

Schupp, Fabian; Geiger, Felix

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With a little help from my friends: Survey-based derivation of euro area short rate expectations at the effective lower bound *

Felix Geiger Fabian Schupp

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Abstract

The estimation of dynamic term structure models (DTSMs) turns out to be challenging in the presence of a small sample. It is exacerbated if the sample is characterized by a prolonged period of low interest rates near a time-varying effective lower bound. These challenges all weigh heavily when estimating a DTSM for the euro area OIS yield curve sample. Against this background, we propose a shadow-rate term structure model (SRTSM) that includes a time-varying effective lower bound accounting for the spread between the policy and short-term OIS rate and it also allows for future changes in the effective lower bound. In addition, it incorporates survey information in order to pin down the level of longer-term rate expectations. The model allows to adequately assess short-term monetary policy rate expectations and it generates far-distant rate expectations that are correlated with an estimated equilibrium nominal short rate derived from a macroeconomic model set-up. Our results also highlight the signaling channel of non-standard monetary policy shocks in the run-up to asset purchases based on high frequency identification approach. Our model outperforms DTSM specifications without above modeling features from a statistical and economic perspective. We confirm our findings employing a Monte Carlo simulation.

Keywords: Term structure modeling, short rate expectations, lower bound, survey information, yield curve decomposition, monetary policy, euro area.

JEL classification: E32, E43, E44, E52

*Contact address: Felix Geiger, Deutsche Bundesbank, Wilhelm-Epstein-Strasse 14, 60431, Frankfurt. Email: felix.geiger@bundesbank.de; Fabian Schupp, Deutsche Bundesbank, Wilhelm-Epstein-Strasse 14, 60431, Frankfurt. Email: fabian.schupp@bundesbank.de. We thank Wolfgang Lemke, Martin Mandler, Emanuel Moench, Jan Scheithauer, Christian Speck, Johannes Tischer, Andreea Vladu, Thomas Werner, Andreas Worms and Dora Xia as well as participants of the Deutsche Bundesbank Research Workshop, March 21, 2018 for valuable discussions and comments. We are indebted to Martin Mandler, Michael Scharnagel and Luzie Thiel for sharing their instrument data set in order to identify monetary policy shocks. Discussion Papers represent the authors' personal opinions and do not necessarily reflect the views of the Deutsche Bundesbank or its staff.

1 Introduction

Dynamic term structure models (DTSMs) provide valuable information for policy makers. Equipped with such models, they can infer market participants' view on the outlook for monetary policy and at the same time assess to what extent risk-averse investors demand a risk premium for holding bond instruments in an environment of interest rate uncertainty. In general, however, inference based on term structure models is accompanied by great econometric challenges (Hamilton and Wu, 2012). In essence, these challenges are related to the high persistence of interest rates which makes the estimation of the model parameters of the underlying data generating process very difficult and sensitive to model specifications. This is especially true in a small sample characterized by low interest rate volatility. With the existence of an effective lower bound (ELB), estimation challenges of term structure models even increase because it introduces non-linearities into the term structure model and estimation process. Due to the absence of closed-form solutions for bond prices, they need to be simulated or approximated analytically within a non-linear filtering framework to extract the risk factors which may impact estimation accuracy (Pribsch, 2013; Wu and Xia, 2016). These considerations all weigh heavily when estimating a term structure model with a euro area data sample which only covers a small sample period and essentially only one complete interest rate cycle (2001-08). Moreover, the sample is characterized by a prolonged period of persistently falling interest rates which approached the ELB in July 2012 when the Eurosystem lowered the deposit facility rate (DFR) to 0 and subsequently adopted negative interest rate policies (NIRP).

Against this background, we develop a DTSM for the euro area OIS yield curve which explicitly accounts for above features and fulfills two criteria, i.e. (i) a good model fit and (ii) plausible short- and long-term rate expectations that can be used for policy analysis. We find that given the severe small sample problem with a protracted period of low interest rates near the time-varying ELB, a shadow short rate model specification that incorporates actual as well as expected changes of the ELB is important from a statistical and economic point of view. Moreover, by incorporating survey information on short- and long-term interest rate expectations, the model is able to better pin down the future path of short rates which is important when decomposing longer-term yields and forward rates.

Our model is able to provide a good model fit of the yield curve across time. In the ELB-period the mean absolute fitting error is less than 1 basis point for the one-month rate and 2 basis points for the ten-year rate, respectively. In contrast to alternative model specifications that do not account for a time-varying effective lower bound, our model is able to replicate the temporarily negative slope of the yield curve which was recorded

during the course of 2016 when market's were expecting further DFR cuts. Accounting for expected policy rate changes, therefore, is important to ensure a good model fit at the short-end of the yield curve, a finding also documented by (Wu and Xia, 2017).

The model generates expected short rate paths that do not violate lower bound restrictions. The most likely path of the short rate follows a trajectory which is in line with survey forecasts and which is consistent with the intended policy rate path of the ECB's Governing Council according to its forward guidance. As the possible lift-off of policy rates is linked to the end of net asset purchases of the extended asset purchase programme (APP), changes in the expected duration of net asset purchases should translate into changes of the most likely short rate path. Our model can replicate this hypothesis and it highlights the signaling channel of APP with the announcement of asset purchases as a commitment device for future short rates. In particular, we estimate the impact of monetary policy shocks on the forward curve and its components based on an external instrument approach. We find that unconventional monetary policy shocks in the run-up to the start of asset purchases in March 2015 considerably contributed to the drop in long-term interest rates. Term premia as well as short rate expectations fell in response to these monetary policy shocks.

In order to pass judgment on the economic plausibility of the level and the variability of expected short rates, we compare our model-implied expectations component with an estimate of the equilibrium nominal short rate in the medium- to long-term derived from a macroeconomic model (Holston, Laubach, and Williams, 2017). Indeed, our model estimates resemble the level as well as the dynamics of the equilibrium nominal rate remarkably close even though the two models do not share any information in the estimation. Thus, long-term forward rates appear to reflect trends in key macroeconomic variables in both real and nominal terms, which play an important role in the formation of longer-term interest rate expectations.

Our benchmark model outperforms alternative specifications in terms of economic plausibility. GATSM estimations generate model-implied short rate expectations that violate lower bound restrictions and imply far-distant short rate expectations that seem too low from an economic perspective or may even become negative. De-meaning the pricing factors in the first place as in Adrian, Crump, and Moench (2013) at least ensures that the unconditional mean of the short rate matches the sample mean which pushes up the level of expected short rates. Still, model estimates fail to range at levels consistent with long-term survey expectations or with far-distant short rate expectations derived from a macro model. Only if surveys are incorporated, GATSMs and SRTSMs generate survey and macro consistent short rate expectations. Interestingly, despite the small euro area sample, our findings indicate that with respect to the considered euro area

yield curve sample, estimated DTSMs always produce a very high persistence of the short rate process under the \mathbb{P} -measure. Therefore, the difference between non-bias- and bias-corrected estimates are not substantially large. Insofar, short rate expectations in a bias-corrected *GATSM* do not exhibit implausible large time variation compared to a non-bias-corrected *GATSM* as partly documented for bias-corrected estimates based on US data (Wright, 2014).

To cross-check our results, we conduct an extensive Monte Carlo exercise in which we simulate 500 yield curve data sets with a sample period of 60 years. We ensure that that these samples include an extended period (more than 12 months) of interest rates at the lower bound. In our analysis we compare performance across models in identifying the true unconditional mean and persistence of the data generating process using the full sample as well as smaller samples comparable to the size of the euro area sample. RESULTS TO BE ADDED.

Our paper is related to various strands in the literature. SRTSMs which focused on US, UK and Japanese yield curve data typically assume a constant ELB set or estimated to be close to zero. For the US, see Krippner (2015b); Christensen and Rudebusch (2015); Bauer and Rudebusch (2016); Wu and Xia (2016); Priebisch (2013). SRTSMs based on Japanese data are Ichiue and Ueno (2013); Kim and Singleton (2012) and for UK data see Andreasen and Meldrum (2015). For the euro area some models likewise implemented SRTSMs based on a fixed, but estimated ELB (see the online implementations of Wu and Xia, 2016; Krippner, 2015b). However, given the NIRP and the subsequent steps of the DFR into negative territory, more recent applications for the euro area implemented a time-varying ELB (Lemke and Vladu, 2016; Kortela, 2016; Wu and Xia, 2017). With respect to the modelling of the time-varying ELB, our model is closely related to Wu and Xia (2017) who allow for time-varying expectations of future DFR cuts in agents' bond pricing.

Our work also relates to the vast amount of research that documents the challenges with respect to the estimation of term structure models. In essence, these challenges are most and foremost related to the very high persistence of interest rates which in combination with small samples impedes the estimation procedure and consequently the robust revelation of the mean-reverting characteristics of the short rate process (Kim, 2008; Duffee, 2011; Duffee and Stanton, 2012). Research has addressed this issue by improving and speeding-up the estimation process (Joslin, Singleton, and Zhu, 2011; Christensen, Diebold, and Rudebusch, 2011; Hamilton and Wu, 2012; Adrian et al., 2013), applying bias correction (Bauer, Rudebusch, and Wu, 2012) for *GATSMs* or incorporating survey information into the estimation process (Kim and Orphanides, 2012).

There are also many studies that study the impact of monetary policy shocks on the

yield curve based on high frequency identification schemes (Kuttner, 2001; Cochrane and Piazzesi, 2002; Gurkaynak, Sack, and Swanson, 2005; Gertler and Karadi, 2015; Abrahams, Adrian, Crump, Moench, and Yu, 2016; Crump, Eusepi, and Moench, 2017). Studies that focus on APP announcements on the euro area yield curve are Motto, Altavilla, and Carboni (2015); Lemke and Werner (2017).

The paper is structured as follows: Section 2 introduces our preferred benchmark model with a focus on modeling the time-varying ELB. Section 3 discusses estimation strategy. In Section 4 we present our main results with a focus on above defined criteria, i.e. (i) model fit and (ii) plausible short- and long-term rate expectations that can be used for policy analysis. Moreover, we compare our model estimates to alternative *DTSM* specifications and check our results in terms of robustness and the impact of modeling choice. Section 4.5.3 presents implications for the various estimation and model variants based on a Monte Carlo analysis using simulated yield curve data sets that are characterized by a protracted period in which the ELB is binding. Section 5 concludes.

2 Model

The class of SRTSMs introduces the concept of a (time-varying) effective lower bound, l_t , together with a shadow short rate, $si_{1,t}$. Similar to standard GATSMs, it is assumed that the pricing factors X_t follow a first-order Gaussian vector autoregressive process both under the risk-neutral (\mathbb{Q}) and the historical (\mathbb{P}) probability measure

$$X_t = \mu^{\mathbb{Q}} + \rho^{\mathbb{Q}} X_{t-1} + \Sigma u_t, \quad u_t \sim N(0, I) \quad (1)$$

$$X_t = \mu^{\mathbb{P}} + \rho^{\mathbb{P}} X_{t-1} + \Sigma u_t, \quad u_t \sim N(0, I). \quad (2)$$

The shadow short rate, $si_{1,t}$, is an affine function of the pricing factors and it holds

$$si_{1,t} = \delta_0 + \delta_1' X_t. \quad (3)$$

The short rate, $i_{1,t}$ is then described as maximum of either $si_{1,t}$ or l_t

$$i_{1,t} = \max(si_{1,t}, l_t). \quad (4)$$

By assumption, the short rate corresponds to the shadow short rate as long as the latter is above the lower bound. If, however, the shadow short rate falls below the lower bound, the short rate is constrained by the lower bound. This setup allows for the possibility that

the expected path of the short rate remains at this lower bound for an extended period of time, provided that the shadow short rate is expected to prevail below l_t .

Under the condition of no-arbitrage, the price of a zero-coupon bond with residual maturity n is defined as

$$P_{n,t} = E_t^{\mathbb{Q}} \left[\exp \left(- \sum_{i=0}^{n-1} i_{1,t+i} \right) \right] \quad (5)$$

and continuously compounded spot rates thus as

$$i_{n,t} = -n^{-1} \ln P_{n,t}. \quad (6)$$

Given the lower bound restriction, the mapping of pricing factors into interest rates is non-linear and in this case no closed-form solutions for bond prices exist. Therefore, we follow Wu and Xia (2017) who show that generally, implied one-period forward rates h periods ahead, $f_{h,t}$, can be expressed as

$$f_{h,t} \approx \int \left(l_{t+h} + \sigma_h^{\mathbb{Q}} g \left(\frac{sf_{h,t} - l_{t+h}}{\sigma_h^{\mathbb{Q}}} \right) \right) P_t^{\mathbb{Q}}(l_{t+h}) dx \quad (7)$$

where $g(x) = x\Phi(x) + \phi(x)$ with $\Phi(x)$ the standard normal cdf, $\phi(x)$ the standard normal pdf and $\sigma_h^{\mathbb{Q}}$ the conditional variance of future shadow short rates. The variable $sf_{h,t}$ is the shadow forward rate h -periods ahead. It is affine in the pricing factors with loadings \tilde{a}_h and \tilde{b}_h and computed as $f_{h,t} = \tilde{a}_h + \tilde{b}_h X_t$. Notice that in this general form, the forward rate is calculated as the average of future short rates with known l_{t+h} weighted by the risk-neutral probability of l_{t+h} .

With respect to the lower bound, we want to account for several stylized facts which can be observed for euro area OIS rates linked to the EONIA. One of these being that the latter can be considered as bound by the DFR.¹ However, it is important to note that the DFR does not necessarily constitute the ELB as typically the EONIA stays a few basis points away from the DFR even in times of very high excess liquidity.² Therefore, the ELB can be thought of as the sum of two elements, the DFR and the minimum spread between EONIA and the DFR. The DFR itself is subject to discrete changes over time as, e.g., documented by subsequent cuts into negative territory in the course of 2014-2016

¹Transactions underlying the computation of EONIA take place between counterparties that all have access to the deposit facility of the Eurosystem. Thus, they are expected to have no incentive to lend below that rate.

²In times without excess liquidity, EONIA closely follows the main refinancing rate set by the Eurosystem. Then, with increasing excess liquidity, however, EONIA moves away from that rate and non-linearly approaches the deposit facility rate offered by the Eurosystem (Deutsche Bundesbank, 2014).

which were to some extent expected as documented by survey evidence (see Lemke and Vladu, 2016). Finally, the dynamics of forward rates during this period hint to the fact that markets might have expected even further DFR cuts over and above the DFR cuts that were largely anticipated for the respective next Governing Council meeting.

To account for these features and to preserve an approximate analytical solution for bond prices, we specify the time-variation in the ELB in the following way:

$$l_{t+h} = \begin{cases} 0 & \text{if prior to ELB-period and } \forall h = 0, 1, 2, \dots \\ \gamma_t i_t^{DFR} + (1 - \gamma_t) i_{t+1}^{DFR} + sp_t & \text{if ELB-period and } h = 0 \\ \min(l_t, \bar{f}_t) & \text{if ELB-period and } \forall h = 1, 2, \dots \end{cases} \quad (8)$$

with $\bar{f}_t = \min(f_{t,h})$ for $h = [1, 2, \dots, N]$. In the period before reaching the ELB, we set the current and expected ELB to zero. Following Wu and Xia (2017), from then onwards, the current ELB, l_t , equals the weighted average of the DFR in period t and the expected DFR in period $t + 1$, which in our specification is treated as known in period t , where γ_t is the fraction of days between the end of month and the next Governing Council meeting in the following month. Moreover, in order to allow for further DFR cuts to be expected by agents in the following months, we approximate the expected ELB as the minimum of the current ELB and the minimum forward rate 1 to N periods ahead observed in period t . Notice that we do not explicitly model the forming of DFR expectations in an internally consistent way as in Wu and Xia (2017).³ However, we think that our modeling approach is a reasonable shortcut to produce a very good fit of the yield curve at shorter tenors during the ELB period and to be able to generate short rate paths that do not violate lower bound restrictions and are broadly in line with survey evidence (see Section 4 on this account). With this deterministic lower bound specification we follow Wu and Xia (2016). Equation 7 can then be approximated analytically as

$$f_{h,t} \approx l_{t+h} + \sigma_h^{\mathbb{Q}} g \left(\frac{sf_{h,t} - l_{t+h}}{\sigma_h^{\mathbb{Q}}} \right). \quad (9)$$

Further, as discussed above, the high persistence of yields which are only available in short samples for the euro area leaves the model with only little information about the true data generating process \mathbb{P} as well as the drift in far-distant short rate expectations. To possibly arrive at more precise estimates of the parameters under the \mathbb{P} -

³In order to preserve an approximative analytical solution, Wu and Xia (2017) specify $P_t^{\mathbb{Q}}(l_{t+h})$ within a regime-switching model in which the lower bound is modelled as two-state Markov chain to describe the persistence and the momentum of the policy lower bound and to allow agents to be forward-looking with respect to future lower bound changes that affect bond pricing.

measure, we link model-implied expectations to survey forecasts on short rate expectations as a further central feature of our model following Kim and Singleton (2012). Given the well known potential drawbacks that may come with incorporating survey forecasts, we add measurement errors when we align model-implied expectations with the corresponding survey forecasts.⁴ For any given survey interest rate forecast with residual maturity n in j -periods ahead, we add the following equation to our model set-up:

$$i_{n,t+j}^{survey} = E_t^{\mathbb{P}} [i_{n,t+j}] + e_{n,t}^{survey} \quad (10)$$

and $e_{n,t}^{survey}$ is the survey expectation measurement error.

3 Estimation

For estimation purposes, we cast our benchmark model $SRTSM_B$ in state space form with the transition equation given by Equation 2

$$X_t = \mu^{\mathbb{P}} + \rho^{\mathbb{P}} X_{t-1} + \Sigma u_t, \quad u_t \sim N(0, I). \quad (11)$$

The measurement equation takes the form of

$$\hat{Y}_t = Y_t + e_t \quad (12)$$

in which Y_t is the J -vector of model-implied interest rates with $Y_t = g(X_t, \mu^Q, \phi^Q, \Sigma, \delta_0, \delta_1, lb_t)$ and \hat{Y}_t corresponds to the J -vector of observed interest rates as well as survey forecasts adjusted for a vector of measurement errors e_t (for GATSMs it holds that $Y_t = A + B'X_t$). As the mapping between interest rates and pricing factors in the measurement equation is non-linear, the state space system needs to rely on a non-linear filter when maximizing the likelihood function. We choose to estimate the model by maximum likelihood based on the extended Kalman filter.⁵ With respect to the model identification, we closely follow Bauer and Rudebusch (2016) and estimate our model with $L = 3$ latent pricing

⁴First, as pointed out by Kim and Orphanides (2012), surveys report average expectations, while market prices are driven by marginal expectations on interest rates - a problem that might be exacerbated by relatively low numbers of participants compared to the number of participants in the market. A further explanation why survey-based expectations may only be an approximate reflection of market expectations may be the potential variation in the information available to participants and the point in time at which they submit their answers. Therefore, it can be assumed that the subjective expectations of survey participants deviate from the objective statistical expectations held under the \mathbb{P} -measure. Second, there might be incentives for survey participants to not reveal their true expectations, leaving surveys biased themselves, making them an inaccurate measure of participants' true expectations (Cochrane and Piazzesi, 2008; Chernov and Mueller, 2012)

⁵Alternative non-linear filters include the iterated extended as well as the unscented Kalman filter (Kim and Singleton, 2012; Priebisch, 2013; Krippner, 2015c).

factors based on the normalization of Joslin et al. (2011) with $\rho^{\mathbb{Q}} = \text{diag}(\rho_1^{\mathbb{Q}}, \rho_2^{\mathbb{Q}}, \rho_3^{\mathbb{Q}})$ and in Jordan form, $\mu^{\mathbb{Q}} = [k_{\infty}^{\mathbb{Q}}, 0, 0]'$, Σ is lower triangular and $\delta_0 = 0, \delta_1 = [1, 1, 1]'$.

In order to make the interpretation of latent pricing factors derived from our model easier, we can also transform the factors to an equivalent representation with new latent pricing factors P_t that resemble principal components in terms of level and dynamics along the procedure sketched out in Lemke and Vladu (2016).⁶ This transformation makes it possible to directly compare estimated parameters with those of estimated *GATSMs* based on principal components used as pricing factors. Therefore, we also report parameter estimates in terms of $\delta_{0,P}, \delta_{1,P}$ and μ_P, ρ_P, Σ_P both under the \mathbb{P} - and \mathbb{Q} -measure.

In our estimation, we use monthly overnight index swap (OIS) rates based on EONIA for the period January 1999 to October 2017 covering the maturities M in 1,3 and 6 months as well as 1,2,3,5,7 and 10 years. Hence, our yield curve data consists of $T = 226$ month for $J = 8$ maturities of interest rates. As these rates are reliably available only from July 2005 onwards, we follow Lemke and Vladu (2016) and augment our data set with spread adjusted zero-coupon rates based on Euribor swaps prior to 2005. Moreover, we follow the authors' specification of defining the ELB period from July 2012 onwards when the DFR hit the zero bound. We focus on the OIS term structure as in our view OIS interest rates represent the yield curve in the euro area with the tightest link to expected monetary policy actions priced in interest rates. First, it is risk-free in the sense that it does not carry sovereign credit risk the pricing of which might change over time and might distort the decomposition of interest rates. Second, as OIS rates are swap contracts in which cash flows are swapped, they do not serve as store of value and thus, should not be influenced by flight-to-safety and -liquidity investors to the same extent as sovereign bonds. And finally, the OIS curve is intrinsically linked to (one of) the monetary policy instrument(s) which the Eurosystem directly controls as one leg of the contract is associated to the EONIA path which usually closely follows the MRO or - in times of large excess liquidity - the DFR of the Eurosystem.

With respect to modeling the time-variation in the ELB, we specify Equation 8 the following way. First, as confirmed by survey and estimation evidence, the DFR cuts in June 14, December 15 and March 16 were largely expected by market participants while the cut in September largely came as a surprise (Lemke and Vladu, 2016; Wu and Xia, 2017). Therefore, we allow the current ELB, l_t , to already incorporate these DFR cuts in the respective months previous to their realization by weighting the DFR cut with the

⁶An affine transformation of the latent factors X_t to the pricing factors P_t implies that $P_t = AW + WBX_t$ where W is the weighting matrix which maps the set of observed yields into the first three principal components; A and B represent the affine loadings from an estimated *GATSM* based on Joslin et al. (2011). It then holds that $\mu_P = WB\mu - WB\rho(WB)^{-1}$, $\rho_P = WB\rho(WB)^{-1}$, $\Sigma_P\Sigma_P' = WB\Sigma\Sigma'(WB)'$, $\delta_{0,P} = \delta_0 - \rho'(WB)^{-1}WA$ and $\delta_{1,P} = ((WB)^{-1})'\rho$. See Joslin et al. (2011).

parameter γ_t . Second, for the dynamics of the ELB h -periods ahead, we choose l_{t+h} to be the minimum observed one-month forward rate in 1 to 24 months.⁷

With respect to the use of survey information, we rely on selected Consensus Economics interest rate forecasts of the 3-month Euribor in 12- and 24-months time (available quarterly and semi-annually). Moreover, we also add to the survey measurement equations the long-horizon forecast for the average 3-month Euribor in 6 to 10 years which is available on a quarterly basis since September 2016. Survey data up to the 2 year horizon are adjusted by the Euribor-OIS spread, respectively. We exclude other available survey information at very short horizons and intermediate horizons. We do so because survey information might only be biased approximations of model-implied expectations and we want to let the yield curve data speak for itself as much as possible on the parameters governing the \mathbb{P} -measure.

4 Results

4.1 Goodness of fit

Overall, our benchmark model ($SRTSM_B$) performs well in terms of model fit (see Table 1, parameter estimates are reported in Table A.1). The mean absolute fitting error (MEA) of yields over the complete sample is 2.8 basis points and 20.3 basis points for surveys which is comparable in size with other SRTSM estimates including survey information (see Priebisch, 2017, for US results). Notice that during the ELB period, both the yield and survey fit improves. The MAE for the one-month rate is 1 basis points and the fit of short-term surveys is between 6 to 10 basis points while long-term surveys range at 25 basis points.

This good average model fit is largely confirmed when depicting the model-implied yield curve at selected dates and comparing it to observed yields (see Figure 1). However, during the ELB-period the model fit somewhat varies depending on the specific observation dates. For instance, in October 2012 and July 2015, when the short-end of the yield curve was very flat, our model is able to replicate this feature to a very good extent. In February 2016, when market participants were broadly expecting a further DFR cut, our model is able to replicate a downward sloping yield curve, but delivers higher fitting errors up to the 2-years maturity horizon.

Our analyses show that with respect to the short rate, already small fitting errors may generate an economically significant impact on the expected and most likely short rate path and, thus, on assessing monetary policy expectations. Therefore, fitting the short

⁷ $\bar{f}_t = \min(f_{t,h})$ for $h = [1, 2, \dots, 24]$ months.

Table 1: In-sample model fit of yields and survey forecasts

maturity in months	1	3	6	12	24	36	60	84	120	avg
yields										
total sample:	4	3	3	3	2	3	3	2	3	3
pre-ELB sample:	5	4	3	4	3	3	3	1	3	3
ELB sample:	1	2	2	1	1	2	3	2	2	2
expected 3-month rate in months	12	24	60 – 120							
surveys										
total sample:	12	26	19							
pre-ELB sample:	15	32	–							
ELB sample:	6	10	25							

Note: This table shows the mean absolute errors (MAE) of model-implied yields and short rate expectations compared to observed yields and survey forecasts for selected sample periods in basis points. The total sample covers the period January 1999 to October 2017 while the pre-ELB sample covers the period January 1999 to June 2012 and the ELB sample the period July 2012 to October 2017.

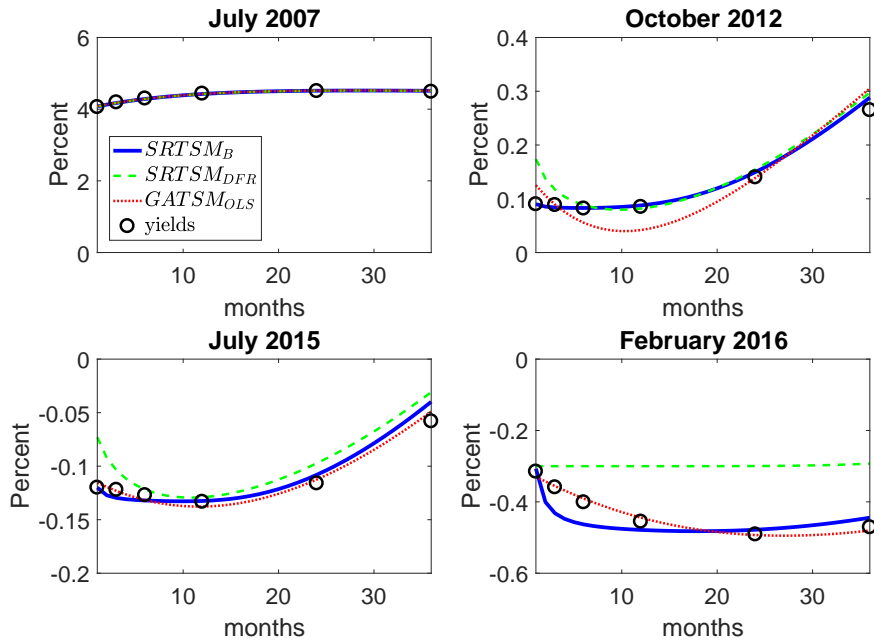
rate is important when evaluating the future short rate distribution over time. In order to do that, we explicitly allow the current spread between EONIA and the DFR in addition to expected DFR shifts to enter the ELB in l_t . This leads to a very good model fit of the short rate during the ELB-period (see Figure 2). Closely related, our model implies a shadow short rate which is less prone to other modeling specifications. This finding is again mostly related to the incorporation of the spread into the ELB definition which ensures that the ELB is binding for the model-implied short rate during the ELB-period by construction. This modeling strategy, thus, makes the timing of when the shadow short rate first moves below the ELB insensitive to other modeling specifications which may affect the dynamics of the pricing factors (see Figure A.1).⁸

4.2 Model-implied short-term rate expectations

We start our analysis on model-implied interest rate expectations by decomposing forward rates into short rate expectations as well as forward premia for selected short-term and long-term maturities (panel(a) of Figure 3) based on our benchmark model. At the 1Y1Y

⁸For a detailed discussion on the impact of model specification on the derivation of a shadow short rate see Krippner (2015a).

Figure 1: Yield curve model fit at selected dates

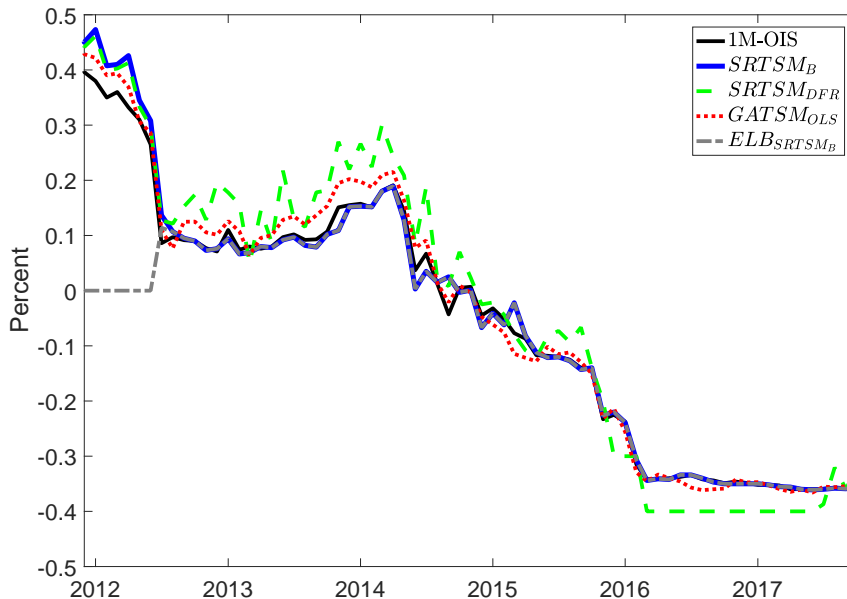


Note: This figure plots the model-implied yield curve based on various term structure model specifications including $SRTSM_B$, $SRTSM_{DFR}$ and $GATSM$ up to 3 years based on selected dates.

forward horizon (see panel a), most of the variation in forward rates stems from changes in short rate expectations. Prior to the ELB-period, forward premia ranged between 0 and 1%. Notice that forward premia turned slightly negative by mid 2011 and remained remarkably anchored at this level from 2012 onwards. The prominent role of short rate expectations can also be identified when conducting a variance decomposition for the variation in the level and the change of the 1Y1Y rate. As shown in Table A.2, about 88% of the variation in the level is due to the expectations component over the total sample. During the ELB-period, it even accounts for over 110% of the variation in the monthly change of the 1Y1Y forward rate.

To add to this finding, we depict 1-month forward premia for the 1, 3, 6-months as well as 1 and 2-year horizon (panel (b) of Figure 3). For comparability, forward premia are scaled to unit per month and reported in basis points. The figure shows that after turning negative in 2011, term premia for shorter maturities reduced to 0 where they have stuck since with the DFR cut to zero. Simultaneously, forward premia up to the one and two year horizon have stayed slightly negative with very low volatility compared to the time before 2011. These model-implied results can be seen against the background of a deterioration of the macroeconomic outlook with severe downside risks to price stability

Figure 2: Model fit of the short rate



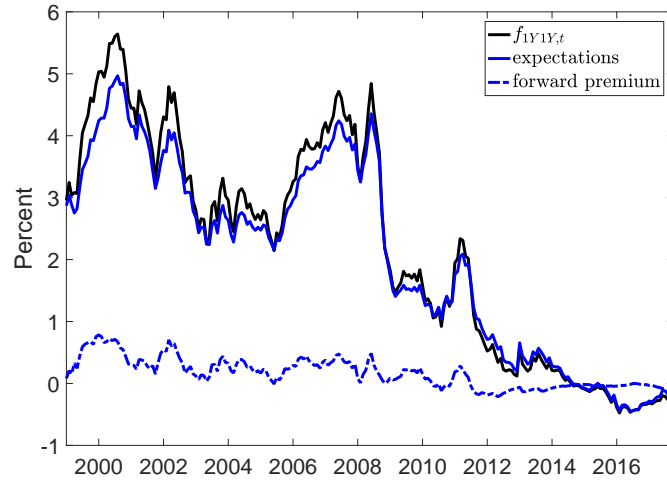
Note: This figure plots the model-implied short rate based on various term structure model specifications including $SRTSM_B$, $SRTSM_{DFR}$ and $GATSM$ together with the effective lower bound (ELB) of the $SRTSM_B$ model.

and an increasing probability of a deflationary scenario. The Eurosystem responded to these risks by introducing NIRP, strengthening its policy rate forward guidance as well as preparing and implementing its various asset purchase programmes. In this context, model-implied forward premia for shorter horizons show that the Eurosystem was able to anchor short-term interest rate expectations extremely well. Moreover, our results also seem to suggest that in addition to policy rate forward guidance which has been in place already since July 2013⁹, signalling its willingness to dive deeper into non-standard monetary policy measures has been also important to steer short-term rate expectations and to reduce interest rate uncertainty priced in forward premia. In this respect, our results also emphasize the signaling channel of non-standard monetary policy measures including asset purchases which affect both short rate expectations as well as risk compensation demanded by market participants (see Bauer and Rudebusch, 2014, for US evidence).

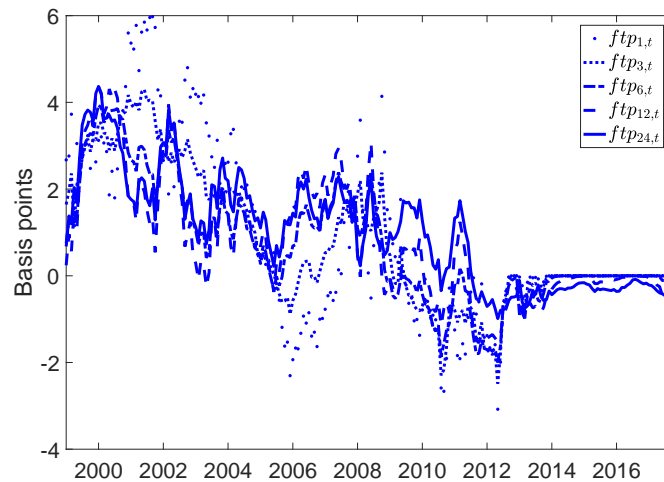
⁹The Eurosystem’s Governing Council introduced its interest rate forward guidance in Juli 2013 by expressing its expectations that “key interest rates will remain at present or lower levels for an extended period of time”. In June 2014, the governing council decided to delete the word “lower” from its forward guidance. This was only reintroduced when the council decided to link its interest rate forward guidance to its expanded asset purchase programme (APP) by stating the expectation that “the Governing Council expects the key ECB interest rates to remain at present or lower levels for an extended period of time, and well past the horizon of our net asset purchases.”

Figure 3: **Short-term forward rate decomposition**

(a) 1Y1Y forward rate



(b) (Normalized) short-term forward premia



Note: Panel (a) plots the time-series of the decomposition of the 1y1y forward rate. Panel (b) plots the time-series of normalized 1-month forward premia at the 1, 3, 6, 12 and 24 months horizons. Forward premia are normalized by maturity in months. End-of-month values for January 1999 to October 2017.

Given the asymmetry of the distribution of future short rates during the ELB-period, our model also accounts for the wedge between the *mean* and the *mode*, i.e. the most likely, future short rate path which is eminent at short- and medium-term horizons where the ELB implies a truncated distribution. The wedge between these two statistical numbers is

important when assessing monetary policy expectations that are priced in the yield curve. The bigger the wedge the tighter the ELB constraint binds for the yield curve (Swanson and Williams, 2014; Bauer and Rudebusch, 2016).¹⁰ To illustrate this point, we plot the dynamics of the mean and the mode of the future short rate for a fixed-horizon forecast in March 2019 together with the corresponding forward rate path and the expected ELB (panel (a) of Figure 4). The figure highlights the bias when relying on the forward rate or the expected short rate path during the ELB period. First, the forward rate path is biased due to the existence of substantial time-variation in forward premia. Second, due to the asymmetry, the expected short rate path shows a constant upward bias. Correct inference with regard to monetary policy expectations can only be drawn from the modal path of the short rate which represents the optimal forecast under absolute error loss (Bauer and Rudebusch, 2016). Closely related, the model can inform on the median of the lift-off distribution of the short rate which measures the time at which the short rate is likely to cross a certain threshold level.¹¹ Panel (b) of Figure 4 depicts the lift-off horizon based on the modal path as well as the lift-off distribution for a specification that gives an idea about the point in time at which market participants regard a first 10 bp rate hike as most likely. Both indicators move fairly close to each other. With the transition to the ELB-period, the crossing-time kept moving further out constantly. For instance, in summer 2016, market participants did not believe to see a first 10 bp rate hike before 2020. Since then and going forward in time, both indicators signaled a gradual reduction in the number of months until a first DFR hike is regarded as most likely. For the end of the sample in October 2017, this assessment implies a first DFR hike in the summer of 2019.

4.3 Model-implied longer-term rate expectations

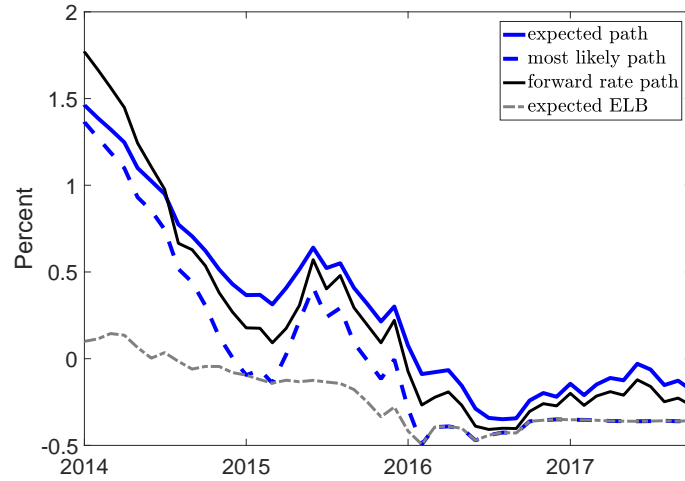
In this Section, we turn to the implications of model-implied intermediate and long-term expectations as well as term premia based on our benchmark model. To start with, in Table 3 we report summary statistics for the (shadow) short rate based on the \mathbb{P} measure. The model is estimated with an unconditional mean of 3.98% and a fairly high

¹⁰In line with Bauer and Rudebusch (2016), we define the *mode* of the short rate path as $\max(E_t[s_{1,t+i}], lb_{t+i})$

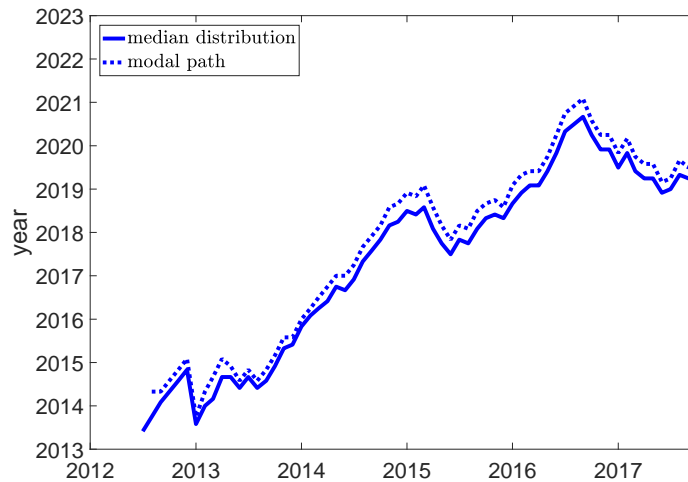
¹¹The lift-off distribution is calculated by simulating a large number of short rate paths under the \mathbb{P} measure and then saving the future horizon at which each single path rises above a certain threshold. While determining these future horizons, it can be accounted for the fact that some paths due to shocks cross the threshold, but then may again fall back below. This is done by requiring a path to stay above the threshold to be chosen for a certain amount of time, e.g. 12 months. This way it is ensured that the inspected path has really lifted off. Ideally, the median of that distribution corresponds to the future point in time at which the modal path crosses the threshold, but it might deviate if enough paths fall back below the threshold too quickly after lifting off for the first time (Bauer and Rudebusch, 2016)

Figure 4: **Distribution of short rates**

(a) Future short rate in March 2019



(b) First +10 BP DFR hike



Note: Panel (a) plots the model-implied dynamics of the expected and most likely path of the short rate together with the forward rate for a fixed horizon in March 2019 based on the $SRTSM_B$ model. Panel (b) plots the timing of the first DFR hike by +10 bp based on the short rate distribution and the modal path of the short rate. End-of-month values for January 1999 to October 2017.

persistence of the pricing factor process of the transition matrix which is expressed by a largest eigenvalue of 0.992 in $\phi^{\mathbb{P}}$. Indeed, a shock to the most persistent pricing factor has a half-life of roughly 7 years. Although our model implies that the short rate will converge

to a constant in the very long run, according to the short rate summary statistics, our model also implies a substantial time-variation of far-distant short rate expectations up to the 10 year horizon.

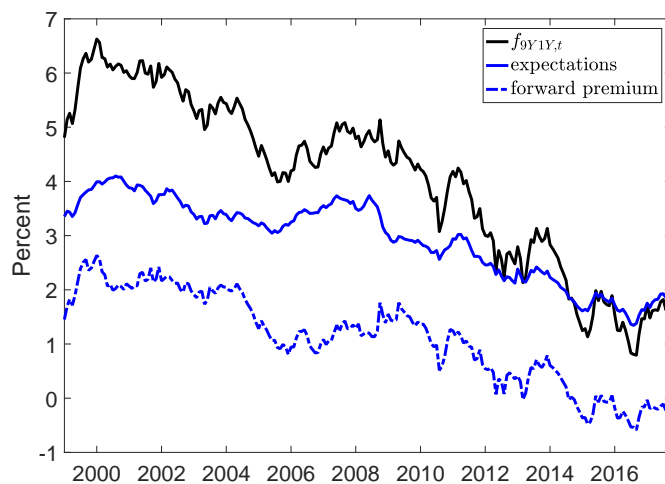
To see this, we plot the 9Y1Y forward rate together with its decomposition into the expected short rate and forward premium component in panel (a) of Figure 5. A high degree of the variability in forward rates can be attributed to the forward premium which exhibits a marked decline over the sample period from close but below 2% into negative territory, standing at about -23 bp at the end of the sample. In particular, a first large drop can be observed in the wake of the Greenspan conundrum between June 2004 and June 2006. Following a short upward movement it then began to follow a lasting downward trend after the outbreak of the financial crisis in 2008. The market's anticipation of widespread asset purchases since the beginning of 2014 then triggered another sharp drop leading the premium into negative territory where it has remained since, although its downward trend came to a halt. This time-variation of forward premia is also reflected at more intermediate horizons (see panel (b) of Figure 5). Notice that at these intermediate to long-term maturities, forward premia co-move more linearly in maturity than at shorter maturities (see panel (b) of Figure 3). However, the decline in the long-term forward rate reflects also the time variation in far-distant short rate expectations which have trended downwards since the height of the financial and economic crisis in 2008.

A variance decomposition for the 9Y1Y forward rate confirms that over the total sample roughly 53% of the variation in the level of the 9Y1Y rate is due to the forward premium component (see Table A.3). In the ELB period, the share of the forward premium variation increases to roughly 56%. In terms of variation in the change in the forward rate, 69% can be attributed to the change in forward premia highlighting their prominent role at longer tenures. At the same time, these numbers imply that the variation of long-term forward rates in terms of level and change is explained by the expectations component, too. Importantly, this also holds true in the run-up to the decisions of the Eurosystem to implement large-scale asset purchases that had been increasingly anticipated since summer 2014. Indeed, our model suggests that roughly one half of the observed decline of the 10Y-OIS rate from September 2014 to March 2015 can be explained by changes in the average path of the expected short rate over the 10-year horizon. This stands in contrast to the findings of Lemke and Werner (2017) who find that almost all of the long-term yield decline during this period was due to the decline in the term premium within the portfolio rebalancing channel.

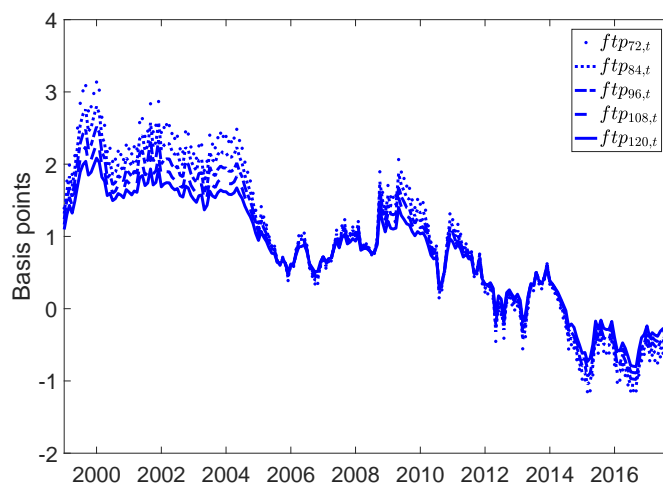
In order to pass judgment on the economic plausibility of the level and the variability of the expected short-term interest rates in intermediate and long-term forward rates (and therefore also on the forward premia), we compare the expectations component with an

Figure 5: Long-term forward rate decomposition

(a) 9Y1Y forward rate



(b) (Normalized) long-term forward premia

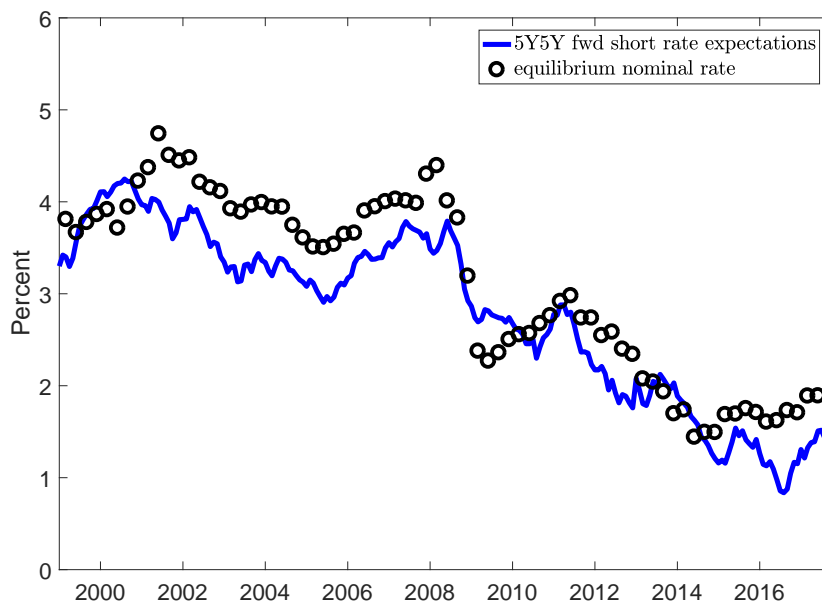


Note: Panel (a) plots the time-series of the decomposition of the 1y1y forward rate. Panel (b) plots the time-series of normalized 1-month forward premia at the 6, 8, 8, 9 and 10-year horizons. Forward premia are normalized by maturity in months. End-of-month values for January 1999 to October 2017.

estimated equilibrium nominal short-term interest rate derived from a macroeconomic model. Interest rate expectations contained in financial market prices at the long end of the term structure should position themselves at this level if it is assumed that the term structure reflects macroeconomic information, particularly with regard to long-term

inflation expectations and the equilibrium real interest rate. The latter is determined by estimating a natural rate of interest which is consistent with a permanently closed output gap and a stable inflation rate in the medium to longer term, after the economy overcomes from all cyclical fluctuations.¹²

Figure 6: **5Y5Y short rate expectations and longer-run equilibrium nominal rate**



Note: This figure plots the time-series of the $SRTSM_B$ model-implied average short rate expectations in 5 to 10 years together with an estimate of the longer-run nominal equilibrium rate based on Holston et al. (2017). The equilibrium nominal rate is derived by adjusting the estimated longer-run real equilibrium rate and adding longer-run inflation expectations based on Consensus forecasts.

Indeed, 5Y5Y interest rate expectations derived from our benchmark model capture the level and path of the nominal natural interest rate quite well. In this period, the latter is primarily driven by the real natural interest rate path while simultaneously longer-term inflation expectations are rather stable. This observation is interesting as the two models do not share any information in the estimations. While $SRTSM_B$ solely contains term

¹²We exemplarily choose the real natural rate estimate based on Holston et al. (2017) and add medium- to long-term inflation expectations based on Consensus forecasts to present the rate in nominal terms. The maturity perspective of the derived natural rate of interest in this model estimation is not explicitly defined, but refers to a longer-term perspective due to the modeling strategy and the definition of the latent variable and shock processes: “Our definition takes a ‘longer-run’ perspective, in that it refers to the level of real interest rates expected to prevail, say, five to ten years in the future, after the economy has emerged from any cyclical fluctuations and is expanding at its trend rate.” (Laubach and Williams, 2016).

structure information, the macroeconomic model only takes the inflation rate, the level of Gross Domestic Product and the ex-ante short-term real interest rate into consideration. Long-term forward rates thus appear to reflect trends in key macroeconomic variables in both real and nominal terms, which play an important role in the formation of far-distant rate expectations (see also Bauer and Rudebusch, 2017; Crump et al., 2017; Cieslak and Povala, 2015; Dijk, Koopman, Wel, and Wright, 2014) on this assessment).

We also compare our intermediate and far-distant forward rate decomposition (5Y5Y fwd) to US estimates based on Kim and Wright (2005) who also incorporate survey information into their term structure model. As shown in Figure A.2, until the beginning of 2013 the expectations component in US and euro area 5y5y forward rates is similar both in terms of level and variation. The high co-movement in US and euro area forward rates during this period of time is also to a significant extent related to US and euro area forward premia.¹³ From summer 2013 onwards however, initiated by the US taper tantrum and followed by a deteriorating economic and inflation outlook as well as increasing expectations of large-scale asset purchases in the euro area, US and euro area far-distant short rate expectations decoupled with the latter significantly falling. The same holds for the dynamics of the euro area forward premium which was much more depressed than its US counterpart in the run-up to the APP decision in January 2015.¹⁴

Finally, we check to what extent the asymmetry of the short rate distribution also matters for long-term interest rates. By comparing interest rates and shadow interest rates both under the \mathbb{Q} - and \mathbb{P} -measure, it is possible to compute a measure of the degree the time-varying ELB exerts influence at the long-end of the term structure of interest rates. Indeed, our findings suggest that it does so, in particular since the beginning of 2014 (Figure A.3). The ELB wedge widened not only under the \mathbb{Q} -measure but also under the \mathbb{P} -measure, though not to the same quantitative extent. A somewhat more nuanced picture can be observed for far-distant forward rate (Figure A.4). While under the \mathbb{Q} -measure the ELB wedge widened up since 2014, the mean and the mode of the short rate under the \mathbb{P} -measure at the 10Y1M-horizon is essentially identical.

4.4 Assessing the impact of monetary policy

To provide evidence on how monetary policy influences the various components of the yield curve, we investigate how forward rates, short rate expectations and term premia respond to monetary policy shocks within our model. As these shocks are not directly

¹³Indeed correlation coefficients during this period are 0.9 and 0.8 for the expectations component and the forward premium.

¹⁴Correlation coefficients declined to 0.4 and 0.6 for the expectations component and the forward premium.

observable, a viable workaround is to assume that high frequency changes of selected interest rates around monetary policy announcement dates are reliable observable proxies for monetary policy shocks which can then be used to study the response of interest rates (Kuttner, 2001; Cochrane and Piazzesi, 2002; Gurkaynak et al., 2005; Piazzesi and Swanson, 2008; Nakamura and Steinsson, 2018, among others). Recent applications to estimated term structure models are Abrahams et al. (2016); Crump et al. (2017).

However, this identification strategy may run the risk only capturing part of the underlying monetary policy shock and they may be measured with error. It therefore typically requires tight windows around monetary policy announcements in order to reduce endogeneity and noise concerns (Gurkaynak et al., 2005; Nakamura and Steinsson, 2018). Moreover, the literature implicitly assumes that the reaction of interest rates to monetary policy (and other) shocks is constant over time by applying linear regression techniques. Our benchmark model challenges this assumption as it convincingly shows that interest rates are actually non-linear functions of the pricing factors and the reaction of interest rates to innovations in the pricing factors crucially depends on the degree to which the ELB is binding along the yield curve at a given point in time (ELB wedge).

To alleviate these concerns, we treat these high frequency changes in interest rates as instrument variables and not as directly observable monetary policy shocks. This approach has been applied in the macroeconomic proxy structural VARs literature that aims to identify the dynamic causal effects of various macroeconomic shocks (Stock and Watson, 2012; Mertens and Ravn, 2012; Gertler and Karadi, 2015).

Following standard terminology, we assume that the L reduced-form innovations u_t of the pricing equation 11 are L linear combinations of structural shocks. Therefore, it holds that

$$u_t = H\epsilon_t = [H_1, \dots, H_L] (\epsilon_{1,t}, \dots, \epsilon_{L,t})' \quad (13)$$

where H_1 is the first column of H and $\epsilon_{1,t}$ is the first structural shock. With $\Omega = \Sigma\Sigma'$, it also holds that $\Omega = H\Omega_\epsilon H'$. Given invertibility of the system, structural shocks can be expressed as linear combinations of reduced-form innovations

$$\epsilon_t = H^{-1}u_t. \quad (14)$$

As discussed in Stock and Watson (2012, 2018), structural shocks and hence H can be recovered by means of a predictive regression of the relevant instrument z_t on the innovations u_t up to scale and sign. The scale and sign of the structural shock, say $\epsilon_{1,t}$ and H_1 , are set by normalizing the shock to have a unit current impact on a specific pricing factor. Most importantly, while the link between the instrument and the innovations

remains linear, the instrument approach allows us to model the reaction of yields and forward rates in a non-linear way in line with Equation 7. Thus, monetary policy shocks may exhibit a different impact on the yield and forward curve at a given point in time depending on the size of the ELB wedge.

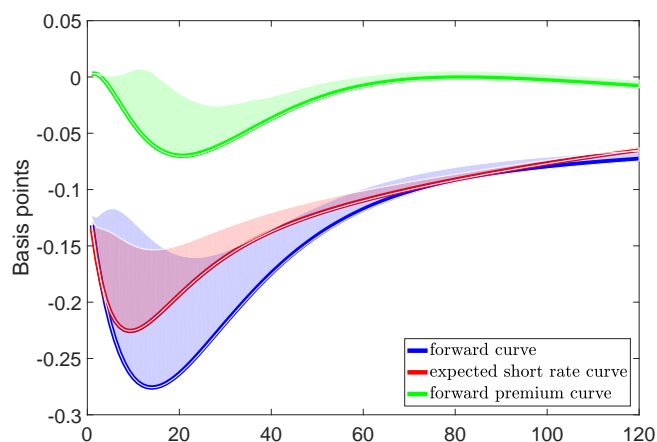
In the following we identify conventional (CMP) and unconventional monetary policy (UMP) shocks based on the instrument data set of Mandler and Scharnagl (2018). They XX. In order to derive economically interpretable impulse response functions based on these shocks, we rotate our benchmark model $SRTSM_B$ as described in Section 3. Specially, we transform the three latent factors in a way that they resemble the 1M, 2Y and 10Y (shadow) rate. We then normalize the CMP shock in sign and size so that a 10 basis point change in this shock implies an equally large change in the (shadow) short rate. An UMP shock is normalized so that on impact this shock triggers a change in the 10Y-shadow rate by the same amount.

Results for the instantaneous response of the various components of the forward curve to an expansionary conventional monetary policy shock during the period January 1999 to June 2014 are provided in panel (a) of Figure 7. Notice that the figure does not depict uncertainty around the impulse responses. It merely shows the distribution of impulse responses to monetary policy shocks at different points in time and highlights the asymmetry of responses depending on the strength of the binding character of the ELB. The nominal forward curve exhibits the largest response at the 1- to 2-year maturity horizon with a negative reaction even at very long-term maturities. Hence, our model implies a very high persistence of conventional monetary policy shocks along the forward curve. Interestingly, at maturities up to 2 years, the decline in the forward curve is due to both changes in the expected short rate and forward premia with the former dominating the overall effect. At longer maturities, the effect on the forward curve can be attributed almost entirely to the expectations component (see also Nakamura and Steinsson (2018) for a similar result based on US data). The figure also illustrates the increasing binding character of the ELB between mid-2012 and mid-2014 which is expressed by the muted response of the forward rate components at the 85%-percentile.

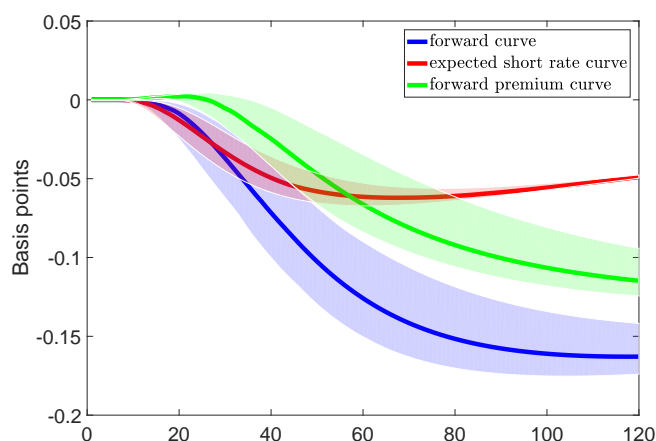
Panel (b) of Figure 7 depicts the corresponding instantaneous response of the forward curve together with the expected short rate and forward premium curve between July 2014 and October 2017 to UMP shocks. The median reaction at the long end is negative and spills over to medium-term maturities. Up to the 2-year horizon, however, there is essentially no reaction as rates are stuck at the ELB. The largest impact on the forward curve stems from the forward premium at the 10-year maturity horizon emphasizing the transmission of non-standard measures through duration extraction. At medium-term maturities our model attributes a more prominent role to the expectations component.

Figure 7: **Instantaneous response to monetary policy shocks**

(a) Conventional MP shock (01/99-06/14)



(b) Unconventional MP shock (07/14-10/17)



Note: Panel (a) plots the median instantaneous response of the forward curve and its components with [15% – 85%]-quantiles to an conventional monetary policy shock (CMP) for the sample period January 1999 to June 2015. Panel (b) plots the median instantaneous response of the forward curve and its components with [15% – 85%]-quantiles to an unconventional monetary policy shock (UMP) for the sample period July 2014 to October 2017.

However, even at very long-term maturities, the expected short rate falls in reaction to an unconventional monetary policy shock. Therefore, our model also highlights the signaling channel of non-standard monetary policy measures.

In Table 2, we perform a historical decomposition of the 10Y-OIS rate for various sample periods in order to assess the contribution of UMP shocks to the change of this

rate. Between June 2014 and the start of asset purchases in March 2015, the rate dropped by 0.77% which is attributed almost entirely to UMP shocks according to our model estimates. Thereby, both the term premium and the expectations component contributed to this decline. In contrast, between March 2015 and September 2016, more than two thirds of the observed decline in the 10Y-OIS rate stems from non-identified shocks affecting mainly the expectations component. From September 2016 onwards, again UMP and others shocks contribute to the rise of the 10Y-OIS rate by 0.6%.

Table 2: **Contribution of unconventional monetary policy shocks to change in interest rates**

10Y-OIS rate		total	expectations	term premium
07/14 – 03/15:	total	-77	-40	-37
	UMP shock	-72	-34	-38
	other	-5	-6	1
03/15 – 09/16:	total	-33	-32	-1
	UMP shock	-10	-11	1
	other	-23	-21	-2
09/16 – 10/17:	total	62	51	11
	UMP shock	32	8	24
	other	31	43	-13

Note: This table shows the contribution of unconventional monetary policy shocks to the change in the 10Y-OIS rate for selected sample periods based on the $SRTSM_B$ model and unconventional monetary policy (UMP) shocks identified with external instruments.

Finally, we take a closer at the shadow short rate and analyze to what extent its dynamics are related to UMP shocks (Figure A.5). It turns out that albeit these shocks increasingly affected the shadow short rate throughout 2015 and the end of 2016 given the high persistence of UMP shocks, much of the variation stems from other, non-identified shocks. Therefore, its move into deep negative territory should not be interpreted as a pure reflection of a sequence of UMP shocks.

4.5 Specification analysis and robustness of model-implied rate expectations

4.5.1 In-sample fit

In this Section, we compare the results of our benchmark model to those of alternative modeling specifications. We run estimations of further DTSMs including GATSMs and SRTSMs that do or do not account for a time-varying ELB or survey information. In particular, we estimate two additional *SRTSM* specifications, one in which the ELB equals the DFR (*SRTSM_{DFR}*) and one in which we implement the same ELB setup as in our benchmark model but in which we exclude survey information (*SRTSM_{woS}*). In addition we estimate three GATSM model variants (*GATSM_{OLS}*, *GATSM_S*, *GATSM_{BC}*) based on Joslin et al. (2011) which differ with respect to the use of surveys and with respect to the application of bias correction to the parameters under the \mathbb{P} -measure in line with (Bauer et al., 2012).

We start by comparing the overall in-sample model fit. As shown in Table A.4, all models generate a similar average model fit, ranging between 2 and 3 basis points based on the mean absolute error. As a result, there is no model specification that performs significantly better in terms of average model fit. However, the comparison of model-implied yield curves with observed yields at selected dates reveals noticeable differences across models (see Figure 1).¹⁵ The following observations stand out: Prior to the ELB-period, all inspected models generate a similar fit of the yield curve. However, this changes with the beginning of the ELB-period. Both *SRTSM_B* and *GATSM_{OLS}* fit the observed data during this period slightly better than *SRTSM_{DFR}*.¹⁶ We show this exemplary for February 2016. At this time, market participants were broadly expecting a further DFR cut at the next meeting of the ECB’s governing council. Given their downward flexibility, both models are able to fit the negative slope of the yield curve. While in *GATSM_{OLS}* this flexibility is ensured by the absence of a lower bound, in *SRTSM_B* accounting for expected ELB shift is crucial to generate a satisfying yield curve fit. In contrast, a specification that does not account for expected DFR shifts as in *SRTSM_{DFR}* fails to re-produce a downward sloping forward curve which trades below the current DFR. This shortcoming has important implications for the distribution of short rates and yield curve decompositions, a finding we will discuss later in Subsection 4.5.2. While the look at the yield curve in February 2016 shows that *GATSM_{OLS}* is best capable of fitting the downward sloping yield curve at the lower bound, *SRTSM_B* plays out its strengths vis-a-vis *GATSM* whenever the short end of the yield curve is flat over an extended period

¹⁵For readability, we do not show the model variants *GATSM_S*, *GATSM_{BC}* and *SRTSM_{woS}* in Figure 1.

¹⁶This in line with findings by (Kortela, 2016; Wu and Xia, 2017)

of time (see October 2012).

4.5.2 Short-rate summary statistics and rate expectations

Comparing the implications of different model specifications for short rate summary statistics, the most notable difference is related to the model-implied unconditional mean of the short rate (see Table 3). While the estimated models without surveys ($SRTMS_{woS}$, $GATSM_{OLS}$, $GATSM_{BC}$) generate an unconditional mean between 0.79 and 1.78, the models with surveys ($SRTSM_B$, $SRTSM_{DFR}$ and $GATSM_S$) imply values between 3.67 and 4.38 for the short rate. Clearly, the inclusion of surveys leads to markedly higher levels of far-distant short rate expectations. To partly overcome the shortcoming of a very low unconditional mean in a data sample that is characterized by a prolonged period of low interest rates such as the one considered in this paper, the pricing factors could also be de-measured as in Adrian et al. (2013). Alternatively, it could be specified that the unconditional mean of the pricing factors $E^{\mathbb{P}}[X_t]$ must equal their sample mean (Bauer et al., 2012). Both approaches ensure that the unconditional mean of the short rate $E^{\mathbb{P}}[i_{1,t}]$ matches its sample mean thereby partly alleviating the small sample problem with respect to the level of far distant expected short rates (see the result for $GATMS_{OLS}$ in brackets as well as $GATMS_{BC}$). Still, based on the short rate summary statistics, far-distant short rate expectations are lower compared to survey-based estimations. Including an ELB-specification, in contrast, does not result in a clear difference with respect to the unconditional mean. While $SRTSM_B$ produces the highest unconditional mean, the second highest level can be found in $GATSM_S$ followed by $SRTSM_{DFR}$ with the DFR as ELB specification.

Turning to the mean reversion characteristics of the pricing factors with its implications for the persistence of the short rate process, interestingly, all estimated models produce a rather slow mean reversion so that far-distant short rate expectations react to shocks to the pricing factors to a significant extent. The maximum eigenvalue of the matrix $\rho^{\mathbb{P}}$ in all model variants is larger than 0.99. $SRTSM_B$ and $SRTSM_{woS}$ imply the lowest half life of the most persistent factor process with 6.3 years. In contrast to the findings of Kim and Priebsch (2013), our estimated $GATSMs$ exhibit an even higher persistence of the short rate process. The half life of a shock to the most persistent pricing factor for the the non-bias corrected $GATSM$ variants is between 11.6 and 10.3 years although the models are estimated over the complete ELB-period. Also, the inclusion of short- and long-term interest rate survey information as in $GATSM_S$ does not change this result. Our findings indicate that with respect to the considered euro area yield curve sample, estimated DTSMs always produce a very high persistence of the short rate process under the \mathbb{P} -measure. Therefore, the estimated difference in the persistence of the pricing

Table 3: (Shadow) short rate summary statistics – \mathbb{P} -estimates

model	$SRTSM_B$	$SRTSM_{DFR}$	$SRTSM_{woS}$
unconditional mean $E^{\mathbb{P}}i_1$:	3.976	3.672	0.743
eigenvalues under \mathbb{P} -measure:	0.992	0.991	0.990
	0.912	0.913	0.895
	0.834	0.860	0.895
half life in years:	6.92	7.00	6.33
model	$GATSM_{OLS}$	$GATSM_S$	$GATSM_{BC}$
unconditional mean $E^{\mathbb{P}}i_1$:	-0.789 (1.778)	4.384	1.778
eigenvalues under \mathbb{P} -measure:	0.996	0.994	0.999
	0.917	0.934	0.918
	0.917	0.812	0.918
half life in years:	11.58	10.33	99.50
sample mean (i_1): 1.78			

factors between non-biased and biased-corrected estimates up to the 10 year horizon are not substantially large which stands in contrast to US evidence Bauer, Rudebusch, and Wu (2014); Wright (2014).

We now turn to the derivation of model-implied near- and far-distant short rate expectations. We start with short-term horizons and check whether the inclusion of an ELB specification has an important impact on the behavior of the short rate path at short-term horizons. Assuming our ELB specification in $SRTSM_B$ to be a reasonable approximation of the true ELB, we first check the number of ELB violations by counting the number of months in which the expected short rate path falls below the (expected) ELB, l_{t+h} , for the various model variants (Table A.5). While ELB violations are excluded by construction in $SRTSM_B$ and $SRTSM_{woS}$, in $SRTSM_{DFR}$ few violations occur in periods in which the DFR is a binding restriction for the short rate while being below the ELB (mainly as there exists a positive spread between the DFR and the short rate). Obviously, all $GATSM$ fail to respect the ELB restrictions observed in the data during the ELB period. The violations in these models amount between 21 and 52 months.

The importance of specifying a DTSM for the euro areas as a $SRTSM$ with an ELB specification can also be highlighted when assessing near-term monetary policy rate expectations. In Figure A.6, we simulate the median lift-off distribution of a +10 BP DFR hike

for the various model variants.¹⁷ Clearly, $GATSMs$ produce a wide spectrum of results. On the one hand, the lift-off measure based on $GATSM_{BC}$ turns out to be highly volatile with month-to-month changes amounting to several years which seems rather unreliable. On the other hand, $GATSM_S$ produces almost no variation in the lift-off measure at all. The $GATSM_{OLS}$ comes out in between those two extreme results, still offering a rather high amount of variation, reacting strongly to movements in interest rates. Interestingly, since fall 2016 variation disappears, so that the model persistently predicts a DFR hike of 10 BP within about half a year.

Compared to $GATSM_{BC}$, but also to $GATSM_{OLS}$, the models $SRTSM_B$, $SRTSM_{DFR}$ and $SRTSM_{woS}$ all produce less volatile lift-off series which are very similar in terms of dynamics, but reveal larger differences in terms of level. The results suggest that survey information on the one hand reduces the degree of stickiness of the short rate at the lower bound in times when forward rates as well as DFR expectations were tilted to the downside. On the other hand this additional information also dampens the reaction of short rate expectations to large swings in interest rates as observed during the Bund tantrum in the beginning of 2015 or in the wake of the global hike in rates in Fall 2016. With respect to the ELB specification and associated fitting errors of the model-implied short rate, both features have a pronounced impact on the median distribution of the most likely short rate path (see the simulation results for $SRTSM_{DFR}$).

Regarding long-term rate expectations, Figure A.7 depicts the 10Y1M expected short rate of our benchmark model (modal path) together with estimated confidence interval bands based on parameter estimation and current state filter uncertainty.¹⁸ All survey-based models lie within the confidence interval bands of $SRTSM_B$, so that we conclude that the results for long-term rate expectation are robust to model specification and economically plausible as long as survey information is included. In contrast, $SRTSM_{woS}$,

¹⁷The lift-off distribution is calculated by simulating a large number of short rate paths under the \mathbb{P} -measure and then saving the future horizon at which each single path rises above a certain threshold. We define the threshold for a +10 BP DFR hike as our benchmark ELB specification plus 10 BP. For example, if currently the short rate was trading at a 5 BP spread above the DFR, the threshold for an expected +10 BP DFR hike would be at -25 BP. Thus, the simulated lift-off horizon partly depends on the observed spread. Alternatively one could assume a constant spread across all times. However, this would not affect results significantly.

¹⁸The Monte Carlo integration approach to simulate parameter and current state filter uncertainty relies on Hamilton (1994, 898) but we exclude forecasting uncertainty with respect to the risk factors. At first hand, what seems surprising is that estimation uncertainty with respect to the expected short rate in 10 years significantly falls during the ELB period. However, this finding originates from the fact that the shadow short rate which embeds both, filter and parameter uncertainty, is way below the ELB in negative territory during the ELB period. The conditional short rate distribution is censored below the ELB (which is itself deterministic), with a point mass of $Prob(i_{1,t+h} \leq l_{t+h})$ at l_{t+h} . This implies that a significant proportion of estimation uncertainty is likewise censored below the ELB and thus is not reflected in long-term expected short rates. Moreover, due to the incorporation of long-term survey information, the unconditional mean of the short rate under the \mathbb{P} -measure of 3.69% is estimated very precisely with a standard deviation of roughly 0.3% based on parameter uncertainty.

$GATSM_{OLS}$ and $GATSM_{BC}$ generate a significantly lower level of short rate expectations at far-distant horizons. Interestingly, up to the ELB period, far-distant short rate expectations in $GATSM_{BC}$ do not exhibit implausible large time variation compared to $GATSM_{OLS}$ as partly documented for bias-corrected estimates based on US data (Wright, 2014).

4.5.3 Monte Carlo exercise

To robustify our results, we conduct a Monte Carlo exercise for which we simulate interest rates with $J = 1, 3, 6, 12, 24, 36, 60, 84, 120$ months of maturities based on a SRTSM with a fixed ELB at 0%. In light of the high persistence of interest rates observed in the euro area sample and given the high computational costs associated with non-linear estimations, in the Monte Carlo exercise we simulate a sample length of $T = 720$ months (compared to $T = 226$ in the euro area sample) in order to check whether our specification analysis can also be confirmed in a much longer data sample. We estimate a total number of 50 simulated samples.¹⁹ Thereby, we ensure that these samples comprise at least 12 and not more than 60 months in which the short rate is stuck at the lower bound. In line with Kim and Singleton (2012), we also simulate survey data by generating model-implied expectations and adding measurement errors similar in size of those estimated in our benchmark model. We add those surveys at quarterly frequency for 3 months rate expectations in 12 and 24 months and in bi-annually frequency for 3 months rate expectations in 6 to 10 years. For each sample we run estimations based on our $SRTSMs$ and $GATSMs$ specifications.

The results of this exercise confirm our finding that survey information is essential to pin down the data generating process (DGP) in an environment of very persistent interest rates and prolonged ELB periods. This result holds despite using long samples comprising 60 years of monthly observations (see Table 4). Indeed, only the models that include survey information ($SRTSM_B, GATMS_S$) are able to pin down the unconditional mean of the DGP fairly close while producing high persistence in model-implied interest rates. $GATSM_{OLS}$ and $GATSM_{BC}$ on the other hand underestimate both the unconditional mean and the persistence of the true DGP. While their estimate for the unconditional mean matches the sample mean of simulated yields, the latter itself is an insufficient proxy for the unconditional mean of the short rate because the ELB period biases the sample mean downward.

¹⁹The small number of simulated samples is due to computational burden in the context of the estimation of non-linear term structure models. If computational costs would be lower, we could also add smaller samples similar in size to the euro area sample to the Monte Carlos exercise.

Table 4: **Simulation results (median) - long samples**

model	<i>DGP</i>	<i>SRTSM_B</i>	<i>SRTSM_{woS}</i>	<i>GATSM_{OLS}</i>
unconditional mean $E^{\mathbb{P}}(i_{1,t})$:	3.693			2.862
sample mean:	2.888			
max eigenvalues under \mathbb{P} :	0.992			0.976

model	<i>DGP</i>	<i>GATSM_S</i>	<i>GATSM_{BC}</i>
median unconditional mean $E^{\mathbb{P}}(i_{1,t})$:	3.693		2.880
median sample mean:	2.888		
median max eigenvalues under \mathbb{P} :	0.992		0.983

5 Concluding remarks

We propose a shadow rate term structure model for the euro area OIS yield curve that preforms well when evaluated against two criteria (i) good model fit and (ii) the derivation of plausible short-and long-term rate expectations which can be used for policy analysis. Our model explicitly accounts for the specific features of the euro area yield curve sample which can be regarded as very small and characterized by highly persistent interest rate dynamics near or at the time-varying effective lower bound for a prolonged period of time. To do so, our model features such a lower bound which is forward looking in the sense that anticipated changes in the DFR are taken into account before their realization and it considers the spread between the policy rate, i.e. the deposit facility rate in times of negative interest rate policies, and the short rate of the OIS yield curve. To better pin down short- and especially long-term expectations embedded in yield curve data, we also inform the model with survey based interest rate forecasts.

We use our model to assess monetary policy expectations derived from the short end of the yield curve by accounting for the asymmetry of the distribution of short rates during the effective lower bound period. The forward curve itself gives an upward biased picture with respect to future monetary policy rate decisions given negative forward premia even at 1-year horizons. Similarly, mean estimates of future monetary policy rates are upward biased given the truncated distribution of future short rates. Correct inference with respect to monetary policy expectations can only be drawn from the modal, i.e. most likely, path of future short rates.

At far-distant horizons our model delivers short rate expectations that are highly correlated with an estimated nominal equilibrium short rate derived from a macroeconomic modeling set-up even though the considered models do not share any information during the estimations. According to our model results, long-term forward rates thus appear to reflect trends in key macroeconomic variables in both real and nominal terms, which play an important role in the formation of far-distant rate expectations. Moreover, non-standard monetary policy measures together with interest rate forward guidance, not only depressed forward premia but also the expectations component embedded in intermediate and long-term forward rate maturities, thereby highlighting the signaling channel of asset purchases. We confirm this narrative by assessing the impact of unconventional monetary policy shocks based on high frequency identification external instrument approach.

We test alternative modeling specifications including shadow short rate models with different effective lower bound definitions. We also exclude survey information from our preferred model and we estimate various Gaussian affine term structure variants. Overall, we find that these alternative models either exhibit an unsatisfying model fit and / or produce implausible short- and long-term rate expectations from an economic perspective in addition to less convincing outcomes when assessing short-term monetary policy rate expectations. We finally confirm our findings by a Monte Carlo analysis comprising simulated yield curve samples including prolonged periods at the effective lower bound. Indeed, we find that when facing such samples, including survey information and an effective lower bound is important to recover the true data generating process.

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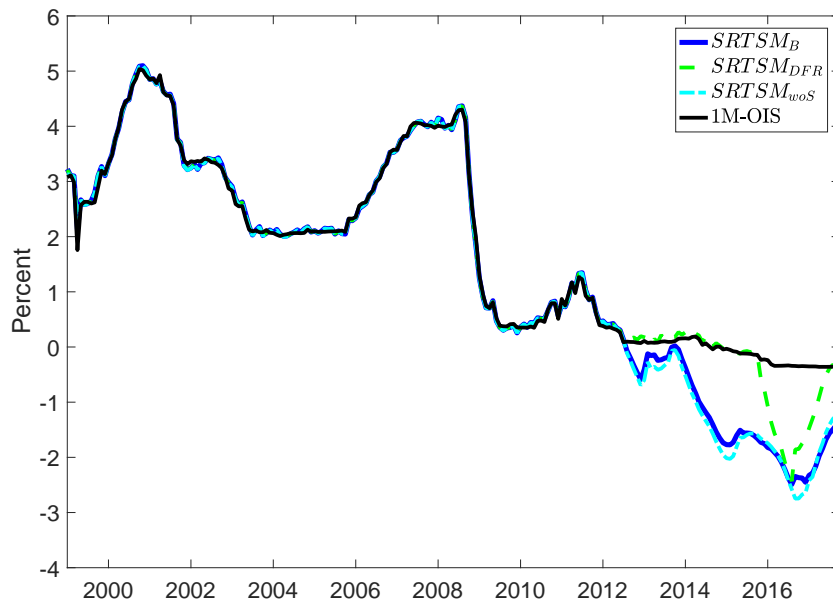
A Appendix

Table A.1: **Parameter estimates for $SRTSM_B$**

	X -factor representation			P -factor representation			
$\mu^{\mathbb{P}}$	0.0322 (0.0091)	-0.2033 (0.1065)	0.2402 (0.0980)	$\mu_P^{\mathbb{P}}$	0.3139 (0.0489)	-0.2583 (0.0580)	0.1494 (0.0405)
$\rho^{\mathbb{P}}$	0.9944 (0.0019)	0.0313 (0.0151)	0.0321 (0.0163)	$\rho_P^{\mathbb{P}}$	0.9923 (0.0022)	-0.00877 (0.0200)	-0.6589 (0.1659)
	0.0300 (0.0331)	0.9640 (0.0340)	0.0920 (0.0358)		-0.0019 (0.0029)	1.0461 (0.0409)	0.6195 (0.1025)
	-0.0431 (0.0374)	-0.0645 (0.0151)	0.7882 (0.0233)		0.0001 (0.0019)	-0.0437 (-0.0109)	0.7082 (0.0675)
$\mu^{\mathbb{Q}}$	0.0277 (0.0012)	0	0	$\mu_P^{\mathbb{Q}}$	0.2924 (1.4163)	-0.1897 (0.7775)	0.0873 (0.1616)
$\rho^{\mathbb{Q}}$	0.9970 (0.0002)	0	0	$\rho_P^{\mathbb{Q}}$	1.0051 (0.0015)	-0.0402 (1.5669)	-0.6686 (6.7540)
	0	0.9398 (0.0012)	0		-0.0045 (0.0001)	1.0485 (0.0003)	0.5011 (0.0011)
	0	0	0.9238 (0.0006)		0.0010 (0.0000)	-0.00291 (-0.0000)	0.8071 (0.0003)
Σ	0.3034 (0.0188)	0	0	Σ_P	0.4581 (0.261)	0	0
	-0.8556 (0.1921)	2.3807 (0.4143)	0		0.0553 (0.0206)	0.2721 (0.0179)	0
	0.55537 (0.1851)	-2.3788 (0.4162)	0.1781 (0.0277)		-0.0233 (0.0071)	-0.0585 (0.0066)	0.0719 (0.0031)
δ_0	0			$\delta_{0,P}$	-0.0690		
δ_1	1	1	1	$\delta_{1,P}$	0.3177	-0.3778	0.5159

Note: Parameter estimates of the $SRTSM_B$ based on the X -factor as well as rotated P -factor representation. Asymptotic standard errors (based on the Hessian matrix) in parentheses.

Figure A.1: Shadow short rates



Note: This figure plots the model-implied shadow short rate based on various term structure model specifications including $SRTSM_B$, $SRTSM_{DFR}$ and $SRTSM_{woS}$. End-of-month values for January 1999 to October 2017.

Table A.2: 1Y1Y forward rate variance decomposition

model		$SRTSM_B$	$SRTSM_{DFR}$	$SRTSM_{woS}$
Level				
total sample:	expectations	0.88	0.87	0.75
	forward premium	0.12	0.13	0.25
pre-ELB sample:	expectations	0.85	0.86	0.75
	forward premium	0.15	0.14	0.25
ELB sample:	expectations	1.09	1.12	0.94
	forward premium	-0.09	-0.12	0.06
Difference				
total sample:	expectations	0.73	0.72	0.95
	forward premium	0.26	-0.28	-0.05
pre-ELB sample:	expectations	0.72	0.71	0.95
	forward premium	0.28	0.29	0.05
ELB sample:	expectations	0.94	0.92	1.05
	forward premium	0.06	0.18	-0.05
model		$GATSM_{OLS}$	$GATSM_S$	$GATSM_{BC}$
Level				
total sample:	expectations	0.82	0.87	0.87
	forward premium	0.18	0.13	0.13
pre-ELB sample:	expectations	0.84	0.88	0.89
	forward premium	0.16	0.12	0.11
ELB sample:	expectations	0.69	0.87	0.73
	forward premium	0.31	0.13	0.27
Difference				
total sample:	expectations	1.03	0.74	1.09
	forward premium	-0.03	0.26	-0.09
pre-ELB sample:	expectations	1.04	0.74	1.10
	forward premium	-0.04	0.26	-0.10
ELB sample:	expectations	0.92	0.73	0.98
	forward premium	0.08	0.27	0.02

Table A.3: **9Y1Y forward rate variance decomposition**

model		$SRTSM_B$	$SRTSM_{DFR}$	$SRTSM_{woS}$
Level				
total sample:				
	expectations	0.47	0.48	0.29
	forward premium	0.53	0.52	0.71
pre-ELB sample:				
	expectations	0.42	0.46	0.26
	forward premium	0.58	0.54	0.74
ELB sample:				
	expectations	0.43	0.45	0.32
	forward premium	0.56	0.55	0.68
Difference				
total sample:				
	expectations	0.31	0.31	0.23
	forward premium	0.69	0.69	0.77
pre-ELB sample:				
	expectations	0.29	0.31	0.22
	forward premium	0.71	0.69	0.78
ELB sample:				
	expectations	0.36	0.40	0.26
	forward premium	0.64	0.60	0.74
model		$GATSM_{OLS}$	$GATSM_S$	$GATSM_{BC}$
Level				
total sample:				
	expectations	0.55	0.56	0.86
	forward premium	0.45	0.44	0.14
pre-ELB sample:				
	expectations	0.57	0.59	0.88
	forward premium	0.43	0.41	0.12
ELB sample:				
	expectations	0.36	0.40	0.56
	forward premium	0.64	0.60	0.44
Difference				
total sample:				
	expectations	0.41	0.64	0.41
	forward premium	0.59	0.36	0.59
pre-ELB sample:				
	expectations	0.44	0.69	0.43
	forward premium	0.56	0.31	0.57
ELB sample:				
	expectations	0.32	0.51	0.36
	forward premium	0.68	0.49	0.64

Table A.4: **In-sample model fit across models**

model	$SRTSM_B$	$SRTSM_{DFR}$	$SRTSM_{woS}$
total sample:	3	3	2
pre-ELB sample:	3	3	3
ELB sample:	2	3	2

model	$GATSM_{OLS}$	$GATSM_S$	$GATSM_{BC}$
total sample:	2	3	2
pre-ELB sample:	3	3	3
ELB sample:	2	2	2

Note: This table shows the mean absolute error of model-implied yields to observed yields for different sample periods. The total sample covers the period 1999M1-2017M10 while the pre-ELB sample covers the period 1999M1-2012M6 and the ELB sample the period 2012M7-2017M10.

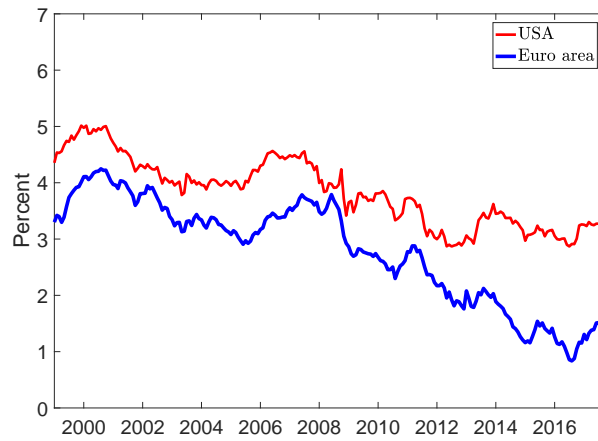
Table A.5: **Lower bound violations of expected short rate paths**

number of months for which $E_t^{\mathbb{P}}[i_{1,t+n}] < l_{t+h}$ for $n = 1, 2, \dots, 120$ and $t = 1, 2, \dots, 226$			
model	$SRTSM_B$	$SRTSM_{DFR}$	$SRTSM_{woS}$
	0	14	0

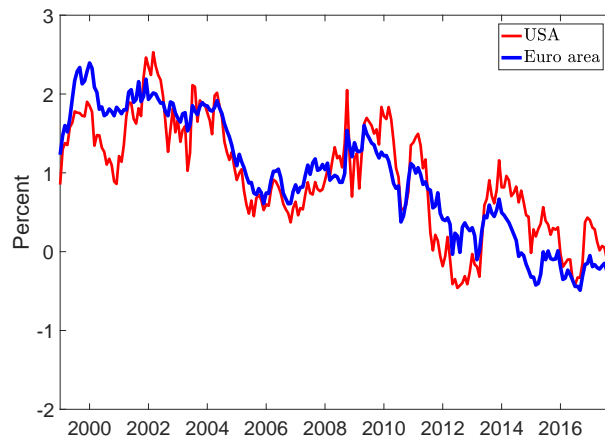
model	$GATSM_{JSZ}$	$GATSM_S$	$GATSM_{BC}$
	21	33	52

Figure A.2: 5Y5Y forward rate decomposition US vs. euro area

(a) Expectations



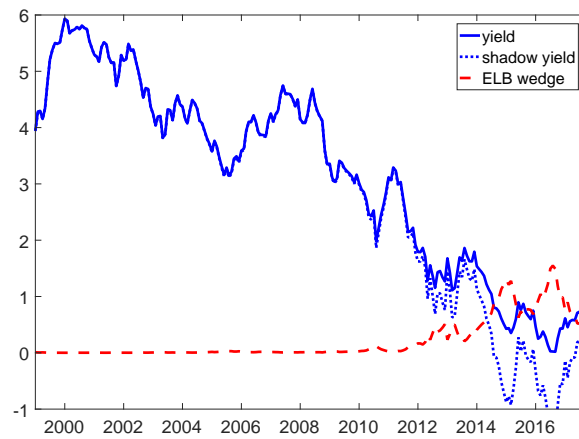
(b) Forward premium



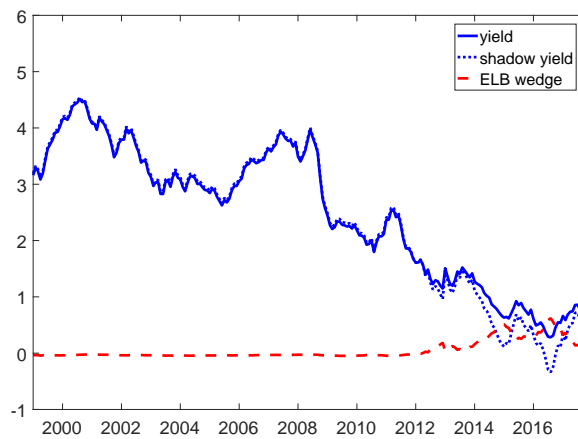
Note: Panel (a) plots the expectations component of the time-series of the decomposition of the 5Y5Y forward rate. Panel (b) plots the corresponding forward premium component. Based on $SRTMS_B$ for the euro area and Kim and Wright (2005) for the US. End-of-month values for January 1999 to October 2017.

Figure A.3: 10Y yield and 10Y shadow yield

(a) \mathbb{Q} -measure



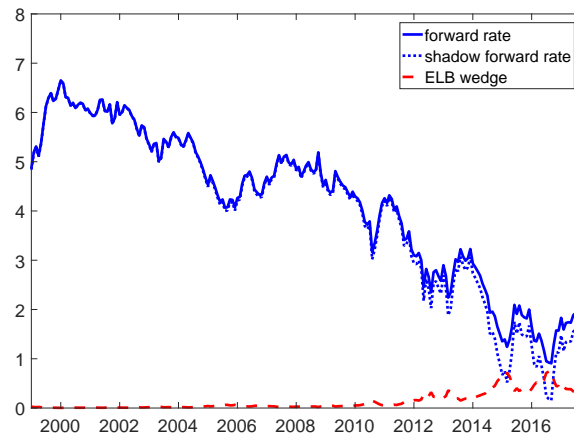
(b) \mathbb{P} -measure



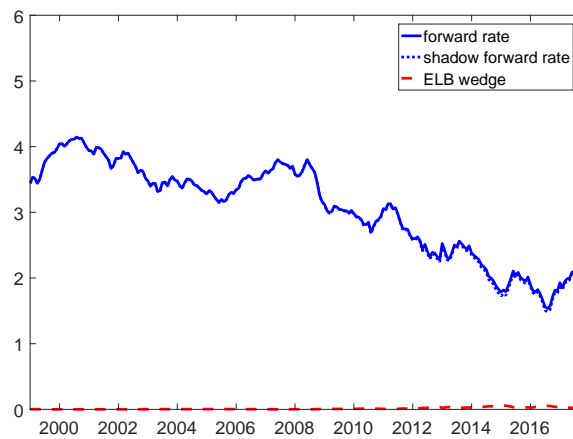
Note: Panel (a) plots the 10Y yield and shadow yield under the \mathbb{Q} -measure and panel (b) under the \mathbb{P} -measure based on $SRTMS_B$. End-of-month values for January 1999 to October 2017.

Figure A.4: 10Y1M forward rate and 10Y1M shadow forward rate

(a) \mathbb{Q} -measure



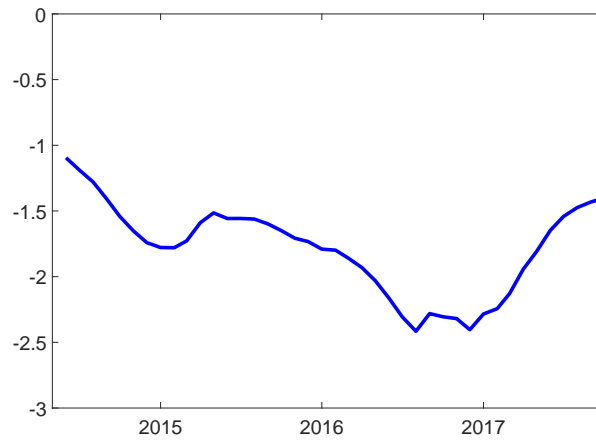
(b) \mathbb{P} -measure



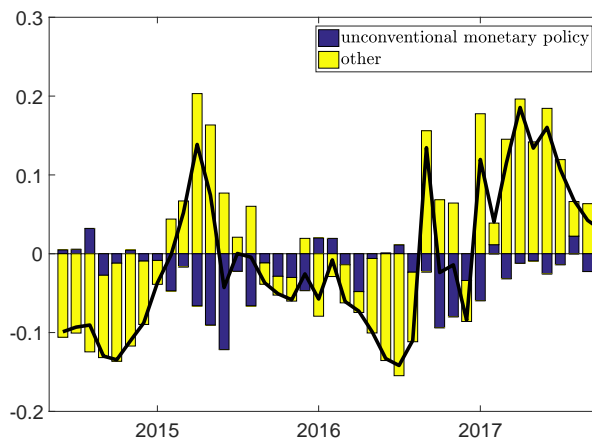
Note: Panel (a) plots the 10Y1M forward rate and shadow forward rate under the \mathbb{Q} -measure and panel (b) under the \mathbb{P} -measure based on $SRTMS_B$. End-of-month values for January 1999 to October 2017.

Figure A.5: **Historical decomposition of the shadow short rate**

(a) level of the shadow short rate

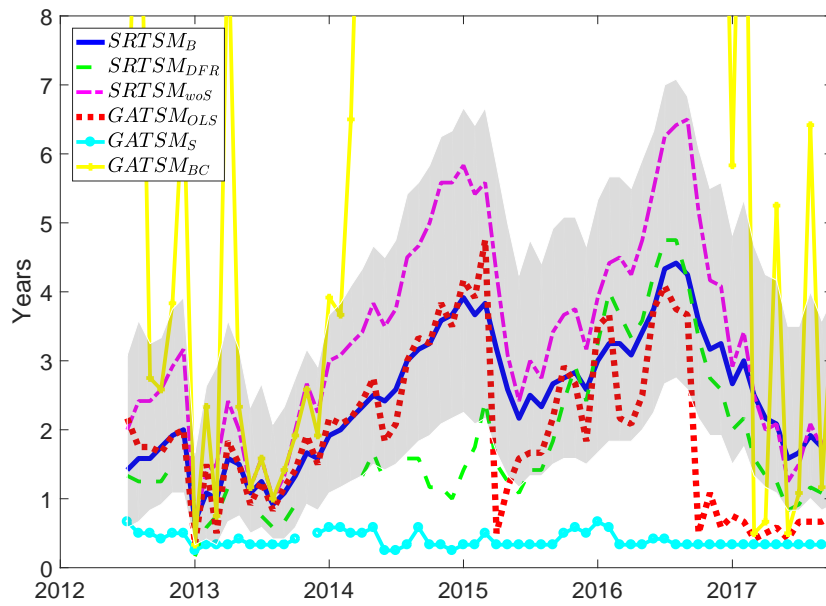


(b) change and contribution of shocks



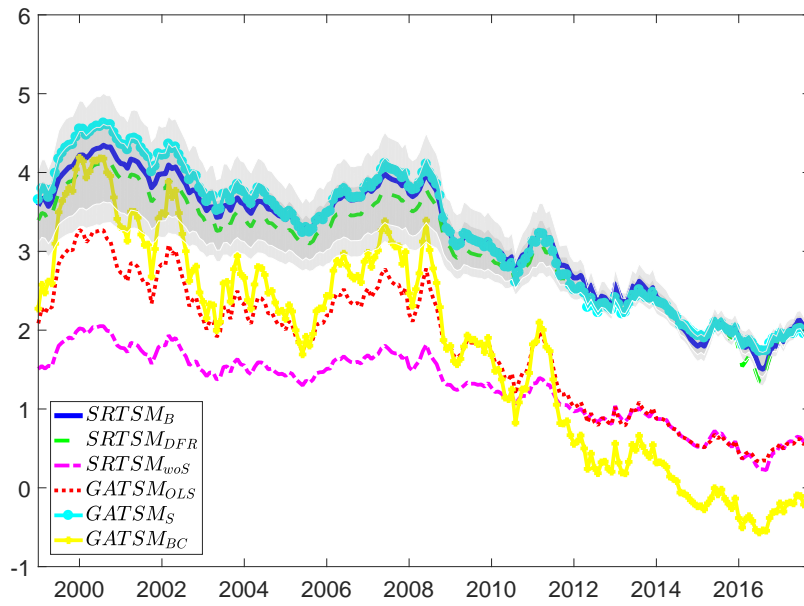
Note: Panel (a) plots the the shadow short rate from June 2014 to October 2017 based on $SRTSM_B$. Panel (b) plots the corresponding historical decomposition with a focus on unconventional monetary policy shocks.

Figure A.6: +10 BP DFR hike (median distribution)



Note: This figure plots the number of months of the median distribution of a +10 BP DFR hike based on various model specification. End-of-month values January 2012 to October 2017. The shaded area lies between the 15%- and 85%-quantile of our benchmark median distribution.

Figure A.7: 10Y1M short rate expectations



Note: This figure plots model-implied expected short rates based on various term structure model specifications together with [5%,15%]- and [85%,95%]-quantile confidence intervals based on our preferred $SRTSM_B$ model. Confidence intervals refer to parameter estimation and current state filter uncertainty. End-of-month values for January 1999 to October 2017.