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# An Elementary Theory of Endogenous Technical Change and Wage Inequality\*

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This paper presents general results on the effect of endogenous technical change on relative wages in a simple general equilibrium model that covers several more elaborate models of technology choice. The results are based on a simple and intuitive notion of complementarity between technology and labor inputs, and characterize necessary and sufficient conditions for central results from the literature on endogenous technical change. The developed theory generalizes existing work to technologies that are not purely factor-augmenting, production functions that are not differentiable in technology, and settings with more than two (and potentially infinitely many) different types of labor. Applying the theory to assignment models of the labor market, the paper derives new results on the endogenous determination of automation technology, both in a closed economy and in a two country setting with international trade, and sketches directions for future research on endogenous technical change in assignment models.

**JEL:** J24, J31, O33, **Keywords:** Endogenous Technical Change, Wage Inequality, Factor Demand, Automation, Assignment Model, Monotone Comparative Statics

## 1. Introduction

It is by now widely accepted in the empirical literature on wage inequality that technical change is an important cause of the changes in wage inequality observed over the last three to four decades in many developed economies.<sup>1</sup> While many economic models treat technology

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<sup>1</sup>Early proponents of this view are Bound and Johnson (1992) and Katz and Murphy (1992), who argue that technical change is the major reason for the increase of the wage gap between US college and high school graduates in the 1980s. More recent work proposes and supports the idea that in many industrialized countries the rise of computer technologies during the last decades reduced wage growth especially for workers in routine intensive occupations, who are mostly located in the middle of the wage distribution (see Autor, Levy and Murnane, 2003; Autor, Katz and Kearney, 2006; Goos and Manning, 2007; Autor and Dorn, 2013; Michaels, Natraj and Van Reenen, 2014). Even more recently, Graetz and Michaels (2017), Acemoglu and Restrepo (2017), and Dauth, Findeisen, Suedekum and Woessner (2017) find effects of robotic technologies on employment and wages.

as exogenous, there are good reasons to believe that it is at least partially endogenous to the economic system: profit incentives likely affect which of the set of already developed technologies a firm uses and which type of new technologies research and development efforts are directed to.

The theory of endogenous technical change starts exactly from the assumption that production technologies are determined by profit incentives. So far, the theory has uncovered several interesting interactions between technology and the structure of labor supply, and applied them in various settings.<sup>2</sup> The central results of the theory, however, rely on highly specific functional forms, most notably factor-augmenting technologies, and other strong assumptions, arguably without a justification other than formal tractability.

Here I propose a general theory of the interaction between labor supply, technology, and relative wages, which generates necessary and sufficient conditions for the central results of the literature in a general class of models.

My results show that the central results of the existing literature hold under much wider conditions than previously thought, lending credibility to existing applications. Moreover, they are based on simple and intuitive concepts, and thus improve our understanding of the basic economic forces behind the central effects of endogenous technical change. Finally and most importantly, by going beyond factor-augmenting technologies my results become applicable to a range of settings that so far have largely been outside the scope of endogenous technical change theory. I demonstrate the last point by making a first step in the exploration of endogenous technical change effects in assignment models of the labor market, which I consider one of the most promising fields for future application of the theory.

A notable exception to the specificity of existing work on endogenous technical change is given by [Acemoglu \(2007\)](#), who presents general results for the effects of endogenous technical change on absolute rather than relative wages. From a theory perspective, one may view my work as the completion of a general theory of the effects of endogenous technical change on wages, with the first part on absolute wages given by [Acemoglu \(2007\)](#).

I analyze a simple static general equilibrium model with endogenous production technology that, as shown by [Acemoglu \(2007\)](#), covers (reduced forms of) many more elaborate models of technology choice.

The first central result of the literature is that with two labor inputs (say, high- and low-skilled labor), purely factor-augmenting technology, and a homotheticity assumption on aggregate production, any local increase in the relative supply of high-skilled labor induces skill-biased technical change. My first result shows that the assumption of factor-augmenting technologies can be dropped without replacement. Indeed, I also allow for non-local (or non-infinitesimal) changes in relative skill supply, superseding differentiability assumptions, which leaves only the homotheticity assumption in place.

This surprisingly general result carries a simple intuition: any increase in relative labor sup-

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<sup>2</sup>The seminal papers on endogenous technical change, [Acemoglu \(1998\)](#) and [Kiley \(1999\)](#), analyze a model with endogenous factor-augmenting technology to propose that the skill biased technical change supposedly responsible for the observed increase in the US skill premium is (partly) driven by the increasing relative supply of skilled labor. Later work explores the interaction of endogenous factor-augmenting technology with international trade ([Acemoglu, 2003](#); [Thoenig and Verdier, 2003](#)) and offshoring ([Acemoglu, Gancia and Zilibotti, 2015](#)).

ply increases the return to technologies that are complementary to the relative labor input, and, by symmetry of complementarity relations, using such technologies more intensively increases the relative wage. Remarkably, the proof of my first main result is almost a one-to-one translation of this intuition into formal math.

The second central existing result says that, again with two labor types and factor-augmenting technologies, the effect of the skill-biased technical change induced by an increase in relative skill supply dominates the direct substitution effect – such that the skill premium increases in response to an increase in relative skill supply – if and only if the elasticity of substitution between high- and low-skilled labor is sufficiently large. Again dropping factor augmentation, I give a generalization of this condition, which sheds new light on the type of environment in which it can be satisfied. In particular, I show that the induced technical change effect dominates if and only if there is a certain form of non-convexity in the aggregate production possibilities set of the economy. Most notably, this requires that the equilibrium technology is not the result of a joint maximization of production over technology and labor. In consequence, the technical change effect cannot dominate in models where firms choose labor demand and production technologies simultaneously and independently of the technology decisions of other firms.

My final set of results gives a natural extension of the previous two results to settings with an arbitrary number of labor types. I identify conditions under which a uniform increase of relative supply over the type space – that is, every relative supply of a higher versus a lower labor type goes up (where the order on types, that is, the definition of which of a given pair of types is higher and which is lower, can be chosen arbitrarily) – induces technical change uniformly biased towards higher labor types – that is, the induced technical change increases every relative wage of a higher versus a lower type. The result is a strict generalization of the corresponding results for the two type case and is based on the same intuitive concepts. It allows to take the perspective of endogenous technical change to changes in wage inequality that occur at a higher level of disaggregation than captured by a two type model. In particular, it applies to supply changes that are potentially non-monotone along the wage distribution, such as polarizing changes.

I use the developed theory to derive new results on endogenous automation technology – defined as technology that allows for a direct substitution of labor by machines in certain tasks – in a setting with many labor types.<sup>3</sup> For this, I incorporate machines as an additional production factor in the canonical continuum type assignment model by [Teulings \(1995\)](#), drawing on recent progress in comparative statics analyses for this type of model by [Costinot and Vogel \(2010\)](#). In line with recent forecasts on the future automation potential of different tasks (see for example [Frey and Osborne, 2017](#)), I assume that machines have comparative advantage in less complex tasks than labor, such that any increase in the set of tasks performed by machines (automation) displaces low-skilled labor from some of its previous tasks.

My first result shows that, with a fully endogenous productivity schedule for machines across tasks, any uniform increase in relative skill supply induces automation, which in turn uni-

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<sup>3</sup>The results complement those of [Acemoglu and Restrepo \(2016\)](#), who analyze the dynamic evolution of automation technology and its response to exogenous technology shocks rather than its response to changes in the structure of labor supply.

formly increases skill premia over the entire type space. This extends the well-known hypothesis proposed by [Acemoglu \(1998\)](#), that the joint upwards trend of relative skill supply and skill premium observed in the US since the 1980s may reflect the response of technology to the increasing relative supply of skill, from models with purely factor-augmenting technologies to a model with endogenous automation technology. The result is important because, as noted for example in [Acemoglu and Autor \(2011\)](#), the model with purely factor-augmenting technology from [Acemoglu \(1998\)](#) is not consistent with another important empirical fact on the evolution of the wage distribution over this period: the stagnation or even decline of low-skilled workers' real wages. The automation model, in contrast, can produce such absolute wage declines for low-skilled workers. Relatedly, I also show that automation can never reduce the average wage across all workers (after adjustment of the machine stock), which gives a general version of the productivity effect of automation discovered in a more specific framework by [Acemoglu and Restrepo \(2016\)](#).

Second, I extend the analysis to the two country setting with trade in tasks studied in [Costinot and Vogel \(2010\)](#). It turns out that only the world supply of labor, obtained by aggregating efficiency units of different labor types across countries, determines the bias of technology. In consequence, a sufficiently strong increase in the relative skill supply in a skill scarce country – for example in an emerging economy like China – induces automation in a skill abundant country linked to the skill scarce country via trade in tasks.

Finally, I discuss two directions for future applications of the developed theory. First, moving beyond the assumption that machines always have comparative advantage in less complex tasks relative to all labor types appears a promising path for future research on endogenous technical change in assignment models.<sup>4</sup> Second, my theoretical results facilitate analyzing the implications of endogenous technical change for redistributive policies such as (non-linear) income taxes, which I pursue in a companion paper.

The structure of the paper is as follows. The next section describes the static model used to derive the main results, states the most general existing results on the effect of endogenous technical change on relative wages (Section 2.1), and derives the main results of the paper, first in the two type setting (Sections 2.2 and 2.3), then for multiple types (Section 2.4). I apply the results to assignment models in Sections 3.1 (closed economy) and 3.2 (two country setting with international trade). Section 4 discusses directions for future applications of the theory and Section 5 concludes.

## 2. A General Theory of Relative Equilibrium Bias of Technology

### 2.1. Setup and Previous Results

I analyze a static general equilibrium model with workers who supply different types of labor and consume a single consumption good, and a firm sector that demands labor and transforms it into the consumption good. The production technology that transforms labor into the consumption good is endogenous.

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<sup>4</sup>This point is due to Daron Acemoglu, with whom I am currently working on an endogenous technical change analysis in assignment models without the fixed comparative advantage assumption.

I take labor supply as exogenous, so workers make no meaningful decisions. Aggregate supply of labor of type  $i \in I$  is denoted by  $L_i$ ,  $I$  being the set of labor types.

I assume that the firm sector admits an aggregate production function  $F(L, \theta)$ , with  $L$  collecting the labor inputs of all different types and  $\theta$  denoting technology. [Acemoglu \(2007\)](#) shows that different models of technology choice in the firm sector lead to the result that the equilibrium technology maximizes aggregate production.<sup>5</sup> Therefore, instead of settling for a specific model of technology choice, I simply assume that the equilibrium technology is given by  $\theta^*(L) := \sup \arg\max_{\theta \in \Theta} F(L, \theta)$ , where  $\Theta$  is the space of all feasible technologies.<sup>6</sup> I present several more detailed models of the firm sector from [Acemoglu \(2007\)](#), all consistent with the aggregate assumptions made here, in Appendix A.

Labor markets are assumed to be perfectly competitive, so wages equal marginal products, that is, denoting the wage of labor type  $i$  by  $w_i$ ,  $w_i(L, \theta) = \partial F(L, \theta) / \partial L_i$ .

Finally, I impose the following assumptions on the aggregate production function to ensure that wages and the equilibrium technology are always well defined.

**Assumption 1.** *The aggregate production function is strictly increasing and continuously differentiable in  $L$ , and continuous in  $(L, \theta)$ . The space of feasible technologies  $\Theta$  is a compact topological space.*

The focus of the paper is on the effects of technical change on relative wages induced by changes in relative labor supply. With two types of labor,  $L_1$  and  $L_2$ , these effects are conveniently summarized by the notions of weak and strong relative equilibrium bias of technology (see [Acemoglu, 2007](#)).

**Definition 1.** There is weak relative equilibrium bias of technology at labor supply  $L$  if

$$\frac{w_1}{w_2}(L', \theta^*(L)) \leq \frac{w_1}{w_2}(L', \theta^*(L'))$$

for any  $L'$  with  $L_1/L_2 \leq L'_1/L'_2$ .

**Definition 2.** There is strong relative equilibrium bias of technology at labor supply  $L$  if

$$\frac{w_1}{w_2}(L, \theta^*(L)) \leq \frac{w_1}{w_2}(L', \theta^*(L'))$$

for any  $L'$  with  $L_1/L_2 \leq L'_1/L'_2$ .

In words, there is weak relative equilibrium bias of technology if any increase in the relative supply of labor type 1 versus labor type 2 induces technical change that increases the relative

<sup>5</sup>[Acemoglu \(2007\)](#) considers three different models of technology choice: (i) each final good firm chooses its technology simultaneously with its labor inputs from a given set of feasible technologies; (ii) a technology monopolist supplies technology embodied in an intermediate good to all final good firms; (iii) oligopolistic technology firms supply technology embodied in differentiated varieties of intermediate goods to all final good firms. For more details see Appendix A.

<sup>6</sup>The supremum is taken for each of the results of the following sections with respect to the order defined in the corresponding section. Whenever no such order is defined,  $\theta^*$  is just an arbitrary unique selection out of the set of maximizers. Having a unique equilibrium technology is convenient for the results of the next sections, as it gives rise to a unique equilibrium wage distribution, but similar results could be obtained for sets of equilibrium technologies and the corresponding sets of relative wages.



wage of labor type 1 versus type 2, that is, technical change biased towards labor type 1. Alternatively, the type 1 bias of technology increases in the relative supply of labor type 1. Strong relative equilibrium bias of technology requires that the effect of the induced technical change dominates the standard substitution effect between the two types, such that the total effect of an increase in relative supply on the relative wage is positive. Alternatively, the endogenous technology relative wage increases in relative supply.<sup>7</sup>

To compare my results directly with previous work, I also define local versions of weak and strong relative bias.<sup>8</sup> These local versions require that technologies are represented by real vectors to make use of derivatives.

**Definition 3.** Suppose  $\Theta \subset \mathbb{R}^N$  and  $\theta^*$  is homogeneous of degree zero in  $L$ . Then, there is local weak relative bias of technology at labor supply  $L$  if

$$\nabla_{\theta} \frac{w_1}{w_2}(L, \theta^*(L)) \cdot \frac{d\theta^*(L)}{dL_1/L_2} \geq 0.$$

**Definition 4.** Suppose  $\Theta \subset \mathbb{R}^N$  and  $w_1/w_2(L, \theta^*(L))$  is homogeneous of degree zero in  $L$ . Then, there is local strong relative equilibrium bias of technology at labor supply  $L$  if

$$\frac{d}{dL_1/L_2} \frac{w_1}{w_2}(L, \theta^*(L)) \geq 0.$$

For the derivatives with respect to relative supply to be well defined, the definitions of local relative bias require that the equilibrium technology and the relative wage are functions of relative labor supply only.<sup>9</sup> Instead of discussing these requirements for each of the results on local relative bias separately, I give a simple sufficient condition for them here.

**Condition 1.** The aggregate production function  $F(L, \theta)$  can be written as the composition of an inner function  $f(L, \theta)$  that is linear homogeneous in  $L$  and an outer function  $g(f, L)$  that is strictly increasing in  $f$ .

Condition 1 ensures that the equilibrium technology does not change with proportional changes in labor supply, that is, the equilibrium technology can be written as a function of relative supply only. If in addition  $L$  does not enter the outer function  $g$  directly, the same holds for the endogenous technology relative wage. Note that homogeneity of  $F$  in  $L$  is sufficient for Condition 1, even if the outer function must be independent of  $L$ .

Equipped with the above definitions, we can recapitulate the most general existing results on relative bias. These result are restricted to settings with factor-augmenting technologies and twice continuously differentiable production function.

<sup>7</sup>I use the term endogenous technology relative wage to refer to the relative wage  $w_1/w_2(L, \theta^*(L))$  as a function of  $L$ , taking into account the total effect of  $L$  on the relative wage (the partial effect plus the effect via technology). Similarly, I refer to the aggregate production function  $F(L, \theta^*(L))$  as a function of  $L$  only (taking into account the total effect of  $L$  on  $F$ ) as the endogenous technology production function.

<sup>8</sup>For brevity I will often refer to weak (strong) relative equilibrium bias of technology as weak (strong) relative bias.

<sup>9</sup>It would also be possible to give a slightly wider definition of local weak relative bias, requiring that the directional derivative of technology in any direction that increases relative labor supply has a positive effect on the relative wage. The gains of doing so, however, do not appear to justify the additional notation. I comment on this possibility at the end of Section 2.2.

**Result 1** (Theorem 1, [Acemoglu 2007](#)). Suppose that  $\Theta = \mathbb{R}_+^2$ ,  $F$  can be written as  $F(L, \theta) = G(\theta_1 L_1, \theta_2 L_2) - C(\theta)$  with  $G$  and  $C$  twice continuously differentiable and homothetic,  $G$  strictly increasing and concave, and  $C$  strictly decreasing and strictly convex. Then:

1. There is local weak relative equilibrium bias of technology at any labor supply  $L$ .
2. There is local strong relative equilibrium bias of technology at labor supply  $L$  if and only if  $\sigma(L) - 2 - \delta(L) > 0$ , where  $\sigma(L)$  is the elasticity of substitution between the two arguments of  $G$  at the point  $(\theta_1^*(L)L_1, \theta_2^*(L)L_2)$  and  $\delta(L)$  is the inverse of the elasticity of substitution between the two arguments of  $C$  at the point  $\theta^*(L)$ .

In the following, I generalize these results to settings without specific functional form restrictions, without differentiability of aggregate production in technology, and provide a natural extension to settings with multiple labor types.

## 2.2. Weak Relative Equilibrium Bias of Technology

The first main result of the paper identifies minimal conditions for weak relative bias according to Definition 1. It is based on the idea that an increase in relative supply of labor type 1 versus type 2 should induce technical change that is complementary to the relative supply increase. Since weak relative bias is defined for supply changes that span the entire labor supply space and complementarity relationships may in principle change over the space, the result will require complementarity relationships to be stable.

To make the notion of stable complementarity relationships precise, I define complementarity by ordering technologies according to their bias towards labor of type 1.

**Definition 5.** For any two technologies  $\theta, \theta' \in \Theta$ ,  $\theta \preceq_s \theta'$  if and only if  $w_1/w_2(L, \theta) \leq w_1/w_2(L, \theta')$  for all labor inputs  $L$ .

Stability of complementarity relationships now requires that a technology  $\theta$  that is more complementary to relative labor supply (equivalently: more biased towards labor type 1) than another technology  $\theta'$  at some labor supply  $L$  is also more complementary to relative supply than  $\theta'$  at any other labor supply  $L'$ . This simply means that any two technologies can be compared under the order  $\preceq_s$ , so  $\preceq_s$  must be a total order.

**Condition 2.** The order  $\preceq_s$  is total on the technology space  $\Theta$ , that is, for any two technologies  $\theta, \theta'$  either  $\theta \preceq_s \theta'$  or  $\theta' \preceq_s \theta$ .

The main result on weak relative bias now says that there is weak relative bias at every labor supply  $L$  whenever complementarity relationships are stable and the relationship between technical change and relative labor supply is well defined.

**Theorem 1.** Suppose  $\theta^*(L)$  can be written as  $\theta^*(L_1/L_2)$  and complementarity relationships are stable as by Condition 2. Then there is weak relative equilibrium bias of technology at any labor supply  $L$ .

Theorem 1 simply states that the function  $\theta^*$  is increasing, that is, any increase in relative supply increases the type 1 bias of technology (increases technology under the order  $\preceq_s$ ). The



result is both surprisingly general and surprisingly simple in view of the restricted scope of existing results.

Remarkably, neither of the two conditions for weak relative bias can be relaxed in a substantial way. When the equilibrium technology is not a function of relative supply only, a proportional change in supply of both labor types may induce biased technical change, which immediately contradicts weak relative bias.<sup>10</sup> When complementarity relationships are not stable, it may be possible to construct a supply change that induces biased technical change in line with weak relative bias when evaluated at the initial labor supply, but not at the eventual labor supply as required by weak relative bias.<sup>11</sup> I give examples for violations of each of the two conditions at the end of this subsection.

The need for stability of complementarity relationships only arises in Theorem 1 because weak relative bias covers changes across the entire labor supply space. Once restricting attention to local changes, complementarity relationships are automatically stable, which means that for local weak relative bias, Condition 2 can be dropped.<sup>12</sup>

**Corollary 1.** *Suppose  $\theta^*(L)$  can be written as  $\theta^*(L_1/L_2)$ ,  $\Theta$  is a subset of  $\mathbb{R}^N$ ,  $F$  is  $C^2$ , and  $\theta^*$  is differentiable. Then there is local weak relative equilibrium bias of technology at any labor supply  $L$ .*

So, there is local weak relative bias whenever it is well defined according to Definition 3.<sup>13</sup>

The results on weak relative bias are closely related to the results on weak absolute bias in Acemoglu (2007), where weak absolute bias means that an increase in a single type's labor supply induces technical change that increases this type's absolute wage. Indeed the absolute bias results can easily be derived along the lines of the preceding discussion of relative bias. In particular, Acemoglu's Theorem 3 states that there is weak absolute bias if there is an order on the technology space such that aggregate production is supermodular in technology and exhibits increasing differences in technology and the relevant labor type.<sup>14</sup> Choosing the order – analogously to the order  $\preceq_s$  above – such that any increase in technology increases the type's absolute wage, we get increasing differences for free (that is, without assuming it explicitly). Then, supermodularity – analogously to quasisupermodularity in Theorem 4 in Appendix B – ensures that complementarity relationships are stable on the set of potential equilibrium

<sup>10</sup>Indeed, the condition that a proportional change in labor supply does not change the equilibrium technology can be replaced by the marginally weaker condition that a proportional change does not induce a biased change in the equilibrium technology.

<sup>11</sup>Instability of complementarity relationships is actually only a problem if it concerns technologies that may be chosen in equilibrium, that is, technologies in the image of  $\theta^*$ . This observation leads to a slightly more general version of Theorem 1, which I present in Appendix B.

<sup>12</sup>Local changes in technology are represented by direction vectors in the technology space. Along these direction vectors the type 1 bias of technology either increases or decreases, so each direction vector (and thus each local change) can be unambiguously classified as complementary to type 1 biased technical change or not.

<sup>13</sup>One may want to adopt a slightly wider definition of local weak relative bias, which drops the requirement that  $\theta^*$  is a function of relative supply only and replaces the derivative of  $\theta^*$  with respect to relative supply with the directional derivative of  $\theta^*$  in any direction  $v$  that increases relative supply. With this definition, we could replace the condition on  $\theta^*$  in Corollary 1 with the marginally weaker condition that a proportional change in labor supply does not induce biased technical change, that is  $w_1/w_2(\bar{L}, \theta^*(L))$  is homogeneous of degree zero in  $L$  (holding  $\bar{L}$  fixed). It is easy to check that this condition would then not only be sufficient but also necessary for local weak relative bias according to the wider definition.

<sup>14</sup>Supermodularity means that  $F(L, \bar{\theta}) + F(L, \underline{\theta}) \geq F(L, \theta) + F(L, \theta')$  for any two technologies  $\theta$  and  $\theta'$ , where  $\bar{\theta}$  and  $\underline{\theta}$  are their supremum and infimum. Moreover,  $F$  has increasing differences in  $(L_i, \theta)$  if  $F(L'_i, L_{-i}, \theta) - F(L_i, L_{-i}, \theta)$  increases in  $\theta$  for any  $L_i \leq L'_i$ .

technologies.<sup>15</sup> One difference between the conditions for weak relative and absolute bias is that those for relative bias are purely ordinal – that is, invariant to monotone transformations of aggregate production – while those for absolute bias in Acemoglu (2007) are not. The reason is clearly that absolute wages respond to such transformations but relative wages do not.

To close the subsection, I consider two examples in each of which one of the two conditions for weak relative bias is violated.

**Example 1** (Capital-Skill Complementarity). Consider a model with skilled labor  $L_1$ , unskilled labor  $L_2$ , and capital  $K$ , and let capital take the role of technology for the moment. Capital is produced at a constant cost of  $\kappa$  units of output, such that aggregate production becomes  $F(L_1, L_2, K) = G(L_1, L_2, K) - \kappa K$ , where we make the standard assumption that  $G$  is linear homogeneous and concave. We assume in addition that capital is complementary to relative skill supply, such that the skill premium  $w_1/w_2$  increases in capital (so,  $K \preceq_s K'$  if and only if  $K \leq K'$ ). Now, linear homogeneity and concavity of  $G$  imply that a proportional increase in  $L_1$  and  $L_2$  must increase the marginal product of capital and hence  $K^*$ , the capital level that maximizes aggregate production. It follows that the equilibrium technology  $K^*$  is not a well defined function of relative labor supply, so the conditions for weak relative bias are violated. Indeed, the capital adjustment induced by a proportional change in labor supply increases the skill premium, which directly contradicts weak relative bias.

Nevertheless, a small change in the setup allows to give an impression of how widely applicable the above results are. Suppose there is another (technology) variable  $\theta$  and aggregate production becomes  $F(L_1, L_2, K, \theta) = G(L_1, L_2, K, \theta) - \kappa K$ . Suppose also that we maintain the assumption that  $G$ , and thereby  $F$ , is linear homogeneous in  $(L_1, L_2, K)$  (for concreteness, think of  $\theta$  as indexing different production functions, each linear homogeneous in labor and capital, that differ in the extent to which capital is skill biased). Now, we can write down a reduced form production function  $\tilde{F}(L_1, L_2, \theta) := F(L_1, L_2, K^*(L_1, L_2, \theta), \theta)$ , which clearly is linear homogeneous in  $(L_1, L_2)$ . Although we have no idea about the specific form of the reduced form production function – and even with a specific function  $G$  to start with, for example a nested CES function, we would often not be able to derive an explicit solution for the reduced form – all results from above are applicable, so we know that there is always local weak relative bias and there is (global) weak relative bias whenever complementarity relationships between technology and relative labor supply are stable.

**Example 2** (Choice of  $\sigma$ ). Second, consider a model in which skilled labor  $L_1$  and unskilled labor  $L_2$  are combined via a CES production function, and let the elasticity of substitution  $\sigma$  take the role of technology. The elasticity of substitution is chosen from  $\mathbb{R}_+$ , with greater values, that is, greater flexibility in production, coming at a higher cost as given by the increasing cost function  $C(\sigma)$ . Aggregate production can then be written as

$$F(L_1, L_2, \sigma) = (1 - C(\sigma)) \left( \alpha L_1^{\frac{\sigma-1}{\sigma}} + (1 - \alpha) L_2^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}.$$

<sup>15</sup>More directly, supermodularity and increasing differences ensure that the equilibrium technologies chosen at different levels of labor supply can be compared with each other under the technology order, which then gives stability of complementarity relationships, analogous to Condition 2, on the set of equilibrium technologies.)

Clearly,  $F$  is linear homogeneous in labor, so we know immediately that there is local weak relative bias everywhere.

Globally however, weak relative bias fails because the complementarity relationship between the elasticity of substitution and relative labor supply is not stable. Concretely, the skill premium  $w_1/w_2$  is given by  $\alpha/(1-\alpha)(L_2/L_1)^{(1/\sigma)}$  such that  $\sigma$  is skill biased (or complementary to relative skill supply) if and only if  $L_1$  is greater than  $L_2$ . It follows that no pair of different elasticities of substitution can be compared under the order  $\preceq_s$ .

Indeed, we can easily construct a change in relative labor supply for which weak relative bias fails. Suppose the relative skill supply is initially far below one and changes to a value close to but above one. The value of flexibility is high in the initial situation, because there are far more unskilled than skilled workers, but low after the change when both types are in similar supply. The change thus induces a fall in the elasticity of substitution, which, at the new labor supply, reduces the skill premium because the relative skill supply is greater than one after the change. So, an increase in relative skill supply induces technical change that, at the new relative supply level, is biased against skilled labor, a contradiction to weak relative bias.

### 2.3. Strong Relative Equilibrium Bias of Technology

The main result on strong relative bias provides a tight link (in terms of necessary and sufficient conditions) between strong relative bias and a certain form of non-convexity in the aggregate production possibilities set. While the condition for strong relative bias may not always be easy to check and thus may not seem a great advance over testing strong relative bias by direct computation in applications, the condition provides interesting insights about the type of model strong relative bias can arise in.

**Theorem 2.** *Suppose  $w_1/w_2(L, \theta^*(L))$  can be written as a function of  $L_1/L_2$  only. Then, there is strong relative equilibrium bias of technology at labor supply  $\bar{L}$  if and only if the endogenous technology production function  $F(L, \theta^*(L))$  is quasiconvex with minimum at  $\bar{L}$  on the line tangent to the isoquant at  $\bar{L}$ .*

The main insight from Theorem 2 is that strong relative bias is not possible when the equilibrium technology is the result of a joint maximization of production over labor and technology. More concretely, it is not possible whenever (i) there are identical firms with production functions  $F$ , which (ii) choose their technologies independently from each other. In particular, the simplest model of technology choice in Appendix A cannot feature strong relative bias, but as soon as allowing for spillovers in technology choices across firms strong relative bias becomes possible (see Appendix A for details). All other models presented in Appendix A allow for this possibility as well. The dependence on non-convexities in the aggregate production possibilities set is not easily anticipated from the existing Result 1, which links strong relative bias to a sufficiently large elasticity of substitution in models with purely factor-augmenting technologies. Since both results provide necessary and sufficient conditions, Theorem 2 implies that an aggregate production function with factor-augmenting technologies becomes (locally) non-concave exactly when the elasticity of substitution crosses the threshold given in Result 1. Interestingly, if the substitution elasticity is constantly above this threshold, firms that maxi-

mize jointly over labor and technology inputs (and have production functions identical to the aggregate) choose corner solutions, relying only on one type of labor.

As the main result for weak relative bias has its local version, also Theorem 2 has a counterpart for local strong relative bias, which expresses the non-convexity condition in terms of derivatives.

**Corollary 2.** *Suppose  $w_1/w_2(L, \theta^*(L))$  can be written as a function of  $L_1/L_2$  only,  $\Theta$  is a subset of  $\mathbb{R}^N$ ,  $F$  is  $C^2$ , and  $\theta^*$  is differentiable. Then, there is strong relative equilibrium bias of technology at labor input  $\bar{L}$  locally if and only if the second derivative of the endogenous technology production function  $F(L, \theta^*(L))$  restricted to the line tangent to the isoquant at  $\bar{L}$  is non-negative.*

Instead of parametrizing the endogenous technology production function along the tangent line to the isoquant and considering its second derivative, the condition for local strong relative bias can also be expressed directly in terms of the Hesse matrix of the (unrestricted, exogenous technology) production function. Remark 2 in Appendix B shows that, for local strong relative bias, the Hesse matrix must fail to be negative semi definite in direction of the isoquant.

As in the case of weak relative bias, the results have a close relation to the results on strong absolute bias from Acemoglu (2007). Acemoglu's central theorem, Theorem 4, says that there is local strong absolute bias if and only if the Hesse matrix fails to be negative semi definite, which is slightly weaker than the condition of Remark 2 because it does not require concavity of aggregate production to fail along the isoquant but in an arbitrary direction. One implication is that a production function with, after endogenous technology adjustment, locally increasing returns to scale in labor provides for (local) strong absolute bias (since concavity fails along rays through the origin) but not necessarily for (local) strong relative bias (since concavity may not fail along the isoquant). More specifically and related to the difference between weak absolute and relative bias, strong absolute bias responds to monotone transformations of aggregate production, whereas strong relative bias does not.

## 2.4. Extension to Multiple Inputs

Consider now a setting with multiple labor types, that is, assume  $L$  is an  $M$ -dimensional vector or a measure on a sigma algebra over the interval  $[0, 1]$ . The central result of the section will give sufficient conditions under which a uniform increase in relative supply over the entire type space induces technical change that leads to a uniform increase in relative wages.<sup>16</sup>

We start again by ordering technologies according to their bias towards higher labor types. More concretely, we construct the order  $\preceq_m$  such that a technology is greater than another if and only if it uniformly increases relative wages compared to the other technology.

**Definition 6.** For any two technologies  $\theta, \theta' \in \Theta$ ,  $\theta \preceq_m \theta'$  if and only if  $w_i/w_j(L, \theta) \leq w_i/w_j(L, \theta')$  for all  $i, j$  with  $j \leq i$  and for all labor inputs  $L$ .

<sup>16</sup>By a uniform increase in relative labor supply (over the entire type space), I mean a change in labor supply such that the ratios  $L_i/L_j$  increase for every  $i > j$ . Similarly, a uniform increase in relative wages (over the entire type space) means a change in the wage distribution such that  $w_i/w_j$  increases for every  $i > j$ . Note that the ordering of types is arbitrary throughout the entire section, so all results hold for any given order on the type space.

The main problem in transferring the results from the two type setting is that the measure of wage inequality by which we assess the effect of technical change is no longer a one-dimensional object. Therefore, even if all complementarity relationships are stable – in the sense that the direction in which a given change in technology affects a given relative wage is the same at all points in the labor supply space – the order  $\preceq_m$  is not necessarily total on the technology space. There may exist, for example, technical changes that increase relative wages at the lower end of the type space, but decrease them at the upper end.

A radical solution to this problem would be to restrict the technology space such that it is totally ordered under  $\preceq_m$ , that is, to assume directly that  $\preceq_m$  is a total order. A less restrictive way, which I pursue in the following, is to impose conditions on aggregate production  $F$  to guarantee that uniform increases in relative supply always lead to changes in technology that can be classified under the order  $\preceq_m$ .<sup>17</sup> For that, we must impose some structure on the complementarity relationships between different technologies, rather than only between technologies and labor supply. We will do so by assuming that aggregate production is quasisupermodular in technology.<sup>18</sup>

To see what this means, consider two technologies  $\theta$  and  $\theta'$  that cannot be ordered under  $\preceq_m$ . For concreteness, think of a three type setting, where  $\theta$  implies a greater relative wage between the high and the middle type, but the relative wage between the middle and the low type is greater under  $\theta'$ . Now, let  $\underline{\theta}$  be the infimum of  $\theta$  and  $\theta'$ , and  $\bar{\theta}$  the supremum, that is, under  $\underline{\theta}$  both relative wages (between the high and the middle and between the middle and the low type) are smaller than under both  $\theta$  and  $\theta'$ , while they are greater under  $\bar{\theta}$ . Quasisupermodularity now says that, whenever moving from  $\underline{\theta}$  to  $\theta$ , that is, increasing the relative wage between the high and the middle type at a low relative wage between middle and low type, increases aggregate production, then it must also increase aggregate production to move from  $\theta'$  to  $\bar{\theta}$ , that is, to increase the relative wage between the high and the middle type at a high relative wage between middle and low type. Intuitively, increases in relative wages in different regions of the type space are (weakly) complementary to each other. In short and generally, quasisupermodularity means that, whenever  $F(L, \theta) \geq F(L, \underline{\theta})$ , then it must also hold that  $F(L, \bar{\theta}) \geq F(L, \theta')$ , for any  $\theta, \theta'$  and any  $L$ .<sup>19</sup>

With this complementarity structure, the technical change induced by a uniform increase in relative supply will always affect relative wages uniformly, so we can restrict attention to such forms of technical change. Then, the logic of complementarity between technical change and relative supply changes from the two type setting easily carries over, and we have the

<sup>17</sup>More formally, if we endow the labor supply space with an order analogous to the technology order, such that one labor supply is ordered above another if and only if it features uniformly greater relative supply over the entire type space, then the imposed conditions will guarantee that the image of any chain in the labor supply space under the equilibrium technology  $\theta^*$  is a chain in the technology space.

<sup>18</sup>To define quasisupermodularity in technology, the technology space  $(\Theta, \preceq_m)$  must be a lattice, that is, any two technologies must have a supremum and an infimum in the technology space. This, however, is hardly restrictive, as it does not say anything about the profitability of the supremum and infimum technologies. In principle, we can extend any given technology space to include suprema and infima of all technology pairs in the space without essential loss of generality, by making all added technologies prohibitively unprofitable.

<sup>19</sup>While, as noted before, the ordering of the type space may not coincide with the types' order in the wage distribution, it may be helpful to think about this special case. Then, quasisupermodularity means intuitively that technical change that increases inequality at the lower end of the wage distribution is weakly complementary to technical change that increases inequality at the upper end.

following theorem.

**Theorem 3.** *Let  $F$  be homogeneous in  $L$  and quasisupermodular in  $\theta$ , and let  $(\Theta, \preceq_m)$  be a lattice. Then,*

$$\frac{w_i}{w_j}(L', \theta^*(L)) \leq \frac{w_i}{w_j}(L', \theta^*(L'))$$

*for all  $i, j$  with  $j \leq i$ , and for all  $L, L'$  with  $L_h/L_k \leq L'_h/L'_k$  for all  $k \leq h$ .*

There are two additional points to note about Theorem 3. First, the theorem is a strict generalization of Theorem 1. Second, unlike in the two type case, the local version that could be obtained for the multiple type case does not allow to drop the quasisupermodularity requirement. The reason is that the wage distribution is multidimensional here, which implies that even locally not all directions in the labor supply space can be classified as either uniformly increasing or decreasing relative wages. Formally, this means that the (partial) order  $\preceq_m$  can, unlike the order  $\preceq_s$  in the two type case, not be turned into a total order locally.

### 3. Application to Assignment Models

#### 3.1. Closed Economy

To apply the developed results, we have to check whether aggregate production in the model of interest is homogeneous in labor (or, more generally, satisfies the homotheticity Condition 1) and whether the endogenous technology of interest maximizes aggregate production in equilibrium. Here, I apply the results to assignment models with machines that can directly replace labor in certain tasks to derive new insights about the factors behind the adoption of automation technology. For that, I incorporate machines as an additional production factor in the assignment model by [Costinot and Vogel \(2010\)](#) and endogenize the machine productivity schedule across tasks.<sup>20</sup>

In particular, the model features a continuum of identical firms that combine tasks to produce a single final good according to

$$Y = \left( \int_{\underline{s}}^{\bar{s}} (B(s)Y(s))^\rho ds \right)^{\frac{1}{\rho}}, \quad (1)$$

where  $Y(s)$  denotes the input of task  $s \in S = [\underline{s}, \bar{s}]$ . The final good can either be consumed or transformed into machines at a conversion rate of  $r$  (so one unit final good yields  $1/r$  units machines).

Firms choose whether to employ machines or labor out of a continuum of labor types to perform each of the tasks. In a given task  $s$ , the productivity of labor type  $i \in [0, 1]$  is

<sup>20</sup>This extends the assignment model with labor-replacing machines from [Acemoglu and Autor \(2011\)](#) to feature a continuum of different labor types and a CES instead of a Cobb-Douglas final good aggregator. [Acemoglu and Autor \(2011\)](#) only analyze the effects of endogenous technology on absolute wages, as they only have general results for absolute bias from [Acemoglu \(2007\)](#). My theory of relative bias now allows to derive corresponding results for relative wages. The model is also similar to the automation model of [Acemoglu and Restrepo \(2016\)](#) but does not impose strong restrictions on the shape of the comparative advantage schedules. Moreover, [Acemoglu and Restrepo \(2016\)](#) do not analyze the effect of changes in relative labor supply on the bias of technical change, but the long-run evolution of technology in a dynamic framework.



given by the (exogenous) labor productivity schedule  $A(s, i) > 0$ .<sup>21</sup> The machine productivity schedule, denoted by  $A_c(s) \geq 0$ , is chosen by firms from the set  $\mathcal{A}$ . I assume that, first, labor productivity  $A(s, i)$  strictly increases in  $i$ , so higher labor types have absolute advantage in all tasks and will therefore earn higher wages in equilibrium. Second, higher labor types have comparative advantage over lower types at higher tasks, while, for all machine productivity schedules  $A_c \in \mathcal{A}$ , all labor types have comparative advantage over machines in higher tasks:

$$\frac{A_c(s')}{A_c(s)} \leq \frac{A(s', i)}{A(s, i)} \leq \frac{A(s', i')}{A(s, i')} \quad \text{for all } s \leq s' \text{ and } i \leq i'.$$

The assumption mirrors the results of recent work on the future automation potential of different job tasks. [Frey and Osborne \(2017\)](#), for example, claim that the likelihood of an occupation being automated within the next decades monotonically declines along the occupational wage distribution, with high-paying occupations having the lowest automation probability. Analogously, the assumed comparative advantage schedule implies that machines perform a range of tasks with the lowest indexes, followed by labor types in increasing order; that is, machines and low labor types perform tasks that are direct neighbors in the task space, such that any increase in the range of tasks performed by machines – which I will refer to as automation – displaces low types from their previous tasks.

More formally, we can describe the assignment of production factors to tasks by a threshold task  $\tilde{s}$  (the automation threshold henceforth) and an assignment function  $M(i)$ , such that all tasks below  $\tilde{s}$  are performed by machines, and  $M(i)$  returns for each labor type the assigned task. Since higher types perform higher tasks, the assignment function is strictly increasing. Assuming perfectly competitive markets, taking the final good as numéraire, and denoting the marginal product of task  $s$  by  $p(s)$ , wages are then given by  $w_i = p(M(i))A(M(i), i)$ . Finally, an equilibrium of the model is given by an automation threshold  $\tilde{s}$ , an assignment  $M$ , a machine stock  $K$ , a machine productivity schedule  $A_c$ , and wages  $w_i$ , such that firms maximize profits and demand exactly the exogenous labor supply  $L$ , taking wages as given.

I will focus on symmetric equilibria in which all firms employ the same amount of labor  $L$ . Besides their labor demand, firms choose the machine productivity schedule  $A_c$ , the automation threshold  $\tilde{s}$  (that is, what tasks to have performed by machines), the machine quantity  $K$ , and the assignment  $M$  of labor to tasks that are not automated. Of these choices, those of machine productivity and the automation threshold are directly related to the development and installment of machines intended to perform tasks previously performed by labor, that is, to automation. I will therefore treat these two objects as the endogenous technology of interest, and analyze the effects of their endogenous adjustment on relative wages in the following; in the notation of the previous section, I set  $(A_c, \tilde{s})$  to  $\theta$ .<sup>22</sup>

The first step towards application of Theorem 3, the appropriate theorem for models with a continuum of labor types, is to derive aggregate production in the form  $F(L, \theta)$ . Accounting for the four choices firms make and assuming symmetric labor inputs across firms, aggregate

<sup>21</sup>The labor productivity schedule is best interpreted as representing the human capital distribution over labor types, which I take here as given to focus on the effects of endogenous technical change.

<sup>22</sup>If instead we focused on only one of the two objects, the main results of the section would go through, but in the case of  $A_c$ , their derivation would become more complicated.

production can here be written as

$$\begin{aligned} & \tilde{F}(L, A_c, \tilde{s}, K, M) \\ & := \left( \int_{\tilde{s}}^{\tilde{s}} (B(s)A_c(s)K(s))^\rho ds + \int_{\tilde{s}}^{\tilde{s}} \left( B(s)A(s, M^{-1}(s))L_{M^{-1}(s)} \frac{dM^{-1}(s)}{ds} \right)^\rho ds \right)^{\frac{1}{\rho}} - rK, \end{aligned}$$

where  $A(s, M^{-1}(s))L_{M^{-1}(s)}dM^{-1}(s)/ds$  are the effective units of labor assigned to task  $s$  under the assignment rule  $M$ .<sup>23</sup> Taking wages as given, firms' choices of  $(A_c, \tilde{s}, K, M)$  will maximize output, such that we can write aggregate production in reduced form as

$$F(L, A_c, \tilde{s}) := \max_{K, M} \tilde{F}(L, A_c, \tilde{s}, K, M).$$

Maintaining  $A_c$  and  $\tilde{s}$ , the endogenous technologies of interest, as arguments sets the notation for comparative statics with respect to their response on changes in labor supply, analogous to the comparative statics with respect to  $\theta$  in the previous section.

In equilibrium firms choose  $A_c$  and  $\tilde{s}$  to maximize output, and it is also easily verified that  $F(L, A_c, \tilde{s})$  is linear homogeneous in labor: scaling  $L$  and  $K$  by the same factor in  $\tilde{F}$  leaves marginal products of all tasks unchanged, such that the optimal assignment  $M$  is unchanged, and the scaling of  $K$  is indeed optimal.

**Observation 1.** *Aggregate production  $F(L, A_c, \tilde{s})$  is linear homogeneous in labor. In equilibrium, it is given by  $F(L, A_c^*(L), \tilde{s}^*(L))$  with  $A_c^*(L)$  and  $\tilde{s}^*(L)$  maximizing aggregate production,  $(A_c^*(L), \tilde{s}^*(L)) = \operatorname{argmax}_{A_c, \tilde{s}} F(L, A_c, \tilde{s})$ .*

To apply Theorem 3 it thus remains to show that  $F$  is quasisupermodular in the endogenous technology  $(A_c, \tilde{s})$  under the order  $\preceq_m$ , which orders technologies according to their uniform bias towards higher labor types.

For that, first write wages in reduced form as  $w_i(L, A_c, \tilde{s})$  (assuming that labor assignment and machine stock are always chosen optimally and can thus be suppressed, as in  $F(L, A_c, \tilde{s})$  above), and note that they must satisfy  $w_i(L, A_c, \tilde{s}) = p(M(i))A(M(i), i) \geq p(s)A(s, i)$  for all  $s$  when the labor assignment  $M$  is chosen to maximize output at  $(L, A_c, \tilde{s})$ . An envelope argument then implies that (see Lemma 2 in Costinot and Vogel, 2010)

$$\frac{d \log w_i}{di} = \frac{\partial \log A(M(i), i)}{\partial i}, \quad (2)$$

such that relative wages are fully determined by the labor assignment function. To order technologies according to their uniform bias, now consider two technologies  $(A_c, \tilde{s}), (A'_c, \tilde{s}')$  with  $\tilde{s} \leq \tilde{s}'$ . While machine productivity  $A_c$  does not affect the labor assignment directly, an increase in the automation threshold from  $\tilde{s}$  to  $\tilde{s}'$  shifts all labor types towards higher tasks – because low types are displaced from their tasks and induced to switch to the next higher tasks, inducing the next higher types to move upwards as well, and so on – such that the assignment function  $M$  increases everywhere (see Lemma 5 in Costinot and Vogel, 2010 for

<sup>23</sup>Effective units of labor supplied by type  $i$  are  $A(M(i), i)L_i$ . The expression for the effective units at task  $s$  follows directly from a change of variables.

the formal argument). According to equation (2), this implies that the change from  $(A_c, \tilde{s})$  to  $(A'_c, \tilde{s}')$  uniformly increases relative wages over the entire type space; intuitively, all labor types are assigned to higher tasks, where higher types have comparative advantage and thus earn higher relative wages.<sup>24</sup> It follows that we can order technologies totally by comparing automation thresholds only.

**Observation 2.** *Technology  $(A'_c, \tilde{s}')$  uniformly increases relative wages compared to technology  $(A_c, \tilde{s})$ ,  $(A'_c, \tilde{s}') \succeq_m (A_c, \tilde{s})$ , if and only if  $\tilde{s}' \geq \tilde{s}$ .*

Observation 2 allows to reduce changes in the (high dimensional) endogenous technology to the change in the automation threshold as the single relevant dimension; the resulting order  $\preceq_m$  is thus total and aggregate production trivially quasisupermodular in technology under  $\preceq_m$ . So, Theorem 3 applies, and we obtain the following result.

**Proposition 1.** *For any uniform increase in relative labor supply, the joint adjustment of machine productivity and the automation threshold uniformly increases relative wages over the type space.*

Formally:

$$\frac{w_i}{w_j}(L', A_c^*(L), \tilde{s}^*(L)) \leq \frac{w_i}{w_j}(L', A_c^*(L'), \tilde{s}^*(L'))$$

for all  $i, j$  with  $j \leq i$ , and for all  $L, L'$  with  $L_h/L_k \leq L'_h/L'_k$  for all  $k \leq h$ .

Moreover, we know from Observation 2 that the increase in relative wages comes indeed from automation, that is, from an increase in the range of tasks performed by machines. Proposition 1 hence implies that any uniform increase in relative labor supply induces automation, which in turn increases relative wages.

A complementary result, which is easily obtained in the developed setting, characterizes the effect of automation on the average wage across labor types. For this result, it does not matter whether automation is induced by a change in labor supply or in any other parameter of the model, so we write aggregate production as a function of some arbitrary parameter  $q$ , machine productivity  $A_c$ , and the automation threshold  $\tilde{s}$ , that is,  $F(q, A_c)$ . The parameter  $q$  can be thought of, for example, as the set of available machine productivity schedules, some parameter of the final good aggregator function, or, of course, labor supply.

Observe now that aggregate production is, by Euler's theorem, equal to aggregate labor income and hence proportional to the average wage  $\bar{w}$ :<sup>25</sup>

$$F(q, A_c) = \int_0^1 w_i(q, A_c) L_i di =: \bar{w} \int_0^1 L_i di.$$

Since the equilibrium technology maximizes aggregate production, it must then also maximize the average wage. This implies immediately that any joint adjustment of machine productivity and the automation threshold, induced by a change in the arbitrary parameter  $q$ , must have a positive effect on the average wage.

<sup>24</sup>Formally, the partial derivative of the log productivity schedule for labor with respect to  $i$ ,  $\partial \log A(s, i) / \partial i$ , increases in  $s$ , so by equation (2), all relative wages increase when the assignment function goes up everywhere.

<sup>25</sup>This relies on the fact that aggregate production is defined here net of machine production, that is,  $F(q, A_c, \tilde{s}) = Y(q, A_c, \tilde{s}) - rK(q, A_c, \tilde{s})$  where  $Y$  is gross production (consumption plus machines).

**Proposition 2.** *Consider a change in some parameter  $q$  – for example a change in labor supply  $L$  or in the set of feasible machine productivity schedules  $\mathcal{A}$  – that causes automation, that is, an increase in the equilibrium automation threshold  $\tilde{s}^*(q)$ . Then, the increase in automation has a positive effect on the average wage.*

Formally:

$$\bar{w}(q', A_c^*(q), \tilde{s}^*(q)) \leq \bar{w}(q', A_c^*(q'), \tilde{s}^*(q'))$$

for all  $q, q'$  such that  $\tilde{s}^*(q) \leq \tilde{s}^*(q')$ .

Proposition 2 gives a general version of the productivity effect of automation discovered in [Acemoglu and Restrepo \(2016\)](#), and provides a simple intuition for it: when technology maximizes aggregate production taking into account the endogenous adjustment of the machine stock, it essentially maximizes aggregate labor income and hence, at fixed labor supply, the average wage. Every endogenous automation step must therefore, after adjustment of the machine stock, have a positive effect on the average wage.<sup>26</sup>

To summarize, Proposition 1 shows that the logic of the factor-augmenting models analyzed in previous work – an increase in the relative supply of some labor type induces technical change biased towards this type – extends to models with automation technologies. This insight is important in light of the fact that one of the most prominent applications of factor-augmenting models with endogenous technologies has been to propose a supply-driven explanation for the joint upwards trend in the relative supply of skilled labor and the skill premium in several advanced economies over the last decades. While factor-augmenting models offer an interesting explanation for this observation, they are inconsistent with the simultaneously observed stagnation, or even decline, of low skilled workers' real wages (see, for example, [Acemoglu and Autor, 2011](#)). Since automation models are capable of producing wage reductions for some workers – although, due to the productivity effect, not for the average worker – Proposition 1 shows that it is possible to align the supply-driven explanation for the evolution of the skill premium with stagnating or declining real wages for some worker groups.<sup>27</sup>

### 3.2. International Trade

Consider now two countries, West and East, that each produce a single final good as in the assignment model discussed above. Following [Costinot and Vogel \(2010\)](#), the countries trade in tasks, such that task prices  $p(s)$ , measured in units of the final good of the West (the numéraire), must be equal in both countries. The comparative advantage schedule for labor is the same in both countries up to a scaling factor  $\gamma_E$ , so the productivity of labor type  $i$  in task

<sup>26</sup>This result crucially depends on the endogenous adjustment of the machine stock  $K$ . With fixed  $K$ , automation may reduce the average wage, as shown in a special case of the above model by [Acemoglu and Restrepo \(2016\)](#).

<sup>27</sup>In the automation model discussed above, it is easy to construct extreme cases of comparative advantage schedules for labor under which increases in automation reduce wages for the lowest worker types even after adjustment of the machine stock. Take for example a case where the productivity of the lowest labor types declines over the task space at an extremely high rate, while productivity for the majority of types is constant or increasing. Then, at a fixed machine stock, automation induces extreme declines in the lowest types' wages whereas reductions for other types are modest or even positive. At some point of "extremism" of the example, it should be clear that the adjustment of the machine stock cannot be strong enough to prevent the lowest types' wages from falling, as it would then have to increase wages for the majority of types enormously, which would require extreme productivity gains from automation.

$s$  is  $A(s, i)$  in the West and  $\gamma_E A(s, i)$  in the East. Labor supply in West and East is denoted by  $L^W$  and  $L^E$ , and the price of the East's final good in units of the West's is  $P^E$ .

Assume now that in each country there are final good firms that transform tasks into final goods and task producers transforming labor and machines into tasks. Task producers in country  $j = W, E$  choose machine productivity  $A_c^j$ , the machine stock  $K^j$ , the assignment of labor to tasks  $M^j$  – all defined as in the closed economy model above – and the automation threshold  $\tilde{s}^j$ , now defined as the highest task performed by machines in country  $j$ . We can therefore write aggregate production again as a function of labor supply and task producers' choice variables,

$$\tilde{F}(L^W, L^E, A_c^W, A_c^E, \tilde{s}^W, \tilde{s}^E, K^W, K^E, M^W, M^E) := Y^W - rK^W + p^E Y^E - p^E rK^E,$$

where  $Y^j$  is country  $j$ 's final output gross of machines as in equation (1) in the closed economy case, and where the  $Y^j$  and  $p^E$  depend on the arguments of  $\tilde{F}$ , which I suppressed for notational convenience. Also note that I already presumed that task producers divide their output between countries to maximize profits, and hence did not include an argument representing trade flows. Since task prices equal marginal products of tasks in both countries, task producer's choices will maximize aggregate production. Thus, focusing on the endogenous adjustment of the automation variables  $A_c^j$  and  $\tilde{s}^j$ , we obtain aggregate production in reduced form as

$$F(L^W, L^E, A_c^W, A_c^E, \tilde{s}^W, \tilde{s}^E) := \max_{K^W, K^E, M^W, M^E} \tilde{F}(L^W, L^E, A_c^W, A_c^E, \tilde{s}^W, \tilde{s}^E, K^W, K^E, M^W, M^E).$$

The central observation is now that the assignment of labor to tasks must be the same in both countries for every given  $(L^j, A_c^j, \tilde{s}^j)_{j=W,E}$ : if the marginal product of labor type  $i$  is highest in task  $s$  in the West (and hence  $i$  is assigned to  $s$  in the West), this must also hold in the East, because task prices are equal across countries and the difference in labor productivity  $\gamma_E$  is the same for all types. It follows directly that wages for each labor type satisfy  $w_i^W = \gamma_E w_i^E$ , that is, the cross-country wage ratios are always equal to  $\gamma_E$  for each type. So first, relative wages (across types) are equal in both countries, and all changes in the endogenous technologies necessarily affect relative wages in East and West in the same way. Second, reduced form aggregate production  $F$  can be written as a function of an appropriately defined world labor supply instead of both countries' individual supplies.<sup>28</sup>

**Observation 3.** *Aggregate production can be written as a function of world labor supply  $L$  and the endogenous technologies,*

$$F(L, A_c^W, A_c^E, \tilde{s}^W, \tilde{s}^E),$$

with world labor supply given by  $L := L^W + \gamma_E L^E$ .

In equilibrium, it is given by  $F(L, A_c^{W*}(L), A_c^{E*}(L), \tilde{s}^{W*}(L), \tilde{s}^{E*}(L))$  with  $A_c^{j*}(L)$  and  $\tilde{s}^{j*}(L)$ ,  $j =$

<sup>28</sup>An alternative route to Observation 3 is to note that the free trade equilibrium in the present model replicates the integrated equilibrium. That is, the model is isomorphic to a closed economy model with total labor supply given by  $L^W + \gamma_E L^E$ . See Costinot and Vogel (2010) for details.

$W, E$ , maximizing aggregate production,

$$(A_c^{W*}(L), A_c^{E*}(L), \tilde{s}^{W*}(L), \tilde{s}^{E*}(L)) = \underset{A_c^W, A_c^E, \tilde{s}^W, \tilde{s}^E}{\operatorname{argmax}} F(L, A_c^W, A_c^E, \tilde{s}^W, \tilde{s}^E).$$

Finally, equal assignment of labor to tasks in both countries implies that the lowest task performed by labor is given by the greater one of the two automation thresholds, that is, by  $\max\{\tilde{s}^W, \tilde{s}^E\}$ .<sup>29</sup> With the same argument as in the closed economy case, we can then order technologies by simply comparing their greatest automation thresholds.

**Observation 4.** *Technology  $(A_c^{W'}, A_c^{E'}, \tilde{s}^{W'}, \tilde{s}^{E'})$  uniformly increases relative wages compared to technology  $(A_c^W, A_c^E, \tilde{s}^W, \tilde{s}^E)$ ,*

$$(A_c^{W'}, A_c^{E'}, \tilde{s}^{W'}, \tilde{s}^{E'}) \succeq_m (A_c^W, A_c^E, \tilde{s}^W, \tilde{s}^E),$$

*if and only if  $\tilde{s}^{max'} \geq \tilde{s}^{max}$ , where  $\tilde{s}^{max} := \max\{\tilde{s}^W, \tilde{s}^E\}$  and  $\tilde{s}^{max'}$  is defined analogously.*

Clearly, the resulting order is again total, such that aggregate production is necessarily quasimodular in the endogenous technologies. Theorem 3 then implies the following.

**Lemma 1.** *For any uniform increase in relative world labor supply, the adjustment of the endogenous technologies,  $(A_c^W, A_c^E, \tilde{s}^W, \tilde{s}^E)$ , uniformly increases relative wages in both countries.*

*Formally:*

$$\frac{w_i^h}{w_j^h} \left( L', A_c^{W*}(L), A_c^{E*}(L), \tilde{s}^{W*}(L), \tilde{s}^{E*}(L) \right) \leq \frac{w_i^h}{w_j^h} \left( L', A_c^{W*}(L'), A_c^{E*}(L'), \tilde{s}^{W*}(L'), \tilde{s}^{E*}(L') \right)$$

*for  $h = W, E$ , for all  $i, j$  with  $j \leq i$ , and for all  $L, L'$  with  $L_h/L_k \leq L'_h/L'_k$  for all  $k \leq h$ .*

One of the most interesting ways by which world labor supply can increase uniformly is a strong uniform increase in relative labor supply in the East.

**Proposition 3.** *A uniform increase in relative labor supply in the East that is sufficiently strong to uniformly increase relative world labor supply induces automation in the West, which increases relative wages uniformly in both countries.*

Proposition 3 is potentially important in light of the recent increases in education levels in China. It predicts that strong improvements in educational attainment in emerging economies may favor automation in advanced economies. The channel for this effect is international trade: when the relative supply of high-educated labor increases in the emerging country, imports of tasks intensive in low-educated labor become more expensive for the advanced economy, creating incentives for (accelerated) automation of those tasks. Whether the positive effect of the induced automation on relative wages dominates the negative direct effect of the reduced import competition for low-skill workers, depends, according to the logic of Theorem 2, on the presence of non-convexities in the aggregate production possibilities set, and hence on the details of the process of technology choice.

<sup>29</sup>Note that, if the two thresholds do not coincide, there is full specialization in some machine tasks.



A second interesting result is obtained by noting that, when labor from the East is not productive at all, that is,  $\gamma_E = 0$ , the two country model boils down to the closed economy model of the West. Thus, changing  $\gamma_E$  from 0 to some strictly positive value replicates the effects of opening international trade on the West. Such a change in  $\gamma_E$  can simply be analyzed as a change in world labor supply  $L$  whenever  $\gamma_E$  enters the aggregate production function  $F$  only via  $L$ . This is in turn the case whenever the two countries have identical final good production functions.<sup>30</sup>

**Proposition 4.** *Suppose the final good production functions of the West and the East are identical. Then, the opening of international trade, represented by a change in the productivity of East labor from  $\gamma_E = 0$  to  $\gamma'_E > 0$ , reduces automation and thereby uniformly decreases relative wages in the West.*

This result does clearly not depend on the exact starting value of zero, and thus applies to every increase of  $\gamma_E$ . Costinot and Vogel (2010) view a surge in  $\gamma_E$  as a representation of an improvement in offshoring opportunities for Western firms, since offshoring allows them to employ foreign labor in their technologically superior production (suppose for this argument that  $\gamma_E < 1$ ). With this interpretation, the logic of Proposition 4 applies to offshoring as well. Proposition 4 recasts the result from Acemoglu (2003) that the opening of trade leads to low-skill biased technical change in the skill abundant country in a framework with a continuum of labor types and automation technology.<sup>31</sup> It suggests that international trade (with skill scarce countries) and offshoring (to skill scarce countries) are substitutes for automation of low-skill intensive tasks.<sup>32</sup>

## 4. Further Applications

Although the central results of the paper are derived in a static model, they can be applied, under some conditions, to steady-states of dynamic models with endogenous technology. Indeed, one can generalize the dynamic directed technical change models from the seminal contributions of Acemoglu (1998) and Kiley (1999) to feature an arbitrary number of labor types and technologies that are not purely factor-augmenting. In such a model, the main results of the present paper, Theorems 1 to 3 and their corollaries, apply in steady-state.

In more detail, the generalized dynamic model features an aggregate production function  $F(L, \theta)$  that is linear homogeneous in the real-valued technology variables  $(\theta_1, \theta_2, \dots, \theta_N) = \theta$ . Each  $\theta_i$  is a CES aggregate of a variety of intermediate goods that embody technologies. As in the endogenous growth literature, investment in research and development can be targeted to

<sup>30</sup>See Costinot and Vogel (2010). They characterize the free trade equilibrium – in a version of the present model with exogenous technology – by two differential equations where  $\gamma_E$  ( $\gamma_F$  in their notation) enters only via world labor supply when final good production functions are identical.

<sup>31</sup>Acemoglu (2003) studies a dynamic model with factor-augmenting technologies and two labor types. He distinguishes between two cases, full and no enforcement of intellectual property rights in the skill scarce country. In the case of full enforcement, the effects of trade opening on the skill bias of the factor-augmenting technology are similar to those in Proposition 4.

<sup>32</sup>This insight may help to understand the apparent lack of correlation between offshoring and import competition from China on the one hand and the use of industrial robots on the other hand, documented in Acemoglu and Restrepo (2017) across commuting zones in the US.

each of the technologies 1 to  $N$  and leads to the innovation of new, more productive versions of existing intermediate goods.<sup>33</sup>

Being granted a patent on the production of the intermediate good embodying the innovation, an innovator receives monopoly profits from the supply of the good at each point in time. The relative sizes of these profits across technologies 1 to  $N$  are determined by the relative marginal returns to the  $\theta_i$ . In consequence, the model admits a steady-state in which all technology variables (and aggregate production) grow at the same rate. Since aggregate production is linear homogeneous in technology, relative wages are then fully determined by the ratios between technology variables, and hence constant in steady-state.

Crucially, the relative steady-state technology terms  $\theta_i/\theta_j$  can be represented as solutions to the static optimization problem  $\arg\max_{\theta \in \Theta} F(L, \theta)$ , where  $\Theta$  is of the form  $\{\theta | g(\theta) \leq R\}$ , with  $g$  being strictly increasing in all variables and its exact form given by assumptions about innovation costs and spillovers between technologies.<sup>34</sup> It is then easy to see that Theorems 1 to 3 apply to this static auxiliary problem, such that all results on relative supply, technical change, and relative wages carry over to the dynamic model's steady-state. Instead of providing formal details on these considerations here, I briefly discuss another, more substantial application in the following.<sup>35</sup>

Since technical change is widely considered an important factor behind medium- and long-run changes in wage inequality, it is natural to ask about the interaction between endogenous technical change and redistributive policies. In a companion paper, I explore the implications of endogenous technical change for the design of labor income taxes, drawing on the theory developed in Section 2. More concretely, I study the design of non-linear income tax schedules in the multiple type model of Section 2.4 with endogenous labor supply decisions. The resulting model is a standard Mirrleesian economy with the exception of the production sector, which incorporates endogenous technology choices by firms.

Since labor supply decisions are endogenous, the application of Theorem 3 is not straightforward in such a model. I solve this problem by invoking a suitably adapted version of the principle of taxation, by which every incentive compatible allocation can be implemented via a non-linear income tax schedule. This allows me to work on the set of incentive compatible allocations and back out the tax schedule that implements a given allocation afterwards.

In more detail, I study two different questions. First, I ask how the welfare implications of a given tax reform are affected by taking into account the endogenous adjustment of technology.<sup>36</sup> Second, I study optimal tax design with endogenous technology. For the first question, I start by analyzing the effects of a change from one incentive compatible allocation to another, and then back out the tax reform that implements this allocation change. For the second question, I characterize the welfare maximizing incentive compatible allocation to derive properties of the tax schedule that implements this allocation. In both cases, the approaches allow for a

<sup>33</sup>This follows the creative destruction version of endogenous growth models developed by [Aghion and Howitt \(1992\)](#). A specification following the increasing varieties version from [Romer \(1990\)](#) would also be possible.

<sup>34</sup>Under the assumption that the probability of an innovation only depends on the amount of research investment targeted to this innovation,  $g$  is linear.

<sup>35</sup>Formal details are available upon request.

<sup>36</sup>More precisely, I compare the welfare implications of a given tax reform as computed by a tax designer who believes that technology is exogenous with those computed by designer who believes in an endogenous technology adjustment.

direct application of Theorem 3 in the first step of the analysis, because, by choosing allocations and therewith labor supply, comparative statics results for the case of exogenous labor supply become applicable.

Regarding the first question, I find that, under some conditions, any progressive tax reform induces technical change that reduces wage inequality before taxes, and the designer who takes into account the adjustment of technology always perceives the maximum welfare gain across all progressive tax reforms to be greater than the designer who believes technology to be exogenous. This anticipates the results of the second part, which indicate that the optimal tax schedule becomes more progressive – that is, features higher marginal tax rates at the top and lower rates at the bottom – when taking into account the endogeneity of technology.

For the purpose of the present paper, the application to non-linear income taxation demonstrates that, although more involved, the developed theory can be applied with gain even to settings with endogenous labor supply.

## 5. Conclusion

The paper admits four conclusions. First, weak relative bias – a positive relationship between the relative bias of technology and relative labor supply – is fairly robust with respect to properties of the aggregate production function. In a static general equilibrium model with exogenous labor supply, its local version only requires a form of homotheticity in labor. The global version needs in addition that complementarity relationships between technical change and relative labor supply are stable.

Second, strong relative bias – a positive relationship between the endogenous technology relative wage and relative labor supply – arises in models with (a certain form of) non-convexities in the aggregate production possibilities set, and only in these.

Third, there is a simple intuition for the effect of endogenous technical change on relative wages, based on complementarity relationships: an increase in relative labor supply induces technical change that is complementary to relative supply, and this in turn, by symmetry of complementarity relationships, is relatively biased towards the labor type that becomes more abundant. This simple logic applies whenever a proportional increase in labor supply does not induce biased technical change.

Fourth, the results of this paper should simplify exploring the implications of endogenous (biased) technical change in a variety of different models related to wage inequality, which may lead to interesting new insights. Examples are, as discussed in Sections 3.1 to 4, models with an endogenous assignment of workers to tasks both with and without international trade, and the theory of redistributive income taxation.

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## A. Models of Technology Choice

In the main text, I simply assumed with reference to [Acemoglu \(2007\)](#) that the equilibrium technology is such that it maximizes aggregate production. The simplest model that gives this result is a model in which a mass one of identical firms produces the final good using labor under perfect competition and chooses the production technology to maximize profits (“Economy D” in [Acemoglu, 2007](#)).

In particular, suppose each firm  $i$  produces according to  $F(L^i, \theta_i)$ ,  $\theta_i \in \Theta$  being the firm specific technology choice. Maximizing profits while taking prices as given, each firm’s technology choice must maximize output at its profit-maximizing labor demand. Thus, in any symmetric equilibrium for exogenous aggregate labor supply  $L$ , each firm’s technology maximizes  $F(L, \theta_i)$ , and hence the collective technology choice  $\theta$  maximizes aggregate output  $F(L, \theta) = \int_0^1 F(L, \theta_i) di$ .

Since firms maximize profits jointly over their labor inputs and technology,  $F$  must be concave in equilibrium, which, by Theorem 2, precludes strong relative bias. A simple way to allow for non-convexities in the aggregate production possibilities set at equilibrium points is to allow for spillovers across firms’ technology choices. In particular, suppose that each firm produces according to  $F(L_i, \theta_i, \theta_{-i})$ , where  $\theta_{-i} = \{\theta_j\}_{j \neq i}$  collects technology choices of all other firms and  $F$  is homogeneous and concave in  $L_i$ . Assume in addition that the spillover effects are such that, at given prices, firms prefer a situation where all other firms choose the same technology

as they do,<sup>37</sup>

$$F(L_i, \theta_i, \theta_{-i}) \geq F(L_i, \theta_i, \theta'_{-i}) \text{ with } \theta_{-i} = \{\theta_j\}_{j \neq i}, \theta_j = \theta_i \forall j \text{ and for all } \theta'_{-i}.$$

Then, with symmetric labor inputs  $L$  across all firms, the technology  $\theta^* = \operatorname{argmax}_{\theta \in \Theta} F(L, \theta, \{\theta\})$  forms an equilibrium.<sup>38,39</sup> Moreover, writing aggregate production as  $\tilde{F}(L, \theta) := F(L, \theta, \{\theta\})$ ,  $\tilde{F}$  does not have to be concave at the equilibrium, because no firm maximizes over labor inputs and all firms' technologies jointly.

An alternative way to model technology choice that is closer to the endogenous growth literature is to assume that technologies are embodied in intermediate goods supplied by technology monopolists ("Economy O" in [Acemoglu, 2007](#)).

For that, suppose aggregate production is given by

$$G(L, \theta)^\alpha \sum_{k=1}^K q_k(\theta_k)^{1-\alpha} - \sum_{k=1}^K \kappa_q q_k,$$

where  $q_k$  is the quantity of intermediate good  $k$ , which embodies technology  $\theta_k$ . Each intermediate good is produced by a monopolist at constant marginal cost  $\kappa_q$  from final output. The monopolist for good  $k$  also chooses technology  $\theta_k$ .

Deriving first final good firms' demand for intermediates and then monopolists' optimal quantity decision at given technologies, one finds that the quantities  $q_k$  are proportional to  $G(L, \theta)$ . It follows that aggregate production is proportional to  $G(L, \theta)$  as well.

In a second step, one can show that the marginal products of labor are proportional to the partial derivatives (with respect to the corresponding labor type) of  $G$ , and hence proportional to the marginal products of labor in aggregate production. Thus, we have an aggregate production function in labor and technology with relative marginal products equal to relative wages. Besides the fact that the equilibrium technology maximizes aggregate production, this is exactly what I used to derive the results of Section 2.

Turning to the equilibrium technology, one can show that monopoly profits are also proportional to  $G$  at any given technology. Hence, the technology  $\theta^* = \operatorname{argmax}_{\theta \in \Theta} G(L, \theta)$ , with  $\Theta = \prod_{k=1}^K \Theta_k$  and  $\Theta_k$  the technology choice sets of the monopolists, is an equilibrium technology (since no monopolist can increase its profits by deviating) and maximizes aggregate output (since it maximizes  $G$  and  $G$  is proportional to aggregate output).

[Acemoglu \(2007\)](#) presents two further models of technology choice that lead to the results

<sup>37</sup>This captures the idea that a given firm benefits from another firm's research and development activities the most when the other firm develops similar technologies. Also note that the assumption only restricts the direct, purely technological effect of spillovers, not the effects mediated via competition between firms. Concretely, the assumption does not preclude that any given firm may wish that its competitors chose other technologies and demand different types of labor than itself, thereby reducing its unit wage costs and allowing for greater profits. Indeed, if the technological effect of spillovers is not very strong, any firm wishes all other firms to choose very unproductive technologies, allowing the firm itself to produce at low labor costs for a large share of the market.

<sup>38</sup>By  $\{\theta\}$  I mean  $\{\theta_j\}_{j \neq i}$  with  $\theta_j = \theta$  for all  $j$ .

<sup>39</sup>To see that, fix  $\theta_{-i}$  at  $\{\theta^*\}$ , and consider the maximization problem  $\max_{L_i, \theta_i} \{F(L_i, \theta_i, \{\theta^*\}) - wL_i\}$ . Suppose there exist wages  $w$  such that  $(\theta', L)$  is a maximizer of this problem for some  $\theta'$  ( $L$  denotes as usual the exogenous aggregate labor supply). Then, the assumption on technology spillovers implies that  $\theta' = \theta^*$ , which guarantees that each individual firm maximizes profits in the proposed symmetric equilibrium.



on aggregate production and equilibrium technology assumed in the main text. The first is a special case of the model with intermediate goods, obtained by setting the number of technology monopolists to 1. In the second model, technology is chosen by a socially run research firm interested in aggregate output maximization.

## B. Proofs and Additional Remarks

The following theorem is a small generalization of Theorem 1, presented in the main text. Theorem 1 requires that complementarity relationships between technologies and relative labor supply are stable over the labor supply space in the setting with two labor types. The conditions of Theorem 4 below guarantee that complementarity relationships are stable over the set of technologies that may actually be chosen in equilibrium, that is, over the image of  $\theta^*$ . Put differently, they guarantee that the image of  $\theta^*$  is totally ordered.

**Theorem 4.** *Suppose  $\theta^*(L)$  can be written as  $\theta^*(L_1/L_2)$ ,  $F$  is quasisupermodular in  $\theta$ , and let  $(\Theta, \preceq_s)$  be a lattice. Then, there is weak relative equilibrium bias of technology at any labor supply  $L$ .*

*Proof of Theorems 1 and 4.* If the order  $\preceq_s$  is total on  $\Theta$ ,  $F$  is quasisupermodular in  $\theta$  and  $(\Theta, \preceq_s)$  is a lattice. Thus, Theorem 4 is more general than Theorem 1, and we will only proof the more general one.

We can restrict the proof to the case where  $\theta^*(L) \neq \theta^*(L')$ . The proof is by contradiction. Suppose  $\theta^*(L) \not\preceq_s \theta^*(L')$  and let  $\underline{\theta}$  denote the meet and  $\bar{\theta}$  the join of  $\theta^*(L)$  and  $\theta^*(L')$ . Then, quasisupermodularity implies that  $F(L, \theta^*(L')) \leq F(L, \bar{\theta})$ . Since  $\theta^*$  is homogeneous of degree zero, we can restrict the proof to labor inputs  $L$  and  $L'$  with  $F(L, \theta^*(L')) = F(L', \theta^*(L'))$ . Then, moving from  $L$  to  $L'$  along the isoquant of the function  $F(L, \theta^*(L'))$  increases the function  $F(L, \bar{\theta})$ , because  $\theta^*(L') \preceq_s \bar{\theta}$  and hence  $w_1/w_2(L, \theta^*(L')) \leq w_1/w_2(L, \bar{\theta})$  for all  $L$ . This implies  $F(L', \theta^*(L')) \leq F(L', \bar{\theta})$ , a contradiction.  $\square$

While Theorems 1, 4, and their proof follow the ideas of the theory of monotone comparative statics by Milgrom and Shannon (1994), they do not make use of it directly. I elaborate on the relationship between Milgrom and Shannon's and my approach in the following remark.

*Remark 1.* The standard approach of monotone comparative statics to the result of Theorem 4 would be to start with an (unspecified) order on the technology space  $\Theta$ , and assume then that the production function  $F$  satisfies single crossing in technology and relative labor under this order when restricted to any given (exogenous technology) isoquant.<sup>40</sup> In contrast, I define the order directly via the relative bias of technology, which then supersedes the single crossing assumption (see Definition 5). In particular, whenever an order exists under which single crossing holds, this order is the one I define; conversely, under my order, single crossing is always satisfied. Given equivalence of the two approaches, I believe my approach to be both simpler and more intuitive, as it directly formalizes the notion of complementarity between technology and relative labor without referring to some unspecified order at first.

<sup>40</sup>Single crossing in  $(\theta, L_1/L_2)$  means here that, with  $F^r$  denoting production restricted to the given isoquant,  $F^r(L'_1/L'_2, \theta) \geq F^r(L_1/L_2, \theta)$  for some  $L'_1/L'_2 \geq L_1/L_2$  implies that  $F^r(L'_1/L'_2, \theta') \geq F^r(L_1/L_2, \theta')$  for any  $\theta' \succeq \theta$ . Put differently, the difference  $F^r(L'_1/L'_2, \theta) - F^r(L_1/L_2, \theta)$  crosses zero at most once and, if so, from below.

*Proof of Corollary 1.* The proof replicates the proof of Theorem 1 with the tools of differential calculus. Denote the set of all direction vectors in  $\Theta$  that (weakly) increase  $w_1/w_2$  at  $(L, \theta^*(L))$  by  $U_1$ . Since  $\theta^*$  is homogeneous of degree zero, we can again restrict the proof to direction vectors in the labor input space that are tangent to the isoquant at  $(L, \theta^*(L))$ , that is, to the vector  $v = (1, -w_1/w_2(L, \theta^*(L)))$ . Now, denote the direction of the technology adjustment to a marginal change of labor inputs in direction  $v$  by  $u := (\nabla \theta^*(L) \cdot v)$ , and suppose that  $u \notin U_1$ . Then:

$$\begin{aligned}
& 0 > \nabla_{\theta} \frac{w_1}{w_2}(L, \theta^*(L)) \cdot u \\
\Rightarrow & 0 > \nabla_{\theta} w_1(L, \theta^*(L)) \cdot u - \nabla_{\theta} w_2(L, \theta^*(L)) \cdot \frac{w_1}{w_2}(L, \theta^*(L)) \cdot u \\
& = \nabla_{L_1, \theta}^2 F(L, \theta^*(L)) \cdot u - \nabla_{L_2, \theta}^2 F(L, \theta^*(L)) \cdot \frac{w_1}{w_2}(L, \theta^*(L)) \cdot u \\
& = \nabla_{\theta, L_1}^2 F(L, \theta^*(L)) \cdot u - \nabla_{\theta, L_2}^2 F(L, \theta^*(L)) \cdot \frac{w_1}{w_2}(L, \theta^*(L)) \cdot u \\
& = \nabla_L (\nabla_{\theta} F(L, \theta^*(L)) \cdot u) \cdot v
\end{aligned}$$

where the last line gives the desired contradiction. Therefore,  $u \in U_1$ , which proves the corollary.  $\square$

*Proof of Theorem 2.* Since  $w_1/w_2(L, \theta^*(L))$  is homogeneous of degree zero in  $L$ , we can focus on input changes from  $\bar{L}$  to  $L'$  with  $L'$  on the line tangent to the isoquant at  $\bar{L}$  (the *tangent line* henceforth).

( $\Rightarrow$ ) The proof is by contradiction. Suppose there is a labor input  $L'$  with  $L'_1/L'_2 > \bar{L}_1/\bar{L}_2$  at which the relative wage of input 1 versus input 2 is smaller than at  $\bar{L}$ , that is:  $w_1/w_2(L', \theta^*(L')) < w_1/w_2(\bar{L}, \theta^*(\bar{L}))$ . Then, there exists an interval around  $L'$  on the tangent line on which the exogenous technology function  $F(L, \theta^*(L'))$  is strictly decreasing in direction of increases in  $L_1$  relative to  $L_2$ . But since this exogenous technology function is a lower bound for the endogenous technology function  $F(L, \theta^*(L))$  and the two are equal at  $L'$ , the endogenous technology function cannot be quasiconvex with minimum at  $\bar{L}$  on the tangent line.

( $\Leftarrow$ ) Take any two labor inputs  $L'$  and  $L''$  on the tangent line such that the relative input  $L_1/L_2$  is greater at  $L'$  than at  $\bar{L}$  and greater at  $L''$  than at  $L'$ . Consider the exogenous technology functions  $F(L, \theta^*(L'''))$  for all  $L'''$  on the line interval between  $L'$  and  $L''$ . Strong relative equilibrium bias requires that the directional derivative of these exogenous technology functions along the tangent line in direction of increases in  $L_1$  relative to  $L_2$  is non-negative at the point where they are equal to the endogenous technology function  $F(L, \theta^*(L))$ . Moreover, the endogenous technology function is the upper envelope of the exogenous technology functions on the line interval. Then, corollary 4 in Milgrom and Segal (2002) implies that the difference between  $F(L', \theta^*(L'))$  and  $F(L'', \theta^*(L''))$  can be written as

$$F(L', \theta^*(L')) - F(L'', \theta^*(L'')) = \int_0^1 \nabla_L F(l(\tau), \theta^*(l(\tau))) \cdot v_l d\tau \geq 0,$$

where  $l(\tau)$  parametrizes the change along the tangent line from  $L'$  to  $L''$ ,  $v_l$  is the direction vector of this change, and the inequality follows from the above noted fact that the integrand

is non-negative everywhere. It follows that the endogenous technology function  $F(L, \theta^*(L))$  is non-decreasing on the ray starting in  $\bar{L}$  and following the tangent line in direction of increases in  $L_1$  relative to  $L_2$ . Analogous arguments apply to the ray starting in  $\bar{L}$  and following the tangent line in the other direction.  $\square$

*Proof of Corollary 2.* Since  $w_1/w_2(L, \theta^*(L))$  is homogeneous of degree zero in  $L$ , we can focus on directional derivatives of the endogenous technology wage ratio  $w_1/w_2(L, \theta^*(L))$  in direction of the line tangent to the isoquant at  $\bar{L}$  (the *tangent line* henceforth) that increase  $L_1$ . At every point on the tangent line, the envelope theorem implies that the derivative of the endogenous technology function  $F(L, \theta^*(L))$  restricted to the tangent line (henceforth the *restricted endogenous technology function*) equals (a multiple of)

$$w_1(L, \theta^*(L)) - \frac{w_1(\bar{L}, \theta^*(\bar{L}))}{w_2(\bar{L}, \theta^*(\bar{L}))} w_2(L, \theta^*(L)).$$

The second derivative of the restricted endogenous technology function is obtained by taking the total derivative along the tangent line:

$$\begin{aligned} & \nabla_L w_1(L, \theta^*(L)) \cdot v + \nabla_\theta w_1(L, \theta^*(L)) \cdot (\nabla_L \theta^*(L))^T \cdot v \\ & - \frac{w_1(\bar{L}, \theta^*(\bar{L}))}{w_2(\bar{L}, \theta^*(\bar{L}))} \nabla_L w_2(L, \theta^*(L)) \cdot v + \nabla_\theta w_2(L, \theta^*(L)) \cdot (\nabla_L \theta^*(L))^T \cdot v, \end{aligned}$$

where  $v$  is the direction vector of the tangent line. Finally, multiplying this expression by  $1/w_2(\bar{L}, \theta^*(\bar{L}))$  and evaluating it at  $\bar{L}$  yields the directional derivative of the endogenous technology wage ratio along the tangent line. Hence, this directional derivative and the second derivative of the restricted endogenous technology production function have the same sign.  $\square$

*Remark 2.* The condition of Corollary 2 can also be expressed in terms of the Hessian matrix of the aggregate production function  $F$ . Let  $H_F(\bar{L}, \theta^*(\bar{L}))$  be the Hessian of  $F$  at the point  $(\bar{L}, \theta^*(\bar{L}))$ . The condition of Corollary 2 is satisfied if and only if the Hessian is not negative semidefinite when restricting direction vectors such that the direction for  $(L_1, L_2)$  is the direction of the line tangent to the isoquant at  $\bar{L}$ . Formally, there is local strong relative bias if and only if

$$v^T H_F(\bar{L}, \theta^*(\bar{L})) v \geq 0$$

for some  $v \in \mathbb{R}^{N+2}$  such that  $v_1/v_2 = -w_2/w_1(\bar{L}, \theta^*(\bar{L}))$  and there exists an  $\epsilon > 0$  such that  $\theta^*(\bar{L}) + \epsilon \cdot (v_3, \dots, v_{N+2}) \in \Theta$ .

To see this, note that the second derivative of the endogenous technology production function restricted to the tangent line along the isoquant is given by (a multiple of)  $v^T H_F(\bar{L}, \theta^*(\bar{L})) v$  for  $v_1 = 1$ ,  $v_2 = w_1/w_2(\bar{L}, \theta^*(\bar{L}))$ , and  $(v_3, \dots, v_{N+2}) = \nabla \theta^*(\bar{L})$ . If this derivative is negative, no other feasible direction  $(v_3, \dots, v_{N+2})$  in the technology space can give a positive result for the quadratic form of the Hesse matrix. Otherwise this other direction would allow to reach a higher production level in a sufficiently small neighborhood of  $\bar{L}$  on the tangent line, but this contradicts  $\theta^*(L)$  maximizing production. Thus, the condition of Corollary 2 is equivalent to the condition on the Hesse matrix presented above.

*Proof of Theorem 3.* We can restrict the proof to the case where  $\theta^*(L) \neq \theta^*(L')$ . The proof is by contradiction. Suppose  $\theta^*(L) \not\preceq_m \theta^*(L')$  and let  $\underline{\theta}$  denote the meet and  $\bar{\theta}$  the join of  $\theta^*(L)$  and  $\theta^*(L')$ . Then, quasisupermodularity implies that  $F(L, \theta^*(L')) \leq F(L, \bar{\theta})$ . By homogeneity, we can restrict the proof to labor inputs  $L$  and  $L'$  with  $F(L, \theta^*(L')) = F(L', \theta^*(L'))$ . Denote by  $q$  the lowest labor type for which  $L_q \leq L'_q$  and let  $l(\tau)$  parametrize a path from  $L$  to  $L'$  that is monotone in each entry  $L_s$  of  $L$  and lies in the isoquant of  $F(L, \theta^*(L'))$ . Then, the following holds: <sup>41</sup>

$$\begin{aligned} 0 &= F(L', \theta^*(L')) - F(L, \theta^*(L')) \\ &= \int_0^1 \nabla_L F(l(\tau), \theta^*(L')) \cdot \frac{dl}{d\tau} d\tau, \end{aligned}$$

and for every  $\tau \in [0, 1]$ :

$$\begin{aligned} 0 &= \nabla_L F(l(\tau), \theta^*(L')) \cdot \frac{dl}{d\tau} \\ &= w(l(\tau), \theta^*(L')) \cdot \frac{dl}{d\tau} \\ &= \frac{w_q(l(\tau), \bar{\theta})}{w_q(l(\tau), \theta^*(L'))} w(l(\tau), \theta^*(L')) \cdot \frac{dl}{d\tau} \\ &\leq w(l(\tau), \bar{\theta}) \cdot \frac{dl}{d\tau} \\ &= \nabla_L F(l(\tau), \bar{\theta}) \cdot \frac{dl}{d\tau}, \end{aligned}$$

where the fourth line follows from the fact that  $(w_q(l(\tau), \bar{\theta})/w_q(l(\tau), \theta^*(L'))))w_i(l(\tau), \theta^*(L')) \leq w_i(l(\tau), \bar{\theta})$  if and only if  $q \leq i$  (because  $\theta^*(L') \preceq_m \bar{\theta}$ ) and the fact that  $0 \leq dl_i/d\tau$  if and only if  $q \leq i$ . It follows that

$$\begin{aligned} 0 &= \int_0^1 \nabla_L F(l(\tau), \theta^*(L')) \cdot \frac{dl}{d\tau} d\tau \\ &\leq \int_0^1 \nabla_L F(l(\tau), \bar{\theta}) \cdot \frac{dl}{d\tau} d\tau \\ &= F(L', \bar{\theta}) - F(L, \bar{\theta}). \end{aligned}$$

This implies  $F(L', \theta^*(L')) \leq F(L', \bar{\theta})$ , a contradiction.

Note that the contradiction relies on the assumption that, in case of multiple maximizers of  $F(L', \theta)$ ,  $\theta^*(L')$  is their supremum. If  $\theta^*$  is an arbitrary selection from the set of maximizers, we could assume a strict version of quasisupermodularity, or quasisupermodularity under the strict version of  $\preceq_m$ . This would preserve the statement of Theorem 3 for strict increases in relative skill supply, that is, for labor supply changes from  $L$  to  $L'$  with  $L_h/L_k < L'_h/L'_k$  for all  $k < h$ .  $\square$

<sup>41</sup>In the continuum case  $\nabla_L F \cdot dl/d\tau$  should be interpreted as the Gateaux derivative of  $F$  in direction of the signed measure  $dl/d\tau$ . By the first order conditions of firms, this Gateaux derivative is equal to  $w \cdot \frac{dl}{d\tau}$ , which means integration of the wage function  $w$ , assumed to be integrable, with the measure  $dl/d\tau$ ,  $\int_1 w d(dl/d\tau)$ .