bank investment behaviour (more government bonds and fewer loans). This implies that banks face higher additional liquidity needs caused by the government bond shock and they have less collateral to obtain liquidity from the LOLR.

The rest of the paper is structured as follows. In Section 2 we provide an overview of the related literature. In Section 3 we explain the institutional background of liquidity requirements within the Basel III Accord. Section 4 describes the model setup. Section 5 analyses the banks’ optimal investment and financing behaviour under different liquidity regulations. Based on these analyses, Section 6 discusses the consequences of the different liquidity requirements for financial stability in case of a sovereign crisis and the importance of the central bank acting as a LOLR in this context. The final section summarises the paper.

2 Related Literature

Our paper contributes to two strands of literature: the literature on financial contagion especially between sovereigns and banks, and the literature dealing with liquidity requirements and their impact on bank behaviour and financial stability. Since the European sovereign debt crisis of 2009 onwards, there has been a growing literature on financial contagion between sovereigns and banks. As main potential contagion channels (i) a direct exposure channel, (ii) a collateral channel, (iii) a sovereign credit rating channel, (iv) a gov-

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4As in Allen and Gale (2000) we will refer to financial contagion if financial linkages imply that a shock, which initially affects only one or a few firms (financial or non-financial), one region or one sector of an economy, spreads to other firms, regions or sectors.
ernment support channel, and (v) a macroeconomic channel have been identified. A huge part of the literature dealing with the sovereign-bank nexus discusses newly implemented or proposed institutions aiming to weaken potential financial contagion channels between sovereigns and banks. In this respect, the European Banking Union is one of the most well-known recent reforms. Referring to the European Banking Union, Covi and Eydam (2018) argue that the second pillar of the Banking Union the Single Resolution Mechanism (SRM) weakens the contagion channel between sovereigns and banks because of a “bail-in” rule, implying that bank insolvencies no longer strain public finances. Farhi and Tirole (2018) argue that the Single Supervisory Mechanism, i.e. the first pillar of the Banking Union, can diminish contagion effects between internationally operating banks and sovereigns as due to a shared supranational banking supervision banks’ adverse risk-shifting incentives are impeded. Acharya and Steffen (2017) stress the need for a complemented banking and fiscal union. Both are necessary to build a functioning capital market union that minimises the probability of sovereign-bank contagion. Brunnermeier et al. (2016) develop a model which illustrates how to isolate banks from sovereign risk via the introduction of European Safe Bonds (“ESBies”) issued by a European debt agency. The idea is that holding these bonds disentangles banks from sovereign distress as “ESBies” are backed by a well-diversified portfolio of euro-area government bonds and are additionally senior on repayments. Neyer and Sterzel (2017) show that the introduction of capital requirements for government bonds can weaken contagion effects from sovereigns to banks in combination with the central bank acting as a LOLR. In the same context, Abad (2018) shows within a dynamic general equilibrium model that the preferential treatment of government bonds in capital regulation amplifies the sovereign-bank nexus. He also suggests backing government bonds with equity capital to disentangle bank and sovereign risks. Buschmann and Schmaltz (2017) point out that the Liquidity Coverage Ratio (LCR) may reinforce contagion effects from sovereign to banks. Within the LCR framework, government bonds are classified as high quality liquid assets irrespective of their inherent liquidity risks. This classification makes sovereign bonds an attractive asset for banks, so that they may increase their sovereign holdings to meet the LCR. Then, in times of sovereign distress banks

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5 For a survey of channels through which sovereign risk can affect the banking sector see for example BCBS (2017), Committee on the Global Financial System (2011) or European Systemic Risk Board (2015).
are exposed to severe liquidity risks associated with their sovereign bond holdings. The authors propose an alternative LCR (LCR+), that incorporates sovereign risk in order to reduce the contagion effects from sovereigns to banks.

In recent years, there has been a growing theoretical literature on the impact of liquidity regulation on bank behaviour and financial stability.\(^6\) Diamond and Kashyap (2016), modify the Diamond and Dybvig (1983) model and show that binding liquidity requirements reduce the bank-run probability and thus increases financial stability. Calomiris et al. (2015) develop a theoretical model which analyses the effectiveness of a liquidity requirement that takes the form of a cash requirement. They show that introducing cash requirements makes financial crises less likely as banks’ default risks are reduced. The reason is that higher holdings of risk-free cash reduces the banks’ portfolio risk, so that they gain market confidence. In times of distress they are thus able to attract and retain deposits, which reduces the probability of liquidity issues. Ratnovski (2013) argues that a liquidity buffer can prevent bank insolvencies only in the case of a small liquidity shock as the size of the liquidity buffer is limited. He points out the importance of banks’ transparency, and accordingly the ability to communicate solvency information to outsiders. This allows banks to gain access to external financing and thus to also withstand large liquidity shocks. Farhi and Tirole (2012) argue from a welfare-theoretical perspective that banks are engaged in excessive maturity transformation by issuing large amounts of short-term debt. This enables banks to increase their leverage, but also exposes banks to potential refinancing risks in the case of a liquidity shock. To reduce the excessive maturity transformation the optimal form of regulation is a liquidity requirement, which reduces banks’ short-term funding. Perotti and Suarez (2011) also emphasise that banks choose a higher amount of short-term funding than is socially optimal. They analyse whether liquidity regulation, and in particular which form of liquidity regulation, is able to restore the socially optimal amount of banks’ short-term funding. It is shown, that both a simple Pigovian tax on short-term debt and a ratio-based liquidity regulation are able to contain

\(^6\)There has also been an increasing number of empirical papers dealing with this issue. For respective papers analysing the impact of liquidity requirements on bank behaviour see, for example: Baker et al. (2017), Banerjee and Mio (2017), Bonner (2012), Bonner (2016), Bonner et al. (2015), De Haan and van den End (2013), DeYoung and Jang (2016), Duijn and Wierts (2014), Gobat et al. (2014), King (2013) and Scalia et al. (2013). For empirical literature dealing with the impact of liquidity requirements on financial stability see, for example, Lallour and Mio (2016) or Hong et al. (2014).
banks’ liquidity risks. However, which of the two regulations is the most efficient depends on banks’ heterogeneity in risk-taking incentives and in their ability to extend credits. Ratnovski (2009) shows that banks will hold insufficient liquid assets if they assume that the central bank acts as a LOLR, providing liquidity in a systemic crisis. Quantitative liquidity regulation forces banks to hold a liquidity buffer, implying that banks do not rely on the support of the central bank. However, this regulation is costly. To reduce these costs Ratnovski supposes a LOLR policy based on information on the banks’ capitalisation. Building on this information the central bank sets repayment conditions to reduce the incentives for banks to gamble for LOLR support. König (2015) develops a theoretical model which shows that bank liquidity regulation may endanger financial stability. Introducing liquidity requirements has two effects: a liquidity effect and a solvency effect. The liquidity effect arises as banks are forced to hold more liquid assets and thereby the probability of becoming illiquid decreases. However, as liquid assets have lower returns than illiquid assets a liquidity buffer induces lower bank returns and therefore increases the banks’ insolvency risk. Hence, liquidity regulation only increases the resilience of the banking sector as long as the liquidity effect exceeds the solvency effect. Referring to the ‘lemon-problem’ introduced by Akerlof (1970), Malherbe (2014) emphasises that liquidity regulation worsens adverse selection in markets for long-term assets which may lead to a market breakdown. In particular, a bank sells long-term assets for two reasons: first, to receive liquidity, and second, to prevent a loss when they realise before maturity that the asset is a “lemon”, i.e. that it will fail. However, the latter is private information. This information asymmetry may lead to adverse selection in the market for the long-term asset. The introduction of bank liquidity regulation induces banks to increase their liquid asset holdings. This means that it becomes more likely that banks will sell a long-term asset because it is a “lemon” rather than to receive liquidity. This regulation-induced change in bank behaviour reinforces the adverse selection problem and therefore increases the probability of a market breakdown. Hartlage (2012) evaluates whether the LCR is a regulatory tool that effectively regulates banks’ liquidity. His main result is that a binding LCR may undermine financial stability as it incentivises banks to engage in regulatory arbitrage. This incentive for banks arises as in the LCR retail deposits are classified as a
less volatile funding source than wholesale funds. As a consequence, banks replace wholesale funding with retail deposits to meet the LCR. Hartlage argues that this undermines financial stability, as retail deposits especially from large-volume depositors, which are not secured by the deposit insurance, are a less stable funding source than assumed by the regulator.

Our contribution to this literature: We show that liquidity requirements actually reinforce the contagion channel from sovereigns to banks due to a regulation-induced change in bank investment behaviour. Furthermore, we show that the contagion effects between sovereigns and banks will be reinforced if a preferential treatment of government bonds in bank liquidity regulation is repealed.

3 Institutional Background

Before the global financial crisis of 2007/2008, bank regulation relied mainly on capital regulation. However, the crisis underlined the importance of sufficient bank liquidity for the proper functioning of the financial system. In response to the financial crisis the BCBS (2008) published principles for a sound bank liquidity risk management. To complement these principles, the BCBS (2010) introduced two minimum standards for funding liquidity within the Basel III framework: the Liquidity Coverage Ratio (LCR) and the Net Stable Funding Ratio (NSFR).

The Liquidity Coverage Ratio

The aim of the LCR is to promote the short-term resilience of banks’ liquidity profiles by ensuring that banks have sufficient unencumbered high-quality liquid assets (HQLA) to withstand a significant stress scenario of a duration of at least one month. Following a consultant period from 2011 onwards, in January 2013 the BCBS published the final version of the LCR framework. In July 2013, the European Commission implemented the Basel LCR framework into European law by way of the fourth Capital Requirement Directive (CRD IV) and the Capital Requirement Regulation (CRR). After an observation period, the LCR was phased in gradually within an implementation period from October 2015 to January 2018. The LCR is defined as:
\[
LCR = \frac{\text{Stock of HQLA}}{\text{Total net cash outflows over the next 30 calendar days}} \geq 100\%. \tag{1}
\]

It consists of two components: the stock of HQLA (numerator) and the total expected net cash outflows over the next 30 calendar days (denominator). HQLA are assets with a high potential to be easily and quickly liquidated at little or no loss of value even in times of stress. There are three categories of HQLA: level 1 assets, level 2A assets and level 2B assets. Level 1 assets consist of coins and banknotes, central bank reserves and a range of sovereigns securities, level 2A assets also include some sovereign securities, corporate debt securities and covered bonds, and the asset class 2B contains lower-rated corporate debt securities, mortgage-backed securities and common equity shares (see BCBS, 2013, paragraph 50, 52 and 54). Whereas there is no limit for level 1 assets, level 2A assets can only comprise up to 40% of the stock of HQLA, and the stock of level 2B assets is limited up to 15%. Furthermore, level 1 assets are also not subject to haircuts. However, a haircut of 15% is applied to level 2A assets, and level 2B assets are subject to haircuts of 25% to 50%. The denominator represents the total expected net cash outflows over the next 30 calendar days. This term is defined as the total expected cash outflows minus the minimum of total expected cash inflows. However, to ensure a minimum level of HQLA holdings, total expected cash inflows are subject to a cap of 75% of the total expected cash outflows.

**The Net Stable Funding Ratio**

The NSFR is designed to supplement the LCR. It requires banks to have a sustainable maturity structure of their assets and liabilities over a one-year time horizon. The BCBS proposed the NSFR framework in 2010. After a consultant period and a repossession (in January 2014) the final version of the NSFR was published in October 2014 (BCBS, 2014). It was scheduled to become a minimum standard for banks by January 2018 (BCBS, 2014). By now (June 2018) the CRR contains only a reporting obligation for banks and the NSFR has not become a binding requirement yet. Formally, the liquidity ratio is defined as:
\[ NSFR = \frac{\text{Available amount of stable funding}}{\text{Required amount of stable funding}} \geq 100\%. \] 

It consists of two components: the available amount of stable funding (numerator) and the required amount of stable funding (denominator). The available amount of stable funding is calculated by the total value of a bank’s capital and liabilities expected to be reliable over the time horizon of one year. In particular, the equity and liability instruments are categorised in one of five categories regarding their expected availability within a stress scenario. The total value of the instruments in each category is then weighted with an available stable funding (ASF) factor and finally summed up. Note that funding instruments which are regarded as stable funding sources receive a high ASF factor and vice versa. The required amount of stable funding is based on the liquidity characteristics of banks’ assets and off-balance-sheet (OBS) exposures. Accordingly, the banks’ assets and OBS exposures are assigned to one of eight required stable funding (RSF) categories. The amount of each category is weighted with an RSF factor and then summed up. Note that the higher the liquidity value of an asset or an OBS exposure, the lower the RSF factor and vice versa.

**Preferential treatment of sovereign exposures within liquidity regulation**

Within the LCR framework as well as within the NSFR framework, government bonds receive a preferential treatment with respect to other asset classes. Considering the LCR, sovereign bonds are eligible to be classified as level 1 assets, and are thereby not subject to haircuts and quantification limits when they satisfy at least one of the following three conditions (see [BCBS 2013](#) paragraph 50): (i) they are assigned a 0% risk-weight under the Basel II Standardised Approach, (ii) they are issued in domestic currencies by the sovereigns in the countries in which the liquidity risk is being taken or the bank’s home country, (iii) sovereign bond holdings which are denominated in foreign currency are eligible up the amount of the bank’s net cash outflows in that foreign currency in times of distress. Moreover, the LCR framework requires that the HQLA should be well diversified within each asset class. However, there is an exception for sovereign bonds (as well as
for cash, central bank reserves and central bank debt securities) of the bank’s jurisdiction in which the bank operates, or of its home jurisdiction (see BCBS 2013, paragraph 44). Also, with respect to the NSFR framework, sovereign bonds are assigned a favourable treatment. As government bonds are classified as level 1 assets in the LCR, they are assigned an RSF factor of 5% within the NSFR. Only coins, banknotes and central bank reserves are assigned a lower RSF factor of 0%, whereas level 2 assets are assigned RSF factors of between 15% and 50%. This privileged treatment makes sovereign securities an attractive asset for banks to meet the LCR as well as the NSFR compared to other securities.

4 Model

The model framework, except for the bank regulation part, and the modelling of the interbank market, corresponds exactly to the framework presented in Neyer and Sterzel (2017).

4.1 Technology

We consider three dates, \( t = 0, 1, 2 \) and a single all-purpose good that can be used for consumption or investment. At date 0, the all-purpose good can be invested in three types of assets: one short-term and two long-term assets. The short-term asset represents a simple storage technology. Investing one unit at date 0 returns one unit at date 1. The two long-term assets are government bonds and loans. Government bonds are not completely safe but yield a random return \( S \). With probability \( \beta \), the investment succeeds and produces \( h > 1 \) units of this good at date 2. With probability \( (1 - \beta) \) the investment fails and one unit invested at date 0 produces only \( l < 1 \) units at date 2. The government bond is a liquid asset. It can be sold on an interbank market at date 1. The loan portfolio yields a random return \( K \). If the loan investment succeeds, one unit invested at date 0 will generate a return of \( H > h > 1 \) units at date 2 with probability \( \alpha < \beta \). If the investment fails, it will produce only \( L < l < 1 \) units of the single good at date 2 with probability \( (1 - \alpha) \). The loan portfolio is the asset with the highest expected return \( (E(K) > E(S) > 1) \), it has the highest risk \( (Var(K) > Var(S)) \), and it is totally illiquid.
at date 1. At date 2 banks learn whether the long-term assets (government bonds and loans) succeed or fail.

4.2 Agents and Preferences

There are three types of agents: a continuum of risk-averse consumers normalised to one, a large number of banks and a large number of risk-neutral investors. Each consumer is endowed with one unit of the single all-purpose good at date 0.

Like in [Diamond and Dybvig (1983)] consumers can be categorised into two groups. One group values consumption only at date 1 (early consumers), the other group only at date 2 (late consumers). We assume both groups are the same size so that the proportion of early consumers is \( \gamma = 0.5 \) and the proportion of late consumers is \( (1 - \gamma) = 0.5 \).

Denoting a consumer’s consumption by \( c \), his utility of consumption is given by

\[
U(c) = \ln(c). \tag{3}
\]

However, at date 0 a consumer does not know whether he is an early or late consumer. Therefore, he concludes a deposit contract with a bank. According to this contract, he will deposit his one unit of the all-purpose good with the bank at date 0 and can withdraw \( c_1^* \) units of the all-purpose good at date 1 or \( c_2^* \) units of this good at date 2. As we have a competitive banking sector, each bank invests in the short-term asset and the two long-term assets in a way that maximises its depositors’ expected utility.

Banks are subject to idiosyncratic liquidity risk but there is no aggregate liquidity risk (the fraction of early consumers is \( \gamma = 0.5 \) for certain). Accordingly, they do not know their individual proportion of early consumers. A bank has a fraction \( \gamma_1 \) of early consumers with probability \( \omega \) and a bank faces a fraction \( \gamma_2 > \gamma_1 \) of early consumers with probability \( (1 - \omega) \), so that \( \gamma = 0.5 = \omega \gamma_1 + (1 - \omega) \gamma_2 \). As in [Allen and Carletti (2006)], we assume for the sake of simplicity the extreme case in which \( \gamma_1 = 0 \) and \( \gamma_2 = 1 \), so that \( \omega = 0.5 \). Because of this strong assumption, we have two types of banks: banks with only early consumers (early banks) and banks with only late consumers (late banks), and the probability of becoming an early or a late bank is 0.5 each.
In addition to the deposits from consumers, banks have the opportunity to raise funds (equity capital) from risk-neutral investors. These investors are endowed with an unbounded amount of capital \( W_0 \) at date 0. The contract concluded between a bank and an investor defines the units of the all-purpose good which are provided at date 0 as equity capital \( e_0^* \geq 0 \) and the units which are repaid to the investors at date 1 and date 2 \( (e_1^* \geq 0 \text{ and } e_2^* \geq 0) \). As in Allen and Carletti (2006), the utility function of a risk-neutral investor is given by

\[
U(e_0, e_1, e_2) = \rho(W_0 - e_0) + e_1 + e_2, \tag{4}
\]

where \( \rho \) presents the investor’s opportunity costs of investing in the banking sector.

### 4.3 Optimisation Problem

At date 0, all banks are identical, so we can consider a representative bank when analysing the banks’ optimal investment and financing behaviour at date 0. Deposits are exogenous and equal to one. The bank has to decide on units \( x \) to be invested in the short-term asset, on units \( y \) to be invested in government bonds, on units \( u \) to be invested in loans and on units \( e_0 \) to be raised from the risk-neutral investors. A bank’s optimal behaviour requires the maximisation of the expected utility of its risk-averse depositors. Consequently, a bank’s optimisation problem reads

\[
\max E(U) = 0.5\ln(c_1) + 0.5[\alpha \beta \ln(c_{2Hh}) + \alpha (1 - \beta) \ln(c_{2Hl})] \\
+ (1 - \alpha) \beta \ln(c_{2Lh}) + (1 - \alpha)(1 - \beta) \ln(c_{2Ll})] \tag{5}
\]

with

\[
c_1 = x + yp, \tag{6}
\]

\[
c_{2Hh} = uH + \left(\frac{x}{p} + y\right)h - e_{2Hh}, \tag{7}
\]

\[
c_{2Hl} = uH + \left(\frac{x}{p} + y\right)l - e_{2Hl}, \tag{8}
\]

\[
c_{2Lh} = uL + \left(\frac{x}{p} + y\right)h - e_{2Lh}, \tag{9}
\]

\[
c_{2Ll} = uL + \left(\frac{x}{p} + y\right)l - e_{2Ll}, \tag{10}
\]

13
\begin{equation}
\text{s.t. } \rho e_0 = 0.5(\alpha e_2 H + (1 - \alpha)e_2 L) + 0.5(\alpha \beta e_{2Hh} \\
+ \alpha(1 - \beta)e_{2Hl} + (1 - \alpha)\beta e_{2Lh} + (1 - \alpha)(1 - \beta)e_{2Ll}),
\end{equation}

Equation (11) represents the investors’ incentive-compatibility constraint. Investors will only be willing to provide equity capital \(e_0\) to the banking sector if at least their opportunity costs \(\rho\) are covered. With probability 0.5 the bank is an early bank. Then
the bank will use its total amount of the short-term asset, including those units obtained in exchange for its total amount of government bonds on the interbank market, to satisfy all its depositors at date 1. From the proceeds of the loan portfolio, which accrue at date 2, early depositors do not benefit, so that the investors receive the total proceeds from this asset (and only from this asset), i.e. $e_{2H} = uH$ or $e_{2L} = uL$. With probability 0.5, the bank is a late bank. Then, at date 2, it will repay its depositors and investors. The investors will receive a residual payment from the proceeds of the bank’s total loan and government bond investment, i.e. those returns not being used to satisfy the bank’s depositors. Note that this residual payment may be zero.

Constraint (12) describes a possible required minimum liquidity ratio $LR_{\min}$. The ratio $LR_{\min}$ captures the LCR, as it requires banks to back potential short-term liquidity withdrawals with a specified amount of liquid assets. In particular, it is expressed as a ratio of banks’ liquid assets (short-term assets and government bonds) weighted with a corresponding liquidity factor ($\kappa_x$ and $\kappa_y$) to the maximum possible deposit withdrawals at date 1, which are equal to one. If $\kappa_x = \kappa_y$, the regulator classifies a short-term asset and a government bond as equally liquid. In this regulation scenario government bonds are treated preferentially to the short-term asset as they have to be sold on an interbank market to obtain liquidity, implying that government bonds are exposed to a potential market liquidity risk unlike the short asset. This privileged treatment will be repealed if $\kappa_y < \kappa_x$. Then, the liquidity factor assigned to government bonds is lower than the factor assigned to the short-term asset i.e. the potential market liquidity risk is taken into account by the regulator. Government bonds are classified as less liquid than the short asset. The budget constraint is represented in equation (13), and the last constraint (14) represents the non-negativity constraint.

4.4 Interbank Market for Government Bonds

Banks use government bonds to balance their idiosyncratic liquidity needs: All banks invest in government bonds at date 0. When each bank has learnt whether it is an early bank or a late bank at date 1, the early banks sell their government bonds to the late banks in exchange for the short-term asset to repay their depositors. We assume that the
consumers’ expected utility of an investment in risky government bonds is higher than that of an investment in the safe short-term asset, i.e.

\[ \beta \ln(h) + (1 - \beta) \ln(l) \geq \ln(1). \]  

(15)

If it were not for this assumption, banks would have no incentive to invest in government bonds at date 0, which means that an interbank market for government bonds with a positive market price would not exist at date 1. In the following, we briefly describe the demand- and the supply-side of the interbank market for government bonds and derive the equilibrium.

Late banks will only buy government bonds if in this case the expected utility of their depositors is at least as high as when they simply store the short-term asset, i.e. if

\[ \beta \ln(h) + (1 - \beta) \ln(l) - \ln(p) \geq \ln(1). \]  

(16)

This implies that there is a maximum price late banks are willing to pay for a government bond given by

\[ p^{\text{max}} = h^\beta l^{(1 - \beta)}. \]  

(17)

All banks are identical and thus solve the same optimisation problem at date 0. Accordingly, for all banks the optimal quantities invested in the short-term asset and the long-term assets are identical. We denote these optimal quantities by \( x^*, y^*, \) and \( u^*. \) Considering the number of depositors is normalised to one, the optimal quantities of each individual bank correspond to the respective aggregate quantities invested in each asset.

\[ \text{If it were not for this assumption, late banks would only be willing to pay a lower price than 1 for a government bond at date 1. However, this would mean that a government bond is worth less than the short-term asset at date 1, so that banks prefer to invest in the short-term asset instead of investing in government bonds at date 0.} \]

\[ \text{For a more detailed description of this government bond market see Neyer and Sterzel (2017).} \]
type. As half of the banks are late banks, aggregate demand for government bonds at
date 1 is

\[ y^D = \begin{cases} 
0.5 \frac{x^*}{p} & \text{if } p \leq p^{max}, \\
0 & \text{if } p > p^{max}.
\end{cases} \]  

(18)

For \( p \leq p^{max} \) the demand curve for government bonds is downward sloping because
late banks want to sell their total amount of the short-term asset which is limited to
0.5\( x^* \). Consequently, a higher price \( p \) implies that fewer government bonds can be bought.
However, early consumers only value consumption at date 1 so that early banks want to
sell all their government bonds at this date independently of the price. The supply of
government bonds is thus perfectly price inelastic:

\[ y^S = 0.5y^*. \]  

(19)

Considering (18) and (19) and denoting the equilibrium price for government bonds \( p^{**} \)
the market clearing condition becomes

\[ \frac{x^*}{p^{**}} = y^*. \]  

(20)

As there is no aggregate liquidity uncertainty and as all banks solve the same optimisation
problem at date 0, the aggregate supply and demand for government bonds and thus
the date-1 equilibrium variables are known at date 0. This implies that the equilibrium
government bond price at date 1 must be

\[ p^{**} = 1. \]  

(21)

If \( p^{**} < 1 \), the return on government bonds would be smaller than on the short-term asset
at date 1, so that no bank would invest in government bonds at date 0. If \( p^{**} > 1 \), a
government bond would be worth more than the short-term asset at date 1, so that no

\[ ^{To be able to distinguish between those quantities optimally invested in the different assets and
those quantities exchanged in equilibrium on the interbank market, we index optimal variables with * and
equilibrium variables with **.} \]
bank would invest in the short-term asset at date 0. In both cases, there would not be an interbank market for government bonds with a positive price. Considering (15) and (17), \( p^{\text{max}} \geq 1 \), which implies that the interbank market is always cleared with the exchanged quantity of government bonds in equilibrium given by

\[
y^{**} = 0.5y^*.
\]

(22)

5 Optimal Bank Investment and Financing Behaviour

This section analyses the impact of different treatments of government bonds in bank liquidity regulation on bank investment and financing behaviour. We start our analysis by determining how banks invest and finance these investments without any regulation. We then analyse how their behaviour will change if a binding required minimum liquidity ratio \( LR^{\text{min}} \) is introduced. In a first regulation scenario the regulator classifies the short-term asset and government bonds as equally liquid (preferential treatment of government bonds). Our analysis shows that compared to the case without any binding required liquidity ratio, bank investment in liquid assets increases at the expense of a decrease in their loan investment. However, the decrease in loans is higher than the increase in liquid assets, i.e. the regulation also implies that banks raise less equity capital. In a second regulation scenario the regulator regards government bonds as less liquid than the short-term asset (repealing the preferential treatment of government bonds). It turns out that then the effects observed in the first regulation scenario are reinforced.

To demonstrate a bank’s optimal investment and financing behaviour in the different scenarios, we make use of the same numerical example as in [Neyer and Sterzel (2017)] which is similar to the one used by [Allen and Carletti (2006)]. The government bond returns \( h = 1.3 \) with probability \( \beta = 0.98 \) and \( l = 0.3 \) with probability \( (1 - \beta) = 0.02 \). Consequently, the investment in government bonds of one unit of the consumption good at date 0 yields the expected return \( E(S) = 1.2746 \) at date 2. Loans are also state-dependent and return at date 2. They return \( H = 1.54 \) with probability \( \alpha = 0.93 \), and they fail and yield \( L = 0.25 \) with probability \( (1 - \alpha) = 0.07 \). Hence, the expected loan return at date 2 is \( E(K) = 1.4497 \). Investors’ opportunity costs are \( \rho = 1.5 \).
5.1 No Liquidity Requirements

If there is no binding required liquidity ratio ($LR_{min}^L = 0$), we will get the solutions given in Table 1 for optimal bank behaviour. With respect to these results, we will comment on

<table>
<thead>
<tr>
<th>Balance Sheet</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x^*$ 0.4544 41.87%</td>
</tr>
<tr>
<td>$y^*$ 0.4544 41.87%</td>
</tr>
<tr>
<td>$u^*$ 0.1765 16.26%</td>
</tr>
<tr>
<td>$\sum$ 1.0853 100%</td>
</tr>
</tbody>
</table>

Contracts with Investors:

- early banks: $e_{2H}^t = 0.2718$ $e_{2L}^t = 0.0441$
- late banks: $e_{2Hh}^t = 0$ $e_{2Hl}^t = 0$ $e_{2Lh}^t = 0$ $e_{2Ll}^t = 0$

Deposit Contracts:

$c_1^* = 0.9088$ $c_{2Hh}^* = 1.4532$ $c_{2Hl}^* = 0.5444$ $c_{2Lh}^* = 1.2256$ $c_{2Ll}^* = 0.3168$ $E(U) = 0.1230$

Proof. See Proof I in Appendix A.

Table 1: Banks’ Optimal Balance Sheet Structure and Repayments to Investors and Depositors when there is no Liquidity Regulation

two aspects in more detail: first, the equally high investment in the short-term asset and government bonds ($x^* = y^*$) and second, that banks raise equity capital ($e_0^t > 0$) although it is costly.

Regarding the result $x^* = y^*$ it is important that half of the banks are early banks whereas the other half are late banks, and that there is idiosyncratic but no aggregate liquidity uncertainty. The latter implies that banks know the equilibrium price $p^{**} = 1$ at date 0 (see Section 4.4 for details). Accordingly, all banks invest an identical amount in government bonds and in the short-term asset, to be able to hedge their idiosyncratic liquidity risks completely by trading government bonds on the interbank market at date 1. This allows us to set $x^* = y^* = 0.5z^*$ in our subsequent analyses. The variable $z^*$ thus
donates a bank’s optimal investment in liquid assets (short-term asset and government bonds).

Furthermore, the results reveal that although there are no capital requirements, banks raise costly equity capital. Equity capital is costly because opportunity costs, and thus the amount banks expect to repay to investors, exceed the expected return even of the banks’ most profitable asset, in our case loans \((\rho > E(K))\). The reason is that equity capital allows the liquidity risk involved with an investment in relatively highly profitable loans to be transferred at least partially from risk-averse depositors to risk-neutral investors, leading to an increase in depositors’ expected utility. In more detail, an investment in highly profitable loans leads to the highest expected consumption of a late consumer. However, as loans are totally illiquid, this investment involves a liquidity risk for a consumer. If it turns out that he is an early consumer, he will not benefit at all from this investment. Without the possibility for banks to raise equity capital, the consumers would bear the total liquidity risk themselves. An investment in highly profitable but totally illiquid loans will increase the expected late consumers’ consumption, but due to the budget constraint \[ \frac{\partial z}{\partial u} \bigg|_{\text{no capital}} = -1, \] so that there is a respective decline in early consumer consumption. With the possibility of raising equity capital the budget constraint is softened and an increase in loans leads to a lower necessary decrease in liquid assets, \[ \frac{\partial z}{\partial u} \bigg|_{\text{with capital}} > -1 = \frac{\partial z}{\partial u} \bigg|_{\text{no capital}}. \] Consequently, an investment in loans, which increases the expected date-2 consumption, only implies a relatively small decrease of consumption at date 1, so that there is an overall increase in depositors’ expected utility. Crucial for this result is that a huge part of the additional loan investment is financed by raising equity capital from risk-neutral investors. Due to their risk-neutrality, they do not mind being repaid either at date 1 or 2, so it is optimal that they bear the liquidity risk involved with the banks’ loan investment. This means that if it turns out that a bank is an early bank, the

\[ \text{\footnotesize For a detailed explanation of banks’ investment and financing behaviour without the possibility to raise equity capital see Neyer and Sterzel (2017, Section 5.2).} \]

\[ \text{\footnotesize In our numerical example, this decline in date-1 consumption and thus in early depositors’ utility would be so strong that banks would not invest (at all) in illiquid loans but only in liquid assets (short-term asset, government bonds).} \]

\[ \text{\footnotesize Note that the possibility to have thus a higher expected consumption at date 2 \((E(c_2^*)) = 1.4191\) implies that the consumers are willing to except a repayment at date 1 of less than 1 \((c_1^* = 0.9088)\).} \]
investors of this bank will receive the total proceeds from the loan investment at date 2
\((e_{2H}, e_{2L}^* > 0)\). However, if it turns out that a bank is a late bank, they will receive nothing
\((e_{2H}, e_{2HI}, e_{2LH}, e_{2LL} = 0)\). Considering investors thus get repaid with the total proceeds
from the bank loan investment but only with probability 0.5, and that their opportunity
costs are higher than the expected return on loans \((\rho > E(K))\), the bank loan investment
must exceed the amount of raised equity capital to be able to satisfy investors’ claims.
This means that it is not possible to finance an additional loan investment exclusively by
raising more equity, i.e. an increase in loan investment is still associated with a decrease
of investment in liquid assets \((-1 < \frac{\partial z}{\partial \epsilon} \text{ with capital} < 0)\).  

5.2 Liquidity Requirements: Preferential Treatment of Government
Bonds

In this section, we analyse bank behaviour when banks face a required minimum liq-
uidity ratio in which government bonds are preferentially treated i.e. the short-term
asset and government bonds are treated as equally liquid. In the constraint (12) we have
\(\kappa_x = \kappa_y = 1\). Government bonds are treated preferentially to the short-term asset as, un-
like the short-term asset, they have to be sold on an interbank market to obtain liquidity.
Hence, government bonds are exposed to a potential market liquidity risk. If banks do
not face binding liquidity requirements (Section 5.1), they will choose an optimal liquidity
ratio of \(LR^{opt} = \frac{2^* + \rho^*}{1} = 0.9088\). In order to analyse the impact of a binding required
liquidity ratio, \(LR^{min} > LR^{opt}\) must hold, so that we set \(LR^{min} = 0.92\). The results for
optimal bank behaviour under this constraint are shown in Table 2. The comparison of
the results for optimal bank behaviour given in Tables 1 and 2 reveals that the binding
liquidity requirement induces banks to increase their liquid asset investment at the ex-
 pense of a decrease in their loan investment. However, the decrease in loans is higher than
the increase in liquid assets, i.e. the regulation also implies that banks raise less equity

\[\frac{\partial z}{\partial \epsilon} \text{ with capital} < 0\]  

Formally, the investors’ incentive-compatibility constraint given by (11) becomes
\(e_{2H}, e_{2L}^* = 0.5^* E(K)\), so that \(\frac{\partial E(K)}{\partial \epsilon} = \frac{\partial E(K)}{\partial \epsilon}\). This means that the loan investment needs to be at least \(\frac{\partial E(K)}{\partial \epsilon}\) times higher than the
amount of raised equity capital. In our numerical example loan investment thus needs to be 2.0692 times
higher than the amount of raised equity.

We want to analyse the impact of a binding required liquidity ratio on bank behaviour. Therefore, we
assume a minimum liquidity ratio which is slightly higher than \(LR^{opt}\). Note, that if 0.9088 < \(LR^{min} < 1\),
the qualitative effects would be the same. However, if \(LR^{min} = 1\), banks were forced to invest their total
deposits in liquid assets. In this case, banks were obsolete.
Table 2: Banks’ Optimal Balance Sheet Structure and Repayments to Investors and Depositors when the Short-term Asset and Government Bonds are Classified as Equally Liquid in Bank Liquidity Regulation

Proof. See Proof II in Appendix A

15 In our numerical example additional loan investment must be more than twice as high as additional equity capital, see footnote 13.

16 Formally: From the budget constraint (13) we have that $dz + du = de_0$. The investors’ incentive-compatibility constraint (11) in combination with bank’s optimal risk-sharing require $du = 2.0692de_0$ (see...
and loan investment reveals that the introduction of a binding minimum liquidity ratio implies an inefficiently low use of the possibility to transfer liquidity risks involved with the investment in highly profitable loans from risk-averse depositors to risk-neutral investors which reduces the depositors’ expected utility.

5.3 Liquidity Requirements: Repealing the Preferential Treatment of Government Bonds

This section analyses bank optimal investment and financing behaviour when the preferential treatment of government bonds is repealed under bank liquidity regulation, i.e. when the regulator considers the potential market liquidity risk of government bonds. Formally, government bonds are assigned a lower liquidity factor than the short-term asset \(\kappa_y < \kappa_x\) in the required minimum liquidity ratio \(LR_{\text{min}}\). Accounting for that we set \(\kappa_x = 1\) and \(\kappa_y = 0.95\). The required minimum liquidity ratio then becomes \(LR_{\text{min}} = \frac{\kappa_x x + \kappa_y y}{1} = x + 0.95y = 0.92\).\(^{17}\) The resulting optimal bank behaviour in this regulation scenario is shown in Table 3.

Comparing the results given in Tables 2 and 3 reveals that classifying government bonds as less liquid than the short-term asset in bank liquidity regulation has qualitatively the same impact on bank behaviour as the introduction of the binding minimum liquidity ratio described in the previous section: Banks increase their liquid asset investment at the expense of a decrease in their loan investment. However, the decrease in loans is higher than the increase in liquid assets, i.e. the regulation also implies that banks raise less equity capital \(z^*\) increases, \(e^*_0\) and \(u^*\) decrease. Consequently, the beneficial liquidity risk transfer will be further restricted, leading to a further reduction in the depositors’ expected utility. A binding minimum liquidity ratio implies that banks are required to hold more liquid assets than they will do if it is not for the regulation. In a regulation also Section 5.1). The introduction of the binding liquidity ratio implies \(dz = 0.0112\). Solving the equations for \(du\) and \(de_0\), we obtain \(du = -0.0217\) and \(de_0 = -0.0105\).

\(^{17}\)The liquidity factor \(\kappa_y\) has been chosen arbitrarily within the interval \([0.84, 1]\), i.e. it may not reflect the exact liquidity risk of government bonds. Considering the exact liquidity risk is not necessary in our analysis as we only want to determine the qualitative effects on bank behaviour and financial stability when repealing the preferential treatment of government bonds in liquidity regulation, and these effects are the same for all \(\kappa_y \geq 0.84\). If \(\kappa_y < 0.84\), banks would no longer invest in government bonds. Banks invest in government bonds to hedge their idiosyncratic liquidity risks which means that \(x^* = y^*\) (see Sections 4.4 and 5.1). However, if \(\kappa_y < 0.84\), hedging the idiosyncratic liquidity by using an interbank market for government bonds will no longer be possible as banks would then have to invest more than their amount of deposits into liquid assets to fulfil the liquidity requirements \((x = y > 1 = D)\).
### Balance Sheet

<table>
<thead>
<tr>
<th>A</th>
<th>L</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x^* = 0.4718$</td>
<td>44.81%</td>
</tr>
<tr>
<td>$y^* = 0.4718$</td>
<td>44.81%</td>
</tr>
<tr>
<td>$u^* = 0.1092$</td>
<td>10.37%</td>
</tr>
<tr>
<td>$\sum = 1.0528$</td>
<td>100%</td>
</tr>
</tbody>
</table>

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$D = 1$</td>
<td>94.98%</td>
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</tbody>
</table>

### Contracts with Investors:

<table>
<thead>
<tr>
<th>early banks:</th>
<th>late banks:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e^*_{2H} = 0.1682$</td>
<td>$e^*_{2L} = 0.0273$</td>
</tr>
<tr>
<td>$e^*_{2Hl} = 0$</td>
<td>$e^*_{2Lh} = 0$</td>
</tr>
<tr>
<td>$e^*_{2Hl} = 0$</td>
<td>$e^*_{2Ll} = 0$</td>
</tr>
</tbody>
</table>

### Deposit Contracts:

- $c^*_1 = 0.9436$
- $c^*_{2H} = 1.3948$
- $c^*_{2Hl} = 0.4512$
- $c^*_{2L} = 1.2540$
- $c^*_{2Ll} = 0.3104$

- $E(U) = 0.1221$

**Proof.** See Proof III in Appendix A

Table 3: Banks’ Optimal Balance Sheet Structure and Repayments to Investors and Depositors when Government Bonds are Classified as Less Liquid than the Short-term Asset in Bank Liquidity Regulation

In the scenario in which government bonds are classified as less liquid than the short-term asset, **banks must hold in total even more liquid assets** to fulfil the requirement compared to a scenario in which both assets are treated as equally liquid.

However, as in the regulation scenario in which both assets are treated as equally liquid, banks can only hold more liquid assets at the expense of lower investment in loans and a reduction in equity capital because of the budget constraint (13) in combination with the investors’ incentive-compatibility constraint (11). The impact of introducing a binding minimum liquidity ratio, in which the short-term asset and government bonds are classified as equally liquid on bank behaviour, will thus be reinforced if government bonds are classified as less liquid in bank liquidity regulation.

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18 Note that the different treatment of government bonds and the short-term asset in bank liquidity regulation has no influence on the result that $x^* = y^*$ as long as $\kappa_y \geq 0.84$ (see footnote 17).
6 Financial Stability

At the beginning of this section we show that increasing doubts about sovereign solvency may lead to liquidity issues in the banking sector triggered by a respective price drop for sovereign bonds. Illiquid but per se solvent banks go bankrupt. Within our model framework we derive that liquidity requirements cannot prevent these bankruptcies. However, a central bank acting as a LOLR can avoid bank insolvencies due to liquidity issues. Against this background, introducing liquidity requirements in general, and repealing the preferential treatment of government bonds in liquidity regulation in particular, undermines financial stability in the case of a sovereign debt crisis. Note that the modelling of the government bond shock and of the LOLR corresponds exactly to the modelling in Neyer and Sterzel (2017).

6.1 Government Bond Shock

After the banks have made their financing and investment decisions at date 0, but before the start of interbank trading at date 1, the economy is hit by a shock in the form of a sudden increase in the default probability of government bonds (we refer to this shock as a government bond shock). This implies a respective decrease of the expected return on government bonds. Denoting after-shock variables with a bar, we thus have $(1-\bar{\beta}) > (1-\beta)$ and $E(\bar{S}) > E(S)$. When investment decisions are made, this government bond shock is assigned a zero probability at date 0, as the liquidity shock in Allen and Gale (2000). The return on the short-term asset and the expected return on the loan portfolio are not affected by the shock.\footnote{To keep the model as simple as possible, we assume that the expected loan return is not affected by the government bond shock. However, there is empirical evidence that there are spillovers going from sovereigns to other sectors of an economy (see e.g. Corsetti et al. 2013) as sovereigns’ ratings normally apply as a ”sovereign floor“ for the ratings assigned to private borrowers. Nevertheless, if we take this correlation into account our results will not qualitatively change. See footnote 22 for details.}

Regarding the interbank trading at date 1, the shock influences the later banks’ demand for sovereign bonds in the interbank market. The decline in the expected return on government bonds implies that the maximum price late banks are willing to pay for a bond decreases (see equations (17) and (18)). The early banks’ supply of government bonds is not affected by the shock. As their depositors only value consumption at date 1,
they want to sell their total holdings of government bonds at the same time, independent of their default probability (see equation \(19\)).

To be able to satisfy the early banks’ depositors according to their contract, the price the bank receives for a government bond must be at least one, i.e. we have a critical price

\[
p^{\text{crit}} = 1. \tag{23}\]

Setting in equation \(17\) \(p^{\text{max}}\) equal to \(p^{\text{crit}}\) and then solving the equation for \((1 - \beta)\) gives the critical default probability

\[
(1 - \beta)^{\text{crit}} = \frac{\ln(h) - \ln(p^{\text{crit}})}{\ln(h) - \ln(l)} = \frac{\ln(h)}{\ln(h) - \ln(l)}. \tag{24}\]

If the aftershock default probability of government bonds exceeds this critical probability, the expected return on government bonds will become so low that the maximum price late banks are willing to pay for a bond will fall below one, early banks will be illiquid and insolvent. Therefore, the threshold \((1 - \beta)^{\text{crit}}\) allows us to distinguish between a small and a large government shock.

A small government shock implies that \((1 - \beta^{\text{small}}) \leq (1 - \beta)^{\text{crit}}\). The increased sovereign default probability induces a decrease in the maximum price late banks are willing to pay for a sovereign bond. However, as it does not fall below one \((1 \leq p^{\text{max small}} < p^{\text{max}})\), the equilibrium price and the equilibrium transaction volume do not change, \(p^{\text{** small}} = p^{*} = 1, \ y^{\text{** small}} = y^{*} = 0.5 y^{*}\). As a result, a small government bond shock does not lead to liquidity issues in the banking sector\(^{20}\).

A large government bond shock means that \((1 - \beta^{\text{large}}) > (1 - \beta)^{\text{crit}}\). The increase in the government bonds’ default probability is so high that their expected return becomes so low that the maximum price late banks are willing to pay for a bond falls below one. Considering equation \(17\), the aftershock equilibrium price becomes

\[
p^{\text{** large}} = p^{\text{max large}} < 1. \tag{25}\]

\(^{20}\)For a broad discussion of who actually bears the costs in the case of a small and a large government bond shock see Neyer and Sterzel (2017).
Note that due to the perfectly price inelastic supply the equilibrium trading volume has not changed, $y^{**\text{large}} = y^{**} = 0.5y^*$. The decrease of the equilibrium price below 1 means that early banks are no longer able to fulfill their deposit contracts:

$$c^{\text{large}}_1 = x^* + y^* p^{**\text{large}} < x^* + y^* p^{**} = x^* + y^* = c^*_1.$$  \hspace{1cm} (26)

Early banks are thus insolvent and are liquidated at date 1.

### 6.2 Central Bank as a Lender of Last Resort

To avoid bankruptcies of illiquid but per se solvent banks we introduce a central bank as a lender of last resort (LOLR) in the sense of Bagehot \cite{1873}. The central bank provides liquidity to troubled banks against adequate collateral. In our model, banks’ loan portfolios serve as collateral. In order to avoid any potential losses for the central bank, the maximum amount of liquidity $\psi$ the central bank is willing to provide to an early bank against its loan portfolio as collateral is

$$\psi = u^* L.$$  \hspace{1cm} (27)

An early bank’s additional liquidity needs after a large government bond shock $\tau$ are determined by the repayment agreed upon in the deposit contract $c^*_1$ and the lower after-shock repayment $c^{\text{large}}_1$ (without a LOLR):

$$\tau = c^*_1 - c^{\text{large}}_1 = y^*(p^{**} - p^{**\text{large}}) = y^*(1 - p^{**\text{large}}).$$  \hspace{1cm} (28)

Comparing the bank’s additional liquidity needs $\tau$ with the maximum amount of liquidity the central bank is willing to provide $\psi$ gives us the critical government bond price

$$p^{\text{critLOLR}} = 1 - \frac{u^* L}{y^*} < 1.$$  \hspace{1cm} (29)

\footnote{Note, that in our model government bonds do not serve as collateral. If this were the case, the central bank would have to buy government bonds for the price of 1, protecting illiquid banks from going bankrupt. This would induce a subsidy by the central bank as the market price for government bonds is lower than 1 after the large shock. Furthermore, the central bank would be exposed to credit risks as in the case of bond failures, the central bank would bear losses ($l < p = 1$).}
If \( p^{\text{large}} < p^{\text{critLOLR}} \) the bank is illiquid and insolvent. Inserting \( p^{\text{critLOLR}} \) for \( p^{\text{large}} \) in equation (25) and then solving the equation for \( (1 - \beta^{\text{large}}) \), gives us the critical default probability

\[
(1 - \beta)^{\text{critLOLR}} = \frac{\ln(h) + \ln(u^L)}{\ln(h) - \ln(l)} = \frac{\ln(h) + \ln(u^L 2L)}{\ln(h) - \ln(l)}.
\] (30)

If the government bond shock is so large that \( (1 - \beta^{\text{large}}) > (1 - \beta)^{\text{critLOLR}} \), the equilibrium price \( p^{\text{large}} \) will fall below \( p^{\text{critLOLR}} \), and early banks will become insolvent, despite the existence of a LOLR. The liquidity issue leads to a solvency issue as the price drop is so huge that the early banks do not have sufficient collateral to obtain enough liquidity from the LOLR to satisfy their depositors.

Comparing the critical default probability with and without a central bank as a LOLR (see equations (24) and (30)) reveals the obvious result that with a LOLR the critical default probability is higher. However, the comparison also shows that with a LOLR the critical default probability not only depends on the possible government bond returns \( h \) and \( l \), as is the case without a LOLR, but, in addition, on the loan portfolio return \( L \) and the banks’ investment in government bonds \( y^* \) and loans \( u^* \).\(^{22}\) This has important implications for the banking sector’s shock-absorbing capacity under the different liquidity regulation approaches as we will see in the next section.

6.3 The Shock Absorbing Capacity of the Banking Sector in Different Liquidity Regulation Scenarios

The above analysis allows us to discuss the (government bond) shock-absorbing capacity of the banking sector, and in this sense its stability\(^{23}\) in different liquidity regulation scenarios. The difference between the critical and the initial default probability of government

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\(^{22}\) We argued at the beginning of this section that considering a possible spillover of the government bond shock to loans would not lead to a qualitative change of our results. If the probability of loan success was negatively affected by the government bond shock, i.e. if \( \alpha > \bar{\alpha} \), the discussed liquidity issues for the early banks would not be affected. The crucial point is that the decrease in \( \alpha \) would neither induce a change in the liquidity provision by the central bank \( (\psi) \) nor would it lead to an additional liquidity demand \( (\tau) \). As these variables determine the shock-absorbing capacity of the banking sector (see Section 6.3), spillover effects from sovereign to loans have no impact on our results.

\(^{23}\) The ECB defines financial stability as a condition in which the financial system – intermediaries, markets and market infrastructures – can withstand shocks without major distribution in financial intermediation and the general supply of financial services.
bonds serves as a measure of the banking sector’s shock-absorbing capacity. The measure shows how large a government bond shock can be without implying the insolvency of early banks and thus of a huge part of the banking sector. Considering equations (24) and (30) and denoting the shock-absorbing capacity by $SAC$ and $SAC^{LOLR}$ respectively, we get for the banking sector’s shock absorbing capacity without a LOLR

$$SAC = (1 - \beta)^{\text{crit}} - (1 - \beta) = \frac{\ln(h)}{\ln(h) - \ln(l)} - (1 - \beta)$$

(31)

and for the banking sector’s shock absorbing capacity with a LOLR

$$SAC^{LOLR} = (1 - \beta)^{\text{critLOLR}} - (1 - \beta) = \frac{\ln(h) + \ln\left(\frac{u^* L}{\tau^2}ight)}{\ln(h) - \ln(l)} - (1 - \beta).$$

(32)

Equation (31) reveals that without a LOLR, the shock-absorbing capacity is not at all influenced by liquidity requirements. The reason is that without a LOLR early banks will become insolvent if the equilibrium price for a government bond falls below 1 i.e. in the case of a large government bond shock. Early banks then will no longer be able to satisfy their customers’ claims. The government bond price drop is only determined by the expected return on a government bond (see equation (17)) which is not affected by liquidity regulation at all. Hence, if there is no LOLR, the sovereign shock-induced liquidity problem cannot be solved by any kind of liquidity requirements i.e. the difference $(1 - \beta)^{\text{crit}} - (1 - \beta) = SAC$ is always the same. This result is illustrated in Figure 1 by the solid line.

However, with a LOLR liquidity requirements influence the banking sector’s shock-absorbing capacity. The reason is that the required minimum liquidity ratios influence bank investment behaviour (see Section 5). In both liquidity regulation scenarios banks increase their government bond investments $y^*$ and decrease their loan investments $u^*$, and both variables have an influence on $SAC^{LOLR}$ as equation (32) shows. The increase in government bond holdings implies an increase in the banks’ additional liquidity needs $\tau$ after the shock (see equation (28)). The decrease in loan investment leads to a decrease in the additional liquidity $\psi$ the central bank is willing to provide (see equation (27)). Both effects induce a decrease of the $SAC^{LOLR}$. As the increase in $y^*$ and the decrease in $u^*$
is the strongest in the liquidity regulation scenario where government bonds are classified as less liquid than the short-term assets, the (government bond) shock-absorbing capacity of the banking sector will be the lowest if the preferential treatment of government bonds within the $LR_{min}$ is repealed. This result is illustrated in Figure 1 by the broken line.

\begin{align*}
\text{No Regulation} & \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \q
treatment of sovereign borrowers in EU banking regulation. Our paper adds to this debate in two ways. First, by analysing the impact of different treatments of government bonds in bank liquidity regulation on bank investment and financing behaviour. Second, by investigating how far liquidity requirements in general and the abolishment of the preferential government bond treatment in liquidity regulation in particular contribute to making the banking sector more resilient against sovereign debt crises.

One important reason for relatively large government bond holdings is that banks use them to manage their everyday business. Capturing this idea, in our model banks hold government bonds to balance their idiosyncratic liquidity needs. Increasing sovereign risk may induce a price drop for government bonds, implying liquidity issues in the banking sector which then leads to the insolvency of a huge number of banks (systemic crisis). This model shows that liquidity requirements, regardless of the government bond treatment, are not able to increase financial stability in case of a sovereign crisis. Preventing banks from going bankrupt due to liquidity issues, a central bank acting as LOLR is necessary. Banks can then obtain additional liquidity from the LOLR against adequate collateral. It is then crucial that the banks’ investment structure determines the resilience of the banking sector in the case of sovereign distress. A required minimum liquidity ratio, and especially repealing the preferential treatment of government bonds in liquidity regulation, induces banks to hold more liquid assets in total (government bonds and the short-term asset) at the expense of a decrease in loan investment. Due to this regulation-induced change in banks’ investments, in a sovereign debt crisis banks face higher liquidity needs in order to fulfil the contracts with their consumers as contractually agreed. However, on the other hand, they have less collateral to obtain additional liquidity from the central bank. As a result, repealing the preferential treatment of government bonds in liquidity regulation does not contribute to a more resilient banking sector in sovereign crises.

A Appendix A

Proof I. Using the Lagrangian $\mathcal{L}$ the bank’s optimisation problem can be formulated as
\[
\max_{x,y,u,c_{2Hh},c_{2Hl},c_{2Lh},c_{2Ll}} \mathcal{L} = 0.5 \ln(c_1) + 0.5 \left[ 0.93 \cdot 0.98 \ln(c_{2Hh}) + 0.93 \cdot 0.02 \ln(c_{2Hl}) + 0.07 \cdot 0.98 \ln(c_{2Lh}) + 0.07 \cdot 0.02 \ln(c_{2Ll}) \right] - \lambda \left( x + y + u \right) \\
-1 \left[ \frac{0.5}{1.5} \left( 1.4497u + 0.9114c_{2Hh} + 0.0186c_{2Hl} + 0.0686c_{2Lh} + 0.0014c_{2Ll} \right) \right] - \mu_x x - \mu_y y - \mu_u u \\
- \mu_{c_{2Hh}}c_{2Hh} - \mu_{c_{2Hl}}c_{2Hl} - \mu_{c_{2Lh}}c_{2Lh} - \mu_{c_{2Ll}}c_{2Ll},
\]

(A.1)

with \( c_1 = x + y \ast \ast \),

\[
c_{2Hh} = 1.54u + \left( \frac{x}{p^{\ast \ast}} + y \right) 1.3 - c_{2Hh}, \\
c_{2Hl} = 1.54u + \left( \frac{x}{p^{\ast \ast}} + y \right) 0.3 - c_{2Hl}, \\
c_{2Lh} = 0.25u + \left( \frac{x}{p^{\ast \ast}} + y \right) 1.3 - c_{2Lh}, \\
c_{2Ll} = 0.25u + \left( \frac{x}{p^{\ast \ast}} + y \right) 0.3 - c_{2Ll},
\]

where \( \lambda \) is the Lagrange multiplier corresponding to the budget constraint (13) and also includes the investors' incentive-compatibility constraint (11), whereas \( \mu_x, \mu_y, \mu_u, \mu_{c_{2Hh}}, \mu_{c_{2Hl}}, \mu_{c_{2Lh}}, \mu_{c_{2Ll}} \) are Lagrange multipliers corresponding to the non-negativity conditions (14). As the same argumentation holds as in Section 4.4 and 5.1 we have \( p^{\ast \ast} = 1 \) and \( x^* = y^* = 0.5z^* \). By differentiating the Lagrange function with respect to \( z, u, c_{2Hh}, c_{2Hl}, c_{2Lh}, c_{2Ll}, \lambda, z, \mu_u, \mu_{c_{2Hh}}, \mu_{c_{2Hl}}, \mu_{c_{2Lh}}, \mu_{c_{2Ll}} \) we obtain

\[
\frac{\partial \mathcal{L}}{\partial z} = \frac{0.5}{z} + \frac{0.5 \cdot 0.93 \cdot 0.98 \cdot 1.3}{1.3z + 1.54u - c_{2Hh}} + \frac{0.5 \cdot 0.93 \cdot 0.02 \cdot 0.3}{0.3z + 1.54u - c_{2Hl}} + \frac{0.5 \cdot 0.07 \cdot 0.98 \cdot 1.3}{1.3z + 0.25u - c_{2Lh}} + \frac{0.5 \cdot 0.07 \cdot 0.02 \cdot 0.3}{0.3z + 0.25u - c_{2Ll}} - \lambda - \mu_z \overset{!}{=} 0,
\]

(A.2)

\[
\frac{\partial \mathcal{L}}{\partial u} = \frac{0.5 \cdot 0.93 \cdot 0.98 \cdot 1.54}{1.3z + 1.54u - c_{2Hh}} + \frac{0.5 \cdot 0.93 \cdot 0.02 \cdot 1.54}{0.3z + 1.54u - c_{2Hl}} + \frac{0.5 \cdot 0.07 \cdot 0.98 \cdot 0.25}{1.3z + 0.25u - c_{2Lh}} + \frac{0.5 \cdot 0.07 \cdot 0.02 \cdot 0.25}{0.3z + 0.25u - c_{2Ll}} - \lambda \left( 1 - \left( \frac{0.5 \cdot 1.4497}{1.5} \right) \right) - \mu_u \overset{!}{=} 0,
\]

(A.3)

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\[
\frac{\partial L}{\partial e^{2Hh}} = 0.5 \cdot 0.93 \cdot 0.98 \cdot (-1) - \lambda \left( \frac{0.5}{1.5} \cdot 0.9114 \right) - \mu e^{2Hh} = 0, \quad (A.4)
\]
\[
\frac{\partial L}{\partial e^{2Hl}} = 0.5 \cdot 0.93 \cdot 0.02 \cdot (-1) - \lambda \left( \frac{0.5}{1.5} \cdot 0.0186 \right) - \mu e^{2Hl} = 0, \quad (A.5)
\]
\[
\frac{\partial L}{\partial e^{2Lh}} = 0.5 \cdot 0.07 \cdot 0.98 \cdot (-1) - \lambda \left( \frac{0.5}{1.5} \cdot 0.0686 \right) - \mu e^{2Lh} = 0, \quad (A.6)
\]
\[
\frac{\partial L}{\partial e^{2Ll}} = 0.5 \cdot 0.07 \cdot 0.02 \cdot (-1) - \lambda \left( \frac{0.5}{1.5} \cdot 0.0014 \right) - \mu e^{2Ll} = 0, \quad (A.7)
\]
\[
\frac{\partial L}{\partial \lambda} = z + u - 1 - \frac{0.5}{1.5}\left[ 1.4497u + 0.9114e^{2Hh} + 0.0186e^{2Ll} \right] = 0, \quad (A.8)
\]
\[
\frac{\partial L}{\partial \mu} = -z = 0, \quad (A.9)
\]
\[
\frac{\partial L}{\partial \mu} = -u = 0, \quad (A.10)
\]
\[
\frac{\partial L}{\partial \mu e^{2Hh}} = -e^{2Hh} = 0, \quad (A.11)
\]
\[
\frac{\partial L}{\partial \mu e^{2Hl}} = -e^{2Hl} = 0, \quad (A.12)
\]
\[
\frac{\partial L}{\partial \mu e^{2Lh}} = -e^{2Lh} = 0, \quad (A.13)
\]
\[
\frac{\partial L}{\partial \mu e^{2Ll}} = -e^{2Ll} = 0. \quad (A.14)
\]

Multiplying both sides of the equations (A.2) with \(z\), (A.3) with \(u\), (A.4) with \(e^{2Hh}\), (A.5) with \(e^{2Hl}\), (A.6) with \(e^{2Lh}\) and (A.7) with \(e^{2Ll}\), adding the six equations and regarding equation (A.8), we obtain \(\lambda = 1\). After testing which non-negativity conditions bind, we derive that the non-negativity conditions for \(e^{Hh}, e^{Hl}, e^{Ll}\) and \(e^{Ll}\) become binding, i.e. \(e^{Hh}_0 = e^{Hl}_0 = e^{Ll}_0 = 0\) and thus \(\mu e^{2Hh} = \mu e^{2Hl} = \mu e^{2Lh} = \mu e^{2Ll} \neq 0\). Solving then for \(z^*\) and \(u^*\) we get \(z^* = 0.9088\) and \(u^* = 0.1765\) and regarding the constraint (11) the optimal amount of equity capital is \(e^*_0 = 0.0853\). ■
Proof II. When a bank faces a required minimum liquidity ratio \( LR^{\text{min}} = 0.92 = \frac{x+y}{1} \), its optimisation problem can be formulated in the form of the Lagrange function

\[
\begin{align*}
\max_{x,y,u,e_{2Hh},e_{2Hl},e_{2Lh},e_{2Ll}} \quad & \mathcal{L} = 0.5\ln(c_1) + 0.5[0.93 \cdot 0.98\ln(c_{2Hh}) + 0.93 \cdot 0.02\ln(c_{2Hl}) + 0.07 \cdot 0.98\ln(c_{2Lh}) + 0.07 \cdot 0.02\ln(c_{2Ll})] \\
& - \lambda \left( x + y + u - 1 - \left[ 0.5 \cdot \frac{1.4497u + 0.9114e_{2Hh} + 0.0186e_{2Hl} + 0.0686e_{2Lh} + 0.0014e_{2Ll}}{1.5} \right] \right) \\
- & \mu_x x - \mu_y y - \mu_u u - \mu_{e_{2Hh}} e_{2Hh} - \mu_{e_{2Hl}} e_{2Hl} - \mu_{e_{2Lh}} e_{2Lh} - \mu_{e_{2Ll}} e_{2Ll} - \mu_{LR} \left( x + y - 0.92 \right),
\end{align*}
\]  

(A.15)

with \( c_1 = x + yp^{**} \),

\[
\begin{align*}
\ c_{2Hh} &= 1.54u + \left( \frac{x}{p^{**}} + y \right) 1.3 - e_{2Hh}, \\
\ c_{2Hl} &= 1.54u + \left( \frac{x}{p^{**}} + y \right) 0.3 - e_{2Hl}, \\
\ c_{2Lh} &= 0.25u + \left( \frac{x}{p^{**}} + y \right) 1.3 - e_{2Lh}, \\
\ c_{2Ll} &= 0.25u + \left( \frac{x}{p^{**}} + y \right) 0.3 - e_{2Ll},
\end{align*}
\]

where \( \lambda \) is the Lagrange multiplier corresponding to the budget constraint (13) and also includes the investors’ incentive-compatibility constraint (11). The variables \( \mu_x, \mu_y, \mu_u, \mu_{e_{2Hh}}, \mu_{e_{2Hl}}, \mu_{e_{2Lh}} \) and \( \mu_{e_{2Ll}} \) are the Lagrange multipliers corresponding to the non-negativity conditions (14) and \( \mu_{LR} \) is the Lagrange multiplier corresponding to the required minimum liquidity ratio (12). Considering that \( p^{**} = 1 \) (see Section 4.4) as well as \( x^* = y^* = 0.5z^* \) (for a detailed explanation see text in Section 5.1) and differentiating \( \mathcal{L} \) with respect to \( z, u, e_{2Hh}, e_{2Hl}, e_{2Lh}, e_{2Ll}, \lambda, \mu_{LR}, \mu_z, \mu_u, \mu_{e_{2Hh}}, \mu_{e_{2Hl}}, \mu_{e_{2Lh}}, \mu_{e_{2Ll}} \) gives
\[ \frac{\partial L}{\partial z} = 0.5 \cdot \frac{0.93 \cdot 0.98 \cdot 1.3}{1.3z + 1.54u - e_{2Hh}} + \frac{0.93 \cdot 0.02 \cdot 0.3}{0.3z + 1.54u - e_{2Hl}} - \lambda - \mu_z - \mu_{LR} = 0, \quad (A.16) \]

\[ \frac{\partial L}{\partial u} = 0.5 \cdot \frac{0.93 \cdot 0.98 \cdot 1.3}{1.3z + 1.54u - e_{2Hh}} + \frac{0.93 \cdot 0.02 \cdot 0.3}{0.3z + 1.54u - e_{2Hl}} - \lambda - \mu_u = 0, \quad (A.17) \]

\[ \frac{\partial L}{\partial e_{2Hh}} = 0.5 \cdot \frac{0.93 \cdot 0.98 \cdot (-1)}{1.3z + 1.54u - e_{2Hh}} - \lambda - \mu_{e_{2Hh}} = 0, \quad (A.18) \]

\[ \frac{\partial L}{\partial e_{2Hl}} = 0.5 \cdot \frac{0.07 \cdot 0.98 \cdot 0.25}{1.3z + 0.25u - e_{2Lh}} - \lambda - \mu_{e_{2Lh}} = 0, \quad (A.19) \]

\[ \frac{\partial L}{\partial e_{2Lh}} = 0.5 \cdot \frac{0.07 \cdot 0.98 \cdot 0.25}{1.3z + 0.25u - e_{2Lh}} - \lambda - \mu_{e_{2Lh}} = 0, \quad (A.20) \]

\[ \frac{\partial L}{\partial e_{2Ll}} = 0.5 \cdot \frac{0.07 \cdot 0.98 \cdot 0.25}{1.3z + 0.25u - e_{2Ll}} - \lambda - \mu_{e_{2Ll}} = 0, \quad (A.21) \]

\[ \frac{\partial L}{\partial \lambda} = 0.5 \cdot \frac{0.93 \cdot 0.98 \cdot (1.3z + 1.54u - e_{2Hh})}{1.3z + 1.54u - e_{2Hh}} + \frac{0.93 \cdot 0.02 \cdot 0.3}{0.3z + 1.54u - e_{2Hl}} - \lambda - \mu_u = 0, \quad (A.22) \]

\[ \frac{\partial L}{\partial \mu_{LR}} = 0.92 \neq 0. \quad (A.23) \]

\[ \frac{\partial L}{\partial \mu_z} = - z \neq 0, \quad (A.24) \]

\[ \frac{\partial L}{\partial \mu_u} = - u \neq 0, \quad (A.25) \]

\[ \frac{\partial L}{\partial \mu_{e_{2Hh}}} = - e_{2Hh} \neq 0, \quad (A.26) \]

\[ \frac{\partial L}{\partial \mu_{e_{2Hl}}} = - e_{2Hl} \neq 0, \quad (A.27) \]

\[ \frac{\partial L}{\partial \mu_{e_{2Lh}}} = - e_{2Lh} \neq 0, \quad (A.28) \]

\[ \frac{\partial L}{\partial \mu_{e_{2Ll}}} = - e_{2Ll} \neq 0. \quad (A.29) \]

Considering that \( x^* = y^* = 0.5z^* \) and \( LR^{\min} = 0.92 = x + y \), we obtain that \( z^* = 0.92 \) \((\mu_{LR} \neq 0)\). After testing which non-negativity conditions bind, we derive that the non-negativity conditions for \( e_{Hh}, e_{Hl}, e_{Ll} \) and \( e_{Ll} \) become binding, i.e. \( e^*_h = e^*_l = e^*_l = \cdots \).
\(e_{Ll}^* = 0\) and thus \(\mu_{e_{2Hh}} = \mu_{e_{2Hl}} = \mu_{e_{2Lh}} = \mu_{e_{2Ll}} \neq 0\). Solving then for \(u^*\) and \(e_0^*\) we get \(u^* = 0.1548\) and \(e_0^* = 0.0748\).

**Proof III.** When banks face a required minimum liquidity ratio and government bonds are applied a lower liquidity factor than the short-term asset \((LR_{min} = \frac{\kappa_x + \kappa_y}{1} = x + 0.95y = 0.92)\), their optimisation problem in the form of a Lagrangian is then

\[
\max_{x, y, u, e_{2Hh}, e_{2Hl}, e_{2Lh}, e_{2Ll}} \mathcal{L} = 0.5 \ln(c_1) + 0.5[0.93 \cdot 0.98 \ln(c_{2Hh}) + 0.93 \cdot 0.02 \ln(c_{2Hl})] + 0.07 \cdot 0.98 \ln(c_{2Lh}) + 0.07 \cdot 0.02 \ln(c_{2Ll})] \\
-\lambda \left( x + y + u - 1 - \left[ \frac{0.5}{1.5} (1.4497u + 0.9114e_{2Hh}) + 0.0186e_{2Hl} + 0.0686e_{2Lh} + 0.0014e_{2Ll} \right] \right) \tag{A.30}
\]

\[-\mu_x x - \mu_y y - \mu_u u - \mu_{e_{2Hh}} e_{2Hh} - \mu_{e_{2Hl}} e_{2Hl} - \mu_{e_{2Lh}} e_{2Lh} - \mu_{e_{2Ll}} e_{2Ll} - \mu_{LR} \\
(x + 0.95y - 0.92),\]

with \(c_1 = x + yp^{**}\),

\[
c_{2Hh} = 1.54u + \left( \frac{x}{p^{**}} + y \right) 1.3 - e_{2Hh},
\]

\[
c_{2Hl} = 1.54u + \left( \frac{x}{p^{**}} + y \right) 0.3 - e_{2Hl},
\]

\[
c_{2Lh} = 0.25u + \left( \frac{x}{p^{**}} + y \right) 1.3 - e_{2Lh},
\]

\[
c_{2Ll} = 0.25u + \left( \frac{x}{p^{**}} + y \right) 0.3 - e_{2Ll},
\]

where \(\lambda\) is the Lagrange multiplier corresponding to the budget constraint \([13]\) and also includes the investors’ incentive-compatibility constraint \([11]\), \(\mu_x, \mu_y, \mu_u, \mu_{e_{2Hh}}, \mu_{e_{2Hl}}, \mu_{e_{2Lh}}, \mu_{e_{2Ll}}\) are the Lagrange multipliers corresponding to the nonnegativity conditions \([14]\) and \(\mu_{LR}\) is the Lagrange multiplier corresponding to the required minimum liquidity ratio \([12]\). Considering that \(p^{**} = 1\) (see Section \([4.4]\)) banks equally split their investment in liquid assets \((x^* = y^* = 0.5z^*)\) also when sovereign bonds are applied a lower liquidity factor than the short-term asset (see footnote \([18]\)). By
differentiating $\mathcal{L}$ with respect to $z$, $u$, $e_{2Hh}$, $e_{2Hl}$, $e_{2Lh}$, $e_{2Li}$, $\lambda$, $\mu_{LR}$, $\mu_{z}$, $\mu_{u}$, $\mu_{e_{2Hh}}$, $\mu_{e_{2Hl}}$, $\mu_{e_{2Lh}}$ and $\mu_{e_{2Li}}$ we obtain

$$
\frac{\partial \mathcal{L}}{\partial z} = \frac{0.5 \cdot 0.93 \cdot 0.98 \cdot 1.3}{1.3z + 1.54u - e_{2Hh}} + \frac{0.5 \cdot 0.93 \cdot 0.02 \cdot 0.3}{1.3z + 1.54u - e_{2Hl}} - \lambda - \mu_{z} - 0.975 \mu_{LR}^! = 0,
$$

(A.31)

$$
\frac{\partial \mathcal{L}}{\partial u} = \frac{0.5 \cdot 0.93 \cdot 0.98 \cdot 1.54}{1.3z + 1.54u - e_{2Hh}} + \frac{0.5 \cdot 0.93 \cdot 0.02 \cdot 1.54}{1.3z + 1.54u - e_{2Hl}} - \lambda \left(1 - \left(\frac{0.5 \cdot 1.4497}{1.5}\right)\right) - \mu_{u}^! = 0,
$$

(A.32)

$$
\frac{\partial \mathcal{L}}{\partial e_{2Hh}} = \frac{0.5 \cdot 0.93 \cdot 0.98 \cdot (-1)}{1.3z + 1.54u - e_{2Hh}} - \lambda \left(\frac{0.5}{1.5} \cdot 0.9114\right) - \mu_{e_{2Hh}}^! = 0,
$$

(A.33)

$$
\frac{\partial \mathcal{L}}{\partial e_{2Hl}} = \frac{0.5 \cdot 0.07 \cdot 0.98 \cdot (-1)}{1.3z + 1.54u - e_{2Hl}} - \lambda \left(\frac{0.5}{1.5} \cdot 0.0186\right) - \mu_{e_{2Hl}}^! = 0,
$$

(A.34)

$$
\frac{\partial \mathcal{L}}{\partial e_{2Lh}} = \frac{0.5 \cdot 0.07 \cdot 0.02 \cdot (-1)}{1.3z + 1.54u - e_{2Lh}} - \lambda \left(\frac{0.5}{1.5} \cdot 0.0014\right) - \mu_{e_{2Lh}}^! = 0,
$$

(A.35)

$$
\frac{\partial \mathcal{L}}{\partial e_{2Li}} = \frac{0.5 \cdot 0.07 \cdot 0.02 \cdot (-1)}{1.3z + 1.54u - e_{2Li}} - \lambda \left(\frac{0.5}{1.5} \cdot 0.0014\right) - \mu_{e_{2Li}}^! = 0,
$$

(A.36)

$$
\frac{\partial \mathcal{L}}{\partial \lambda} = z + u - 1 - \left[\frac{0.5 \cdot 1.4497u + 0.9114e_{2Hh} + 0.0186e_{2Hl} + 0.0686e_{2Lh} + 0.0014e_{2Li}}{1.5}\right] = 0.
$$

(A.37)

$$
\frac{\partial \mathcal{L}}{\partial \mu_{LR}} = 0.975z - 0.92 \quad \frac{1}{!} = 0.
$$

(A.38)

$$
\frac{\partial \mathcal{L}}{\partial \mu_{z}} = -z \quad \frac{1}{!} = 0,
$$

(A.39)

$$
\frac{\partial \mathcal{L}}{\partial \mu_{u}} = -u \quad \frac{1}{!} = 0,
$$

(A.40)

$$
\frac{\partial \mathcal{L}}{\partial \mu_{e_{2Hh}}} = -e_{2Hh} \quad \frac{1}{!} = 0,
$$

(A.41)

$$
\frac{\partial \mathcal{L}}{\partial \mu_{e_{2Hl}}} = -e_{2Hl} \quad \frac{1}{!} = 0,
$$

(A.42)

$$
\frac{\partial \mathcal{L}}{\partial \mu_{e_{2Lh}}} = -e_{2Lh} \quad \frac{1}{!} = 0,
$$

(A.43)

$$
\frac{\partial \mathcal{L}}{\partial \mu_{e_{2Li}}} = -e_{2Li} \quad \frac{1}{!} = 0.
$$

(A.44)
Considering \( x^* = y^* = 0.5z^* \) and \( LR_{\text{min}}^n = 0.92 = x + 0.95y \), we obtain that \( z^* = 0.9436 \) \( (\mu^*_{LR} \neq 0) \). After testing which non-negativity conditions bind, we derive that the non-negativity conditions for \( e_{Hh}, e_{HI}, e_{LI} \) and \( e_{LL} \) become binding, i.e. \( e^*_{Hh} = e^*_{HI} = e^*_{LI} = e^*_{LL} = 0 \) and thus \( \mu_{e_{Hh}} = \mu_{e_{HI}} = \mu_{e_{LI}} = \mu_{e_{LL}} = 0 \). Solving then for \( u^* \) and \( e^*_{0} \) we get \( u^* = 0.1092 \) and \( e^*_{0} = 0.0528 \).

\[ \blacksquare \]

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