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**Conference Paper**

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Beiträge zur Jahrestagung des Vereins für Socialpolitik 2018: Digitale Wirtschaft - Session: Development Economics III, No. G14-V1

**Provided in Cooperation with:**

Verein für Socialpolitik / German Economic Association

*Suggested Citation:* Ait Lahcen, Mohammed (2018) : Money, Inflation, and Unemployment in the Presence of Informality, Beiträge zur Jahrestagung des Vereins für Socialpolitik 2018: Digitale Wirtschaft - Session: Development Economics III, No. G14-V1, ZBW - Leibniz-Informationszentrum Wirtschaft, Kiel, Hamburg

This Version is available at:

<https://hdl.handle.net/10419/181578>

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# Money, Inflation, and Unemployment in the Presence of Informality<sup>☆</sup>

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## Abstract

This paper studies the impact of informality on the long-run relationship between inflation and unemployment in developing economies. I present a dynamic general equilibrium model with informality in both labor and goods markets and where money and credit coexist. An increase in inflation affects unemployment through two channels: the entry channel (size) and the hiring channel (composition). On one hand, higher inflation reduces the surplus of monetary trades thus lowering firms entry and increasing unemployment. On the other hand, it shifts firms hiring decision from high separation informal jobs to low separation formal jobs thus reducing unemployment. The net effect depends on the difference in separation rates and the availability of credit in formal transactions. The model is calibrated to match certain long-run statistics of the Brazilian economy. Numerical results indicate that inflation has a small negative effect on unemployment while producing a significant impact on labor allocation between formal and informal jobs. These results point to the importance of accounting for informality when considering the inflation-unemployment trade-off in the conduct of monetary policy.

*Keywords:* informality, Phillips curve, money, labor, search and matching

*JEL classification:* E26, E41, J64, H26, O17

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## 1. Introduction

The long-run relationship between inflation and unemployment in developed economies has been studied extensively. Several authors found compelling evidence against a vertical long-run Philips curve (Karanassou et al., 2003; Beyer and Farmer, 2007; Schreiber and Wolters, 2007; Berentsen et al., 2011) which stands at odds with the assumption of long-run neutrality of money and makes this issue relevant for welfare and policy analysis. Despite this evidence, the literature on the shape of the long-run Philips curve (LRPC, henceforth) in developing economies is almost non-existent. Interestingly enough, the relationship between inflation and unemployment in the context of a developing economy might be affected by specific institutional failures not present in developed economies such as strong informality, weak labor institutions

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<sup>☆</sup>I am grateful to my advisor Aleksander Berentsen and co-advisor Pedro Gomis-Porqueras for their useful comments that greatly improved the paper and to my colleagues Lukas Altermatt and Florian Madison for many insightful discussions. I thank Borağan Aruoba, Kenza Benhima, Mariano Bosch, Pierre Cahuc, Gabriele Camera, Francesco Carli, Fernando Martin, Cyril Monnet, Randall Wright and seminar participants at the Universities of Basel, Bern, and Zurich, the Cadi Ayyad University of Marrakech, the ASSET conference 2017, the Gerzensee alumni conference 2016 and the 10th RGS doctoral conference in economics for valuable comments and suggestions.

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and limited financial inclusion. This work aims to help fill this gap in the literature by studying the impact of informality on the shape of the LRPC.

The contribution of this paper consists in developing a theoretical framework that captures the relationship between inflation and unemployment in the presence of informality in both labor and goods markets. To this purpose, I introduce informality into a monetary dynamic general equilibrium model with frictional labor and goods markets based on Berentsen, Menzio, and Wright (2011). I extend the model of Berentsen et al. (2011) mainly in three directions. First, I add a hiring choice between formal and informal jobs which allows for an endogenously determined size of the informal sector. Second, I add credit as a means of payment alongside money in decentralized goods market transactions. Third, I link the ability of firms to offer credit to their hiring decision in the labor market. These three ingredients allow for interesting interactions between inflation, unemployment and informality.

In the model, inflation affects unemployment through two channels: the entry channel and the hiring channel. On one hand, higher inflation reduces the surplus of monetary trades thus lowering firms entry and increasing unemployment. On the other hand, the lower impact of inflation on formal transactions where credit is partially available shifts the hiring decision towards formal jobs with a lower separation rate thus reducing unemployment. The matching technology in the goods market provides an amplification mechanism which can strengthen both channels. The equilibrium effect is ambiguous and depends on the size of the informal sector and the availability of credit in formal transactions. I calibrate the model to match some long-run statistics of the Brazilian economy. Numerical results show that, with a sizable informal sector, inflation has a significant effect on labor reallocation between formal and informal jobs but the net effect on unemployment and output is very small. This results points to the importance for central banks to take into consideration the size of the informal sector in the conduct of monetary policy and in particular with regard to the trade-off between inflation and unemployment.

There is a strong disagreement in the academic literature and among international institutions about the definition of informality. Some of the main ongoing issues concern both the relationship between legal and illegal informal activities and the distinction between informality and the household sector (ILO, 2013). In this paper, I define informality as the market-based production of legal goods and services which is unreported to the government. This definition excludes the household sector as well as criminal activities. The deliberate effort to conceal informal activities from the government is an important aspect in the distinction between the informal sector and the household sector which encompasses all domestically produced goods and services that are exempt from taxation and labor regulation. In the same way, the inherent legal nature of informal activities stands in contrast to the illegal nature of criminal activities even if both are characterized by the active avoidance of government monitoring.

Despite the lack of consensus on its definition, informality has been one of the main issues studied by development macroeconomists. Within this literature, one can distinguish between two main currents. The first one is empirical and is chiefly concerned with the measurement of informality using reduced-form econometric models. This approach faces considerable methodological challenges to the extent that agents operating within the informal economy are relentlessly trying to conceal any traces

of their activities. To overcome these challenges, economists resort to indirect measurement methods using electricity consumption, money demand and cash transactions, mismatches in national accounts or household surveys. Schneider et al. (2011) provide an extensive survey of this literature.

The second current proposes theoretical models capable of replicating some aspects of agents behavior in order to provide policy recommendations. Early work on this literature focused on the dualistic view of the economy by modeling the presence of an informal sector as the result of barriers to entry into the formal sector facing firms and workers. In more recent models, the informal sector comes to existence as an optimal response to regulatory pressure such as high income tax or stringent labor regulations.

Within the second current, there exists a growing strand studying the effect of different labor institutions and policies on the labor market in the presence of informal employment (Fugazza and Jacques, 2004; Zenou, 2008; Albrecht et al., 2009; Ulyssea, 2010; Bosch and Esteban-Pretel, 2012, 2014; Charlot et al., 2015; Meghir et al., 2015). Another strand focuses on explaining the allocation choices of workers and firms between formal and informal jobs (García, 2013; Almeida and Carneiro, 2012; Boeri and Garibaldi, 2005). This literature also covers other aspects of informal employment such as cyclical flows and transition dynamics between formal and informal labor markets (Bosch and Maloney, 2008, 2010), the wage differentials between formal and informal jobs (El Badaoui et al., 2010) and informal self-employment (Narita, 2014; Albrecht et al., 2009; Satchi and Temple, 2009).

Several authors studied the relationship between taxation and informality (Gomis-Porqueras and Peralta-Alva, 2008; Prado, 2011; Ihrig and Moe, 2004; Aruoba, 2010). These authors usually assume that entering the informal economy is mainly the result of a decision to avoid taxes. The trade-off is often between paying taxes and being part of the formal economy or avoiding taxes and paying a penalty in case of detection by the fiscal authority. The degree of monitoring, legal enforcement and punishment against informality play a strong role in this trade-off. Those parameters can be proxies for the rule of law and the presence of strong institutions which provides broader explanations for the size of the informal sector. One recurrent finding is that the size of the informal sector is negatively affected by an increased level of enforcement or a reduction in the tax rate while weak institutions are often associated with high levels of informality.

This paper stands close to the informality literature dealing with money and inflation. Most of the cross-country empirical evidence points towards a positive relationship between inflation and the size of the informal economy. One explanation is the use of seigniorage as a source of government income in some developing countries suffering from low tax revenues due to a large informal sector. Koreshkova (2006) develops a cash-in-advance model that relates the size of the informal sector to the trade-off between inflation and the ensuing share of seigniorage in the government revenues on the one hand and the income tax on the other. She shows that inflation works to smooth the tax burden between the formal and informal parts of the economy. Gomis-Porqueras et al. (2014) introduce a money search model where agents can avoid taxes on part of their income by using cash transactions. They derive a model-based measure of informality and produce country-level estimates which tend to be on the lower range of the more traditional reduced-form estimates such as those reported by Enste and Schneider

(2000). This can be explained by the stronger restrictions their theoretical model imposes on data as well as the exclusion of illegal activities from their measurements. Bittencourt et al. (2014) use a monetary overlapping generations model with endogenous tax evasion to study the effect of both financial sector development and inflation on the size of the informal economy. They find that a lower level of financial development provides agents with a higher incentive in participating in tax evasion activities.

The rest of the paper is organized as follows: in the next section I present the theoretical model. The equilibrium solution is characterized in section 3. Section 4 summarizes some theoretical results derived from the model. The model's calibration as well as some numerical results are presented in sections 5 and 6. Section 7 concludes.

## 2. Model

I consider a setting with discrete time and infinite horizon. In every period, three markets take place sequentially: a decentralized labor market à la Mortensen and Pissarides (1994), called LM, a decentralized goods market following Kiyotaki and Wright (1993) and Lagos and Wright (2005), called DM, and a centralized Walrasian market, called CM. In the CM market, trade is a frictionless process and agents can buy or sell the numéraire good at the equilibrium market price. In the LM and DM, agents must search for matching opportunities and use bargaining to share the match surplus.

Two types of agents live infinitely in this model, firms and households, indexed by  $f$  and  $h$  respectively and each of measure 1<sup>1</sup>. Households work, buy and consume goods. Firms maximize profits by hiring workers to produce goods and then sell the goods to households. Agents discount between periods using the same time preference rate  $\beta$ . There is no discounting between markets within the period.

An agent (firm or worker) can be in one of three states depending on the type of work arrangement they are taking part in: formal employment,  $e$ , informal employment,  $i$  and unemployment,  $u$ . I define the value functions  $U$ ,  $V$  and  $W$  for the LM, DM and CM respectively. These value functions depend on the agent's type  $t \in \{f, h\}$ , on their current employment status  $j \in \{e, i, u\}$ , on their real money holdings  $z$  as well as on the loans  $\ell$  they extended or received.

Firms and households meet in the LM and try to form bilateral work relationships. I assume random matching based on a matching function,  $\mathcal{M} = \mathcal{M}(u, v)$ , where the number of matches is a function of  $u$ , the measure of unemployed workers, and  $v$ , the measure of posted vacancies. The matching function describes the number of new matches resulting from contacts between unemployed workers and firms seeking to fill open vacancies. As is standard in the labor search literature,  $\mathcal{M}$  is increasing, concave and homogeneous of degree one. On the one hand, a firm with a vacancy finds a worker with probability  $\alpha_f = \mathcal{M}(u, v)/v = \mathcal{M}(1/\theta, 1)$  where  $\theta = v/u$  is the labor market tightness. On the other hand, an unemployed worker finds a job with probability  $\alpha_h = \mathcal{M}(u, v)/u = \mathcal{M}(1, \theta)$ . Firms and workers take the aggregate matching probabilities as given.

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<sup>1</sup>In what follows, the terms *household*, *worker* and *buyer* are used interchangeably. One could also think of a household as being composed of one worker sent to the LM and one buyer sent to the DM. With unit measures of households and firms, the probability that the buyer meets the firm employing his kin worker is 0.

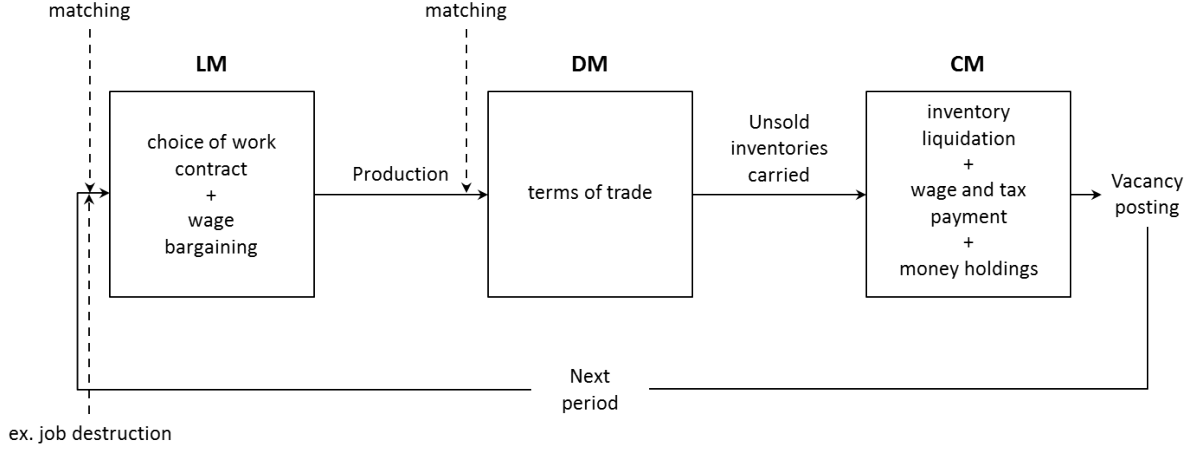


Figure 1: Model structure

All workers and firms are ex-ante identical. Unmatched firms can enter the next period's LM by posting a generic job vacancy at the end of the CM. Once an unmatched firm meets a worker the idiosyncratic productivity of the match,  $\varepsilon$ , is revealed. This productivity is match specific and reflects the quality of the match. It is drawn from a distribution  $F(\varepsilon)$  with bounded support  $[\underline{\varepsilon}, \bar{\varepsilon}]$ . Based on  $\varepsilon$ , the firm and the worker decide whether to engage in a formal or an informal work relationship. I define  $n_e$  as the measure of formally employed workers and  $n_i$  as the measure of informally employed workers. Since each firm corresponds to one job, these measures are also those of formal and informal firms in the economy.

Both formal and informal firms produce  $y + \varepsilon$  units of a good that is storable within the period. The produced good can be sold to households either in the DM or the CM. Selling a quantity  $q$  of the good in the DM costs  $c(q)$ . The remaining inventories are sold in the CM at price  $p$  normalized to 1.

Wages are negotiated in the LM and paid in the following CM in terms of the general good. Firms employing formal workers are subject to a lump-sum tax  $\tau$  which reflects the costs of formal employment.

Formal and informal jobs are destroyed with probabilities  $\delta_e$  and  $\delta_i$  respectively. I assume  $\delta_i \geq \delta_e$ . The higher separation rate of informal jobs can be rationalized through government monitoring or weak enforcement of informal contracts.

Next, firms (sellers) and households (buyers) enter the DM where they can trade the search good  $q$  pairwise. Matching in the DM is based on a random matching function,  $\mathcal{N} = \mathcal{N}(B, S)$ , where  $B$  and  $S$  are the measures of active buyers and sellers respectively.  $\mathcal{N}$  is increasing, concave and homogeneous of degree one. On the one hand, all households take part in the DM as buyers provided they are matched hence  $B = 1$ . On the other hand, I follow Berentsen et al. (2011) by assuming a firm can take part in the DM market as a seller only if it is able to produce. This is only possible if the firm has managed to recruit a worker in the LM which implies  $S = 1 - u$ .

A seller meets a buyer with probability  $\sigma_f = \mathcal{N}(B, S)/S$ . In the same manner, a buyer meets a seller with probability  $\sigma_h = \mathcal{N}(B, S)/B$ . Since there are formal and informal firms operating in this economy, buyers can be randomly matched with either type of firms. With probability  $\frac{n_e}{1-u}$  the encountered seller is

a formal firm and with probability  $\frac{n_i}{1-u}$  an informal firm. To simplify notation, I define the unconditional probability of meeting a formal seller as  $\sigma_e = \sigma_h \frac{n_e}{1-u}$  and the unconditional probability of meeting an informal seller  $\sigma_i = \sigma_h \frac{n_i}{1-u}$ . Notice that the LM and DM are related not only through the measure of active firms but also through the ratio of informal to formal firms. Indeed, for a given measure of active firms, an increase in the share of formal (informal) employment in the labor market increases the probability for buyers to meet a formal (informal) seller.

The search good  $q$  is produced by firms and only households want to consume it during the DM such that there is no double coincidence of wants. Commitment is limited, bilateral meetings are anonymous and agents cannot store the general good in the CM market to use it in subsequent periods as a medium of exchange. I assume the existence of an imperfect record keeping technology which makes contract enforcement by the government possible. Acquiring the record keeping technology is costless but the enforcement of the recorded financial obligations requires compliance with government regulation. Hence, only firms employing formal labor and paying taxes can benefit from government's enforcement and therefore make use of the record keeping technology. On the opposite, informal firms cannot benefit enforce their contracts and as a consequence resort to money as the only mean of exchange for their transactions in the DM. In contrast, in DM formal matches, formal firms can offer loans to buyers. These loans are a form of differed payment or supplier credit to be repaid in terms of the general good in the subsequent CM. However, I assume that credit is available only with probability  $\eta$  which is equivalent to imperfect record keeping. This insures the co-existence of credit and money in formal DM transactions. In addition, government punishment is arbitrarily harsh such that default is not an option for buyers.

The CM is a frictionless Walrasian market where the general good is traded at the equilibrium price  $p$  normalized to 1. In the CM, firms liquidate what remains of their production, post vacancies, pay wages and taxes and distribute their profits as dividends to the households. The latter buy and consume the general good, repay their loans and decide how much money to take to the next period.

Finally, I adopt the following convention regarding money. I define real balances  $z$  such as  $z = m/p = \phi m$  where  $\phi$  is the price of money in terms of the general good. The aggregate quantity of money in the economy is given by  $M$  and grows by the rate  $\gamma = M'/M$ . In the steady state, I assume that the real value of money supply is constant over time such that  $\phi M = \phi' M'$ . This implies that inflation is determined by the growth rate of the quantity of money available in the economy.

### 2.1. Households

At the beginning of the LM, the value function of an unemployed worker is:

$$U_u^h(z) = \alpha_h \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \max \{V_e^h(\varepsilon, z), V_i^h(\varepsilon, z)\} dF(\varepsilon) + (1 - \alpha_h) V_u^h(z)$$

With probability  $\alpha_h$ , an unemployed worker entering the LM with real money holdings  $z$  is matched with a firm and the match productivity  $\varepsilon$  is then revealed. Depending on  $\varepsilon$  the worker is offered a formal job with value  $V_e^h$  or an informal job with value  $V_i^h$ . With probability  $1 - \alpha_h$ , the unemployed worker is not matched and keeps searching for at least another period which yields the same value  $V_u^h$ .

If a worker was employed during the previous period, he enters the LM with his work arrangement still effective. The value functions of formal and informal jobs for a worker with idiosyncratic productivity  $\varepsilon$  are:

$$\begin{aligned} U_e^h(\varepsilon, z) &= (1 - \delta_e)V_e^h(\varepsilon, z) + \delta_e V_u^h(z) \\ U_i^h(\varepsilon, z) &= (1 - \delta_i)V_i^h(\varepsilon, z) + \delta_i V_u^h(z) \end{aligned}$$

which state that a formal (informal) worker might lose his current job with probability  $\delta_e$  ( $\delta_i$ ). In such a case, he becomes unemployed and can be matched with a new firm only starting from the next period.

In the DM, a buyer (household) is randomly matched with a formal or an informal firm from which he can buy quantity  $q$  of the search good which the firm already produced in the previous LM. Both types of firms accept money as a means of payment. However, only formal firms can extend a loan with probability  $\eta$  to be repaid in the next CM. The type of firm a buyer is matched with in the DM is independent of her work status in the LM. The search good provides utility  $v(q)$  with  $v(0) = 0$ ,  $v' > 0$  and  $v'' < 0$ . The value function of a buyer entering the DM with employment status  $j \in \{e, i, u\}$  is:

$$\begin{aligned} V_j^h(\varepsilon, z) &= \sigma_e \eta (v(q_c) + W_j^h(z - \phi d, \ell)) + \sigma_e (1 - \eta) (v(q_m) + W_j^h(z - \phi d)) \\ &\quad + \sigma_i (v(q_m) + W_j^h(z - \phi d)) + (1 - \sigma_e - \sigma_i) W_j^h(z) \end{aligned}$$

The buyer is matched with a formal firms with probability  $\sigma_e$ . If obtaining credit is possible, which occurs with probability  $\eta$ , the buyer can either use money  $d$ , credit  $\ell$  or both to buy quantity  $q_c$ . Otherwise, the buyer has no choice but to use his previously acquired money in which case he buys quantity  $q_m$ . With probability  $\sigma_i$ , the buyer meets an informal seller in which case he is again limited to the use of his money holdings and acquires the same quantity  $q_m$ . With probability  $1 - \sigma_e - \sigma_i$ , the buyer doesn't take part in the DM and so takes her money holdings directly to the CM.

The quantity acquired in formal monetary matches and in informal matches is the same since uncertainty about DM match type resolves long after the buyer had acquired his money holdings and the buyer has no option to adjust his holdings once he learns which type of matches he will face.

When an unemployed household enters the CM, he solves the following maximization problem:

$$W_u^h = \max_{x, z'} \{x + b + \beta U_u^h(z')\}$$

subject to:

$$\begin{aligned} x + \phi m' &= z - \ell + \Delta + T \\ z' &= \phi' m' = \frac{\phi m'}{\gamma} \end{aligned}$$

where  $x$  is consumption,  $b$  is the flow value of unemployment,  $\Delta$  are the dividends distributed by firms



and  $T$  are lump sum transfers by the government.  $z$  are the real balances that the household brings with it to the CM market and  $z'$  are the amount of real balances it takes with it to the next period's LM. Any loans  $\ell$  that the household obtained in the previous DM are settled in the CM.

Inserting the budget constraint in the objective function and making use of the linearity of the value function in current money holdings, I get:

$$W_u^h(z, \ell) = b + \Delta + T + z - \ell + \max_{z'} \{-\gamma z' + \beta U_u^h(z')\}$$

As we can see, the choice of  $z'$  is independent of  $z$ . We will see later that it is also independent of the household's employment status.

In the same way, employed households entering the CM face the following maximization problems:

$$W_j^h(\varepsilon, z, \ell) = w_j(\varepsilon) + \Delta + T + z - \ell + \max_{z'} \{-\gamma z' + \beta U_j^h(\varepsilon, z')\} \quad j \in \{e, i\}.$$

Using the linearity of  $W_j^h(z, \ell)$  in  $z$  and  $\ell$ ,  $V_j^h$  can be written as:

$$V_j^h(\varepsilon, z) = \sigma_e \eta (v(q_c) - \phi d - \ell) + \sigma_e (1 - \eta) (v(q_m) - \phi d) + \sigma_i (v(q_m) - \phi d) + z + W_j^h(\varepsilon, 0) \quad j \in \{e, i, u\}$$

where  $W_j^h(\varepsilon, 0)$  is the value function of a household entering the CM with no money holdings and no outstanding loans. In order to simplify notation, I drop  $z$  and  $\ell$  from the arguments of the value function in the absence of money or loans.

Plugging this in the expression for  $U_u^h$  and inserting its next period expression into  $W_u^h$  I get the following recursive formulation:

$$W_u^h(z, \ell) = b + \Delta + T + z - \ell + Z + \beta \left[ \alpha_h \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \max \{W_e^h(\varepsilon'), W_i^h(\varepsilon')\} dF(\varepsilon') + (1 - \alpha_h) W_u^h \right] \quad (1)$$

where

$$Z \equiv \max_{z'} \{(\beta - \gamma)z' + \beta (\sigma_e \eta (v(q'_c) - \phi' d' - \ell') + \sigma_e (1 - \eta) (v(q'_m) - \phi' d') + \sigma_i (v(q'_m) - \phi' d'))\}$$

In the same fashion, I get the following recursive formulations for  $W_e^h$  and  $W_i^h$ :

$$\begin{aligned} W_e^h(\varepsilon, z, \ell) &= w_e(\varepsilon) + \Delta + T + z - \ell + Z + \beta \delta_e W_u^h + \beta (1 - \delta_e) W_e^h(\varepsilon) \\ W_i^h(\varepsilon, z, \ell) &= w_i(\varepsilon) + \Delta + T + z - \ell + Z + \beta \delta_i W_u^h + \beta (1 - \delta_i) W_i^h(\varepsilon) \end{aligned}$$

From the expression of  $Z$  we clearly see that the choice of real money holdings  $z'$  is independent of the match productivity  $\varepsilon$ . The intuition behind this is that money holdings are useful only in the DM where firms treat all households the same without consideration for their employment status.

In what follows I focus the analysis on non-stochastic stationary equilibria which simplifies the above

expressions to:

$$W_e^h(\varepsilon, z, \ell) = \frac{w_e(\varepsilon) + \Delta + T + z - \ell + Z + \beta\delta_e W_u^h}{1 - \beta(1 - \delta_e)}$$

$$W_i^h(\varepsilon, z, \ell) = \frac{w_i(\varepsilon) + \Delta + T + z - \ell + Z + \beta\delta_i W_u^h}{1 - \beta(1 - \delta_i)}$$

## 2.2. Firms

The value of a firm entering the LM with a vacancy is:

$$U_u^f = \alpha_f \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \max \left\{ V_e^f(\varepsilon), V_i^f(\varepsilon) \right\} dF(\varepsilon) + (1 - \alpha_f) V_u^f$$

with probability  $\alpha_f$  the firm is matched with a worker. The match productivity level  $\varepsilon$  is then revealed and depending on it the firm will decide whether to enter a formal work contract which gives the value  $V_e^f(\varepsilon)$  or an informal work arrangement with value  $V_i^f(\varepsilon)$ . With probability  $1 - \alpha_f$  the firm is not matched and is left with the continuation value  $V_u^f$ .

The value of a firm entering the LM with a formal job with productivity  $\varepsilon$  is:

$$U_e^f(\varepsilon) = (1 - \delta_e) V_e^f(\varepsilon) + \delta_e V_u^f$$

where  $\delta$  is the probability of job destruction. Similarly, the value of firm entering with an informal job is:

$$U_i^f(\varepsilon) = (1 - \delta_i) V_i^f(\varepsilon) + \delta_i V_u^f$$

Matched firms produce the general good in the LM. The output of the match is the sum of the general productivity  $y$  and the match specific productivity  $\varepsilon$ . Firms take their output to the DM and CM for sale. At the beginning of the DM, a firm which had produced in the LM is matched with probability  $\sigma_f$  to a household, supplies him with quantity  $q$  of the special good at cost  $c(q)$  in terms of the general good with  $c' > 0$  and  $c'' \geq 0$  against a payment of money  $\phi d$  or a loan  $\ell$  if the firm is formal. Formal firms can only offer credit with probability  $\eta$  whereas with probability  $1 - \eta$  they can only sell against money. The remaining output  $y + \varepsilon - c(q)$  is carried to the CM and sold at price  $p$  normalized to 1. With probability  $1 - \sigma_f$ , the firm is not matched and carries all of its output  $y + \varepsilon$  to be sold at the CM. This leaves us with the following value functions at the beginning of the DM:

$$V_e^f(\varepsilon) = \sigma_f [\eta W_e^f(y + \varepsilon - c(q_c), \phi d, \ell) + (1 - \eta) W_e^f(y + \varepsilon - c(q_m), \phi d)] + (1 - \sigma_f) W_e^f(y + \varepsilon)$$

$$V_i^f(\varepsilon) = \sigma_f W_i^f(y + \varepsilon - c(q_m), \phi d) + (1 - \sigma_f) W_i^f(y + \varepsilon)$$

In the CM, firms liquidate their unsold inventories and pay wages to their workers. Since holding money is costly and firms have no use for it they do not need to carry any balances to the next period. As a consequence, the value of a firm with productivity  $\varepsilon$  which carries inventory  $x$ , real balances  $z$  and

loans  $\ell$  into the CM simplifies to:

$$W_e^f(\varepsilon, x, z, \ell) = x - w_e(\varepsilon) - \tau + z + \ell + \beta U_e^f(\varepsilon)$$

$$W_i^f(\varepsilon, x, z) = x - w_i(\varepsilon) + z + \beta U_i^f(\varepsilon)$$

Firms employing formal workers have to pay a lump-sum tax in the CM. One can think of  $\tau$  as a proxy for the fiscal burden on formal firms and which informal firms are able to avoid.

As in the DM, an unfilled firm has nothing to sell and thus cannot take part in the CM. However, it can decide to enter the market by posting a vacancy at the end of the CM with the value:

$$W_u^f = \max \{0, -k + \beta U_u^f\}$$

where  $k$  is the cost paid in the CM for posting a vacancy. Assuming free entry, firms keep entering until the benefits of entry are exhausted i.e.  $W_u^f = 0$ , which results in the following free entry condition:

$$\frac{k}{\beta \alpha_f} = \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \max \{V_e^f(\varepsilon), V_i^f(\varepsilon)\} dF(\varepsilon) \quad (2)$$

The structure of the problem of the firm is slightly different from the household's since the firm's payoff in the DM depends on its employment status in the previous LM. Hence I write a recursive formulation in terms of the value function  $V$  at the beginning of DM. Using the linearity of  $W^f$ , I rewrite  $V_e^f$  as:

$$V_e^f(\varepsilon) = \sigma_f \eta (\phi d + \ell - c(q_c)) + \sigma_f (1 - \eta) (\phi d - c(q_m)) + y + \varepsilon - w_e(\varepsilon) - \tau + \beta U_e^f(\varepsilon)$$

Inserting the expression for  $U_e^f$  and making use of the free entry condition I get:

$$V_e^f(\varepsilon) = R_e(\varepsilon) - w_e(\varepsilon) - \tau + \beta (1 - \delta_e) V_e^f(\varepsilon)$$

where  $R_e(\varepsilon) = y + \varepsilon + \sigma_f \eta (\phi d_c + \ell - c(q_c)) + \sigma_f (1 - \eta) (\phi d_m - c(q_m))$  is the current period expected revenue of a formal firm. In the same way, I write the value of an informal firm entering the CM as:

$$V_i^f(\varepsilon) = R_i(\varepsilon) - w_i(\varepsilon) + \beta (1 - \delta_i) V_i^f(\varepsilon)$$

where  $R_i(\varepsilon) = y + \varepsilon + \sigma_f (\phi d - c(q_m))$  represents the expected current period revenues for an informal firm. Since I focus on non-stochastic stationary equilibria, the equations above can be rewritten as:

$$V_e^f(\varepsilon) = \frac{R_e(\varepsilon) - w_e(\varepsilon) - \tau}{1 - \beta(1 - \delta_e)}$$

$$V_i^f(\varepsilon) = \frac{R_i(\varepsilon) - w_i(\varepsilon)}{1 - \beta(1 - \delta_i)}$$

### 2.3. Government

Government prints money at a rate  $\gamma$  and collects lump sum tax  $T^2$ . and payroll tax  $\tau$  which are used to finance and some public spending  $G$ . The government budget constraint can be written as follows:

$$G = T + \tau n_e + (\gamma - 1)\phi M$$

where  $\tau$  is levied on all formal firms, of measure  $n_e$ . The last right hand side term represents seigniorage income. I assume that  $G$  adjusts such that the budget is balanced.

## 3. Equilibrium

In the following, I solve the problems facing agents in each market separately and then put everything together to solve for the general equilibrium of the model. First is the LM where firms and households face the decision of which job contract to choose and which wage to set. Second is the DM where firms and households decide on the terms of trade in both credit and monetary matches. Finally the CM where households decide how much money to carry to the next period and new firms decide whether to enter the labor market.

### 3.1. Optimal decisions in the LM

The choice of the optimal work arrangement, i.e. the arrangement offering the highest present discounted value, depends on the idiosyncratic productivity of the match  $\varepsilon$ . When  $\varepsilon$  is revealed at the beginning of the LM, the matched firm and worker are each faced with two decisions: first, which type of work arrangement to choose given that keeping the match is worthwhile and second, what should the wage be. The first decision determines the total match surplus and the second determines the way the total surplus will be split. The two decisions combined determine the surplus of each party. I start by solving the wage bargaining problem for each job type and then, given the wage bargaining solution, I solve for the optimal work arrangement as a function of the match productivity  $\varepsilon$ .

The surplus of a LM match, after choosing the work arrangement, is defined as the total gain from the match by both the firm and the worker:

$$\begin{aligned} S_e(\varepsilon) &= V_e^f(\varepsilon) - V_u^f + V_e^h(\varepsilon, z, \ell) - V_u^h(z, \ell) \\ &= V_e^f(\varepsilon) + W_e^h(\varepsilon) - W_u^h \end{aligned}$$

$$\begin{aligned} S_i(\varepsilon) &= V_i^f(\varepsilon) - V_u^f + V_i^h(\varepsilon, z) - V_u^h(z) \\ &= V_i^f(\varepsilon) + W_i^h(\varepsilon) - W_u^h \end{aligned}$$

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<sup>2</sup>As discussed by Gomis-Porqueras (2016), under the assumption that taxes are lump-sum, the implied effect of inflation on firms entry and unemployment is less distortive and hence presents a lower bound compared to a proportional tax

The total surplus in both types of jobs is independent of money holdings and outstanding loan obligations which simplifies the problem greatly.

In what follows, I use Nash bargaining with termination threat points to decide the wage when a firm and a worker are matched. This leads to the sharing rules:

$$\omega_e V_e^f(\varepsilon) = (1 - \omega_e) (W_e^h(\varepsilon) - W_u^h) \quad (3)$$

$$\omega_i V_i^f(\varepsilon) = (1 - \omega_i) (W_i^h(\varepsilon) - W_u^h) \quad (4)$$

where  $\omega_e$  and  $\omega_i$  are the bargaining power of formal and informal workers respectively. Notice that in the presence of a proportional wage tax  $\tau_w$ , the sharing rule in formal jobs changes to

$$\omega_e V_e^f(\varepsilon) = (1 + \tau_w)(1 - \omega_e) (W_e^h(\varepsilon) - W_u^h) \quad (5)$$

In this case identical Nash bargaining power for workers in both types of jobs is not enough to guarantee identical surplus sharing rules. The marginal payroll tax reduces the match surplus and creates a wedge which distorts the way the surplus is split between the worker and the firm. Furthermore, since  $\tau_w$  is proportional to the wage, the worker and the firm might find it optimal to reduce the wage in order to increase the total surplus of the match even if it reduces the surplus going to the worker. The presence of a wage tax results also in a difference in sharing rules under formal and informal jobs which creates a discontinuity in the way the surplus is shared rising the possibility that for the same productivity level the firm and worker disagree on what is the optimal choice of contract. Intuitively, going from an informal to a formal contract for the same productivity level might simultaneously change the total surplus and decrease the share of one of the parties.

In order to rule out any possible disagreement, one way is to make the simplifying assumption

$$\omega_i = \frac{\omega_e}{1 + \tau(1 - \omega_e)}$$

resulting in the surplus being shared in the same way independently of the choice of the work contract (Bosch and Esteban-Pretel, 2012). This implies  $\omega_i \leq \omega_e$ , in line with the empirical fact that formal workers are usually more organized and enjoy more legal protection compared to informal workers.

For the sake of simplicity I rule out the proportional wage tax and focus on the lump-sum tax on formal firms  $\tau$ . I also assume  $\omega_e = \omega_i = \omega$  to simply guarantee identical surplus sharing rules between the two types of jobs.

Using the model's equations and the bargaining solutions (5) and (4), I derive the following wage equations:

$$w_e(\varepsilon) = \omega (R_e(\varepsilon) - \tau) - (1 - \omega) (\Delta + T + Z - (1 - \beta)W_u^h)$$

$$w_i(\varepsilon) = \omega R_i(\varepsilon) - (1 - \omega) (\Delta + T + Z - (1 - \beta)W_u^h)$$

I can combine the surplus sharing rules and the free entry condition (2) to rewrite equation (1) as:

$$W_u^h = \frac{b + \Delta + T + Z + \frac{\omega}{1-\omega}\theta k}{1 - \beta}$$

which I use to simplify the wage equations above to:

$$\begin{aligned} w_e(\varepsilon) &= \omega (R_e(\varepsilon) + \theta k - \tau) + (1 - \omega)b \\ w_i(\varepsilon) &= \omega (R_i(\varepsilon) + \theta k) + (1 - \omega)b \end{aligned}$$

These wage equations imply that the value functions of both firms and households are linear and increasing in  $\varepsilon$ . This monotonicity result leads to the following proposition:

**Proposition 1.** *For given parameter values there exists a productivity threshold  $\tilde{\varepsilon}$  above which matched firms and workers will always choose a formal work arrangement and below which matched firms and workers will choose an informal work arrangement.*

The previous proposition follows from the linearity of the model's solution in  $\varepsilon$  and in particular the fact that the household's optimal money holdings are independent of its LM status. Proposition 1 is illustrated in figure 2. In the segment  $[\tilde{\varepsilon}, \bar{\varepsilon}]$  of the support of  $F(\varepsilon)$ , the formal firm's value  $V_e^f(\varepsilon)$  lies above the informal firm's value  $V_i^f(\varepsilon)$ . In the segment  $[\underline{\varepsilon}, \tilde{\varepsilon}]$  this order is reversed.

In what follows I assume the existence of equilibria where the threshold  $\tilde{\varepsilon}$  satisfies the condition  $\tilde{\varepsilon} \in [\underline{\varepsilon}, \bar{\varepsilon}]$ . The existence conditions of such equilibria will be discussed later.

More formally,  $\tilde{\varepsilon}$  is the productivity level for which the matched firm and worker are indifferent between an informal and a formal work contract:

$$V_e^f(\tilde{\varepsilon}) = V_i^f(\tilde{\varepsilon})$$

After some algebra, one can easily derive the expression for  $\tilde{\varepsilon}$

$$\tilde{\varepsilon} = b + \frac{\omega}{1 - \omega}\theta k - y - \sigma_f(g(q_m) - c(q_m)) - \frac{1 - \beta(1 - \delta_i)}{\beta(\delta_i - \delta_e)} [\sigma_f \eta ((g(q_c) - c(q_c)) - (g(q_m) - c(q_m))) - \tau] \quad (6)$$

From equation (6), we have the following partial equilibrium results:

$$\frac{\partial \tilde{\varepsilon}}{\partial \tau} > 0 \quad ; \quad \frac{\partial \tilde{\varepsilon}}{\partial b} > 0 \quad ; \quad \frac{\partial \tilde{\varepsilon}}{\partial k} > 0 \quad ; \quad \frac{\partial \tilde{\varepsilon}}{\partial y} < 0$$

Using the productivity threshold  $\tilde{\varepsilon}$ , I define the job finding rate in the formal sector as  $JFR_e \equiv \alpha_h(1 - F(\tilde{\varepsilon}))$  and in the informal sector as  $JFR_i \equiv \alpha_h F(\tilde{\varepsilon})$ .

I denote employment in the formal and informal sectors as  $n_e$  and  $n_i$  and the economy-wide unemployment rate as  $u$ . The job finding rates above imply that employment measures evolve according to

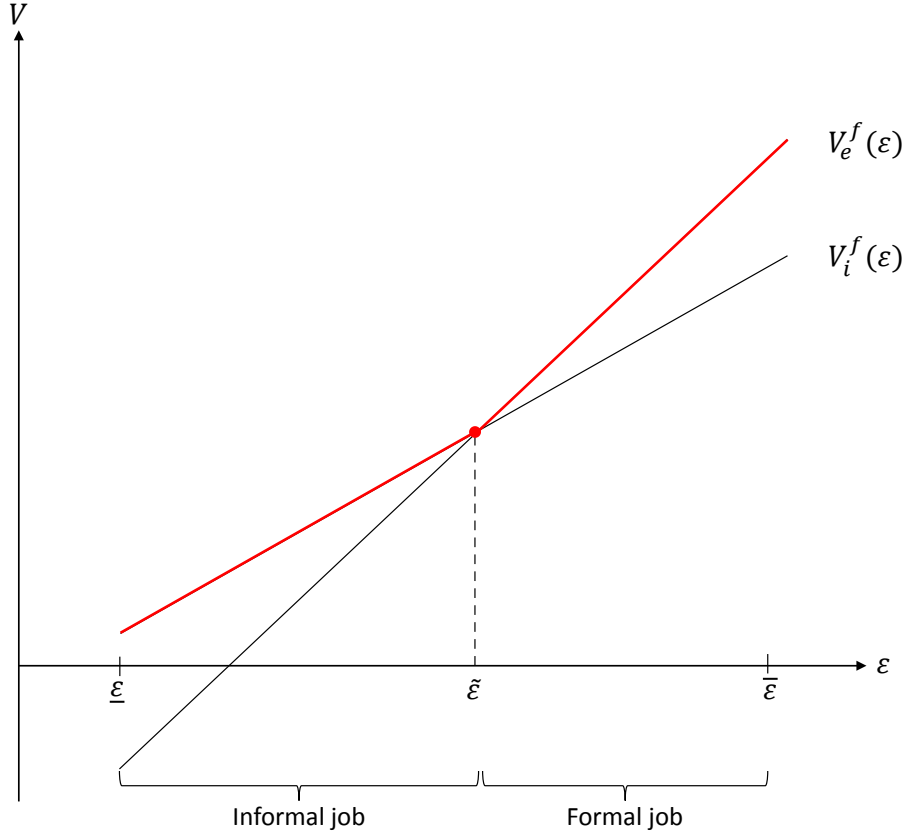


Figure 2: Hiring decision thresholds

the following laws of motion:

$$\begin{aligned}
 u' &= (1 - \alpha_h)u + \delta_e n_e + \delta_i n_i \\
 n_e' &= (1 - \delta_e)n_e + \alpha_h(1 - F(\tilde{\varepsilon}))u \\
 n_i' &= (1 - \delta_i)n_i + \alpha_h F(\tilde{\varepsilon})u
 \end{aligned}$$

which depend on  $\theta$  and  $\tilde{\varepsilon}$  and where  $u + n_e + n_i = 1$  holds for every period.

For stationary equilibria  $\Delta u = \Delta n_i = \Delta n_e = 0$  holds which results in the following steady state distribution of the labor force:

$$\begin{aligned}
 \bar{u} &= \frac{\delta_e \bar{n}_e + \delta_i \bar{n}_i}{\alpha_h} \\
 \bar{n}_e &= \frac{\alpha_h(1 - F(\tilde{\varepsilon}))\bar{u}}{\delta_e} \\
 \bar{n}_i &= \frac{\alpha_h F(\tilde{\varepsilon})\bar{u}}{\delta_i}
 \end{aligned}$$

From the system of equations above, it is easy to solve for unemployment  $u$  as a function of  $\theta$  and  $\tilde{\varepsilon}$ :

$$\bar{u} = \frac{\delta_e + \delta_i \rho(\tilde{\varepsilon})}{(1 + \rho(\tilde{\varepsilon}))\alpha_h(\theta) + \delta_e + \delta_i \rho(\tilde{\varepsilon})} \quad (7)$$

where  $\rho(\tilde{\varepsilon})$  is the steady state ratio of informal to formal employment defined as

$$\rho(\tilde{\varepsilon}) \equiv \frac{n_i}{n_e} = \frac{\delta_e}{\delta_i} \frac{F(\tilde{\varepsilon})}{1 - F(\tilde{\varepsilon})} \quad (8)$$

Notice that this ratio is independent of unemployment and labor market tightness and depends only on the ratio of the separation rates, the match productivity distribution  $F$  and the informality threshold  $\tilde{\varepsilon}$ .

### 3.2. Optimal decisions in the DM

The terms of trade in the DM are determined using the proportional bargaining solution due to Kalai (1977)<sup>3</sup>. I assume that the bargaining power of buyers  $\varphi$  is the same for both types of DM matches. The proportional solution implies that each party receives a constant share of the total surplus  $v(q) - c(q)$  proportional to their bargaining power. For the buyer this share is

$$v(q_m) - \phi d_m = \varphi(v(q_m) - c(q_m))$$

and for the firm it is

$$\phi d_m - c(q_m) = (1 - \varphi)(v(q_m) - c(q_m))$$

where  $\varphi \in [0, 1]$  is the buyer's bargaining power and  $(q_m, \phi d_m)$  the terms of trade (quantity and price) that solve

$$\max_{q_m, d_m} v(q_m) - \phi d_m$$

subject to the seller's participation constraint

$$\phi d_m - c(q_m) \geq (1 - \varphi)(v(q_m) - c(q_m))$$

and the feasibility constraints

$$\begin{aligned} \phi d_m &\leq z \\ c(q_m) &\leq y + \varepsilon \end{aligned}$$

The seller's participation constraint states that the seller will require at least the share of the total surplus resulting from his bargaining power  $1 - \varphi$  to participate in the trade. Since the buyer always gets

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<sup>3</sup>I use the proportional bargaining solution instead of the generalized Nash solution in order to avoid the non-monotonicity resulting from the latter which complicates the comparative statics of the model. However I provide a sketch of the Nash solution in the appendix. For a discussion of different pricing mechanisms in monetary economies see Rocheteau and Nosal (2017).



a positive utility from consuming more he will offer the seller just enough to make him participate. As a consequence, the seller's participation constraint will always be binding. The first feasibility constraint states that the buyer cannot spend more money than he is carrying and the second feasibility constraint states that the firm cannot sell more goods than it produced in the LM. Since we assume  $\gamma > \beta$  carrying money across periods is costly and therefore households don't have an incentive to carry more money than they intend to spend in the DM market. This makes the second constraint always binding. In addition, I assume that the second constraint is never binding. Substituting the binding constraints simplifies the problem to

$$\max_{q_m} \varphi(v(q_m) - c(q_m))$$

subject to

$$(1 - \varphi)v(q_m) + \varphi c(q_m) = z$$

As a consequence, the proportional bargaining solution is a pair  $(q_m, d_m)$  that satisfies  $q_m = g^{-1}(z)$  and  $\phi d_m = z$  with

$$g(q_m) \equiv \varphi c(q_m) + (1 - \varphi)v(q_m) = z$$

Notice that  $\partial q_m / \partial z = \partial g^{-1}(z) / \partial z = 1/g'(q_m) \geq 0$  meaning that more money holdings increases  $q_m$ .

Following the same procedure, the terms of trade in credit matches satisfy

$$\max_{q_c} \varphi(v(q_c) - c(q_c))$$

subject to

$$(1 - \varphi)v(q_c) + \varphi c(q_c) = z + \ell$$

Since there is no restriction on the amount of credit  $\ell$  buyers can get from firms the quantity consumed in credit matches is independent of buyers' money holdings and is always equal to the first bet quantity  $q_c = q^*$  where  $q^*$  solves

$$v'(q^*) = c'(q^*) \tag{9}$$

As a consequence, the proportional bargaining solution in credit matches is a pair  $(q_c, g(q_c))$  that satisfies  $q_c = q^*$ . Whether the quantity  $q^*$  will be acquired using credit or money is irrelevant since buyers are indifferent between the two at this point. Because  $q_m \leq q^*$  always holds, this offers formal firms a higher trading surplus in the DM compared to informal firms as long as  $\eta > 0$ .

### 3.3. Optimal decisions in the CM

Given the terms of trade they expect to face in the next DM, households decide on the optimal amount of money holdings to carry from the CM to the next period:

$$\max_z \left\{ (\beta - \gamma)z + \beta [\sigma_e \eta (v(q_c) - g(q_c)) + (\sigma_e(1 - \eta) + \sigma_i) (v(g^{-1}(z)) - z)] \right\}$$

Using the independence of  $q_c$  from  $z$ , the above problem can be simplified to:

$$\max_z \{(\beta - \gamma - \beta\sigma_e(1 - \eta) - \beta\sigma_i)z + \beta(\sigma_e(1 - \eta) + \sigma_i)v(g^{-1}(z))\}$$

Assuming an interior solution, the first order condition is:

$$\frac{v'(q_m)}{g'(q_m)} = \frac{\gamma - \beta}{\beta(\sigma_e(1 - \eta) + \sigma_i)} + 1$$

Using the Fisher equation  $\frac{\gamma}{\beta} = 1 + i$ , I get the following expression:

$$\frac{v'(q_m)}{g'(q_m)} = \frac{i}{\sigma_e(1 - \eta) + \sigma_i} + 1$$

where  $i$  is the nominal interest rate and  $\sigma_e$  and  $\sigma_i$  depend on the measure of formal and informal firms in the LM. I use the definitions

$$\sigma_e = \sigma_h \frac{n_e}{n_e + n_i} \quad \text{and} \quad \sigma_i = \sigma_h \frac{n_i}{n_e + n_i}$$

to rewrite the equation above in terms of unemployment and the ratio of informal to formal employment  $\rho(\tilde{\varepsilon})$ :

$$\frac{v'(q_m)}{g'(q_m)} = \frac{i}{\sigma_h(u) \left(1 - \frac{\eta}{1 + \rho(\tilde{\varepsilon})}\right)} + 1 \quad (10)$$

where  $\rho(\tilde{\varepsilon})$  is given by (8) and  $\sigma_h = \mathcal{N}(1, 1 - u)$ .

I label the equation above as the DM curve. For a given fraction of credit matches  $\eta$  and interest rate  $i$ , equation (10) works as a transmission channel which determines the quantity traded in monetary matches  $q_m$  as a function of the size and composition of employment. For a given composition of employment and firms<sup>4</sup>

If  $\eta = 0$ , the DM curve (10) collapses to

$$\frac{v'(q_m)}{g'(q_m)} = \frac{i}{\sigma_h(u)} + 1$$

where  $q_m$  doesn't depend directly on informality. In this case, the DM transmission channel depends only on the level of unemployment.

An increase in  $\sigma_h$  or a decrease in  $\eta$  or  $i$  pushes  $q_m$  towards  $q^*$ . Intuitively buyers will carry just enough money to trade in informal transactions and monetary formal transactions such that the higher the probability of monetary matches the higher the liquidity value of money and the more interesting it is to carry it across periods.

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<sup>4</sup>In this model, as in Mortensen and Pissarides (1994), talking about the composition of firms or employment is identical since one firm corresponds to one employer. Because the production function exhibits constant returns to scale the number of workers per firm is irrelevant for the results.

### 3.4. Non-Stochastic Steady State Equilibria

In what follows, I solve for the non-stochastic steady state equilibrium of the model. The steady state definition used here is in terms of both constant real value of money supply,  $\phi M = \phi' M'$ , and equal in and out-flows in the labor market.

Using the definition of the productivity threshold  $\tilde{\varepsilon}$  I restate the free entry condition (2) as

$$\frac{k}{\beta\alpha_f(\theta)} = \int_{\tilde{\varepsilon}}^{\bar{\varepsilon}} V_e^f(\varepsilon) dF(\varepsilon) + \int_{\underline{\varepsilon}}^{\tilde{\varepsilon}} V_i^f(\varepsilon) dF(\varepsilon) \quad (11)$$

where  $\tilde{\varepsilon}$  satisfies the condition

$$V_e^f(\tilde{\varepsilon}) = V_i^f(\tilde{\varepsilon})$$

Equation (11) is the job creation (JC) condition which determines how many firms enter by posting vacancies and hence determines the LM tightness  $\theta$ . The left hand side represents the expected cost of entry while the right hand side represents its expected benefit. Firms enter the LM up to the point where the two are equalized.

**Proposition 2.** *In the JC equation (11), the cost of entry (LHS) is increasing in  $\theta$  and the profits from entry (RHS) are decreasing in  $\theta$ . Hence there is a unique steady state  $\theta$  that satisfies equation (11) for a given  $\{q_c, q_m, \tilde{\varepsilon}\}$ .*

This proposition is a standard result in labor search models. Its proof is shown in appendix B.2. The intuition behind it is the following: the cost of entry is increasing in  $\theta$  since posted vacancies take a longer time to be filled when  $\theta$  increases. This results from the congestion externality in the LM. The profits from entry are decreasing in  $\theta$  since a tighter market increases the outside opportunity for workers which increases wages and reduces the net expected profit for firms. This comes from the thick market externality facing workers in the LM. In addition, a higher  $\theta$  exacerbates the congestion externality for firms in the DM which lowers their profits but at the same time increases their profits through a higher quantity traded in the DM's monetary matches. Hence there is a unique  $\theta$  that satisfies equation (11).

Finally, under the assumption that each household holds a share of a market portfolio composed of all active firms in the economy, the equilibrium dividend income  $\Delta$  is equal to aggregate profits  $\Pi$ :

$$\Pi = n_e \int_{\varepsilon_e}^{\bar{\varepsilon}} R_e(\varepsilon) - w_e(\varepsilon) - \tau dF(\varepsilon) + n_i \int_{\varepsilon_i}^{\varepsilon_e} R_i(\varepsilon) - w_i(\varepsilon) dF(\varepsilon) - u\theta k$$

Next, I define the steady state equilibrium of this economy.

**Definition 1.** *A steady-state equilibrium in this economy is defined by: (I) a productivity threshold  $\tilde{\varepsilon}$ , (II) a level of LM tightness  $\theta$ , (III) a level of unemployment  $u$ , (IV) and quantities  $\{q_m, q_c\}$  traded in the DM, which together satisfy*

- The Job Creation condition (11);
- The informality threshold equation (6);

- Optimal consumption in credit and monetary DM matches (9) and (10) ;
- The Beveridge curve (7).

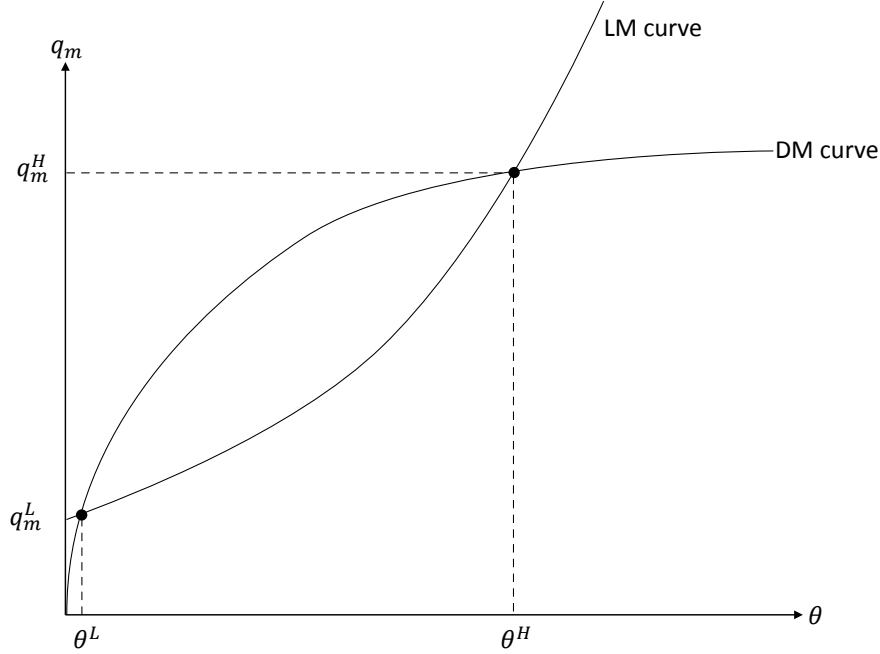


Figure 3: Steady State Equilibria in  $(\theta, q_m)$  space

As usual in monetary models, equilibrium is not unique. Since fiat money has no fundamental value and its liquidity value depends on expectations of agents about its future value, there is always a non-monetary equilibrium where agents don't value money because they expect other agents to not value it. There could also be multiple monetary equilibria because of the strategic complementarity between entry decision of firms and the money holdings decision of households. Figure 3 depicts the DM curve (10) and the JC curve (11) in the  $(\theta, q_m)$  space. Notice that the JC curve and the DM curve intersect in two points: a high monetary equilibrium point  $(\theta^H, q_m^H)$  and a low monetary equilibrium point  $(\theta^L, q_m^L)$ .

In the high equilibrium  $(\theta^H, q_m^H)$ , households expect more frequent trading in the DM because of a higher level of firms entry  $\theta^H$ . For that reason, households bring a higher amount of money holdings and end up consuming a higher quantity of the DM good  $q_m^H$ . At the same time, firms expect a high level of demand in the DM and hence higher trading surplus which results in higher firms entry and job creation. The intuition is reversed for the low equilibrium  $(\theta^L, q_m^L)$ . In the rest of the paper I focus on the high employment equilibrium  $(\theta^H, q_m^H)$ .

#### 4. Theoretical Results and Comparative Statics

The model presented in the previous sections offers a relatively simple and tractable framework to think about issues related to money, inflation and the labor market in a context where informality matters.

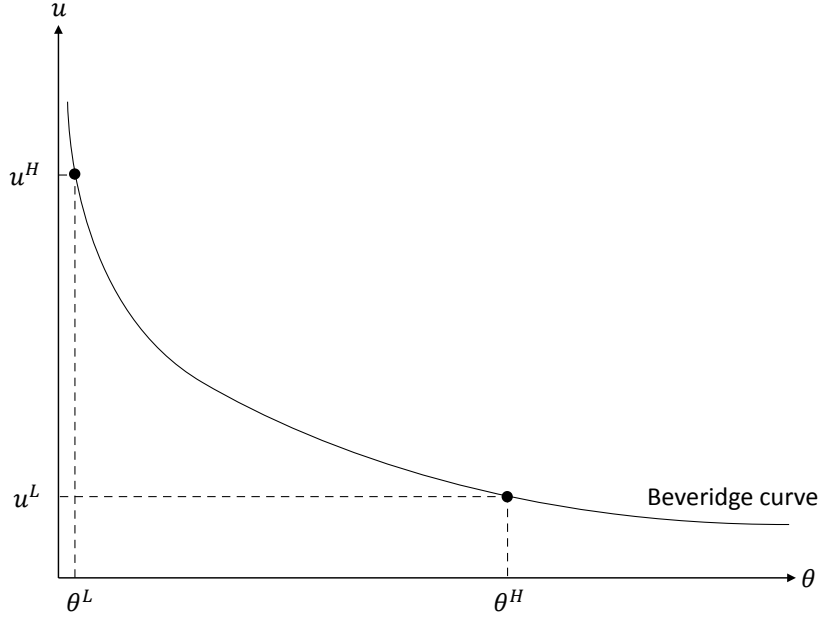


Figure 4: Steady state equilibria in  $(\theta, u)$  space

Although the model is rich enough to discuss issues of fiscal policy or labor market institutions, my focus will be on monetary policy and in particular on the effect of informality on the long-run relationship between inflation and unemployment. In this section, I discuss several theoretical results from the model and conduct some comparative statics to address the issue at hand.

#### 4.1. Unemployment and Informality

Does the level of informality matter for unemployment? The standard textbook Beveridge Curve (Pissarides, 2000) describes the long-run steady state unemployment level as a function of firms entry  $\theta$  and the exogenous separation rate. Firms entry affects unemployment through the LM matching probability  $\alpha_h$  which is an increasing function of  $\theta$ . The more firms enter the labor market and post vacancies the higher the probability of finding a job for workers and the lower the level of unemployment. In the presence of an informal sector, unemployment not only depends on firms entry decision but also on their hiring decision which determines the relative size of the informal sector. Equation (7) depicts the “informality-augmented” Beveridge Curve as a function of both  $\theta$  and the ratio of informal to formal employment  $\mu$ , itself a function of the informality threshold  $\tilde{\varepsilon}$  as shown in equation (8). However, this dependence holds only when the two types of jobs exhibit different separation rates. By setting  $\delta_e = \delta_i = \delta$  in (7), we revert back to the standard Beveridge Curve

$$\bar{u} = \frac{\delta}{\alpha_h(\theta) + \delta}$$

where the relative size of the informal sector doesn't affect unemployment. When the job separation rates between formal and informal jobs are different, the labor steady state allocation between the two types of jobs matters for the level of unemployment. In particular, we have the following result:

**Proposition 3.** *From the Beveridge Curve (7), if  $\delta_i > \delta_e$  we have the following partial equilibrium results:*

$$\frac{\partial \bar{u}}{\partial \theta} < 0 \quad ; \quad \frac{\partial \bar{u}}{\partial \tilde{\varepsilon}} > 0$$

Proposition 3 follows directly from equation (7). The first partial derivative means that an increase in  $\theta$  increases the matching probability for workers which in turn increases the job finding rate for both types of jobs and reduces unemployment. For the second result, an increase in  $\tilde{\varepsilon}$  for a given  $\theta$  increases the ratio of informal to formal jobs. Since informal jobs have a higher separation rate ( $\delta_i > \delta_e$ ), changing the composition of employment from formal jobs to informal jobs for a given  $\theta$  will increase  $\bar{u}$ .

#### 4.2. Unemployment and Inflation: the Entry Channel

As discussed by Berentsen et al. (2011) in a monetary model where only formal firms are active, inflation affects unemployment by reducing the quantities traded in monetary matches  $q_m$  through equation (10). This is because higher inflation reduces the return on holding money which induces households to carry less money across periods. A lower  $q_m$  results in a lower trading surplus in the DM which necessary results in lower profits for firms<sup>5</sup>. Less expected profits mean lower firms entry and lower job creation which results in higher unemployment. Because the matching probability in the DM  $\sigma_h(\theta)$  depends on firms entry the impact of inflation on money holdings is amplified. I label this size effect of inflation on unemployment the *Entry Channel*.

**Proposition 4.** *In equilibrium, an increase in inflation lowers the entry of firms and hence the LM tightness  $\theta$ .*

The proof of this proposition is presented in appendix B.4. Intuitively, an increase in  $i$  reduces the expected gain from vacancy posting which results in less firms entering the LM and lowers market tightness  $\theta$ . Figure 5 depicts the impact of an increase in inflation and hence the nominal interest rate  $i$  on  $\theta$  in  $(\theta, q_m)$  space. An increase in inflation increases the cost of carrying money across periods which translates lower quantity traded in monetary DM matches ( $\frac{\partial q_m}{\partial i} < 0$ ) and a downward shift of the DM curve. Since the JC curve is not directly affected, the two curves will cross at a lower point  $(\theta', q'_m)$ . As depicted in figure 4, a lower  $\theta$ , everything else being equal, results in a higher level of unemployment.

#### 4.3. Unemployment and Inflation: the Hiring Channel

As a result of the partial ability of formal firms to offer credit in the DM, the inflation tax on money holdings affects differently the expected profits of formal and informal firms and by extension the surplus of formal and informal jobs. As shown in figure 6, increasing  $i$  shifts both  $V_e^f$  and  $V_i^f$  downward. Since

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<sup>5</sup>The size of the effect will depend on the trading protocol used to share the DM surplus (Rocheteau and Wright, 2005; Aruoba et al., 2007; Craig and Rocheteau, 2008).

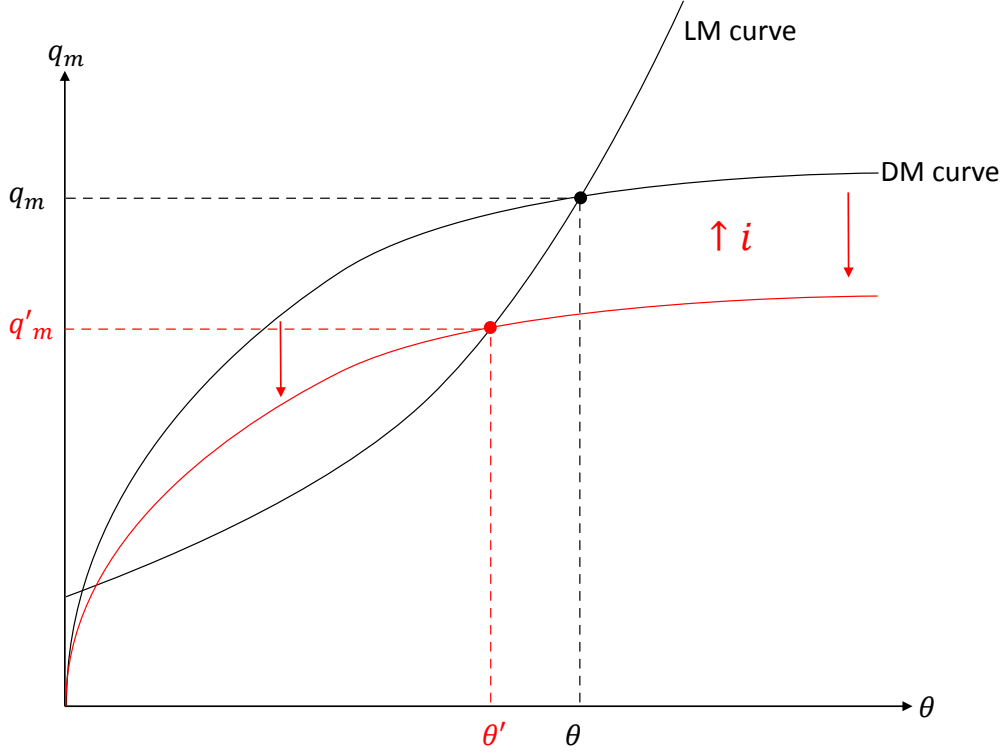


Figure 5: Effect of an increase in  $i$  on firms entry in  $(\theta, q_m)$  space

formal firms are able to offer credit with probability  $\eta$ , the resulting average traded quantity in formal DM matches is higher

$$(1 - \eta)q_m + \eta q_c > \eta q_m$$

for  $\gamma > \beta$ . In addition, we have that  $\frac{\partial q_c}{\partial i} = 0$  from (9) and  $\frac{\partial q_m}{\partial i} < 0$  from (??). It follows that when inflation increases the expected value of informal firms decreases more than the expected value of formal firms

$$\frac{\partial V_e^f(\varepsilon)}{\partial i} < \frac{\partial V_i^f(\varepsilon)}{\partial i}$$

for any  $\varepsilon$ . On figure 6, this translates into  $V_i^f(\varepsilon)$  shifting downward more than  $V_e^f(\varepsilon)$  which automatically shifts  $\tilde{\varepsilon}$  to the left. This means that the hiring standards of firms change as a consequence of changes in monetary conditions and in particular the productivity interval  $[\underline{\varepsilon}, \tilde{\varepsilon}]$  for which informal jobs are profitable is decreasing in inflation. From the definition of the steady state ratio of informal to formal jobs (8) we know that  $\frac{\partial \rho(\tilde{\varepsilon})}{\partial \tilde{\varepsilon}} > 0$  which means a decrease in  $\tilde{\varepsilon}$  will automatically lower the size of the informal sector.

**Proposition 5.** *An increase in inflation, for a given  $\theta$ , lowers the informality threshold  $\tilde{\varepsilon}$  and the ratio of informal to formal employment  $\rho(\tilde{\varepsilon})$  and shifts the Beveridge Curve downward.*

The proof of this proposition is given in appendix B.3.

Now remember from the discussion in section 4.1 that changes in  $\rho(\bar{\varepsilon})$  can have consequences for unemployment when the separation rates of the two types of jobs are different. In particular, if  $\delta_i > \delta_e$  an increase in inflation lowers  $\rho(\bar{\varepsilon})$  and drives down the steady state unemployment level. I call this composition effect of inflation on unemployment the *Hiring Channel*.

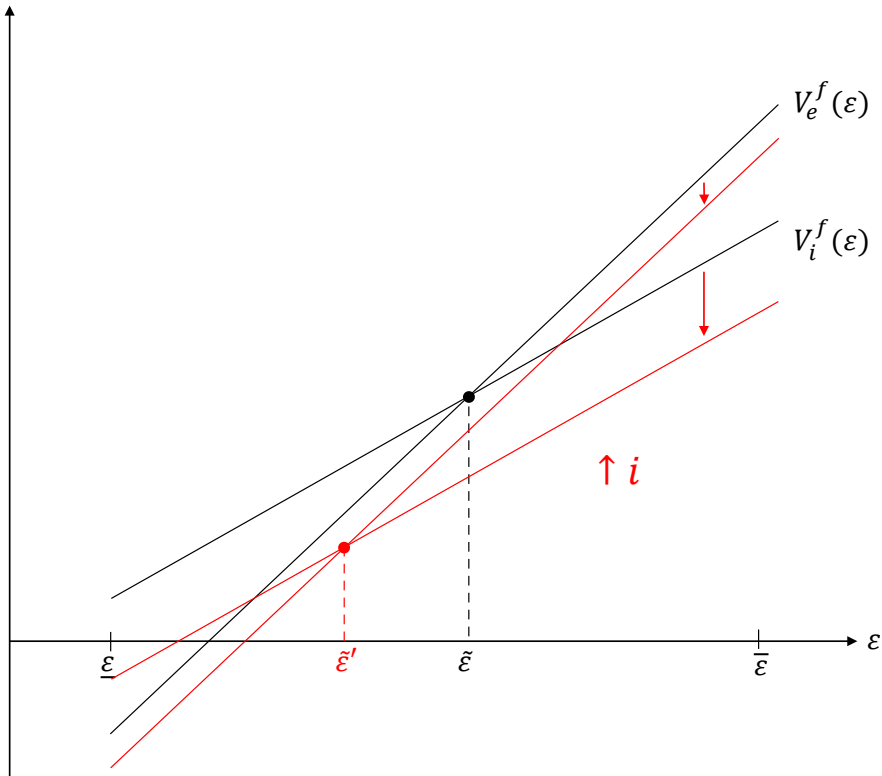


Figure 6: Effect of an increase in  $i$  on  $\bar{\varepsilon}$

#### 4.4. Inflation and Unemployment: the General Equilibrium Effect

The general equilibrium effect of inflation on unemployment when there is a large informal sector depends on the relative strength of the entry and the hiring channels. Remember that an increase in inflation puts an upward pressure on unemployment through the entry channel and a downward pressure on it through the hiring channel.

**Proposition 6.** *An increase in inflation reduces the DM surplus in monetary matches which transmits to employment through two channels:*

1. *As size effect through a change in the entry of firms  $\theta$  (entry channel);*
2. *A composition effect through a change in the hiring decision of firms  $\bar{\varepsilon}$  (hiring channel).*

*The general equilibrium effect on unemployment is ambiguous and depends on the interaction of these two channels.*

The general equilibrium effect can result in three cases:



**Case 1: an increase in inflation increases unemployment.** This case happens when the entry channel dominates the hiring channel. As shown in figure 7, an increase in inflation reduces the relative size of the informal sector which shifts the Beveridge Curve downward. If  $\theta$  stays constant, this downward shift will result in a lower level of unemployment. However, we know from the discussion in section 4.2 that the entry channel of inflation will shift  $\theta$  to the left. When the entry channel dominates,  $\theta$  moves to the left so much that the resulting unemployment  $u''$  is higher than the previous level. More formally we have

$$\frac{\partial \bar{u}}{\partial \theta} \cdot \frac{d\theta}{di} > \frac{\partial \bar{u}}{\partial \tilde{\varepsilon}} \cdot \frac{d\tilde{\varepsilon}}{di}$$

**Case 2: an increase in inflation decreases unemployment.** This case results when the hiring channel dominates the entry channel. In figure 7 this occurs when the effect of the downward shift in the Beveridge Curve on unemployment is stronger than the decrease in unemployment resulting from the leftward shift in  $\theta$  or in other words

$$\frac{\partial \bar{u}}{\partial \theta} \cdot \frac{d\theta}{di} < \frac{\partial \bar{u}}{\partial \tilde{\varepsilon}} \cdot \frac{d\tilde{\varepsilon}}{di}$$

**Case 3: an increase in inflation doesn't affect unemployment.** This case occurs when the impact of inflation through the two channels cancels out such that

$$\frac{\partial \bar{u}}{\partial \theta} \cdot \frac{d\theta}{di} = \frac{\partial \bar{u}}{\partial \tilde{\varepsilon}} \cdot \frac{d\tilde{\varepsilon}}{di}$$

Which case will occur depends on parameter values and in particular on the degree of availability of credit and the difference between the separation rates of the two types of jobs. In order to determine which effect dominates in equilibrium, I calibrate the model and conduct some numerical simulations which I present in the next section.

## 5. Quantitative Analysis

The calibration exercise focuses on the long-run properties of the data by matching certain model-based statistics with their equivalent long-run empirical averages. The model is calibrated using Brazilian data. Each period in the model corresponds to a quarter. Limited data availability restricts the sample to the period from 1996q1 to 2014q4. The reason I focus my numerical analysis on Brazil is that it has a sizable informal sector, close to the average level in Latin American countries (Perry et al., 2007), and relatively good statistics about it. This is because the distinction in Brazil between formal and informal work is quite straightforward (Gerard and Gonzaga, 2016).

In order to conduct numerical exercises, I choose the following functional forms: the utility function in the DM is  $v(q) = Aq^{1-a}/(1-a)$ . Utility in the CM is linear  $U(x) = x$ . Firms' cost function in the DM has the form:  $c(q) = Cq^\zeta$ . The matching functions in the LM and DM are described by  $\mathcal{M}(u, v) = \xi u^{1-\sigma} v^\sigma$  and  $\mathcal{N}(B, S) = BS/(B+S)$  respectively. The LM matching function results in the matching probabilities  $\alpha_h(\theta) =$

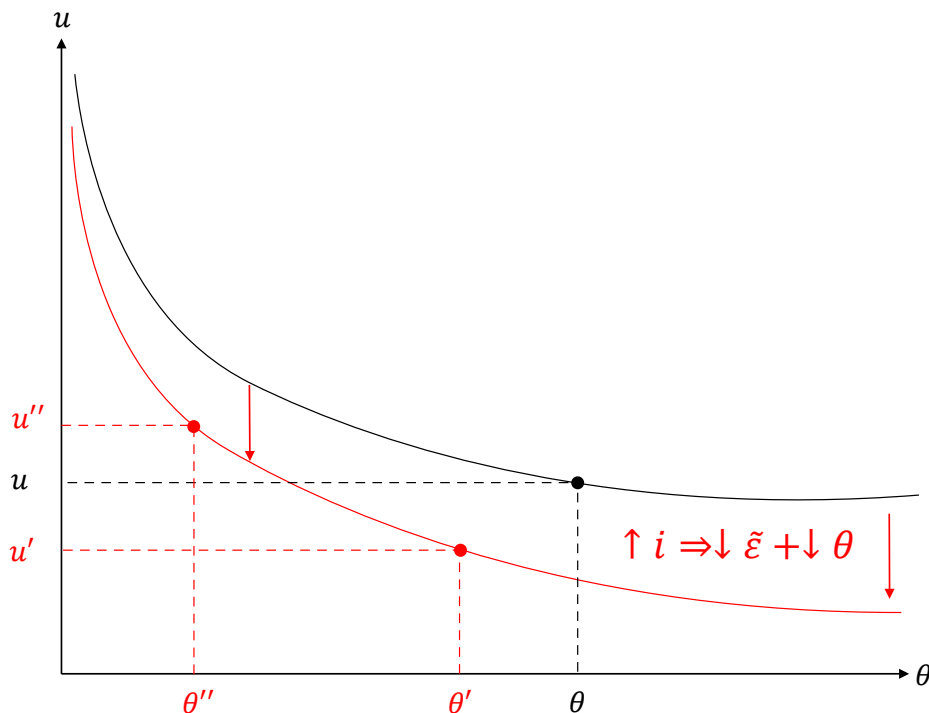


Figure 7: Net effect of an increase in  $i$  on unemployment in  $(\theta, u)$  space

As argued by Lazaryan and Lubik (2017) and others, in discrete time the LM matching probabilities  $\alpha_h$  and  $\alpha_f$  can take values above one. Intuitively, if the time period is long enough, everyone can exit unemployment at least once. This violates the idea that the matching in the labor market is probabilistic. Restricting the probabilities to lie on the unit interval requires

$$\left(\frac{1}{\xi}\right)^{1/(\sigma-1)} < \theta < \left(\frac{1}{\xi}\right)^{1/(1-\sigma)}$$

which is a non-empty interval for  $\theta$  when  $\xi \in (0, 1)$ .

I separate the model's parameters into two groups: exogenous parameters and jointly calibrated parameters. The first group consists of parameters which I directly set to a specific value while the second group is jointly calibrated with the equilibrium solution such that the model matches certain targets from the data.

The chosen values for the first group of parameters are listed in table 1.  $\beta$  is set such that the real interest in the model matches the average of the difference between the risk-free nominal interest rate and the rate of inflation in the data. There are several possibilities when it comes to the calibration of the flow value of unemployment  $b$  which captures the value of leisure, unemployment benefits and home production. The standard one is to target a given percentage of the average productivity which

Table 1: Exogenously set parameters

Parameter	Description	Value
$\beta$	Discount factor	0.99
$i$	Nominal interest rate	0.02
$b$	Labor disutility	0.40
$y$	General productivity level	1.00
$C$	Parameter of DM cost function	1.00
$\zeta$	Parameter of DM cost function	1.00
$\eta$	Credit availability in formal DM matches	0.20
$\delta_e$	Formal jobs separation rate	0.03
$\delta_i$	Informal jobs separation rate	0.10
$\sigma$	Elasticity of LM matching function	0.50
$\omega$	LM workers bargaining power	0.50
$F(\varepsilon)$	Distribution of match productivity	Uniform $[0,1]$

corresponds in this model to the average productivity in both formal and informal matches. Shimer (2005) sets the value to 40%, Hall and Milgrom (2008) choose 0.71 while Hagedorn and Manovskii (2008) argue for a value of 95%. Another way is to define  $b$  as unemployment insurance benefits and calibrate it such that the average replacement ratio in the model matches the empirical replacement rate. In Brazil, the benefit level ranges between 100% to 187% of the minimum wage which means that the replacement rate is very high for formal workers at the bottom of the wage distribution (Gerard and Gonzaga, 2016). These benefits are only received by unemployed workers and informal workers who were formally employed. Trying to take this into account will complicate the calibration without necessarily much added value. To keep things relatively simple I calibrate  $b$  at 40% of the lowest productivity level such that:

$$\frac{b}{y + \underline{\varepsilon}} = 0.4$$

The elasticity of the LM matching  $\sigma$  is set to 0.5 as is standard in the literature. The LM bargaining power of workers  $\omega$  is set equal to  $\sigma$  in order to satisfy the Hosios efficiency condition. I set  $\delta_e$  and  $\delta_i$  to match observed quarterly job separation rates of formal and informal jobs of 3% and 10% respectively (Bosch and Esteban-Pretel, 2012). The parameters of the cost function  $C$  and  $\zeta$  are set to 1 such that  $c(q) = q$ . The lump-sum tax on formal firms  $\tau$  is a measure of the tax burden on formal firms. The World Bank's Doing Business reports a tax rate of 68% of before-tax commercial profits in Brazil in 2016. This measure includes all taxes and mandatory contributions payable by the firm after accounting for allowable deductions and exemptions. These taxes include the corporate income tax, all social contributions and labor taxes paid by the employer, property taxes, turnover taxes and other taxes. However, this measure doesn't include the value-added tax (VAT) which does not affect formal firms profits but is effectively a tax on formal goods consumption. I set  $\tau$  such that

$$\int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \frac{\tau}{R_e(\varepsilon) - w_e(\varepsilon)} dF(\varepsilon) = 68\%$$

The share of credit formal matches  $\eta$  is set at 20% which corresponds roughly to the average share of

credit card transactions in total consumer spending transactions in Brazil<sup>6</sup>.

Once all the exogenous parameters are set I proceed to jointly calibrate the second group of parameters. This group comprises utility function parameters  $A$  and  $a$ , cost function parameter  $\zeta$ , DM bargaining power  $\varphi$ , the matching function efficiency  $\xi$ , and the cost of posting vacancies  $k$ . The joint calibration procedure consists in solving for the vector of calibrated parameters  $\mathcal{P} = \{A, a, \varphi, k, \xi\}$  and the equilibrium solution  $X = \{\tilde{\varepsilon}, \theta, u, q_m, q_c\}$  which together reduce the distance (squared percentage difference) between the targeted moments  $\mathcal{S}_{data}$  and the corresponding theoretical moments  $\mathcal{S}_{model}$  while taking the system of steady state equilibrium equations (6) and (7), (9), (11), and (10), as equality constraints and imposing interval bounds  $\mathcal{I}$  on the value of some parameters:

$$\begin{aligned} & \min_{\mathcal{P}, \theta, \varepsilon_i, \varepsilon_e} (\mathcal{S}_{model}(X; \mathcal{P}) - \mathcal{S}_{data})^2 \\ & \text{s. t. } \mathbf{EC}(X; \mathcal{P}) = 0 \quad ; \quad \mathcal{P} \in \mathcal{I} \end{aligned}$$

In order to specify the targets of the calibration procedure, I define how the model maps into the data. In terms of output I have:

$$\begin{aligned} Y_{e_{DM}} &= n_e \sigma_f [\eta(g(q_e^c) - c(q_e^c)) + (1 - \eta)(g(q_e^m) - c(q_e^m))] \\ Y_{i_{DM}} &= n_i \sigma_f (g(q_i) - c(q_i)) \\ Y_{e_{CM}} &= n_e \left( y + \int_{\tilde{\varepsilon}}^{\bar{\varepsilon}} \varepsilon \, d \frac{F(\varepsilon)}{F(\bar{\varepsilon}) - F(\tilde{\varepsilon})} \right) - (1 - F(\tilde{\varepsilon})) u \theta k \\ Y_{i_{CM}} &= n_i \left( y + \int_{\underline{\varepsilon}}^{\tilde{\varepsilon}} \varepsilon \, d \frac{F(\varepsilon)}{F(\tilde{\varepsilon}) - F(\underline{\varepsilon})} \right) - F(\tilde{\varepsilon}) u \theta k \end{aligned}$$

where  $Y_{e_{DM}}$  and  $Y_{i_{DM}}$  are the net aggregate output of formal and informal firms sold in the DM and  $Y_{e_{CM}}$  and  $Y_{i_{CM}}$  are their net aggregate output sold in the CM respectively.

Depending on the country, the real GDP in the data might or might not take into account informal activities (Andrews et al., 2011). In some countries, GDP accounts only for formal activities which is the case for example in the US. In some others, GDP includes formal and some of the informal activities. This is increasingly the case in EU countries. For the purposes of calibration I assume that a share of informal activities is included in Brazil's GDP data. This allows me to define recorded activities in the model as the total of formal firms output sold in both DM and CM and informal firms' output sold in the CM.  $M$  is defined in the model as the amount of cash carried by households to be spent in both formal and informal monetary transactions in the DM. In the data  $M$  corresponds to either M1 or sweep-adjusted M1 (Aruoba et al., 2011). As a consequence, I define the size of the informal sector as a share of observed GDP as:

$$\frac{Y_i}{Y} = \frac{Y_{i_{DM}} + Y_{i_{CM}}}{Y_{e_{DM}} + Y_{e_{CM}} + Y_{i_{CM}}}$$

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<sup>6</sup>The figures are readily available at the website of the Associação Brasileira das Empresas de Cartões de Crédito e Serviços.

and the model-based money demand equation as:

$$L(i) = \frac{g(q_m)}{4 * (Y_{eDM} + Y_{eCM} + Y_{iCM})}$$

where  $L(i)$  depends on  $i$  directly through real balances  $g(q_m)$  and indirectly through the different GDP measures.

In the tradition of Lucas (2000) and following Lagos and Wright (2005),  $A$  and  $a$  are included in the joint-calibration in order to fit the model-based money demand function  $L(i) \equiv \frac{M}{pY}$  to the data. The idea is to match two moments: the average real money balances at the average nominal interest rate and the elasticity of money demand to the nominal interest rate. To estimate the interest elasticity of money demand that I will target in the calibration, I use a log-log specification:

$$\log M/PY_t = \beta_1 + \beta_2 \log i_t + \nu_t$$

The OLS estimate of  $\beta_2$  is used as a point estimate for the interest elasticity of money demand  $\epsilon$ :

$$\epsilon = \frac{\partial L(i)}{\partial i} \frac{i}{L(i)}$$

Following Aruoba et al. (2011), I use  $\varphi$ , the bargaining power of buyers in the DM, to target an average markup in the DM trades of 30%. Since the CM is a competitive market, the markup there is 0. The markup in monetary transactions is defined as

$$\mu_m = \frac{g(q_m)/q_m}{c'(q_m)} - 1$$

and in credit transactions it is defined as

$$\mu_c = \frac{g(q_c)/q_c}{c'(q_c)} - 1$$

The average markup in DM trades can be written as

$$\mu_{DM} = \frac{n_e(\eta\mu_c + (1-\eta)\mu_m) + n_i\mu_m}{n_e + n_i}$$

Taking the economy wide average, I get:

$$\bar{\mu} = \mu_{DM} \frac{Y_{eDM} + Y_{iDM}}{Y}$$

In addition to the average markup and the money demand moments, I target the average unemployment rate in Brazil over the period 1996-2014 of 9.5%, the estimated size of the informal employment of around 30% of the labor force, and the average size of the retail sector of 20%.

Table 4 summarizes the result of the joint calibration. The model is able to match all of the targets based on reasonable values for the calibrated parameters and in particular the model is able to match

Table 2: Calibration results

			Data	Model
<b>Calibrated parameters</b>				
$k$	Cost of vacancy posting		-	1.448
$\tau$	Lump-sum tax on formal firms		-	0.258
$\varphi$	Bargaining power in DM		-	0.258
$A$	Parameter of DM utility		-	2.124
$a$	Parameter of DM utility		-	0.083
$\xi$	LM matching efficiency		-	1.447
<b>Calibration targets</b>				
$u$	Unemployment		0.095	0.095
$n_i$	Informal employment		0.300	0.300
$\int_{\bar{\varepsilon}}^{\bar{\varepsilon}} \tau / \Pi$	Average tax on formal firms		0.68	0.300
$\mu_{DM}$	Average markup in retail sector		0.300	0.300
$L(\bar{i})$	Average Money Demand		0.215	0.215
$\epsilon$	Elasticity of MD to $i$		-0.418	-0.418
Sum of squared residuals			-	0.000

the Brazilian money demand data as shown in figure 8.

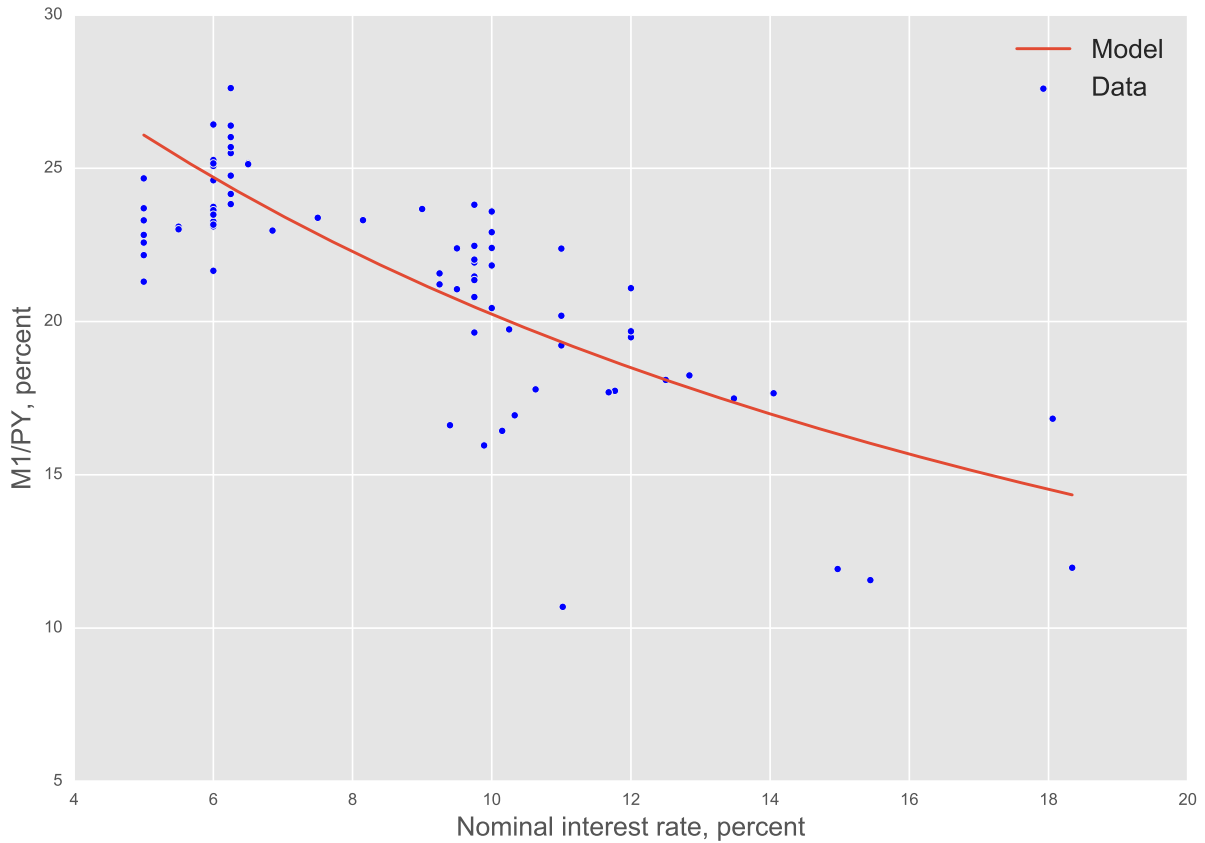


Figure 8: Money demand: model vs. data

In order to test the validity of the model I compare the model results with some empirical statistics which I did not specifically include in the calibration procedure. In table 3, I report the resulting values for different measures of the size of the informal sector using the model-based GDP measures introduced in the previous section. The different informality measures are based on different assumptions regarding

the inclusion of informal sector in official GDP figures and are within reasonable range of the literature estimates of the size of the informal sector in Brazil, notably Schneider et al. (2011) who report an estimated average size of the informal sector in Brazil of 39% over the period 1999-2007 using the MIMIC (multiple indicators-multiple causes) model, a linear latent variables econometric approach.

Table 3: Different possible measures of the informal sector

Measure	$\frac{Y_i}{Y_e}$	$\frac{Y_i}{Y_e + Y_{iCM}}$	$\frac{Y_i}{Y_e + Y_i}$
	Value	41.8%	31.0%

Formal wages are on average 40% higher than informal wages which is in line with the literature on informality. The labor share of total income in the model economy is 71.7% whereas this share amounts to around 60% in Brazil.

Table 4: Model validation

		<b>Data</b>	<b>Model</b>
$q_m/q_c$	Ratio of monetary to credit consumption	-	0.429
$\ell\%$	Share of credit in retail sales (value)	0.150	0.217
$\mu$	Economy average markup	-	0.060
$\int w(\varepsilon)/Y$	Share of labor income	0.600	0.717
$(1 + \tau)\bar{w}_e(\varepsilon)/\bar{w}_i(\varepsilon)$	Average wage difference	-	1.400

In table 5, I report elasticities of various measures to changes in some policy parameters. A 1% increase in inflation around the steady state equilibrium reduces both informal employment and unemployment by 0.42% and 0.03% respectively. This means that the hiring channel of inflation dominates slightly the entry channel. To see that, figure 9 shows the effect of changing the nominal interest rate and hence inflation on the composition of the labor force. An increase in quarterly nominal interest rates from 5% to 18% drives informality down from 35% to around 20% with a slightly negative impact on unemployment. Regarding the effect on output, increasing inflation by 1% reduces the aggregate output by 0.04%. This is because output is mostly affected through the entry channel of inflation.

Increasing the availability of credit for formal firms by 1% reduces unemployment and informal employment by 0.1% and 0.33% respectively. Interestingly enough, financial development is effective at reducing unemployment while inflation is better at fighting informality. This implies that a combination of the two instruments might be a desirable policy.

In line with the literature, an increase of 1% in the corporate tax increases unemployment and informal employment by 0.48% and 1.34% respectively. A higher separation rate of informal jobs, a proxy for government monitoring and the quality of a country's institutions such as the rule of law and the strength of the state, is the best way to reduce the size of the informal sector. Increasing  $\delta_i$  reduces both unemployment and informality without hurting output.

## 6. Conclusion

In this paper, I presented a monetary dynamic general equilibrium model with unemployment and informality. This model provides a flexible and relatively simple framework to understand interactions

Table 5: Elasticities w.r.t. changes in different policy parameters

	$i$	$\eta$	$\tau$	$\delta_i$
$u$	<b>-0.03</b>	-0.10	0.48	-1.14
$n_i$	<b>-0.42</b>	-0.33	1.34	-4.83
$Y$	<b>-0.04</b>	0.02	-0.06	0.14
$Y_i/Y$	<b>-0.45</b>	-0.37	1.45	-5.28

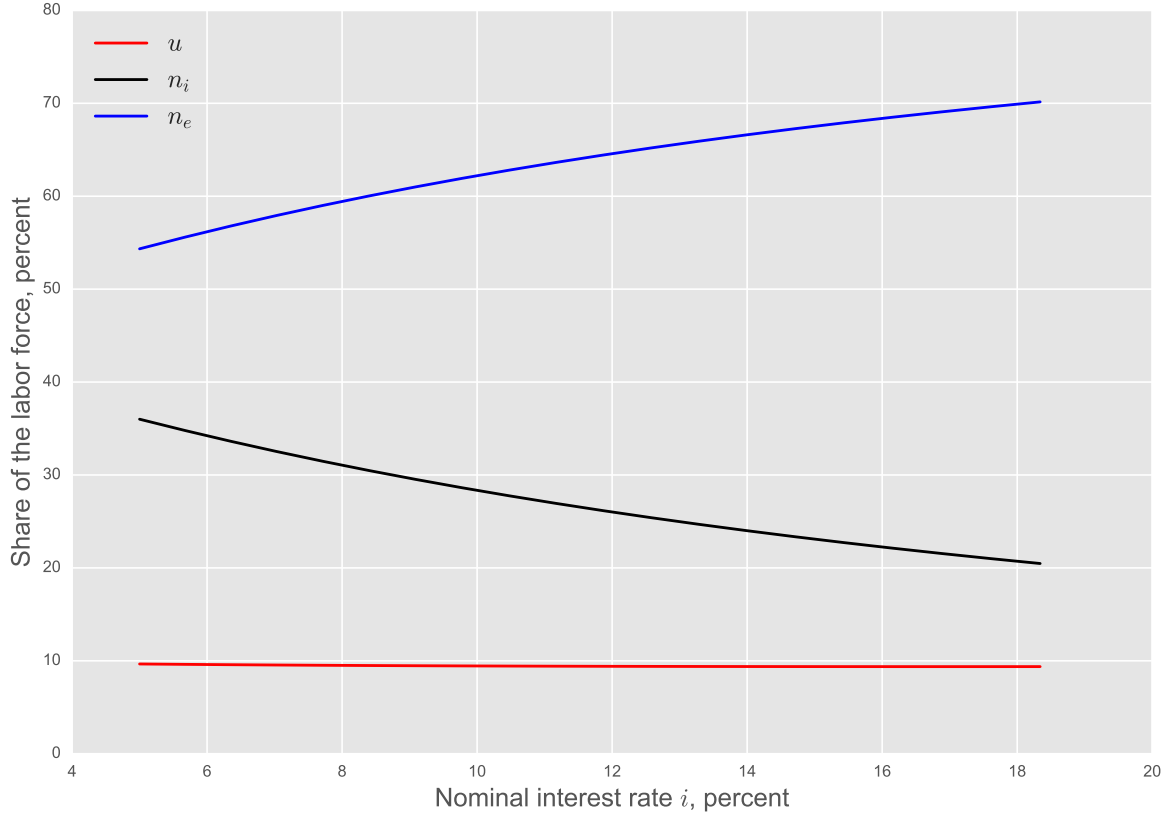


Figure 9: Effect of a change in  $i$  on steady state labor force composition

between the informal sector and the rest of the economy. In the model, informality results from the optimal choices of firms and households given the frictions they face when deciding how to allocate labor, how much to consume and how to pay for their consumption. A main innovation of this model is to connect informality in both labor and goods markets by limiting buyers to the use of money as a means of payment when trading with firms employing informal workers. Inflation affects unemployment and informality by taxing monetary balances carried by households from one period to the next. The higher cash intensity of informal transactions makes informal firms and jobs more vulnerable to the inflation tax compared to the formal sector.

One would think that a bigger informal sector will make the positive long run relation between inflation and unemployment stronger. However, if we consider the differences in separation rates between formal and informal jobs then the long-run Philips curve might become vertical or slightly negatively sloped. This is because the composition of the labor force in terms of formal and informal jobs is affected by monetary conditions prevailing in the economy. An increase in inflation reduces firms profits and hence affects



unemployment through two channels: through the entry channel it lowers the benefits of vacancy posting reducing in turn entry of firms and increasing unemployment; through the hiring channel it shifts job creation from high separation informal jobs to low separation formal jobs which reduces unemployment. The net effect of these two channels is ambiguous and depends on the levels of financial development and the ability of the government to monitor informal firms.

I presented the results of a calibration exercise of the model to the Brazilian economy. The model is able to replicate most of the stylized facts found in the empirical literature on informality. Numerical results show that the hiring effect slightly dominates and inflation has a small negative effect on unemployment. However, the impact of inflation on labor allocation between formal and informal jobs is significant. These results indicate that the composition of the labor force between formal and informal employment matters for the inflation-unemployment trade-off which points to the importance of understanding and measuring informality for the conduit of monetary policy.

## Appendix A Nash Bargaining in the DM

Here I solve for the terms of trade in the DM using the generalized Nash bargaining solution. I assume that the bargaining power of buyers  $\varphi$  is the same for all types of DM matches.

When a buyer is part of a pure monetary match, the bargaining problem can be formulated as follows:

$$\max_{q_m, d_m} [v(q_m) - \phi d_m]^\varphi [\phi d_m - c(q_m)]^{1-\varphi}$$

subject to:

$$\begin{aligned}\phi d_m &\leq z \\ c(q_m) &\leq y + \varepsilon\end{aligned}$$

Since money is costly, households don't have an incentive to carry more money than they intend to spend in the DM market which makes the first constraint binding. In addition, I assume that the second constraint is never binding which allows us to write the problem as an unconstrained optimization problem. This results in the following first order condition:

$$z = \frac{\varphi v'(q_m)c(q_m) + (1-\varphi)v(q_m)c'(q_m)}{\varphi v'(q_m) + (1-\varphi)c'(q_m)} \equiv g(q_m)$$

As a consequence, the Nash bargaining solution is a pair  $(q_m, d)$  that satisfies  $q_m = g^{-1}(z)$  and  $\phi d = z$ . Notice that  $\partial q_m / \partial z = \partial g^{-1}(z) / \partial z = 1/g'(q_m) \geq 0$  meaning that more money holdings increases  $q$ .

Next, I solve for the terms of trade in credit formal matches:

$$\max_{q_c, d, \ell} [v(q_c) - \phi d - \ell]^\varphi [\phi d + \ell - c(q_c)]^{1-\varphi}$$

subject to:

$$\begin{aligned}\phi d &\leq z \\ c(q_c) &\leq y + \varepsilon\end{aligned}$$

Using the same assumptions as before, the bargaining problem results in the following first order conditions:

$$\begin{aligned}z + \ell &= \frac{\varphi v'(q_c)c(q_c) + (1-\varphi)v(q_c)c'(q_c)}{\varphi v'(q_c) + (1-\varphi)c'(q_c)} \equiv g(q_c) \\ z + \ell &= (1-\varphi)v(q_c) + \varphi c(q_c)\end{aligned}$$

Combining both first order conditions we get the following optimality condition:

$$v'(q_c) = c'(q_c)$$

Hence the optimal solution is  $q_c = q^*$  the efficient quantity which solves  $v'(q) = c'(q)$ . As a consequence, the Nash bargaining solution is a pair  $(q_c, g(q_c))$  that satisfies  $q_c = q^*$ . Whether the quantity  $q^*$  will be acquired using credit or money is irrelevant since buyers are indifferent between the two at this point. Because  $q_m \leq q^*$  always holds, this offers formal firms a higher trading surplus in the DM compared to informal firms as long as  $\eta > 0$ .

## Appendix B Proofs

### B.1 Effect of an increase in $i$ on $q_m$

To measure the effect of an increase in  $i$  on  $q_m$  I proceed by totally differentiating equation (10):

$$\begin{aligned} \frac{v''(q_m)g'(q_m) - v'(q_m)g''(q_m)}{g'(q_m)^2} dq_m = & \frac{1}{\sigma_h \left(1 - \frac{\eta}{1 + \rho(\tilde{\varepsilon})}\right)} di - \frac{i}{\left(1 - \frac{\eta}{1 + \rho(\tilde{\varepsilon})}\right) (\sigma_h)^2} d\sigma_h \\ & - \frac{i \frac{\eta}{(1 + \rho(\tilde{\varepsilon}))^2}}{\sigma_h \left(1 - \frac{\eta}{1 + \rho(\tilde{\varepsilon})}\right)^2} d\rho(\tilde{\varepsilon}) \end{aligned}$$

Evaluating the previous expression at the Friedman rule ( $i = 0$ ) and rearranging the terms, I get:

$$\frac{dq_m}{di} = \frac{1}{\sigma_h \left(1 - \frac{\eta}{1 + \rho(\tilde{\varepsilon})}\right)} \left( \frac{g'(q_m)^2}{v''(q_m)g'(q_m) - v'(q_m)g''(q_m)} \right) < 0$$

The effect of  $i$  away from the Friedman rule depends on the effect of inflation on LM conditions which affects the matching probabilities in the DM.

### B.2 Proof of Proposition 2

From the JC condition (11) we have

$$\frac{k}{\beta\alpha_f(\theta)} = \int_{\tilde{\varepsilon}}^{\bar{\varepsilon}} \frac{(1 - \omega)[R_e(\varepsilon) - b - \tau] - \omega\theta k}{1 - \beta(1 - \delta_e)} dF(\varepsilon) + \int_{\underline{\varepsilon}}^{\tilde{\varepsilon}} \frac{(1 - \omega)[R_i(\varepsilon) - b] - \omega\theta k}{1 - \beta(1 - \delta_i)} dF(\varepsilon)$$

Using Leibniz rule<sup>7</sup> to differentiate the expression above w.r.t.  $\theta$  and then using the envelope property of  $\tilde{\varepsilon}$  yields

$$-\frac{k}{\beta\alpha_f^2} \frac{\partial\alpha_f}{\partial\theta} = \frac{(1 - \omega)}{1 - \beta(1 - \delta_e)} \int_{\tilde{\varepsilon}}^{\bar{\varepsilon}} \frac{\partial R_e(\varepsilon)}{\partial\theta} - \omega k dF(\varepsilon) + \frac{(1 - \omega)}{1 - \beta(1 - \delta_i)} \int_{\underline{\varepsilon}}^{\tilde{\varepsilon}} \frac{\partial R_i(\varepsilon)}{\partial\theta} - \omega k dF(\varepsilon)$$

It is easy to show that when the Friedman rule ( $i = 0$ ) applies we get:  $\frac{\partial R_e(\varepsilon)}{\partial\theta} = \frac{\partial R_i(\varepsilon)}{\partial\theta} \leq 0$ . From the definition of the matching function, we also have  $\frac{\partial\alpha_f}{\partial\theta} < 0$ . Hence, it can be easily shown that the

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<sup>7</sup>  $\frac{d}{dx} \left( \int_{a(x)}^{b(x)} f(x, t) dt \right) = f(x, b(x)) \frac{d}{dx} b(x) - f(x, a(x)) \frac{d}{dx} a(x) + \int_{a(x)}^{b(x)} \frac{\partial}{\partial x} f(x, t) dt$  where  $-\infty < a(x), b(x) < +\infty$

LHS of the equation above is positive while the RHS is negative. This implies that the LHS of the JC equation (11) is increasing in  $\theta$  while the RHS is decreasing in  $\theta$ . Hence  $\theta$  is unique.

*B.3 Proof of Proposition 5*

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*B.4 Proof of Proposition 4*

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