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Conference Paper
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Provided in Cooperation with:
Verein für Socialpolitik / German Economic Association

Suggested Citation: Drechsel-Grau, Moritz; Greimel, Fabian (2018) : Falling Behind: Has Rising Inequality Fueled the American Debt Boom?, Beiträge zur Jahrestagung des Vereins für Socialpolitik 2018: Digitale Wirtschaft - Session: Heterogenous Agents, No. G06-V3, ZBW - Leibniz-Informationszentrum Wirtschaft, Kiel, Hamburg

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Falling Behind: Has Rising Inequality Fueled the American Debt Boom?

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February 22, 2018

Preliminary draft. Please do not cite without permission.

Abstract

We want to explain the rise of household debt in the US since 1980. We present a mechanism that is consistent with the following stylized facts: (i) Real mortgage debt, (ii) debt-service-to-income ratios and (iii) house sizes (in sqft) have increased since the 1980 across all income quintiles. This is despite (iv) real incomes have stagnated for the bottom 50% since the 1980s. Our mechanism is based on other-regarding preferences. Rich agents upgrade their houses to match their risen incomes. Poorer agents want to substitute future consumption for a bigger house today (in order to “keep up with the richer Joneses”). Holding prices constant, debt will necessarily increase since houses are durable and require large payment upfront, but only low maintenance costs in the future. We build a tractable model consistent with these facts and extend it to quantitative a model to show that the partial equilibrium results go through in general equilibrium.

Keywords: household debt, debt boom, social comparisons, consumption network, keeping up with the Joneses

JEL Codes: D14, D31, E21, E44, E70
1 Introduction

In the decades leading up to the Great Recession, the distribution of national income growth has become ever more skewed. Average post-tax incomes of the top ten percent have grown more than twice as fast as incomes of the “middle forty” (P50-P90), and more than five times as fast as those in the bottom half of the distribution (Piketty et al., 2018a). This divergence in incomes has lead to increasing inequality in living standards (Aguirar and Bils, 2015; Bellet, 2017). Relative to the rich, everyone has fallen behind.

It has been argued (e.g. Rajan, 2010; Frank, 2013) that this was one of the main drivers of the American debt boom – the rise in US household debt across the entire distribution since the 1980s (see section 3 and Kuhn et al., 2017). There is plenty of evidence that people compare themselves with others and suffer from relative deprivation (e.g. Luttmer, 2005; Card et al., 2012). Non-rich households will then attempt to keep up with the living standards set by the rich. To this end, they shift expenditures towards status-enhancing goods, cutting down on both present and future status-neutral consumption (Frank, 2013; Bertrand and Morse, 2016; De Giorgi et al., 2016). Housing is arguably one of the most visible and status-enhancing goods and accounts for the largest share in households’ expenditures. As housing is largely debt-financed, household debt rises with growing housing aspirations.

In this paper we explore the extent to which rising inequality and status concerns have fueled the American debt boom prior to the Great Recession. This question is important for two reasons. First, the secular surge in inequality warrants a deeper understanding of how it may affect the aggregate economy. Second, understanding the determinants of household debt and its distribution is paramount for future policies as they were among the main drivers of the Great Recession (Mian et al., 2013, 2017; Martin and Philippon, 2017).

We first develop a tractable infinite-horizon consumption network model in order to illustrate the mechanism analytically. We then introduce social comparisons into a heterogeneous agents macroeconomic model with housing and heterogeneous income profiles for a quantitative analysis.

Our tractable infinite-horizon network model (in the spirit of Ballester et al., 2006) illustrates how other-regarding preferences can rationalize the afore-mentioned evidence. Agents can spend their lifetime wealth on a non-durable and status-neutral consumption good and on durable and status-enhancing housing. Given prices, agents’ optimal houses and debt are linear functions of one’s own income and that of the reference group. If the incomes of households in the reference group rise, these households will upgrade their houses. Hence, reference housing (a weighted average of all houses in the reference group) will increase. In order to keep up with its reference group, the households substitute consumption for housing which requires taking out additional debt. Agents substitute future consumption flows with debt service payments. We provide conditions on the comparison weights of the network, under which rising income inequality raises aggregate debt. This is the case if comparisons are upward-looking and each member of the reference group is equally important.

In the quantitative part of the paper, we analyze an incomplete market model with heterogeneous income profiles and upward-looking comparisons in housing. The rich type-dependent income process allows for tight control over the income distribution and its

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1We abstract from the possibility that non-durable goods, such as an expensive drink in a bar, may induce consumption externalities. However, we posit that most typical status goods (cars, clothing) are durable to some degree.
evolution – a key ingredient to our quantitative analysis.\footnote{The assumption of heterogeneous income types/profiles is both intuitive (different education, innate skill, etc.) and supported by the data (Guvenen, 2007, 2009; Guvenen et al., 2016)} The key difference to the tractable model is the presence of precautionary savings to insure against income risk. Our preliminary results indicate that increasing top incomes lead nonrich households to increase own housing in response to increased houses at the top. To that end, households reduce expenditures on nondurable consumption and take on more debt. House prices increase and interest rates decrease.

Our paper makes three contributions. First, we provide a systematic quantitative investigation of the explanatory power of the popular hypothesis that rising inequality has driven the surge in household debt due to status concerns of non-rich households. Second, while there is a growing literature documenting a steady rise in income and wealth inequality, we know rather little about potential consequences of this distributional shift (Guvenen et al., 2017; Piketty et al., 2018a). Third, psychologists as well as applied microeconomists and experimental economists have amassed extensive evidence on the importance of social comparisons for people’s economic choices (Fehr and Schmidt, 1999; Luttmer, 2005; Bursztyn et al., 2014; De Giorgi et al., 2016; Bursztyn et al., 2017; Bellet, 2017). This evidence has yet to find its way into macroeconomic models leaving us with little knowledge on the aggregate effects of relative income and consumption concerns.

**Outlook** In ongoing and future work, we will

- allow for income growth over the lifecycle
- estimate a rather flexible income process for the years 1980 and 2007 using data provided by Guvenen et al. (2016) and Piketty et al. (2018a)
- introduce a labor-leisure choice
- add firms, in particular a construction sector to the supply of housing
- introduce spatial heterogeneity in amenities

## 2 Social Comparisons

*A house may be large or small; as long as the neighboring houses are likewise small, it satisfies all social requirement for a residence. But let there arise next to the little house a palace, and the little house shrinks to a hut. [...] and however high it may shoot up in the course of civilization, if the neighboring palace rises in equal or even in greater measure, the occupant of the relatively little house will always find himself more uncomfortable, more dissatisfied, more cramped within his four walls.*

– Karl Marx, 1849\footnote{We borrowed this quote from Bellet (2017).}

The idea that people care about how our belongings compare to those of our neighbors is certainly not new. Veblen (1899) noticed that the purchase and consumption of conspicuous goods contributes to our social status. Duesenberry (1949) argued that the consumption-savings decision is mainly driven by habits and social comparisons. Following the seminal findings of Easterlin (1974), who first documented that the link between happiness and own income is rather weak, social scientists have amassed extensive evidence on the importance of social comparisons for people’s well-being (e.g. Fehr and Schmidt, 1999; Ferrer-i-Carbonell, 2005; Luttmer, 2005; Clark and Senik, 2010; Card et al., 2012).\footnote{See Clark et al. (2008) for a review and discussion of this literature.}
Besides their importance for individual well-being, there is also correlational and sound (quasi-)experimental evidence that social comparisons matter for economic behavior and that conspicuous consumption is used to enhance one’s social status. Angelucci and De Giorgi (2009) find that cash transfers to eligible households indirectly increase the consumption of ineligible households living in the same villages via increased debt loans and reduced savings. Kuhn et al. (2011) show that the neighbors of lottery winners spend more on cars. In a field experiment with Brazilian bankers, Bursztyn et al. (2014) find substantial peer effects in investment decisions due to keeping-up-motives. In another field experiment, Bursztyn et al. (2017) show that platinum cards are more likely to be used in social contexts, implying social image motivations. In addition, they provide evidence of positional externalities from the consumption of status goods.

On a broader level, De Giorgi et al. (2016) combine matched employer-employee data and Danish wealth data to estimate a sizable elasticity of own with respect to peers’ consumption by exploiting partially overlapping networks. Bertrand and Morse (2016) use state-year variation and detailed expenditure data to document that nonrich households consume a larger share of their current income when exposed to higher top income and consumption levels. Competing explanations such as income expectations, wealth effects or upward price pressure cannot account for this pattern. The authors further provide correlational evidence suggesting that households used credit in order to keep up with rising expenditures at the top. This is in line with the findings of Georgarakos et al. (2014) who show that higher incomes of a person’s (richer) peers increases borrowing and the likelihood of personal bankruptcy.

In the context of housing, Bellet (2017) analyzes the effect of a relative downscaling of a person’s house using data from the American Housing Survey in addition to Zillow-data on three million suburban houses built between 1920 and 2009. He finds that relative downscaling due to the construction of bigger houses in the same area leads to lower satisfaction levels with one’s own house. Affected homeowners are also more likely to upgrade their home and take on more debt. Importantly, the effects are highly asymmetric in the sense that only houses at the top of the distribution matter for social comparisons. These findings are consistent with upward-looking comparison behavior.

Interestingly, social comparisons seem to be upward-looking. Households care about what happens above them while paying little attention to relative gains of those below one’s own position. Besides the just-mentioned evidence by Bellet (2017), this asymmetry is also present in the context of self-reported well-being (Ferrer-i-Carbonell, 2005; Card et al., 2012) and born out by direct survey evidence on the strength and direction of comparisons (Clark and Senik, 2010). Bertrand and Morse (2016) also find an asymmetric relationship consistent with upward-looking comparisons: non-rich consumption reacts to top income levels but not to median or low income levels in a state.

The fact that comparisons are upward-looking is highly relevant for the interplay of income inequality and aggregate variables as it may give rise to expenditure and debt cascades. Despite the conclusive evidence on the role of social comparisons in shaping consumption-savings decisions, they have yet to find their way into macroeconomic models leaving us with little knowledge on the aggregate effects of (asymmetric) social comparisons.

While they do not find differential effects between goods of different levels of visibility (Heffetz, 2011), they do not distinguish between durable and non-durable goods. In addition, housing expenditures (rents) are treated as non-visible.

Frank et al. (2014) find similar evidence.

We are not aware of a study where asymmetry was tested for but not discovered.
3 The Facts

Since the 1980s inequality and household debt have grown in lockstep. Figure 1 shows the time series of household debt to GDP for the US from the Macrohistory database (Jordà et al., 2017a,b) and the top 10% income share for the US (from the World Wealth & Income Database, Alvaredo et al., 2016). Both time series take off around 1980 and rise until the Great Recession in 2007.

Mortgages constitute the biggest chunk of household debt, and account for much of the rise since 1980. The upper panel of figure 2 shows the average composition of household debt into different debt categories using data come from the SCF (1983–2017). Mortgages have always accounted for the biggest share. The lower panel shows the growth rates for each category. We needed to choose 1989 as the base year because the early SCFs are not detailed enough. Since 1989 housing-related debt has grown faster than other debt.

In fact, mortgage debt is the main debt category across all income groups. Figure 3 shows average household debt and its composition for different income quintiles for 1983–2016. Since the SCF (1983–2017) is a household survey, we correct the income by household size before constructing the quintiles. Mortgage debt is not only the biggest chunk in all income groups, but it has also grown at about the same rate. Any explanation of the rise in household debt should be consistent with the fact that mortgages grew across all income groups.

The largest fraction of debt is held by richer households. Figure 4 in the appendix shows the composition of average household debt into income quintiles. Since 1983, however, the real growth in average debt has been remarkably similar across income groups.

Incomes at the top grew substantially faster than in the middle and lower part of the distribution. Figure 5 (taken from Piketty et al., 2018a) shows that the share of incomes

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8We use the OECD-modified equivalence scale where the household head has weight 1, the spouse 0.5 and each child counts has weight 0.3 (OECD, ????, see)
Keeping prices (CPI) and population constant. Data from the Surveys of Consumer Finances. HELOCs and other residential debt are part of other debt in 1983 and 1986.

**Figure 2:** Real growth of household debt in the US by debt categories.

Keeping prices (CPI) and population constant. Construction of income quintiles using *OECD-modified equivalence scale*. Data from the Surveys of Consumer Finances. HELOCs and other residential debt are part of other debt in 1983 and 1986.

**Figure 3:** Real growth of household debt in the US by debt categories for each income quintile.
Figure 4: Real growth of household debt in the US by income quintiles.

Going to the bottom 50% has stagnated since 1980. Thus, half the population have not participated at all in the economic growth of the past four decades. Incomes in the middle forty increased only half as fast as those in the top 10%. Hence, rising top incomes have resulted in relative losses for everyone else.

Debt service payments have increased for all income groups. Figure 6 shows the growth of median debt-service-to-income per income quantile. This is despite a steady decline in real interest rates. It shows that households spend an ever larger fraction of their income on housing. This holds even for those households with stagnating incomes. So, any explanation should address this substitution effect for households with stagnating incomes.

Moreover, we see a similar increase in housing inequality. Importantly, houses in the top 10% have become much bigger reflecting the rise in top incomes (see figure 7). Even houses in the bottom half have grown despite stagnating or declining incomes for half the population and house price growth. Note that figure 7 shows the increase in house sizes directly which rules out effects stemming from the increase in house prices.

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9We focus on the intensive margin, so all household without debt were omitted.
Figure 5: Real average pre-tax income growth from 1962 to 2014 (taken from Piketty et al., 2018b)

Group medians conditional on debt service > 0. Construction of income quintiles using OECD-modified equivalence scale. Data from the Surveys of Consumer Finances.

Figure 6: Median growth of debt-service-to-income ratios in the US per income quintile.

Figure 7: Growth in house size (taken from Bellet, 2017)
4 Relation to the literature

Our paper combines ideas of network model with methods from continuous time macroeconomics. We build a tractable model that is a two-good, infinite-horizon, general equilibrium version of the static network game by Ballester et al. (2006). While they analyze the Nash equilibrium of a static game with strategic complementarities, we solve for general equilibrium—adding houses as a second good, budget constraints and market clearing conditions.

We formulate our quantitative model in continuous time building on Achdou et al. (2015), who provide a framework to efficiently solve heterogeneous agents models numerically. Our model adds type-dependent income processes (as in Gabaix et al., 2016; Guvenen et al., 2016) and other-regarding preferences. To the best of our knowledge, this is the first incomplete markets model with interdependent preferences such that agents influence each other not only via prices but also directly via their choices.\footnote{Despite the added complexity we can still solve for general equilibrium in a few seconds on a standard laptop computer.}

We provide a specific case where inequality matters for macroeconomic aggregates (see Ahn et al., 2017).

America has seen a boom in household debt since 1980 (see figure 1).\footnote{Despite the added complexity we can still solve for general equilibrium in a few seconds on a standard laptop computer.} There appear to be three explanations for this debt boom: (i) financial liberalization, (ii) rising demand for safe assets and (iii) rising demand for credit.

Financial innovation might have allowed banks to lower the lending standards for their credit products and offer these products to less credit-worthy households. Livshits et al. (2016) investigate the boom in credit card debt in the US since the 1980s. They argue that financial innovation (that is, better monitoring technologies) allowed banks to issue credit cards to ever lower income households. Indeed, Livshits et al. attribute 20% of the increase in credit card debt to new, less credit-worthy credit card holders. As we show in figure 2, credit card debt accounts only for a very small fraction of household debt (it is shown as part of “other debt” in the figure). Our focus is on mortgages, by far the most important debt category.

Favilukis et al. (2017) show how hard it is to generate a mortgage debt boom from laxer borrowing constraints in a macroeconomic model. They need the majority of the population to sit at or near the borrowing constraint initially, in order to get a sizable effect on house prices (which are their main focus) and mortgages. Their model has two types, one of which has a very strong bequest motive. Agents of this type are born rich with a bequest and pass it on to their children. The majority of the population is of the other, poor, type who never get far away from the borrowing constraint. Kiyotaki et al. (2011) and Sommer et al. (2013) have documented the same difficulties of generating a debt boom from loser borrowing constraints in standard heterogeneous agent macroeconomic framework.

What is more, the financial innovation doesn’t seem to be a consequence of greater productivity. Philippon (2015) finds constant unit costs of financial intermediation over time—despite large historical variations in the ratio of intermediated assets and GDP.

The debt boom might also be the financial sector’s reaction to an ever bigger demand for safe assets. Gorton (2016) views the production of safe assets as one of the major roles of the financial sector in an economy. Financial innovation allowed intermediaries to create safe assets from bundling and tranching mortgages. To satisfy the high demand for their product, they had to accept ever lower quality inputs—that is, mortgages with lower lending standards. So, where has the demand for safe assets come from? Gorton (2016) argues that the demand was created in emerging markets like China and India. As these
economies have grown richer, they wanted to invest their wealth in safe assets. Since their immature domestic financial markets could not provide them, investors resorted to the US.

Not only emerging markets have grown richer, but so have the top income groups in the US (we show that in figure 5). Kumhof et al. (2015) build a macroeconomic model around this fact. In their model the rich have preferences for financial wealth. As they grow richer, they demand ever more financial assets. This demand needs to be matched by credit taken out by the rest of the population. So, the surge in inequality causes an increase debt through higher credit supply.

We add to the literature on household debt by formalizing and testing the demand-centered theory that inequality has led to rising debt because non-rich households want to keep up with the living standard set by the rich (Rajan, 2010; Frank, 2013). We also do not limit our analysis to the period from 2001 to 2007 but recognize that debt started to grow much earlier.

Alvarez-Cuadrado and Japaridze (2017) analyze this mechanism in a very stylized setting with one good (no houses!), three income types without idiosyncratic risk and three periods. Absent durable goods, their model is unable to capture what we believe is the essential mechanism: substitution from non-durable consumption to durable housing automatically increases debt.
5 A Tractable Model of Mortgage Debt

We illustrate the mechanism in a simple infinite-horizon model with durable houses, but without uncertainty. We solve analytically for an equilibrium with constant policies. We will show that if agents substitute houses for consumption, their debt increases. Then we augment our model with other-regarding preferences. What pops out is a dynamic network game, that we solve for constant policies. We show that debt and houses are affine in the incomes of the reference group’s members. Under upward social comparisons, rising income of the rich will lead to higher aggregate debt in partial equilibrium.

5.1 Setup

We analyze an economy with a finite number of types of agents $i \in \{1, \ldots, N\}$.$^{11}$ Households live forever and have discount factor $\beta$. In each period they consume non-durable consumption $c$ (price 1), durable housing $h$ (price $p$) and can lend or borrow in a risk-free asset $a$ at interest rate $r$. Houses depreciate at rate $\delta$. Agents vary by their initial endowments $w_0$ and constant income streams $y$, which are deterministic and constant over time. Agents’ utility depends on reference housing, $\bar{h}$, which is a function of the housing distribution. They maximize life-time utility

$$\sum_{t=0}^{\infty} \beta^t u(c_t, h_t; \bar{h}_t)$$

subject to the period budget constraints

$$c_0 + ph_0 + a_1 = y + w_0,$$
$$c_t + p(h_t - (1 - \delta)h_{t-1} + a_{t+1} = y + (1 + r)a_t \text{ for all } t \geq 1,$$

and a life-time budget constraint

$$c_0 + ph_0 - y + \sum_{t=1}^{\infty} \left(\frac{1}{1 + r}\right)^t (c_t + p(h_t - (1 - \delta)h_{t-1}) - y) = w_0.$$

We assume that preferences are twice continuously differentiable and strictly concave.

**Assumption 1.** $\partial_x u > 0$, $\partial_{xx} u < 0$ for $x \in \{c, h\}$

In addition, we assume that the economy is at its steady state, where all policies and prices are constant over time.

**Assumption 2 (Steady state).** $c_t = c$, $h_t = h$ and $a_{t+1} = a$ for all $t$. Prices $p$ and $r$ are time constant as well.

Since the problem is deterministic, stationarity requires $(1 + r)\beta = 1$ leaving $p$ as the only price to be determined in general equilibrium. Hence, we can rewrite the problem as

$$\max_{c,h,a} u(c, h; \bar{h})$$

s.t. $c + ph + a = y + w_0$
$$c + p\delta h - ra = y$$
$$(1 + \frac{1}{r})c + ph + \frac{1}{r}\delta ph = \frac{y}{r} + w_0.$$  

Finally, we assume a linear relationship between optimal consumption $c$ and housing $h$.

$^{11}$When turning to the general equilibrium analysis, we will assume a continuum of agents for each type, so that no agent’s choices affect prices.
Assumption 3. \( c = \alpha_1 h - \alpha_0 \), where \( \alpha_1, \alpha_0 \geq 0 \).

For example, assumption 3 is satisfied for any utility function \( u \) that satisfies \( u(c, h, \bar{h}) = f(c^{1-\xi}(\phi_1 h - \phi_2 \bar{h})^\xi) \).

Under these assumptions we can solve the household’s problem for given prices. The solution is summarized in the lemma below. Lemma 1 gives explicit expressions for optimal housing and loans (or savings) which we can use to analyze optimal debt holdings of households.

Lemma 1. Under assumptions 1, 2 and 3

\[
h = \frac{(1 + r)(y + \alpha_0) + rw_0}{p(r + \delta) + \alpha_1(1 + r)}, \quad a = \frac{w_0(\alpha_1 + p\delta) - (y + \alpha_0)p(1 - \delta)}{p(r + \delta) + \alpha_1(1 + r)}
\]

Proof. See appendix A.1.

5.2 Debt with conventional preferences

Lifetime wealth consists of two components: initial wealth and income flows. Proposition 1 shows that agents are indebted if their initial wealth is sufficiently low relative to life-time earnings.

Proposition 1. Under assumptions 1, 2 and 3 an agent borrows \((a < 0)\) iff

\[
w_0 < (y + \alpha_0)p(1 - \delta) \frac{1}{\alpha_1 + p\delta}
\]

Proof. This follows directly from the explicit expression for \( a \) in lemma 1.

Since housing is a durable good, it may make sense to use credit in order to buy a big house right away and use future incomes to repay the debt while still enjoying the house. Hence, if agents are poor initially but expect to earn a lot over their lifetime, they will borrow in order to buy an appropriately sized house. This is because the expenditures for the house are relatively high \((ph)\) in the first period and relatively low \((p\delta h)\) in all subsequent periods.

In particular, all agents with zero initial wealth will hold debt. This is illustrated in figure 8.

![Figure 8: Agents without initial wealth hold debt.](image)

Cash flows is not constant over time. In the first period, the household pays

\[
\text{payment}_0 = c + ph, \quad \text{payment}_t = c + p\delta h \text{ for } t \geq 1.
\]
The full house price is due in the initial period, and only “maintenance costs” to cover depreciation thereafter. In each period $t > 0$, income $y$ will exceed expenditures. There are excess funds

monthly rate $= y - c - \delta ph > 0$.

(This follows directly from the life-time budget constraint.) The household takes out a loan worth the present discounted value of all these funds

$$a = \sum_{t=1}^{\infty} \left( \frac{1}{1+r} \right)^t (c + \delta ph - y) = \frac{1}{r} (c + \delta ph - y) < 0,$$

in the initial period. This loan is just so high that it can be repaid until the infinite future. The household will spend the loan and his remaining funds on the house,

$$ph = a + y - c = \frac{1}{r} (c + \delta ph - y) + y - c.$$

**Debt rises if agents substitute houses for consumption** Assume now two different agents, who take prices as given. They have the same exogenous income stream and initial wealth. One agent values houses more than the other. Given assumption 3 this means that

$$\alpha^H_1 < \alpha^C_1, \quad \text{or} \quad \alpha^H_0 > \alpha^C_0.$$

Then agent $H$, who values houses more, will be more indebted.

**Proposition 2.** Let assumptions 1, 2 and 3 be given. If agents value houses relatively more ($\alpha_1$ goes up or $\alpha_0$ goes down), borrowing increases (or savings decrease),

$$\frac{\partial a}{\partial \alpha_1} > 0 \quad \text{and} \quad \frac{\partial a}{\partial \alpha_0} < 0.$$

**Proof.** See appendix A.2.

If for some reason (and given prices) agents substitute consumption for houses their debt will increase. They lower their consumption in every period until the infinite future and need to reshuffle their funds to pay for housing. The biggest chunk has to be paid upfront (downpayment), and only maintenance work has to be paid later to make up for the depreciation. To balance these unequal payment flows they increase their credit.

### 5.3 Debt with other-regarding preferences

We now assume that agents have social status concerns. They value their houses relative to the houses of their reference group. For tractability, we choose the following utility function which results in the above affine relationship between optimal consumption and housing.

**Assumption 4** (Social comparisons). $u(c, h, \bar{h}) = c^{\xi} (h - \phi \bar{h})^{1-\xi}$ and $\bar{h}_i = \sum_{j \neq i} \sigma_{ij} h_j$ is a weighted sum of other agent’s consumption. We can write the vector of housing references as

$$\bar{h} = (\bar{h}^1, \ldots, \bar{h}^N)^T = \Sigma \cdot h := (\sigma_{ij})(h^i).$$

where the matrix $\Sigma$ can be interpreted as the adjacency matrix of the network of types capturing the comparison-links between agents of each type.

We further require the comparisons to satisfy the following condition.
Assumption 5. The Leontief inverse \((I - \phi\Sigma)^{-1}\) exists and is equal to \(\sum_{i=0}^{\infty} \phi^i \Sigma^i\).\(^{12}\)

We show that under assumption 4 an agent A’s debt increases if another agent B’s lifetime income increases—as long as there is a direct or indirect link from A to B (proposition 3 below). That link exists, if agent A cares about agent B, or if agent A cares about some agent C who cares about agent B.

Under the stronger assumption of upward social comparisons, we can link aggregate debt to rising inequality.

**Assumption 4’ (Upward social comparisons).** We assume further that agents are ordered by their lifetime income \(rw_{i0} + y^i\). Agents compare themselves only with those with a higher income.

\[\sigma_{ij} = 0 \text{ if } i \text{ richer than } j \,(\text{i.e. } i \geq j), \quad \sigma_{ij} \geq 0 \text{ if } i \text{ poorer than } j \,(\text{i.e. } i > j)\]

That is, \(\Sigma\) is lower triangular with zeros on the diagonal. This is illustrated in figure 9. Note that \(\Sigma^N = 0 \in \mathbb{R}^{N \times N}\), so assumption 5 is always satisfied.

If incomes of the lifetime income richest type rises—that is, top income and wealth inequality grows—aggregate debt will rise as well (proposition 4 below).

5.3.1 General social comparisons

We have to solve the optimization problems of all agent jointly. For each agent we get an equation of the form

\[h_i = C_2(y_i + rw_{i0}) + C_1\phi h_i.\]

These can be stacked into a system of linear equations and solved jointly. For given house price \(p\), lemma 2 provides analytical solutions for agents’ equilibrium choices.

**Lemma 2.** Under assumptions 2 (steady state), 4 (social comparisons) and 5 (existence of Leontief inverse; assumptions 1 and 3 are implied)

\[h = C_2 \sum_{i=0}^{\infty} (C_1\phi\Sigma)^i (y + rw_0)\]

\[a = \frac{C_2}{1 + r} ((\alpha_1 + p\delta)w_0 - p(1 - \delta)y - \frac{p^2(1 - \delta)\phi^2}{1 - \xi} \sum_{i=0}^{\infty} (C_1\phi\Sigma)^i - I)(y + rw_0))\]

**Proof.** See appendix A.3. \(\square\)

Agents choices depend on a weighted average of all other agents’ lifetime incomes. The weights depend on the (direct and indirect) social links, captured by the income-weighted Bonacich centrality, \(\sum_{i=0}^{\infty} (C_1\phi\Sigma)^i (y + rw_0)\). Our results show that with social comparisons (and for given prices) the agents decisions will depend on other agents decisions, an thus on their incomes and initial wealths.\(^{13}\)

These results are summarized in the following proposition.

**Proposition 3.** Under assumptions 2 (steady state) and 4 (social comparisons; assumptions 1 and 3 are implied)

(i) house size is linearly increasing in own lifetime income and the lifetime income of the comparison group.

\(^{12}\)This assumption is satisfied whenever the sequence \(\Sigma^i \to \Sigma^\infty\). For example, if \(\Sigma\) is a stochastic matrix with a stationary distribution.

\(^{13}\)These results are reminiscent of those in Ballester et al. (2006). They showed that the unique Nash equilibrium in a large class of network games is proportional to the (standard) Bonacich centrality.
(ii) household debt is linearly increasing in the lifetime income of the comparison group and own income and linearly decreasing in own initial wealth.

Proof. This follows directly from lemma 2. 

5.3.2 Upward social comparisons

Now, we consider the case where agents only compare themselves with richer peers. We order agents by their lifetime income $y_i + r w_0$. Type 1 is the poorest and type $N$ is the richest. The corresponding graph and adjacency matrix are shown in figure 9.

![Graph](image)

**Figure 9:** The network under upward looking comparisons. Types are ordered by their permanent income. For poorest (1) to richest (N). Each type $i$ has edges only to richer types $j > i$.

**Proposition 4.** With upward social comparisons (assumption 4'),

(i) rising top incomes lead to rising debt for everybody, and thus to rising aggregate debt.

(ii) debt-service-to-income ratios rise. (because debt rises and income is constant)

**Conjecture 1.** (i) A mean preserving spread in the income distribution leads to rising aggregate debt.

The question will be if the response to the poor is weaker than the response to the rich. And the response is weaker! The “direct effect” will exactly be offset between the poor and the rich. The “indirect effect” will only affect the poor. So, it should be easy to show that total debt increases after redistribution from rich to poor.

5.3.3 Example: Upward comparisons with three agents

We now illustrate the results for the simple case of three types of agents, poor $P$, middle class $M$, and rich $R$. The poor type compares himself with both other types, the middle type compares himself only with the rich type, and the rich type not at all. Figure 10 shows the corresponding graph and its adjacency matrix.

We want to explain the general results from above. So, as a first step, we n As always under upward comparisons, the adjacency matrix is convergent with $\Sigma^3 = 0$. 

$$
\Sigma = \begin{pmatrix}
0 & \sigma_{12} & \sigma_{13} & \cdots & \sigma_{1N} \\
0 & 0 & \sigma_{23} & \cdots & \sigma_{2N} \\
\vdots & \vdots & \ddots & \ddots & \vdots \\
0 & 0 & 0 & \ddots & \sigma_{N-1,N} \\
0 & 0 & 0 & \cdots & 0
\end{pmatrix}
$$

(a) Graph

(b) Adjacency matrix
The matrix $\Sigma^2$ counts the paths of length 2. In our example there is only one such path—from type $P$ to type $R$. Defining $\phi = C_1\phi$, the vector of Bonacich centralities is given by

$$\sum_{i=0}^{\infty} \alpha^i \Sigma^i = I + \sum_{i=1}^{2} \alpha^i \Sigma^i = I + \begin{pmatrix} 0 & \alpha \cdot \sigma_{PM} & \alpha \cdot \sigma_{PR} + \alpha^2 \cdot \sigma_{PM} \cdot \sigma_{MR} \\ 0 & 0 & \alpha \cdot \sigma_{MR} \\ 0 & 0 & 0 \end{pmatrix}$$

The partial equilibrium choices for housing and assets are now given by

$$\begin{pmatrix} h_P \\ h_M \\ h_R \end{pmatrix} = C_2(y + rw_0) + C_2 \begin{pmatrix} 0 & \tilde{\phi} \cdot \sigma_{PM} & \tilde{\phi} \cdot \sigma_{PR} + \tilde{\phi}^2 \cdot \sigma_{PM} \cdot \sigma_{MR} \\ 0 & 0 & \tilde{\phi} \cdot \sigma_{MR} \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} y_P + rw_0^P \\ y_M + rw_0^M \\ y_R + rw_0^R \end{pmatrix}$$

$$a = \frac{C_2}{1+r} (\alpha_1 + p\delta)w_0 - p(1-\delta)y - \frac{p^2(1-\delta)\xi C_2}{1-\xi} \phi \left( \sum_{i=0}^{\infty} C_1^i \phi^i \Sigma^i - I \right) (y + rw_0)$$

An agent’s housing choice increases linearly in own permanent income, $y + rw_0$, and on the permanent income of agents in the reference group. The poor agent’s consumption increases through the direct links, but also indirect links. These are discounted stronger ($\phi^2$ instead of $\phi$). Agents’ decisions to save or borrow depend on the ratio of initial wealth $w_0$ and income $y$. The higher the income relative to initial wealth, the greater the need to borrow.

5.4 General Equilibrium

To be completed.
6 A quantitative model

In this section, we set up a quantitative incomplete markets model with heterogeneous households who value social status derived from relative housing/consumption. The model is cast in continuous time.

6.1 Households

Our economy features a continuum of households who vary in their human capital and face idiosyncratic, income risk. They receive income \( y \) which can be used to purchase a non-durable consumption good \( c \) at price 1, durable housing \( h \) at price \( p \) and invest in a safe asset \( a \) which pays interest \( r \). As we are interested in long-run trends, we assume that housing and assets can be traded frictionlessly. This allows us to work with net wealth, \( w = a + ph \), as the only endogenous state variable. The state of the economy is then the joint distribution of wealth and income, \( G(w, y) \), with joint density \( g(w, y) \).

Income  Human capital is assumed to be permanent and drawn from a finite set giving rise to discrete income types \( j = 1, \ldots, J \) with exogenous population share \( \pi^j \). The assumption of different fixed income types is firmly rooted in recent empirical studies of the US earnings process and the stationary analogue of heterogeneous income profiles (Guvenen, 2007, 2009; Guvenen et al., 2016). We model income \( y^j \) of household \( i \) with permanent income type \( j \) as a diffusion process with type-specific parameters

\[
d y^j_i = \mu^j(y^j_i)dt + \sigma^j(y^j_i)dW^j_i
\]

where \( W \) is a Wiener process with normally distributed increments. Assuming that income follows an Ornstein-Uhlenbeck (OU) process gives rise to a Gaussian stationary income distribution. We thus choose to model log income as an OU-Process.\(^{14}\)

\[
d \log y^j_i = \theta^j(\mu^j - \log y^j_i)dt + \sigma^j dW^j_i
\]

Within each permanent income group, income is log-normally distributed.

Preferences  Households derive utility from non-durable consumption \( c \) and value social status derived from relative housing. That is, a given level of housing quality \( h \) induces utility and the excess housing of richer households, \( \bar{h} - h \), leads to disutility. Thereby, \( \bar{h} \) is a reference measure such as the average quality of all better houses. At the current stage of the project, we assume CRRA preferences over consumption and a linear combination of own housing and excess deprivation, \( s(h, \bar{h}) \).

\[
u(c, h, \bar{h}) = \left( \frac{c^{1-\xi}s(h, \bar{h})^\xi}{1 - \gamma} \right)^{1-\gamma}
\]

\[
s(h, \bar{h}) = \phi_1 h - \phi_2 \bar{h}
\]

Reference Housing  The reference measure of housing, \( \bar{h} \), is the social benchmark to which households compare their own house. It is a function of the housing choices of other households and hence of the joint distribution of income and wealth, \( \bar{h} = \bar{h}(G) \). In general, one can express \( \bar{h} \) as a weighted average of all houses in the population

\[
\bar{h} = \int x(h(w, y))h(w, y)dG(w, y)
\]

\(^{14}\)The OU-process is the continuous time analogue of a Gaussian AR1 process. The parameter \( \theta \) governs the speed of mean reversion. Using Ito’s Lemma, this implies the following process for the level of income: \( dy = (\theta(\mu - \log y) + \frac{\sigma^2}{2}y)dt + \sigma y dW \).
Building on evidence from economics and psychology, we assume that comparisons are upward-looking. Households only care about how their house stacks up against bigger/better houses. The asymmetric nature of interpersonal comparisons makes reference housing also a function of own housing, $\bar{h} = \bar{h}(h, G)$.

$$\bar{h}(h) = \int x(h, h(w, y)) h(w, y) dG(w, y)$$  \hspace{1cm} (6)

A natural assumption is to define reference housing as the average housing quality of all bigger houses:

$$\bar{h}(h) = \mathbb{E}[h'|h > h] = \frac{1}{1 - F(h)} \int_{h}^{\infty} h' dF(h')$$  \hspace{1cm} (7)

where $F_h$ is the marginal distribution of houses in the economy.

**Endogenous Reference Groups** Specifying upward-looking comparisons as in 7 where the reference group depends on the household’s choice of housing imply that marginal utility of housing may not be positive since more housing also increases the reference measure.

$$u_h(c, s(h)) = \frac{\partial u}{\partial s} \begin{cases} 1 + \phi - \phi \bar{h}'(h) > 0 \quad \Rightarrow \quad \bar{h}'(h) < \frac{1 + \phi}{\phi} \end{cases}$$  \hspace{1cm} (8)

While assuming instantaneous updating of the reference group seems like a questionable assumption in light of evidence from behavioral economics and psychology, the main challenge is computational. In particular, one has to iteratively solve the household problem to find a schedule of reference housing that is consistent with the housing choices induced by it.

**Exogenous Reference Groups** At the current stage of the project, we opt for an alternative, simpler specification of reference groups that is both closely connected to the tractable model and will allow us to solve the household problem recursively. The above assumption of permanent income types splits the household problem into $J$ disjoint subproblems – one for each income type. We now assume that households only compare themselves to households of a higher type.

$$\bar{h}^j = \mathbb{E}[h^k | k > j]$$  \hspace{1cm} (9)

Comparisons are hence quasi-upward-looking as, on average, these households will have bigger houses. Assuming that the highest income type does not have interpersonal preferences, we can now solve the household problems recursively, starting with the highest income type.

**Budget Constraint** The budget constraint is given by the drift of wealth

$$\dot{a}_{t} + p_{t} \dot{h}_{t} = y_{t}^d + r_{t} a_{t} - \delta p_{t} h_{t} - c_{t}$$

$$\dot{w}_{t} = y_{t}^d + r_{t} w_{t} - (r_{t} + \delta) p_{t} h_{t} - c_{t}$$  \hspace{1cm} (10)

**Collateral Constraint** Households can take on collateralized debt in order to buy housing. The household’s debt constraint is a function of its collateral/ home value, $p_t h_t$. 18
In particular, we impose that the initial mortgage cannot exceed a fixed fraction of the collateral value:

$$-a_t \leq \psi p_t h_t \iff p_t h_t \leq \frac{w_t}{1-\psi}$$ (11)

Thereby \( \psi \in [0,1] \) is the maximum admissible loan-to-value (LTV) ratio. Denote the set of feasible housing choices by

$$\mathcal{H}(w) = \{ h : (1-\psi)ph \leq w \}$$ (12)

### 6.2 Stationary Equilibrium

A stationary equilibrium in our economy is characterized by a set of value and policy functions and stationary distributions as well as a schedule of reference housing that solve the income-type-specific HJB- and KF-equations as well as market clearing conditions for housing and assets. For better readability, we present them without the \( j \)-superscript for income types.

**HJB Equations** The household’s value and policy functions and the schedule of reference housing have to satisfy the following set of partial differential equations:

$$\rho v(w,y) = \max_{c,h \in \mathcal{H}(w)} u(c,h,h) + \partial_w v(w,y)(y + rw - (r + \delta)ph - c)$$

$$+ \partial_y v(w,y)\mu(y) - \frac{\sigma(y)^2}{2}\partial_{yy} v(w,y)$$ (13)

**Optimal Choices** The policies for housing and consumption have to satisfy the following set of optimality conditions

$$\partial_c u(c,h;\tilde{h}) = \partial_w v(w,y)$$

$$\partial_h u(c,h;\tilde{h}) = \partial_w v(w,y)(r + \delta)p + \lambda(1-\psi)p$$

$$0 = \lambda(w - (1-\psi)ph)$$ (16)

where \( \lambda \) is the multiplier on the collateral constraint. Let \( c^*(w,y), h^*(w,y) \) be the optimal policies satisfying these conditions. The formulas are given in appendix. Define the optimal drift of wealth as \( d^* = y + rw - (r + \delta)ph^* - c^* \).

**Reference Housing** The vector of reference housing measures \((\tilde{h}^j)_{j=1,...,J-1}\) is consistent with the optimal choices of households:

$$\tilde{h}^j = \frac{1}{\sum_{k>j} \pi^k} \sum_{k>j} \pi^k \int h^{k*}(w,y) dG(w,y) \quad \forall j < J$$ (17)

**Kolmogorov Forward Equations** The stationary distributions and policy functions of each type have to satisfy the following differential equations:

$$0 = -\partial_w \left(d^*(w,y)g(w,y)\right) - \partial_y \left(\mu(y)g(w,y)\right) + \frac{1}{2}\partial_{yy} \left(\sigma(y)^2g(w,y)\right)$$ (18)

---

15This specification follows Favilukis et al. (2017) and Kaplan et al. (2017).
Market Clearing  We assume that financial assets are in zero net supply and fully inelastic housing supply $H^S$. The house price $p$ and the interest rate $r$ then have to solve

$$0 = \sum_{j=1}^{J} \pi^j \int a^{j*}(w, y^j; r, p) dG^j(w, y^j)$$  \hspace{1cm} (19)$$

$$H^S = \sum_{j=1}^{J} \pi^j \int h^{j*}(w, y^j; r, p) dG^j(w, y^j)$$  \hspace{1cm} (20)$$

In the future, we will make the supply of housing elastic with respect to housing demand by introducing construction firms who use labor and land permits to produce housing.

6.3 Solution Method

We solve the HJB- and KF-equations using the finite difference methods proposed in Achdou et al. (2015). Assuming exogenous reference groups enables us to solve the partial equilibrium recursively starting with the highest income type (instead of iteratively finding a reference housing schedule that is consistent with the HJB- and KF-equations).

One such equilibrium—the stationary equilibrium and policies—is shown in figure 11.

6.4 Preliminary Quantitative Results

Below we present some numerical exercises. We use conventional parameter values. In a future version, these will be chosen more carefully. In particular, we want to estimate the parameters of the type-dependent income process.

The results show that in partial equilibrium, agents react to an increase in reference housing through lower consumption, more houses and higher debt.

Further, rising inequality drives up aggregate debt in general equilibrium under social comparisons.

The role of comparisons  The intuition from the tractable model carries over to the quantitative model. If reference housing increases, agents substitute housing for consumption. Figure 12 illustrates how agents react (on average) to changes in reference
consumption—holding prices constant. When the houses in the reference group increase, agents reduce consumption and increase housing. As a consequence aggregate credit increases.

Rising top income inequality drives up debt in General Equilibrium  

We compare two steady states, that roughly correspond to the years 1980 and 2013 in the US. The models have four types, that loosely correspond to the following income groups: (1) the bottom 20%, (2) P20-P50, (3) P50-P90, and (4) the top 10% of income earners. We solve for a baseline general equilibrium and then double the income of the top-10% (which is close to the observed increase, see figure 5). We solve these general equilibria with ($\phi = 0.3$) and without ($\phi = 0$) comparisons.

The results are shown in table 1. The table shows the average demands for each type and for the total economy. We see that in the case with comparisons, total debt grows by 29.2%, almost 10 percentage points more than without comparisons (18.4%). While the bottom group (type 1) actually reduces debt in the case without comparisons ($-45.9\%$), all other types’ debt increases stronger with comparisons.

Figures 13 and 14 show the marginal distributions of income, wealth, assets and housing for each type. Most notably, income of the richest type doubles and as a consequence the housing demand shifts to the right in figure 14.
Table 1: Numerical example: Comparison of steady states

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<th></th>
<th>without comparisons</th>
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<td>growth</td>
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<tr>
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Figure 13: Baseline Scenario

Figure 14: Richer Rich
References


Part I
Appendix
A Proofs

A.1 Proof of lemma 1
From the period \( t > 0 \) budget constraint and the assumed form of \( c \) we obtain an expression for \( a \).
\[
a = \frac{\alpha_1 h - \alpha_0 + p\delta h - z}{r} = \frac{(\alpha_1 + p\delta) h - (z + \alpha_0)}{r}
\]
Then we plug \( a \) into the period-0 budget constraint to obtain the desired expression for \( h \).
\[
(z + \alpha_0) + w_0 = (\alpha_1 + p)h + \frac{1}{r}((\alpha_1 + p\delta)h - z)
\]
\[
\iff r(z + w_0) + \bar{z} = (r(\alpha_1 + p) + \alpha_1 + p\delta)h
\]
\[
\iff h = \frac{r(z + w_0) + \bar{z}}{r(\alpha_1 + p) + \alpha_1 + p\delta} = \frac{(1 + r)\bar{z} + rw_0}{p(r + \delta) + \alpha_1(1 + r)}
\]
Plugging in for \( h \) in the first equation gives the desired expression for \( a \).
\[
a = \frac{(\alpha_1 + p\delta)(1 + r)\bar{z} + rw_0}{r(\alpha_1 + p) + \alpha_1 + p\delta} - \bar{z}
\]
\[
= \frac{\alpha_1 + p\delta}{p(r + \delta) + \alpha_1(1 + r)}((1 + r)\bar{z} + rw_0) - \bar{z}
\]
\[
= \bar{z}\left(\frac{\alpha_1 + p\delta}{p(r + \delta) + \alpha_1(1 + r)}(1 + r) - 1\right) + w_0r\frac{\alpha_1 + p\delta}{p(r + \delta) + \alpha_1(1 + r)}
\]
\[
= \bar{z}\left(\frac{(1 + r)(\alpha_1 + p\delta) - (p(r + \delta) + \alpha_1(1 + r))}{p(r + \delta) + \alpha_1(1 + r)}\right) + w_0r\frac{\alpha_1 + p\delta}{p(r + \delta) + \alpha_1(1 + r)}
\]
\[
= \bar{z}\frac{rp(\delta - 1)}{p(r + \delta) + \alpha_1(1 + r)} + w_0r\frac{\alpha_1 + p\delta}{r(\alpha_1 + p) + \alpha_1 + p\delta}\in(0, 1)
\]
\[
\Rightarrow a = \frac{w_0(\alpha_1 + p\delta) - \bar{z}p(1 - \delta)}{p(r + \delta) + \alpha_1(1 + r)} = \frac{w_0(\alpha_1 + p\delta) - (z + \alpha_0)p(1 - \delta)}{p(r + \delta) + \alpha_1(1 + r)}
\]

A.2 Proof of proposition 2
Using the explicit solution from lemma 1,
\[
\frac{\partial a}{\partial \alpha_1} = -\frac{\bar{z}p(\delta - 1)}{(p(r + \delta) + \alpha_1(1 + r))^2} + w_0\frac{r(\alpha_1 + p) + \alpha_1 + p\delta - (\alpha_1 + p\delta)(1 + r)}{(p(r + \delta) + \alpha_1(1 + r))^2}
\]
\[
= \frac{(z + \alpha_0)p(1 - \delta) + w_0rp(1 - \delta)}{(p(r + \delta) + \alpha_1(1 + r))^2} > 0 \quad \text{if } z > 0 \text{ and } w_0 \geq 0 \text{ or vice versa.}
\]
\[
\frac{\partial a}{\partial \alpha_0} = -\frac{p(1 - \delta)}{p(r + \delta) + \alpha_1(1 + r)} < 0.
\]

A.3 Proof of lemma 2
We need two more simple lemmas.
Lemma 3. With social comparisons (assumption 4)

\[ c = p \frac{\xi}{1 - \xi} h - p \frac{\xi}{1 - \xi} \phi \bar{h} = \alpha_1 h - \alpha_0 \]

Proof. The first order conditions are

\[ \lambda = \xi \frac{u}{c} \text{ and } \lambda p = (1 - \xi) \frac{u}{h - \phi h} \]

So, \( c = p \frac{\xi}{1 - \xi} h - p \frac{\xi}{1 - \xi} \phi \bar{h} \).

Lemma 4. If \( \sum_{i=0}^{\infty} a_i W^i \) converges, then

\[ W(I - aW)^{-1} = \frac{1}{a} \left( \sum_{i=0}^{\infty} a_i W^i - I \right) = \frac{1}{a} \left( \sum_{i=1}^{\infty} a_i W^i \right). \]

Proof. Two lines of algebra that have to be written up.

Using the previous two lemmas we can tackle the proof that we are after. From lemma 3 we know that

\[ c = p \frac{\xi}{1 - \xi} h - p \frac{\xi}{1 - \xi} \phi \bar{h} = \alpha_1 h - \alpha_0 \]

From lemma 1 we know that

\[ h = (1 + r)(z + \alpha_0) + rw_0 \]

\[ \frac{p}{r + \delta} + \alpha_1 (1 + r) \]

Stacking these equations for all \( N \) households we get

\[ h = C_2 \]

using that \( \bar{h} = \Sigma h \) we get

\[ h = \frac{(1 + r)(z + \alpha_0) + rw_0}{p(r + \delta) + \alpha_1 (1 + r)} \]

Similarly, we stack the expressions for \( a \) (from lemma 1)

\[ a = \frac{1}{p(r + \delta) + \alpha_1 (1 + r)} ((\alpha_1 + p\delta)w_0 - p(1 - \delta)(z + \alpha_0) \]

\[ = \frac{C_2}{1 + r} ((\alpha_1 + p\delta)w_0 - p(1 - \delta)(z + p \frac{\xi}{1 - \xi} \phi \bar{h}) \]

\[ = \frac{C_2}{1 + r} ((\alpha_1 + p\delta)w_0 - p(1 - \delta)(z + p \frac{\xi}{1 - \xi} \phi \Sigma C_2 (I - C_1 \phi \Sigma)^{-1}(z + rw_0)) \]

using lemma 4

\[ = \frac{C_2}{1 + r} ((\alpha_1 + p\delta)w_0 - p(1 - \delta)z \]

\[ - \frac{p^2(1 - \delta)\xi C_2}{1 - \xi} \phi \left( \sum_{i=1}^{\infty} C_i \phi^i \Sigma^i \right)(z + rw_0)). \]
B Language of networks

A network is a collection of nodes \( N = \{1, \ldots, N\} \) and edges connecting these nodes. The network can be represented by its adjacency matrix \( \Sigma = (\sigma_{ij}) \in \mathbb{R}^{N \times N} \). Each entry \( \sigma_{ij} \) stands for the link from node \( j \) to node \( i \). Node \( j \) is linked to node \( i \) if and only if \( \sigma_{ij} \neq 0 \).

In our setting the network consists of \( N \) consumers (nodes). Consumers are linked if they care about one another. That is, if agent \( i \)'s takes agent \( j \)'s choices into account, then \( \sigma_{ij} \) is positive.

Our network is weighted (consumers care differentially about others) and directed (\( j \) might care about \( i \) while \( i \) does not care about \( j \)).

Paths The adjacency matrix shows which nodes are connected with paths of length one. In addition, agents \( i \) and \( j \) might be connected via a third agent \( k \) (if agent \( i \) cares about \( k \) and \( k \) cares about \( j \)). In this case there is a path of length 2 from \( i \) to \( j \). The weighted number of length-2-paths between to nodes is given by the squared adjacency matrix \( \Sigma^2 \). More generally, the \( k \)-th power of \( \Sigma \) “count” the weighted paths of length \( k \).

Centrality In order to analyse a network is useful to look at measures of centrality. Agent \( i \)'s Bonacich centrality measures how many paths (of varying length) lead to his node, that is, it measures how much agent \( i \) cares about other agents. Longer path get a smaller weight.

The total (weighted) number of paths is given by the infinite series

\[
\sum_{k=0}^{\infty} v^k \Sigma^k = (I - v\Sigma)^{-1},
\]

which is called the Leontief inverse (if the series converges). The \((i, j)\) component of this matrix is the discounted weighted number of paths from \( j \) to \( i \).

Using the Leontief inverse we can define Bonacich centrality.

Definition 1 (Bonacich centrality). For a network with adjacency matrix \( \Sigma \) and a scalar \( v \in [0, 1] \) the vector of Bonacich centralities is given by

\[
b(\Sigma, v) = (I - v\Sigma)^{-1} 1.
\]

For an agent \( i \) the Bonacich centrality counts the number of paths of any length from all other agents to \( i \). That is, Bonacich centrality measures how much agent \( i \) cares about others.

Definition 1’ (Weighted Bonacich centrality). For a an adjacency matrix \( \Sigma \in \mathbb{R}_{+}^{N \times N} \), a scalar \( v > 0 \) and a row vector \( w \in \mathbb{R}_{+}^N \) define the vector of weighted Bonacich centralities as

\[
b(\Sigma, v, w) = (I - v\Sigma)^{-1} w,
\]

The standard Bonacich centrality is obtained by \( b(\Sigma, v) = b(\Sigma, v, 1) \).

C Quantitative Model

In this section, we provide the optimal choice of consumption and housing for a given value function \( v(w, y) \). We suppress the \( j \)-superscript for distinct income types. The optimality conditions are:

\[
\frac{1 - \xi}{c} \left( c^{1-\xi} ((1 + \phi) h - \phi h)^{1-\gamma} \right) = \partial_w v(w, y) \tag{21}
\]

\[
\frac{\xi(1 + \phi)}{(1 + \phi) h - \phi h} \left( c^{1-\xi} ((1 + \phi) h - \phi h)^{1-\gamma} \right) = \partial_w v(w, y) (r + \delta) p + \lambda(1 - \psi) p \tag{22}
\]

\[
\lambda(w - (1 - \psi) ph) = 0 \tag{23}
\]
Case 1: Unconstrained  When the housing choice is unconstrained, i.e. $\lambda = 0$, we can express relative housing as a function of consumption

$$(1 + \phi)h - \phi \bar{h} = \frac{\xi (1 + \phi)c}{(1 - \xi)(r + \delta)p}$$

(24)

Using this in the first optimality condition gives optimal consumption as

$$c^*(w, y; \bar{h}) = \left( \frac{\partial w v(w, y)}{1 - \xi} \left( \frac{(1 - \xi)(r + \delta)p}{\xi (1 + \phi)} \right)^{\xi (1 - \gamma)} \right)^{-1/\gamma}$$

(25)

and thus optimal unconstrained housing as:

$$h^*(w, y; \bar{h}, v) = \frac{\xi c^*(w, y; \bar{h})}{(1 - \xi)(r + \delta)p} + \frac{\phi \bar{h}}{1 + \phi}$$

(26)

Case 2: Constrained  If the collateral constraint binds, housing is simply given by

$$h^*(w) = \frac{w}{(1 - \psi)p}$$

(27)

Since the collateral constraint is not a function of consumption, the first optimality condition still holds such that optimal consumption is now:

$$c^*(w, y; \bar{h}, v) = \left( \frac{\partial w v(w, y)}{1 - \xi} \left( (1 + \phi)h^*(w) - \phi \bar{h} \right)^{\xi (\gamma - 1)} \right)^{1/\gamma}$$

(28)