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Capitalist income and hierarchical power: A gradient hypothesis

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Working Papers on CAPITAL AS POWER

No. 2018/06

Capitalist Income and Hierarchical Power A Gradient Hypothesis

Blair Fix

July 2018

Capitalist Income and Hierarchical Power: A Gradient Hypothesis

Blair Fix*

July 16, 2018

Abstract

This paper offers a new approach to the study of capitalist income. Building on the 'capital as power' framework, I propose that capitalists earn their income not from any productive asset, but from the legal right to command a corporate hierarchy. In short, I hypothesize that capitalist income stems from *hierarchical power*. Based on this thinking, I hypothesize that the capitalist fraction of an individual's income is a gradient function of hierarchical power (which I define as the number of subordinates under one's control). Using data from US CEOs, I find evidence that this is true. Furthermore, a hierarchical model of the United States that generalizes this data accurately reproduces many aspects of the US distribution of capitalist income, including the relation between income size and capitalist income fraction. This evidence suggests that the ownership structure of US society is closely linked to the hierarchical structure of firms. This has important implications for the study of income distribution.

JEL Subject Codes: D31 (Personal Income, Wealth, and Their Distributions); D33 (Factor Income Distribution); B5 (Current Heterodox Approaches);

Keywords: capitalist income; hierarchy; power; functional income distribution; personal income distribution; inequality; capital as power

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Introduction 2

1 Introduction

What is the source of capitalist income? There is perhaps no question in political economy that is more contested. While there is a consensus among political economists that capitalists earn income because they own capital, this merely begs the question — what is capital? And why do capitalists earn income from owning it?

Ideas about the source of capitalist income inform the major divisions in political economic theory. According to neoclassical theory, capitalists earn income because their capital is inherently productive. Marxist theory, on the other hand, proposes that capitalists earn income by exploiting workers. But, as Nitzan and Bichler (2009) observe, both approaches have a fundamental problem — they posit theories of value based on *unobservable* units. Neoclassical theory claims that value stems from *utility*, while Marxist theory claims that value stems from labor, as measured by *socially necessary abstract labor time*. But, neither utility nor socially necessary abstract labor time can be observed (even in principle).

Given this fundamental problem, Nitzan and Bichler take the radical step of proposing an entirely new approach to political economy that they call *capital as power*. As the name implies, Nitzan and Bichler hypothesize that capital should be treated not as a physical thing, but as a "symbolic quantification of power". The 'capital as power' framework has opened the door to many new forms of empirical analysis (for a sample, see Baines, 2014a,b; Cochrane, 2015; Di Muzio, 2015, 2017; Hager, 2013, 2016; McMahon, 2013, 2018; Park, 2016). This paper builds on the 'capital as power' approach to explore the relation between firm hierarchy and capitalist income.

Put succinctly, I propose that capitalist income is derived from *hierarchical power* (which I define as the number of subordinates under an individual's control). By owning firms, capitalists earn the legal right to command firm hierarchies. From this position of power, capitalists can partition firm income streams as they see fit (Fix, 2015). But a hierarchy does not have a *single* position of power. Rather, there is a *gradient* of power from top to bottom. Perhaps, along with this gradient of power, there is a gradient of ownership and a gradient of capitalist income? I call this the 'capitalist gradient' hypothesis. The idea is that the proportion of income individuals earn from capitalist sources tends to increase with hierarchical power. In other words, we can predict (in statistical terms) someone's capitalist income fraction simply by knowing their position within a firm hierarchy.

This is a bold and very much exploratory idea, but one worth testing. Sur-

prisingly (from a mainstream perspective), the capitalist gradient hypothesis has empirical support. First, I find that the capitalist income fraction of US CEOs scales with hierarchical power. Second, I use a hierarchical model of the United States economy (developed in Fix, 2018a) to test if this CEO relation can be generalized to the whole US population. The model suggests that it does. This capitalist gradient model reproduces the US distribution of capitalist income as well as the scaling relation between income size and capitalist income fraction.

While these results are contingent on the limited data that is available, they suggest that a systemic relation exists between hierarchical power and both functional and personal income. Put simply, hierarchical elites, top earners, and capitalists are *all the same people*. This has important theoretical and empirical implications for the study of income distribution. On the theoretical front, it supports the idea that capitalist income stems from hierarchical power. On the empirical front, it opens new avenues for understanding the relation between income inequality and the class-based distribution of income.

This paper is laid out as follows. In section 2, I briefly review the major theories of capitalist income and discuss their shortcomings. In section 3, I outline my 'capitalist gradient' hypothesis, which proposes that capitalist income is a function of hierarchical power. In section 4, I test this hypothesis using data on CEO pay in the United States. In section 5, I use a hierarchical model of the United States to test if the CEO findings generalize to the US population as a whole. Lastly, in sections 6 and 7, I discuss the significance of these results and outline avenues for future research.

2 Theories of Capitalist Income

I review here three theories of capitalist income: neoclassical, Marxist, and capital as power. The first two are the great pillars of political economic theory that have dominated 20th century thinking. Capital as power, on the other hand, has been developed only recently and is not widely known. My aim is to review the basic tenets of neoclassical and Marxist theory, and outline the problems that Nitzan and Bichler attempt to solve (or avoid) with their 'capital as power' approach.

Neoclassical Theory

Let's begin with the neoclassical vision of capital. In neoclassical theory, capital is a 'factor of production'. Capital consists of all the tools, technology, and in-

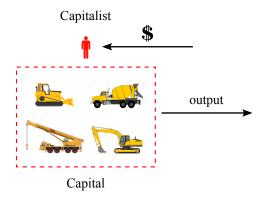


Figure 1: The Neoclassical Vision of Capitalist Income

In neoclassical theory, capitalists earn income because their capital is inherently productive. Capitalists earn the 'marginal product' of their capital — the incremental increase in output per incremental increase in capital input.

frastructure that are used to produce economic output. Capitalists earn income because their capital is *productive* — it contributes to economic output. This thinking was formalized by Clark (1899) and Wicksteed (1894) in the form of marginal productivity theory. In this theory, capitalists earn the marginal product of their capital — the incremental increase in output per incremental increase in capital input.

The neoclassical vision of capitalist income is illustrated in Figure 1, where capital is shown as machinery. Admittedly, neoclassical economists have significantly broadened their definition of 'capital' over the years. For instance, there is now 'human capital' (Becker, 1962; Mincer, 1958; Schultz, 1961), 'knowledge capital' (Carr et al., 2001; Loof and Heshmati, 2002), and even 'cultural capital' (Throsby, 1999). However, what has *not* changed is the insistence that 'capital' (in all its forms) is inherently *productive*.

While neoclassical marginal productivity has a seductive simplicity, it is beset by insurmountable measurement problems. First, to measure the marginal product of capital, we must be able to isolate changes in output that are due to changes in capital *alone* (and not due to changes in labor, or anything else). In practice, this is virtually impossible to do (Pullen, 2009). Furthermore, to measure the incremental change in capital input, we must be able to unambiguously measure the size of the capital stock itself. But as the Cambridge capital controversy demonstrated, there is no way to measure the quantity of capital independently of prices and income distribution (Cohen and Harcourt, 2003; Felipe and Fisher, 2003; Harcourt, 2015; Hodgson, 2005). This makes the measurement of

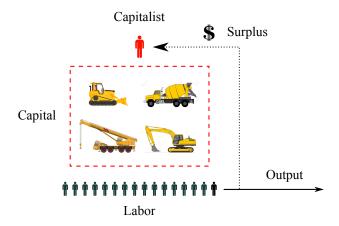


Figure 2: The Marxist Vision of Capitalist Income

In Marxist theory, capitalists earn income because they own the 'means of production'. Unlike neoclassical theory, Marxists see labor as the source of all value, and capitalists as parasites. Because capitalists control the means of production (capital), they are able to extract a surplus from labor.

capital circularly dependent on the very thing we are trying to explain — the income of capitalists. Furthermore, prices change all the time, which means that the quantity of capital must be tied to something more fundamental. In neoclassical theory, this elementary particle is the 'utility' created by capital. The problem, as Nitzan and Bichler (2009) note, is that utility is unobservable (see also Hodgson, 2012). As a result, the neoclassical theory of capitalist income is untestable.

Marxist Theory

Like neoclassical theory, Marxists start with a physical understanding of capital. According to Marx (1867), capital is the 'means of production' — the tools, technology, and infrastructure that are used by society to create economic output. But unlike the neoclassical vision of harmony between workers and capitalists, the Marxist twist is to assert that capitalist income is *parasitic*. Marxists believe that labor is the source of all value. Capitalists earn income because they own the means of production, and are able to extract a surplus from labor. This thinking is illustrated in Figure 2.

The task for Marxists is to measure the surplus value created by labor, and to show that this is equal to the profit earned by capitalists. Nitzan and Bichler identify three basic problems with this approach. First, Marxist theory requires a

consistent unit of human labor that allows qualitatively different workers (such as doctors and farmers) to be compared. Marx proposed 'socially necessary abstract labor' as his basic unit. Unfortunately, like the neoclassical notion of utility, this unit is unobservable. Second, Marx differentiated between 'productive' and 'unproductive' labor, with only productive labor creating value. But how are we to differentiate between the two? Nitzan and Bichler argue that there is no way to do so. Lastly, there is the problem of transforming labor time into prices. Again, we run into insurmountable difficulties. In his exhaustive review of the topic, Paul Samuelson (1971) notes that the two accounting systems of labor time and price are mutually exclusive and cannot be equated to one another. Like neoclassical theory, these problems mean that the Marxist vision of capitalist income is untestable.

Capital as Power

Both neoclassical and Marxist theories of capital keep their eyes firmly on the 'real' sphere of production — on the ownership of *things*. This means that both approaches must show how the 'real' sphere of production accounts for the 'nominal' sphere of prices. But this gives rise to all of the problems discussed above. In the 'capital as power' approach, Nitzan and Bichler attempt to avoid this problem by focusing on the *act* of ownership itself, and not on *what* is owned. Nitzan and Bichler summarize: "ownership is wholly and only an institution of exclusion, and institutional exclusion is a matter of organized power". According to the 'capital as power' hypothesis, capital is not a thing, but an *act*. It is a commodification of property rights — a vendible form of power.

Intellectual property represents the simplest form of this exclusionary power. By purchasing a patent, the owner gains the legal right to exclude all others from making use of the patented idea. Because of this legal monopoly, the owner has the power to set prices. For instance, in 2014 the US company Retrophin purchased the rights to market Thiola, a drug used to treat the rare disease cystinuria. Promptly thereafter, Retrophin raised the price by a factor of 20 - because they could (Lowe, 2014). Furthermore, the owner of intellectual property need not produce anything or provide any service to earn income. As the practice of 'patent trolling' attests, the owner of a patent may simply enforce his right of exclusion by using the legal system to extract 'damages' from non-compliant firms. It is clear, in this case, that the patent owner's income derives from institutionalized exclusionary power, and not from any productive asset. This focus on the institutional act of ownership is, in my view, the key strength of the 'capital as power' approach.

3 Capitalist Income and Hierarchy: A Gradient Hypothesis

Like Nitzan and Bichler, I begin my inquiry into capitalist income by reflecting on the act of ownership. Consider what it means to purchase all the shares in a company. What is it that you are buying? True, you are buying whatever assets (tangible or intangible) the company owns. But these change with time. I argue that what you are really purchasing is *legal control* over the company. From this position of power, you have legal authority to divide up the firm's income stream as you see fit. You could slash wages and pay yourself a magnificent profit, or raise wages and earn no profit at all. From this perspective, capitalist income stems from one's power as owner.

For the present argument regarding the basis of capitalist income, I set aside the question of how the firm's income stream is derived. Instead, I am interested in how an owner wields power to partition a firm's income stream. The central hypothesis in this paper is that firms are hierarchically organized. This hypothesis implies that ownership confers the right to sit at the top of the firm hierarchy. From this position of hierarchical power (as owner), the capitalist has the authority to divide up the firm's income stream. This suggests that capitalist income stems from *hierarchical power*, as illustrated in Figure 3.

While intuitive, this vision of capitalists sitting at the tops of corporate hierarchies is almost certainly too simplistic. The problem is that it is based on a 19th century, all-or-nothing concept of ownership. In Figure 3, there is only *one* owner of the firm. Unfortunately, the rise of joint-stock companies muddies this tidy theory. Joint-stock companies allow ownership to by divided among many people. In the modern world, the dominance of public corporations means that *partial* ownership is the rule, not the exception. This realization led to the famous 'separation thesis' posited by Berle and Means (1932). The idea is that ownership has become so diffuse that capitalists no longer control the corporate hierarchy. Instead, control is ceded to managers, who are *employees*.

The problem with the separation thesis is that it acknowledges the rise of

¹Based purely on worldly experience, most people would agree that firms are hierarchically organized. Surprisingly, this structure has not been widely studied. However the handful of studies that do exist confirm our basic intuition. Firms tend to have a pyramid employment structure and an inverse pyramid pay structure. For case studies, see Audas et al. (2004); Baker et al. (1993); Dohmen et al. (2004); Grund (2005); Lima (2000); Morais and Kakabadse (2014); Treble et al. (2001). For aggregate studies, see Ariga et al. (1992); Bell and Van Reenen (2012); Eriksson (1999); Heyman (2005); Leonard (1990); Main et al. (1993); Mueller et al. (2016); Rajan and Wulf (2006); Tao and Chen (2009). For a meta analysis of these studies, see Fix (2018b).

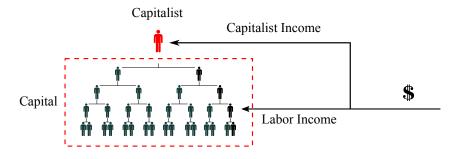


Figure 3: A Hierarchical Power Vision of Capitalist Income

This figure shows my interpretation of the 'capital as power' framework, when applied to a hierarchically organized firm. Unlike in neoclassical and Marxist visions of capital (Fig. 1 and 2, respectively) I do not show physical capital. This is not to say that physical capital does not exist — we simply do not focus on it. Rather, we focus on ownership of *institutions*. Capital is conceived solely in terms of property rights. By purchasing a firm, a capitalist purchases the legal right to sit at the top of the firm hierarchy. From this position of power, the capitalist has the right to divide up the firm's income stream as he sees fit. Under this vision, hierarchical power is the source of capitalist income.

partial ownership, but insists on a traditional dichotomy between capitalists and laborers. The truth is that the line between being a capitalist and being a laborer has been blurred. Top managers often earn a large portion of their income from stock options. Conversely, owners of firms often pay themselves some form of salary. Instead of a capitalist-laborer *dichotomy*, what we need is a capitalist-laborer *gradient*. This implies that there is a steady range between being purely a capitalist and being purely a laborer. Figure 4 shows what this might look like when applied to a hierarchy. As one moves up the hierarchy, individuals become increasingly more capitalistic.

This capitalist gradient hypothesis can be interpreted a number of ways. The simplest interpretation is to assume a gradient of ownership within a *single* firm. However, this is realistic only for firms that are 100% employee owned. While such firms do exist (and can become quite large), they are not the norm. It is more common for a firm to have *partial* employee ownership via an employee stock ownership plan. In 2017, about 14 million Americans were enrolled in employee stock ownership plans (ESOP) (NCEO, 2017). This represents about 9% of the workforce. It is quite plausible that these employee stock options are preferentially rewarded to the top tiers of the hierarchy. However, ESOP assets constitute a small minority (roughly 4%) of total US market capitalization.² This

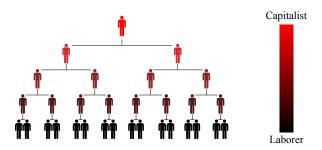


Figure 4: A Gradient Vision of Capitalist Income

This figure shows a more nuanced (than Fig. 3) interpretation of the relation between capitalist income and firm hierarchy. In this model, there is a smooth gradient between being 100% capitalist (earning all your income from capitalist sources) and being 100% laborer (earning all your income from labor sources). I hypothesize that the capitalist share of individual income tends to increase with hierarchical power.

means they are probably not the main source of capitalist income.

Therefore, it is most realistic to interpret the gradient model as a statistical phenomenon that occurs at the *societal* level. We admit that the ownership structure of any given firm is likely complex. Similarly, we admit that individuals who earn capitalist income may receive it from a variety of firms. But at the *aggregate* level, we hypothesize that earning capitalist income is related to hierarchical class structure. This is the hypothesis that I test.

Measuring Hierarchical Power

To test the capitalist gradient hypothesis, we need to measure hierarchical power. What is *hierarchical power*? I define it as the ability to control subordinates within a hierarchical chain of command Fix (2018b). Unlike the more general concept of 'social power', hierarchical power is easier to pin down and quantify. This is because the chain of command structure of a hierarchy clearly delineates *who has control over whom*. A hierarchy is nothing but a nested set of power relations between superior and subordinates (*ruler* and *ruled*). It is a control structure that concentrates power at the top (Mones et al., 2012).

² In 2017, ESOPs had total assets of roughly \$1.3 trillion (NCEO, 2017), while total US market capitalization was roughly \$30 trillion, according to the Russel 3000 index.

I propose that one's power within a social hierarchy is proportional to the *number of subordinates under one's control*. I put this in formula form as:

hierarchical power = number of subordinates
$$+ 1$$
 (1)

The logic of this equation is that all individuals start at a baseline power of 1, indicating that they have control over themselves. Power then increases linearly with the number of subordinates.

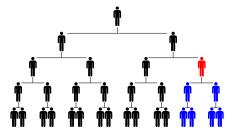


Figure 5: Measuring Hierarchical Power

As an example, suppose we want to find the hierarchical power of the red individual in Figure 5. This person has two direct subordinates, each of whom have 2 subordinates. Thus the red individual has control over 6 subordinates in total, mean his/her hierarchical power is 7. The general form of a branching hierarchy means that hierarchical power increases *exponentially* with rank.

4 Testing the Capitalist Gradient Hypothesis Using CEO Pay

If the capitalist gradient hypothesis is correct, we should be able to find evidence that capitalist income fraction increases with hierarchical power. I test the capitalist gradient hypothesis using US CEO income data. This data is convenient for two reasons. First, CEO income data is easy to obtain. US regulation requires that public companies disclose CEO compensation. Second, we can estimate a CEO's hierarchical power without any knowledge of the firm's hierarchical structure. Under the assumption that the CEO holds the top hierarchical position in a firm, it follows that their hierarchical power is equivalent to the *number of employees in the firm*.

This thinking is visualized in Figure 6. If a firm has x employees, x-1 of them will be subordinate to the CEO. Since hierarchical power is defined as the number of subordinates plus one, the CEO's hierarchical power is simply firm

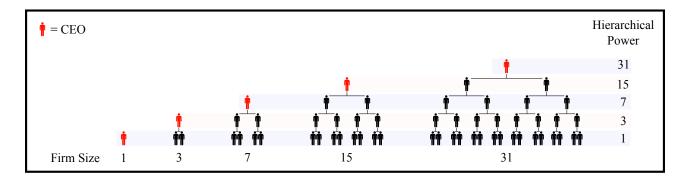


Figure 6: CEO Hierarchical Power

This figure shows the relation between firm size and CEO hierarchical power. Each hierarchy represents a different firm, with the CEO at the top (red). If hierarchical power is defined as the number of subordinates + 1 (Eq. 1), CEOs have hierarchical power equal to firm size.

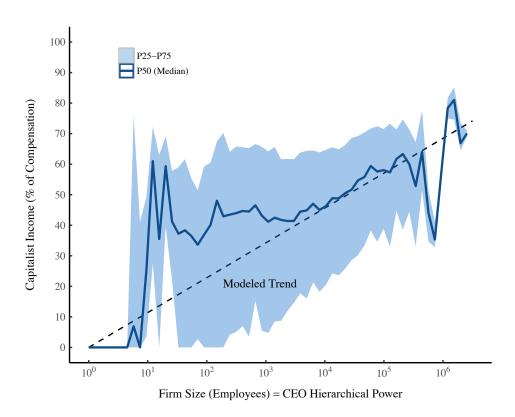


Figure 7: Capitalist Income Fraction of US CEOs

This figure plots the relation between capitalist income fraction and firm size for roughly 40,000 American CEOs over the years 1992-2015. Assuming that CEOs sit at the top of the corporate hierarchy, firm size is a direct indicator of CEO hierarchical power. The median (P50) and interquartile range (P25-P50) for capitalist income fraction are calculated using logarithmically spaced firm-size bins. The dashed line indicates the linear regression used for modeling purposes. Data comes from Execucomp and Compustat. For sources and methods, see the Appendix.

size x. Thus, if we have data for firm size, we automatically have data for CEO hierarchical power.

So how do we calculate the 'capitalist' component of CEO income? I define the CEO capitalist income fraction as the portion of total income received from stock options:

CEO Capitalist Income Fraction =
$$\frac{\text{Income from Stock Options}}{\text{Total Compensation}}$$
 (2)

Unlike cash compensation, there are many different ways to value stock options (Carr and Madan, 1999; Core and Guay, 2002; Jarrow and Rudd, 1982). This means that CEO capitalist income fraction has some inherent ambiguity. However, the *nuances* of stock option valuation do not concern me here. Instead, I am interested in *general trends* in CEO compensation. For this task, the standard methods for stock option valuation will do just fine. I use CEO income data from the Execucomp database. The data series and their underlying methods are discussed in the Appendix.

Figure 7 shows the resulting relation between capitalist income fraction and firm size for roughly 40,000 American CEOs over the years 1992-2015. Two important findings emerge. Firstly, the capitalist fraction of CEO income tends to *increase* with firm size (and hence hierarchical power). Secondly, capitalist income fraction *tends towards zero* for CEOs in very small firms (fewer than 10 employees). These results are consistent with the capitalist gradient hypothesis. They support the idea that earning capitalist income is a gradient function of hierarchical power.

5 Testing the Capitalist Gradient Hypothesis Using a Hierarchy Model of the United States

The evidence from US CEOs begs a question: does the relation between CEO capitalist income fraction and hierarchical power generalize to the broader US population? While data constraints stop us from answering this question directly (which is why we turned to CEO data in the first place), we can answer it *indirectly* by using a hierarchy model of the United States.

The US Hierarchy Model

In Fix (2018a), I develop and test a large-scale model that simulates the hierarchical structure of the US economy. The model is essentially an extrapolation. It

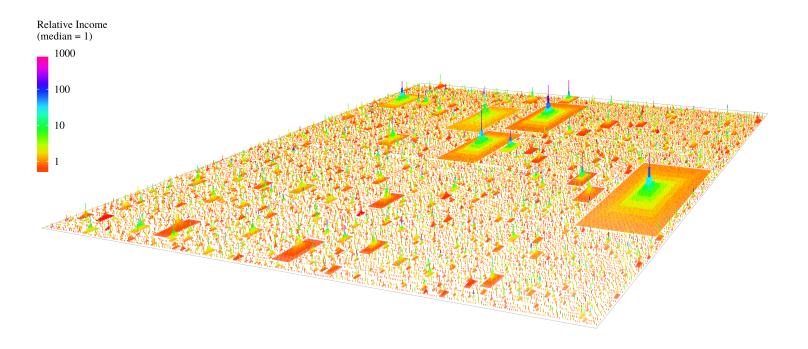


Figure 8: A Landscape View of the US Hierarchy Model

This figure visualizes the US hierarchy model as a landscape of three dimensional firms. Each pyramid represents a single firm, with size indicating the number of employees and height corresponding to the number of hierarchical levels. If you look closely, you will see vertical lines corresponding to individuals. Income (relative to the median) is indicated by color. This visualization has 20,000 firms — a small sample of the actual model, which uses 1 million firms.

takes the available case-study data on firm hierarchy and fits trends to it. Using these trends, as well as data for a large sample of US firms from the Compustat database, the model extrapolates to simulate the hierarchical employment and pay structure of the US private sector. The model produces results that are consistent with the US distribution of income, especially the tail.

I do not review the technical details of the model here (see Fix, 2018a). However, to give an intuitive understanding of what the hierarchy model 'looks' like, Figure 8 shows a landscape view of the model's structure. Each pyramid represents a different hierarchically-organized firm. The size of each pyramid corresponds to the number of employees, height represents hierarchical level, and color represents relative income.

Modeling Capitalist Income

The US hierarchy model can be used to indirectly test if the CEO capitalist income trends in Figure 7 generalize to the US population as a whole. I do this by using the CEO data to create a simple function relating capitalist income fraction to hierarchical power. Once I have this function, I plug it into the hierarchy model and endow each individual with a capitalist income. I then check the model's results against US data. If the model produces results that are way off the mark, we know that the CEO results *do not* generalize to the whole population. However, if the model produces results that are consistent with US data, this is *indirect* evidence that capitalist income fraction increases with hierarchical power in the wider US population.

The first step is to idealize the Figure 7 trend between CEO capitalist income fraction and hierarchical power. The simplest interpretation of this trend is that CEO income fraction increases linearly with the logarithm of hierarchical power. I fit the CEO data with a one-parameter logarithmic function, resulting in the 'Modeled Trend' line shown in Figure 7. This gives the following function relating capitalist income fraction (K_{frac}) to hierarchical power (P):³

$$K_{\text{frac}} = 0.05 \ln(P) \tag{3}$$

This function is naive in the sense that it implies a *deterministic* relation between hierarchical power and capitalist income fraction — something that certainly does not exist in the real world. However, models are always simplifications, and it is often useful to simplify a noisy (stochastic) trend with a deterministic one. If the results are good, we can add more realism later. If the results are bad we throw away the model.

The next step is to plug this equation into the hierarchy model. We calculate the hierarchical power of each individual in the model (see the Appendix) and then use Eq. 3 to calculate the capitalist fraction of their income. The resulting US capitalist gradient model is visualized in landscape form in Figure 9. As expected, capitalist income fraction is tightly related to hierarchical rank.

If the CEO capitalist income trend is generalizable, the capitalist gradient

³ The discerning reader may note that, since a logarithmic function is uniformly increasing, Eq. 3 permits capitalist income fraction greater than 1. In practice, such results do not occur because the model does not permit firm sizes greater than 2.3 million — the largest US firm that has ever existed (Walmart, circa 2015). For this maximum hierarchical power of 2.3 million, Eq. 3 yields a capitalist income fraction of about 0.7.

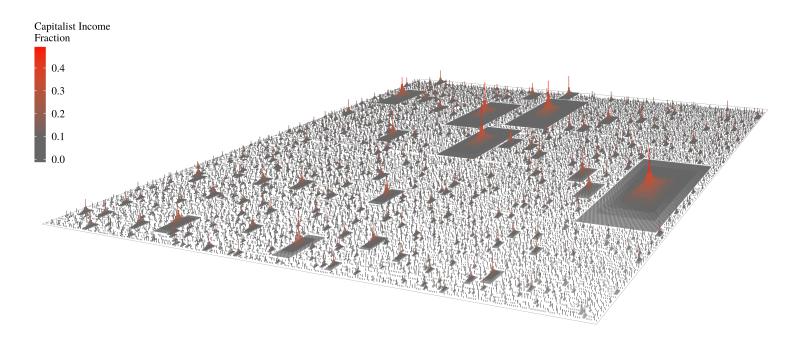


Figure 9: A Landscape View of the US Capitalist Gradient Model

This figure visualizes the US capitalist gradient model as a landscape of firms. Each pyramid represents a firm, with size indicating the number of employees. Hierarchical rank is indicated by height, and capitalist income fraction by color.

model should produces results that match US data. So does it? Figure 10 compares the model to the United States.

Let's begin with the relation between capitalist income fraction and individuals' total income size (Fig. 10A). Here I plot Thomas Piketty's (2014) data showing how US capitalist income fraction increases with income percentile. As illustrated by the inset plot (which uses a linear *x*-axis scale), there is an *explosion* of capitalist income that occurs in the topmost income percentiles. Evidently, those who earn very large incomes are overwhelmingly capitalists (and vice versa). The main panel spreads out this explosion by using an inverted logarithmic *x*-axis scale. Two different US trend-lines are shown. The upper line includes *capital gains* in the calculation of capitalist income, while the lower line does not. (The step-wise nature of these curves reflects Piketty's income bins.) Like the US data, the capitalist gradient model predicts an explosion in capitalist income amongst top earners.

Moving on, Figure 10B shows the size distribution of US capitalist income.

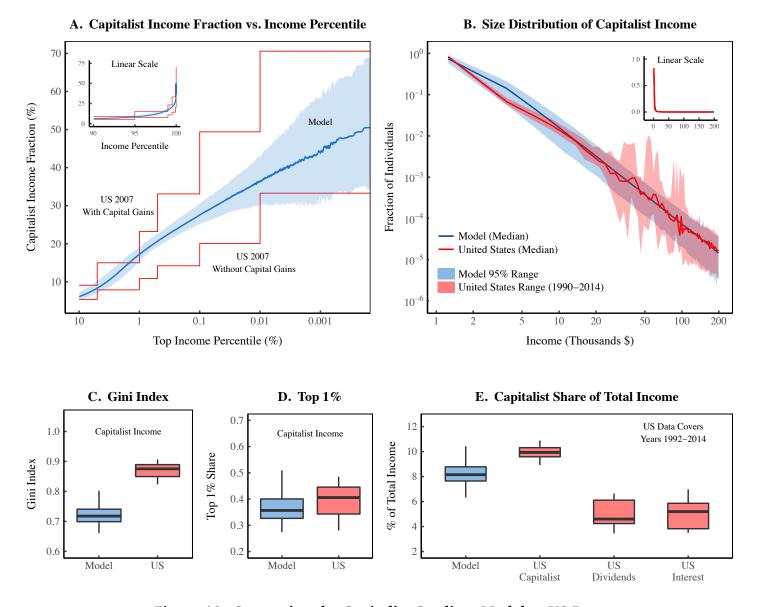


Figure 10: Comparing the Capitalist Gradient Model to US Data

This figure compares the income distribution generated by the capitalist gradient model to US data. Panel A shows how capitalist income fraction increases with income percentile (ranked by *total* income). The inset plot uses a linear *x*-axis scale, while the main plot uses an inverted logarithmic scale of top incomes. Note that US empirical data has 'steps' that correspond to the bins in the source data. The blue line and shaded regions indicate the model's median and 95% range, respectively. For Panels B, C and D, US capitalist income is defined as the sum of income from dividends and interest. Data covers the years 1990 - 2014. Panel B shows the size distribution of capitalist income. The model data is normalized to have mean income in the same range as the US data. Panel C shows the inequality of capitalist income, as measured by the Gini index, while Panel D shows capitalist income inequality as measured by the income share of the top 1%. Panel E shows the capitalist share of total (national) income. For comparison, I also show the dividend and net interest share of US income. For sources and methods, see the Appendix.

For this graph (as well as Fig. 10C, D and E), I define capitalist income as the sum of income from dividends and interest. Although many people do earn *some* capitalist income (i.e. from interest on savings), the amount is usually *inconsequentially* small. This fact is reflected in the inset panel, which plots the capitalist income distribution on a *linear* scale. Nearly all reported capitalist incomes are lower than \$5000. In order to see the tail of the distribution, the main plot uses a log-log scale. Again, the model is consistent with US data. To get these results, I do nothing but index the model data so it has the same mean as US data. Without tuning it to do so, the model effectively reproduces the tail of US capitalist income distribution.

How about capitalist income inequality? Figure 10C and D show the Gini index and top 1% share of capitalist income, respectively. Just to be clear, the latter metric captures the share of total *capitalist* income held by the top 1% of reported *capitalist* incomes. First off, note how *unequal* US capitalist income is. The Gini index hovers around 0.9 (the maximum is 1), while the top 1% of capitalists earn about 40% of total capitalist income. The model reproduces this staggering income share of the top 1%, but falls short with the Gini index. Why? Part of the problem can be seen in Figure 10B — the model produces slightly too many capitalist incomes between \$2000 to \$5000.

However, the primary problem has to do with the function used to determine capitalist income (Eq. 3). Capitalist income is assumed to increase linearly with the logarithm of hierarchical power. Since log(1) = 0, all individuals with a hierarchical power of 1 (the lowest amount possible) will have *exactly* zero capitalist income. When calculating inequality, these null incomes are (by convention) *excluded*. If we adjust the model slightly so that instead of having *no* income, these individuals have a *tiny* capitalist income, we get Gini index results that match US data. See Appendix B for more details of this adjustment.

Lastly, Figure 10E shows the capitalist share of total (national) income. The model produces a capitalist income share that is slightly lower than (but in a similar range as) the US data (from 1992-2014).

To summarize, the US capitalist gradient model takes trends between capitalist income and hierarchical power found in CEO pay, and generalizes them to the US population as a whole. The resulting model produces results that closely match US empirical data. This is indirect evidence suggesting that capitalist income fraction scales with hierarchical power in the general US population.

Discussion 18

6 Discussion

The results shown in Figure 7 and 10 are preliminary, and should be treated with appropriate uncertainty. That being said, I want to reflect on their potential significance. In effect, the capitalist gradient model connects three things. It suggests that hierarchical class structure, ownership class structure, and personal income distribution are all related. Put another way, hierarchical elites, capitalists, and top earners *are all the same people*.

What are we to make of this hypothesized relation between authority, property rights, and income? One interpretation is that it is nothing new. Suppose, when speaking about a feudal society, I stated that hierarchical elites, aristocrats, and the very rich are all the same people. This would be nothing particularly controversial. We are quite comfortable concluding that historical societies had a ruling class (Mosca, 1939). But many would bristle at that thought in our own society. Yet consider Reinhard Bendix's description of the relation between authority, property rights, and income in German feudal society. He writes:

... governmental functions were usable rights which could be sold or leased at will. For example, judicial authority was a type of property. The person who bought or leased that property was entitled to adjudicate disputes and receive the fees and penalties incident to such adjudication. (Bendix, 1980, p. 149)

If we paraphrase Bendix, we arrive at the same reasoning that I used to derive the capitalist gradient hypothesis. Building on the work of Nitzan and Bichler, I suggested that 'capitalist authority' is a 'type of property'. The person who buys this property is 'entitled' to wield hierarchical power and 'receive income' in return. From this reasoning came the hypothesis that capitalist income should be related to hierarchical rank and power.

From the perspective of mainstream economic theory, this hypothesis is quite radical. It undermines the ubiquitous assumption that capitalists earn income from a productive asset. But given Bendix's comments on feudal society, the capitalist gradient hypothesis may be quite conservative. Why? Conservatism implies a *lack of change* — a maintenance of the same order. The capitalist gradient hypothesis may be conservative because it suggests that income distribution in modern capitalist societies might not be as different from past feudal societies as we would like to think.

Conclusions 19

7 Conclusions

This paper has outlined a new approach to the study of capitalist income that focuses on firm hierarchy. Drawing on the capital as power framework (Nitzan and Bichler, 2009), I have sought to understand capitalist income in terms of the institutional *act* of ownership. When a capitalist purchases a firm, I have argued that they purchase the right to rule a corporate hierarchy. From this position of hierarchical power, the capitalist has the legal right to partition the firm's income stream as they see fit. In this approach, capitalist income stems from *hierarchical power*.

Because of the rise of partial ownership, which has led to the blurring of the distinction between capitalists and laborers, I have proposed that capitalist income is a *gradient function* of hierarchical power. The idea is that individuals with more hierarchical power will tend to (on average) derive a greater portion of their income from capitalist sources. Evidence from US CEO data supports this hypothesis. Furthermore, a hierarchy model of the US that generalizes this CEO relation accurately reproduces many aspects of the US distribution of capitalist income. This includes the scaling of capitalist income fraction with income size.

Together, this evidence suggests a connection between hierarchical status, ownership and income, at least in the United States. In a long-term historical context, this connection is not particularly surprising. It is exactly what we would expect to find when investigating a feudal aristocracy. However, it runs counter to the prevailing doctrines in mainstream economic theory that deny the existence of concentrated power (Peach, 1987).

Of course, more research is needed to understand the generality of the results presented here. For instance, do they hold in other countries and during different time periods? Because of the importance of replication (and further analysis), I have made available all raw data and code used in this paper.

To conclude, the last century of political economy has been marked by a prolonged ideological struggle between neoclassical and Marxist thought. Neoclassical theory paints a picture of harmony in which capitalists and laborers are each paid their fair share. In contrast, Marxist theory paints a picture of conflict, in which capitalists derive their income by exploiting workers. Unfortunately, both theories are mired by deep epistemological problems that make them untestable.

Conclusions 20

The solution, Nitzan and Bichler have argued, is to focus on the institutional act of ownership (and not on the things that are owned). I have taken Nitzan and Bichler's work, and added to it the study of firm hierarchy. While this approach certainly does not hold all the answers, the evidence presented here suggests that it is a fruitful way to study capitalist income.

Sources and Methods 21

Appendix

Supplementary materials for this paper are available at the Open Science Framework:

The supplementary materials include:

- 1. Raw source data;
- 2. R code for all analysis;
- 3. Hierarchy model code.

A Sources and Methods

Fig. 7: Capitalist Income Fraction of US CEOs

I define the capitalist income share of executives (K_{frac}) as the ratio of stock-options income to total income:

$$K_{\text{frac}} = \frac{\text{Stock Options}}{\text{Total Income}} \tag{4}$$

Executive compensation data comes from the Execucomp database. Execucomp contains two main accounting methods for valuing stock options: a '1992' reporting format that applies from 1992 to 2005', and a '2006' reporting format that applies from 2006 onward. These series are summarized in Table 1. For both reporting formats, the relevant total income series (TDC1) remains the same. I calculate the capitalist income fraction of executives using the following two formulas for 1992 format and 2006 format, respectively:

$$K_{\rm frac_1992} = \frac{\rm RSTKGRNT + OPTION_AWARDS_BLK_VALUE}{\rm TDC1} \tag{5}$$

$$K_{\rm frac_2006} = \frac{\rm STOCK_AWARDS_FV + OPTION_AWARDS_FV}{\rm TDC1} \tag{6}$$

Firm employment data is from Compustat series EMP.

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Table 1: Data Used to Calculate Executive Capitalist Income Fraction

Series	Description	Reporting Format
RSTKGRNT	The value of restricted stock granted during the year (determined as of the date of the grant).	1992
OPTION_AWARDS_BLK_VALUE	The aggregate value of stock options granted to the executive during the year as valued using Standard & Poor's Black-Scholes methodology.	1992
TDC1	Salary, Bonus, Other Annual, Total Value of Restricted Stock Granted, Total Value of Stock Options Granted (using BlackScholes), Long-Term Incentive Payouts, and All Other	1992
STOCK_AWARDS_FV	Fair value of all stock awards during the year as detailed in the Plan Based Awards table. Valuation is based upon the grant-date fair value as detailed in FAS 123R.	2006
OPTION_AWARDS_FV	Fair value of all options awarded during the year as detailed in the Plan Based Awards table. Valuation is based upon the grant-date fair value as detailed in FAS 123R.	2006
TDC1	Salary, Bonus, Non-Equity Incentive Plan Compensation, Grant-Date Fair Value of Option Awards, Grant-Date Fair Value of Stock Awards, Deferred Compensation Earnings Reported as Compensation, and Other Compensation.	2006

Fig. 10: Comparing the Capitalist Gradient Model to US Data

Capitalist Income Fraction vs. Income Percentile

US data is for the year 2007 and comes from Piketty (2014), Fig. 8.10. Data is available at piketty.pse.ens.fr/en/capital21c2.

Capitalist Income Gini Index, Top 1% Share, and Size Distribution

Data for US capitalist income Gini index, top 1% share, and size distribution all come from the IPUMS database. I define capitalist income as the sum of income from dividends and interest. (Dividends = series *INCDIVID*, Interest = series *INCINT*).

The main challenge with this dataset is that it *censors* income above \$100,000. All incomes above this threshold are replaced with a 'topcode' value. To deal with this censoring, I use the method proposed by Jenkins et al. (2011). The gist of this method is that you fit the uncensored data with a parametric distribution. You then replace the censored (topcoded) data with stochastic values drawn from the fitted parametric distribution (above the censor threshold). This gives a partially synthetic dataset on which you compute whatever statistic you desire. Because the process is stochastic, you repeat it many times, giving a range of values for the given statistic.

I follow Jenkins et al. by using the GB2 distribution (generalized beta distribution of the second kind) to fit uncensored data. I use the R GB2 package (Graf and Nedyalkova, 2012) to fit both the dividends and interest data with a GB2 distribution. After replacing topcoded values with synthetic data, I sum dividends and interest income to estimate capitalist income. Figure 11 shows the resulting estimates for the Gini index and top 1% share of capitalist income. Although there is uncertainty in each annual estimate, the actual *range* of inequality values is *dominated* by the secular trend.

All code and data used for this analysis are provided in the Supplementary Material.

Capitalist Share of Total (National) Income

US Capitalist income share data comes from the Bureau of Economic Analysis, Table 1.12. (National Income by Type of Income). Capitalist income is defined as the sum of net dividends and net interest.

Sources and Methods

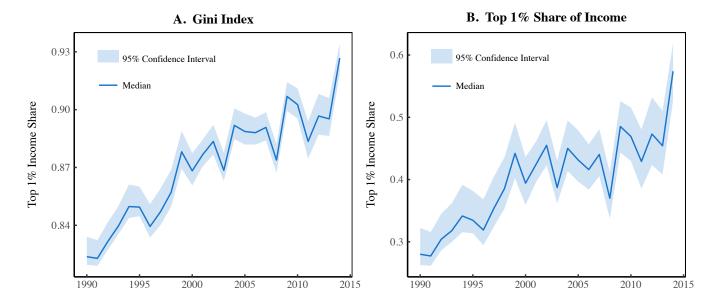


Figure 11: US Capitalist Income Inequality Estimates

This figure shows estimates of inequality in US capitalist income distribution. Data comes from the IPUMS database CPS public micro data. Capitalist income is the sum of dividends (series INCDIVID) and interest (series INCINT). Confidence intervals indicate the uncertainty in the estimate that arises from the stochastic method used to replace topcoded values.

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Calculating Hierarchical Power in the US Hierarchy Model

I define an individual's hierarchical power as the number of subordinates (*S*) under their control, plus 1:

$$P = S + 1 \tag{7}$$

Because the hierarchy model simulates only the *aggregate* structure of firms (employment by hierarchical level), hierarchical power is calculated as an *average* per rank. For hierarchical rank h, the average hierarchical power (\bar{P}_h) is defined as the average number of subordinates (\bar{S}_h) plus 1:

$$\bar{P}_h = \bar{S}_h + 1 \tag{8}$$

Each individual with rank h is assigned the average power \bar{P}_h . The average number of subordinates \bar{S}_h is equal to the sum of employment (E) in all subordinate levels, divided by employment in the level in question:

$$\bar{S}_h = \sum_{i=1}^{h-1} \frac{E_i}{E_h} \tag{9}$$

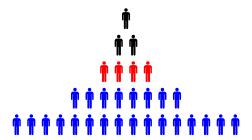


Figure 12: Calculating the Average Number of Subordinates

As an example, consider the hierarchy in Figure 12. The average number of subordinates below each individual in hierarchical level 3 (red) would be:

$$\bar{S}_3 = \frac{E_1 + E_2}{E_3} = \frac{16 + 8}{4} = 6$$
 (10)

Therefore, these individuals would all be assigned a hierarchical power of 7.

B Adjusting the Capitalist Gradient Model

The capitalist gradient model is built on the following functional relation between hierarchical power (P) and capitalist income fraction (K_{frac}):

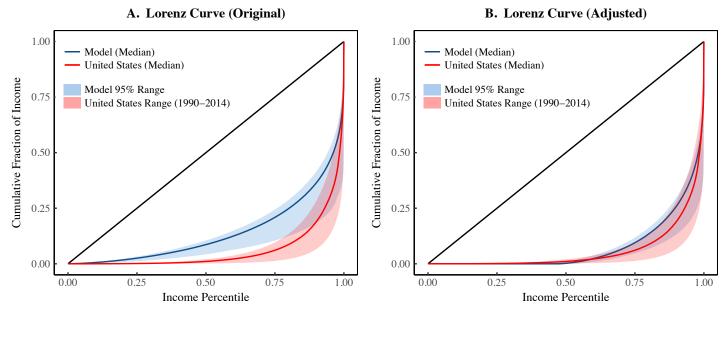
$$K_{\text{frac}} = 0.05 \ln(P) \tag{11}$$

Recall that hierarchical power is defined as the number of subordinates + 1. All individuals with no subordinates therefore have hierarchical power P = 0. Since ln(1) = 0, all these individuals will have *exactly* zero capitalist income. By convention, income distribution is usually only tabulated for non-zero incomes. Thus these individuals are excluded.

In the adjusted capitalist gradient model, I introduce an adjustment to the capitalist income fraction equation:

$$K_{\text{frac}} = 0.05 \ln(P) + \epsilon \tag{12}$$

Here ϵ is a constant very close to zero. Its effect is only felt when P=1. Instead of getting $K_{\rm frac}=0$, we get $K_{\rm frac}=\epsilon$. What does this do? It effectively endows individuals who previously had zero capitalist income with a tiny amount of capitalist income (a few dollars). The effect may seem insignificant, but it has an important impact on the capitalist income distribution. As shown in Figure 13, the adjusted model better matches the US data.



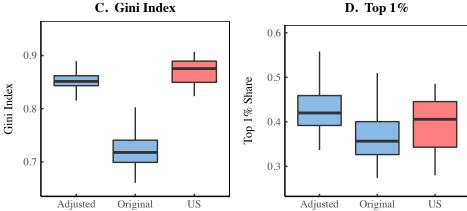


Figure 13: The Adjusted Capitalist Gradient Model vs. US Data

This figure shows the results of an adjusted capitalist gradient model. The adjusted model allows individuals with a hierarchical power of 1 to have a small capitalist income. This significantly changes the model's Lorenz curve and Gini index. Panel A shows the original capitalist gradient model's Lorenz curve plotted against US data. Panel B shows the adjusted capitalist gradient model's Lorenz curve. Panel C compares the Gini indexes of the original and adjusted models to US data. Panel D shows the top 1% income share. US data is from the IPUMS database. See Appendix A.

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