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Globalization and the Concentration of Talent

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Globalization and the Concentration of Talent*

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Abstract

We analyze the allocation of talent across teams in large matching markets with competition for rank. We show that under general conditions globalization in the form of a convex transformation of payoffs promotes the concentration of talent, i.e. it makes positive assortative matching more likely and negative assortative matching less likely. This is in line with recent trends in European football (soccer) leagues.

Keywords: globalization, inequality, matching, rank competition, strategic interaction

JEL Classification: D00, D02, D3, D4, D62, D85, J01, P40, Z2

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1 Introduction

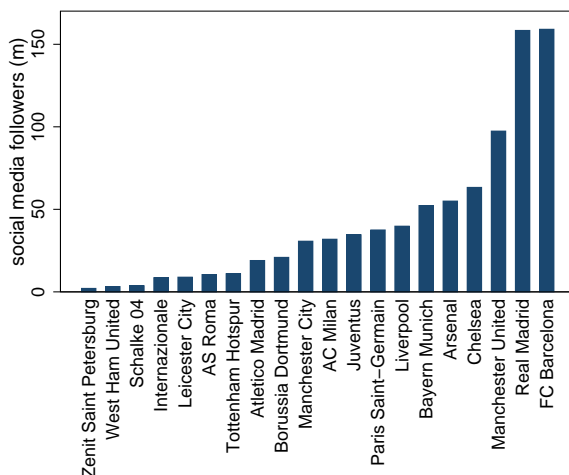
The distributional consequences of globalization are a constant concern for policy makers worldwide. Globalization has been shown to potentially increase inequality through various channels: It may harm workers in import-competing industries, support higher CEO pay, and amplify superstar effects, for example. In this paper, we examine yet another channel of how globalization impacts inequality that has heretofore received little attention in the literature: the concentration of talent.

Globalization provides firms with access to new markets. These opportunities, however, do not benefit all firms alike. The most productive firms are the ones who export (Melitz, 2003; Bernard et al., 2007). Similarly, only the best artists or sport teams are able to reach out to global audiences. Social media followers of football (soccer) clubs, for instance, are highly concentrated on the most famous clubs even among the select group of clubs with highest revenues worldwide—see Figure 1. Teams (or, in general, firms) will reach out to foreign markets only if this is profitable. The fact that the best teams—i.e., those with the highest payoffs prior to globalization—serve global markets therefore implies that after globalization total payoffs are more concentrated in the hands of market leaders. We argue that, as a consequence, globalization promotes the concentration of talent.

To study the underlying mechanisms, we consider an economy where workers with heterogeneous skills form competing teams. Competition among teams results in a rank distribution for each team, which is a function of its own skill level (i.e., of the skill levels of its team members) as well as of the skill levels of all other teams. A team’s rank in the market determines its payoff. Globalization acts then as an amplifier of ‘superstar-effects’ à-la Rosen (1981) by increasing the (relative) rewards to being ranked high in the market. This is in line both with the above reasoning and with the stylized facts on European football presented below. Mathematically, we represent globalization as a convex transformation of payoffs over ranks.

Our work belongs to the extensive literatures analyzing matching markets and their relationship to superstar effects and globalization in general—see literature review below. The key novelty of our work is that while the ballpark of this literature operates under the assumption of positive assortative matching, we ask how globalization impacts matching outcomes themselves. We show that under very mild restrictions on the mode of competition between teams, globalization promotes the concentration of talent, i.e. it makes positive assortative matching more likely and negative assortative matching less

Figure 1: Social media followers of top football teams



Notes: Own illustration, based on Deloitte (2017). This figure shows for each of the 20 football clubs with highest revenues worldwide in 2015/16 the total number of social media followers. This total refers to the sum of followers on Twitter and Instagram, and ‘likes’ on Facebook.²

likely. Specifically, we show that in any economy where positive assortative matching is an equilibrium prior to globalization, it must also be an equilibrium after globalization. Moreover, there are economies where positive assortative matching is an equilibrium before globalization but not after. The opposite is true for equilibria with negative assortative matching. The basic intuition is simple: positively assortatively matched teams are more likely to reach the highest ranks, the ones that benefit (most) from globalization.

This increase in the concentration of talent has important distributional effects. In particular, we show that when globalization causes the economy to switch to an equilibrium with positive assortative matching, its effect on wage inequality is larger when compared to a case with always positive assortative matching, as is typically considered in the literature.¹

Our main result—globalization promotes positive assortative matching—is remarkably general. With regard to the competition mode among teams, we simply assume that *mixed* teams formed by one high- and one low-skilled individual are more likely to be ranked in the mid range, with teams formed only by high-skilled (resp. low-skilled) individuals more likely to be ranked in the upper range (resp. lower range). This is a very minimal restriction on the mode of competition in our set-up, and one which renders the

¹An increased concentration of talent may well have distributional consequences above and beyond any immediate effect on wages, e.g. in the presence of knowledge spillovers. We briefly allude to such matters in the conclusion and leave a thorough investigation for future research.

²The basic pattern is the same when considering just ‘likes’ on Facebook.

relationship between skills and performance meaningful. As to the payoff scheme, we apply the normalization that higher ranks are better, i.e., we assume that the payoff scheme is increasing over ranks. Other than these two considerations, we impose no restrictions on how teams' skills are reflected in their respective rank distributions and, hence, in their (expected) payoffs. In this respect, it is worth noting that while in our baseline scenario we consider two types of workers (low- and high-skilled) who form teams of two, we also show that our main results readily extend to the case of $S \geq 2$ types *and* $N \geq 2$ team members. In other words, globalization increases the concentration of talent in such markets as well. Focusing for the main part of the paper on the case of two types and players facilitates the exposition and makes our results more transparent.

Motivating facts from European football

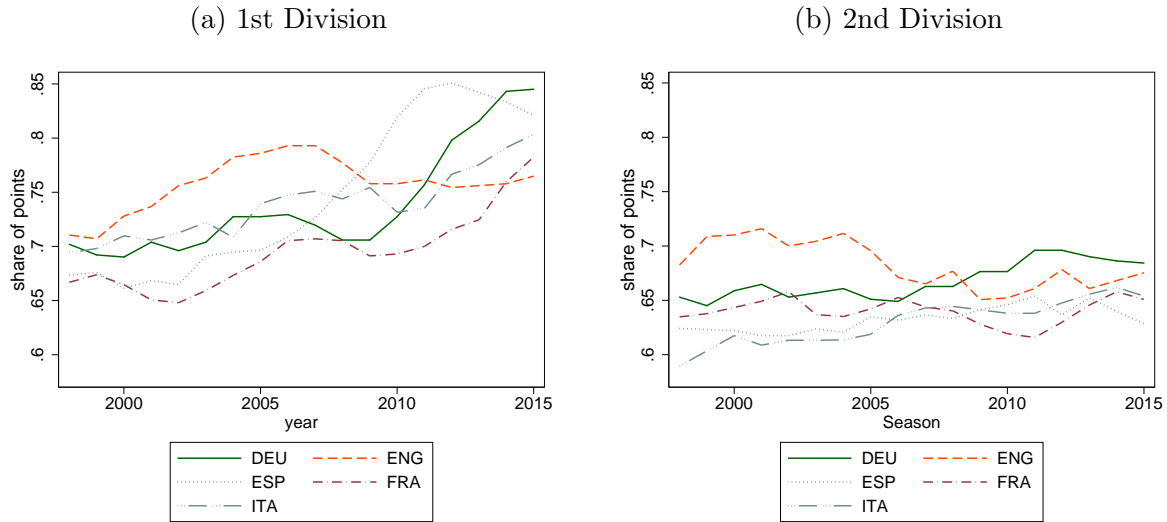
Our paper can be motivated by reference to the competition in European football leagues. European football is particularly well suited for the purpose of investigating the effect of globalization on markets with competition for rank between teams of different skill levels. First, football teams compete in their national leagues for rank. Second, a team's performance in these leagues is a direct measure for its skill level. Third, the increasing importance of UEFA Champions League is a prime example for our concept of globalization as we now explain.

Football clubs in Europe compete in national leagues. Each year the clubs ranked highest in their respective national league qualify to participate in the UEFA Champions League, a pan-European competition that was transformed in the course of the 1990s from a knock-out competition between national champions into something close to a European super-league today, and that has witnessed enormous growth in the past. Annual payouts to participating teams, for example, have more than thirteenfolded since the late 1990s reaching more than EUR 1.3bn in 2017/18, and the final of the UEFA Champions League is broadcasted in over 200 countries with up to 400m people tuning in, making it the biggest annual sports-event worldwide.³ The Champions League is so dominant today that qualification for it is sometimes considered to be as important as winning even the most important national leagues.⁴

³See https://www.footballbenchmark.com/uefa_champions_league_non_big_five_participation, <https://www.uefa.com/uefachampionsleague/news/newsid=2493261.html#/>, and <http://fr.uefa.com/uefachampionsleague/news/newsid=2111684.html#/> (retrieved on 31 December 2017).

⁴See for example <http://www.telegraph.co.uk/sport/football/teams/arsenal/9633456/Arsenal-manager-Arsene-Wenger-says-qualifying-for-Champions-League-on-a-par-with-winning-a-trophy.html> (retrieved on 31 December 2017).

Figure 2: Concentration of talent in European football leagues



Notes: Own illustration, based on kicker.de and wikipedia.org. The share of points refers to the centered 5-year moving average of the ratio of end-of-season points of the national champion over the maximum achievable number of points. The left-hand-side figure refers to the respective first-division league for each of the five countries, the right-hand-side panel to the corresponding second-division league. We thank Stefan Legge for generously sharing the data on second-division outcomes.

The establishment and growth of the UEFA Champions League is a prime example of globalization in a market. How does globalization feed back into competition in national leagues? The Champions League provides participating teams with large direct payouts and, in addition, with a global platform—in particular those teams that frequently proceed to the final rounds in this competition.⁵ Hence, as a consequence of the increasing importance of the UEFA Champions League, the returns to being ranked high in a national league have increased, reflecting amplified ‘superstar effects’. The key observation is that this development went hand in hand with an increased concentration of talent, as may be seen from Figure 2(a). This figure shows the share of the maximal achievable points won by the respective national champion for each of the ‘big 5’ European football leagues—England, France, Germany, Italy, and Spain. This share has been steadily increasing over time, strongly suggesting that the players with the highest talent are increasingly con-

⁵See Figure 1. Real Madrid and FC Barcelona, for example, are the teams with most semi-final appearances from 2000 to 2017 (11 and 10, respectively).

centrated in a few teams (viz., those who win).^{6,7} Notably, the increased concentration of talent is now also a matter of concern for officials at the Fédération Internationale de Football Association (FIFA).⁸

To substantiate further our conjecture that this increase in the concentration of talent is attributable to globalization, we contrast our ‘treatment group’ of first-division leagues with the ‘control group’ of corresponding second-division leagues—see Figure 2(b). Teams in these leagues cannot qualify for the UEFA Champions League (or any other European competition) via their national leagues, implying that globalization is by definition less important for competition in these leagues. If globalization was indeed a key driver for the concentration of talent, then we should not expect the same upward trend to have occurred in these second-division leagues. As shown in Figure 2(b), this is exactly what we observed in the past.

We conclude this discussion with emphasizing that while competition in European football is very well suited to illustrate the mechanisms that we consider, similar mechanisms apply in different contexts, and, in particular, in any set-up where pooling of talent helps to achieve highest ranks and where globalization favors market leaders. Hence, we may expect globalization to promote concentration of talent more generally. A standard Melitz (2003)-model, for example, predicts a linear relationship between firm productivity and profits in a closed economy, and a piecewise linear and convex relationship in an open economy. This means that we may expect trade liberalizations to increase the concentration of talent.⁹ Indeed, Krishna et al. (2014) document that following trade liberalizations exporters tend to attract more able workers.

⁶The increased concentration of talent is also witnessed at the level of the Champions League itself. In the period from 2008 to 2017, for example, 15 different clubs made it to the semi-finals with a Herfindahl Index of 0.1125 and 3 teams only—FC Barcelona, Real Madrid, and Bayern Munich—accounted for more than 50% of all semi-finals appearances. As opposed to that, in the period from 1998 to 2007 20 different clubs made it to the semi-finals with a Herfindahl Index of 0.0675.

⁷In principle, the increased concentration of talent might of course have also fueled into an increase in the concentration of revenues. Note, however, that revenues of football teams are largely dependent on their ranking in national and supranational leagues and less on their team strength per se. For instance, TV licensing income is typically rank-dependent as is admission to the UEFA Champions League and sponsoring income is at least partly success-based.

⁸See, e.g. <http://www.espn.com/soccer/fifa-world-cup/story/3383510/fifa-president-gianni-infantino-backs-sweeping-transfer-market-reform> (retrieved on February 18, 2018).

⁹To be precise, the relationship is (piecewise) linear for adjusted productivity $a\sigma^{-1}$, where a is productivity and $\sigma > 1$ is the elasticity of substitution between varieties.

Relation to the literature

Globalization in our set-up increases the gains from being ranked high in a market, i.e. we think of globalization as an amplifier of superstar-effects. In his seminal contribution, Rosen (1981) shows how small differences in talent of entertainers can result in large heterogeneity in their income if revenues are a convex function of talent. He argues that this is particularly true in markets with imperfect substitutability between artists of different quality and when the marginal cost of reaching out to additional customers is low or even zero as, for example, with performances broadcasted on TV. In the original work by Rosen (1981), superstars benefit from being able to reach broader audiences.¹⁰ As long as consumption is indivisible in the sense that an increase in quantity cannot compensate for a lower quality, similar effects can, however, also arise if suppliers can serve a fixed number of clients only. Then, increased income inequality on the side of the buyers can translate into income inequality for suppliers. Such mechanisms can explain the increased dispersion in house prices (Määttänen and Terviö, 2014), imply that inequality can spill over across occupations (Clemens et al., 2017), and they give rise to higher CEO pay in a globalized world with larger firm sales (Gabaix and Landier, 2008; Terviö, 2008; Gersbach and Schmutzler, 2014; Ma and Ruzic, 2017). These papers have in common that there is always positive assortative matching between buyers' income (or firm size) and suppliers' quality. They then perform comparative statics conditional on this matching, where these comparative statics can be linked to globalization. We also consider comparative statics with regard to globalization that strengthens superstar effects. Our set-up and the main focus of our analysis are, however, very different. We consider competition between teams in a market and study the conditions under which positive assortative matching arises. We show that globalization increases the concentration of talent, which, in turn, will fuel into (top-)income inequality. We thus analyze a complementary channel of how globalization can add to income inequality via superstar effects.¹¹

We build on the literature characterizing matching equilibria. In his seminal contribution, Becker (1973) showed that there will be positive assortative matching in a marriage market whenever a couple's payoff function is supermodular in the partners' types (characteris-

¹⁰Haskel et al. (2012) discuss how globalization can amplify superstar-effects and argue based on an augmented Heckscher-Ohlin model that superstar-effects à la Rosen (1981) may well have contributed to recent trends in US wage inequality.

¹¹Our work thus also relates to the broader literature analyzing different drivers of (top-)income inequality, see e.g. Piketty et al. (2014); Bénabou and Tirole (2016); Jones and Kim (2015); Geerolf (2017) for recent theoretical contributions.

tics).¹² In his paper, payoffs depend exclusively on the own matching and in particular not on the matching of all other men and women in the market. As opposed to that, we consider a case where teams compete against each other and, hence, each team’s (expected) payoff depends on the skill levels of all other teams. In that we follow Chade and Eeckhout (2015) who also study conditions for positive and negative assortative matching in a one-sided market with competition between teams.¹³ They limit attention to 1:1 competition between randomly drawn pairs of teams and show that in their set-up, multiple equilibria may exist and equilibria need not be efficient. Our focus is different: We allow for generic forms of competition between teams and summarize this competition in a rank distribution function for teams that depends on a team’s own skill level and the skill levels of all other teams in the market. We then show that under general conditions globalization promotes positive assortative matching and discuss the distributional consequences.¹⁴

Finally, our work is also related to the literature analyzing the distributional consequences of globalization more generally. A large literature is concerned with the distributional consequences of international trade. In recent work, trade has been shown to have heterogeneous effects across regional labor markets (e.g. Autor et al. 2013; Dauth et al. 2014; Dix-Carneiro and Kovak 2015), across (types of) workers (e.g. Autor et al. 2014; Galle et al. 2015; Lee 2016; Helpman et al. 2017), and across (types of) consumers (e.g. Faber 2014; Fajgelbaum and Khandelwal 2016). Somewhat closer to our paper is the work by Costinot and Vogel (2010), who consider an assignment model of heterogeneous workers to tasks to study the distributional consequences of international trade. In their model, however, there is always positive assortative matching of workers to tasks. Grossman et al. (2016) consider two-sided matching between managers and workers of different skills that sort into different industries and analyze the distributional effects of changes in the trade environment. While in their set-up workers and managers always match positively assortatively within industries, they may or may not sort positively assortatively across industries, i.e. talent may or may not concentrate in one industry. Grossman et al. (2016) do not consider, however, how this concentration itself is affected by globalization which is our main focus. In that regard, our paper is closest to Helpman et al. (2010) and Porzio

¹²See Kremer (1993); Shimer and Smith (2000); Legros and Newman (2002, 2007); Eeckhout and Kircher (2018) for extensions of these ideas and conditions for positive assortative matching in different contexts.

¹³Another strand of this literature studies existence conditions for stable matchings and efficiency of these matchings in two-sided markets with externalities (Sasaki and Toda, 1996; Hafalir, 2008; Mumcu and Saglam, 2010; Pycia and Yenmez, 2017).

¹⁴We consider the case of pure competition for rank. With total payoff in the market being independent of the matching, equilibrium outcomes are trivially efficient.

(2017). These papers are, however, very different from our work both in terms of the economic environment and in terms of the main mechanisms of interest.¹⁵

Organisation of this paper

The remainder of the paper is organized as follows. In Section 2 we introduce our model. In Section 3 we analyze equilibria in our economy. In Section 4 we investigate the effect of globalization on equilibrium outcomes. In Section 5 we consider an extension with several types and team members. Section 6 concludes. The proofs of all results are in the Appendix.

2 Model

To analyze how globalization impacts the concentration of talent, we will consider an economy where workers of different skills form teams that compete for rank and each rank is awarded a certain payoff. In such a setting, we then apply a convex transformation of the payoff scheme—to be interpreted as a reduced form of globalization—and analyze whether as a result of such transformation, positive assortative matching is an equilibrium for a broader set of competitions. In the present section we introduce the model.

2.1 Economic environment

The economy is populated by a continuum of workers, denoted by \mathcal{W} . Workers receive an increasing utility in money and differ in their skills: they are either high-skilled or low-skilled. Accordingly, we assume that $\mathcal{W} = \mathcal{W}^h \cup \mathcal{W}^l$, where $\mathcal{W}^h = \mathcal{W}^l \equiv [0, 1]$, \mathcal{W}^h and \mathcal{W}^l denoting the set of high-skilled and low-skilled workers, respectively.

Workers match to other workers and form *teams*, which consist of a pair of workers and are generally denoted by t . A team may be of three types: two high-skilled workers may match, and we use t^h to denote such a team; two low-skilled workers may match, and we use t^l to denote such a team; one worker of each type may match, and we use t^m to denote such a team. Teams potentially differ in their overall skill level, which is increasing

¹⁵Helpman et al. (2010) show how after a trade-liberalization the concentration of talent in the most productive firms increases in a Melitz (2003)-model with search frictions in the labor market. Porzio (2017) shows how globalization—in his case the availability of state-of-the-art technologies to developing countries—can give rise to a dual economy where high-skilled individuals concentrate in the sectors that adopt the state-of-the-art technology.

in the skills of each team member. Side-payments are possible, and thus we consider an environment with transferable utility.

A *matching* μ is the collection of all teams. That is, each worker belongs to exactly one team, and thus a matching partitions the set \mathcal{W} . We denote the set of all possible matchings of \mathcal{W} by \mathcal{M} .¹⁶ Note that except for changes that affect sets of workers of measure zero, \mathcal{M} can be indexed by $\mu(\alpha)$ with $\alpha \in [0, 1]$, where $1 - \alpha$ denotes the share of teams t^m over all teams in $\mu(\alpha)$. Note that $\alpha/2$ then denotes both the share of teams t^h and t^l over all teams of the corresponding matching $\mu(\alpha)$. Later on, we shall pay particular attention to the case where $\alpha = 1$, and we shall accordingly refer to $\mu(1)$ as the *positive assortative matching (PAM)*. Similarly, we shall refer to $\mu(0)$ as the *negative assortative matching (NAM)*.

2.2 Competition between teams

Teams compete for rank, since their payoff depends only on their rank $y \in [0, 1]$. We apply the convention that rank $y = 1$ ($y = 0$) is the best (the worst). The exact nature of this competition will not matter for our purpose, and for now it suffices to assume that competition results in some rank distribution for each team. This rank distribution depends on a team's own skill level, as well as on the skill levels of all other teams in the market. In our symmetric set-up, the latter can be summarized by the share of teams that are positively assortatively matched, namely α . We will accordingly use $F^{\alpha,k}(y)$ to denote the probability that given a matching $\mu(\alpha)$, with $\alpha \in [0, 1]$, a team t^k will reach rank y or lower, with $k \in \{l, m, h\}$. In other words, $F^{\alpha,k}(y)$ is the CDF of team t^k over rank $y \in [0, 1]$ when all teams are arranged according to $\mu(\alpha)$. For simplicity, we assume that for all $\alpha \in [0, 1]$, $F^{\alpha,k}(\cdot)$ is continuously differentiable, and denote the corresponding PDF by $f^{\alpha,k}(y) = \frac{dF^{\alpha,k}(y)}{dy}$. Because all positions in the ranking have to be filled, we must have that for all $y \in [0, 1]$

$$1 = \frac{\alpha}{2} \cdot f^{\alpha,l}(y) + \frac{\alpha}{2} \cdot f^{\alpha,h}(y) + (1 - \alpha) \cdot f^{\alpha,m}(y). \quad (1)$$

The rank distributions for teams are the outcomes of the underlying competition. In particular, we can conceive of the competition as determining the rank distribution of each team in the economy as a function of the distribution of skills among teams. Accordingly, we will henceforth have a set of rank distributions for each type of team and each matching $\mu(\alpha)$, $\mathcal{F} = \{(F^{\alpha,l}, F^{\alpha,m}, F^{\alpha,h})\}_{\alpha \in [0,1]}$, denote the *competition* in our economy. To render

¹⁶For the sake of notation, we drop the dependency of the set of all possible matchings on \mathcal{W} .

the notion of skill meaningful, we define $\mathcal{A}_\alpha^{\mathcal{F}} := \{y \in [0, 1] : f^{\alpha, m}(y) \geq 1\}$ and $\tilde{\mathcal{A}}_\alpha^{\mathcal{F}} := \{y \in [0, 1] : f^{\alpha, l}(y) + f^{\alpha, h}(y) \leq 2\}$ for all $\alpha \in [0, 1]$, and then consider the following two conditions:

Assumption 1 (Competition)

- (i) (PAM version) The set $\mathcal{A}_1^{\mathcal{F}}$ is convex and compact.
- (ii) (NAM version) The set $\tilde{\mathcal{A}}_0^{\mathcal{F}}$ is convex and compact.

Note that for all $\alpha \in (0, 1)$, we have $\mathcal{A}_\alpha^{\mathcal{F}} = \tilde{\mathcal{A}}_\alpha^{\mathcal{F}}$ by Equation (1). Further, if rank distributions are continuous in the distributions of skills in the entire economy, the boundaries of the sets $\mathcal{A}_\alpha^{\mathcal{F}}$ and $\tilde{\mathcal{A}}_\alpha^{\mathcal{F}}$ are continuous in α .¹⁷ Any restrictions on the sets $\mathcal{A}_\alpha^{\mathcal{F}}$ and $\tilde{\mathcal{A}}_\alpha^{\mathcal{F}}$, respectively, are then a reflection of the same underlying rationale, and we introduce this distinction to avoid technical difficulties pertaining to the limiting behavior of the various rank distributions when analyzing equilibria with positive and negative assortative matching, respectively.

To get some further understanding of the economic content of Assumption 1, consider the case of some $\alpha \in [0, 1]$, with the interpretation of Assumption 1(i) and Assumption 1(ii) following. Clearly, since $f^{\alpha, m}(y)$ is continuous in $[0, 1]$ for all $\alpha \in [0, 1]$, the set $\mathcal{A}_\alpha^{\mathcal{F}}$ is compact also for all $\alpha \in [0, 1]$. This is a standard assumption of rather technical nature. We thus focus now on $\mathcal{A}_\alpha^{\mathcal{F}}$ for some $\alpha \in [0, 1]$, and in particular for $\alpha = 1$. In essence, assuming that this set is convex can be seen as a minimal definition of skills in our economy. For one thing, it is satisfied if a mixed team is likely to achieve ranks in the mid range, although it is more general than that. Indeed, consider that matching $\mu(\alpha)$ has formed, i.e. there are shares $1 - \alpha$ of mixed teams and $\alpha/2$ of low- and high-skilled teams. What should each of the mixed teams expect in terms of its ranking in the competition? One possibility is that $f^{\alpha, m}(y) = 1$ for all $y \in [0, 1]$, in which case $\mathcal{A}_\alpha^{\mathcal{F}} = [0, 1]$ and all such teams expect to be ranked in any position with the same probability. In general, $\mathcal{A}_\alpha^{\mathcal{F}}$ is the set of ranking positions whose associated probability is at least that of the former uniform case. This determines in itself a plausible threshold that separates ranks $y \in [0, 1]$ into *low-probability ranks* ($f^{\alpha, m}(y) < 1$) and *high-probability ranks* ($f^{\alpha, m}(y) \geq 1$). Assuming that $\mathcal{A}_\alpha^{\mathcal{F}}$ is a convex set, we rule out competitions whose outcome does not behave naturally with regard to the rank of the mixed teams. Indeed, if skills of the members of a team are to give an advantage in the ranking in the competition, mixed teams should expect to be ranked in mid-range positions, with low-quality teams being ranked in low positions

¹⁷Note, however, that our subsequent analyses do not hinge on this restriction.

and high-quality teams being ranked in high positions. A weak way of implementing this rationale is to preclude the possibility that a low-probability rank exists for a mixed team between two high-probability ranks. This is captured in Assumption 1.

It is immediate to verify that the set $\mathcal{A}_\alpha^{\mathcal{F}}$ is convex when $f^{\alpha,m}(\cdot)$ is quasi-concave, in particular when it is single-peaked. The set $\mathcal{A}_\alpha^{\mathcal{F}}$ is also convex if, e.g., all teams compete among themselves head to head—with the probability of winning each one-to-one game being proportional to each team’s skills—and are then ranked according to the number of victories. For instance, assume that each time two teams meet, there is a fixed probability $p \geq \frac{1}{2}$ that the higher skilled team wins, with that probability being $\frac{1}{2}$ if two equally skilled teams compete. Then, as the number of games that a particular team is involved in goes to infinity, the ratio of victories for this team will converge to its expected value by the law of large numbers. As a consequence, the rank distribution of a low-skilled team will be uniform on $[0, \frac{\alpha}{2}]$, that of a mixed team will be uniform on $[\frac{\alpha}{2}, 1 - \frac{\alpha}{2}]$ and that of a high skilled team uniform on $[1 - \frac{\alpha}{2}, 1]$.¹⁸

2.3 Payoffs

Teams receive rank-dependent payoffs. In particular, we let

$$\begin{aligned} h : [0, 1] &\rightarrow \mathbb{R}_+ \\ y &\rightarrow h(y) \end{aligned} \tag{2}$$

denote the *payoff scheme* that assigns a certain amount of money $h(y)$ to a team that has been ranked in position y . Consistent with our normalization that higher ranks are better, we will assume that payoffs are weakly increasing in rank.

Assumption 2 (Payoffs)

(i) $h(0) = 0$, (ii) $h'(\cdot) \geq 0$.

The normalization $h(0) = 0$ can be made without loss of generality. With $h(\cdot)$ constant, in particular, there would be no effect of workers’ skills on (expected) outcomes. In what follows, we will therefore limit attention to the economically interesting cases where $h(\cdot)$ is non-constant and let

$$\mathcal{H} = \{h : [0, 1] \rightarrow \mathbb{R}_+ \mid h \in C^1([0, 1]), h'(\cdot) \geq 0, 0 = h(0) < h(1)\} \tag{3}$$

¹⁸To see this note that the expected share of victories for a low skilled team will be $p^l = \frac{\alpha}{4} + (1 - \frac{\alpha}{2}) \cdot (1 - p)$, that of a mixed team will be $p^m = \frac{\alpha}{2} \cdot p + \frac{\alpha}{2} \cdot (1 - p) + \frac{1-\alpha}{2} = \frac{1}{2}$, and that of a high skilled team will be $p^h = \frac{\alpha}{4} + (1 - \frac{\alpha}{2}) \cdot p$. It is then straightforward to verify that for every $p > \frac{1}{2}$, we must have $p^l < p^m < p^h$.

denote the set of admissible payoff schemes, where $C^1([0, 1])$ denotes the set of continuously differentiable functions on $[0, 1]$.

The main focus of the paper is on the redistributive aspects of globalization and on how these interact with the concentration of talent. To that end, we will consider a reduced form modeling choice for globalization. Specifically, we will represent (the effect of) globalization by a twice continuously differentiable function

$$\begin{aligned} g : \mathbb{R}_+ &\rightarrow \mathbb{R}_+ \\ x &\rightarrow g(x), \end{aligned} \tag{4}$$

which will modify the payoff scheme and transform it from h to $g \circ h$.¹⁹ We will assume that $g(\cdot)$ satisfies the following restrictions:

Assumption 3 (Globalization)

$$(i) \ g'(\cdot) \geq 0, \quad (ii) \ g''(\cdot) \geq 0, \quad (iii) \ g(0) = 0, \quad (iv) \ g'(0) > 0.$$

On the one hand, assumptions 3(iii) and 3(iv) are of technical nature, and they are immaterial to our results. They simply guarantee that $g \circ h \in \mathcal{H}$ for all $h \in \mathcal{H}$ and all $g \in \mathcal{G}$, where \mathcal{G} is the set of functions satisfying Assumption 3. Assumptions 3(i) and 3(ii), on the other hand, are more substantive. According to them, we will represent globalization by an increasing and convex transformation of the payoff scheme. While Assumption 3(i) is made to preserve the normalization that higher ranks are better, the increased convexity of payoffs (Assumption 3(ii)) will amplify “superstar-effects”, as already noted by Rosen (1981).

One simple case when Assumptions 3(i) and 3(ii) are naturally satisfied is a situation where teams have the opportunity to access foreign markets, albeit at a fixed cost, similar to a Melitz (2003)-model of international trade. In particular, suppose that the gains from entering this market are increasing in a team’s rank in its domestic economy. In the case of European football considered in the introduction, we may think of entering a foreign market as actively trying to increase a team’s fan base in these markets with the goal of raising revenues via sponsoring, merchandising, or licensing, for example. Such endeavors

¹⁹Our approach to modelling globalization is similar to the approach taken e.g. by Grossman et al. (2016) who model globalization—a change in the trade environment in their case—as a change in relative output prices.

are naturally more promising for teams that perform well in their domestic leagues.²⁰ In particular, suppose for simplicity that payoffs generated abroad, $\tilde{h}(y)$, are proportional to the domestic payoffs $h(y)$, i.e.,

$$\tilde{h}(y) = \lambda \cdot h(y)$$

for some constant $\lambda > 0$. Teams will enter the foreign market only if this is profitable, implying that total payoffs of a team ranked y will be

$$g(h(y)) = h(y) + \max\{0, \lambda \cdot h(y) - c\} .$$

It is straightforward to verify that $g(\cdot)$ is increasing and convex.²¹

3 Equilibrium

In the previous section we outlined our model. In summary, we are considering an *economy* that is characterized by a triple $e = (\mathcal{W}, h, \mathcal{F})$, composed of a set of workers \mathcal{W} , a payoff scheme $h(\cdot)$, and a competition \mathcal{F} . We now proceed to the analysis of equilibria in each economy $e \in \mathcal{E}$, where we use \mathcal{E} to denote the set of all possible economies that satisfy our previous restrictions.

Given $e = (\mathcal{W}, h, \mathcal{F}) \in \mathcal{E}$, it will come in handy to define, for $k \in \{l, m, h\}$,

$$V(t^k | \mu(\alpha)) = \int_0^1 h(y) dF^{\alpha, k}(y). \tag{5}$$

That is, $V(t^l | \mu(\alpha))$, $V(t^m | \mu(\alpha))$, and $V(t^h | \mu(\alpha))$ denote the expected payoff of a team t^l , t^m , and t^h , respectively, in economy e when teams are arranged according to $\mu(\alpha)$. Now, remember that we are assuming linear utility in money, i.e. agents care about their expected payoffs only. Further, with a continuum of workers, we can conceive of them as operating in a competitive labor market. This means that workers can decide to form teams with other workers in the economy, taking as given the wage rates for skilled and unskilled workers, which we denote by \bar{w} and \underline{w} , respectively.²² An equilibrium in an economy e is therefore defined as follows:

²⁰In Germany, for example, Bayern Munich and Borussia Dortmund, the biggest and most successful football clubs in recent years, are most actively promoting their teams abroad and they are the only clubs running foreign offices (see <https://www.welt.de/sport/article157261763/Das-Millionenspiel-der-Bundesligaklubs-in-Uebersee.html>). They also have by far the most facebook likes outside of Germany (see <http://meedia.de/2015/09/23/bundesliga-bis-3-liga-das-grosse-facebook-ranking-der-fussballclubs/>).

²¹Note that in this example $h(\cdot)$ is not differentiable for all $y \in [0, 1]$. Nevertheless, our results do not hinge crucially on this regularity assumption, which we have only imposed for simplicity.

²²In our economy, teams' payoffs, and hence, payoffs to team members, are random. Since utility is linear in money, workers are risk-neutral and we can therefore conceive of \bar{w} (\underline{w}) as the expected payoff that a high-skilled (low-skilled) worker receives.

Definition 1 (Equilibrium)

An equilibrium of economy $e \in \mathcal{E}$ is a triple $(\mu(\alpha), \underline{w}, \bar{w}) \in \mathcal{M} \times \mathbb{R} \times \mathbb{R}$ such that, for $k \in \{l, m, h\}$,

$$t^k \in \mu(\alpha) \Rightarrow \begin{cases} V(t^h|\mu(\alpha)) - \bar{w} \geq V(t^m|\mu(\alpha)) - \underline{w} & \text{if } k = h \\ V(t^m|\mu(\alpha)) - \bar{w} \geq V(t^l|\mu(\alpha)) - \underline{w} \text{ and} \\ V(t^m|\mu(\alpha)) - \underline{w} \geq V(t^h|\mu(\alpha)) - \bar{w} & \text{if } k = m \\ V(t^l|\mu(\alpha)) - \underline{w} \geq V(t^m|\mu(\alpha)) - \bar{w} & \text{if } k = l \end{cases} \quad (6)$$

That is, within any team matched under $\mu(\alpha)$, none of its members expects a higher payoff by forming another team with a worker of a type different from his/her current match. In other words, in an equilibrium there are no incentives for workers to break away from the current pair and form a new pair. In the case where $\alpha = 1$, Conditions (6) reduce to

$$V(t^h|\mu(1)) - \bar{w} \geq V(t^m|\mu(1)) - \underline{w}, \text{ and} \quad (7)$$

$$V(t^l|\mu(1)) - \underline{w} \geq V(t^m|\mu(1)) - \bar{w}. \quad (8)$$

The following result characterizes equilibria in our economies whenever they exist.

Proposition 1 (Equilibrium)

Let $e = (\mathcal{W}, h, \mathcal{F}) \in \mathcal{E}$. Then, the following statements hold:

- (i) There is an equilibrium with positive assortative matching (PAM, $\alpha = 1$) if and only if

$$\int_0^1 h(y)dy \geq \int_0^1 h(y)dF^{1,m}(y). \quad (9)$$

In this case, the equilibrium wages are

$$\bar{w} = \frac{1}{2} \cdot \int_0^1 h(y)dF^{1,h}(y) \quad (10)$$

$$\underline{w} = \frac{1}{2} \cdot \int_0^1 h(y)dF^{1,l}(y). \quad (11)$$

- (ii) There is an equilibrium with $\alpha \in (0, 1)$ if and only if

$$\int_0^1 h(y)dy = \int_0^1 h(y)dF^{\alpha,m}(y). \quad (12)$$

In this case, the equilibrium wages are

$$\bar{w} = \frac{1}{2} \cdot \int_0^1 h(y)dF^{\alpha,h}(y) \quad (13)$$

$$\underline{w} = \frac{1}{2} \cdot \int_0^1 h(y)dF^{\alpha,l}(y). \quad (14)$$

(iii) There is an equilibrium with negative assortative matching (NAM, $\alpha = 0$) if and only if

$$\int_0^1 h(y)dy \geq \int_0^1 h(y) \cdot \left(\frac{1}{2}dF^{0,h}(y) + \frac{1}{2}dF^{0,l}(y) \right). \quad (15)$$

In this case, the equilibrium wages are

$$\bar{w} \in \left[\frac{1}{2} \cdot \int_0^1 h(y)dF^{0,h}(y), \int_0^1 h(y)dF^{0,m}(y) - \frac{1}{2} \cdot \int_0^1 h(y)dF^{0,l}(y) \right] \quad (16)$$

$$\underline{w} = \int_0^1 h(y)dF^{0,m}(y) - \bar{w}. \quad (17)$$

Proof:

See Appendix. □

Subsequently, our main focus will be on the equilibrium with positive assortative matching, and hence our analyses will be centered around Condition (9). Because in an equilibrium with PAM there are only high- and low-skilled teams in equal shares and they must on average earn the expected payoff across all teams in the economy, Condition (9) can be rewritten as

$$\int_0^1 h(y) \cdot \frac{1}{2} \cdot [dF^{1,l}(y) + dF^{1,h}(y)] \geq \int_0^1 h(y)dF^{1,m}(y),$$

which is the familiar supermodularity condition. The key observation is, however, that in our case this supermodularity refers to expected payoffs which, in turn, are determined by non-trivial interactions between the mode of competition—as reflected in $dF^{1,k}(y)$ —and the payoff scheme, $h(y)$. The latter will be affected by globalization.

After characterizing equilibria when they exist, an ensuing question is whether an equilibrium exists at all. In this regard, we shall proceed on the following assumption in the remainder of the paper:

Assumption 4 (Existence of equilibrium)

$V(t^k|\mu(\alpha))$ is continuous in α , for $k \in \{l, m, h\}$ and every $h \in \mathcal{H}$.

We stress that throughout the paper, and in particular in the above condition, the dependence of $V(t^k|\mu(\alpha))$ on $h \in \mathcal{H}$ has been dropped for the sake of exposition. Imposing Assumption 4 guarantees that the (expected) payoff of each team depends continuously on changes in the workers' matching, both before and after globalization. A sufficient, but not necessary, condition for this to happen is that the competition outcome does not

change abruptly when small changes occur in the way workers match in the economy, i.e. when $f^{\alpha,k}$ is continuous in α . Under this mild assumption, mixed teams' (expected) payoffs also depend continuously on changes in the workers' matching, both before and after globalization. Moreover, we claim that an equilibrium in this scenario always exists. To see this, observe on the one hand, that for no equilibrium with PAM to exist, we must have

$$\begin{aligned} \frac{1}{2} \cdot V(t^l|\mu(1)) + \frac{1}{2} \cdot V(t^h|\mu(1)) &= \int_0^1 h(y) \cdot \left(\frac{1}{2} dF^{1,h}(y) + \frac{1}{2} dF^{1,l}(y) \right) = \int_0^1 h(y) dy \\ &< \int_0^1 h(y) dF^{1,m}(y) = V(t^m|\mu(1)), \end{aligned}$$

where the second equality follows from Equation (1). On the other hand, for no equilibrium with NAM to exist, we must have

$$\begin{aligned} V(t^m|\mu(0)) &= \int_0^1 h(y) dF^{0,m}(y) = \int_0^1 h(y) dy \\ &< \int_0^1 h(y) \cdot \left(\frac{1}{2} dF^{0,h}(y) + \frac{1}{2} dF^{0,l}(y) \right) = \frac{1}{2} \cdot V(t^l|\mu(0)) + \frac{1}{2} \cdot V(t^h|\mu(0)). \end{aligned}$$

Due to Assumption 4, there must exist an $\alpha^* \in (0, 1)$ such that

$$V(t^m|\mu(\alpha^*)) = \frac{1}{2} \cdot V(t^l|\mu(\alpha^*)) + \frac{1}{2} \cdot V(t^h|\mu(\alpha^*)).$$

Using Equation (1) as well as the above equation, we obtain

$$\begin{aligned} \int_0^1 h(y) dy &= \frac{\alpha^*}{2} \cdot V(t^l|\mu(\alpha^*)) + \frac{\alpha^*}{2} \cdot V(t^h|\mu(\alpha^*)) + (1 - \alpha^*) \cdot V(t^m|\mu(\alpha^*)) = V(t^m|\mu(\alpha^*)) \\ &= \int_0^1 h(y) dF^{\alpha^*,m}(y), \end{aligned}$$

and hence there is an equilibrium where workers are arranged in $\mu(\alpha^*)$. This proves our claim that an equilibrium always exists under Assumption 4.

Accordingly, in our set-up the existence of an equilibrium is guaranteed, in which case it is described by Proposition 1. The equilibrium, however, need not be unique. It turns out that this will nevertheless be the case under either of the following assumptions:

Assumption 5 (Uniqueness of equilibrium)

Let $h \in \mathcal{H}$ be given. Then, either

$$V(t^m|\mu(\alpha)) > V(t^m|\mu(0)) \text{ for all } \alpha \in (0, 1] \tag{18}$$

or

$$V(t^l|\mu(\alpha)) + V(t^h|\mu(\alpha)) > V(t^l|\mu(1)) + V(t^h|\mu(1)) \text{ for all } \alpha \in [0, 1). \tag{19}$$

Equation (18) requires that mixed teams expect the lowest payoff when all teams are mixed, while Equation (19) requires that the average payoff of a high- and a low-skilled team is lowest when no team is mixed. Indeed, note first as a preliminary observation that in our symmetric set-up in which only skills are relevant for ranking, in the case where all teams are mixed, they should all expect to be ranked in any position with equal probability. In fact, from Equation (1), it follows that

$$\int_0^1 h(y)dy = \int_0^1 h(y)dF^{0,m}(y) = V(t^m|\mu(0)).$$

On the one hand, assume Equation (18). Then, it must hold that for all $\alpha \in (0, 1]$,

$$\int_0^1 h(y)dy < V(t^m|\mu(\alpha)) = \int_0^1 h(y)dF^{\alpha,m}(y).$$

This means that there cannot be an equilibrium with $\alpha \in (0, 1]$ —see Proposition 1—, and hence an equilibrium with NAM ($\alpha = 0$) is the only possibility. On the other hand, assume Equation (19). Then, it must hold that, for all $\alpha \in [0, 1)$,

$$\int_0^1 h(y)dy < \frac{1}{2} \cdot [V(t^l|\mu(\alpha)) + V(t^h|\mu(\alpha))] = \int_0^1 h(y) \cdot \frac{1}{2} \cdot [dF^{\alpha,l}(y) + dF^{\alpha,h}(y)].$$

Accordingly, there cannot be an equilibrium with $\alpha \in [0, 1)$, and an equilibrium with PAM ($\alpha = 1$) is the only possibility.²³ To sum up, if Equation (18) holds then an equilibrium with NAM ($\alpha = 0$) is the only equilibrium matching, while if Equation (19) holds then PAM ($\alpha = 1$) is the only equilibrium matching. In either case there cannot exist an equilibrium with $\mu(\alpha)$, where $0 < \alpha < 1$. We shall not impose Assumption 5 in what follows unless we state it otherwise.

4 Globalization

We will now analyze how globalization impacts the concentration of talent. Here and below we shall say that an economy $e \in \mathcal{E}$ *satisfies PAM* (resp. *satisfies NAM*) if there are wages

²³The fact that there cannot be an equilibrium with NAM follows immediately from Proposition 1. The fact that there cannot be an equilibrium with $\alpha \in (0, 1)$ follows from the fact that for $\alpha \in (0, 1)$

$$\int_0^1 h(y)dy < \int_0^1 h(y) \cdot \frac{1}{2} \cdot [dF^{\alpha,l}(y) + dF^{\alpha,h}(y)]$$

implies that

$$\int_0^1 h(y)dy > \int_0^1 h(y)dF^{\alpha,m}(y)$$

by Equation (1) and from Proposition 1.

\underline{w} and \bar{w} such that $(\mu(1), \underline{w}, \bar{w})$ (resp. $(\mu(0), \underline{w}, \bar{w})$) is an equilibrium. From Proposition 1 we know that this will be the case if and only if $h(\cdot)$ and \mathcal{F} satisfy Condition (9) (resp. Condition (15)). Because both these conditions, as well as the corresponding conditions for equilibria with $\alpha \in (0, 1)$, are invariant with respect to affine transformations of the payoff scheme, we will henceforth normalize payoffs under globalization such that

$$\int_0^1 h(y)dy = \int_0^1 g(h(y))dy.$$

The above equation highlights the importance of the redistributive aspects of globalization with regard to the concentration of talent, which is what we are interested in. The following technical auxiliary result will be central for our understanding of this topic.

Lemma 1

Let $h \in \mathcal{H}$ and $g \in \mathcal{G}$, and consider a continuously differentiable CDF, $F(\cdot)$, with support $[0, 1]$ such that

$$\{y \in [0, 1] : f(y) \geq 1\}$$

is a convex set, with $f(y) = \frac{dF(y)}{dy}$. Then,

$$\int_0^1 h(y)dy \geq \int_0^1 h(y)dF(y) \Rightarrow \int_0^1 g(h(y))dy \geq \int_0^1 g(h(y))dF(y).$$

Proof:

See Appendix. □

According to Lemma 1, whenever a team that expects to be ranked according to $F(\cdot)$ earns prior to globalization less than the average payoff, this will also be the case after globalization. In our setup, this implies in turn that globalization promotes the concentration of talent, as the following theorem shows:

Theorem 1

Let $(\mathcal{W}, h, \mathcal{F}) \in \mathcal{E}$ and $g \in \mathcal{G}$. Then, provided that Assumption 1(i) holds,

- (i) if $(\mathcal{W}, h, \mathcal{F})$ satisfies PAM, so does $(\mathcal{W}, g \circ h, \mathcal{F})$;
- (ii) sometimes $(\mathcal{W}, g \circ h, \mathcal{F})$ satisfies PAM and $(\mathcal{W}, h, \mathcal{F})$ does not.

Proof:

See Appendix. □

Theorem 1 states that whenever there is an equilibrium with positive assortative matching prior to globalization, such an equilibrium will also exist with globalization. Moreover, in some cases such an equilibrium exists with globalization but not without. The basic intuition is the following: Globalization rewards teams ranked high in the market. When compared to a mixed team, positively assortatively matched teams are more likely to achieve extreme ranks and, in particular, to be ranked at the top. This follows from the fact that the rank distribution of the mixed teams is biased towards achieving mid-range ranks. In other words, positively assortatively matched teams will benefit more (on average) from an amplified ‘superstar effect’ after globalization than before.

Two remarks are in order. First, a potential caveat pertaining to our above result is that we limit attention to the existence of equilibria with positive assortative matching without considering their uniqueness.²⁴ In principle, it may be that there is a unique equilibrium with positive assortative matching prior to globalization, but that such an equilibrium is no longer unique with globalization. It turns out, however, that this will not be the case: Whenever an equilibrium with positive assortative matching is unique prior to globalization it is also unique after globalization.²⁵

Second, in Theorem 1 we concentrated on the existence of equilibria with positive assortative matching. Of course, the concentration of talent also increases when moving out of an equilibrium with negative assortative matching. It turns out that globalization also promotes the concentration of talent in this latter sense. In particular, we can show the following result:

Theorem 2

Let $(\mathcal{W}, h, \mathcal{F}) \in \mathcal{E}$ and $g \in \mathcal{G}$. Then, provided that Assumption 1(ii) holds,

- (i) if $(\mathcal{W}, g \circ h, \mathcal{F})$ satisfies NAM, so does $(\mathcal{W}, h, \mathcal{F})$;
- (ii) sometimes $(\mathcal{W}, h, \mathcal{F})$ satisfies NAM and $(\mathcal{W}, g \circ h, \mathcal{F})$ does not.

²⁴Remember that we do not make Assumption 5.

²⁵This follows from Proposition 1, Lemma 1 and Theorem 2 below. First, Theorem 2 immediately implies that if NAM is an equilibrium with globalization, it has to be an equilibrium without globalization as well. Second, using Condition (12) and the same logic as in the proof of Lemma 1 we can obtain that with globalization there can only be an equilibrium with $\alpha \in (0, 1)$ if it holds that

$$\int_0^1 h(y)dy \leq \int_0^1 h(y)dF^{\alpha,m}(y).$$

By continuity of $\int_0^1 h(y)dF^{\alpha,m}(y)$ in α , it then follows that prior to globalization either there must be an equilibrium with $\alpha \in (0, 1)$ or PAM cannot be an equilibrium.

Proof:

See Appendix.

□

It is important to point out that PAM ($\alpha = 1$) and NAM ($\alpha = 0$) represent two extremes of the one-dimensional space $\{\mu(\alpha)\}_{\alpha \in [0,1]}$. More generally, one could ask whether as a result of globalization, the concentration of talent will also increase by shifting from an equilibrium with $\mu(\alpha)$ to an equilibrium with $\mu(\alpha')$, where $\alpha, \alpha' \in [0, 1]$ satisfy $\alpha \leq \alpha'$. To answer this question, however, further assumptions need to be imposed on the (local) behavior of $V(t^m|\mu(\alpha))$ under payoff schemes $h(\cdot)$ and $g(h(\cdot))$, in particular on whether such functions are (locally) monotone or not. While this can be done, it would not add substance to the message of the paper.

To sum up, our theory—as described by Theorems 1 and 2—predicts strong implications of globalization for the concentration of talent. In turn, this has important distributional consequences. It is well known that conditional on positive assortative matching globalization can increase relative wages of high-skilled workers via amplified superstar effects.²⁶ This need not be the case in our set-up. In principle, it is possible that globalization shifts the economy from an equilibrium with negative assortative matching into one with positive assortative matching and, at the same time, increases the (relative) wages of the low skilled workers. Consider, for example, a case where for all $\alpha \in [0, 1]$:

$$\begin{aligned}
 f^{\alpha,l} &= \begin{cases} 2 & \text{if } 0 \leq y \leq \frac{1}{3} \text{ or } \frac{5}{6} \leq y \leq 1 \\ 0 & \text{otherwise} \end{cases} \\
 f^{\alpha,m} &= \begin{cases} 3 & \text{if } \frac{1}{3} \leq y \leq \frac{2}{3} \\ 0 & \text{otherwise} \end{cases} \\
 f^{\alpha,h} &= \begin{cases} 1 & \text{if } 0 \leq y \leq \frac{1}{3} \text{ or } \frac{5}{6} \leq y \leq 1 \\ 3 & \text{if } \frac{2}{3} \leq y \leq \frac{5}{6} \\ 0 & \text{otherwise} \end{cases} .
 \end{aligned}$$

It is straightforward to verify that $f^{\alpha,k}$ satisfies either variant of Assumption 1. First, there will be no equilibrium with positive assortative matching whenever $h(\cdot)$ is concave, e.g. when $h(y) = y^{1/2}$. Second, any globalization $g(y) = g_1\left(\frac{4}{3}y^2\right)$, where $g_1(\cdot)$ is a mean-preserving convex transformation that increases payoffs for ranks $y \geq \frac{5}{6}$ only, will promote positive assortative matching and at the same time benefit low-skilled workers.

²⁶Cf. the literature review at the beginning of this paper.

Yet, the most realistic scenario is one where, conditional on positive assortative matching, globalization increases the (relative) wage of the high-skilled workers.²⁷ In this scenario, the concentration of talent represents an additional channel of how globalization increases (top-)income inequality. In particular, this will be the case under the standard assumption of complementarity between skills that ensures that a low-skilled team is better off competing in a market where every other team is mixed (i.e. NAM) than in a market where some (or all) teams are positively assortatively matched. In this scenario, the wage of the low-skilled workers is decreasing in the share of positively assortatively matched teams α . In turn, this means that the shift to an equilibrium with PAM harms the low-skilled workers above and beyond any harm caused by globalization conditional on the matching.^{28,29}

5 Several types and team members

In this section we briefly discuss how our main result can readily be extended to the case with several skill types and several team members. First, suppose there are $S \geq 2$ types that can be identified by their skill level $s \in \mathcal{S}$, where \mathcal{S} denotes the set of skills available in the economy. Second, let $N \geq 2$ denote the number of workers in each team, with \mathcal{W}_t denoting the set of workers in team t , s_t^i the skill level of the i^{th} worker in team t and $\mathcal{S}_t = \{s_t^1, s_t^2, \dots, s_t^N\}$ the *multiset* of skill levels in team t . Then, there is an equilibrium with positive assortative matching if and only if for every type of worker $s \in \mathcal{S}$ it holds

$$V(t^{\{s\}^N} | PAM) - (N - 1) \cdot w^s \geq \max_{\hat{\mathcal{S}} \in \mathcal{S}^{N-1}} V(s \cup \hat{\mathcal{S}} | PAM) - \sum_{s \in \hat{\mathcal{S}}} w^s.$$

As in the previous sections, w^s denotes the equilibrium wage for a worker with skill level s , $t^{\mathcal{S}_t}$ identifies a team with workers of skill levels \mathcal{S}_t , and $(\cdot | PAM)$ indicates that all other workers are matched positively assortatively. It can be verified that wages in an

²⁷Conditional on positive assortative matching, globalization increases the (relative) wage of the high-skilled workers if it increases the expected payoff of a high-skilled team. This will be the case if, for example, $f^{1,h}(\cdot)$ is weakly increasing while $f^{1,l}(\cdot)$ is weakly decreasing. Note that in the above example, the lowest skilled team is most likely to achieve highest ranks. While our set-up is general enough to accommodate this case, it would contradict a very weak notion of the role of skills in the competition.

²⁸Recall from Proposition 1 that the wage that a low-skilled worker could earn when forming a low-skilled team is the lower bound of his wage in an equilibrium with negative assortative matching.

²⁹Of course, any potential learning externalities between team members may increase the distributional effects of globalization even further. A careful analysis of such spillovers is a promising avenue for future research.

equilibrium with positive assortative matching satisfy

$$w^s = \frac{1}{N} \cdot V(t^{\{s\}^N} | PAM) , \text{ for all } s \in \mathcal{S}.$$

Using the above equilibrium wages and rearranging terms, we obtain that there is an equilibrium with positive assortative matching if and only if for every \mathcal{S}_t it holds

$$\frac{1}{N} \cdot \sum_{s \in \mathcal{S}_t} V(t^{\{s\}^N} | PAM) \geq V(t^{\mathcal{S}_t} | PAM). \quad (20)$$

Now, suppose that for all \mathcal{S}_t , it holds that

$$\left\{ y \in [0, 1] : f^{PAM, \mathcal{S}_t}(y) \geq \frac{1}{N} \cdot \sum_{s \in \mathcal{S}_t} f^{PAM, \{s\}^N}(y) \right\} \quad (21)$$

is a convex set. Here, f^{PAM, \mathcal{S}_t} stands for the PDF associated with the ranking of a team with skill levels \mathcal{S}_t when competing against teams that are arranged according to PAM. Analogously to the case of two types and two team members, the most natural interpretation of Condition (21) is that the rank distributions of mixed teams are biased towards achieving mid-range ranks when compared to assortatively matched teams of the types of its team members. Note that we can re-write Condition (20) as

$$\int_0^1 h(y) dy \geq \int_0^1 h(y) \cdot \left[f^{PAM, \mathcal{S}_t}(y) + 1 - \frac{1}{N} \cdot \sum_{s \in \mathcal{S}_t} f^{PAM, \{s\}^N}(y) \right] dy.$$

Then, by defining

$$\tilde{f}(y) := f^{PAM, \mathcal{S}_t}(y) + 1 - \frac{1}{N} \cdot \sum_{s \in \mathcal{S}_t} f^{PAM, \{s\}^N}(y)$$

we can apply Lemma 1 to generalize Theorem 1 to the case with many types and team members.

6 Conclusion

We have investigated the effect of a convex transformation of payoffs on the concentration of talent in large matching markets with rank competition. We have argued that globalization can attest for this type of transformation of the payoff scheme, which implies that in relative terms, high ranks are rewarded higher prizes *after* globalization than *before*. We have chosen a reduced-form approach to modelling competition that rests on minimal assumptions pertaining to the relationship between skills and market outcomes, and we

may expect the mechanisms we consider to be relevant for many actual markets in which matching externalities are paramount and payoffs are rank-dependent. Because payoffs in such markets may be—and often are—influenced by exogenous elements (namely, globalization), our research question seems of evident theoretical and empirical concern.

Our main insight is that globalization promotes the concentration of talent, i.e. the emergence of positive assortative matching becomes likelier. This has important distributional consequences, as it tends to further increase income inequality in modern societies. Potentially adverse effects of globalization above and beyond a direct effect on income inequality have received increasing attention in recent years (Autor et al., 2014; Che et al., 2016; McManus and Schaur, 2016; Pierce and Schott, 2016). An increased concentration of talent might be an important factor in this regard, as in the presence of learning externalities it may harm low-skilled workers and perpetuate—or even increase—skill differences and, more generally, it contributes to social segregation. Future work may set out to study such effects and their welfare implications more carefully.

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Appendix

In this Appendix, we first prove Proposition 1. Then we prove a series of intermediate results, the combination of which shows Lemma 1, and finally proof Theorem 1 and Theorem 2.

Proof of Proposition 1

In the following, we show each part of Proposition 1.

Proof of Part (i): In an equilibrium $(\mu(1), \underline{w}, \bar{w})$ with positive assortative matching, Conditions (6) reduce to Conditions (7) and (8). Because there is a continuum of workers—and hence every worker can always find another worker with whom to match—, it follows that

$$\bar{w} = \frac{1}{2} \cdot \int_0^1 h(y) dF^{1,h}(y)$$

and

$$\underline{w} = \frac{1}{2} \cdot \int_0^1 h(y) dF^{1,l}(y),$$

which yields Equations (10) and (11). With these expressions for wages, both Conditions (7) and (8) then reduce to the same condition, namely

$$\frac{1}{2} \cdot V(t^h|\mu(1)) + \frac{1}{2} \cdot V(t^l|\mu(1)) \geq V(t^m|\mu(1)). \quad (22)$$

Accordingly, there is an equilibrium with positive assortative matching, $(\mu(1), \underline{w}, \bar{w})$, if and only if (22) is satisfied. Now, note that for $\alpha = 1$, Equation (1) reduces to

$$1 = \frac{1}{2} \cdot \frac{dF^{1,l}(y)}{dy} + \frac{1}{2} \cdot \frac{dF^{1,h}(y)}{dy}.$$

Using the above equation and Equation (5), we can write

$$\frac{1}{2} \cdot V(t^h|\mu(1)) + \frac{1}{2} \cdot V(t^l|\mu(1)) = \int_0^1 h(y) \cdot \left(\frac{1}{2} \cdot dF^{1,l}(y) + \frac{1}{2} \cdot dF^{1,h}(y) \right) = \int_0^1 h(y) dy,$$

and

$$V(t^m|\mu(1)) = \int_0^1 h(y) \cdot dF^{1,m}(y).$$

Finally, using the above two equations in (22), we obtain Condition (9). This concludes the proof of Part (i).

Proof of Part (ii): First of all, note that the same argument as in Part (i) implies that in an equilibrium $(\mu(\alpha), \underline{w}, \bar{w})$ with $\alpha \in (0, 1)$, wages are indeed given by (13) and (14). Further, with $\alpha \in (0, 1)$, Conditions (6) reduce to

$$\begin{aligned} V(t^h|\mu(\alpha)) - \bar{w} &= V(t^m|\mu(\alpha)) - \underline{w}, \\ V(t^l|\mu(\alpha)) - \underline{w} &= V(t^m|\mu(\alpha)) - \bar{w}. \end{aligned}$$

Using the equilibrium wages given by Equations (13) and (14) as well as Equation (5), the above two equations can both be rearranged to

$$\frac{1}{2} \cdot V(t^h|\mu(\alpha)) + \frac{1}{2} \cdot V(t^l|\mu(\alpha)) = V(t^m|\mu(\alpha)).$$

Finally, Equation (12) is easily obtained if we use Equation (5) again in combination with Equation (1). This finishes the proof of Part (ii).

Proof of Part (iii): With negative assortative matching, Conditions (6) reduce to

$$V(t^m|\mu(0)) - \bar{w} \geq V(t^l|\mu(0)) - \underline{w}, \quad (23)$$

$$V(t^m|\mu(0)) - \underline{w} \geq V(t^h|\mu(0)) - \bar{w}. \quad (24)$$

On the one hand, if there is indeed an equilibrium $(\mu(0), \underline{w}, \bar{w})$, it must be that

$$\underline{w} = V(t^m|\mu(0)) - \bar{w}. \quad (25)$$

The reason is that wages of both members of a mixed team must add up to total expected payoffs of the team. Using Equation (5) on Equation (25), we obtain Equation (17), which is necessary for $(\mu(0), \underline{w}, \bar{w})$ to be an equilibrium. On the other hand, adding Conditions (23) and (24) yields the following further necessary condition

$$\frac{1}{2} \cdot V(t^h|\mu(0)) + \frac{1}{2} \cdot V(t^l|\mu(0)) \leq V(t^m|\mu(0)). \quad (26)$$

Condition (15) then follows from using Equations (1) and (5) on (26). Finally, Condition (16) follows from noting that (i) if $\bar{w} < \frac{1}{2}V(t^h|\mu(0))$, high-skilled workers would benefit from breaking apart from a mixed team and then matching another high-skilled worker, (ii) if $\underline{w} < \frac{1}{2}V(t^l|\mu(0))$, low-skilled workers would benefit from breaking apart from a mixed team and then matching another low-skilled worker. Conversely, for any wages \underline{w} and \bar{w} that satisfy these two latter constraints plus Equation (25), no worker can profitably deviate from his current match, implying that the negative assortative matching $(\mu(0), \underline{w}, \bar{w})$ is indeed an equilibrium.³⁰ This finishes the proof of Part (iii). □

Proof of Lemma 1

We start with a preliminary observation and then prove a series of lemmas which jointly imply Lemma 1. For convenience, we rewrite here the statement that we want to prove. Throughout this part of the appendix, we have $h \in \mathcal{H}$ and $g \in \mathcal{G}$ fixed. We shall consider continuously differentiable CDFs on $[0, 1]$, denoted by $F(\cdot)$, such that

$$\mathcal{A} := \{y \in [0, 1] : f(y) \geq 1\} \quad (27)$$

is a convex and compact set, with $f(y) = \frac{dF(y)}{dy}$. For simplicity, we assume that $f(y) > 0$ for all $y \in [0, 1]$.³¹ We shall show that for any such $F(\cdot)$

$$\int_0^1 h(y)dy \geq \int_0^1 h(y)dF(y) \Rightarrow \int_0^1 g(h(y))dy \geq \int_0^1 g(h(y))dF(y). \quad (28)$$

³⁰Note that the set defined in (16) is non-empty by Equation (15).

³¹This assumption facilitates the analysis but we could easily dispense of it.

We can accordingly assume henceforth that

$$\int_0^1 h(y)dy \geq \int_0^1 h(y)dF(y). \quad (29)$$

It will come in handy to denote $\mathcal{A} = [\underline{a}, \bar{a}]$. We assume that $\mathcal{A} \neq \emptyset$, for if not the desired result holds trivially. It will suffice to consider the cases of $\underline{a} = 0$, $\bar{a} = 1$, and $0 < \underline{a} \leq \bar{a} < 1$ separately.

Lemma 2

Assume that $\underline{a} = 0$ and let $F(\cdot)$ be a CDF with domain $[0, 1]$. Then

$$\int_0^1 g(h(y))dy \geq \int_0^1 g(h(y))dF(y).$$

Proof:

The result follows immediately once we note that

$$\begin{aligned} \int_0^1 g(h(y)) \cdot [f(y) - 1] dy &= \int_0^{\bar{a}} g(h(y)) \cdot [f(y) - 1] dy + \int_{\bar{a}}^1 g(h(y)) \cdot [f(y) - 1] dy \\ &\leq g(h(\bar{a})) \cdot \left[\int_0^{\bar{a}} [f(y) - 1] dy + \int_{\bar{a}}^1 [f(y) - 1] dy \right] = 0. \end{aligned}$$

The inequality holds from the definition of set \mathcal{A} —see (27)—because $g(h(\cdot))$ is increasing. The last equality is a consequence of $f(\cdot)$ being a PDF with domain $[0, 1]$. □

Lemma 2 implies that the logical implication in (28) holds true when $\underline{a} = 0$. We next consider the case of $\bar{a} = 1$.

Lemma 3

Assume that $\bar{a} = 1$ and let $F(\cdot)$ be a CDF with domain $[0, 1]$. Then

$$\int_0^1 h(y)dy < \int_0^1 h(y)dF.$$

Proof:

The result follows immediately once we note that

$$\begin{aligned} \int_0^1 h(y) \cdot [f(y) - 1] dy &= \int_0^{\underline{a}} h(y) \cdot [f(y) - 1] dy + \int_{\underline{a}}^1 h(y) \cdot [f(y) - 1] dy \\ &> h(\underline{a}) \cdot \left[\int_0^{\underline{a}} [f(y) - 1] dy + \int_{\underline{a}}^1 [f(y) - 1] dy \right] = 0. \end{aligned}$$

The inequality holds from the definition of set \mathcal{A} because $h(\cdot)$ is increasing and non-constant. The last equality is a consequence of $f(\cdot)$ being a PDF with domain $[0, 1]$. □

Lemma 3 shows that the logical implication in (28) must hold true when $\bar{a} = 1$. In what follows, we restrict our attention to CDFs $F(\cdot)$ such that $0 < \underline{a} \leq \bar{a} < 1$. We initially focus on the particular case where Condition (29) holds with an equality, and then we analyze the general case.

Lemma 4

Assume that $0 < \underline{a} \leq \bar{a} < 1$ and let $F(\cdot)$ be a CDF with domain $[0, 1]$ such that

$$\int_0^1 h(y) dy = \int_0^1 h(y) dF(y). \quad (30)$$

Then

$$\int_0^1 g(h(y)) dy \geq \int_0^1 g(h(y)) dF(y). \quad (31)$$

Proof:

Using Equation (30), we can rewrite Equation (31) as follows

$$\int_0^1 r(y) \cdot [f(y) - 1] dy \geq 0, \quad (32)$$

where we let

$$r(y) = h(y) - g(h(y)). \quad (33)$$

The remainder of the proof is devoted to show (32). We start by noting that by definition of set \mathcal{A} —see (27)—, there must exist b , with $\underline{a} \leq b \leq \bar{a}$, such that:

$$F(y) \begin{cases} < y & \text{if } 0 < y < b \\ = y & \text{if } y = b \\ > y & \text{if } b < y < 1 \end{cases}. \quad (34)$$

Splitting the integral on the LHS of (32), we get

$$\begin{aligned} & \int_0^1 r(y) \cdot [f(y) - 1] dy \\ &= \int_0^{\underline{a}} r(y) \cdot [f(y) - 1] dy + \int_{\underline{a}}^b r(z) \cdot [f(z) - 1] dz + \int_b^{\bar{a}} r(y) \cdot [f(y) - 1] dy + \int_{\bar{a}}^1 r(z) \cdot [f(z) - 1] dz. \end{aligned} \quad (35)$$

Note that the variable in the above integrals is either y or z . The reason for this will become apparent in a moment. Indeed, let us for a moment focus on the second integral on the RHS of (35), where we apply the following change of variable

$$F(z) - F(\underline{a}) - [z - \underline{a}] = y - F(y).$$

This change of variable implicitly defines z as a function of y , $z = z_1(y)$, for $z \in [\underline{a}, b]$. It is a matter of simple technical arguments to check that this change of variable is well-defined—note that $\underline{a} > 0$ —, and that function $z_1(y)$ is strictly increasing and satisfies $z_1(0) = \underline{a}$ and $z_1(\underline{a}) = b$. Moreover, by the Implicit Function Theorem, we have

$$[f(z_1(y)) - 1] dz = [1 - f(y)] dy .$$

Hence, we have

$$\int_{\underline{a}}^b r(z) \cdot [f(z) - 1] dz = \int_0^{\underline{a}} r(z_1(y)) \cdot [1 - f(y)] dy. \quad (36)$$

Similarly, we next apply the change of variables

$$[z - \bar{a}] - [F(z) - F(\bar{a})] = F(y) - y,$$

to the fourth integral on the RHS of (35). This defines z implicitly as a function of y , $z = z_2(y)$, that is strictly increasing and satisfies $z_2(b) = \bar{a}$ and $z_2(\bar{a}) = 1$, thereby yielding

$$\int_{\bar{a}}^1 r(z) \cdot [f(z) - 1] dz = \int_b^{\bar{a}} r(z_2(y)) \cdot [1 - f(y)] dy. \quad (37)$$

Using (36) and (37) in (35), we obtain

$$\begin{aligned} & \int_0^1 r(y) \cdot [f(y) - 1] dy \\ &= \int_0^{\underline{a}} [r(z_1(y)) - r(y)] \cdot [1 - f(y)] dy + \int_b^{\bar{a}} [r(z_2(y)) - r(y)] \cdot [1 - f(y)] dy. \end{aligned}$$

Now, by the Fundamental Theorem of Calculus, we can rewrite the above expression as

$$\begin{aligned} & \int_0^1 r(y) \cdot [f(y) - 1] dy \\ &= \int_0^{\underline{a}} \int_y^{z_1(y)} r'(x) dx [1 - f(y)] dy + \int_b^{\bar{a}} \int_y^{z_2(y)} r'(x) dx [1 - f(y)] dy \\ &= \int_0^{\underline{a}} \int_y^{z_1(y)} h'(x) \cdot [1 - g'(h(x))] dx [1 - f(y)] dy + \int_b^{\bar{a}} \int_y^{z_2(y)} h'(x) \cdot [1 - g'(h(x))] dx [1 - f(y)] dy \\ &\geq [1 - g'(h(b))] \cdot \left[\int_0^{\underline{a}} \int_y^{z_1(y)} h'(x) dx [1 - f(y)] dy + \int_b^{\underline{a}} \int_y^{z_2(y)} h'(x) dx [1 - f(y)] dy \right] \\ &= 0. \end{aligned}$$

This can be explained as follows. The second equality follows from (33), which yields $r'(x) = h'(x)(1 - g'(h(x)))$. The inequality then follows from the facts that (i) $g'(h(\cdot))$ is increasing—because $g(\cdot)$ is convex—, (ii) $h'(\cdot) \geq 0$, and (iii) $f(y) \leq 1$ for all $y \in [0, \underline{a}]$ and $f(y) \geq 1$ for all $y \in [b, \bar{a}]$ —see (27) and (34). Finally, the last equality follows from Equation (30), which implies

$$\begin{aligned} & \int_0^{\underline{a}} \int_y^{z_1(y)} h'(x) dx [1 - f(y)] dy + \int_b^{\bar{a}} \int_y^{z_2(y)} h'(x) dx [1 - f(y)] dy \\ &= \int_0^{\underline{a}} [h(z_1(y)) - h(y)] \cdot [1 - f(y)] dy + \int_b^{\bar{a}} [h(z_2(y)) - h(y)] \cdot [1 - f(y)] dy \\ &= \int_0^1 h(y) \cdot [f(y) - 1] dy = 0. \end{aligned}$$

□

According to Lemma 4, the desired result—i.e. the logical implication in (28)—holds when the CDF $F(\cdot)$ is such that

$$\int_0^1 h(y) dy = \int_0^1 h(y) dF(y).$$

By means of the following two lemmas, we extend this result to the case when

$$\int_0^1 h(y) dy > \int_0^1 h(y) dF(y).$$

Lemma 5

Let $F(\cdot)$ and $\bar{F}(\cdot)$ be two CDFs with domain $[0, 1]$, and let $f(\cdot)$ and $\bar{f}(\cdot)$ denote the corresponding PDFs. If $F(\cdot)$ and $\bar{F}(\cdot)$ satisfy the Monotone Likelihood Ratio Property (MLRP), i.e. if

$$\frac{f(y)}{\bar{f}(y)}$$

is increasing in y , it then holds that

$$\int_0^1 g(h(y))dF(y) > \int_0^1 g(h(y))d\bar{F}(y).$$

Proof:

The result follows immediately from

$$\begin{aligned} \int_0^1 g(h(y))dF(y) &= \int_0^1 g(h(y)) \cdot \frac{dF(y)}{d\bar{F}(y)}d\bar{F}(y) = \int_0^1 g(h(y)) \cdot \frac{f(y)}{\bar{f}(y)}d\bar{F}(y) \\ &> \int_0^1 g(h(y))d\bar{F}(y) \cdot \int_0^1 \frac{f(y)}{\bar{f}(y)}d\bar{F}(y) = \int_0^1 g(h(y))d\bar{F}(y), \end{aligned} \quad (38)$$

where the first equality follows from multiplying and dividing by $d\bar{F}(y)$, the inequality follows from Chebyshev's inequality (Mitrinović et al., 1993, p. 239) because

$$g(h(y)) \text{ and } \frac{f(y)}{\bar{f}(y)}$$

are both non-constant and non-decreasing functions, and the last equality follows from the fact that $F(\cdot)$ is a CDF with domain $[0, 1]$, and hence

$$\int_0^1 \frac{f(y)}{\bar{f}(y)}d\bar{F}(y) = \int_0^1 dF(y) = 1.$$

□

Lemma 6

Assume that $0 < \underline{a} \leq \bar{a} < 1$ and that $\bar{F}(\cdot)$ is a CDF with domain $[0, 1]$ such that

$$\int_0^1 h(y)dy \geq \int_0^1 h(y)d\bar{F}(y). \quad (39)$$

Then, there exists another CDF with domain $[0, 1]$, $F(\cdot)$, such that $f(y) = \lambda(y)\bar{f}(y)$, with $\lambda'(\cdot) \geq 0$, satisfying

$$\int_0^1 h(y)dy = \int_0^1 h(y)dF(y),$$

where $f(\cdot)$ and $\bar{f}(\cdot)$ are the corresponding PDFs.

Proof:

We assume that the inequality in (39) is strict, for if not the result follows trivially from taking $F = \bar{F}$. That is, we assume that

$$\int_0^1 h(y)dy > \int_0^1 h(y)d\bar{F}(y).$$

Because $h(\cdot)$ is non-constant in $[0, 1]$, there must be $y^* \in [0, 1]$ such that

$$h(y^*) > \int_0^1 h(y)dy.$$

Now, for each $k \in [0, \infty)$, define

$$d(k) := \int_0^1 h(y) \cdot \frac{\beta(k)}{1 + e^{-k(y-y^*)}} d\bar{F}(y),$$

where $\beta(k)$ is (uniquely) chosen such that

$$\beta(k) \cdot \int_0^1 \frac{1}{1 + e^{-k(y-y^*)}} d\bar{F}(y) = 1. \quad (40)$$

Clearly, $d(\cdot)$ is a continuous, increasing function on $[0, \infty)$. Moreover,

$$d(0) = \int_0^1 h(y) d\bar{F}(y) < \int_0^1 h(y) dy$$

and

$$\lim_{k \rightarrow \infty} d(k) = h(y^*) > \int_0^1 h(y) dy.$$

Hence, there must be $k^* \in (0, +\infty)$ such that

$$d(k^*) = \int_0^1 h(y) dy.$$

Taking

$$F(y) = \int_0^y \frac{\beta(k^*)}{1 + e^{-k^*(y-y^*)}} d\bar{F}(y) \quad (41)$$

completes the proof of the lemma.³²

□

We are now in a position to prove the following result:

Lemma 7

Assume that $0 < \underline{a} \leq \bar{a} < 1$ and that

$$\int_0^1 h(y) dy > \int_0^1 h(y) d\bar{F}(y). \quad (42)$$

Then

$$\int_0^1 g(h(y)) dy \geq \int_0^1 g(h(y)) d\bar{F}(y). \quad (43)$$

³²Note that $F(y)$ as defined in (41) is a CDF by (40) and the fact that $\frac{\beta(k^*)}{1+e^{-k^*(y-y^*)}}$ is positive valued.

Proof:

By Lemma 6, there exists a $F(\cdot)$ such that $f(y) = \lambda(y)\bar{f}(y)$, with $\lambda'(\cdot) \geq 0$, satisfying

$$\int_0^1 h(y)dy = \int_0^1 h(y)dF(y) .$$

By Lemma 4, we have

$$\int_0^1 g(h(y))dy \geq \int_0^1 g(h(y))dF(y) .$$

From Lemma 5 it follows that

$$\int_0^1 g(h(y))dF(y) \geq \int_0^1 g(h(y))d\bar{F}(y) .$$

Finally, combining the two above inequalities we obtain

$$\int_0^1 g(h(y))dy \geq \int_0^1 g(h(y))d\bar{F}(y),$$

which completes the proof of the lemma. □

Finally, the combination of Lemmas 2, 3, 4, and 7 proves Lemma 1. □

Proof of Theorem 1

Part (i) follows immediately from Lemma 1, which implies that whenever $h(y)$ and $F^{1,m}(y)$ satisfy Condition (9), so will $g(h(y))$ and $F^{1,m}(y)$. We prove part (ii) by means of an example with the desired property. Accordingly, let $h(x) = x^{1/2}$ and $g(x) = \frac{4}{3} \cdot x^2$. Clearly, $h \in \mathcal{H}$ and $g \in \mathcal{G}$. It is then a matter of simple algebra to verify that

$$\int_0^1 h(y)dy = \int_0^1 g(h(y))dy = \frac{2}{3} .$$

Next, consider a competition mode $F^{1,m}(\cdot)$ defined for every $\varepsilon > 0$ as follows

$$dF^{1,m}(y) = \begin{cases} 1/\varepsilon & \text{if } 1/2 - \varepsilon/2 \leq y \leq 1/2 + \varepsilon/2 \\ 0 & \text{otherwise} \end{cases} .$$

On the one hand,

$$\int_0^1 g(h(y)) \cdot dF^{1,m}(y) = \frac{2}{3} .$$

On the other hand, if $\varepsilon > 0$ small enough, we obtain

$$\int_0^1 h(y) \cdot dF^{1,m}(y) \approx \sqrt{\frac{1}{2}} > \frac{2}{3} .$$

Accordingly, $(\mathcal{W}, g \circ h, \bar{F})$ satisfies PAM, but $(\mathcal{W}, h, \bar{F})$ does not. □

Proof of Theorem 2

The formal proof of Theorem 2 closely follows the arguments in the proofs of Lemma 1 and Theorem 1. We therefore present a brief summary of the main arguments. In doing so, it will be convenient to define $\tilde{f}(y) = \frac{1}{2} [f^{0,l}(y) + f^{0,h}(y)]$.

The strategy for the proof of part (i) is to show that

$$\int_0^1 h(y)dy < \int_0^1 h(y) \cdot \tilde{f}(y)dy \Rightarrow \int_0^1 g(h(y))dy < \int_0^1 g(h(y)) \cdot \tilde{f}(y)dy,$$

i.e. if NAM is no equilibrium prior to globalization, it cannot be an equilibrium with globalization. This result follows from first noting that if either $\underline{a} = 0$ or $\bar{a} = 1$ the result holds, where \underline{a} and \bar{a} denote the lower and the upper bound, respectively, of the set $\tilde{\mathcal{A}}_0^{\mathcal{F}} := \{y \in [0, 1] : \tilde{f}(y) \leq 1\}$. This follows from arguments similar to those of the proofs of Lemmas 2 and 3. Hence, we can again limit attention to the case where $\underline{a}, \bar{a} \in (0, 1)$. Second, from the fact that $\tilde{\mathcal{A}}_0^{\mathcal{F}}$ is a convex set and following arguments along the lines of the proof of Lemma 4 it can be verified that

$$\int_0^1 h(y)dy = \int_0^1 h(y) \cdot \tilde{f}(y)dy \Rightarrow \int_0^1 g(h(y))dy \leq \int_0^1 g(h(y)) \cdot \tilde{f}(y)dy.$$

Third, if $\int_0^1 h(y)dy < \int_0^1 h(y)\tilde{f}(y)dy$ then $\tilde{f}(y)$ monotone likelihood ratio dominates some other PDF $f(y)$ that satisfies $\int_0^1 h(y)dy = \int_0^1 h(y)f(y)dy$ by Lemma 6. This implies that

$$\int_0^1 g(h(y)) \cdot f(y)dy < \int_0^1 g(h(y)) \cdot \tilde{f}(y)dy$$

by Lemma 5 and the desired result follows again from combining the above.

Finally, Part (ii) can again be shown by means of an example with the desired property. In particular, the statement is true if $h(x) = x^{1/2}$, $g(x) = 2 \cdot x^4$, and

$$f^{0,l}(y) = \begin{cases} \frac{1}{\epsilon} & \text{if } 0 \leq y \leq \epsilon \\ 0 & \text{otherwise} \end{cases},$$

and

$$f^{0,h}(y) = \begin{cases} \frac{1}{\epsilon} & \text{if } 1 - \epsilon \leq y \leq 1 \\ 0 & \text{otherwise} \end{cases}.$$

□