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## Climate Policy and Optimal Public Debt

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# Climate Policy and Optimal Public Debt<sup>1</sup>

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preliminary version

**Abstract:** This paper analyzes the optimal level of public debt when taxes are used not only for funding public expenditures but also for correcting externalities from climate change. Taking into account externalities may imply the optimal policy to deviate from tax smoothing. Provided accumulated marginal damages from today's consumption are larger than those from tomorrow's consumption, the inclusion of environmental externalities decreases (increases) optimal public debt if tax rates are on the increasing (decreasing) side of the Laffer curve. The reversed holds if the accumulated marginal damage increases over time. Allowing for endogenous adaptation investments reduces the deviation from tax smoothing.

JEL classification: H23, H63, Q54, Q58

Keywords: environmental externality, public debt, tax smoothing

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# 1 Introduction

The substantial social and economic costs of environmental degradation through climate change have been thoroughly detailed, amongst others, by Tol (2002a, 2002b) and Stern (2008). For this matter, it should be of little surprise that nowadays politics are considerably influenced by environmental concerns. It has become apparent that governments have to take measures to both, decelerate climate change and find ways to cope with the consequences of altered, less favorable environmental conditions. This is also reflected in the Paris Agreement on climate change which states the intent of more than 170 involved parties to limit greenhouse gas emissions and the rise of the global temperature but also acknowledges the need to adapt to the fallout from climate change (United Nations, 2015). Moreover, an intensively and controversially discussed side-effect of climate policy is its fiscal impact on public budgets. On the one hand, there is the hope of co-benefits like, for instance, the possible double dividend when revenues from carbon taxation are recycled in order to lower other distortionary taxes (e.g., Goulder (1995) or Proost and Van Regemorter (1995)). On the other hand, variations in tax revenues over time and investing in adaption to climate change may put additional strain on the public budget. This effect is of particular importance, since many countries, not only in Europe but worldwide, still struggle with austerity in the wake of the financial crisis of 2007–2010 and try to limit (or entirely halt) the accumulation of new debt.

This paper contributes to the discussion on the fiscal implications of climate policy. We take a look at the rather unexplored but - as explained above - highly policy relevant relation between emission taxation and optimal public debt: When a tax is implemented not only to provide resources for spending requirements, but also to lower greenhouse gas emissions, will it create an economic reason to increase or decrease optimal public debt? To answer this question, we commence our analysis from the tax smoothing approach of Barro (1979, 1989) and extend this approach by including environmental externalities. We develop a two-period model where in each period a representative household consumes two goods, one of which pollutes the environment. Emissions are assumed to accumulate over time and cause environmental damage. They are taxed by an emission tax on the polluting good. The tax plays a dual role since it may be used for both funding a given time path of public expenditures and correcting the environmental externality. In addition, in the first period the spending requirements may be financed by issuing public debt which

has to be repaid in the second period. The time path of tax rates and public debt are set such that welfare of the household is maximized. The basic insight we derive from this model is that the impact of the environmental externality on optimal public debt depends on whether the optimal tax rates are on the increasing or decreasing side of the Laffer curve and whether the accumulated marginal environmental damages from first period consumption are higher or lower than those from second period consumption.

More specific, we first consider the case without environmental externality as benchmark. Optimal tax rates are then constant over time in order to minimize the present value of the excess burden associated with taxation and optimal public debt smoothes out variations in public expenditures. Hence, optimal public debt is positive only if the expenditure requirement is larger in the first period than in the second period; with constant spending requirements optimal public debt is zero. This is the classical tax smoothing argument. We then turn to the case with environmental externality and show that the externality may imply the optimal policy to deviate from tax smoothing. If the accumulated marginal environmental damages from first-period consumption are larger (smaller) than those from second-period consumption, then the role of the tax as a correcting instrument renders the optimal tax rate in the first period larger (smaller) than that in the second period, in contrast to tax smoothing. If in addition both tax rates are on the increasing side of the Laffer curve, tax revenues in the first period are larger (smaller) than those in the second period and for constant spending requirements public debt is negative (positive). These results are reversed if the tax rates are on the decreasing side of the Laffer curve. Then, tax revenues are larger (lower) in the second period and for constant expenditures public debt becomes positive (negative). We show that these results hold independently of whether tax revenues from the Pigouvian (first best) internalization of the externality are already sufficient to finance the spending requirements.

As an extension of our analysis, we take into account endogenous adaptation investments that are of increasing interest and importance in the public debate, as argued in the beginning. We introduce an opportunity for the decision maker to invest in a technology that requires upfront effort in the first period and will adapt the economy to better cope with pollution in the second period. Thereby, we further extend the model by adding an endogenous margin to public spendings. This is in contrast to the standard tax smoothing analysis where spending requirements are exogenously given and influence neither the

household's utility nor any technology. Since engagement in adaptation effort alleviates the damage from pollution in the second period, we find that the adaptation will shrink the wedge between the tax rates induced by the environmental externality. Hence, taking into account adaptation to climate change reduces the deviation from the tax smoothing principle caused by the environmental externality. If previously it had been optimal to accumulate public savings (debt), adaptation would reduce the optimal level of public savings (debt). Nevertheless, even with adaptation the inclusion of the environmental externality impacts optimal public debt qualitatively in the same way as described above for the case without adaptation, though to a lesser extent.

The diversity of cases for which we derive results raises the question which case is the most relevant one from an empirical point of view. To answer this question, two arguments may be of importance. First, in the context of climate change it seems plausible that greenhouse gas emissions from today's consumption lead to higher accumulated marginal damages than those from tomorrow's consumption, simply because the decay of greenhouse gas emissions in the atmosphere is negligible. Hence, it seems to be likely that the optimal emission taxes decrease over time, starting with relatively high levels and then turning to lower levels. This conclusion is also consistent with the optimal tax path in the context of the green paradox (Sinn, 2012). Second, empirical research on the Laffer curve suggests that tax rates usually are on the increasing side of the Laffer curve, at least for labor and capital taxes, see e.g. Trabandt and Uhlig (2011). Putting both arguments together, in our model we are in the case where the inclusion of externalities leads to higher taxes rates and revenues in the first period compared to the second period and, thus, to a fall in the optimal debt level. Allowing for adaption investments mitigates this negative impact of the externality on public debt. Nevertheless, we sell this implication with caution since the question for the most relevant case is in the end an empirical one and we cannot be sure that previous empirical findings are applicable to our framework. For example, to the best of our knowledge there is no empirical study showing that emission tax rates are on the increasing side of the Laffer curve, too. If their primary function is to internalize pollution damages, environmental taxes might also be located on the decreasing side. Hence, the question for the most relevant case deserves a thorough empirical analysis that is beyond the scope of our paper.

We contribute to the literature in two ways. First, we add a further dimension to the

discussion on the fiscal implications of climate policy. As already mentioned above, the double dividend is a prominent topic in this strand of literature, see e.g. [Bovenberg and De Mooij \(1994\)](#), [Proost and Van Regemorter \(1995\)](#), [Parry \(1995\)](#) and [Goulder \(1995\)](#). The basic question addressed is whether an emission tax has not only the positive effect of increasing environmental quality by reducing emissions, but also the potential to improve efficiency of the tax systems by reducing other distortionary taxes. A related topic is discussed in the recent study of [Franks et al. \(2017\)](#). In a dynamic general equilibrium model, these authors investigate whether emission taxation yields higher welfare than taxation of mobile capital, even if environmental externalities are ignored. However, all these papers ignore the link between emission taxation and optimal public debt, which is the main contribution of our analysis.

Second, our paper brings the climate change problem into the literature on public debt. This literature can basically be divided into positive studies explaining the accumulation of public debt, like the political economy models of, e.g., [Persson and Svensson \(1989\)](#), [Tabellini and Alesina \(1990\)](#) and [Woo \(2003\)](#), and normative studies investigating optimal public debt, like the tax smoothing theory of Barro ([1979](#), [1989](#)). Our analysis relates to the normative approach and, in contrast to the tax smoothing analysis, shows that in the presence of a taxable, polluting consumption good, optimal public deficits may be non-zero, even if spending requirements are constant over time. To the best of our knowledge, the only paper that discusses the relation between environmental policy and public debt is [Fodha and Seegmuller \(2014\)](#). They examine the welfare effect of an environmental abatement policy which may either be funded by tax revenues or public debt in a fully dynamic model. They find that pollution abatement should not be conducted on the cost of increased debt when the capital stock is low. However, in their model the tax is modeled as lump sum tax and not as a distortionary tax on pollution or on a polluting consumption good. Hence, in contrast to our analysis, they cannot investigate the implication of climate policy for the tax smoothing role of public debt.

The paper is organized as follows. In Section 2 we introduce the basic framework. In Section 3, we determine the optimal climate and debt policy in case without adaptation. Section 4 provides an examples to buttress our results from the general analysis. In Section 5, we investigate how our findings will be affected when the economy can adapt to pollution by means of investing in an adaptation technology. The final section concludes.

## 2 The model

### 2.1 Private Sector

We consider an economy with a representative household which lives for two periods 1 and 2. In period  $t = 1, 2$  the household consumes a composite good  $Y$  in quantity  $y_t$  and a polluting good  $X$  in quantity  $x_t$ . The household's utility in period  $t$  is given by

$$u_t = y_t + V(x_t), \quad (1)$$

with  $V' > 0$  and  $V'' < 0$ . Without loss of generality, we normalize the discount rate of the household to zero, so the present value of the household's utility reads  $w = u_1 + u_2$ .

In each period, the household dispose of an exogenous endowment normalized to one. Moreover, we assume that the goods  $Y$  and  $X$  can be produced from the endowment by an one-to-one- technology. Hence, prices of both goods are equal to one, too. Good  $Y$  is untaxed, whereas good  $X$  is taxed by a unit tax with tax rate  $\tau_t$  in period  $t$ . The household may receive a lump-sum transfer  $z_t$  from the government in period  $t$ . For simplicity, we ignore private savings. Hence, the private budget constraint in period  $t$  reads

$$y_t + (1 + \tau_t)x_t = 1 + z_t. \quad (2)$$

Tax rates and the lump-sum transfer is taken as given by the household.

The household chooses consumption in order to maximize the present value of its utility. Inserting (2) in (1), the maximization problem can be written as

$$\max_{x_1, x_2} w = \sum_{t=1,2} \left\{ V(x_t) + 1 + z_t - (1 + \tau_t)x_t \right\}.$$

The first-order conditions can be written as

$$V'(x_t) = 1 + \tau_t, \quad t = 1, 2. \quad (3)$$

This condition equates the household's marginal utility to the after-tax price of good  $X$  in period  $t$ . It determines the household's optimal consumption of good  $X$  in period  $t$  as function of the tax rate in period  $t$ . Formally, we have  $x_t = X(\tau_t)$  with  $X'(\tau_t) = 1/V'' < 0$  and  $X''(\tau) = -V'''/V''^3 \geq 0$ .<sup>1</sup>

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<sup>1</sup>Here, we implicitly assume  $V''' \geq 0$ , which is satisfied, for example, if  $V$  is quadratic or if  $V$  is monotone and has monotone derivatives. In the latter case,  $V''' > 0$  is implied by  $V' > 0$  and  $V'' < 0$ .

## 2.2 Government

Beside the tax rates  $\tau_1$  and  $\tau_2$  and the lump-sum transfers  $z_1$  and  $z_2$ , the government may raise revenues in the first period by public debt  $b$  which it has to repay in the second period. Without loss of generality we normalize the interest rate equal to zero. With its policy, the government pursues two goals. First, in period  $t$  it has an exogenous revenue requirement  $g_t \geq 0$  which has to be met by taxation and public debt. Second, the government may use its policy in order to internalize the externalities caused by pollution of good  $X$ . We assume that pollution in the first period causes an environmental damage given by  $D_1(x_1)$  with  $D'_1 > 0$  and  $D''_1 \geq 0$ . In period 2, the environmental damage is given by  $D_2(x_2 + \gamma x_1)$  with  $D'_2 > 0$ ,  $D''_2 \geq 0$  and  $\gamma \geq 0$ . The parameter  $\gamma$  allows distinguishing between flow pollution ( $\gamma = 0$ ) and stock pollution ( $\gamma > 0$ ). An example for the latter case are greenhouse gases and climate change.

In choosing its fiscal policy, the government takes into account the reaction of the private sector captured by the optimal consumption  $x_t = X(\tau_t)$ . The government's maximization problem can be written as

$$\max_{\{b, \tau_t, z_t\}_{t=1,2}} w = \sum_{t=1,2} \left\{ V[X(\tau_t)] + 1 + z_t - (1 + \tau_t)X(\tau_t) \right\} - D_1[X(\tau_1)] - D_2[X(\tau_2) + \gamma X(\tau_1)], \quad (4)$$

subject to

$$\tau_1 X(\tau_1) + b = g_1 + z_1, \quad \tau_2 X(\tau_2) - b = g_2 + z_2, \quad (5)$$

$$z_1 \geq 0, \quad z_2 \geq 0. \quad (6)$$

According to (4), the government maximizes the present value of the household's utility net of the environmental damage. It takes into account the public budget constraints given in (5). Moreover, due to (6) we restrict the policy space to non-negative lump-sum transfers.<sup>2</sup> The reason is that we follow the tax smoothing literature referred to in the introduction and focus on the case where the government has to use distortionary taxation in order to meet its revenue requirement. Hence, the sole role of the transfer is to redistribute back excessive revenues in a non-distortionary way. In contrast to the tax

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<sup>2</sup>Note that the tax rates have to be strictly positive since otherwise the government cannot meet its revenue requirement. Therefore, we implicitly assume  $\tau_1, \tau_2 > 0$ . Public debt  $b$  may take any sign.



smoothing literature, in our framework tax revenues may be above the revenue requirement since the government pursues a second goal, i.e. the internalization of externalities.

In order to characterize the solution to the problem above, we use the Lagrangian

$$L = \sum_{t=1,2} \left\{ V[X(\tau_t)] + 1 + z_t - (1 + \tau_t)X(\tau_t) \right\} - D_1[X(\tau_1)] - D_2[X(\tau_2) + \gamma X(\tau_1)] \\ + \lambda_1[\tau_1 X(\tau_1) + b - g_1 - z_1] + \lambda_2[\tau_2 X(\tau_2) - b - g_2 - z_2],$$

where  $\lambda_1$  and  $\lambda_2$  are Lagrange multipliers associated with the public budget constraints. As Kuhn-Tucker first-order conditions we obtain

$$L_b = \lambda_1 - \lambda_2 = 0, \quad (7)$$

$$L_{\tau_1} = -X(\tau_1) - \left\{ D'_1[X(\tau_1)] + \gamma D'_2[X(\tau_2) + \gamma X(\tau_1)] \right\} X'(\tau_1) \\ + \lambda_1 \left[ X(\tau_1) + \tau_1 X'(\tau_1) \right] = 0, \quad (8)$$

$$L_{\tau_2} = -X(\tau_2) - D'_2[X(\tau_2) + \gamma X(\tau_1)] X'(\tau_2) + \lambda_2 \left[ X(\tau_2) + \tau_2 X'(\tau_2) \right] = 0, \quad (9)$$

and the slackness conditions

$$L_{\lambda_1} = \tau_1 X(\tau_1) + b - g_1 - z_1 \geq 0, \quad \lambda_1 \geq 0, \quad \lambda_1 L_{\lambda_1} = 0, \quad (10)$$

$$L_{\lambda_2} = \tau_2 X(\tau_2) - b - g_2 - z_2 \geq 0, \quad \lambda_2 \geq 0, \quad \lambda_2 L_{\lambda_2} = 0, \quad (11)$$

$$L_{z_1} = 1 - \lambda_1 \leq 0, \quad z_1 \geq 0, \quad z_1 L_{z_1} = 0, \quad (12)$$

$$L_{z_2} = 1 - \lambda_2 \leq 0, \quad z_2 \geq 0, \quad z_2 L_{z_2} = 0, \quad (13)$$

where in (8) and (9) we used (3). Note that from (12) and (13) it follows  $\lambda_1, \lambda_2 \geq 1 > 0$ . Hence, (10) and (11) imply that both budget constraints are always satisfied with equality. Finally, when  $z_1$  and  $z_2$  are zero, the second-order conditions require that the determinant of the bordered Hessian  $|H|$  of the remaining system is negative. In [Appendix](#) we show

$$|H| = (x_1 + \tau_1 X'_1)^2 L_{\tau_2 \tau_2} + (x_2 + \tau_2 X'_1)^2 L_{\tau_1 \tau_1} + 2\gamma(x_1 + \tau_1 X'_1)(x_2 + \tau_2 X'_2) D''_2 X'_1 X'_2, \quad (14)$$

with

$$L_{\tau_1 \tau_1} = -X'_1 - (D'_1 + \gamma D'_2) X''_1 - (D''_1 + \gamma^2 D''_2) X'^2_1 + \lambda_1 (2X'_1 + \tau_1 X''_1),$$

$$L_{\tau_2 \tau_2} = -X'_2 - D'_2 X''_2 - D''_2 X'^2_2 + \lambda_2 (2X'_2 + \tau_2 X''_2),$$

and  $x_t = X(\tau_t), X'_t := X'(\tau_t), X''_t := X''(\tau_t), D'_t := D'(x_t)$  and  $D''_t := D''(x_t)$ . In all relevant cases considered below we will check whether  $|H| < 0$  is satisfied.

### 3 General Analysis

In this section we characterize the optimal policy of the government without referring to specific functional forms. Central to our main results are the public budget constraints in equation (5). Adding the two constraints gives

$$\tau_1 X(\tau_1) + \tau_2 X(\tau_2) = g_1 + g_2 + z_1 + z_2. \quad (15)$$

Hence, the present value of tax revenues (LHS) has to be equal to the present value of public spending and transfers (RHS). In the subsequent analysis, this intertemporal budget constraint will be helpful in determining the optimal tax rates. Subtracting the second budget constraint in (5) from the first gives

$$b = \frac{g_1 - g_2}{2} - \frac{\tau_1 X(\tau_1) - \tau_2 X(\tau_2)}{2} + \frac{z_1 - z_2}{2}. \quad (16)$$

Once we have determined the optimal tax rates  $\tau_1$  and  $\tau_2$  and the optimal transfers  $z_1$  and  $z_2$ , we will use (16) in order to compute the optimal level of public debt. Basically, the equation has the same meaning as in previous studies of tax smoothing without pollution. The first term on the RHS shows the central tax smoothing argument in the absence of externalities: Public debt is used to equalize variations in exogenous public spending. The reason is that without externalities the optimal tax policy minimizes the excess burden of taxation and, thus, requires constant tax revenues over time with the consequence that the second term on the RHS vanishes. In contrast, in our analysis with environmental externalities it will turn out that tax revenues may vary over time and, thus, may impact the optimal debt policy via the second term on the RHS of (16).<sup>3</sup>

More specific, if tax revenues in the second period,  $\tau_2 X(\tau_2)$ , are larger than tax revenues in the first period,  $\tau_1 X(\tau_1)$ , then the second term of the RHS of (16) inclusive of the minus sign in front of it is positive and yields an additional rationale for public debt. Crucial for such a result is the Laffer curve in period  $t$  given by

$$R(\tau_t) = \tau_t X(\tau_t). \quad (17)$$

We impose the following quite general assumption on the shape of the Laffer curve.

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<sup>3</sup>In most cases of interest the transfers  $z_1$  and  $z_2$  are zero, so they do not further impact public debt. In the cases with positive optimal transfer levels only the sum of transfers,  $z_1 + z_2$ , is determined. Since we introduced these transfers only to redistribute back excessive tax revenues, also in case with positive transfers it is natural to assume  $z_1 = z_2$  in order to abstract from further effects on public debt.

**Assumption A** *The Laffer curve  $R(\tau)$  is twice continuously differentiable and satisfies  $R'(\tau) = X(\tau) + \tau X'(\tau) \gtrless 0$  if and only if  $\tau \gtrless \bar{\tau}$  with  $\bar{\tau} > 0$ ,  $R''(\tau) = 2X'(\tau) + \tau X''(\tau) < 0$ ,  $R(0) = 0$  and  $\lim_{\tau \rightarrow \infty} R(\tau) < (g_1 + g_2)/2 < R(\bar{\tau})$ .*

These assumptions state that the Laffer curve is inverted U-shaped with a unique maximum at the positive tax rate  $\hat{\tau}$  and vanishing tax revenues at a zero tax rate. The latter properties in Assumption A ensure that maximal revenues at  $\bar{\tau}$  are more than enough to meet the spending requirements. This together with the U-shape of the Laffer curve, in turn, implies that on both sides of the Laffer curve there are further tax rates that generate tax revenues sufficiently high to meet the spending requirements.

Next let us rewrite the first-order conditions in order to identify conditions under which revenue requirements are larger in the second period than in the first period. From (7) we obtain  $\lambda_1 = \lambda_2 =: \lambda$ . Using this in (8) and (9) yields

$$\lambda = \frac{X(\tau_1) + \left\{ D'_1[X(\tau_1)] + \gamma D'_2[X(\tau_2) + \gamma X(\tau_1)] \right\} X'(\tau_1)}{X(\tau_1) + \tau_1 X'(\tau_1)}, \quad (18)$$

$$\lambda = \frac{X(\tau_2) + D'_2[X(\tau_2) + \gamma X(\tau_1)] X'(\tau_2)}{X(\tau_2) + \tau_2 X'(\tau_2)}, \quad (19)$$

Since  $\lambda \geq 1 > 0$  from the slackness conditions (12) and (13), the nominators and denominators on the RHS of (18) and (19) must have the same sign. They may be either both positive or both negative, in each of these equations. The implications, however, depends on whether the slackness conditions are binding or not.

In a special case we can unambiguously state that slackness conditions are binding.

**Proposition 1.** *If  $D_1 \equiv D_2 \equiv 0$ , then the optimal policy is characterized by  $z_1 = z_2 = 0$ ,  $\tau_1 = \tau_2 = \tau$  and  $b = (g_1 - g_2)/2$ , where  $\tau$  is implicitly determined by  $\tau X(\tau) = (g_1 + g_2)/2$  and lies on the increasing side of the Laffer curve  $R(\tau)$ .*

**Proof:** From  $D_1 \equiv D_2 \equiv 0$  and (18) and (19) we obtain

$$\lambda = X(\tau_1)/[X(\tau_1) + \tau_1 X'(\tau_1)], \quad \lambda = X(\tau_2)/[X(\tau_2) + \tau_2 X'(\tau_2)]. \quad (20)$$

Since  $\lambda \geq 1 > 0$  and  $X(\tau_t) > 0$ , it follows  $X(\tau_t) + \tau_t X'(\tau_t) > 0$  for  $t = 1, 2$ . Hence, in each period the optimal tax rate is on the increasing sides of the Laffer curve. As  $X'(\cdot) < 0$ , we have  $X(\tau_t) + \tau_t X'(\tau_t) < X(\tau_t)$  and therefore (20) implies  $\lambda > 1$  and  $z_1 = z_2 = 0$  by the

slackness conditions (12) and (13). Moreover, (20) shows that  $\tau_1$  and  $\tau_2$  are determined by the same equation which implies  $\tau_1 = \tau_2 = \tau$ . Inserting this into the intertemporal budget constraint (15) gives  $\tau X(\tau) = (g_1 + g_2)/2$ . Finally, inserting  $\tau_1 = \tau_2 = \tau$  and  $z_1 = z_2 = 0$  into (16) implies  $b = (g_1 - g_2)/2$ . ■

Proposition 1 replicates the results from the previous literature: When good  $X$  does not cause externalities, the only purpose of taxation is to meet the spending requirement. Since taxation is distortionary, the government sets tax rates such that the excess burden is minimized. The minimum is obtained if the tax rates and, thus, tax revenues are constant over time ( $\tau_1 X(\tau_1) = \tau_2 X(\tau_2)$ ). Of course, due to the excess burden of taxation the government will not generate more revenues than required for exogenous spending, so the transfers are zero in both periods ( $z_1 = z_2$ ). What remains in equation (16) as determinant of public debt is the difference in the spending requirement, i.e.  $b = (g_1 - g_2)/2$ . Hence, a positive debt level may be optimal if spendings requirement vary. For example, if spending is higher in the first period than in the second ( $g_1 > g_2$ ), then it is optimal for an efficient tax policy to issue debt in the first period and to repay it in the second period.

More interesting for our purpose is the case with externalities. Due to (18) and (19), for  $D_1, D_2 \neq 0$  there are two important differences to the case without externalities. First,  $\lambda$  may be equal to one such that the slackness conditions are not binding and, second, tax rates may be on the decreasing part of the Laffer curve. We start with the case where the slackness conditions are still binding and later discuss the case where they are not.

**Proposition 2.** *If  $D_1, D_2 \neq 0$  and  $\lambda > 1$ , then the optimal policy is characterized by*

(i)  $z_1 = z_2 = 0$ .

(ii)  $\tau_1 \gtrless \tau_2$  if and only if  $D'_1 + \gamma D'_2 \gtrless D'_2$ .

(iii) If  $D'_1 + \gamma D'_2 < -x_1/X'_1$  and  $D'_2 < -x_2/X'_2$ , then  $\tau_1$  and  $\tau_2$  are both on the increasing side of the Laffer curve and  $\tau_1 > D'_1 + \gamma D'_2$  and  $\tau_2 > D'_2$ . Moreover

$$b \gtrless \frac{g_1 - g_2}{2} \Leftrightarrow D'_1 + \gamma D'_2 \lesseqgtr D'_2.$$

(iv) If  $D'_1 + \gamma D'_2 > -x_1/X'_1$  and  $D'_2 > -x_2/X'_2$ , then  $\tau_1$  and  $\tau_2$  are both on the decreasing side of the Laffer curve and  $\tau_1 < D'_1 + \gamma D'_2$  and  $\tau_2 < D'_2$ . Moreover

$$b \gtrless \frac{g_1 - g_2}{2} \Leftrightarrow D'_1 + \gamma D'_2 \gtrless D'_2.$$

**Proof:** Part (i) immediately follows from  $\lambda > 1$ , (12) and (13). In order to prove part (ii) rewrite (18) and (19) as

$$F(\tau_1) = \frac{D'_1[X(\tau_1)] + \gamma D'_2[X(\tau_2) + \gamma X(\tau_1)]}{\lambda}, \quad F(\tau_2) = \frac{D'_2[X(\tau_2) + \gamma X(\tau_1)]}{\lambda}, \quad (21)$$

with

$$F(\tau) := \tau - \frac{1 - \lambda}{\lambda} \frac{X(\tau)}{X'(\tau)}, \quad F'(\tau) := 1 - \frac{1 - \lambda}{\lambda} \frac{[X'(\tau)]^2 - X(\tau)X''(\tau)}{[X'(\tau)]^2} > 0, \quad (22)$$

where the sign of  $F'(\tau)$  follows from  $\lambda > 1$  and  $[X'(\tau)]^2 - X(\tau)X''(\tau) > 0$  which, in turn, follows from  $X' < 0$ ,  $X'' \geq 0$  and, thus,  $X''' \leq 0$ . Since  $F(\tau)$  on the LHS in each of the equations of (21) is an increasing function of the tax rates and the RHS in each of the equations of (21) is a decreasing function of the tax rates, we have proven part (ii).

Next turn to part (iii). If  $D'_1 + \gamma D'_2 < -x_1/X'_1$  and  $D'_2 < -x_2/X'_2$ , equations (18) and (19) imply  $x_1 + \tau_1 X'(\tau_1) > 0$  and  $x_2 + \tau_2 X'(\tau_2) > 0$ , i.e. both tax rates lie on the increasing side of the Laffer curve. Moreover, rearranging (18) and (19) in this case gives  $\tau_1 > D'_1 + \gamma D'_2$  and  $\tau_2 > D'_2$ . Taking into account part (ii) and that both tax rates are on the increasing side of the Laffer curve we obtain  $\tau_1 X(\tau_1) \stackrel{\leq}{\geq} \tau_2 X(\tau_2)$  if and only if  $D'_1 + \gamma D'_2 \stackrel{\leq}{\geq} D'_2$ . Using this property together with  $z_1 = z_2 = 0$  in (16) proves the result with respect to optimal debt  $b$  in part (iii). Finally, the proof of part (iv) is perfectly analogous to that of part (iii). ■

Since the slackness conditions are still binding, the transfers are zero as shown in part (i) of Proposition 2. The reason is that taxing good  $X$  according to the marginal environmental damage (Pigouvian taxes) will not generate enough tax revenues to satisfy the spending requirements. This property is also expressed in part (iii) [part (iv)], where the optimal tax rates are above [below] the marginal environmental damage and Pigouvian tax rates would generate lower tax revenues than the optimal tax rates, since the latter are on the increasing [decreasing] side of the Laffer curve. Nevertheless, according to part (ii) of Proposition 2 the optimal tax rates are positively correlated with the marginal environmental damage in the sense that the tax rate is always higher in the period in which good  $X$  causes the larger marginal environmental damage, even though the optimal tax rates are not equal to the marginal environmental damage.

The most important insight of Proposition 2, however, is with respect to optimal debt. As we can see from part (iii) and (iv), the presence of environmental externalities

may influence the optimal debt level as  $b$  may deviate from  $(g_1 - g_2)/2$ , which is the optimal debt level in the absence of externalities. For instance, consider first part (iii) of Proposition 2 where the marginal environmental damage is relatively low, so that in both periods optimal tax rates are on the increasing side of the Laffer curve. If, additionally, the marginal damage is larger (smaller) in period 1 than in period 2, then the optimal debt level is lower (larger) than in the absence of externalities. The reason is that according to part (ii) the optimal tax rate and the optimal tax revenues are larger (smaller) in period 1 than in period 2. In part (iv) we obtain the reversed argument. The marginal environmental damage is relatively large and the optimal tax rates are both on the decreasing part of the Laffer curve. If now the marginal damage is larger (smaller) in period 1 than in period 2, the optimal tax rate is larger (smaller) in period 1 than in period 2 and the tax revenues are lower (larger) in period 1 than in period 2. As consequence, optimal debt exceeds (falls short of) the optimal level in the absence of externalities.

Put differently, there may be a positive debt level even if spending requirements are constant over time ( $g_1 = g_2$ ) and, thus, the tax smoothing argument identified in the previous literature is absent. According to part (iii) of Proposition 2 this is the case if tax rates are on the increasing side of the Laffer curve and the environmental damage is smaller in period 1 than in period 2, i.e.  $D'_1 + \gamma D'_2 < D'_2$ . Admittedly, in the context of climate change this condition may be satisfied when we take into account tipping points for pollution above which environmental damages are huge. But for climate change  $\gamma$  is pretty close to one such that  $D'_1 + \gamma D'_2 < D'_2$  is hardly satisfied and environmental externalities are likely to lower the optimal debt level. As shown by part (iv) of Proposition 2, however, this argument may be reversed if the marginal damage is rather large in both periods. Both tax rates are then on the decreasing side of the Laffer curve. If, additionally, the marginal damage is larger in period 1 than in period 2, i.e.  $D'_1 + \gamma D'_2 > D'_2$ , the optimal tax rate is larger and the tax revenues are lower in period 1 than in period 2. Optimal debt is then positive even if spending requirements are constant over time ( $g_1 = g_2$ ). This case seems to be quite relevant in the case of climate change, since we then expect large environmental damages and  $\gamma$  is close or even almost identical to one.

Next, turn to the case where the slackness conditions are no longer binding and  $\lambda = 1$ . We then obtain

**Proposition 3.** *If  $D_1, D_2 \neq 0$  and  $\lambda = 1$ , then the optimal policy is characterized by*

$$(i) \ z_1 = z_2 = \frac{(D'_1 + \gamma D'_2)X[D'_1 + \gamma D'_2] + D'_2 X(D'_2)}{2} - \frac{g_1 + g_2}{2} > 0.$$

$$(ii) \ \tau_1 = D'_1 + \gamma D'_2 \begin{matrix} \geq \\ \leq \end{matrix} D'_2 = \tau_2 \text{ if and only if } D'_1 + \gamma D'_2 \begin{matrix} \geq \\ \leq \end{matrix} D'_2.$$

(iii) If  $D'_1 + \gamma D'_2 < -x_1/X'_1$  and  $D'_2 < -x_2/X'_2$ , then  $\tau_1$  and  $\tau_2$  are both on the increasing side of the Laffer curve. Moreover

$$b \begin{matrix} \geq \\ \leq \end{matrix} \frac{g_1 - g_2}{2} \Leftrightarrow D'_1 + \gamma D'_2 \begin{matrix} \leq \\ \geq \end{matrix} D'_2.$$

(iv) If  $D'_1 + \gamma D'_2 > -x_1/X'_1$  and  $D'_2 > -x_2/X'_2$ , then  $\tau_1$  and  $\tau_2$  are both on the decreasing side of the Laffer curve. Moreover

$$b \begin{matrix} \geq \\ \leq \end{matrix} \frac{g_1 - g_2}{2} \Leftrightarrow D'_1 + \gamma D'_2 \begin{matrix} \geq \\ \leq \end{matrix} D'_2.$$

**Proof:** For  $\lambda = 1$  equation (18) and (19) can be written as

$$\frac{x_1 + (D'_1 + \gamma D'_2)X'_1}{x_1 + \tau_1 X'_1} = 1 = \frac{x_2 + D'_2 X'_2}{x_2 + \tau_2 X'_2}, \quad (23)$$

It follows that the optimal tax rates are  $\tau_1 = D'_1 + \gamma D'_2$  and  $\tau_2 = D'_2$  which proves part (ii) of Proposition 3. For  $D'_1 + \gamma D'_2 < -x_1/X'_1$  and  $D'_2 < -x_2/X'_2$ , equation (23) implies that  $\tau_1$  and  $\tau_2$  are on the increasing side of the Laffer curve. As we assume  $z_1 = z_2$ , the optimal debt level in (16) becomes

$$b = \frac{g_1 - g_2}{2} - \frac{(D'_1 + \gamma D'_2)X[D'_1 + \gamma D'_2] - D'_2 X(D'_2)}{2}. \quad (24)$$

Since the tax rates are on the increasing side of the Laffer curve, we immediately obtain the result for  $b$  in part (iii), which completes the proof of part (iii). The proof of part (iv) is analogous. Finally, inserting (24) into (10) and (11), where both  $L_{\lambda_1}$  and  $L_{\lambda_2}$  have to be zero, and solving with respect to  $z_1$  and  $z_2$  shows part (i) of Proposition 3. ■

As becomes obvious from part (iii) and (iv) of Proposition 3, with respect to the optimal debt level we obtain exactly the same results as in Proposition 2: Depending on whether the optimal tax rates are on the increasing or decreasing part of the Laffer curve and depending on the relation of the marginal damage in the two periods, introducing externalities into the tax smoothing analysis may increase or decrease the optimal debt level. The difference to Proposition 2 is that the optimal tax rates are on their Pigouvian level (marginal damage) and that the transfers are positive in both periods. The reason is

that the spending requirements are so low that the tax rates which satisfy these requirements are not high enough to internalize the externalities. The transfers thus reflect the difference between tax revenues and spending requirement and become positive.

## 4 An Example

Suppose a linear-quadratic specification of our model. The utility function for good  $X$  in period  $t$  is given by  $V(x_t) = (\alpha + 1)x_t - \beta x_t^2/2$  with  $\alpha, \beta > 0$ . The marginal damage in period 1 and 2 reads, respectively,  $D_1(x_1) = \delta_1 x_1$  and  $D_2(x_2 + \gamma x_1) = \delta_2(x_2 + \gamma x_1)$  with  $\delta_1, \delta_2 > 0$ . The household's first-order condition (3) then imply  $X(\tau_t) = (\alpha - \tau_t)/\beta$ . The Laffer curve in period  $t$  is  $R(\tau_t) = (\alpha\tau_t - \tau_t^2)/\beta$  with a maximum at  $\tau_t = \alpha/2$  and zero tax revenues at  $\tau_t = 0$  and  $\tau_t = \alpha$ . We assume  $0 < \tau_t < \alpha$  in order to ensure positive tax revenues. The central equations (18) and (19) can be written as

$$\lambda = \frac{\alpha - \tau_1 - \delta_1 - \gamma\delta_2}{\alpha - 2\tau_1}, \quad \lambda = \frac{\alpha - \tau_2 - \delta_2}{\alpha - 2\tau_2}. \quad (25)$$

The intertemporal budget constraint (15) becomes

$$\frac{\alpha\tau_1 - \tau_1^2}{\beta} + \frac{\alpha\tau_2 - \tau_2^2}{\beta} = g_1 + g_2 + z_1 + z_2. \quad (26)$$

The optimal debt (16) is

$$b = \frac{g_1 - g_2}{2} - \frac{\alpha\tau_1 - \tau_1^2 - (\alpha\tau_2 - \tau_2^2)}{2\beta} + \frac{z_1 - z_2}{2}. \quad (27)$$

We first consider the case where  $\lambda > 1$  and  $\delta_1 = \delta_2 = \delta$ . We then have  $z_1 = z_2 = 0$  and the bordered Hessian (14) reads

$$|H| = \frac{1 - 2\lambda}{\beta^3} \left[ (\alpha - 2\tau_2)^2 + (\alpha - 2\tau_1)^2 \right] < 0. \quad (28)$$

Start with the benchmark case  $\delta = \gamma = 0$ . From (25) follows  $\tau_1 = \tau_2 =: \tau^*$ . Inserting together with  $z_1 = z_2 = 0$  into (27) gives  $b^* = (g_1 - g_2)/2$ . The tax rate  $\tau^*$  is determined by setting  $\tau_1 = \tau_2 =: \tau^*$  in (26). We obtain a quadratic equation with the two solutions

$$\tau^- = \frac{\alpha}{2} - \sqrt{\left(\frac{\alpha}{2}\right)^2 - \frac{\beta(g_1 + g_2)}{2}} \in ]0, \alpha/2[, \quad (29)$$

$$\tau^+ = \frac{\alpha}{2} + \sqrt{\left(\frac{\alpha}{2}\right)^2 - \frac{\beta(g_1 + g_2)}{2}} \in ]\alpha/2, \alpha[. \quad (30)$$



Using  $\delta = \gamma = 0$  and  $\tau_1 = \tau_2 =: \tau^*$  in (25) gives  $\lambda^* = (\alpha - \tau^*)/(\alpha - 2\tau^*)$ . Since  $\lambda^* > 1 > 0$  and  $\alpha > \tau^*$  we also must have  $\alpha > 2\tau^*$ . The second-order condition requires  $|H|$  from (28) to be negative and, thus,  $1 - 2\lambda^* < 0$ . Using the expression for  $\lambda^*$  we can show that this condition is satisfied iff  $\tau^* < \alpha/2$ . Hence, the solution for the tax rate is  $\tau^* = \tau^-$  and lies on the increasing side of the Laffer curve. In sum, without environmental externalities, the example confirms all results obtained for the general case in Proposition 1.

As next case consider  $\delta > 0$  and  $\gamma = 0$ . Hence, period 1 consumption of good  $X$  does not cause environmental damage in period 2 (flow pollution) and the marginal damage is the same in both periods. From (25) it again follows that both tax rates are equal, i.e. follows  $\tau_1 = \tau_2 =: \tau_\delta$ . Also optimal debt is the same as in the benchmark. This can be seen by inserting  $\tau_1 = \tau_2 =: \tau_\delta$  together with  $z_1 = z_2 = 0$  into (27) which yields  $b_\delta = (g_1 - g_2)/2$ . The tax rate  $\tau_\delta$  is again determined by (26), which gives  $\tau^-$  and  $\tau^+$  from (29) and (30). However, the difference to the benchmark case is that now  $\lambda_\delta = (\alpha - \tau_\delta - \delta)/(\alpha - 2\tau_\delta)$ . Hence, now the numerator and, thus, the denominator may both be negative. This has consequences for the question of whether  $\tau^-$  or  $\tau^+$  is the optimal tax rate. From (28) we have  $1 - 2\lambda_\delta < 0$ . Inserting the expression for  $\lambda_\delta$  and rearranging gives  $(2\delta - \alpha)/(\alpha - 2\tau_\delta) < 0$ . Therefore, if  $\delta < \alpha/2$ , then  $\tau_\delta < \alpha/2$  and  $\tau_\delta = \tau^-$ . In contrast, if  $\delta > \alpha/2$ , then  $\tau_\delta > \alpha/2$  and  $\tau_\delta = \tau^+$ . In sum, when we introduce symmetrically an externality in both periods, this will not affect public debt, which is consistent with our general analysis (since for  $\delta > 0$  and  $\gamma = 0$  we have  $D'_1 + \gamma D'_2 = D'_2$ ). Moreover, in our example we get the additional insight that increasing the marginal damage  $\delta$  in both periods to the same extent will not change the optimal tax rates as long as the marginal damage stays below  $\alpha/2$ . Only if  $\delta$  jumps from values below  $\alpha/2$  to values above  $\alpha/2$ , the optimal tax rate  $\tau_\delta$  jumps once from  $\tau^-$  to  $\tau^+$ .

Next turn to the case  $\delta, \gamma > 0$ . We confine ourself to a numerical example in order to illustrate to our result from the general analysis that the environmental externality may influence public debt. Suppose  $\alpha = \beta = 1$ ,  $\delta = 0.9$ ,  $\gamma = 0.1$  and  $g_1 = g_2 = 0.2$ . Even though both tax rates are no longer equal, we get two solutions. The first one is  $\tau_1^- = 0.255$  and  $\tau_2^- = 0.300$ . However, at this solution we have  $|H| = 0.800 > 0$  which violates the second-order condition. The second solution is  $\tau_1^+ = 0.745 > 0.700 = \tau_2^+$ . The second-order condition is satisfied since  $|H| = -0.800 < 0$ . Tax revenues in the two periods are  $R_1^+ = 0.190 < 0.210 = R_2^+$ . Public debt is positive ( $b^+ = 0.010 > 0$ ) even

though spending requirements are equal in both periods. This case again confirms our results from the general analysis.

## 5 The Case of Adaptation

In this section, we turn to analyze how the optimal policy is affected when adaptation technology becomes available to the social planner. Suppose that the government now has the opportunity to invest effort  $e$  in the first period to reduce damage from pollution in the second period. Essentially, we add an endogenous margin to public spendings which increases social welfare. In the context of climate change, we could think of the construction of a dam that protects from rising sea-levels. Due to the scale of such a project, the economy will not benefit from the investment immediately but, when finished, it will reduce damage from the entire stock and flows of pollution,  $\gamma x_1 + x_2$ . We consider the adaptation technology in our model by adding  $e$  to the budget requirements in the first period. Let the updated period two damage function,  $D_2(\gamma x_1 + x_2, e)$ , be additive separable in the quantity of the polluting consumption good and effort, with  $D_{2,e} < 0$ ,  $D_{2,ee} > 0$  and the cross-derivatives equal to zero.

We want to determine how the optimal tax rates and level of debt established under proposition 2 change when the government engages in adaptation. Therefore, assume that a solution to the government's maximization problem exists, with  $e > 0$  and  $z_1 = z_2 = 0$ . In ?? we derive the marginal effect of introducing adaptation effort on the tax rates as

$$\frac{\partial \tau_1}{\partial e} = \frac{\Gamma_{\tau_2} \Delta_e - \Delta_{\tau_2} \Gamma_e}{\Delta_{\tau_2} \Gamma_{\tau_1} - \Gamma_{\tau_2} \Delta_{\tau_1}}, \quad \frac{\partial \tau_2}{\partial e} = \frac{\Delta_{\tau_1} \Gamma_e - \Gamma_{\tau_1} \Delta_e}{\Delta_{\tau_2} \Gamma_{\tau_1} - \Gamma_{\tau_2} \Delta_{\tau_1}}, \quad (31)$$

where

$$\Gamma = X_1 + (D_{1,x} + \gamma D_{2,x}) X'_1 + D_{2,e} (X_1 + \tau_1 X'_1) = 0, \quad (32)$$

$$\Delta = X_2 + D_{2,x} X'_2 + D_{2,e} (X_2 + \tau_2 X'_2) = 0. \quad (33)$$

Because valid solutions to the maximization problem have to satisfy  $|H| < 0$ , we know that the denominator in equations (31) has to be positive.<sup>4</sup> The optimal level of public

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<sup>4</sup>In fact, the denominator is equal to the second principal minor,  $|H_2|$ , of the associated Hessian. For the Hessian to be negative definite with  $|H| < 0$ , we know that  $|H_2|$  will always be positive.

debt in the presence of adaptation effort is

$$b = \frac{g_1 - g_2}{2} - \frac{R(\tau_1) - R(\tau_2)}{2} + \frac{e}{2}, \quad (34)$$

so that even for constant spending requirements,  $g_1 = g_2$ , a positive level of public debt, i.e.,  $b^* = e/2$ , becomes optimal under the tax smoothing regime. To analyze how the environmental externality influences optimal debt, we take the total differential of (34)

$$\frac{db}{de} = -\frac{1}{2} \left[ \frac{\partial \tau_1}{\partial e} R'(\tau_1) - \frac{\partial \tau_2}{\partial e} R'(\tau_2) \right] + \frac{1}{2}. \quad (35)$$

From the intertemporal budget constraint it is also clear that at least one of the tax rates has to be raised (lowered) if both rates are on the increasing (decreasing) side of the Laffer curve, to maintain a balanced budget when we allow for non-zero effort spendings,  $e > 0$ . The general analysis of equation (35) shows that, in the presence of a taxable environmental externality, the change in  $b$  will typically deviate from  $1/2$ . Hence, the additional spending requirements from increasing  $e$  will not be distributed equally across the two periods.

However, it proves difficult to draw any further conclusions without assuming a specific functional form for  $X(\tau_t)$  or  $D_t(X, e)$ . Therefore, consider again  $X(\tau_t) = (\alpha - \tau_t)/\beta$  from our previous example and suppose  $D_2(\gamma x_1 + x_2, e) = \delta_2[\gamma x_1 + x_2 - \ln(1 + e)]$  while we still have  $D_1(x_1) = \delta_1 x_1$ . Consequently, adaptation effort will lower the damage from pollution at a decreasing rate. Employing these expressions and the respective derivatives, equations (31) reduce to

$$\frac{\partial \tau_t}{\partial e} = -\frac{\delta_2(\alpha - 2\tau_t)}{(2\delta_2 - (1 + e))(1 + e)}. \quad (36)$$

Substituting equations (36) and  $R'(\tau_t) = (\alpha - 2\tau_t)/\beta$  into (35) we receive

$$\frac{db}{de} = \frac{\delta_2}{2\beta(2\delta_2 - (1 + e))(1 + e)} \left[ (\alpha - 2\tau_1)^2 - (\alpha - 2\tau_2)^2 \right] + \frac{1}{2}. \quad (37)$$

**Corollary 1.** *If  $D_1, D_2 \neq 0$  and  $\lambda > 1$ , the availability of an adaptation technology will affect optimal policy*

(i) *if  $\tau_1$  and  $\tau_2$  are both on the increasing side of the Laffer curve by*

$$\frac{db}{de} \begin{matrix} \geq \\ \leq \end{matrix} \frac{1}{2} \quad \Leftrightarrow \quad D'_1 + \gamma D'_2 \begin{matrix} \geq \\ \leq \end{matrix} D'_2,$$

(ii) *if  $\tau_1$  and  $\tau_2$  are both on the decreasing side of the Laffer curve by*

$$\frac{db}{de} \begin{matrix} \geq \\ \leq \end{matrix} \frac{1}{2} \quad \Leftrightarrow \quad D'_1 + \gamma D'_2 \begin{matrix} \leq \\ \geq \end{matrix} D'_2.$$

**Proof:** To prove corollary 1, we draw on the results from proposition 2. In part (i) both tax rates are on the increasing side of the Laffer curve, i.e.,  $R'(\tau_t) = (\alpha - 2\tau_t)/\beta > 0$ . Since  $\beta > 0$ , this condition implies  $\alpha - 2\tau_t > 0$ . The total differential of the intertemporal budget constraint

$$R'(\tau_1)d\tau_1 + R'(\tau_2)d\tau_2 - de = 0, \quad (38)$$

requires  $\exists d\tau_t > 0$  when  $de > 0$  for the budget to be balanced. Hence, at least one of equations (36) has to be positive. This will only hold for  $2\delta_2 - 1 < e$ , but then the denominator of the first summand in (37) becomes negative. From part (ii) of proposition 2 we know that for the initial tax rates  $\tau_1 \gtrless \tau_2$  iff  $D'_1 + \gamma D'_2 \gtrless D'_2$ . Hence, the numerator of the first summand in (37) will also be negative when the marginal damage is larger in the first period and  $db/de$  will be larger than 1/2 in total (and vice versa). The proof of part (ii) is conducted analogously. ■

If initially on the same side of the Laffer curve, both tax rates have to move in the same direction when the government can invest in the adaptation technology. Since we assumed that the Laffer curve is inversely u-shaped, the lower (higher) tax rate has to increase (decrease) more if the tax rates are on the increasing (decreasing) side of the Laffer curve (equation (36) confirms this for the linear case). That means, resorting to the adaptation technology will reduce the wedge which was driven between the tax rates to internalize the environmental damage.<sup>5</sup>

The most important insight from this analysis is that the effect of adaptation on taxes and public debt seems to run counter the implications from the general analysis. When the government spends effort on adapting to the consequences of climate change, the detrimental effect of pollution on welfare will be alleviated. Hence, the emphasis of environmental taxes shifts from preventing consumption of the polluting good towards funding public expenditures. As elaborated above, empirical evidence suggests that tax rates on the increasing side of the Laffer curve may be more relevant in practice. Thus, consider first the cases with  $\tau_2 < \tau_1 \cap R'(\tau) > 0$ , where optimal public debt was positive before the introduction of the technology. Then debt may still grow but always by less than the increase in effort spendings. This result is contingent on a higher marginal

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<sup>5</sup>Comparative statics show that a comparable shock to exogenous spendings,  $g_1$ , would cause the opposite effect on optimal debt. Since exogenous spendings are not productive, i.e., do not reduce environmental damages, they do not lower the incentive to internalize pollution.

damage in the second period which, as stated before, can hold when the environment reaches a critical tipping point. Yet, especially when thinking about greenhouse gases, we might rather assume  $D'_1 + \gamma D'_2 > D'_2$  because their longevity once emitted to the atmosphere will drive  $\gamma$  close to one. But then our findings from section 3 require a lower level of public debt than when there is no environmental externality. Even if the exogenous spending requirements were higher in the first period,  $g_1 > g_2$ , we would have to conclude that environmental concerns restrict the public deficit or even mandate the accumulation of savings. However, when we introduce the adaptation technology, the government may again maintain a higher optimal level of debt because (i) the investment requirements for effort  $e$  will be equally distributed across both periods and (ii) the difference between tax revenues,  $R(\tau_1) - R(\tau_2)$ , becomes smaller again. The insight gained from this case is especially appealing not just because it depends on credible assumptions about the relationship between marginal damages from pollution and empirically relevant properties of the Laffer curve. Even more so, when the tax is lower in the later period,  $\tau_1 > \tau_2$ , environmental policy can effectively reduce pollution because households do not want to 'overconsume' the polluting good  $x$  today in anticipation of future tax increases.

In summary, we find that public investments in an adaptation technology will reduce the spread between the optimal tax rates which was initially created by internalizing the environmental damages. The better the technology, the more our results may resemble the optimal tax structure under the tax smoothing regime.

## 6 Conclusion

to be completed

## Appendix

**Determining (14).** If  $z_1 = z_2 = 0$ , the bordered Hessian is

$$H = \begin{pmatrix} L_{\lambda_1\lambda_1} & L_{\lambda_1\lambda_2} & L_{\lambda_1b} & L_{\lambda_1\tau_1} & L_{\lambda_1\tau_2} \\ L_{\lambda_2\lambda_1} & L_{\lambda_2\lambda_2} & L_{\lambda_2b} & L_{\lambda_2\tau_1} & L_{\lambda_2\tau_2} \\ L_{b\lambda_1} & L_{b\lambda_2} & L_{bb} & L_{b\tau_1} & L_{b\tau_2} \\ L_{\tau_1\lambda_1} & L_{\tau_1\lambda_2} & L_{\tau_1b} & L_{\tau_1\tau_1} & L_{\tau_1\tau_2} \\ L_{\tau_2\lambda_1} & L_{\tau_2\lambda_2} & L_{\tau_2b} & L_{\tau_2\tau_1} & L_{\tau_2\tau_2} \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & 1 & x_1 + \tau_1 X'_1 & 0 \\ 0 & 0 & -1 & 0 & x_2 + \tau_2 X'_2 \\ 1 & -1 & 0 & 0 & 0 \\ x_1 + \tau_1 X'_1 & 0 & 0 & L_{\tau_1\tau_1} & -\gamma D''_2 X'_1 X'_2 \\ 0 & x_2 + \tau_2 X'_2 & 0 & -\gamma D''_2 X'_1 X'_2 & L_{\tau_2\tau_2} \end{pmatrix}$$

where in the second row we calculate the second derivatives of the Lagrangian from (7)–(11). Calculating the determinant of  $H$  with standard methods gives (14).

**Derivation of (31).** We reformulate the social planner's new Lagrangian as

$$L = \sum_{t=1,2} \left\{ V[X(\tau_t)] + 1 + z_t - (1 + \tau_t)X(\tau_t) \right\} - D_1[X(\tau_1)] - D_2[\gamma X(\tau_1) + X(\tau_2), e]$$

$$+ \lambda_1[\tau_1 X(\tau_1) + b - g_1 - z_1 - e] + \lambda_2[\tau_2 X(\tau_2) - b - g_2 - z_2]. \quad (39)$$

The first order conditions are then given by

$$L_{\tau_1} = -X(\tau_1) - \left\{ D_x^1[X(\tau_1)] + \gamma D_x^2[\gamma X(\tau_1) + X(\tau_2), e] \right\} X'(\tau_1)$$

$$+ \lambda_1 \left[ X(\tau_1) + \tau_1 X'(\tau_1) \right] = 0, \quad (40)$$

$$L_{\tau_2} = -X(\tau_2) - D_x^2[\gamma X(\tau_1) + X(\tau_2), e] X'(\tau_2) + \lambda_2 \left[ X(\tau_2) + \tau_2 X'(\tau_2) \right] = 0, \quad (41)$$

$$L_e = -D_e^2[\gamma X(\tau_1) + X(\tau_2), e] - \lambda_1 \leq 0, \quad e \geq 0, \quad e L_e = 0, \quad (42)$$

$$L_{\lambda_1} = \tau_1 X(\tau_1) + b - g_1 - z_1 - e \geq 0, \quad \lambda_1 \geq 0, \quad \lambda_1 L_{\lambda_1} = 0. \quad (43)$$

Again, restricting attention to interior solutions with  $e$  strictly greater than zero, we may substitute  $\lambda = -D_e^2$  and rewrite equations (40) and (41) as

$$\Gamma = X_1 + (D_x^1 + \gamma D_x^2) X'_1 + D_e^2 (X_1 + \tau_1 X'_1) = 0, \quad (44)$$

$$\Delta = X_2 + D_x^2 X_2' + D_e^2 (X_2 + \tau_2 X_2') = 0. \quad (45)$$

Differentiating equations (44) and (45), we find

$$\Gamma_{\tau_1} = X_1' + (D_x^1 + \gamma D_x^2) X_1'' + (D_{xx}^1 + \gamma^2 D_{xx}^2) X_1'^2 + D_e^2 (2X_1' + \tau_1 X_1''), \quad (46)$$

$$\Gamma_{\tau_2} = \gamma D_{xx}^2 X_1' X_2', \quad (47)$$

$$\Gamma_e = D_{ee}^2 (X_1 + \tau_1 X_1') = D_{ee}^2 R'(\tau_1), \quad (48)$$

$$\Delta_{\tau_1} = \gamma D_{xx}^2 X_1' X_2', \quad (49)$$

$$\Delta_{\tau_2} = X_2' + D_x^2 X_2'' + D_{xx}^2 X_2'^2 + D_e^2 (2X_2' + \tau_2 X_2''), \quad (50)$$

$$\Delta_e = D_{ee}^2 (X_2 + \tau_2 X_2') = D_{ee}^2 R'(\tau_2). \quad (51)$$

From the definition of the consumption and damage functions we know that  $\Gamma_{\tau_2}$  and  $\Delta_{\tau_1}$  have to be (weakly) larger than zero. The signs of the derivatives with regard to  $e$  follow directly from which side of the Laffer curve we want to analyze. Finally, to receive the marginal effects of an increase in effort on the tax rates we take the total differentials of (44) and (45)

$$0 = \Gamma_{\tau_1} d\tau_1 + \Gamma_{\tau_2} d\tau_2 + \Gamma_e de, \quad (52)$$

$$0 = \Delta_{\tau_1} d\tau_1 + \Delta_{\tau_2} d\tau_2 + \Delta_e de. \quad (53)$$

Rearranging a substituting these two equations, we receive

$$\frac{\partial \tau_1}{\partial e} = \frac{\Gamma_{\tau_2} \Delta_e - \Delta_{\tau_2} \Gamma_e}{\Delta_{\tau_2} \Gamma_{\tau_1} - \Gamma_{\tau_2} \Delta_{\tau_1}}, \quad \frac{\partial \tau_2}{\partial e} = \frac{\Delta_{\tau_1} \Gamma_e - \Gamma_{\tau_1} \Delta_e}{\Delta_{\tau_2} \Gamma_{\tau_1} - \Gamma_{\tau_2} \Delta_{\tau_1}}. \quad (54)$$

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