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**Working Paper**

## Optimal taxation under different concepts of justness

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## Optimal Taxation Under Different Concepts of Justness

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Robin Jessen, Maria Metzging, and Davud Rostam-Afschar<sup>1</sup>

## Optimal Taxation Under Different Concepts of Justness

### Abstract

*A common assumption in the optimal taxation literature is that the social planner maximizes a welfarist social welfare function with weights decreasing with income. However, high transfer withdrawal rates in many countries imply very low weights for the working poor in practice. We extend the optimal taxation framework by Saez (2002) to allow for alternatives to welfarism. We calculate weights of a social planner's function as implied by the German tax and transfer system based on the concepts of welfarism, minimum absolute and minimum relative sacrifice. We find that the minimum absolute sacrifice principle is in line with social weights that decline with net income.*

*JEL Classification: D63, D60, H21, H23, I38*

*Keywords: Justness; optimal taxation; income redistribution; equal sacrifice; inequality; subjective preferences*

*August 2018*

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# 1 Introduction

Fairness of taxation of incomes and redistribution is a controversial public policy issue, since different concepts of justness may lead to different optimal tax schedules. In fact, the political process may have compromised on a tax policy that respects criteria that cannot be captured by assuming a social planner who weights utility functions. Therefore, we analyze in this study different concepts of justness in an adjusted optimal taxation model to reconcile it with observed tax transfer practices. The standard approach in the welfarist optimal taxation literature is that the social planner maximizes a weighted sum of utilities, were the social weights decrease with income (e.g., [Saez 2001, 2002](#); [Blundell et al. 2009](#)) because this pattern lies within the bounds confined by the two extreme cases of Rawlsian and Benthamite objective functions. Intuitively, the hypothesis of decreasing welfarist weights expresses the idea that the social planner values an increase of net income of the poor by one Euro more than an increase of net income of higher income groups by one Euro. [Saez and Stantcheva \(2016\)](#) describe welfarism with decreasing weights as one of their two polar cases of interest. In contrast, tax transfer systems in many countries can only be optimal if the social planner had chosen weights in a non-decreasing way. As we show, a major reason for this lies in high transfer withdrawal rates for the working poor.<sup>1</sup>

The first main contribution of our paper is an extension of the [Saez \(2002\)](#) model to non-welfarist aims of the social planner. In a recent study, [Saez and Stantcheva \(2016\)](#) propose generalized marginal welfare weights that may depend on characteristics that do not enter utility.<sup>2</sup> In our approach, the social planner maximizes an objective function that allows for non-welfarist concepts of justness. In particular, we define the implicit weights of the social welfare function in terms of justness functions instead of utility functions. These functions impose a penalty on deviations of net income from a specific reference point. This implies that even though individuals maximize utility, the social planner does not necessarily maximize a weighted sum of utility but a function potentially including other criteria. The approach in our paper offers the advantage that we can directly quantify the value the social planner puts on a marginal improvement in a specific justness criterion for a given group compared to other groups. Thus, we can show which criterion is in line with social weights that decrease with income.

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<sup>1</sup>[Lockwood \(2016\)](#) shows that under present bias and with job search, optimal marginal tax rates are even lower than conventionally calculated. This might be especially relevant for marginal tax rates for the working poor.

<sup>2</sup>Similar to [Saez and Stantcheva \(2016\)](#), we take society's preferences as given and do not analyze how they could arise through the political process.

The second main contribution is the operationalization of an alternative specific idea of justness: minimum sacrifice.<sup>3</sup> Minimum sacrifice is related to the equal sacrifice principle (see [Mill 1871](#); [Musgrave and Musgrave 1973](#); [Richter 1983](#); [Young 1988](#)), which stipulates that all individuals should suffer the same ‘sacrifice’ through taxes. Assuming quasi-linear preferences, we define sacrifice as the burden of taxes in terms of utility and specify a loss function that puts an increasing penalty on deviations from minimum sacrifice.

[Berliant and Gouveia \(1993\)](#) show that incentive compatible equal sacrifice tax systems exist. [da Costa and Pereira \(2014\)](#) derive tax schedules that imply equal sacrifice and show that they inhibit inefficiencies for relatively high levels of government consumption as tax rates for high income earners become very large. In contrast, in our application the social planner considers the trade-off between loss in tax revenue due to mobility across income groups and the maximization of justness functions for groups with a positive social weight. Evidence that the equal sacrifice concept is likely to capture the preferences of a majority is only documented for the U.S: [Weinzierl \(2014\)](#) shows based on a survey that around 60 percent preferred the equal sacrifice tax schedule to a welfarist optimal tax schedule. While equal sacrifice equalizes the sacrifice due to taxes across a population, minimum sacrifice minimizes the (weighted) sum of these utility losses. The concept of minimum sacrifice is very close to the libertarian concept studied in [Saez and Stantcheva \(2016\)](#).<sup>4</sup>

The third main contribution is to illustrate how the model can be used with survey data. For this, we use a novel question on the just amount of income from the German Socio-Economic Panel and apply the model to the 2015 German tax and transfer system. In addition, we estimate labor supply elasticities using microsimulation and a structural labor supply model (e.g. [Aaberge and Colombino 2014](#)). Our study thus adds to the literature on empirical optimal taxation (e.g., [Aaberge et al. 2000](#); [Colombino and del Boca 1990](#)).

Our main result is that the concept of *minimum sacrifice* is in line with positive, declining social weights. The explanation for this finding is that the marginal gain of justness increases with the amount of taxes paid and the working poor pay only a low amount of taxes. Although the efficiency costs of redistributing one Euro to this group are relatively small, the reduction in the loss function is small too. In contrast, the increase in utility is high in the welfarist case. A second

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<sup>3</sup>See [Liebig and Sauer \(2016\)](#) for alternative concepts of justness and an overview of empirical justness research.

<sup>4</sup>[Saez and Stantcheva \(2016\)](#) allow for *welfarist* weights to increase with the amount of taxes paid. Thus decreasing taxes for those with a high tax burden is a high priority for the social planner.



finding is a confirmation of previous studies: the welfarist approach implies very low weights for the working poor under the 2015 German tax and transfer system.

In the previous literature, researchers used optimal income taxation frameworks that incorporate labor supply responses to obtain “tax-benefit revealed social preferences” (Bourguignon and Spadaro 2012), i.e., they calculate the social weights under which the current tax and transfer system is optimal. Blundell et al. (2009) apply the Saez (2002) framework to single mothers in Germany and the UK to calculate implied social weights. They find that working mothers with low incomes have low weights compared to the unemployed and most other income groups. For Germany, social weights for working poor single mothers with children under school-age can even become negative, thus implying a non-paretian social welfare function.

Bourguignon and Spadaro (2012) apply positive optimal taxation to the French redistribution system. They find negative social weights for the highest income earners and equally for the working poor if participation elasticities are high. In general, social weights for the working poor are much lower than those for the unemployed or the middle class. Bargain et al. (2014) calculate social weights for 17 European countries and the United States. For all analyzed countries, they find the highest social weights for the unemployed and substantially lower weights for the working poor, i.e., the group with the lowest net income apart from the unemployed. In Belgium, France, Germany, the Netherlands, Portugal, the UK, and Sweden the tax-transfer system implies the lowest social weights for this group.

Lockwood and Weinzierl (2016) perform inverse optimal taxation for the US from 1979 to 2010. They find that, if the standard welfarist model is correct, either perceived elasticities of taxable income or value judgments have changed considerably over time. This is interpreted as evidence that conventional assumptions of the benchmark model of optimal taxation should be questioned. Immervoll et al. (2007) find for several European countries that expanding redistribution to the working poor would be very cost effective and would virtually imply no deadweight burden.

The next section introduces our optimal taxation model for different concepts of justness, Section 3 describes how we calculate actual and just incomes as well as how we estimate extensive and intensive labor supply elasticities for Germany. In Section 4, we describe the resulting weights for different concepts of justness, while Section 5 concludes.

## 2 A Model of Optimal Taxation for Concepts of Justness

### 2.1 The General Framework

We adjust the canonical model by [Saez \(2002\)](#), which combines the pioneering work by [Mirrlees \(1971\)](#) and [Diamond \(1980\)](#), to capture non-welfarist objective functions. See [Appendix A](#) for a formal derivation. The key difference between the [Saez \(2002\)](#) model and our extension is that in [Saez \(2002\)](#) the social planner maximizes the weighted sum of utility. The main advantage of our approach is that we allow for the social planner to maximize the weighted sum of ‘justness functions’  $F_m$ . These functions can depend on various variables and incorporate different concepts of justness. We show that welfarism as in [Saez \(2002\)](#) is a special case.

Net income equals consumption and is given by  $c = y - T$ , where  $y$  denotes gross income and  $T$  denotes total taxes paid by the individual to finance a public good  $G$ .

The social planner chooses tax liabilities  $T$  to optimize a weighted sum  $L$  based on individual justness functions  $F_m$ . We describe this function in [Subsection 2.2](#) in more detail. For now it is sufficient to know that it may depend on  $c_{i^*}$ , for instance it could be  $F_m(c_{i^*}, c_{i^*}^{\text{ref}})$ , where  $c_{i^*}$  is optimal consumption and  $c_{i^*}^{\text{ref}}$  is a reference point, or on other factors even if they do not enter the utility function of individuals. Individuals are indexed by  $m \in M$ , where  $M$  is a set of measure one. Individuals’ utility,  $u_m(c_{i^*}, i^*)$ , depends on job choice  $i^*$  and net income  $c_{i^*}$ , where  $i = 0, \dots, I$  income groups are defined through the group’s gross income  $y_i$ .<sup>5</sup> The optimization is subject to the government budget constraint:

$$\max_{T_0, \dots, T_I} L = \int_M \mu_m F_m dV(m) \quad \text{s.t.} \quad \sum_{i=0}^I h_i T_i = G, \quad (1)$$

where  $\mu_m$  are primitive social weights.<sup>6</sup> Together with the Lagrange multiplier  $\lambda$ , they define the explicit weights  $e_m \equiv \frac{\mu_m}{\lambda}$ , which we focus on in this study. Each income group has the share  $h_i$  of the total population. These shares are endogenous as individuals adjust their labor supply to the tax-transfer system. We assume that the first derivative of  $F_m$  with respect to  $c_i$  is the same for all individuals in a given group  $i$ , i.e.,

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<sup>5</sup>The number of income groups is assumed to be fixed. In the empirical application, we define groups  $1, \dots, I$  as quintiles of the positive gross income distribution. [Bargain et al. \(2014\)](#) show that changing the cut-off points does not affect the results substantially.

<sup>6</sup>Positive values of  $\mu_m$  imply that the social planner aims at ‘improving’  $F_m$ .

$$f_i = f_m \equiv \frac{\partial F_m}{\partial c_i} \quad \text{for all individuals } m \text{ in group } i. \quad (2)$$

Average explicit social weights in a given income group are defined as:<sup>7</sup>

$$e_i = \frac{1}{\lambda h_i} \int_{M_i} \mu^m d\nu(m). \quad (3)$$

We consider the benchmark case with no income effects, where  $\sum_{i=0}^I \partial h_j / \partial c_i = 0$  following Saez (2002). Summing the first order conditions (equation (19) in the appendix) over all  $i = 0, \dots, I$  we obtain the normalization of weights such that:

$$\sum_{i=0}^I h_i e_i f_i = 1. \quad (4)$$

Following Saez (2002), we assume that labor supply adjustment is restricted to intensive changes to “neighbor” income groups and extensive changes out of or into the labor force. Thus  $h_i$  depends only on differences in after-tax income between “neighbor groups” ( $c_{i+1} - c_i$ ,  $c_i - c_{i-1}$ ) and differences between group  $i$  and the non-working group ( $c_i - c_0$ ). The intensive mobility elasticity is

$$\zeta_i = \frac{c_i - c_{i-1}}{h_i} \frac{\partial h_i}{\partial (c_i - c_{i-1})} \quad (5)$$

and the extensive elasticity is given by

$$\eta_i = \frac{c_i - c_0}{h_i} \frac{\partial h_i}{\partial (c_i - c_0)}. \quad (6)$$

The main result is that the optimal tax formula for group  $i$  expressed in terms of the participation elasticities  $\eta_j$  and the intensive elasticity  $\zeta_i$  is

$$\frac{T_i - T_{i-1}}{c_i - c_{i-1}} = \frac{1}{\zeta_i h_i} \sum_{j=i}^I \left[ 1 - e_j f_j - \eta_j \frac{T_j - T_0}{c_j - c_0} \right] h_j. \quad (7)$$

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<sup>7</sup>For welfarist applications it is common in the literature to report implicit weights,  $g_i \equiv \frac{1}{\lambda h_i} \int_{M_i} \mu^m \frac{\partial F_m}{\partial c_i} d\nu(m)$ , which equals  $e_i f_i$ , i.e., the product of the explicit weights and the marginal utility of consumption, if  $\frac{\partial F_m}{\partial c_i}$  is the same for all individuals in group  $i$ . That approach offers the advantage to remain agnostic about utility functions. We calculate *relative* explicit social weights  $e_i/e_0$  as in Blundell et al. (2009). As will be made clear, relative explicit social welfare weights equal relative implicit weights under the welfarist approach with neither income effects nor preference heterogeneity. Thus, social weights of all approaches are comparable.

The intuition of this can be seen by considering an increase of the same amount  $dT$  in all  $T_j$  for income groups  $j = i, i + 1, \dots, I$ . A small increase in taxes mechanically increases tax revenues but induces individuals to move to a lower income class or to unemployment, which reduces tax revenues. After multiplying equation (7) with  $dT \zeta_i h_i$ , the left hand side shows the amount by which tax revenue is reduced due to individuals switching from job  $i$  to  $i - 1$ .<sup>8</sup> At the optimum, this must equal the mechanical tax gains, which are valued at  $[\sum_{j=i}^I (1 - e_j f_j) h_j] dT$ , minus tax losses due to individuals moving to group 0,  $-dT \sum_{j=i}^I \eta_j h_j \frac{T_j - T_0}{c_j - c_0}$ .<sup>9</sup>

The difference to the standard model in Saez (2002) is that we replace the implicit weights  $g_i$  with  $e_i f_i$ . This implies that even though individuals maximize utility, the social planner does not necessarily maximize a weighted sum of utility but a function potentially including other criteria. The optimal tax schedule in Saez (2002) depends on elasticities and weights  $g_i$ , whereas in the adjusted model, it additionally depend on the derivative of the justness function  $f_i$ .

The system of equations defining the optimal tax schedule consists of equation (4) and  $I$  equations like (7). In our application, we use the 2015 German tax system, i.e. we calculate the actual tax liability  $T_i$  of each income group, and solve for  $e_0, \dots, e_I$ . Alternatively, one could assume social weights and calculate the optimal tax schedule that maximizes equation (1) (as done in Appendix C).

## 2.2 Operationalization of Justness Concepts

The key advantage of our approach is that the justness function can be defined very generally, thus allowing us to capture a broader set of concepts of justness than the standard approach. In principle, the function can depend on individual and aggregate variables. The variables included in the justness function determine the dimensions along which the social planner considers a redistribution to be just. These variables do not need to be included in the utility function. For instance, utility is defined on after-tax income  $c_i$  and the choice of income group  $i$  in the standard welfarist approach. Our approach allows considering non-welfarist concepts of justness that rely, e.g., on before-tax income  $y_i$ .

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<sup>8</sup>Due to the assumption of no income effects and because the differences in net income between groups  $i, i + 1, \dots, I$  are unchanged, groups  $i + 1, i + 2, \dots, I$  will only adjust at the extensive margin.

<sup>9</sup>At the optimum, individual  $m$  moving into different jobs due to a slight tax increase does not impact the objective function if the change in  $\mu_m$  offsets the difference in the justness function between the two groups. In the welfarist case, where the envelope theorem applies, this implies that  $\mu_m$  does not change, when a marginal individual changes job.

Our approach nests the welfarist approach with quasi-linear preferences.<sup>10</sup> This special case is given if

$$F_m = u_m(c_i, i) = l_m(i) + b \times c_i, \quad (8)$$

where  $l_m(i)$  denotes the disutility of work in income group  $i$  and  $b \times c_i$  is the linear utility of consumption, such that  $f_i = \frac{\partial u_m(c_i^*, i^*)}{\partial c_i} = b$ .

Although we only require the first derivative of  $F_m$  to be identical for all individuals in an income group, for simplification, we specify the function  $F_m$  itself as  $F_i$ , i.e. to be identical for all members of income group  $i$ . By introducing this general justness function  $F_i$ , we may operationalize other moral judgments that depend directly on variables that do not enter the utility function as in the concept of *minimum sacrifice*. We operationalize two forms of minimum sacrifice: Minimum absolute sacrifice based on the absolute tax liability and minimum relative sacrifice based on the tax liability relative to the net income. The justness function then depends on  $F_i(c_i^*, c_i^{\text{ref}})$ , where  $c_i^*$  is net income at optimally chosen labor supply and  $c_i^{\text{ref}}$  is a reference point.

With  $c_i^{\text{ref}} = y_i$ , sacrifice is defined as the difference in utility derived from net income and the hypothetical utility derived from gross income, i.e., if there were no taxes:

$$\text{Sacrifice} \equiv u(y_i, i^*) - u(c_i^*, i^*). \quad (9)$$

In the case of quasi-linear preferences, where we assume *without loss of generality*, that  $b = 1$ , the sacrifice simplifies to  $y_i - c_i^*$ . We formulate a loss function that captures the penalty to the objective function of the social planner if individuals pay taxes, i.e., if there is a positive sacrifice. This loss function is the justness function associated with minimum sacrifice.

In the case of minimum *absolute* sacrifice the loss that captures deviations of  $c_i^*$  from gross income  $y_i$  is determined by the parameters  $\gamma$  and  $\delta$ :<sup>11</sup>

$$\begin{aligned} F_i &= -(y_i - c_i^*)^\gamma \text{ if } y_i \geq c_i^*, \\ F_i &= (c_i^* - y_i)^\delta \text{ if } c_i^* > y_i, \\ \gamma &> 1, \delta \leq 1. \end{aligned} \quad (10)$$

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<sup>10</sup>The absence of income effects, i.e. the assumption of quasi-linear preferences, is common in the optimal taxation literature following Saez (2002). In this case relative explicit welfare weights equal relative implicit welfare weights:  $f_i = b$  cancels out, i.e.  $\frac{g_i}{g_0} = \frac{e_i}{e_0} \frac{\partial u(c_i^*, i^*) / \partial c_i}{\partial u(c_0^*, i_0^*) / \partial c_0} = \frac{e_i}{e_0}$ .

<sup>11</sup>We leave for future research empirical identification of penalty functions. Note however, that this is only possible if the social weights are known.

The first line gives the penalty of paid taxes.  $\gamma > 1$  implies that the penalty increases more than proportionally with the amount of taxes paid. The second line captures the gains of individuals who receive transfers. If  $\delta$  is smaller than one, the marginal benefits of transfers are decreasing. With positive  $e_i$ , the social planner never chooses points on the right hand side of the Laffer curve (which are not Pareto optimal).<sup>12</sup> This justness function respects two properties. First, losses due to negative deviations from zero sacrifice, i.e., from positive tax liabilities, increase more than proportionally with the size of the deviation. Second, positive deviations, i.e., transfers, of the same size do not offset these losses.<sup>13</sup> It is important to understand that the social welfare weights are the parameters to be estimated, not  $\gamma$  and  $\delta$ . The choice of these parameters determines the loss function analogously to the decision of an econometrician whether to use a loss function on residuals leading to least squares regression or to least absolute deviations regression. In our main application, we set  $\gamma$  to two and  $\delta$  to one. The latter parameter affects mainly the unemployed, the only group that receives net transfers in our application and thus has a ‘positive sacrifice’. The aim of this paper is to show which concepts of justness are in line with declining social weights under a reasonable calibration. See Subsection 4.3 for variations of  $\delta$  and  $\gamma$ . Note that the resulting *absolute* weights from an inverse optimal taxation simulation with different justness functions differ in magnitude because derivatives of the  $F_i$  functions differ. To make the comparison of weights between concepts of justness easier, we therefore calculate relative weights by dividing the obtained absolute weights  $e_i$  through the absolute weight of group 0 as in [Blundell et al. \(2009\)](#).

Similarly, we also consider minimum *relative* sacrifice where the function includes deviations of consumption  $c_i$  from gross income  $y_i$  relative to the level of consumption such that

$$\begin{aligned}
 F_i &= - \left( \frac{y_i - c_{i^*}}{c_{i^*}} \right)^\gamma \quad \text{if } y_i \geq c_{i^*}, \\
 F_i &= \left( \frac{c_{i^*} - y_i}{c_{i^*}} \right)^\delta \quad \text{if } c_{i^*} > y_i > 0, \\
 &\quad \gamma > 1, \delta \leq 1.
 \end{aligned} \tag{11}$$

Although we are interested in the relative relations of the weights of the groups with positive gross income, with the relative sacrifice specification, we need to take a stance on the marginal

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<sup>12</sup>Starting from a point on the right-hand side of the Laffer curve for group  $i$ , improvements in the objective function of the social planner are possible by decreasing taxes  $T_i$ . This would increase  $F_i$  and increase tax revenues. This would, in turn, allow reducing taxes for some other group  $j \neq i$ . This increase in the objective function of the social planner would be a Pareto improvement as long as individual utility increases with net income.

<sup>13</sup>As noted in [Weinzierl \(2014\)](#), this is consistent with loss aversion ([Kahneman and Tversky 1979](#)).

gain in justness for unemployed, because  $f_i$  would equal zero and therefore be meaningless if the same function as that for individuals with positive gross incomes was applied for  $y = 0$ . A straightforward calibration is to set the value of the derivative of the justness function for income group 0 to equal that of group 1, i.e.  $f_0 = f_1$ . This does not change the relations of weights for groups with positive gross income, which are the focus of this study.

### 3 Empirical Calibration

#### 3.1 The Data

We use data from the 2015 wave of the German Socio-Economic Panel (SOEP), a representative annual household panel survey. [Wagner et al. \(2007\)](#) and [Britzke and Schupp \(2017\)](#) provide a detailed description of the data.<sup>14</sup> As the model does not cover spousal labor supply, we restrict the analysis to working-age singles. We exclude individuals with children, heavily disabled and people who receive Unemployment Benefit I,<sup>15</sup> because their budget constraints and labor supply behavior differ substantially. Group 0 consists of the unemployed receiving Unemployment Benefit II.<sup>16</sup> We exclude the long-term unemployed with transfer non-take up, as they differ substantially from the standard case and face a different budget constraint.

Table 1: Summary Statistics

	Mean	Std. Dev.	Obs.
<b>Monetary variables</b>			
Monthly Gross Income	2,626.75	1,925.41	1,119
Monthly Net Income	1,766.18	991.86	1,119
<b>Demographics</b>			
Sex (1=men, 2=women)	1.41	0.49	1,119
Weekly Hours of Work*	41.66	9.51	990
Age	43.97	10.47	1,119
East Germany Dummy	0.27	0.45	1,119

Source: Own calculations based on the SOEP

\*Excluding the unemployed

Table 1 shows summary statistics for our sample. Net incomes equal gross incomes and transfers minus income taxes and social security contributions.

<sup>14</sup>Socio-Economic Panel (SOEP) data for the year 2015, version 32, 2016, doi:10.5684/soep.v32.

<sup>15</sup>This transfer is targeted to the short-term unemployed and depends on the previous labor income.

<sup>16</sup>This transfer is targeted at the long-term unemployed and covers the social existence minimum.

### 3.2 Labor Supply Elasticities

Similar to [Blundell et al. \(2009\)](#) and [Haan and Wrohlich \(2010\)](#), we calibrate the optimal taxation analysis with labor supply estimates obtained from the same microdata (the SOEP), which we used to generate income groups. To this end we specify a random utility discrete choice labor supply model following [van Soest \(1995\)](#); [Aaberge et al. \(1995\)](#); [Aaberge and Colombino \(2014\)](#). We flexibly specify the transcendental logarithmic utility function  $V_{mj}$ , which is “a local second-order approximation to any utility function” ([Christensen et al. 1975](#)). While the highest value of  $V_{mj}$  over the  $j$  hours alternatives non-stochastically determines the choice of labor supply, additionally an independently and identically distributed random term  $\varepsilon_{mj}$  captures an idiosyncratic component.

Gross income is defined as the product of wages and hours of work. Of course, we do not observe potential wages for unemployed. Therefore, we predict potential hourly wages of the unemployed using a selectivity-corrected wage regression (results available on request). The selectivity correction follows the two-step [Heckman \(1979\)](#) approach with binary variables for young children of four age groups, marital status, non-labor income, and indicators for health as exclusion restriction.

Given their hourly wage, individuals make a discrete choice of weekly working hours to maximize utility, which depends on leisure  $L_{mj}$  and after-tax and transfer income  $C_{mj}$ . We discretize hours of work into five alternatives and unemployment (weekly working hours  $\in \{0, 10, 20, 30, 40, 50\}$ ) for the precise calculation of net incomes associated with labor supply decisions using the STSM (see [Jessen et al. 2017](#); [Steiner et al. 2012](#)). In contrast to continuous labor supply models this does not require convexity of the budget set.

If the error terms  $\varepsilon_{mj}$  are assumed to be distributed according to the Extreme-Value type I distribution, the probability that alternative  $k$  is chosen by person  $m$  is given by a conditional logit model ([McFadden 1974](#)):

$$P_{mk} = Pr(V_{mk} > V_{mj}, \forall j = 1 \dots J) = \frac{\exp(U_{mk})}{\sum_{j=1}^J \exp(U_{mj})}, k \in J, \quad (12)$$

where the deterministic component is

$$U_{mj} = \beta_1 \ln(C_{mj}) + \beta_2 \ln(C_{mj})^2 + \beta_3 \ln(L_{mj}) + \beta_4 \ln(L_{mj})^2 + \beta_5 \ln(C_{mj}) \ln(L_{mj}). \quad (13)$$

Observed individual characteristics  $X_1$ ,  $X_2$  and  $X_3$  including age, disability indicators, whether the observed person is German citizen, and number and age of children are allowed to shift tastes



for leisure and consumption  $\beta_1 = \alpha_0^C + X_1' \alpha_1^C$ ,  $\beta_2 = \alpha_0^{C^2} + X_1' \alpha_1^{C^2}$ ,  $\beta_3 = \alpha_0^L + X_3' \alpha_1^L$ ,  $\beta_4 = \alpha_0^{L^2} + X_1' \alpha_1^{L^2}$ ,  $\beta_5 = \alpha_0^{C \times L} + X_1' \alpha_1^{C \times L}$ .

To obtain mobility elasticities we first assign each individual  $m$  to an income group  $i = 1, \dots, I$  based on the wage-hours combination observed in the data. For instance, a person with an hourly wage of 20 Euros earns a gross income of approximately 860 Euros per month, if she works 10 hours per week and about 1720 Euros if she works 20 hours. If she works 10 hours, she is assigned to group I,  $C_{m=20,k=10} = c_{i=1}$ . If she works 20 hours, she is assigned to group II,  $C_{m=20,k=20} = c_{i=2}$ . In contrast, a person with an hourly wage of 50 Euros is assigned to income group II if she works 10 hours, earning about 2150 Euro per month,  $C_{m=50,k=10} = c_{i=2}$ .

Changes in net income associated with specific hours points lead to changes in the choice probabilities given by equation (12). These allow for the calculation of aggregate labor supply effects of an hypothetical increase in income. We simulate these effects by the *Probability or expectation method*, i.e. we assign to each individual probabilities for each hours category (see [Creedy and Duncan 2002](#)) and thus for different income groups assuming that the income group with the highest probability is chosen.

Then we predict changes in relative employment shares of income groups due to changes in relative net incomes  $c_i - c_{i-1}$  and  $c_i - c_0$  (in practice we increase annual net income by 10%) and calculate the mobility elasticities given by equations (5) and (6). The elasticities are reported in the tables in the next section.

## 4 Results

### 4.1 Main Results

Table 2 shows average monthly individual gross incomes (column I) and corresponding average net incomes (column II) for six income groups. As is apparent from the increase in net incomes from group 0 to group 1, the marginal transfer withdrawal rate is substantial in the status quo.

Column III shows the population share of each income group and columns IV and V display the estimated extensive and intensive mobility elasticities. For group 1, there is only one elasticity, see equations (5) and (6). The last three columns show relative explicit social weights for the welfarist and minimum sacrifice approach. The welfarist approach (column VI) is an application of [Saez \(2002\)](#) as in [Blundell et al. \(2009\)](#). Group 0 has the highest social weight, the working poor (group 1) have the lowest weight in line with previous studies described in the introduction.

Table 2: Resulting Relative Weights for Different Justness Concepts

Group	I	II	III	IV	V	VI	VII		VIII
	Gross Income	Net Income	Share	$\eta$	$\zeta$	Welfarist	Minimum Sacrifice		
							Abs.	Rel.	
0	0	625	0.11	—	—	1	1	1	1
1	1,137	949	0.19	0.08*	0.08*	0.29068	0.00077	0.29068	
2	2,082	1,452	0.17	0.10	0.08	0.37729	0.00030	0.22023	
3	2,697	1,755	0.19	0.09	0.07	0.37342	0.00020	0.19871	
4	3,472	2,170	0.17	0.07	0.06	0.40902	0.00016	0.23123	
5	5,458	3,257	0.18	0.05	0.08	0.38680	0.00009	0.27823	

*Note:* German single households; own calculations based on the SOEP and the STSM.

\*Overall elasticity of group one is 0.16.

At the optimum, the welfarist weights show the costs of redistributing one Euro from individuals in group 0 to individuals in other groups. For instance, an increase in income for individuals in group 1 would reduce income in group 0 by only 0.29 Euro because individuals would move from group 0 to group 1, reducing the transfer burden of the state. Equivalently, the social planner values increasing the income for group 1 by one Euro 0.29 times as much as increasing the income of group 0 by one Euro. The low weights for the working poor are related to the high marginal tax rate for individuals moving from group 0 to group 1.<sup>17</sup> Relative weights of the upper four income groups are close to each other, in line with previous findings for Germany by [Bargain et al. \(2014\)](#).

Table C.1 in Appendix C shows the optimal welfarist tax schedule with weights decreasing with income. The resulting optimal tax schedule implies a substantially lower marginal transfer withdrawal rate for the working poor than in the status quo and higher net incomes for groups 1, 2, and 3. This underlines our finding that decreasing welfarist weights would imply lower transfer withdrawal rates.

Column VII of Table 2 displays optimal weights for the minimum absolute sacrifice approach. These weights show how much it costs in terms of the loss function of group 0 to reduce the loss function for members of a particular group as defined in equation (10). We focus the interpretation on the working groups as the unemployed are net recipients of transfers and thus ‘pay a positive sacrifice’, see Section 2.2. A comparison of the weights of tax-paying groups shows the highest

<sup>17</sup>*Ceteris paribus*, higher elasticities and higher marginal tax rates imply a position further to the right of the Laffer curve and thus lower social weights.

weight for the working poor, 0.00077, and decreasing weights with income. The social planner is indifferent between imposing a slightly higher increase in the loss function on the working poor and imposing four times this increase on the middle class (group 3). As the loss function increases quadratically with taxes paid, the first derivative of the loss function with respect to consumption for the working poor is relatively small. Consider the benchmark case with fixed incomes and the same derivative of the loss function for all groups. In this case, all weights would be the same. In our analysis the derivative of the loss function is lower for the working poor. Therefore, weights are higher for this group.<sup>18</sup> A similar reasoning applies to the other groups, which results in declining social weights. Consequently, the minimum absolute sacrifice principle is in line with the 2015 German tax and transfer system.

Column VIII shows results for the minimum *relative* sacrifice principle. Note that the relative weight of the group 1 is the same as for the welfarist approach. The reason is that we calibrated  $f_0$  to equal  $f_1$ , because  $y_0 = 0$ . For the welfarist case with quasi-linear utility  $f_1$  always equals  $f_0$ . Therefore, the costs of increasing  $F_1$  by one relative to the costs of increasing  $F_0$  by one are the same for the two concepts of justness. This is directly reflected by the relative weights, whereas the absolute weights (not reported) differ. This calibration does not change the relations of weights for groups 1-5.

Again, the working poor have the highest weight of the groups with a positive tax burden. However, in contrast to the absolute sacrifice principle, weights are not decreasing with income but U-shaped. Top income earners have relatively high weights according to the *relative* sacrifice principle, because the tax paid is divided through a high consumption level. Thus a small increase in taxes would not increase the loss function of this group by much. In fact, the middle class (group 3) has the lowest weight according to this principle as one would have to redistribute less to members of this group than to members of other groups in order to reduce their loss function by a given amount. Thus, the 2015 German tax and transfer system does not imply decreasing social weights under the minimum relative sacrifice principle.

In sum, we find that the minimum absolute sacrifice principle is in accordance with declining social weights in the status quo. Thus, the minimization of absolute sacrifice is a good description of the aims of the German society regarding the tax and transfer system.

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<sup>18</sup>As the welfarist weights indicate, the deadweight loss of increasing taxes for group 1 is very high. If it was lower, this group's minimum sacrifice weight would be even higher.

## 4.2 Results for Subsamples

To explore whether the 2015 tax transfer schedule was designed according to a particular concept of justness with focus on a specific group in mind, we split the sample into different groups. These groups differ substantially regarding the income distribution and elasticities, which might lead to different social weights.

First, the sample is split into females and males. We find that women have a more elastic labor supply than men and lower incomes. Then, we present our results for East Germans and West Germans, respectively. These two groups lived under different political systems for more than 40 years. We show that West Germans have higher incomes and less unemployment than East Germans.

### 4.2.1 Results for Men and Women

In Table 3 we report results for the subsample of women without children, which we compare, in the following, with the results for the main sample and, later, to men.

Table 3: Resulting Relative Weights for Different Justness Concepts for Women without Children

Group	I		II	III	IV	V	VI	VII	VIII
	Gross	Net	Share	$\eta$	$\zeta$	Welfarist	Minimum Sacrifice		
	Income	Income					Abs.	Rel.	
0	0	615	0.05	—	—	1	1	1	
1	976	872	0.19	0.09*	0.09*	0.13061	0.00063	0.13061	
2	1,903	1,331	0.20	0.12	0.10	0.16613	0.00015	0.05509	
3	2,548	1,705	0.19	0.10	0.10	0.20113	0.00012	0.07105	
4	3,342	2,079	0.23	0.07	0.10	0.18486	0.00007	0.06025	
5	4,948	3,011	0.15	0.06	0.12	0.18374	0.00005	0.08011	

*Note:* German single households; own calculations based on the SOEP and the STSM.

\*Overall elasticity of group one is 0.18.

As expected, gross and net incomes in all income groups are lower and labor supply elasticities are slightly higher. For the welfarist case, the working groups have smaller weights relative to the unemployed than in the main sample. As before, we find that the working poor have the lowest weight. The finding that social weights for the minimum absolute sacrifice concept are decreasing with income is robust for this subsample. As before, in the relative sacrifice case, the working poor

have the highest weights among working groups and top income earners have the second highest weights.

Table 4: Resulting Relative Weights for Different Justness Concepts for Men without Children

	I	II	III	IV	V	VI	VII	VIII
Group	Gross Income	Net Income	Share	$\eta$	$\zeta$	Welfarist	Minimum Sacrifice	
							Abs.	Rel.
0	0	627	0.15	—	—	1	1	1
1	1,265	1,038	0.17	0.05*	0.05*	0.49473	0.00109	0.49473
2	2,228	1,520	0.18	0.08	0.04	0.52023	0.00037	0.29738
3	2,875	1,837	0.16	0.07	0.04	0.52845	0.00025	0.28185
4	3,622	2,279	0.17	0.06	0.04	0.56493	0.00021	0.35296
5	6,124	3,581	0.16	0.05	0.06	0.52826	0.00010	0.39995

*Note:* German single households; own calculations based on the SOEP and the STSM.

\*Overall elasticity of group one is 0.10.

Table 4 shows results for the subsample of men. Incomes are higher and elasticities are lower than for women. In the welfarist case, weights of working groups are higher than for women. This is caused by lower elasticities, which lead to men being further on the left of the Laffer curve. Nevertheless, the working poor again have the lowest weight. The finding that weights in the absolute sacrifice case decrease with income holds for men as well. As in the welfarist case, the weight of the working poor is higher for men than for women because male elasticities are lower. Again, in the relative minimum sacrifice case, the working poor have the highest weight and the middle class has the lowest weight of working groups.

#### 4.2.2 Results for East and West Germany

Gross and net incomes are higher across all groups in West Germany (see Table 6) compared to East Germany (see Table 5). In contrast to the main sample and the previously analyzed subsamples, in the sample of East Germans the working poor are net transfer recipients and the marginal withdrawal rate when moving from group 1 to group 2 is still substantial.

The welfarist weights show highest social weights for the unemployed and lowest for the working poor (group 1 in the West, groups 1 and 2 in the East). An increase in income for individuals in group 1 by one Euro would reduce income in group 0 by only 0.27 Euro in West Germany and

by about 0.30 in East Germany. The relative weights of the four (three for East Germany) higher income groups are very similar and higher than the weights for the working poor.

Table 5: Resulting Relative Weights for Different Justness Concepts for East Germany

	I	II	III	IV	V	VI	VII	VIII
Group	Gross Income	Net Income	Share	$\eta$	$\zeta$	Welfarist	Minimum Sacrifice	
							Abs.	Rel.
0	0	596	0.18	—	—	1	1	1
1	774	851	0.17	0.10*	0.10*	0.30249	0.30249	0.30249
2	1,581	1,222	0.18	0.16	0.08	0.33926	0.00044	0.54047
3	2,200	1,594	0.17	0.13	0.08	0.40821	0.00032	0.63570
4	2,808	1,920	0.14	0.11	0.07	0.42652	0.00022	0.59342
5	4,039	2,625	0.16	0.09	0.08	0.40168	0.00013	0.58436

*Note:* German single households; own calculations based on the SOEP and the STSM.

\*Overall elasticity of group one is 0.20.

Table 6: Resulting Relative Weights for Different Justness Concepts for West Germany

	I	II	III	IV	V	VI	VII	VIII
Group	Gross Income	Net Income	Share	$\eta$	$\zeta$	Welfarist	Minimum Sacrifice	
							Abs.	Rel.
0	0	653	0.08	—	—	1	1	1
1	1,408	1,072	0.21	0.07*	0.07*	0.26746	0.00040	0.26746
2	2,324	1,549	0.16	0.09	0.08	0.31984	0.00021	0.25346
3	2,907	1,857	0.19	0.08	0.08	0.31618	0.00015	0.25474
4	3,699	2,321	0.19	0.06	0.06	0.35053	0.00013	0.33020
5	6,010	3,519	0.17	0.05	0.08	0.32097	0.00006	0.35878

*Note:* German single households; own calculations based on the SOEP and the STSM.

\*Overall elasticity of group one is 0.14.

As in our main findings, optimal weights under minimum absolute sacrifice are decreasing in both samples, though the weight of group 1 is closer to the weight of group 0 than is the case for West Germany as group 1 in the East are net transfer recipients and thus enjoy a ‘positive tax sacrifice’. Note that as group 1 in East Germany consists of transfer net recipients,  $f_0 = f_1$  (see equation (10)) for this group and thus the relative weight of group 1 is the same as in the welfarist and relative sacrifice cases. Regarding groups with a positive tax burden, the weights imply that

the social planner is roughly indifferent between imposing a slight increase in the loss function on the working poor (group 1) in West Germany and imposing five times this increase in the loss function on group 2. For East Germany, the social planner is indifferent between imposing a slight increase in the loss function for individuals in group 2 and an about 38 percent higher increase in the loss function for individuals in group 3. This shows that the minimum absolute sacrifice principle is in line with the 2015 German tax and transfer system for East and West Germans.

Results for the minimum *relative* sacrifice principle show that group 3 has the highest weight of the groups with a positive tax burden in East Germany, while in West Germany weights for the top income group are highest. The difference arises because top income earners in West Germany earn considerably more than their East German counterparts. As explained in Section 4.1, this implies higher weights for this justness concept because the denominator of the loss function is higher. Thus, the German tax and transfer system does not result in decreasing social weights under the minimum relative sacrifice principle.

## 4.3 Robustness and Extensions

### 4.3.1 Subjective Justness

Our framework allows using information on the level of taxes that is considered just by individuals in the optimal tax formulae. We specify the justness functions similarly to the case of minimum sacrifice and set as reference point the level of just after-tax income taken from the survey. Thus the absolute formulation of the justness function is

$$\begin{aligned}
 F_i &= -(c_i^{\text{just}} - c_{i^*})^\gamma \text{ if } c_i^{\text{just}} \geq c_{i^*}, \\
 F_i &= (c_{i^*} - c_i^{\text{just}})^\delta \text{ if } c_{i^*} > c_i^{\text{just}}, \\
 &\gamma > 1, \delta \leq 1
 \end{aligned} \tag{14}$$

and the relative one is

$$\begin{aligned}
 F_i &= -\left(\frac{c_i^{\text{just}} - c_{i^*}}{c_{i^*}}\right)^\gamma \text{ if } c_i^{\text{just}} \geq c_{i^*}, \\
 F_i &= \left(\frac{c_{i^*} - c_i^{\text{just}}}{c_{i^*}}\right)^\delta \text{ if } c_{i^*} > c_i^{\text{just}} > 0, \\
 &\gamma > 1, \delta \leq 1.
 \end{aligned} \tag{15}$$

The parameters are calibrated as for minimum sacrifice.

In the 2015 wave, the SOEP introduced new questions that ask what amount of income respondents would consider just in their current occupation. In particular, individuals state how high their gross income and net income would have to be in order to be just. A screenshot of this part of the questionnaire is provided in Appendix B.<sup>19</sup> Only the currently employed are asked questions about what income they would consider as just.<sup>20</sup>

Compared to other approaches to obtain information about individuals' ideas of justness, the advantage of the question is that individuals do not need to have a worked out theory of just taxation in mind to answer the question. Moreover, interviewees do not need a thorough understanding of tax schedules.

Figure 1 shows the status quo of the German tax and transfer system and the just tax and transfer system based on our sample. The first segment of the actual budget line is almost horizontal at a net income of about 600 Euro due to the high transfer withdrawal rate. The slope of the budget line is steeper further to the right, representing individuals who do not receive transfers, but pay income taxes and social security contributions.

Gray circles represent the actual net incomes for given gross incomes. Some circles are crossed by x. This means either that an individual considers his or her actual income just or the actual income of another person. The 45 degree line marks the points where no taxes are paid. Points above this line represent actual transfer recipients or those who deem receiving transfers as just. However, most individuals perceive net incomes to be just, where taxes have to be paid. It is likely that status quo bias explains this pattern. Nonetheless, the answers of the respondents reflect actual perceptions of just incomes. The solid blue and the dashed red lines summarize this information. The solid blue line depicts the average actual budget constraint for six income groups that we use in the main analysis. The dashed red line shows the just budget constraint for the same groups. The budget lines are based on averages for the groups. The *just* budget line is defined only for those with

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<sup>19</sup>Since 2005 the SOEP includes a question on just income "Is the income that you earn at your current job just, from your point of view?". If respondents answer "No", they are asked "How high would your net income have to be in order to be just?" and since 2009 additionally "How high would your gross income have to be in order to be just?". The introduced justness question in 2005 was inspired by a perceived justness of earnings formula developed by Jasso (1978). In 2015 these questions are revised to allow for more research topics and now all respondents are asked about just gross and net income (see Appendix B).

<sup>20</sup>For the working poor, we add actual transfers to stated just net incomes, as these do not include transfers. Transfers include Unemployment Benefit II, housing benefits.



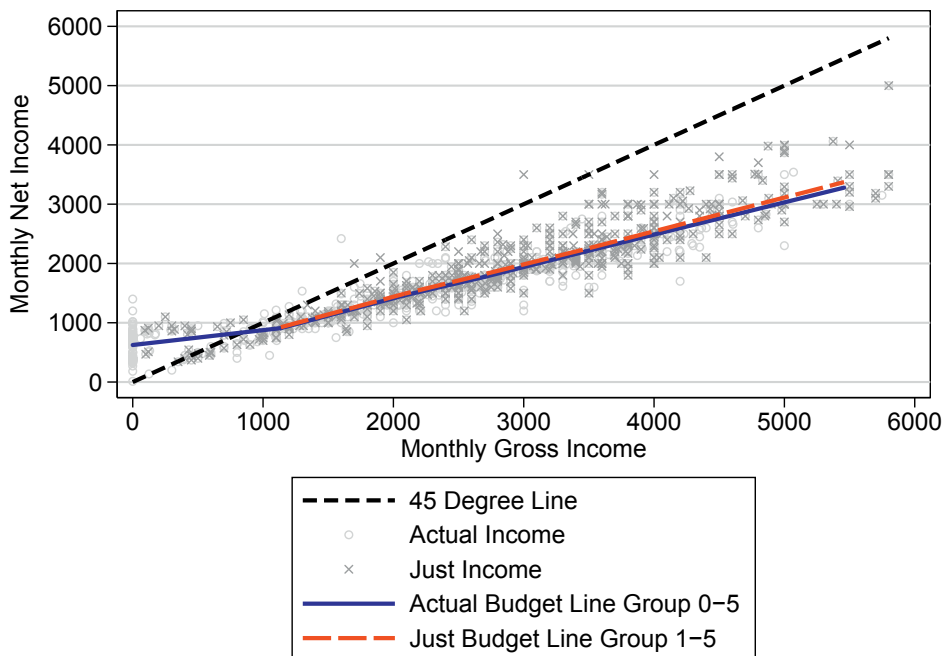


Figure 1: Just Net and Gross Incomes. Source: Own calculations based on SOEP

positive labor income and lies slightly above the actual budget line. This reflects the preferences for paying less taxes. The distribution of net incomes for a given value of gross income is skewed toward the no tax line. Deviations in this direction can be explained with allowances. The positive skew of just net incomes is due to more people perceiving substantially higher net incomes as just than substantially lower net incomes. The incidence of crossed circles, i.e., persons who perceive their current income as just is higher below and around the average budget lines.

As only employed persons respond to the SOEP question about just net income, just net income is set marginally above the actual average transfer income of group 0.<sup>21</sup>

<sup>21</sup>We experimented with different values for this number. While changing the just net income of group 0 has a substantial impact on this group's subjective social justness weights relative to other groups, the weights of other groups relative to one another remain virtually the same.

Table 7: Resulting Relative Weights for Subjective Justness

	I	II	III	IV	V	VI	VII
Group	Net Income	Just Net Income	Difference	$\eta$	$\zeta$	Subjective Justness	
						Abs.	Rel.
0	779	784*	5			1	1
1	1,024	1,036	12	0.08	0.08	0.15341	0.26369
2	1,420	1,461	41	0.10	0.07	0.05241	0.17035
3	1,741	1,778	37	0.09	0.07	0.06424	0.31620
4	2,195	2,243	48	0.07	0.07	0.05207	0.40717
5	3,317	3,415	98	0.05	0.08	0.02475	0.43868

*Note:* German single households; own calculations based on the SOEP and the STSM.

\*Just net income for this group is set as explained in the text.

Table 7 shows social weights according to the absolute and relative subjective justness principles respectively. The sample—and thus means for current gross and net incomes—differs from the main one as only individuals who report that their current gross income is just are used. The subjective justness principle implies penalties for the deviation of net incomes from perceived just net incomes. As discussed above, there is no information on perceived just net incomes of the unemployed, so we focus on the interpretation of the social weights of working groups. For the absolute justness principle, the working poor have the highest social weights of the working population because their average net income deviates from just net income by only 12 Euros. Social weights are decreasing except for group 3, where the just net income is closer to actual net income than is the case for groups 2 and 4. When considering *relative* deviations from just net income, group 5 has the highest social weights of all working groups since the deviation from just income is small relative to the high consumption level of this group.

### 4.3.2 Different Reference Points of Justness Functions

For the *subjective justness* approach, reference points of loss functions were taken directly from survey data. It is interesting to see how the resulting weights for reference points taken from the data compare with weights resulting from three classical scenarios of redistribution, where reference points of all income groups are higher than net income: First, the social planner assumes that low income groups ask for less redistribution relative to the status quo than the high income groups. Second, the low income groups ask for more redistribution than the high income groups. Third,

each income group wants to keep up with the next higher income group and asks for redistribution to achieve the net incomes of the next higher group.

The absolute and relative loss functions are then given by

$$\begin{aligned}
 F_i &= -(c_i^{\text{ref}} - c_i^*)^\gamma \text{ if } c_i^{\text{ref}} \geq c_i^*, \\
 F_i &= (c_i^* - c_i^{\text{ref}})^\delta \text{ if } c_i^* > c_i^{\text{ref}}, \\
 &\gamma > 1, \delta \leq 1
 \end{aligned}
 \tag{16}$$

and

$$\begin{aligned}
 F_i &= -\left(\frac{c_i^{\text{ref}} - c_i^*}{c_i^*}\right)^\gamma \text{ if } c_i^{\text{ref}} \geq c_i^*, \\
 F_i &= \left(\frac{c_i^* - c_i^{\text{ref}}}{c_i^*}\right)^\delta \text{ if } c_i^* > c_i^{\text{ref}} > 0, \\
 &\gamma > 1, \delta \leq 1,
 \end{aligned}
 \tag{17}$$

where  $c_i^{\text{ref}}$  is a calibrated reference point. Tables 8-10 show results for this exercise for the three different cases.

Table 8: Case 1: Difference to Reference Points Increasing with Income

	I	II	III	IV	V
Group	Net Income	Reference Point	Difference	Abs.	Rel.
0	625	675	50	1	1
1	949	1,049	100	0.14534	0.32739
2	1,452	1,602	150	0.12576	0.66444
3	1,755	1,955	200	0.09335	0.71364
4	2,170	2,420	250	0.08180	0.9550
5	3,257	3,557	300	0.06447	1.73131

*Note:* German single households; own calculations based on the SOEP and the STSM.

In Table 8 the reference point is set 50 Euros above the actual net income for group 0 and the difference between actual income and the reference point increases by 50 Euros for every income group until it reaches 300 Euros for group 5. For the absolute loss function, this results in continuously decreasing social weights. In contrast, using the relative loss function, social weights

are increasing starting from group 1 as the increase in the denominator of the loss function more than offsets the increase in the nominator with increasing net income. Nonetheless, the social weight of group 0 is highest because reducing transfers for this group would be associated with a substantial efficiency gain. Not reducing the transfer for this group is only reconcilable with a high social weight. The exercise reported in Table 8 is related to the minimum sacrifice principle; if the designer of the tax and transfer system followed the principle of minimum sacrifice, implicitly an income-increasing difference between net income and reference point was chosen. Therefore, the pattern for the minimum absolute sacrifice case in Table 2 is similar to this stylized case.

Table 9: Case 2: Difference to Reference Points Decreasing with Income

	I	II	III	IV	V
Group	Net Income	Reference Point	Difference	Abs.	Rel.
0	625	1,625	1,000	1	1
1	949	1,119	170	1.70986	8.69247
2	1,452	1,612	160	2.35808	29.8062
3	1,755	1,905	150	2.48944	47.0166
4	2,170	2,310	140	2.92157	86.0196
5	3,257	3,387	130	2.97542	202.023

*Note:* German single households; own calculations based on the SOEP and the STSM.

Table 10: Case 3: Reference Point Next Higher Income Group

	I	II	III	IV	V
Group	Net Income	Reference Point	Difference	Abs.	Rel.
0	625	949	324	1	1
1	949	1,452	503	0.18724	0.42840
2	1,452	1,755	303	0.40344	2.73546
3	1,755	2,170	415	0.29153	2.82285
4	2,170	3,257	1,087	0.12192	1.48678
5	3,257	4,757	1,500	0.08355	2.35881

*Note:* German single households; own calculations based on the SOEP and the STSM.

Table 9 is the counterpart of Table 8 and shows resulting weights, where the difference between actual net income and the reference points decreases with income. For group 0 this difference is

set to 1000 Euro in order to obtain continuously increasing social weights. Again, the efficiency gain of reducing transfers for this group is very large and only when the reference point is far away from actual income would the increase in the loss function offset this efficiency gain. This results in a low relative social weight for this group. The relative loss function weights are increasing with income too—to a much stronger degree than in case 1.

Finally, in Table 10, the net income of the next higher income group is taken as a reference point, except for the highest income earners, where we set the reference point 1500 Euro above the current net income. In this “Keeping up with the Joneses” scenario groups 2 and 3 have relatively high weights for both absolute and relative loss functions as their net income is close to that of the respective next higher income group.

### 4.3.3 Robustness

As for any loss function, results may differ depending on the properties of the function that is to be minimized. We analyze the robustness of the obtained social weights for absolute minimum sacrifice to different values of  $\gamma$  and  $\delta$  (Tables 11 and 12). The result that social weights decline with income is robust to a wide range of calibrations. This shows that the main result is not driven by the parameter choice. Second, we set the intensive and extensive elasticities of all groups to 0.1 and show the results for all concepts of justness (Table D.1 in the appendix). The results are very close to the main results. This shows that slight variations in the elasticities do not change the results substantially.

Table 11: Resulting Relative Weights for Absolute Minimum Sacrifice for Different Values of  $\gamma$  ( $\delta = 1$ )

	I	II	III	IV
Group	$\gamma = 1.5$	$\gamma = 2$	$\gamma = 3$	$\gamma = 5$
0	1	1	1	1
1	0.01413	0.00077	$2.74 \times 10^{-6}$	$4.65 \times 10^{-11}$
2	0.01002	0.00030	$3.17 \times 10^{-7}$	$4.79 \times 10^{-13}$
3	0.00811	0.00020	$1.40 \times 10^{-7}$	$9.48 \times 10^{-14}$
4	0.00756	0.00016	$8.04 \times 10^{-8}$	$2.85 \times 10^{-14}$
5	0.00550	0.00009	$2.66 \times 10^{-8}$	$3.30 \times 10^{-15}$

*Note:* German single households; own calculations based on the SOEP and the STSM.

Table 12: Resulting Relative Weights for Absolute Minimum Sacrifice for Different Values of  $\delta$  ( $\gamma = 2$ )

	I	II	III	IV
Group	$\delta = 0.1$	$\delta = 0.3$	$\delta = 0.5$	$\delta = 1$
0	1	1	1	1
1	$2.35 \times 10^{-7}$	$2.56 \times 10^{-6}$	$1.55 \times 10^{-5}$	0.00077
2	$9.12 \times 10^{-8}$	$9.92 \times 10^{-7}$	$5.99 \times 10^{-6}$	0.00030
3	$6.04 \times 10^{-8}$	$6.56 \times 10^{-7}$	$3.96 \times 10^{-6}$	0.00020
4	$4.78 \times 10^{-8}$	$5.20 \times 10^{-7}$	$3.14 \times 10^{-6}$	0.00016
5	$2.68 \times 10^{-8}$	$2.91 \times 10^{-7}$	$1.76 \times 10^{-6}$	0.00009

*Note:* German single households; own calculations based on the SOEP and the STSM.

## 5 Conclusion

In this paper, we reconcile a puzzling contrast between current tax transfer practice in many countries and the common approach in the optimal taxation literature. While the literature commonly assumes that the social planner values an additional unit of income for poor households more than an additional unit of income for higher income households, commonly observed high transfer withdrawal rates are only optimal if social weights of the working poor are very small. Therefore, we compare alternative approaches to welfarism and calculate the implied social weights. We formulate the problem of a social planner for two distinct concepts of justness: the welfarist approach, where the social planner maximizes the weighted sum of utility; alternatively, the minimum sacrifice concept where the social planner minimizes the weighted sum of absolute or relative (tax-)sacrifice. Moreover, we illustrate how subjective justness can be used in our model where the social planner minimizes absolute or relative deviations from perceived just net income. Of course, all approaches maintain budget neutrality and account for labor supply reactions.

Like the existing literature, we find that the 2015 German tax and transfer system implies very low social weights for the working poor according to the welfarist criterion. The social planner values increasing the income for the working poor by one Euro 0.75 times as much as increasing the income of top earners by one Euro. This implies that an additional Euro of consumption for the working poor is valued less than marginal consumption of top income earners.

In contrast, the current tax-transfer practice can be reconciled as optimal and in line with decreasing social weights under the minimum absolute sacrifice criterion, under which the social planner minimizes a function that puts an increasing penalty on deviations from zero sacrifice. In

this case, the social planner is indifferent between a slight increase in the loss function for the working poor and imposing four times this additional increase in the loss function on the middle class.

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## Appendix

### A Optimal Tax Formulae in the General Model

Behavioral reactions imply that  $h_i$  changes in case of a change in  $T_i$ . Using the product rule and assuming that marginal movers do not impact the objective function, the first order condition with respect to  $T_i$  is obtained as

$$\int_M \mu_m f_m d\nu(m) = \lambda \left( h_i - \sum_{j=0}^I T_j \frac{\partial h_j}{\partial c_i} \right), \quad (18)$$

where  $\lambda$  is the multiplier of the budget constraint. The first order condition with respect to  $\lambda$  is the budget constraint. Assuming  $f_i = f_m$ , reorganizing (18), and defining the average explicit social weights as  $e_i = \mu_i / \lambda h_i$  yields

$$(1 - e_i f_i) h_i = \sum_{j=0}^I T_j \frac{\partial h_j}{\partial c_i}. \quad (19)$$

The assumption of no income effects implies that only  $h_{i-1}$ ,  $h_i$ ,  $h_{i+1}$ , and  $h_0$  change when  $T_i$  changes. If we assume that  $h_i$  can be expressed as a function depending on the difference to the the adjacent income groups and the unemployed  $h_i(c_{i+1} - c_i, c_i - c_{i-1}, c_i - c_0)$ , equation (19) simplifies to

$$\begin{aligned} (1 - e_i f_i) h_i &= T_0 \frac{\partial h_0}{\partial (c_i - c_0)} + T_i \frac{\partial h_i}{\partial (c_i - c_0)} - T_{i+1} \frac{\partial h_{i+1}}{\partial (c_{i+1} - c_i)} - T_i \frac{\partial h_i}{\partial (c_{i+1} - c_i)} \\ &+ T_i \frac{\partial h_i}{\partial (c_i - c_{i-1})} + T_{i-1} \frac{\partial h_{i-1}}{\partial (c_i - c_{i-1})}. \end{aligned} \quad (20)$$

Using the facts that  $\frac{\partial h_i}{\partial (c_i - c_0)} = -\frac{\partial h_0}{\partial (c_i - c_0)}$ ,  $\frac{\partial h_{i+1}}{\partial (c_{i+1} - c_i)} = -\frac{\partial h_i}{\partial (c_{i+1} - c_i)}$ ,  $\frac{\partial h_i}{\partial (c_i - c_{i-1})} = -\frac{\partial h_{i-1}}{\partial (c_i - c_{i-1})}$ , we can write after rearranging

$$(1 - e_i f_i) h_i = (T_i - T_0) \frac{\partial h_i}{\partial (c_i - c_0)} - (T_{i+1} - T_i) \frac{\partial h_{i+1}}{\partial (c_{i+1} - c_i)} + (T_i - T_{i-1}) \frac{\partial h_i}{\partial (c_i - c_{i-1})} \quad (21)$$

Using the definition of the elasticities (5) and (6), we obtain for each group after reorganizing

$$\frac{T_i - T_{i-1}}{c_i - c_{i-1}} = \frac{1}{\zeta_i h_i} \left\{ (1 - e_i f_i) h_i - \eta_i h_i \frac{T_i - T_0}{c_i - c_0} + \zeta_{i+1} h_{i+1} \frac{T_{i+1} - T_i}{c_{i+1} - c_i} \right\}. \quad (22)$$

Note that, by setting  $e_0 = e_i = 0$ , we obtain the Laffer-condition

$$\frac{T_i - T_{i-1}}{c_i - c_{i-1}} = \frac{1}{\zeta_i} + \frac{\zeta_{i+1} h_{i+1}}{\zeta_i h_i} \frac{T_{i+1} - T_i}{c_{i+1} - c_i} - \frac{\eta_i}{\zeta_i} \frac{T_i - T_0}{c_i - c_0}. \quad (23)$$

Substituting the equivalent of (22) for the next group  $i + 1$  in (22) and simplifying gives

$$\begin{aligned} \frac{T_i - T_{i-1}}{c_i - c_{i-1}} = \frac{1}{\zeta_i h_i} \left\{ (1 - e_i f_i) h_i + (1 - e_{i+1} f_{i+1}) h_{i+1} \right. \\ \left. - \eta_i h_i \frac{T_i - T_0}{c_i - c_0} - \eta_{i+1} h_{i+1} \frac{T_{i+1} - T_0}{c_{i+1} - c_0} + \zeta_{i+2} h_{i+2} \frac{T_{i+2} - T_{i+1}}{c_{i+2} - c_{i+1}} \right\}. \end{aligned} \quad (24)$$

Recursive insertion and simplifying gives the  $I$  formulae (7) that must hold if function (1) is optimized.

## B Questionnaire

67. Is the gross income that you earn at your current job just, from your point of view?  
 No .....  Yes .....  → Question 69!  
 ↓
68. How high would your gross income have to be in order to be just?  
 Gross:  euros per month Don't know .....
69. Is the net income that you earn at your current job just, from your point of your view?  
 No .....  Yes .....  → Question 71!  
 ↓
70. How high would your net income have to be in order to be just?  
 Net:  euros per month Don't know .....

Figure B.1: The Question for Justness. Source: Official SOEP Questionnaire

## C Optimal Welfarist Tax Schedule

Table C.1 shows the optimal welfarist tax schedule, where, following Saez (2002), implicit welfare weights are set according to the formula

$$g_i = \frac{1}{\lambda c_i^{0,25}} \quad (25)$$

and the shares of income groups are determined endogenously by

$$h_i = h_i^0 \left( \frac{c_i - c_0}{c_i^0 - c_0^0} \right)^{\eta_i}, \quad (26)$$

where the superscript 0 denotes values in the status quo. The simulation was done achieving budget neutrality and setting net income of group 0 to the status quo, as a deviation from this is not politically feasible.

Table C.1: Optimal Welfarist Tax Schedule

Group	Gross Income	Net Income	Share	Optimal Net Income	Optimal Share	Relative Weight
0	0	625	0.11	625	0.09	1
1	1,137	949	0.19	1,260	0.20	0.84
2	2,082	1,452	0.17	1,629	0.17	0.79
3	2,697	1,755	0.19	1,837	0.19	0.76
4	3,472	2,170	0.17	2,047	0.17	0.74
5	5,458	3,257	0.18	2,826	0.18	0.69

*Note:* German single households; own calculations based on the SOEP and the STSM.

## D Further Sensitivity Checks

Table D.1: Resulting Relative Weights for Different Justness Concepts with Elasticities set to 0.1

Group	I	II	III	IV	V
	$\eta$	$\zeta$	Welfarist	Minimum Sacrifice	
				Abs.	Rel.
0	—	—	1	1	1
1	0.1*	0.1*	0.22027	0.00059	0.22027
2	0.1	0.1	0.33434	0.00027	0.19516
3	0.1	0.1	0.31926	0.00017	0.16989
4	0.1	0.1	0.33459	0.00013	0.18916
5	0.1	0.1	0.30932	0.00007	0.22250

*Note:* German single households; own calculations based on the SOEP and the STSM.

\*Overall elasticity of group one is 0.2.