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**Optimal Public Investment in Economic Centers and the Periphery**

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Optimal Public Investment in Economic Centers and the Periphery*

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Abstract

We analyze productivity enhancing public expenditure in a spatial economic model with labor mobility, firm-specific increasing returns to scale, and transport costs. Building on Krugman (1991), Fujita, Krugman and Venables (1999) and Redding (2016), we compare optimal investment and tax policies of fiscally autonomous regions to those of a benevolent central planer. We find transport costs and the size of scale effects to influence optimal tax and spending rates under both regimes. For sufficiently low transport costs and low substitutability between manufactured goods, regional fiscal autonomy leads to underinvestment: The lower the transport costs, the lower the local investment, and the higher the potential welfare gain through centralized policies. Our results challenge the view that local public goods should be financed entirely by local governments. They also help explain the recent decline of public investment at the municipal level in fiscally decentralized countries like Germany.

Keywords: quantitative spatial economics, public investment, fiscal federalism.

JEL Codes: R12, R53, H72, H77

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1 Introduction

In recent years, concerns have been rising about low public investment in several industrial
countries.\(^1\) In Germany, for example, survey results indicate an accumulated investment
backlog at the municipal level of 126 billion Euro, roughly equivalent to 4% of Germany’s
GDP.\(^2\) Observers blame this backlog on misguided policies, namely enduring fiscal consol-
idation in recent years.\(^3\) As a remedy, they prescribe both devoting additional financial
resources to lower levels of government and helping them to rebuild planning and managing
capacities for investment projects so that they can spend those funds effectively.

In this paper we analyze how geography – transport costs and agglomeration effects
– influence public investment decisions by municipalities. Our results contradict the above
policy prescriptions: Transport costs and agglomeration effects create pecuniary externalities
and coordination failure among different levels of government.\(^4\) These provide incentives for
local governments to underinvest.

Our model thus helps explain the existing investment backlog. It also identifies two fall-
lacies: First, if this backlog existed due to fiscal consolidation, municipalities should resume
to “normal” levels of public investment once this consolidation process is finished. However,
in our model underinvestment arises endogenously and permanently, so even after consoli-
dation is finished, it predicts that underinvestment will persist. Second, municipalities know
best what to spend their money on as they have superior information on local conditions.
The results of our model and calibration exercise show substantial efficiency losses when
public investment is under local control. Consequently, the model advocates central control
of public investment. As such, it requires scrutinizing whether local information advantages

\(^1\)See e.g. OECD (2016), pp. 42; IWF (2014), pp. 75.
\(^2\)Kreditanstalt für Wiederaufbau (2017), which surveyed municipal financial managers
\(^3\)Expertenkommission “Stärkung von Investitionen in Deutschland” (2015).
\(^4\)As Ottaviano and Thisse (2001) point out, pecuniary externalities might be particularly suitable to
explain spatial economic patterns on a larger geographical scale. This is in contrast to technological exter-
nalities, which have limited spatial reach such as within cities or business districts, and which might occur
in complex and diffuse ways, p. 159-160.
outweigh efficiency gains from central control.

Within the European Union, public investment is decentralized and fragmented: on average, subnational government bodies spend roughly two-thirds of public investment funds, ranging from almost 90% in Belgium to 25% in Greece.\(^5\) Decentralization strengthens pecuniary externalities of local fiscal policies making it a likely candidate for explaining the observed weakness in public investment.

Our study requires introducing the New Economic Geography perspective into fiscal policy analysis in federal systems. We amend the standard quantitative spatial economic modeling approach of Eaton and Kortum (2002), Redding (2016), Redding and Rossi-Hansberg (2016) by adding optimizing local and central governments to the model. We assume public investment increases total factor productivity of local firms, which is supported by a wide range of empirical studies.\(^6\)

Previous economic geography models of public investment have focused on how exogenous factors change the spatial allocation of economic activity, not the optimality of policy outcomes such as local infrastructure investment. Examples are the analysis of transportation infrastructure (Felbermayr and Tarasov (2015), Gruber and Marattin (2009), Allen and Arkolakis (2014)), exogenous variations of local infrastructure investment (Brakman, Garretsen and Marrewijk (2008) or of both as in Martin and Rogers (1995).

The economic geography literature on endogenous public investment and taxation is sparse\(^7\), and despite its political relevance, empirically testable models without corner solutions are nonexistent. Our model closes this gap: it delivers interior solutions and allows for many interdependent constituencies. As such, it reflects main components of fiscal federalism and therefore allows for empirical testing.

\(^5\)\(\)See Blöchliger H. and K. Junghun (2016), p. 19-20. In Germany, 11,269 municipalities control roughly 40% of all public investment, the Länder (states) and the federal level control roughly 30% each.

\(^6\)\(\)See e.g. Bom und Ligthart (2013).

\(^7\)\(\)A notable exception is Fenge, von Ehrlich and Wrede (2009) who analyze endogenous productivity enhancing public investment in the ‘footloose capital’ model of Baldwin et al. (2003). They show that trade costs determine the regional provision of public goods, and study the implications for efficiency and agglomeration. A drawback of their two-region-model is, however, that it limits the analysis to either symmetric or concentrated investment in a single region.
The paper develops in four steps. In section 2, we sketch our augmented version of the standard core-periphery model of Krugman (1991) and Helpman (1998). It features immobile workers as a fixed regionally specific factor of production and introduces productivity-enhancing public investment and taxation. In section 3, we discuss the properties of the model’s equilibrium and the role of trade costs and economies of scale. In section 4, we analyze optimal levels of public investment in stylized fiscal regimes. In our model, pecuniary externalities drive sub-optimal investment levels due to regionally autonomous tax and spending polices. In section 5 we discuss the model’s implications and show that its predictions are consistent with county-level data for Germany in 2013.

2 The Model

Our spatial model builds on the core-periphery model in Redding (2016). It describes a closed economy with $R$ regions and two sectors of production, manufacturing and services: The manufacturing sector produces varieties of goods tradable across regions. It is constrained by the supply of mobile workers that can be attracted to the region. The service sector supplies its services only to its own region. It is constrained by local immobile labor supply in a similar fashion as productive land constraints output in Helpman (1998). Immobile workers are firmly attached to the region of their residency, e.g. for social reasons. These workers supply their labor inelastically as they accept the going wage rate firms offer in their region.8

Mobile workers relocate if the real (after tax) wage rate is higher in another region than at their current residence. In combination with increasing returns to scale in manufacturing and transport costs, labor mobility increases agglomeration. Fixed immobile labor supply raises local prices for services, reducing real wages and the attractiveness of a region for mobile labor, thus dampening agglomeration. Therefore, given transport costs, economies of

8Immobile labor could be assumed to be less skilled as in Forslid and Ottaviano (2003), but this is not a crucial assumption for the implications of our model.
scale, and the distribution of immobile labor in geographical space, the model’s equilibrium is determined by the endogenous location choice of mobile labor, which is influenced by local public investment.

To see this, consider a region with higher real wages as compared to other regions. This region attracts mobile workers, which creates additional local demand and promotes further real-wage gains in this region as economies of scale lower prices of manufactured goods relative to wages. The influx of mobile workers also increases demand for services, which are constrained by local immobile labor supply. Therefore, prices for services rise, reducing real income and counterbalancing the agglomeration effect from the manufacturing sector. This dispersion effect limits the attractiveness of a region. This process continues until real wages for mobile labor are the same across regions.

As in Pfüger and Tabuchi (2011) immobile labor is employed in both sectors. This ensures that both types of labor benefit from improvements in total factor productivity in the manufacturing sector. It also allows us to analyze the effects of spending and taxation policies on income distribution across different types of labor and regions. In a nutshell, part of the workforce is purely industrial, immobile across sectors, but mobile across regions. The other part of the workforce is mobile across sectors but immobile across regions.

We add public investment in regions to this standard spatial economic model as follows: Pay-as-you-go taxation finances public investment in local infrastructure, which increases total factor productivity of regional manufacturing. Now consider an economy with a given spatial distribution of mobile and immobile labor before mobile labor moves. As manufacturing employs both types of labor, productivity changes in this sector will not directly lead to a change of the wage differential between the two types. However, increased total factor productivity implies higher wages, which attracts mobile workers, and induces them to move. This second order effect through the reallocation of mobile workers changes local relative labor supply and thus the relative wages of mobile and immobile labor.

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9If immobile labor was not employed in the manufacturing sector, government policies to boost productivity would automatically bring about an equity/efficiency trade-off that is not our interest in this paper.
2.1 Consumer demand

Consumers in region $i \in R$ spend their income $Y_i$ on tradable manufactures $M_i$, henceforth referred to as goods, and locally produced services $S_i$. Consumers maximize utility as follows:

\begin{equation}
U_i = M_i^\gamma S_i^{1-\gamma}, \quad 0 < \gamma < 1.
\end{equation}

Utility maximization and – as we will see shortly – a corresponding spending behavior of local governments leads to the following demand for goods and services:\(^{10}\)

\begin{align*}
(2) \quad & M_i = \gamma \frac{Y_i}{PM_i}, \\
(3) \quad & S_i = (1 - \gamma) \frac{Y_i}{PS_i}.
\end{align*}

We can express the corresponding minimum expenditure price index for total consumption $P_i$ as a function of goods and service prices $PM_i$ and $PS_i$ respectively:\(^{11}\)

\begin{equation}
P_i = PM_i^\gamma PS_i^{1-\gamma}.
\end{equation}

Each region $j \in R$ produces $N_j$ varieties of goods. Consumers love variety, so a representative consumer in region $i$ consumes a basket of globally available varieties $M_i = \left( \sum_{j=1}^{R} \sum_{k=1}^{N_j} m_{i,j,k}\rho \right)^{\frac{1}{\rho}}$, including the varieties produced in her home region $i$. The demand in region $i$ for a specific variety $k$ produced in region $j$ equals

\begin{equation}
m_{i,j,k} = \frac{M_i}{PM_i^{-\epsilon} (pm_{j,k} \cdot T_{i,j})^{\epsilon}}, \quad \epsilon \geq 1.
\end{equation}

\(^{10}\)The parameter $\gamma$ determines the share of total income $Y$ spent on manufactured goods. We use this property to obtain a parameter estimate when we calibrate the model.

\(^{11}\)The price index $P_i$ buys one unit of utility taking substitution effects of relative price changes into account. See Obstfeld, Rogoff (1996), pp. 226.
The parameter $\epsilon$ represents the elasticity of substitution between varieties, and $\rho = (\epsilon - 1)/\epsilon$ is the inverse of the degree of love of variety. The prices of varieties differ between regions because of different local costs of production and differing trade costs between regions. $pm_{j,k}$ is the price of varieties produced in region $j$ (the mill price) and $T_{i,j}$ is an iceberg trade-cost reflecting the value share of the good expended in its shipping from region $j$ to region $i$. Varieties produced and consumed in the same region require no transport, so $T_{i,i} = 1$. The minimum expenditure price index for the basket of manufactured goods $M_i$ consumed in region $i$ is then

\[
PM_i = \left[ \sum_{j=1}^{R} \sum_{k=1}^{N_j} (pm_{j,k} \cdot T_{i,j})^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}}.
\]

2.2 Government Activity

Regional governments provide productivity enhancing local infrastructure $G_i$ financed by taxing local incomes at a rate $t_i$. We do not allow borrowing to ensure that the regional budget is always balanced: $G_i - t_i \cdot Y_i = 0$.

Regional governments buy goods and services to provide infrastructure. They spend their revenues in the same proportion on goods and services as private consumers do. Consequently, relative goods prices are not distorted by government activity.

Total factor productivity $f_i$ of regional manufacturing depends on some base value $\bar{f}_i$ and government infrastructure expenditure relative to regional income $g_i = \frac{G_i}{Y_i}$.

\[
f_i(g_i) = \bar{f}_i \cdot (1 + a_i g_i)^\beta, \quad a_i > 0, \quad \beta > 0.
\]

The elasticity of total factor productivity with respect to public investment converges

\footnote{Note that if $\rho = 1$, the varieties are perfect substitutes for one another.}
\footnote{This assumption is equivalent to assuming a production function for government infrastructure $G_i = M_i^\gamma S_i^{1-\gamma}$, and a cost minimizing government.}
towards $\beta$ with increasing levels of spending.\textsuperscript{14} We want public investment to have the same impact on productivity for the same value of $f_i$ in all regions:

$$a_i = a_f^\frac{1}{\beta}, \quad a > 0.$$  

This scaling of $a_i$ ensures that regions with initial low levels of productivity can partly catch-up through public investment.

\textbf{2.3 Production and Labor Demand}

As in Caliendo and Parro (2015), manufacturing production takes place in a two-stage process. In the first stage, perfectly competitive firms produce homogenous non-tradable intermediate goods $X_i$ with constant returns to scale Cobb-Douglas technology from mobile and immobile labor:

\begin{equation}
X_i = Lmo_i^\alpha Lim_i^{1-\alpha} f_i (g_i), \quad 1 > \alpha > 0.
\end{equation}

$f_i$ captures the total factor productivity in intermediate goods production, which depends positively on regional public infrastructure expenditure $g_i$. Profit maximization leads to demand for immobile and mobile workers that negatively depends on their nominal wages $Wim_i$ and $Wmo_i$:

\begin{equation}
Lim_i^x = (1 - \alpha) X_i \cdot \frac{P_{x_i}}{Wim_i},
\end{equation}
\begin{equation}
Lmo_i^x = \alpha X_i \cdot \frac{P_{x_i}}{Wmo_i}.
\end{equation}

In the second stage, firms produce manufacturing varieties from local intermediate goods with increasing returns to scale. This implies that each variety $k$ is supplied by a single firm.

\textsuperscript{14}The equivalent to $\beta$ in our model can be found in empirical studies which usually suppose a constant elasticity of output with respect to public capital, see e.g. Boarnet (1998).
that acts as a monopolistic price-setter. Firms have identical production functions leading to the following demand function for local intermediate goods:

\[(11) \quad x_{i,k} = F + \omega m_{i,k}.\]

\(F\) is a constant fixed cost needed to set up production, and \(\omega m_{i,k}\) are variable costs. Profit maximization of each regional manufacturer leads to a monopolistic Dixit-Stiglitz (1977) price-setting rule which is given by a constant mark-up over the local intermediate good input price:\(^{15}\)

\[(12) \quad pm_{i,k} = \frac{\omega}{\rho} P x_i.\]

Potential firm entry enforces zero-profits for the single surviving producer who then produces a fixed quantity

\[(13) \quad m_{i,k} = \frac{\rho}{(1 - \rho)} F = \frac{\epsilon - 1}{\omega} F.\]

Each firm employs equation (11) and demands quantity \(x_{i,k} = \epsilon F\) of local intermediate goods. The number of varieties produced equals the number of firms in each region \(i\) given by:

\[(14) \quad N_i = \frac{X_i}{\epsilon F}.\]

Substituting (12) and (14) into the index formula for manufacturing varieties (6), we can restate the price index for the basket of manufactures consumed in location \(i\) as a function

\(^{15}\)Note that the lower case \(pm_{i,k}\) denotes the producer-price index for manufacturing varieties produced in region \(i\). In contrast, the upper case \(Pm_i\) denotes the consumption price index for all consumed manufacturing varieties (from own production and imported from other regions) in region \(i\).
of firm fundamentals, intermediate goods production, and intermediate goods prices:

\[
PM_i = \frac{\omega}{\rho} \left( \frac{1}{\epsilon F} \right)^{\frac{1}{1-\epsilon}} \left[ \sum_{j=1}^{R} X_j (P_{x_j} \cdot T_{i,j})^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}}.
\]  

(15)

Firms in the service sector produce with constant returns technology employing only immobile labor: \( S_i = Lim^s_i \). Profit maximization under perfect competition implies prices for local services to be equal to \( PS_i = Wim_i \). Substituting \( S_i \) and \( PS_i \) in the consumer demand function for local services (3) leads to the following function for immobile labor demand in the service sector

\[
Lim^s_i = (1 - \gamma) \frac{Y_i}{Wim_i}.
\]  

(16)

2.4 Equilibrium Wages, Incomes, and Prices

Using the labor demand functions (9), (10) and (16), we can determine a preliminary labor market equilibrium for any given allocation of mobile labor in each region. Let \( \lambda_i \) be the mobile labor supply in each region as a fraction of the nationwide supply \( \overline{Lmo} \). In this equilibrium before mobile labor moves, we calculate nominal wages for mobile labor in region \( i \) from (10):

\[
Wmo_i = \alpha \frac{P_{x_i} \cdot X_i}{\lambda_i \overline{Lmo}}.
\]  

(17)

To obtain total demand for immobile labor, we add labor demand from both sectors (9) and the (16). Let \( \theta_i \) be the immobile labor supply in each region as a fraction of the nationwide supply \( \overline{Lim} \). Equating total regional demand for immobile labor with its supply determines the equilibrium wages for immobile workers in each region, which are identical
to the local service prices:\textsuperscript{16}

\begin{equation}
Wim_i = PS_i = (1 - \alpha \gamma) \frac{Y_i}{\theta_i \bar{L}_i}.
\end{equation}

Given the wage rates for mobile and immobile labor, regional pre-tax income is

\begin{equation}
Y_i = \frac{1}{\gamma} Px_i \cdot X_i.
\end{equation}

The equilibrium price for intermediate goods in each region is a function of the prices for manufacturing varieties that firms in region \( i \) can set in the interregional markets. The equilibrium price \textit{for each} manufactured variety is determined by the supply of that variety by location \( i \) (13) and the sum of its demand (5) from all regions \( R \), including \( i \)’s own demand. Using the monopolistic pricing-rule (12), total demand for variety \( m_{i,k} \) is:\textsuperscript{17}

\[
m_{i,k} = \gamma \left( \frac{\rho}{\omega} \right)^{1 - \epsilon} Px_i \cdot \sum_{j=1}^{R} Y_j \cdot PM_j^{\epsilon - 1} \cdot T_{j,i}^{1 - \epsilon}.
\]

We use income definition (19) and equate total demand for each variety with the manufactured goods supply in region \( i \) (13) to arrive at the equilibrium f.o.b. price for intermediate goods in region \( i \):

\begin{equation}
Px_i = \frac{\rho}{\omega} \left[ \frac{1}{(\epsilon - 1) F} \right]^\frac{1}{2} \left[ \sum_{j=1}^{R} Px_j \cdot X_j \cdot PM_j^{\epsilon - 1} \cdot T_{j,i}^{1 - \epsilon} \right]^\frac{1}{2}.
\end{equation}

\textbf{2.5 Equilibrium with Labor Mobility}

Our preliminary equilibrium generally leads to real wage differentials of mobile labor across regions, which provide them with incentives to move. After tax real-wage arbitrage leads to a new allocation of mobile labor, in which mobile workers’ real wages across regions are equalized. We employ the equilibrium conditions for wages, incomes, and prices to state the

\textsuperscript{16}1 - \alpha \gamma \) is the income share of immobile labor, which can be estimated from statistical data and used for calibrating the model.

\textsuperscript{17}Consumers in region \( j \) pay the delivery price for the varieties including trade costs. In the context of the model, they effectively demand the quantity they wish to consume plus the iceberg component that melts away in transit from region \( i \) to \( j \). This is why the trade cost factor is multiplied with the demand in destination \( i \).
condition for an interregional equilibrium as follows:

\[
Wmo_i^* = \overline{Z} \lambda_i^{\alpha \gamma - 1} \theta_i^{1 - \alpha \gamma} \left[ \frac{P_{x_i}}{P_{M_i}} f_i(g_i) \right]^{\gamma} (1 - t_i) \equiv Wmo^* \quad \forall i \in R, \text{ with } \overline{Z} > 0.
\]

The relative price index \( \frac{P_{x_i}}{P_{M_i}} \) represents the terms of trade of each region \( i \).\(^{19}\) We will analyze its dependence on fiscal policies below.

Redding (2016) demonstrates that real net-wage equalization across all regions implies normalized real-net wages in all regions \( i \) to be equal to some equilibrium value \( \xi \):

\[
(1 - t_i) \left[ \frac{P_{x_i}}{P_{M_i}} f_i(g_i) \right]^{\gamma} \lambda_i^{\alpha \gamma - 1} \theta_i^{1 - \alpha \gamma} = \xi.
\]

We solve this equation for \( \lambda_i \) and use \( \sum_{i=1}^{R} \lambda_i = 1 \) to arrive at the equilibrium distribution of mobile workers across regions:

\[
\lambda_i = \frac{\theta_i (1 - t_i)^{\frac{1}{1 - \alpha \gamma}} \left[ \frac{P_{x_i}}{P_{M_i}} f_i(g_i) \right]^{\frac{\gamma}{1 - \alpha \gamma}}}{\sum_{j=1}^{R} \theta_j (1 - t_j)^{\frac{1}{1 - \alpha \gamma}} \left[ \frac{P_{x_j}}{P_{M_j}} f_j(g_j) \right]^{\frac{\gamma}{1 - \alpha \gamma}}}.
\]

As equation (21) shows, \( \lambda_i \) is a composite indicator of region \( i \)’s relative economic fundamentals: the regional share of immobile labor, the regional total factor productivity (depending on public investment), the inverse regional effective tax rate, and the regional terms of trade. This follows from mobile workers moving to regions that are attractive economically. \( \lambda_i \) also equals the share of regions’ income in total national income.\(^{19}\)

We substitute these labor shares into the no-arbitrage condition. This yields the following

\[\text{Net real regional income is given by } Y_i^r = \frac{\overline{Z} \lambda_i Wmo^*}{\alpha \gamma}, \quad \text{such that } \sum_{i=1}^{R} Y_i^r = \frac{\overline{Z} \lambda Wmo^*}{\alpha \gamma} \text{ and } \lambda_i = Y_i^r / \sum_{i=1}^{R} Y_i^r.\]
equilibrium real wage rate, which is identical for all regions $i$:

$$W^{m^r} = W \left( \sum_{j=1}^{R} \theta_j (1 - t_j)^{\frac{1}{1-\alpha \gamma}} \left[ \frac{P_{x_j}}{PM_j} f_j (g_j) \right]^{\frac{\gamma}{1-\alpha \gamma}} \right)^{1-\alpha \gamma}.$$  \hspace{1cm} (22)$$

Thus, the equilibrium wage rate positively depends on the composite of the economic fundamentals of all regions. Altogether, we need a system of $1 + 4 \times R$ equations to characterize the interregional equilibrium: the equilibrium mobile workers’ real wage rate (22), the allocation of mobile workers across the $R$ regions (21), the intermediate goods production (8), the intermediate goods price (20) and the manufacturing price index (15) in all $R$ regions.

3 Properties of the Interregional Equilibrium

3.1 Stability and Uniqueness

New economic geography models are inherently unstable. They frequently lead to extreme outcomes for certain combinations of parameters, such as complete agglomeration of production in a single region vis-à-vis a depopulation of other regions. Since these corner solutions are rarely found in reality, we concentrate on parameter values that are within the realm of empirical findings and lead to a unique stable equilibrium.

We follow Pflüger and Tabuchi (2011) who develop a similar model as ours. In particular, they also assume that manufacturing uses the immobile factor – housing – in their example – in production. We can state their sufficient condition for a unique and stable equilibrium in terms of the parameters of our model as $\rho > \alpha \gamma$. Since $1 > \alpha > 0$, this is less strict than the traditional “no-black-hole”-condition $\rho > \gamma$ of Fujita, Krugmann, Venables (1999). Our condition allows for stronger agglomeration (smaller $\rho$) as the limiting value of $\gamma$ is scaled down by the production elasticity $\alpha$ of the mobile factor.\textsuperscript{20} As long as this weaker condition

\textsuperscript{20}If this condition is met, symmetry of all the characteristics of each region such as the share of immobile
is met, the dispersion of economic activity in space will be a long-run equilibrium irrespective of the size of trade costs.

### 3.2 The Impact of Trade Costs and Love of Variety

The parameters that capture the essence of the New Economic Geography, namely $\rho$ for the degree of market imperfection, and $T_{j,i}$ for the size of trade costs, enter the equilibrium conditions indirectly via the terms of trade for each region $P_{x_i}/P_{M_i}$. In equilibrium, the share of mobile workers (21) and hence output in a region $i$ positively depend on the region’s own terms of trade $P_{x_i}/P_{M_i}$. The terms of trade are mainly driven by the distribution and size of markets for manufactured goods in geographic space and indicate relative proximity to these markets (including region $i$’s own production and demand). It can therefore be viewed as a summary statistic of the economic geography advantage of each region.

Next, we simulate a simplified two-region version of our model to illustrate the effects of parameter variations of $\rho$ and $T_{j,i}$ on the equilibrium wage and the allocation of mobile labor via the terms of trade. To avoid a symmetric allocation of mobile laborers $\lambda_i$, we assume that region 1 has a total factor productivity advantage: $f_1 = 3 > f_2 = 1.5$ while leaving $\theta_1 = \theta_2$. At this stage we do not yet consider government policies and set tax and expenditure rates to zero.

We use estimates from previous studies for $\gamma$, and $\bar{L}_{mo}/\bar{L}_{im}$. Peneder and Streicher (2015) estimate the share of manufactures in total consumption to range from 0.3 to 0.4% for the EU as a whole. We therefore set $\gamma = 0.4$. Reichelt and Abraham (2015) find that roughly 20% of the workers in their study move to another region in Germany to find a job in either the same or another occupation. We therefore set the degree of mobility in the population, workers and the level of total factor productivity, will produce a symmetric equilibrium in terms of the allocation of mobile labor, production and per-capita income across regions.

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21See Redding (2016), pp. 156.

22Alternatively, one could assume that region 1 has a higher share of immobile labor, which would also strengthen this region so that it would benefit more from scale effects of production than region 2.

23Granato et al. (2009) find slightly higher mobility ratios for Germany over the period 2000-2005. According to their calculations, the percentage of employed persons that moves from one district (Kreise) to
defined as \( \text{mob} = \frac{Lmo}{(Lmo + Lim)} \) to 0.2., which implies a ratio of \( \frac{Lmo}{Lim} = 0.25 \). In the above Cobb-Douglas setup, \( \alpha \gamma \) is the share of mobile workers’ income in total labor income (see equations (17) and (19)). We assume an average 20% wage premium of mobile workers over the wages of immobile workers \( \left( \sum_{i=1}^{R} \lambda_i \cdot Wmo_i / \sum_{i=1}^{R} \theta_i \cdot Wim_i \right) \), which allows us to derive the production elasticity of mobile workers \( \alpha \). With \( \gamma = 0.4 \) this yields a value of \( \alpha = 0.6 \).

With these assumptions our simulations demonstrate that stronger love of variety (a lower elasticity of substitution \( \epsilon \)) leads to higher real wages of mobile workers and the higher the trade costs, the higher the concentration of economic activity in the region with stronger fundamentals (see Figure 1).

**Figure 1:** Mobile Labor Allocation and Real Wages for Variations of \( \epsilon \) and \( T_{1,2} \)

Notes: Region 1 absorbs between 59% (white) and 72% (black) of all mobile labor. If real net wages of mobile workers are scaled to 1 in the case of perfectly substitutable varieties (white), then they rise by a maximum of 240% with falling trade costs and elasticity of substitution (black). Parameter values are as stated in the text: \( \gamma = 0.4 \); \( \alpha = 0.6 \); \( f_1 = 3 > f_2 = 1.5 \); \( \theta_1 = \theta_2 = 0.5 \).

Love of variety explains the increase in real wages and thus overall consumer welfare as it reduces the amount of income required to achieve the same degree of consumer satisfaction. This is reflected in a fall of the minimum expenditure manufacturing price index in all regions.\(^{24}\) A real-wage increase through a fall of manufacturing prices would also occur in another is about 22% in East Germany and about 27% in West Germany.

\(^{24}\)At a given income, stronger love of variety (falling \( \epsilon \)) leads to the consumption of more varieties, which – all else equal – reduces the minimum expenditure price index for manufactures as defined by (15).
a completely symmetric parameterization with $\lambda_i$ staying unchanged. But with asymmetric productivity and finite trade costs, an increase of love of variety amplifies economic differences and leads to a higher regional concentration of mobile workers. This is because the rise in demand for manufacturing varieties is stronger in the region that is economically stronger from the onset. Finite trade-costs result in a home market effect, as a higher portion of this demand is concentrated on domestic varieties, leading to a rise in intermediate goods prices in the economically stronger region 1. As a result, wages in region 1 are temporarily higher, leading to inward migration and further rising domestic demand. Conversely, production and demand fall in region 2, depressing intermediate goods prices there. Consequently, the terms of trade rise in the fundamentally stronger region relative to the weaker one, leading to a higher spatial concentration of economic activity (higher $\lambda_i$) and an amplification of regional differences.\footnote{The reaction of the intermediate goods prices $P_x$ to changes in $\epsilon$ can be ambiguous for very small trade costs. Our simulations show that if e.g. $T_{1,2}=1.01)$, the domestic market size effect becomes almost irrelevant, since domestic varieties can be easily substituted by imports from other regions. It can even happen that the weaker economic region receives a net-benefit from higher demand from the stronger regions. This can even lead to $P_x$ rising in the economically weaker region and falling in the stronger region. As trade costs fall to zero ($T_{1,2}=1$), there are no differences in regional terms of trade and no impact of changing levels of $\epsilon$ on the equilibrium allocation of mobile labor.}

**Figure 2:** Terms of Trade in both Regions for Variations of $\epsilon$ and $T_{1,2}$

Notes: Terms of trade range in both regions from 1.3 (white) to 12.1 (black). Parameter values as stated in the text: $\gamma=0.4$; $\alpha=0.6$; $f_1 = 3 > f_2 = 1.5$; $\theta_1 = \theta_2 = 0.5$.

For any given elasticity of substitution between manufacturing varieties, rising trade
costs increase the prices for imported goods in both regions and reduce real wages. The economically weaker region, however, is worse off, as it imports a higher share of varieties. The impact on intermediate goods prices $P_x_i$ also differs in the two regions according to their economic strengths and can be explained as follows: Both regions incur a shift in their domestic demand towards domestic varieties but also face lower demand from other regions (substitution effect). In addition, there is a negative real income effect in both regions from rising trade costs. The economically stronger region again suffers less from this negative real income effect as it produces more domestic varieties. This allows firms in the stronger region to keep its intermediate goods price higher than in the weaker region and offer higher wages, which induces inward migration of mobile labor, pushes up local demand and further sustains the domestic prices for its intermediate goods. Conversely, wages and intermediate goods prices are depressed in the economically weaker region. In our simulations, the terms of trade fall in both regions with rising trade cost (see Figure 2), which points to a dominance of rising manufacturing prices in both regions. In the economically stronger region the terms of trade fall relatively less as compared to the weaker region. Rising trade costs will thus amplify regional economic differences and foster spatial concentration of activity while a decline of trade costs reduces them as in Helpman (1998) and favors dispersion of economic activity.\footnote{Consistent with the analysis of Redding (2016), rising trade costs of a region $i$ lead to a higher trade share of region $i$ with itself, which given that the trade balance is always zero implies that the terms of trade fall. Hence, falling trade costs lead to welfare gains through higher real wages of mobile labor.}

### 3.3 Implications for Income Distribution

Our model also offers a prediction for the impact of parameter changes on the distribution of regional and personal income, that is, real wages of mobile and immobile workers. It is important to note that in our model \underline{regional} per capita income differences as well as \underline{personal} income differences across regions depend on the spatial distribution of mobile labor. Immobile workers’ wages positively depend on the number of mobile workers in a region as
this determines the relative scarcity of immobile workers. Thus, the more dispersed mobile workers are across regions (the more equal the distribution of $\lambda_i$) the smaller are regional and personal income differences.\footnote{The average per-capita income spread between mobile and immobile workers across the country in our model is exogenously given by $\frac{1-\alpha\gamma}{\alpha\gamma} \frac{\overline{L}_{\text{mo}}}{L_{\text{im}}}$ \textit{.} It is therefore independent of taxation and public investment policies.}

Against this backdrop, lower trade costs will reduce income differences between immobile workers across regions and also regional per-capita income differences. Thus, our model paints a positive picture of globalization, as a reduction of trade costs is both positive for welfare and equality. In contrast, an increase in love of variety raises real income of mobile labor, but it enhances regional as well as personal income differences. Economically stronger regions benefit more from scale effects than weaker regions and attract additional mobile labor. This influx of mobile workers also raises the wages of immobile workers in the stronger region. Thus, an increase of love of variety increases overall welfare but also income inequality.

4 Spatial Equilibrium with Local Public Investment

We employ our model to analyze welfare-maximizing local public investment and taxation. A central issue is whether the fiscal decision powers are assigned to the central level or devoted to the individual regions. Instead of local taxation to finance investment, also a central government can raise taxes and redistribute resources to regions through vertical transfers.\footnote{Centralized policies would allow for regional budget deficits and surpluses, and an equalization by horizontal transfers. However, our primary interest is to study the differences between a regime that internalizes regional spill-overs to one with a purely regional focus. We therefore require regional budgets to balance even for centralized policies and leave the explicit modeling of optimal transfers to further extensions of this work.}

Without counterbalancing factors, a central planner always weakly improves overall welfare whenever she internalizes an externality. We therefore attempt to answer the following four questions: (1) Under which conditions do centralized and localized decisions lead to equally good welfare outcomes? (2) Under which conditions does public investment differ
between the two regimes? (3) Do the two decision regimes affect regional and personal income inequality, and if yes, how? (4) Can a calibrated version of our model help explain the observed public investment backlog in Germany?

To capture the implications of fiscal centralization versus decentralization, we analyze two stylized regimes. First, a central government (‘central planner’) sets all regional tax and spending rates simultaneously to maximize national per-capita income $Y_{pc}$. Second, regional governments maximize regional per-capita income $Y_{pc_i}$ by setting regional tax and spending rates in a non-cooperative fashion (Nash equilibrium). Both regimes keep regional budgets balanced. The central government internalizes regional spillovers, while the regional governments do not.

Per-capita income is highest when the central government maximizes real wages of mobile labor. She therefore solves the following Lagrange-function:

$$
\mathcal{L} [t_1, \ldots, t_i; g_1, \ldots, g_i; \mu_1, \ldots, \mu_i] = \frac{\lambda_{mo}}{\alpha \gamma} W_{mo} \left[ t_1, \ldots, t_i; g_1, \ldots, g_i \right] - \sum_i \mu_i (g_i - t_i),
$$

with $\mu_i$ as the Lagrange multipliers in all regions $i \in R$. Maximization of $\mathcal{L}$ implies the following optimality conditions:

$$
(23) \quad -\frac{\partial W_{mo}}{\partial t_i} = \frac{\partial W_{mo}}{\partial f_i} \frac{\partial f_i}{\partial g_i}.
$$

In the decentralized regime, the Lagrange-function for each regional government is

$$
\mathcal{L}_i [t_i, g_i] = Y_{pc_i} \left[ t_i, g_i \right] - \mu_i (g_i - t_i). \quad \text{Maximization of } \mathcal{L}_i \text{ by regional governments results in the optimality condition for each region } i:
$$

$$
(24) \quad -\frac{\partial Y_{pc_i}}{\partial t_i} = \frac{\partial Y_{pc_i}}{\partial f_i} \frac{\partial f_i}{\partial g_i}.
$$

As $Y_{pc_i} \left[ t_i, g_i \right] = \frac{\lambda_{[t_i, g_i]} \lambda_{mo}}{\alpha \gamma (\lambda_{im} + \lambda_{[t_i, g_i]})} W_{mo} \left[ t_i, g_i \right] = \frac{\sigma_i(\lambda_i)}{\alpha \gamma} W_{mo}$, with $\sigma_i(\lambda_i)$ as the share of mobile workers in the population of region $i$, the optimality condition in the decentralized
regime can be rewritten as:

\[- \left( \frac{\partial \lambda_i}{\partial t_i} + \frac{\partial \lambda_i}{\partial f_i} \frac{\partial f_i}{\partial g_i} \right) \frac{\partial \sigma_i W_{mo^r}}{\partial \lambda_i} \sigma_i(\lambda_i) = \frac{\partial W_{mo^r}}{\partial t_i} + \frac{\partial W_{mo^r}}{\partial f_i} \frac{\partial f_i}{\partial g_i}.\]

Since \( \rho > \alpha \gamma \) prevents extreme agglomeration such that \( \lambda_i > 0 \) and \( \theta_i > 0 \) in all regions, this implies that \( 1 > \sigma_i > 0 \) and \( \frac{\partial \sigma_i}{\partial \lambda_i} > 0 \). Therefore, regional policies will be identical to those of a central planer if and only if \( \frac{\partial \lambda_i}{\partial t_i} + \frac{\partial \lambda_i}{\partial f_i} \frac{\partial f_i}{\partial g_i} = 0 \). This means that regional and central policies are identical if the marginal effects of taxation and investment expenditure on the location of mobile labor neutralize each other.\(^{29}\)

The central planner is indifferent to the impact of its policies on the regional allocation of mobile labor as long as they maximize real net-wages of mobile workers and with this national welfare. In contrast, regional governments always care about the effect of their policies on the stock of mobile laborers in their region. They trade-off the effect of their policies on mobile versus their local immobile workers’ wages \( (W_{mo^r}) \) with the latter positively depending on the stock of mobile labor \( (\lambda_i) \) in the region. With few exceptions identified below, this leads to socially sub-optimal levels of investment expenditure.

We gain additional insight into the implications of the two regimes on policy outcomes when we detail the marginal effects of policy changes on mobile labor allocation and real wages. Productivity levels and the rate of taxation affect both directly. But there is also an indirect effect on both via the terms of trade. A policy change in any region \( i \) changes the terms of trade not only in region \( i \) itself but also in all other regions \( Px_j/PM_j \). This affects the partial derivative of the real wage rate and the share of mobile labors with respect to investment and taxes. Both partial derivatives depend on the sum of marginal terms-of-trade changes in all regions \( j \) with respect to policy changes in region \( i \), which can be expressed in terms of elasticities as \( \varepsilon_{Px_j,1-t_i} - \varepsilon_{PM_j,1-t_i} \) and \( \varepsilon_{Px_j,f_i} - \varepsilon_{PM_i,f_i} \).

Using the definitions: \( \Delta_{1-t_i} = \sum_{j=1}^{R} \lambda_j (\varepsilon_{Px_j,1-t_i} - \varepsilon_{PM_j,1-t_i}); \Delta_{f_i} = \sum_{j=1}^{R} \lambda_j (\varepsilon_{Px_j,f_i} - \varepsilon_{PM_i,f_i}) \).

\(^{29}\) This condition would be met automatically without labor mobility, or both \( \frac{\partial \lambda_i}{\partial \sigma_i} = 0 \) and \( \frac{\partial \lambda_i}{\partial f_i} = 0 \). One could argue that that those conditions are fulfilled in places of specifically beautiful environment, such as certain places in California.
\(\varepsilon_{PM_i,f_i}\), we can write the partial derivatives of equilibrium wages and mobile labor allocation as:

\[
\frac{\partial W_{mo}}{\partial f_i} = \frac{W_{mo}}{f_i} \gamma (\lambda_i + \Delta f_i),
\]

\[
\frac{\partial W_{mo}}{\partial t_i} = -\frac{W_{mo}}{1 - t_i} (\lambda_i + \gamma \Delta_{1-t_i}) = -\frac{\partial W_{mo}}{\partial (1-t_i)},
\]

\[
\frac{\partial \lambda_i}{\partial f_i} = \frac{\lambda_i}{f_i} \left( \frac{\gamma}{1 - \alpha \gamma} \right) (1 - \lambda_i + \varepsilon_{P_x,f_i} - \varepsilon_{PM_i,f_i} - \Delta f_i),
\]

\[
\frac{\partial \lambda_i}{\partial t_i} = -\frac{\lambda_i}{1 - t_i} \left( \frac{1}{1 - \alpha \gamma} \right) [1 - \lambda_i + \gamma (\varepsilon_{P_x,1-t_i} - \varepsilon_{PM_i,1-t_i}) - \gamma \Delta_{1-t_i}] = -\frac{\partial \lambda_i}{\partial (1-t_i)}.
\]

Hence, we can now write the optimality condition of the central planner (23) as

\[
\frac{\partial f_i}{\partial g_i} \frac{\gamma}{f_i} (1 - t_i) = \frac{\lambda_i + \gamma \Delta_{1-t_i}}{\lambda_i + \Delta f_i}.
\]

We define \(c_i^{CP} (\lambda_i, \gamma \Delta_{1-t_i}, \Delta f_i) = \frac{\lambda_i + \gamma \Delta_{1-t_i}}{\lambda_i + \Delta f_i}\), which allows to write optimal policies of the central planner as:

\[
(25) \quad t_i^{CP} = g_i^{CP} = \frac{a_i \beta \gamma - c_i^{CP}}{a_i (c_i^{CP} + \beta \gamma)}.
\]

We use the above definitions to write the optimality conditions of autonomous regional governments (24) as follows:

\[
\frac{\partial f_i}{\partial g_i} \frac{\gamma}{f_i} (1 - t_i) = \frac{[1 - \sigma_i (\lambda_i)] [1 + \gamma (\varepsilon_{P_x,1-t_i} - \varepsilon_{PM_i,1-t_i})] + [\sigma_i (\lambda_i) - \alpha \gamma] (\lambda_i + \gamma \Delta_{1-t_i})}{[1 - \sigma_i (\lambda_i)] [1 + \varepsilon_{P_x,f_i} - \varepsilon_{PM,f_i}] + [\sigma_i (\lambda_i) - \alpha \gamma] (\lambda_i + \Delta f_i)} \equiv c_i^{RP}.
\]

This allows us to state optimal regional policies as:

\[
(26) \quad t_i^{RP} = g_i^{RP} = \frac{a_i \beta \gamma - c_i^{RP}}{a_i (c_i^{RP} + \beta \gamma)}.
\]

To highlight the implications of labor mobility for optimal policies under the two different regimes, we proceed in two steps. In the first step, we study three corner solutions – perfect
competition, infinite, and zero trade costs – in the generalized framework for an arbitrary
number of regions. In the second step, we employ simulation analysis in a two-region frame-
work to analyze the responsiveness of optimal policies to arbitrary degrees of love-of-variety
and trade costs.

4.1 Corner Solutions in the General Framework

4.1.1 Perfect Competition

Perfect competition in our model emerges when the substitution elasticity between varieties
of manufactured goods is infinite, \( \epsilon \to \infty \). It implies that firms produce a single homogeneous
good under perfect competition with marginal cost pricing \( pm_{j,k} = \omega Px_j \). Consequently,
regions do no trade varieties anymore and the price index \( PM_i \) collapses to \( \omega Px_i \). The terms
of trade are identical and fixed across regions at \( Px_i / PM_i = \omega \) and irresponsive to changes
in government policy \( (\Delta_1 - t_i = 0, \Delta f_i = 0) \). Optimal policy for the central government (23)
requires

\[
\frac{\partial f_i \gamma}{\partial g_i} f_i (1-t_i) = 1.
\]

We use the regional budget balance condition \( t_i = g_i \) to solve for

\[
(27) \quad t_i = g_i = \frac{a_i \beta \gamma - 1}{a_i (1 + \beta \gamma)}.
\]

Under perfect competition, regional tax and expenditure rates are therefore both indepen-
dent from the share of mobile labor in each region and invariant to trade costs. Expenditure
rates differ across regions according to the initial level of their governments’ productivity.
By virtue of our assumption about the \( a_i \), the smaller the initial level (the larger \( a_i \)), the
higher will be the optimal regional public spending and tax rates, leading to catch-up of weaker regions.
Does regional governments’ optimization lead to the same level of spending and tax rates? Substituting the partial derivatives of the equilibrium net-real wage rate with respect to taxes and government spending into the regional optimality condition (24) yields:

\[
\frac{\partial f_i}{\partial g_i} \gamma (1 - t_i) = - \left[ (1 - \sigma_i(\lambda_i)) \frac{Wm_{o_r}}{\partial \lambda_i} \frac{1}{\lambda_i} + \frac{\partial Wm_{o_r}}{\partial f_i} \right] \frac{\gamma}{f_i} (1 - t_i) \frac{1}{1 - \sigma_i(\lambda_i)} \frac{Wm_{o_r}}{\partial \lambda_i} \frac{1}{\lambda_i} + \frac{\partial Wm_{o_r}}{\partial f_i}.
\]

Comparing both optimality conditions for the central planner and autonomous regional policies leads to the following proposition:

**Proposition 1** If manufacturing varieties are perfect substitutes for each other, autonomous regional governments choose the same optimal level of investment \( g_i = \frac{a_i \beta \gamma - 1}{a_i (1 + \beta \gamma)} \) as a benevolent welfare maximizing central government.

**Proof.** The sufficient condition for identical regional and central policies \( \frac{\partial \lambda_i}{\partial f_i} + \frac{\partial \lambda_i}{\partial f_i} \frac{\partial f_i}{\partial g_i} = 0 \) holds, as \( \frac{\partial \lambda_i}{\partial f_i} = \frac{\lambda_i}{f_i} \left( \frac{\gamma}{1 - \alpha \gamma} \right) (1 - \lambda_i) \), \( \frac{\partial \lambda_i}{\partial f_i} = - \frac{\lambda_i}{1 - t_i} \left( \frac{1}{1 - \alpha \gamma} \right) (1 - \lambda_i) \), and \( \frac{\partial f_i}{\partial g_i} |_{g_i = \frac{a_i \beta \gamma - 1}{a_i (1 + \beta \gamma)}} = \frac{f_i (1 + \beta \gamma) a_i \beta}{(1 + \beta \gamma) a_i + \alpha \beta \gamma - 1} = \frac{(1 + \beta \gamma) a_i}{(a_i + 1)} \frac{f_i}{\gamma(1 - t_i)} \).

This result is consistent with conventional theories of fiscal federalism. For example, Tiebout (1956) and Oates (1972) conclude that local governments can efficiently provide local public goods without direct (technological) spillovers to other regions and should be financed by local taxation. The effect of government taxation and investment on real wages and the allocation of mobile labor hinges crucially on the assumption of catch-up via productive government spending in equation (7). This leads to higher real wages in all regions and induces mobile labor to spread. This results in smaller income differences between immobile workers across regions.

4.1.2 Infinite Trade Costs

In our next step towards comparing the policy implications of the new trade theory with the frictionless world of perfect competition, we now assume infinite interregional trade costs,
which imply regional autarky of production and consumption.

For \( T_{i,j} \to \infty \) \( \forall i \neq j \) and \( T_{i,i} = 1 \) the manufacturing price index (15) collapses to \( PM_i = \frac{\omega}{\rho} \left( \frac{1}{\epsilon F} \right)^{\frac{1}{1-\epsilon}} X_i^{-\frac{1}{1-\epsilon}} P x_i \). Hence, regional consumers consume only local varieties of goods and the local intermediate good is the single cost factor in the regional manufacturing price index. Moreover, the terms of trade in each region depend only on the level of regional intermediate goods production \( X_i \).  

\[
(28) \quad \frac{P x_i}{P M_i} = \frac{\rho}{\omega} \left( \frac{1}{\epsilon F} \right)^{\frac{1}{1-\epsilon}} X_i^{\frac{1}{1-\epsilon}} = \frac{\rho}{\omega} \left( \frac{Q}{\epsilon F} \right)^{\frac{1}{1-\epsilon}} \lambda_i^{\frac{\alpha}{(\alpha + \beta)}} \theta_i^{\frac{1-\alpha}{1-\epsilon}} f_i (g_i)^{\frac{1}{1-\epsilon}}.
\]

Starting from this definition we derive expressions for the elasticities of the terms of trade of regions \( j \) to policy changes in region \( i \). Inserting these elasticities into the optimality conditions for central and regional autonomous governments, we arrive at the following proposition:

**Proposition 2**  If manufacturing varieties are imperfect substitutes for each other, in regional autarky both a central planner as well as autonomous regional governments choose optimal investment and tax rates \( g_i^{CP} = g_i^{RP} = \frac{a_i \beta \gamma - \rho}{a_i (\rho + \beta \gamma)} \).

**Proof.** See appendix A.

Since \( \rho < 1 \), expenditure and tax rates are higher in a setting with increasing returns to scale and monopolistic competition than under perfect competition. This is because public investment benefits increasing returns production more than perfectly competitive production through increases in total factor productivity through public investment. Again, as under perfect competition, regional policies are sufficient to achieve a nationally optimal outcome when trade costs are infinite.

\[30\]The intermediate goods production (8) can be written in a more compact form: \( X_i = Q \lambda_i^{\alpha} \theta_i^{1-\alpha} f_i \) if one substitutes the equilibrium wages (17) and (18) into demand for immobile and mobile labor of intermediate goods production (9) and (10), with \( Q = \tilde{L} \sigma \tilde{L} \sigma m \tilde{m}^{1-\alpha} \left[ \frac{(1-\alpha) \gamma}{1-\alpha \gamma} \right]^{1-\alpha} \).

\[31\]See appendix A.
4.1.3 Zero Trade Costs

Both previous scenarios led to autarky either because trade barriers are too high or because gains from trading the same variety are zero. What happens when substitutability between variants and trade costs are both low? They both increase incentives to trade across regions, and therefore spillover effects occur. As the central planner has the power to internalize external effects, we expect her to set policies that are superior to regional policies for the economy as a whole. Is the central planners’ public investment choice higher or lower with trade than under autarky? Under regional authority, will regional investment in some regions outpace national investment as set by a central planner?

To answer these questions, we study the case of no trade costs, where $T_{i,j} = 1 \forall i, j$. In this case, intermediate goods and manufacturing price indices (20) and (15) are identical for all regions. Consequently, the terms of trade are also identical and simplify to a function of the sum of the regional production of intermediate goods:

$$\frac{P_x}{P_M} = \frac{\rho}{\omega} \left( \frac{1}{\epsilon F} \right)^{\frac{1}{\epsilon-1}} \left[ \sum_{j=1}^{R} X_j \right]^{\frac{1}{\epsilon-1}}.$$

An isolated increase of the intermediate goods production in one region therefore improves the terms of trade in all regions symmetrically. An increase in intermediate goods production leads to an increase in the number of manufacturing varieties, and without trade costs, these additional varieties are available for the same price in all regions. This increase in varieties leads to a decline in the consumption based manufacturing price index $P_M$. As expected, zero trade costs maximize positive spillovers form one region to all others.

To learn about relative investment levels under the two regimes, we need to gain additional insight on the elasticity of the terms of trade with respect to policy changes (see appendix B). The elasticities with respect to productivity changes and tax reductions in region $i$ are:

$$\varepsilon_{P_x,f_i} - \varepsilon_{P_M,f_i} = \frac{1}{\epsilon-1} \lambda_i + \frac{1}{(\epsilon-1)(1-\alpha\gamma)} \left[ \frac{X_i}{\sum_{j} X_j} - \lambda_i \right],$$
\[
\varepsilon_{P_{x_{1-t_i}}} - \varepsilon_{P_{x_{1-t_i}}} = \frac{\alpha}{(\varepsilon-1)(1-\alpha)} \left[ \frac{X_i}{\sum_j X_j} - \lambda_i \right].
\]

Both elasticities depend on the difference between the industry share of a region \(X_i/\sum_j X_j\) and the mobile labor share \(\lambda_i\). It follows that this relationship also determines optimal investment policies under both fiscal regimes. Recall that we defined \(c^C_P\) and \(c^R_P\) as the optimality parameters for the central and regional governments, respectively. The elasticities allow us to solve for the optimality parameters (c-values) for the different regimes. For the central planner, we obtain:

\[
c^C_P = \rho \frac{\lambda_i (1-\alpha \gamma) + \frac{\alpha \gamma}{(\varepsilon-1)}}{\lambda_i (1-\alpha \gamma) + \frac{1}{\varepsilon}} \left[ \frac{X_i}{\sum_j X_j} - \lambda_i \right],
\]

and for regional policies:

\[
c^R_P = \rho \frac{[1-\sigma_i(\lambda_i)](1-\lambda_i) + \lambda_i (1-\alpha \gamma) + \frac{\alpha \gamma}{(\varepsilon-1)}}{[1-\sigma_i(\lambda_i)](1-\lambda_i) + \lambda_i (1-\alpha \gamma) + \frac{1}{\varepsilon}} \left[ \frac{X_i}{\sum_j X_j} - \lambda_i \right].
\]

**Proposition 3** With zero trade costs and finite elasticity of substitution, the central planner sets public investment in each region above the autarky level if a region’s share of manufacturing is higher than its share of mobile workers and vice versa. Autonomous regional governments choose investment shares above the autarky levels for regions with a very large share in national manufacturing, a high mobile to immobile labor share ratio \(\lambda_i/\theta_i\), combined with a relative high degree of overall labor mobility, a low share of manufacturing \(\gamma\), or a low productivity of mobile labor \(\alpha\). They choose below autarky levels in all other cases.

**Proof of Part 1 for the central planner.**

A unique stable equilibrium with \(\rho > \alpha \gamma\) implies that \(\frac{\alpha \gamma}{(\varepsilon-1)} < \frac{1}{\varepsilon}\). Therefore, if \(X_i/\sum_j X_j \geq \lambda_i\), then \(c^C_P \leq \rho\) and the expenditure ratio chosen by the central planner is (weakly) greater than in autarky. If \(X_i/\sum_j X_j < \lambda_i\), the expenditure ratio is smaller than in autarky.

**Proof of Part 2 for regional policies.**
\( c_{i}^{RP} > \rho \), the spending ratio is below the autarky level if 
\[
[1 - \sigma_i(\lambda_i)](1 - \lambda_i) + \frac{\alpha \gamma}{(\varepsilon - 1)} \left( \frac{X_i}{\sum_j X_j} - \lambda_i \right) > [1 - \sigma_i(\lambda_i)](1 - \lambda_i) \rho + \frac{1}{\varepsilon} \left( \frac{X_i}{\sum_j X_j} - \lambda_i \right),
\]
which can be simplified to 
\[
\frac{[1 - \sigma_i(\lambda_i)](1 - \lambda_i)}{\rho - \alpha \gamma} > \left( \frac{X_i}{\sum_j X_j} - \lambda_i \right). \]
Thus, even when region \( i \) has a relatively large manufacturing share (right-hand side square bracket is positive), the optimal investment share can still be below the autarky level.

The necessary condition for \( c_{i}^{RP} < \rho \), assuming a complete concentration of manufacturing in region \( i \), is: 
\[
\frac{\rho - \alpha \gamma}{\rho} > 1 - \sigma_i(\lambda_i) \Leftrightarrow \sigma_i(\lambda_i) > \frac{\alpha \gamma}{\rho}.
\]
Defining the degree of overall mobility as \( mob = \frac{L_{mo}}{L_{mo} + L_{im}} \), the share of mobile workers in the regional workforce \( \sigma_i(\lambda_i) = \lambda_i L_{mo}/(\theta_i L_{im} + \lambda_i L_{mo}) = 1/(1 + \frac{\theta_i L_{im}}{\lambda_i L_{mo}}) \) can be written as \( \sigma_i(\lambda_i) = 1/(1 + \frac{\theta_i}{\lambda_i} \frac{(1-mob)}{mob}) \). Thus, the necessary condition can be restated as:
\[
\frac{\lambda_i}{\theta_i} > \frac{\alpha \gamma}{\rho - \alpha \gamma} \frac{(1-mob)}{mob}. \]

The rationale underlying this proposition is as follows: The central planner uses real resources for public investment in every region such that a maximum of varieties of goods can be produced with the given national labor force. If there is symmetry in fundamentals, she chooses the same investment ratios as in autarky, although there are beneficial spillovers from additional investment in one region. The explanation for this is that spillovers lead to welfare improvements in other regions, but they do not change the marginal calculus of costs and benefits of a central planner that already maximizes the real wages of mobile workers.

The case is different when there are differences in mobile labor productivity across regions. Here the central planner can achieve a higher level of overall consumer satisfaction by adjusting investment rates compared to the autarky level and allowing a further reallocation of mobile labor. This is due to the fact that average productivity levels are a proxy for marginal productivity of mobile labor.\(^{32}\)

In a region with labor productivity above the national average, there is an additional positive net-impact of public investment on the number of varieties. In this case, the gain of varieties in the region with further investment overcompensates the loss of varieties in other regions as mobile labor emigrates from there. The opposite is true for regions with below

\(^{32}\)This can be seen from the production function (8), written in compact form \( X_i = Q\lambda_i^{\alpha-1}f_i \), where 
\[
\frac{\partial X_i}{\partial \lambda_i} = \alpha Q\lambda_i^{\alpha-1}f_i = \alpha \frac{X_i}{\lambda_i}.
\]
average labor productivity. There is a net benefit from reducing investment, which crowds away workers but which is overcompensated by the gain of varieties through the emigrated mobile labor employed in other regions.

In contrast, autonomous regional governments simultaneously attempt to maximize per capita income of mobile workers and improve per-capita incomes of immobile workers in their region. Thus, they are particularly sensitive to changes in the share of mobile workers in their regional population \( \sigma_i (\lambda_i) \). Because of the positive spillovers to other regions, the marginal response of \( \lambda_i \) to additional expenditure and taxation in region \( i \) becomes negative for lower levels of investment compared to autarky. This biases the incentives of autonomous regional policy makers to invest downward.

**Proposition 4** With zero trade costs and a finite elasticity of substitution, autonomous regional fiscal policies always lead to lower tax and investment rates compared to centralized policies. The investment gap is the larger the smaller the economic size of a region in terms of \( \lambda_i \). An increase in the share of mobile workers in the national population and/or a rising elasticity of substitution \( \epsilon \) reduces the investment gap.

**Proof.** Since all regional terms of trade are identical, so are the responses to policy changes. Hence for all \( j \in R \), we can write \( \varepsilon_{P_{x_j},1-t_i} - \varepsilon_{P_{M_j},1-t_i} = \varepsilon_{P_{x_j},1-t_i} - \varepsilon_{P_{M_j},1-t_i} \), and \( \varepsilon_{P_{x_i},f_i} - \varepsilon_{P_{M_i},f_i} = \varepsilon_{P_{x_i},f_i} - \varepsilon_{P_{M_i},f_i} \). For the central planner this implies that

\[
\begin{align*}
C_{CP}^i &= \frac{\lambda_i + \gamma (\varepsilon_{P_{x_i},1-t_i} - \varepsilon_{P_{M_i},1-t_i})}{\lambda_i + \varepsilon_{P_{x_i},f_i} - \varepsilon_{P_{M_i},f_i}}.
\end{align*}
\]

As tax changes only shift intermediate goods production from one region to another, whereas public investment in a region will additionally raise intermediate goods production: \( \gamma (\varepsilon_{P_{x_i},1-t_i} - \varepsilon_{P_{M_i},1-t_i}) < \varepsilon_{P_{x_i},f_i} - \varepsilon_{P_{M_i},f_i} \). It follows, that \( C_{CP}^i < 1 \). The identity of the terms of trade elasticities implies for regional policy decisions that

\[
C_{RP}^i = \frac{[1 - \sigma_i(\lambda_i)] [1 + \gamma (\varepsilon_{P_{x_i},1-t_i} - \varepsilon_{P_{M_i},1-t_i})] + [\sigma_i(\lambda_i) \alpha \gamma] [\lambda_i + \gamma (\varepsilon_{P_{x_i},1-t_i} - \varepsilon_{P_{M_i},1-t_i})]}{[1 - \sigma_i(\lambda_i)] [1 + \varepsilon_{P_{x_i},f_i} - \varepsilon_{P_{M_i},f_i}] + [\sigma_i(\lambda_i) \alpha \gamma] (\lambda_i + \varepsilon_{P_{x_i},f_i} - \varepsilon_{P_{M_i},f_i})} \Leftrightarrow
\]

\[
C_{RP}^i = \frac{[1 - \sigma_i(\lambda_i)] [1 - \lambda_i] + [\lambda_i + \gamma (\varepsilon_{P_{x_i},1-t_i} - \varepsilon_{P_{M_i},1-t_i})]}{[1 - \sigma_i(\lambda_i)] [1 - \lambda_i] + (\lambda_i + \varepsilon_{P_{x_i},f_i} - \varepsilon_{P_{M_i},f_i})}.
\]

As \( 1 > \sigma_i(\lambda_i) > 0, 1 > \lambda_i > 0, \) and \( 1 > \alpha \gamma > 0 \), the term \( \frac{[1 - \sigma_i(\lambda_i)] [1 - \lambda_i]}{[1 - \alpha \gamma]} \) will be always
greater than zero. Together with \( \gamma (\varepsilon_{P_{x,1-t_{i}}} - \varepsilon_{P_{M,1-t_{i}}}) < \varepsilon_{P_{x,1}} - \varepsilon_{P_{M,1}} \), this implies that \( c_{i}^{CP} < c_{i}^{RP} \), and therefore that \( g_{i}^{CP} > g_{i}^{RP} \).

Since \([1 - \sigma_{i}(\lambda_{i})](1 - \lambda_{i})\) declines with \( \lambda_{i} \), the gap between regional policies and those of the central planner \( g_{i}^{CP} > g_{i}^{RP} \) will be larger the smaller \( \lambda_{i} \) (the economically weaker region \( i \)). Since for given levels of \( \lambda_{i} \), \( c_{i}^{RP} \) falls and \( g_{i}^{RP} \) rises with a higher share of mobile workers \( \sigma_{i}(\lambda_{i}) \) in each region, a nationwide increase in the share of mobile workers in total workers also reduces the investment gap between regional and central policies. Finally, the investment gap gets larger, the smaller the ratio \( \frac{\lambda_{i} + \gamma(\varepsilon_{P_{x,1-t_{i}}} - \varepsilon_{P_{M,1-t_{i}}})}{\lambda_{i} + \varepsilon_{P_{x,1}} - \varepsilon_{P_{M,1}}} \), and hence, the larger \( g_{i}^{CP} \) is. As \( g_{i}^{CP} \) falls with rising \( \rho \), it follows that the investment gap also shrinks with rising \( \rho \).  

The reason regional policy makers choose too low investment rates from a national welfare perspective is that investment benefits spill over to other regions as follows: Public investment in region \( i \) leads to a higher number of manufacturing varieties. No trade costs implies that these additional varieties are available in all other regions at the same price. This leads to a decrease in the consumer based manufacturing price indices and a subsequent increase in the terms of trade elsewhere.

These spillovers together with love of variety drive this pecuniary externality as they induce regional policy makers to extend investment and taxation beyond a point that attracts additional mobile workers \( \left( \frac{\partial \lambda}{\partial t_{i}} + \frac{\partial \lambda}{\partial f_{i}} \frac{\partial f}{\partial g_{i}} < 0 \right) \). Regional policy makers maximize regional per-capita income. Therefore, they trade-off the benefits of a further increase in real-incomes through more varieties (for both mobile and immobile workers) against the isolated reduction of immobile workers’ wages due to their loss of relative local scarcity. Weighting these factors against each other leads to public investment and taxation below a level that maximizes national per capita income, which is achieved by solely maximizing the net-wages of mobile workers.

The lower the economic strength of a region, the larger the investment gap. This can be explained as follows: Economically weaker regions have a higher share of immobile workers and mobile labor is relatively scarce. Thus, the marginal cost of losing mobile workers
through higher taxation is relatively high. Local governments therefore have lower incentives
to increase taxes and investment. We can test this prediction of the model as regions with
weaker economic fundamentals, such as low total factor productivity, should also see weak
public investment.

4.2 General Comparison of Optimal Regional and Centralized Policies

If $c_{RP_i} \geq c_{CP_i}$, the central planner always invests at least as much or more than autonomous
regions, $g_{RP_i} \leq g_{CP_i}$. Comparing the $c$-values from the two optimality conditions, this
requires that

$$\frac{[1-\sigma_i(\lambda_i)](1+\gamma(\epsilon_{PX_i,1-t_i}-\epsilon_{PM_i,1-t_i})] + [\sigma_i(\lambda_i)-\alpha\gamma](\lambda_i+\gamma\Delta_1-t_i)}{[1-\sigma_i(\lambda_i)](1+\epsilon_{PX_i,f_i}+\epsilon_{PM_i,f_i}) + [\sigma_i(\lambda_i)-\alpha\gamma](\lambda_i+\Delta_f_i)} \geq \frac{\lambda_i+\gamma\Delta_1-t_i}{\lambda_i+\Delta_f_i} \iff$$

$$\frac{1+\gamma(\epsilon_{PX_i,1-t_i}-\epsilon_{PM_i,1-t_i})}{1+\epsilon_{PX_i,f_i}+\epsilon_{PM_i,f_i}} \geq \frac{\lambda_i+\gamma\Delta_1-t_i}{\lambda_i+\Delta_f_i}.$$ 

This condition allows us to derive the following generalized proposition for the relative
size of regional versus centralized public investment for arbitrary levels of non-zero trade
costs.

**Proposition 5** For any finite trade costs and elasticity of substitution between manufac-
turing varieties, regionally determined tax and investment rates are lower than those of a
welfare maximizing central government. The investment gap between regional and centralized
policies declines with rising trade costs.

**Proof.** Defining $\Delta_{1-t_i}^* = \sum_{j\neq i}^R \lambda_j(\epsilon_{PX_j,1-t_i} - \epsilon_{PM_j,1-t_i})$ and $\Delta_{f_i}^* = \sum_{j\neq i}^R \lambda_j(\epsilon_{PX_j,f_i} - \epsilon_{PM_j,f_i})$ we can write the above condition as

$$\frac{1+\gamma(\epsilon_{PX_i,1-t_i}-\epsilon_{PM_i,1-t_i})}{1+\epsilon_{PX_i,f_i}+\epsilon_{PM_i,f_i}} \geq \frac{1+\gamma(\epsilon_{PX_i,1-t_i}-\epsilon_{PM_i,1-t_i})+\frac{\gamma_i}{\lambda_i}\Delta_{1-t_i}^*}{1+\epsilon_{PX_i,f_i}+\epsilon_{PM_i,f_i}+\frac{\gamma_i}{\lambda_i}\Delta_{f_i}^*}.$$ We can reformulate this as: $\frac{a}{b} \geq \frac{a+c}{b+d}$. From $\epsilon_{PX_i,f_i} - \epsilon_{PM_i,f_i} \geq \gamma(\epsilon_{PX_i,1-t_i} - \epsilon_{PM_i,1-t_i})$ follows $b \geq a$. Furthermore, tax cuts
in region $i$ will always have a negative impact on the terms of trade in the other regions $j$,
such that $\Delta_{1-t_i^*} \leq 0$, $e \leq 0$. $e$ will be the smaller, the lower the trade costs are, as relocation of mobile labor to region $i$ becomes more attractive. $\Delta_{f_i^*}$ is positive with small trade costs when the positive spillovers from the availability of more varieties are large, but it will become negative with high trade costs. But again, positive (even fading) spillovers from productivity changes will insure that $\Delta_{f_i^*} > \gamma \Delta_{1-t_i^*}$ and hence, $f > e$. Finally, stability requires that the absolute value of the weighted sum of the effects in the other regions should never be larger than the initial impact in region $i$, such that $\frac{a}{b}$ is an upper ceiling for $\frac{e}{f}$. This implies that $a \cdot f > b \cdot e \Leftrightarrow \frac{a}{b} \geq \frac{a+e}{b+f}$. Since falling trade costs reduce $e$ and increase $f$, the investment gap $g_{CP}^i - g_{RP}^i$ also will get larger.

The fundamental insight of the last two propositions is not the well-known result that the central planner’s policies are welfare optimal. It is rather that location and trade costs between regions are key determinants of productivity enhancing public policies. Specifically, local governments underinvest when trade costs are finite. Our model suggests that decentralized expenditure combined with location economics contributes to the observed weakness in public investment of regional governments. Propositions 3 and 4 establish this weakness to be more pronounced in smaller or weaker economic regions close to economic centers. In our model, regional underinvestment and corresponding taxation are the result of their governments’ attempts to attract mobile labor. As a consequence, decentralized policies inhibit agglomeration, but also reduce income inequality compared to a welfare optimum.

If globalization continues to reduce trade costs, our model predicts public investment to decrease further in countries with fiscal decentralization. Moreover, the decline is expected to more pronounced in the regions that are smaller and more distant from economic centers. If an increase in labor mobility accompanies this reduction in trade cost, smaller numbers of immobile workers have lower influence on regional policies. This automatically aligns regional investment policies more closely to those of a benevolent central policy maker. There is no analytical solution to find out which effect dominates. To study this issue, we therefore turn to simulating our model next.
\section*{4.3 Simulations for a Two-region Setup with Finite Trade Costs}

To learn how optimal investment varies with changes in the elasticity of substitution and trade costs, we employ simulations in a two-region setting. As we have shown before, the local terms of trade are the transmission vehicle for those changes. We therefore analyze the elasticities of the terms of trade in all regions with respect to policy changes in a particular region. We formalize them in four systems (two regions, two policy variables) with eight equations each. For example, marginal productivity changes in region 1 have the following effect:

\[
\begin{pmatrix}
u_1 - 1 & u_2 & u_1(\epsilon - 1) & u_2(\epsilon - 1) & u_1 & u_2 & 0 & 0 \\
u_3 & u_4 - 1 & u_3(\epsilon - 1) & u_4(\epsilon - 1) & u_3 & u_4 & 0 & 0 \\
v_1(1 - \epsilon) & v_2(1 - \epsilon) & -1 & 0 & v_1 & v_2 & 0 & 0 \\
v_3(1 - \epsilon) & v_4(1 - \epsilon) & 0 & -1 & v_3 & v_4 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & -\alpha \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & -\alpha \\
-w_1 & w_1 & -w_2 & w_1 & 0 & 0 & 1 & 0 \\
w_2 & -w_2 & -w_2 & w_2 & 0 & 0 & 0 & 1 \\
\end{pmatrix}
\begin{pmatrix}
\varepsilon_{P11} \\
\varepsilon_{P21} \\
\varepsilon_{PM1} \\
\varepsilon_{PM2} \\
\varepsilon_{X1} \\
\varepsilon_{X2} \\
\varepsilon_{\lambda1} \\
\varepsilon_{\lambda2} \\
\end{pmatrix}
= 
\begin{pmatrix}
0 \\
0 \\
0 \\
0 \\
w_1 \\
0 \\
1 \\
-w_2 \\
\end{pmatrix}
\]

\[
\varphi_1 = \frac{\beta}{\rho} \left( \frac{1}{\epsilon F} \right)^{\frac{1}{\epsilon}},
\]
\[
\varphi_2 = \frac{\rho}{\beta \epsilon} \left[ \frac{1}{(\epsilon - 1)F} \right]^\frac{1}{\epsilon},
\]
\[
u_1 = \varphi_2 \frac{PM_1}{P_{x1}} \left( PM_1 \right)^{1-\epsilon} X_1,
\]
\[
u_2 = \left( \frac{P_{x2}}{P_{x1}} \right)^{\epsilon} T_{2,1}^{1-\epsilon} u_4,
\]
\[
u_3 = \varphi_1 \frac{1-\epsilon}{1-\epsilon} \left( \frac{P_{x1}}{PM_1} \right)^{1-\epsilon} X_1,
\]
\[
u_4 = \left( \frac{PM_1}{PM_2} \right)^{1-\epsilon} T_{2,1}^{1-\epsilon} v_2,
\]
\[
\lambda_1 = \frac{\gamma}{1 - \alpha \gamma} (1 - \lambda_1),
\]
\[
\lambda_2 = \left( \frac{\gamma}{1 - \alpha \gamma} \right) \lambda_1.
\]

The 8x8 matrix \( A \) comprises all simultaneous relations between the endogenous variables in the two-region model. The right-hand side vector captures the direct effect of a marginal productivity change in region 1 on prices, intermediate goods production, and mobile labor allocation in the two regions. The entries in the 5\textsuperscript{th}, 7\textsuperscript{th} and the 8\textsuperscript{th} row show that the initial effect is a higher output of intermediate goods production \( X_1 \) and a shift of mobile labor towards region 1 (a rise of \( \lambda_1 \), a respective decline of \( \lambda_2 \)). Similarly, the system of responses to a tax change in region 1 is:
\[
\begin{pmatrix}
  u_1 - 1 & u_2 & u_1(\epsilon - 1) & u_2(\epsilon - 1) & u_1 & u_2 & 0 & 0 \\
  u_3 & u_4 - 1 & u_3(\epsilon - 1) & u_4(\epsilon - 1) & u_3 & u_4 & 0 & 0 \\
  v_1(1 - \epsilon) & v_2(1 - \epsilon) & -1 & 0 & v_1 & v_2 & 0 & 0 \\
  v_3(1 - \epsilon) & v_4(1 - \epsilon) & 0 & -1 & v_3 & v_4 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
  -w_1 & w_1 & w_1 & -w_1 & 0 & 0 & 1 & 0 \\
  w_2 & -w_2 & -w_2 & w_2 & 0 & 0 & 0 & 1 
\end{pmatrix}
\begin{pmatrix}
  \varepsilon_{P_{X_1,1-t_1}} \\
  \varepsilon_{P_{X_2,1-t_1}} \\
  \varepsilon_{P_{M_1,1-t_1}} \\
  \varepsilon_{P_{M_2,1-t_1}} \\
  \varepsilon_{X_1,1-t_1} \\
  \varepsilon_{X_2,1-t_1} \\
  \varepsilon_{\lambda_1,1-t_1} \\
  \varepsilon_{\lambda_2,1-t_1}
\end{pmatrix}
= \begin{pmatrix}
  0 \\
  0 \\
  0 \\
  0 \\
  0 \\
  0 \\
  w_1/\gamma \\
  -w_2/\gamma
\end{pmatrix}.
\]

The matrix of simultaneous responses \(A\) stays the same, only the impact vector looks slightly different. There is no direct impact on \(X_1\), and the effect of tax reductions on mobile labor is larger than that of productivity changes by the factor \(1/\gamma\). Policy changes in region 2 can be analyzed analogously.\(^{33}\)

The matrices are too complex to derive interpretable reduced forms for the terms-of-trade changes or the optimal tax and spending rates. Therefore, we use the following simulation routine to derive spatial equilibria and optimal policies simultaneously for fixed values of \(\epsilon\) and varying trade costs \(T_{2,1}\).\(^{34}\)

1. Initially, we set policies in both regions to the limiting case of infinite trade costs, hence \(g_1 = t_1 = \frac{a_1 \beta \gamma - \rho}{a_1 (\rho + \beta \gamma)}\), \(g_2 = t_2 = \frac{a_2 \beta \gamma - \rho}{a_2 (\rho + \beta \gamma)}\). This allows calculating equilibrium values for \(\lambda_1\) and \(Wmo^*\) according to equations (21) and (22), using the terms-of-trade definition in (28).

2. Next, we calculate a spatial equilibrium for finite trade costs \(T_{2,1}\) with given policies \(g_1\) and \(g_2\). We find the terms of trade iteratively, using the previously calculated \(\lambda_1\) and terms of trade as starting values. We repeat iterations of the terms of trade, updating the definitions for intermediate goods and manufacturing price indices (20) and (15) and intermediate goods production (8), until we achieve convergence for \(\lambda_1\).

3. We adjust optimal polices \(g_1\) and \(g_2\) according to either (25) or (26) by calculating all the required terms-of-trade elasticities through inversion of the response matrix \(A\) at given values for the terms of trade and \(\lambda_1\).

4. We repeat steps 2 and 3 until we achieve convergence of \(Wmo^*\).

\(^{33}\)Also in this case the matrix of simultaneous responses \(A\) stays the same, only the impact vector on the right-hand side has to be adjusted accordingly.

\(^{34}\)The source code for the STATA do-file can be obtained from the authors upon request.
We run our simulations with the parameter calibration discussed earlier: we set the output elasticity of mobile labor $\alpha = 0.6$, the share spent on manufactures $\gamma = 0.4$, and the ratio of mobile to immobile labor in the economy $L_{mo}/L_{im} = 0.25$. Regions in our simulations are of different economic strength and have different public investment needs. We implement this by assuming different levels of total factor productivity, $f_1 = 3 > f_2 = 1.5$. For the fraction of immobile workers in national immobile worker supply we assume symmetry: $\theta_1 = \theta_2 = 0.5$.

In our model, the elasticity of output with respect to public capital $\beta$ and the marginal impact of public investment on productivity $a_i$ mainly determine optimal public investment. Stephan (2003) estimates output elasticities $\beta$ between 0.38 and 0.65, such that we plausibly can set $\beta = 0.5$. To reflect our catch-up hypothesis we choose a value of $a$ such that the investment rate of the stronger region 1 is zero in the case of perfect competition in manufacturing ($\rho = 1$). At the same time, this implies a relatively high minimum investment rate of 12.5% in region 2, which reflects the large initial productivity gap in combination with our catch-up hypothesis.35 In a first step, we hold the elasticity of substitution fixed at $\epsilon = 3.5$.36 We analyze centralized and autonomous policies separately and focus on variations of trade costs. In a second step, we compare the outcomes of both regimes directly for a broad range of trade costs and elasticities of substitution between varieties.

4.3.1 Results for the Benevolent Central Government

Figure 3 displays the central planner’s optimal investment rates for different trade costs. In our baseline calibration (left panel), spending rates in the economically stronger region 1 (high total factor productivity) are less than half the spending rates than in region 2, reflecting the initial productivity disadvantage of region 2 in combination with our strong assumptions on catching-up by public investment. While optimal investment rates change with trade costs, the deviations from policies under autarky (horizontal lines) are rather

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35Note that these ‘minimum investment rates’ are also lower bounds in the world of imperfect competition and zero trade costs. In this case $c_{K}^{R} \rightarrow 1$ as $\lambda_{i} \rightarrow 0$.

36Brakman, Garretsen and Schramm (2004) estimate the Helpman-Model for Germany and obtain values of 3.8-3.9 which allows to plausibly set $\epsilon = 3.5$. 

33
small in both regions (less than 0.1% points). As predicted by proposition 5, investment rates differ the most from autarky levels when trade costs are zero ($T_{i,j} = 1$).

**Figure 3:** The central planner’s optimal polices

![Figure 3](image)

(a) below average labor productivity in region 1 (b) above average labor productivity in region 1

Notes: Figure 3 displays simulation results for optimal investment as share of GDP for different productivity levels of mobile labor in both regions. The central planner sets public investment for both regions. Trade costs are plotted on the horizontal axes, government spending for region 1 is plotted on the left vertical axes, and government spending for region 2 on the right vertical axes. The baseline parameter calibration in panel (a) implies $X_1/(\lambda_1 L_m) < (X_1 + X_2)/L_m$. For panel (b) we calibrate $\alpha$ and $\gamma$ such that $X_1/(\lambda_1 L_m) > (X_1 + X_2)/L_m$.

In our baseline calibration (left panel), spending rates converge to autarky levels from below in the stronger region 1 and from above in region 2. According to proposition 3, this indicates that in region 2 the equilibrium level of labor productivity in manufacturing lies above the national average. It is the case despite its lower initial total factor productivity $\bar{f}_2$ and is driven by the interaction of productivity changes and mobile labor migration. In particular, the given total factor productivity disadvantage of the weaker region 2 drives labor migration from the weaker region 2 to the stronger region 1. The declining marginal productivity of labor in the Cobb-Douglas production function then forces a boost in the average labor productivity in region 2. This endogenous productivity increase due to a reduction of mobile labor employment in region 2 overcompensates the initial total factor productivity disadvantage in our baseline scenario.

As can be shown for the case of zero trade costs, the elasticity of mobile labor productivity

34
in region $i$ with respect to productivity changes in region $j$ is \( \frac{d(X_i/\lambda_i)}{df_j} \frac{f_j}{X_i/\lambda_i} = \frac{(1-\alpha)\gamma_j}{1-\alpha\gamma} \lambda_j \). Thus, for higher $\alpha$ and/or lower $\gamma$ the endogenous change of mobile labor productivity is smaller. Therefore, the local productivity level in equilibrium is lower than the national average. In our second simulation (right hand panel of Figure 3), we choose a higher output elasticity of mobile labor $\alpha = 0.8$, and a lower income share spent on manufactured goods $\gamma = 0.3$. As expected, we obtain an optimal investment rate for the weaker region 2 that lies below and for the stronger region 1 that lies above the autarky level for small trade costs.\(^{37}\) Since optimal investment rates positively depend on $\gamma$, they drop by roughly 5 percentage points compared to the first scenario.\(^{38}\)

4.3.2 Results for Autonomous Regional Policies

The simulations of optimal policies of autonomous governments are displayed in figure 4. As under central policies shown in figure 3, investment rates in the stronger region in the baseline scenario (left panel) are again less than half than in the weaker region. As in the case of central polices, optimal investment as a share of local GDP varies with the size of trade costs. However, regional governments’ investment choices structurally differ from the central planner’s in three respects. First, when trade costs fall, investment rates rapidly decline in both regions (strong and weak). As predicted by propositions 4 and 5, regional governments choose significantly lower investments for small trade costs than the central planner. Second, optimal investments vary substantially stronger with trade costs under regional policies than under central policies. Third, as predicted by proposition 3, investment rates are lower than

\(^{37}\)Note that this follows from our catch-up hypothesis. If we assume equal marginal productivity of public investment in both regions (e.g. $a_1 = a_2=10$), optimal investment rates for the baseline calibration are also higher than in autarky in the stronger region. More generally, an important determinant of mobile labor productivity vis-à-vis the national average is the productivity of public investment. Under the catch-up hypothesis, public investment is more productive in weaker regions, which leads to higher public investment rates in weaker than in stronger regions. This requires weaker regions to set higher tax rates, which incentivizes mobile workers to relocate to other regions. Thus, strong catch-up crowds away mobile workers and leads to above average productivity of mobile workers in the respective region.

\(^{38}\)In the baseline scenario optimal investment in autarky amounts to $g_1 = \frac{a_1 \beta_1 - \rho}{\alpha_1 (\rho+\beta_1)} \approx 5.6 \cdot 0.5 \cdot 0.4 \cdot 0.71 = 7.8\%$. In the second scenario with above average labor productivity in region 1 optimal investment is $g_1 = \frac{5.6 \cdot 0.5 \cdot 0.3 \cdot 0.71}{5.6 \cdot (0.71+0.5 \cdot 0.3)} \approx 2.5\%$. 

35
in autarky for finite trade costs in both regions.

**Figure 4: Optimal Policies of Regional Governments**

(a) roughly equal economic size of both regions

(b) economically relatively large region 1

**Notes:** Figure 4 displays simulation results for optimal investment as share of GDP for regional decision making and for different relative economic sizes of both regions. Trade costs are plotted on the horizontal axes, government spending for region 1 is plotted on the left vertical axes, and government spending for region 2 on the right vertical axes. The baseline parameter calibration in panel (a) implies $\lambda_1 \approx 0.55$. For panel (b) we calibrate $\theta$ such that $\lambda_1 \approx 0.85$.

In our baseline calibration (left panel of Figure 4) regional governments choose lower investment shares as long as trade costs are not prohibitive. The gap between the investment shares of regional and central policies is highest without trade costs when spillover effects are the largest. The size of the gap is of relatively equal size at around 3%-points of regional GDP in both regions. This translates into an about 20% lower investment level under regional policies for the weaker region, but an about 40% lower investment level in the economically stronger region, as the central planner also achieves higher total output.

The almost equal percentage-points investment gap in both regions is the result of a relatively equal distribution of mobile labor in equilibrium ($\lambda_1 \approx 0.55$ only slightly varies with trade costs). However, if we increase $\lambda_1$, e.g. by setting the share of immobile labor $\theta_1=0.8$ (right panel of Figure 4), the maximum gap increases in the weaker region 2 to around 4.5%-points of regional GDP, translating into at 35% drop in the investment level. It falls in the stronger region 1 to around 1%-point of GDP, which translates into a 15% lower investment level. This inverse relationship between the investment gap and the share
of mobile workers has been suggested by proposition 4.

### 4.3.3 Comparison of Policies in Both Regimes

We now compare the outcomes of optimal policies of the central planner and regional autonomous governments directly. To display the impact of a simultaneous variations of trade costs and substitution elasticities we again utilize isoquant curves (figure 5). The upper two panels in figure 5 display relative investment under regional versus central policy regimes.

**Figure 5: Outcomes of Regional and Central Policies for Baseline Parameters**

Notes: Figure 5 displays the ratio of optimal investment (upper panels) and differences in mobile labor and net real wages of mobile labor (left and right lower panels, respectively) under decentralized and centralized policies. Trade costs are plotted on the vertical axes, $\epsilon$, the elasticity of substitution, is plotted on the horizontal axes. Isoquants mark 10 linear steps between maximum and minimum levels. E.g. in the upper two graphs for relative investment levels each isoquant represents a step of 4 percentage points.

Proposition 4 states that an increase in the degree of labor mobility reduces the investment gap between regional and central policies. If we increase labor mobility from our baseline 0.2 to 0.5 ($Lmo/Lim = 1$), the investment gap for zero trade costs declines by approximately one percentage point of GDP.
Each line represents the same relative investment for combinations of trade costs and elasticities of substitution. Darker shaded areas indicate lower investment under regional relative to central policy regime. The two panels show that there exists a threshold combination of trade costs and substitution elasticities. Above this threshold combination, investment under both policy regimes is the same (white areas in the graphs). Below this threshold combination, the lower trade costs or the elasticity of substitution, the lower are investment levels in both regions under regional relative to central policies. In our calibration, this gap can reach up to 43% lower investment if autonomous regions set policy as compared to the central planers’ investment in region 1. This reflects our propositions 4 and 5.

As the upper panels of figure 5 show, regional policies lead to lower investment. This lower investment leads to lower tax rates, attracting more mobile labor. As a consequence, the concentration of mobile labor is higher in the weaker region, and analogous to investment, the concentration rises with falling trade costs and lower elasticity of substitution (lower left-hand panel). From an overall welfare perspective, this dispersed allocation of mobile labor together with low levels of taxation and investment is suboptimal. This is reflected in net-real wages of mobile labor (a sufficient proxy for national per-capita income) that are lower under the regional policy regime than under centralized policies. The wage differences between the two (lower right-hand panel) follow the same pattern as the three other panels.

5 Conclusion and Empirical Relevance

Our spatial economic model gives rise to pecuniary externalities that help explain differences of local public investment across regions in decentralized fiscal systems. The strength of the effects depends on the distance to economic centers and the degree of competition in markets. Due to this externality, regions reduce their public investment and charge lower taxes in order to keep mobile workers in the region. This causes inefficiently low public investment, too much agglomeration in the economically weaker regions and amplifies differences in regional
productivity.

As this externality increases with falling transportation costs, the model offers a variety of testable hypotheses. According to the model, regions that are close to large economic centers invest smaller amounts as they have an incentive to avoid higher tax rates and instead enjoy positive spill-overs from the centers. The smaller the economic size of these peripheral regions, the higher the share of immobile workers in the region and the higher the disincentive to invest because of the crowding-away of mobile workers.

This effect is present in the data, as an analysis of the distribution of public investment across the 401 districts (Kreise) in Germany for 2013 shows. Figure 6 (left panel) depicts public investment as a share of local GDP. Public investment significantly varies within state borders (red lines) and is concentrated in the south-east. To assess the impact of closeness to economic centers on local public investment we construct an indicator for the relative real market potential ($RMP$):\(^{40}\)

\[
RRMP_i = \frac{RMP_i}{\sum_i RMP_i}, \quad \text{with } RMP_i = \gamma \sum_j R \cdot Y_j \cdot PM_j^{\epsilon - 1} \cdot T_{j,i}^{1-\epsilon}.
\]

We compute the RRMP-indicator (see right panel) using nominal GDP and employment data as starting values and simulate $\lambda_i$ and $PM_i$.

$RRMP$ displays the well-known corridor of urbanization spreading over Western and Central Europe (so-called ‘blue banana’).

We complement this visual inspection with a small regression analysis. With our data, we can test our main hypothesis from propositions 3 to 5 that proximity to large markets reduces investment as a share of local GDP. We therefore expect a negative sign on our real market potential variable. We can also test our catching-up assumption, which implies a negative sign on local GDP. We therefore regress the share of local public investment on our indicator for real market potential and GDP first. The results are displayed in the first column in table 1. Both variables have the expected negative sign. In columns II to IV, we repeat the exercise and include factors which might influence local public investment, namely the location in East Germany, local productivity in manufacturing, dependence on

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\(^{40}\)See Head and Mayer (2004), p. 2620.
Figure 6: Spatial Distribution of Local Public Investment and Market Potential

Notes: Local Public investment data is taken from DIW (2015). Real Market Potential is obtained by simulating the equilibrium allocation for $\lambda_i$ according to equation (21) by the procedure described in section 4.3. As starting values, the shares of mobile workers $\lambda_i$ are set to the shares of regional nominal GDP ($Y_i$) in total GDP in 2015. The shares of immobile workers $\theta_i$ are estimated assuming that 80% of the total labor force ($L$) of 43.057 Mio. is immobile and using regional employment data by: $\theta_i = (L_i - \lambda_i \cdot 20\% \cdot L)/ (80\% \cdot L)$. Regional productivity levels are estimated by $\gamma_i / [L \cdot (\lambda_i \cdot 20\%)^\alpha \cdot (\theta_i \cdot 80\%)^{1-\alpha}]$. Ice-berg trade costs are computed as: $T_{j,i} = 1 + [\text{Distance}(i,j)^{0.35}]$, with internal Distance(i,i) = 2/3 $\sqrt{\text{Area}_i / \pi}$. Regional GDP and employment Data is obtained from Statistisches Bundesamt (2017). Spatial Data is obtained from http://www.geodatenzentrum.de.
Table 1: Determinants of Local Public Investment

<table>
<thead>
<tr>
<th>Local Public Investment as % of GDP:</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>$RRMP$</td>
<td>-0.937***</td>
<td>-1.192***</td>
<td>-0.958***</td>
<td>-0.790***</td>
</tr>
<tr>
<td></td>
<td>[0.132]</td>
<td>[0.158]</td>
<td>[0.133]</td>
<td>[0.141]</td>
</tr>
<tr>
<td>$Y$</td>
<td>-0.132***</td>
<td>-0.009***</td>
<td>-0.006***</td>
<td>-0.006***</td>
</tr>
<tr>
<td></td>
<td>[0.002]</td>
<td>[0.002]</td>
<td>[0.002]</td>
<td>[0.002]</td>
</tr>
<tr>
<td>constant</td>
<td>1.893***</td>
<td>2.192***</td>
<td>2.048***</td>
<td>2.048***</td>
</tr>
<tr>
<td></td>
<td>[0.134]</td>
<td>[0.168]</td>
<td>[0.133]</td>
<td>[0.140]</td>
</tr>
<tr>
<td>EAST</td>
<td>-0.212***</td>
<td>0.176***</td>
<td>0.176***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[0.051]</td>
<td>[0.065]</td>
<td>[0.059]</td>
<td></td>
</tr>
<tr>
<td>Long-term unemployed</td>
<td>-0.006***</td>
<td>-0.006***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>[0.000]</td>
<td>[0.001]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Transfers from state</td>
<td>0.286***</td>
<td>0.286***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>[0.092]</td>
<td>[0.094]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Municipal Debt</td>
<td>0.000</td>
<td>0.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>[0.014]</td>
<td>[0.014]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Adjusted R$^2$</td>
<td>0.16</td>
<td>0.20</td>
<td>0.45</td>
<td>0.48</td>
</tr>
<tr>
<td>No. of observations</td>
<td>397</td>
<td>397</td>
<td>397</td>
<td>397</td>
</tr>
</tbody>
</table>

Notes: robust standard errors in parenthesis. *** denotes significance at the 1% and 5% levels respectively. EAST is a dummy variable that is set to 1 if the district belongs to the former Democratic Republic of Germany and 0 otherwise. Long-term unemployed (as a share of residents under age of 65), transfers from state (per resident), municipal debt (per resident) are for the year 2014 and taken from the INKAR-database of the Bundesinstitut für Bau-, Stadt- und Raumforschung (BBSR).

financial transfers from the state, financial obligations to care about long-term unemployed, and indebtedness. The results are robust to the inclusion of our control variables, which have plausible signs and apart from the municipal debt are also highly significant. Municipalities are responsible for expenses for the long-term unemployed, which is a well-known indicator of fiscal tightness. As such, we expect it to enter our results with a negative sign, which it does. Designated state transfers relieve fiscal tightness and free resources for public investment. They therefore enter our regression with a positive sign. Finally, our dummy variable for municipalities in the former Democratic Republic of Germany (EAST) is significant but displays collinearity with other variables. In regression II it seems to capture to some extend the remoteness of Eastern regions from economic centers, while in III and IV it also seems
to represent the strong fiscal support of Eastern states for their municipalities resulting from reconstruction transfers after reunification.

The results from our preliminary empirical analysis gives us confidence that the theoretical considerations of our paper bear empirical relevance. In future research we will analyze the dependence of local public investment on proximity to economic centers over time and for different countries. In such panel analysis we will be able to test whether reductions of trade costs over time (captured e.g. by traveling times) coincide with lower investment, smaller regional budgets, and a shift of taxation onto immobile factors. From our model, we expect sub-optimally low investment to be more pronounced in strongly decentralized federal systems or in regions with high discretion on taxes and investment spending. We will investigate this hypothesis with data from other countries that exhibit different degrees of fiscal decentralization.

References


International Monetary Fund (2014), World Economic Outlook, October 2014, Chapter 3.


Organization for Economic Cooperation and Development (2016), Economic Outlook,
Appendix

A. Optimal spending and tax ratios with infinite trade costs

From the terms of trade in equation (28) we can derive the following elasticities of the terms of trade of a region \( j \) with respect to policy changes in region \( i \):

\[
\frac{\partial P_{ij}}{\partial T_i} = \frac{\partial Q_{ij}}{\partial T_i} = \frac{\partial R_{ij}}{\partial T_i} = \frac{\partial S_{ij}}{\partial T_i} = \frac{\partial T_{ij}}{\partial T_i} = 0.
\]

\[
\frac{\partial P_{ij}}{\partial T_i} = \frac{\partial Q_{ij}}{\partial T_i} = \frac{\partial R_{ij}}{\partial T_i} = \frac{\partial S_{ij}}{\partial T_i} = \frac{\partial T_{ij}}{\partial T_i} = 0.
\]
\begin{align*}
\varepsilon_{P_x,1-t_i} - \varepsilon_{P_M,1-t_i} &= \frac{1}{\varepsilon-1} \varepsilon_{X,1-t_i} = \frac{\alpha}{\varepsilon-1} \varepsilon_{\lambda,1-t_i}, \\
\varepsilon_{P_x,f_i} - \varepsilon_{P_M,f_i} &= \frac{1}{\varepsilon-1} \varepsilon_{X,f_i} = \begin{cases} \\
\frac{1}{\varepsilon-1} (\alpha \varepsilon_{\lambda,f_i} + 1), & j = i \\
\frac{\alpha}{\varepsilon-1} \varepsilon_{\lambda,f_i}, & j \neq i \end{cases}.
\end{align*}

This implies that:

\begin{align*}
\Delta_{1-t_i} &= \sum_{j=1}^{R} \lambda_j (\varepsilon_{P_x,1-t_i} - \varepsilon_{P_M,1-t_i}) = \frac{\alpha}{\varepsilon-1} \sum_{j=1}^{R} \lambda_j \varepsilon_{\lambda,1-t_i}, \\
\Delta_{f_i} &= \sum_{j=1}^{R} \lambda_j (\varepsilon_{P_x,f_i} - \varepsilon_{P_M,f_i}) = \frac{1}{\varepsilon-1} \sum_{j=1}^{R} \lambda_j \varepsilon_{X,f_i} = \frac{\lambda_i}{\varepsilon-1} + \frac{\alpha}{\varepsilon-1} \sum_{j=1}^{R} \lambda_j \varepsilon_{\lambda,f_i}.
\end{align*}

Since

\begin{align*}
\varepsilon_{\lambda,1-t_i} &= \begin{cases} \\
\left(\frac{1}{1-\alpha \gamma}\right) \left[1 - \lambda_i + \gamma (\varepsilon_{P_x,1-t_i} - \varepsilon_{P_M,1-t_i}) - \gamma \Delta_{1-t_i} \right], & j = i \\
\left(\frac{1}{1-\alpha \gamma}\right) \left[-\lambda_i + \gamma (\varepsilon_{P_x,1-t_i} - \varepsilon_{P_M,1-t_i}) - \gamma \Delta_{1-t_i} \right], & j \neq i \end{cases},
\end{align*}

and

\begin{align*}
\varepsilon_{\lambda,f_i} &= \begin{cases} \\
\left(\frac{\gamma}{1-\alpha \gamma}\right) \left[1 - \lambda_i + \varepsilon_{P_x,f_i} - \varepsilon_{P_M,f_i} - \Delta_{f_i} \right], & j = i \\
\left(\frac{\gamma}{1-\alpha \gamma}\right) \left[\varepsilon_{P_x,f_i} - \varepsilon_{P_M,f_i} - \lambda_i - \Delta_{f_i} \right], & j \neq i \end{cases},
\end{align*}

the sum of all weighted terms-of-trade responses to tax changes is zero, \(\Delta_{1-t_i} = 0\), and the sum of weighted terms-of-trade responses to productivity changes is \(\Delta_{f_i} = \frac{\lambda_i}{\varepsilon-1}\). Therefore, the optimality condition for the central planner \(c_{i,CP}^{\text{CP}} = \frac{\lambda_i + \gamma \Delta_{1-t_i}}{\lambda_i + \Delta_{f_i}}\) directly reduces to \(c_{i,CP}^{\text{CP}} = \rho\).

The optimality condition for regional governments simplifies to

\begin{align*}
c_{i,RP}^{\text{RP}} &= \frac{[1 - \sigma_i (\lambda_i)] [1 + \gamma (\varepsilon_{P_x,1-t_i} - \varepsilon_{P_M,1-t_i})] + [\sigma_i (\lambda_i) - \alpha \gamma] \lambda_i}{[1 - \sigma_i (\lambda_i)] [1 + \varepsilon_{P_x,f_i} - \varepsilon_{P_M,f_i}] + [\sigma_i (\lambda_i) - \alpha \gamma] \lambda_i}.
\end{align*}

Solving the following two equations for the terms of trade response to tax changes:

\begin{align*}
\varepsilon_{P_x,1-t_i} - \varepsilon_{P_M,1-t_i} &= \frac{\alpha}{\varepsilon-1} \varepsilon_{\lambda,1-t_i}, \\
\varepsilon_{\lambda,1-t_i} &= \left(\frac{1}{1-\alpha \gamma}\right) \left[1 - \lambda_i + \gamma (\varepsilon_{P_x,1-t_i} - \varepsilon_{P_M,1-t_i}) - \gamma \Delta_{1-t_i} \right], \quad \text{leads to}
\end{align*}

(i) \(\varepsilon_{P_x,1-t_i} - \varepsilon_{P_M,1-t_i} = \frac{\alpha}{\varepsilon(1-\alpha \gamma)-1} (1 - \lambda_i)\).

Solving the following two equations for the terms of trade response to productivity changes:

\begin{align*}
\varepsilon_{P_x,f_i} - \varepsilon_{P_M,f_i} &= \frac{1}{\varepsilon-1} (\alpha \varepsilon_{\lambda,f_i} + 1), \\
\varepsilon_{\lambda,f_i} &= \left(\frac{\gamma}{1-\alpha \gamma}\right) \left[1 - \lambda_i + \varepsilon_{P_x,f_i} - \varepsilon_{P_M,f_i} - \Delta_{f_i} \right], \quad \text{leads to}
\end{align*}

46
(ii) \( \varepsilon_{P_{x,f_i}} - \varepsilon_{P_{M,f_i}} = \frac{1}{\varepsilon-1} + \frac{\alpha \gamma}{(\varepsilon(1-\alpha \gamma)-1)(\varepsilon-1)} (1 - \lambda_i) \).

Substituting (i) and (ii) into the square brackets, the optimality condition for the regime with regionally autonomous fiscal policies can be restated as

\[
C_{RP}^i = \frac{[1 - \sigma_i(\lambda_i)] \left[ 1 + \frac{\alpha \gamma}{(\varepsilon(1-\alpha \gamma)-1)} (1 - \lambda_i) \right] + [\sigma_i(\lambda_i) - \alpha \gamma] \lambda_i}{\left[ \frac{1 - \sigma_i(\lambda_i)}{\rho} \right] \left[ 1 + \frac{\alpha \gamma}{(\varepsilon(1-\alpha \gamma)-1)} (1 - \lambda_i) \right] + \frac{[\sigma_i(\lambda_i) - \alpha \gamma]}{\rho} \lambda_i} = \rho \iff g_{RP}^i = g_{CP}^i.
\]
B. Derivation of terms-of-trade elasticities for zero trade costs

As highlighted in the main text, with zero trade costs the terms of trade depend on the aggregate industrial production across all regions: \( \frac{P_x}{P_M} = \frac{\rho}{\omega} \left( \frac{1}{\epsilon F} \right)^{\frac{1}{\epsilon - 1}} \left[ \sum_{j=1}^{R} X_j \right]^{\frac{1}{\epsilon - 1}} \). It follows that:

\[
\frac{dP_x}{d\rho} = \frac{1}{\epsilon - 1} \frac{P_x}{PM} \left[ \sum_{j=1}^{R} X_j \right] \sum_{j=1}^{R} \frac{dX_j}{d\rho}, \quad \frac{dP_x}{d(1-t_i)} = \frac{1}{\epsilon - 1} \frac{P_x}{PM} \left[ \sum_{j=1}^{R} X_j \right] \sum_{j=1}^{R} \frac{dX_j}{d(1-t_i)}.
\]

Writing the production function for intermediate goods (8) in a more compact form \( X_i = \bar{R}_i \lambda_i \alpha f_i \), the partial derivatives with respect to \( f_i \) and \( 1 - t_i \) follow as:

\[
\frac{dX_i}{df_i} = \frac{1}{\alpha} \left\{ X_i \cdot (\alpha \varepsilon_{\lambda_i, f_i} + 1), \quad j = i \right\}, \quad \text{and} \quad \frac{dX_i}{d(1-t_i)} = \frac{\alpha}{\gamma} \left\{ X_j \cdot \varepsilon_{\lambda_j, 1-t_i}, \quad j \neq i \right\}.
\]

Substituting this back into \( \frac{dP_x}{df_i} \) and \( \frac{dP_x}{d(1-t_i)} \) leads to:

\[
\varepsilon_{P_x, f_i} - \varepsilon_{P_M, f_i} = \frac{1}{\epsilon - 1} \left[ \alpha \sum_{j=1}^{R} \frac{X_j}{\sum_j X_j} \varepsilon_{\lambda_j, f_i} + \frac{X_i}{\sum_j X_j} \varepsilon_{\lambda_i, 1-t_i} \right], \quad \text{and}
\]

\[
\varepsilon_{P_{x, 1-t_i}} - \varepsilon_{P_{M, 1-t_i}} = \frac{\alpha}{\epsilon - 1} \left[ \sum_{j=1}^{R} \frac{X_j}{\sum_j X_j} \lambda_j \varepsilon_{\lambda_j, 1-t_i} \right].
\]

With zero trade costs \( \varepsilon_{P_{x, f_i}} - \varepsilon_{P_{M, f_i}} = \Delta_f_i \), and \( \gamma (\varepsilon_{P_{x, 1-t_i}} - \varepsilon_{P_{M, 1-t_i}}) = \gamma \Delta_{1-t_i} \), the elasticities of mobile labor with respect to \( f_i \) and \( 1 - t_i \) collapse to:

\[
\varepsilon_{\lambda_j, f_i} = \begin{cases} \left( \frac{\gamma}{1-\alpha \gamma} \right) (1 - \lambda_i), & j = i \\ - \left( \frac{\gamma}{1-\alpha \gamma} \right) \lambda_i, & j \neq i \end{cases},
\]

\[
\varepsilon_{\lambda_j, 1-t_i} = \begin{cases} \left( \frac{1}{1-\alpha \gamma} \right) (1 - \lambda_i), & j = i \\ - \left( \frac{1}{1-\alpha \gamma} \right) \lambda_i, & j \neq i \end{cases}.
\]

Substituting this into the above expression for the terms-of-trade elasticities yields the expressions used in the main text:

\[
\varepsilon_{P_x, f_i} - \varepsilon_{P_M, f_i} = \frac{1}{\epsilon - 1} \lambda_i + \frac{1}{(\epsilon - 1)(1-\alpha \gamma)} \left[ \frac{X_i}{\sum_j X_j} - \lambda_i \right], \quad \text{and}
\]

\[
\varepsilon_{P_{x, 1-t_i}} - \varepsilon_{P_{M, 1-t_i}} = \frac{\alpha}{(\epsilon - 1)(1-\alpha \gamma)} \left[ \frac{X_i}{\sum_j X_j} - \lambda_i \right].
\]