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# Forward Guidance at the Zero Lower Bound: Curse and Blessing of Time-Inconsistency

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## Abstract

Forward guidance as a tool of unconventional monetary policy can be highly efficient to support aggregate demand and to steer the economy out of the zero lower bound (ZLB). However, the effect that stimulates the economy can give rise to a time-inconsistency problem: if the central bank promises to keep interest rates at the ZLB for long, the subsequent increase in inflation and economic activity may create a motive for the central bank to forego its promise and to exit the ZLB earlier than announced. We illustrate the time-inconsistency problem in a New Keynesian model with hand-to-mouth consumers. Using a novel and analytically tractable method for handling occasionally binding constraints, we contrast the case of commitment to forward guidance with the case in which monetary policy allows for an early exit of the ZLB. Our method is able to provide results on uniqueness and existence of (ZLB) equilibria. We study the equilibrium selection given different scenarios and conclude that central bankers should be careful when choosing the number of periods with low interest rates in order to avoid the inconsistency problem. Furthermore, we calculate government spending multipliers and argue that the multiplier is even larger if combined with forward guidance.

*Keywords:* Forward Guidance, zero lower bound, occasionally binding constraints, government spending multiplier

*JEL:* E63, C63, E58, E32, C62

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## 1 Introduction

In recent years, the US as well as European economies have been gradually recovering from the great recession and the accompanying liquidity trap. With the recovery in sight, controversially debated questions are, when and how to return to "normal" monetary policy, i.e. to raise interest rates from the so-called effective zero-lower bound on nominal interest rates (ZLB), returning to a state of affairs in which adjustments of the policy interest rate reflect changes in economic fundamentals. During the Great Recession, central banks have started to use forward guidance for their monetary policy actions.

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We argue that by announcing a future path of interest rates, forward guidance creates a time-inconsistency problem that surfaces in times of economic recovery.<sup>1</sup>

This work has three central contributions. First, we illustrate the effects of *Odyssian Forward Guidance*, contrasting the corresponding equilibrium time path with the case in which the central bank simply follows the Taylor rule whenever possible and sets rates to zero if not. Odyssian Forward Guidance (OFG) is the concept of publishing an interest rate path for the near future, and committing to it independently of the economic conditions at the respective time. Using a New Keynesian model with hand-to-mouth consumers, we show that OFG can significantly dampen the negative impact of a strong demand shock. The longer the central bank promises to keep interest rates at the ZLB, the more effective is OFG in stimulating inflation and aggregate demand, and, as a consequence, the earlier the economy returns to a state, in which a Taylor rule based policy warrants a switch to positive interest rates. This is where the issue of time-inconsistency arises: while inflation and output start to rise, policy makers might be incentivised to raise interest rates. If such a deviation from the OFG path happens early, it might directly lead back to the ZLB and is not optimal. If it occurs later, but before the exit period initially announced, such response is beneficial given the central banks loss function, but undermines the central banks credibility in the future.

We are not the first to study the effects of forward guidance. The beneficial effects of forward guidance in the presence of the ZLB were first pointed out by (Eggertson and Woodford, 2003) in the context of a theoretical model. (Del Negro et al., 2015) find empirically meaningful effects of forward guidance on macroeconomic dynamics, but raise the issue that the effects of forward guidance are grossly overstated in theoretical models (the so-called "Forward Guidance Puzzle"). They attribute this to an oversensitivity of consumption spending to future interest rates and propose a model with Perpetual Youth households as in (Blanchard, 1985) to reduce the link between current consumption and future interest rates. We address this issue, by introducing hand-to-mouth consumers into our model. While the literature on forward guidance is growing, to our knowledge, we are the first to illustrate the issue of time-inconsistency related to OFG at the zero lower bound.

Our second, major contribution is a novel method that allows us to study the different equilibria induced by an occasionally binding constraint, to find the rational expectations equilibrium and provide conditions for its existence and uniqueness. Occasionally binding constraints are an important part of economic modelling and in particular gained importance since major central banks have been restricted in their interest policy by the ZLB. Solution methods currently prominent in the literature either have to be solved on a grid or are subject to partially numerically unstable numerical methods. Solutions on the grid can normally provide a reasonably high accuracy, but are subject to the "curse of dimensionality", meaning that computation time increases exponentially with the size of the state space, and high dimensional models are either only solvable with very high computational costs (and time) or require considerable computational expertise, that is normally not at hand. Typically both is implied and, additionally, the set-up costs are quite high even for simpler problems. Other methods such as Guerrieri and Iacoviello

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<sup>1</sup>Forward Guidance has been applied to the use of unconventional and conventional policy instruments. While we focus here on the short-term interest rate and the associated zero lower bound issue, the gist of our argument is likely to hold for announcements of quantitative easing as well.

(2015) and Holden (2016) rely on different perturbation strategies whereas finding the correct number of periods at the ZLB might be numerically unstable and hence may render the results unreliable. The analytical method proposed here, when in the constrained system, iterates forward in time until the system is unconstrained and then uses the solution of the unconstrained system to solve for the endogenous variables when constrained. The problem then boils down to finding the minimal distance, i.e. the time path until the system is unconstrained. We further provide a simple condition that implies both existence and uniqueness of the equilibrium path from the constrained to the unconstrained state.

Our last contribution is to study government spending multipliers across different scenarios. We hereby confirm previous findings that, due to the absence of a *crowding out* effect, the spending multiplier is larger than one when the economy is at the ZLB and can potentially become very large. We further document that it becomes even slightly larger in combination with OFG. To the extent that the values of the multipliers generated in our analysis are in line with the existing literature,<sup>2</sup> this exercise underscores the reliability of the novel solution method used in our analysis.

The rest of this paper is structured as follows. In the next section we set up a small-scale New-Keynesian model with hand-to-mouth consumers. In the section that follows we provide the conditions under which the rational expectations equilibrium is unique and introduce the solution method. In Section 4 we apply this method to the ZLB problem in a simple New Keynesian framework and study the effects of forward guidance and government spending. Section 5 concludes.

## 2 Model

For our analysis, we employ a small scale New Keynesian model, extended with hand-to-mouth consumers as in (Galí et al., 2007). This particular extension is suited to address the forward guidance puzzle, which was first raised in (Del Negro et al., 2015). While these authors find positive effects of forward guidance announcements by the Federal Reserve, they point out that standard DSGE models tend to grossly overestimate the effects of forward guidance on macroeconomic dynamics. One of the reasons that rational expectations models yield very large effects of forward guidance is the strong response of consumption spending to future interest rates. To moderate this link, we introduce hand-to-mouth consumers into our model. With a fraction of agents in our model being standard Ricardian consumers and another fraction being hand-to-mouth consumers, the response of aggregate demand to changes in future interest rates, and thus to forward guidance is more in line with empirical results. Furthermore, the feature of hand-to-mouth agents itself is empirically appealing, as micro data suggests that a large share of households only has limited access to financial markets and is thus constrained in its capability to smooth consumption via borrowing and lending as Ricardian consumers do.

This section gives a brief description of the model. Detailed derivations of the equilibrium equations are delegated to the appendix. The model consists of the two household types, a monopolistically competitive firm that faces nominal rigidities as in (Calvo,

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<sup>2</sup>see, e.g., (Christiano et al., 2011), (Eggertson, 2011), (Eggertson and Krugman, 2012)

1983a), a stylized government sector and a monetary authority that sets the short-term nominal interest rate according to a Taylor Rule.

Both household types have separable utility in consumption and leisure. The utility function reads

$$U_t = E_0 \sum_{t=0}^{\infty} \beta^t e^{V_t} \left( \log(C_t) - \frac{N_t^{1+\psi}}{1+\psi} \right),$$

where parameters  $\beta$  and  $\psi$  are, respectively, the discount factor and the inverse of the Frisch elasticity.  $C_t$  is consumption,  $N_t$  are the hours worked and  $V_t$  is a shock to the discount factor of households. Within our analysis, a shock to  $V_t$  that increases the patience of households triggers zero lower bounds episodes. The optimizing Ricardian households earn wages  $W_t$  from their labor supply  $N_t^o$  and receive income from one-period riskless zero coupon bonds  $B_t^o$  that yield an interest rate  $R_t$ . They spend their funds on consumption  $C_t^o$  and the purchase of bonds. Their budget constraint thus reads

$$P_t C_t^o + B_{t+1}^o = B_t^o (1 + R_t) + W_t N_t^o + T_t.$$

Here,  $P_t$  denotes the price level at time and  $T_t$  are lump sum taxes raised by the government to finance government spending. In contrast to Ricardian consumers, hand-to-mouth consumers do not have access to the bond market. Their budget constraint is

$$P_t C_t^r = W_t N_t^r,$$

where  $C_t^r$  is consumption by hand-to-mouth consumers and  $N_t^r$  their hours worked. Aggregate consumption and labor hours are weighted sums of the variables of the two household types, where  $\lambda$  is the share of hand-to-mouth consumers in the economy.

$$\hat{C} = \lambda \hat{C}_t^r + (1 - \lambda) \hat{C}_t^o, \quad (1)$$

$$\hat{N} = \lambda \hat{N}_t^r + (1 - \lambda) \hat{N}_t^o. \quad (2)$$

Firms in the good market are in monopolistic competition, and can charge a markup over their marginal cost. The production function of firm  $i$  reads simply  $Y_t(i) = N_t(i)$ . The relation between the production of the individual firm and aggregate output is summarised in the Dixit-Stiglitz aggregator  $Y_t \equiv \left( \int_0^1 Y_t(i)^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}$ , with  $\epsilon > 1$ . Each period, firms face a constant probability of being able to optimally adjust their prices,  $\theta$ . The assumption of Calvo pricing gives rise to the price index  $P_t \equiv ((1 - \theta)(P_t^*)^{1-\epsilon} + (P_{t-1})^{1-\epsilon})^{\frac{1}{1-\epsilon}}$  and yields a standard New Keynesian inflation equation.

The model features government spending  $G_t$ , which - for simplicity - is financed via lump sum taxes from Ricardian consumers. Government spending is exogenous and described by an AR(1) process. The good market clearing condition reads

$$Y_t = C_t + G_t.$$

Lastly, monetary policy is modelled as a Taylor-Rule on short-term nominal interest

rates, and takes account of the occasionally binding nature of the zero lower bound

$$R_{t+1} = \max \left\{ 0, \rho_R R_t + (1 - \rho_R) \left( R + \frac{\phi_1}{\beta} \pi_t + \frac{\phi_2}{\beta} \hat{Y}_t \right) \right\}.$$

$\rho_R$  is the interest smoothing parameter, and  $\phi_1$  and  $\phi_2$  are the coefficients, which govern the strength of the response of interest rate policy to deviations of inflation and output from their target level.

The derivation of the linearized equilibrium conditions is delegated to the appendix. The equilibrium conditions, expressed in  $Y_t, R_t, \pi_t, \hat{G}_t$ , and  $\hat{V}_t$  can be summarized in the following systems of equations

$$\hat{Y}_t - \frac{g}{\Theta} \hat{G}_t = E_t[\hat{Y}_{t+1} - \frac{g}{\Theta} \hat{G}_{t+1}] - \frac{(1-g)}{\Theta} [(R_{t+1} - R) - \pi_{t+1} + V_{t+1} - V_t], \quad (3)$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \left( \left( \frac{1}{1-g} + \psi \right) \hat{Y}_t - \frac{g}{1-g} \hat{G}_t \right) + \hat{Z}_t, \quad (4)$$

$$R_{t+1} = \max \left\{ 0, \rho_R R_t + (1 - \rho_R) \left( R + \frac{\phi_1}{\beta} \pi_t + \frac{\phi_2}{\beta} \hat{Y}_t \right) \right\} \quad (5)$$

$$\hat{G}_t = \rho_g \hat{G}_{t-1} + \epsilon_t^g, \quad (6)$$

$$\hat{V}_t = \rho_v \hat{V}_{t-1} + \epsilon_t^v. \quad (7)$$

$$\hat{Z}_t = \rho_z \hat{Z}_{t-1} + \epsilon_t^z. \quad (8)$$

In this system of equations,  $g$  is the steady state share of government spending in aggregate output.  $\Theta = \left( 1 - \frac{\lambda \psi (1-g)}{(1-\lambda)} \right)$  is derived in the appendix. It is through this coefficient that the share of hand-to-mouth in the economy affects equilibrium dynamics. In an economy without hand-to-mouth consumers (i.e.  $\lambda = 0$ ),  $\Theta$  is equal to one. Equation (3) is the New Keynesian IS Equation, (4) is the New Keynesian Phillips Curve, (5) is the interest rate rule, and (6) and (7) are the stochastic driving forces of the model.

$\beta$	$g$	$\kappa$	$\lambda$	$\phi_\pi$	$\phi_y$
0.99	0.2	0.08	1/3	1.5	0.125

Table 1: Baseline calibration

The baseline calibration can be found in Table 1. The parameters of autocorrelation ( $\rho_v, \rho_z, \rho_g$ ) are given by 0.9, 0.9 and 0.7 respectively. Note that instead of modelling a “classic” discount factor shock, we make use of the ad-hoc demand shock  $\hat{V}_t$ . Since this shock does not enter the optimality conditions a less stronger shock is necessary to push the economy to the zero lower bound.

### 3 Solution Strategy

Let  $\mathbf{x}_t$  be the  $n \times 1$  vector of forward looking variables and let  $\mathbf{v}_t$  be the  $m \times 1$  vector of state variables. If a variable  $x_{1,t}$  is both, depending on its past value  $x_{1,t-1}$  and on its expected future value  $E_t x_{1,t+1}$  let us define an auxiliary variable  $x_{2,t} = x_{1,t-1}$  to keep

the form outlined before. Every system of linear difference equations (LDE) with an occasionally binding constraint (OBC) can then be put in the form

$$\mathbf{M} \begin{bmatrix} \mathbf{x}_t \\ \mathbf{v}_t \end{bmatrix} = \mathbf{P} E_t \begin{bmatrix} \mathbf{x}_{t+1} \\ \mathbf{v}_{t+1} \end{bmatrix} + \mathbf{c} \max \left\{ \mathbf{b} \begin{bmatrix} \mathbf{x}_t \\ \mathbf{v}_t \end{bmatrix}, \bar{x} \right\} + \boldsymbol{\varepsilon}_t \quad (9)$$

where  $\mathbf{b}$  is a  $1 \times (n+m)$  row vector that together with  $\bar{x}$  defines the constraint and  $\boldsymbol{\varepsilon}_t$  is a vector of iid. shocks. The system can be constructed such that the matrix  $\mathbf{P} = \begin{bmatrix} \mathbf{P}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix}$ , apart from the expectation system  $\mathbf{P}_1$  on  $\mathbf{x}_t$  only contains zeroes and a  $m \times m$  identity matrix.<sup>3</sup> Let us define the unconstrained system as the system that contains the steady state and satisfies the Blanchard-Kahn conditions (Blanchard and Kahn, 1980), i.e. the system where a rational expectations solution is determined. Alternatively, the above system can be rewritten as

$$\begin{bmatrix} \mathbf{x}_t \\ \mathbf{v}_t \end{bmatrix} = \begin{cases} (\mathbf{M} - \mathbf{c}\mathbf{b})^{-1} \mathbf{P} E_t \begin{bmatrix} \mathbf{x}_{t+1} \\ \mathbf{v}_{t+1} \end{bmatrix} + \boldsymbol{\varepsilon}_t & \forall \mathbf{b} \begin{bmatrix} \mathbf{x}_t \\ \mathbf{v}_t \end{bmatrix} - \bar{x} \geq 0 \\ \mathbf{M}^{-1} \mathbf{P} E_t \begin{bmatrix} \mathbf{x}_{t+1} \\ \mathbf{v}_{t+1} \end{bmatrix} + \mathbf{M}^{-1} \mathbf{c} \bar{x} + \boldsymbol{\varepsilon}_t & \forall \mathbf{b} \begin{bmatrix} \mathbf{x}_t \\ \mathbf{v}_t \end{bmatrix} - \bar{x} < 0. \end{cases} \quad (10)$$

Hence,  $(\mathbf{M} - \mathbf{c}\mathbf{b})^{-1} \mathbf{P}$  is the system matrix of the unconstrained system and  $\mathbf{N} = \mathbf{M}^{-1} \mathbf{P}$  is likewise is the system matrix of the constraint system. The system  $\mathbf{M} - \mathbf{c}\mathbf{b}$  can be obtained readily by perturbation of the respective system around its steady state and separating the dependence of the constrained input into the  $(n+m) \times 1$  column vector  $\mathbf{c}$ , which is always nonzero unless the constraint is at the zero steady state, which would render the model unsolvable. In mathematical terms, this is equivalent with assuming *continuity* of the system mapping or *consistency at the constraint*. It is easy to see that the above system satisfies this condition if it holds that  $\mathbf{b} \begin{bmatrix} \mathbf{x}_t \\ \mathbf{v}_t \end{bmatrix} = \bar{x}$  and hence  $\max\{\bar{x}, \bar{x}\} = \bar{x}$  and that in both systems the expectations are equal because the transition to next period's state vector  $E_t \mathbf{v}_{t+1}$  is equal.

Put differently, both systems are perturbed around the (same) unconstrained steady state.<sup>4</sup>

Let  $E_t k$  be the *expected number of periods at the constraint* (i.e. in the constrained system) in period  $t$  and denote a rational expectations solution to (9) depending on  $E_t k$  and as

$$\mathbf{x}_t = S(E_t k, \mathbf{v}_t). \quad (12)$$

<sup>3</sup>In what follows we will suppress the subscript whenever the context reveals the dimensionality of  $\mathbf{I}$ .

<sup>4</sup>Taken separately, the constrained system generally does have a *different* steady state  $(\mathbf{x}_c^*, \mathbf{v}_c^*)$ , i.e. the steady state of the constrained system is a nonzero deviation from the steady state of the unconstrained system with

$$\begin{bmatrix} \mathbf{x}_c^* \\ \mathbf{v}_c^* \end{bmatrix} = (\mathbf{M} - \mathbf{P})^{-1} \mathbf{c} \bar{x} \neq \mathbf{0} = \begin{bmatrix} \mathbf{x}_u^* \\ \mathbf{v}_u^* \end{bmatrix}. \quad (11)$$

In the rest of this paper we will suppress the expectations operator for better readability and we will also occasionally use  $\mathbf{y}_t = \begin{bmatrix} \mathbf{x}_t \\ \mathbf{v}_t \end{bmatrix}$  as a shortcut for the stacked vector of endogenous and state variables.

**Definition 1.** A rational expectation solution for a expected number of periods  $k^*$  at the constraint is also a rational expectations equilibrium if, in expectations, the system is constrained for exactly  $k^*$  periods, i.e. if

$$\mathbf{b}E_t \{ \mathbf{y}_{t+k^*} | S(k^*, \mathbf{v}_t), \mathbf{v}_t \} - \bar{x} \geq 0 > \mathbf{b}E_t \{ \mathbf{y}_{t+k} | S(k^*, \mathbf{v}_t), \mathbf{v}_t \} - \bar{x} \quad (13)$$

for  $k \in \{0, 1, \dots, k^* - 1\}$ .

It is straightforward that the system implied by Equation (9) has a rational expectation equilibrium for a given  $\mathbf{v}_t$  if the constraint is not a repeller, i.e. if

$$\mathbf{b}S(0, \mathbf{v}_t) < \bar{x} \implies \mathbf{b}S(1, \mathbf{v}_t) < \bar{x}. \quad (14)$$

A repeller would imply that the mapping is inverted at the constraint. In that case no solution for  $\mathbf{v}_t$  would be defined.

Let us now find a solution  $S$  and then study under which conditions an equilibrium can exist and when such equilibrium is unique. Assume further that The forecasting error implied by the triangle inequality is marginal, i.e. if we rewrite (9) as  $\mathbf{x}_t = f(\mathbf{v}_t, \boldsymbol{\varepsilon}_t)$ , then  $E_t \mathbf{x}_{t+1} \approx f(\mathbf{v}_{t+1}, \mathbf{0})$ . It can be the case that this assumption holds true, for example due to quite small variances in the error terms. We can however not assert that in general.

### 3.1 Main method

The unconstrained system with the system matrix  $(\mathbf{M} - \mathbf{c}\mathbf{b})^{-1}\mathbf{P}$  can be solved using familiar methods, as for instance Eigenvalue-Eigenvector decomposition. Let us denote this solution by the matrix  $\boldsymbol{\Omega}$ , where it holds that

$$\mathbf{x}_t = \boldsymbol{\Omega}\mathbf{v}_t \quad \forall \mathbf{b} \begin{bmatrix} \boldsymbol{\Omega} \\ \mathbf{I} \end{bmatrix} \mathbf{v}_t - \bar{x} \geq 0. \quad (15)$$

Setting  $\mathbf{x}_t = \boldsymbol{\Omega}\mathbf{v}_t$  in the condition as well is fine since for now we are only looking for an equilibrium without any judgement on its uniqueness. If there exists only one equilibrium, this condition will obviously also yield the unique equilibrium.

For  $\mathbf{Q} = \begin{bmatrix} \mathbf{I} & -\boldsymbol{\Omega} \end{bmatrix}$  Equation (15) implies that

$$\mathbf{Q}E_t \begin{bmatrix} \mathbf{x}_{t+k} \\ \mathbf{v}_{t+k} \end{bmatrix} = \mathbf{0} \quad \forall \mathbf{b} \begin{bmatrix} \boldsymbol{\Omega} \\ \mathbf{I} \end{bmatrix} E_t \mathbf{v}_{t+k} - \bar{x} \geq 0, \quad (16)$$

i.e. for every future period  $t+k$  in which the system is expected to be unconstrained.

Assume that we are in the constrained system  $\mathbf{N}$  at time  $t$  and will continue to until at least in period  $t+k-1$ . Iterating System (10) forward tells us that

$$\begin{bmatrix} \mathbf{x}_t \\ \mathbf{v}_t \end{bmatrix} = \mathbf{N}^k E_t \begin{bmatrix} \mathbf{x}_{t+k} \\ \mathbf{v}_{t+k} \end{bmatrix} + (\mathbf{I} - \mathbf{N})^{-1}(\mathbf{I} - \mathbf{N}^k)\mathbf{M}^{-1}\mathbf{c}\bar{x} \quad \forall \mathbf{b}E_t \begin{bmatrix} \mathbf{x}_{t+k-1} \\ \mathbf{v}_{t+k-1} \end{bmatrix} - \bar{x} < 0, \quad (17)$$



where  $(\mathbf{I} - \mathbf{N})^{-1}(\mathbf{I} - \mathbf{N}^k) = \sum_{i=0}^{k-1} \mathbf{N}^i$  is the transformation for a geometric series of matrices. Assuming that the system is unconstrained at  $t + k$  we can use Equation (16) to rewrite (10) as

$$\mathbf{QN}^{-k} \begin{bmatrix} \mathbf{x}_t \\ \mathbf{v}_t \end{bmatrix} = \mathbf{QN}^{-k}(\mathbf{I} - \mathbf{N})^{-1}(\mathbf{I} - \mathbf{N}^k)\mathbf{M}^{-1}\mathbf{c}\bar{x}, \quad (18)$$

$$k = 0 \quad \text{iff.} \quad \mathbf{b}E_t \begin{bmatrix} \mathbf{x}_t \\ \mathbf{v}_t \end{bmatrix} - \bar{x} \geq 0, \quad (19)$$

$$k = 1 \quad \text{iff.} \quad \mathbf{b}E_t \begin{bmatrix} \mathbf{x}_{t+1} \\ \mathbf{v}_{t+1} \end{bmatrix} - \bar{x} \geq 0 > \mathbf{b}E_t \begin{bmatrix} \mathbf{x}_t \\ \mathbf{v}_t \end{bmatrix} - \bar{x}, \quad (20)$$

$$\dots \quad (21)$$

$$k = j \quad \text{iff.} \quad \mathbf{b}E_t \begin{bmatrix} \mathbf{x}_{t+j} \\ \mathbf{v}_{t+j} \end{bmatrix} - \bar{x} \geq 0 > \mathbf{b}E_t \begin{bmatrix} \mathbf{x}_{t+j-1} \\ \mathbf{v}_{t+j-1} \end{bmatrix} - \bar{x}, \quad (22)$$

which implies a solution of the endogenous variables  $\mathbf{x}_t$  in terms of the state variables  $\mathbf{v}_t$  given the expectation on  $k$ , i.e. in which period the system will be unconstrained again. Since  $\mathbf{c}$  is a vector of (known) constants, the whole RHS of (18) is a (known) vector. Let us denote a solution implied by solving Equation (18) as suggested in Equation (12) by

$$S(k, \mathbf{v}_t) = \left\{ \begin{bmatrix} \mathbf{x}_t \\ \mathbf{v}_t \end{bmatrix} : \mathbf{QN}^{-k} \begin{bmatrix} \mathbf{x}_t \\ \mathbf{v}_t \end{bmatrix} = \mathbf{QN}^{-k}(\mathbf{I} - \mathbf{N})^{-1}(\mathbf{I} - \mathbf{N}^k)\mathbf{M}^{-1}\mathbf{c}\bar{x} \right\}, \quad (23)$$

which is a  $n \times n$  matrix multiplied by  $\mathbf{v}_t$  plus a  $n \times 1$  vector of constants. From Equations (17) and (13) and given the corresponding restriction we can express the expectations of period  $t + j$  given the expectation that the system is at the constraint until period  $k$  as

$$L_j(k, \mathbf{v}_t) = \mathbf{N}^{-j}S(k, \mathbf{v}_t) - \mathbf{N}^{-j}(\mathbf{I} - \mathbf{N})^{-1}(\mathbf{I} - \mathbf{N}^j)\mathbf{M}^{-1}\mathbf{c} \quad (24)$$

$$= E_t^k \begin{bmatrix} \mathbf{x}_{t+j} \\ \mathbf{v}_{t+j} \end{bmatrix}. \quad (25)$$

we will also use  $L(k, \mathbf{v}_t)$  and, in the proofs,  $L(k)$  as a shortcut for  $L_k(k, \mathbf{v}_t)$ . Note that  $L(0, \mathbf{v}_t) = S(0, \mathbf{v}_t) = \mathbf{Q}\mathbf{v}_t$  and in general  $L_0(k, \mathbf{v}_t) = S(k, \mathbf{v}_t)$  since this means nothing else than  $L_0(k) = E_t^k \mathbf{y}_t$  being the expectation on the current system state.

### 3.2 Existence and uniqueness

Let us start with a short wrap-up.

**Lemma 1.** *For any given vector of state variables  $\mathbf{v}_t$  a rational expectations equilibrium  $\mathbf{x}_t = S(k^*, \mathbf{v}_t)$  to the system in 9 with  $k^* : \mathbf{b}L(k^*, \mathbf{v}_t) \geq \bar{x}$  is unique if*

$$\mathbf{b}\mathbf{N}^l L(k^*, \mathbf{v}_t) + \mathbf{b} \sum_{0 \leq i < l} \mathbf{N}^i < \bar{x} \quad \forall l \in \{0, 1, \dots, k^* - 1\}, \quad (26)$$

*i.e. that the trajectory of the  $k^*$  equilibrium has not crossed the constraint before period*

$k^*$ , and

$$\forall k' > k^*. \mathbf{b}L(k', \mathbf{v}_t) \geq \bar{x} \quad \exists k < k'. \mathbf{b}\mathbf{N}^k L(k^*, \mathbf{v}_t) + \mathbf{b} \sum_{0 \leq i < k} \mathbf{N}^i \geq \bar{x}, \quad (27)$$

i.e. that expectations are inconsistent when expecting any  $k$  larger than  $k^*$ .

*Proof.* See Appendix A.1 ■

In order to obtain a sharper condition of existence and uniqueness it is helpful to look at the system matrices in closer detail. Let us denote

$$\mathbf{M} = \begin{bmatrix} \mathbf{M}_1 & \mathbf{M}_2 \\ \mathbf{M}_x & \mathbf{M}_v \end{bmatrix}, \quad \mathbf{c} = \begin{bmatrix} \mathbf{c}_x \\ \mathbf{c}_v \end{bmatrix} \quad \text{and} \quad \mathbf{b} = \begin{bmatrix} \mathbf{b}_x \\ \mathbf{b}_v \end{bmatrix}. \quad (28)$$

**Theorem 1.** For any given vector of state variables  $\mathbf{v}_t$  the equilibrium  $\mathbf{x}_t = S(k^*, \mathbf{v}_t)$  of the system in 9 exists and is unique if

$$\mathbf{b}_x (\mathbf{M}_1 - \Omega \mathbf{M}_x)^{-1} (\mathbf{P}_1 \Omega \mathbf{c}_v - \mathbf{c}_x) \geq 0 \quad (29)$$

and the trajectory of  $k^*$  does not return to the constraint after period  $k^*$ .

*Proof.* See Appendix A.2 ■

Theorem 1 states the main result concerning existence and uniqueness. At the constraint, a force  $f_u : \mathbf{c}\mathbf{b}\mathbf{y}_t$  is switched off. If the Blanchard-Kahn conditions are not satisfied in the constrained system this implies that this force is essential to return to the original steady state. It also means that *at* all coordinates of the constraint, force  $f_c : \mathbf{c}\mathbf{b}\bar{x}$  is the exact force necessary to return to the steady state. Constraining this force to remain at this level has two implications: in the *unconstrained* region,  $f_c$  is too high to let the system return to the steady state while in the *constrained* region it is too weak. For that reason every  $y_t$  in the unconstrained region will accelerate away from the constraint. Since this also holds in anticipation, we can establish uniqueness of the outlined solution method under the given condition: in terms of the distance spanned by  $\mathbf{b}$ , the matrix  $\mathbf{M}_1$  has to be such that the expectation dynamics more than set-off the transition dynamics of the state space, and the force induced by  $\mathbf{c}_x$  needs to more-than set-off the impact of the constraint on the state space.

The remaining task is to actually find the equilibrium  $k$ . This is trivial since the function  $L$  can be used to rephrase the conditions in (18) - (22) as

$$k^* = \min_k k \quad \text{s.t.} \quad \mathbf{b}L(k, \mathbf{v}_t) - \bar{x} \geq 0, \quad (30)$$

which, written more compactly, allows to specify the complete solution to the OBC problem as

$$k^* = \operatorname{argmax}_k \{f(k) \operatorname{sign}(\mathbf{b}L(k, \mathbf{v}_t) - \bar{x})\}, \quad (31)$$

where  $f : \frac{\partial |f(k)|}{\partial k} < 0$  can be any function that in its absolute is decreasing in  $k$ . One example would be  $f(k) = \exp(-k)$ . A solution to the problem of finding  $k$  can be implemented quite straightforwardly and more elegantly than implied by the algebra. A suggestion is sketched in Figure B.3 in the Appendix.

## 4 Results

Let us first abstract from forward guidance and have a look at the impulse response functions to a positive/negative productivity shock and a positive/negative demand shock, which is captured by the shock on the discount factor in Figure 1. All four shocks have a size of a one percent deviation from their respective steady state values. As for the positive demand and the negative productivity shock the constraint is not binding, the respective dynamics look familiar since they just represent the standard rational expectations solution. Due to the nature of the solution method, the IRFs for positive and negative shocks are symmetric around zero in the regions where the ZLB is not binding.

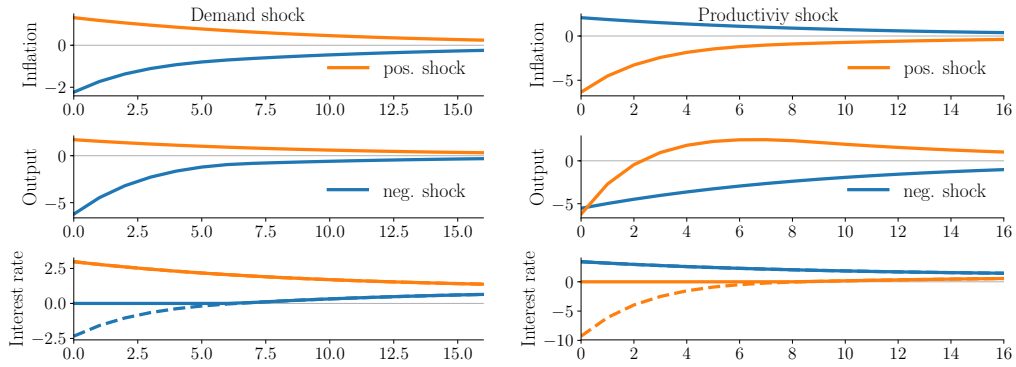


Figure 1: Impulse response functions with respect to a 1% productivity and demand shocks.

When a negative demand shock lowers the interest rate sufficiently for the ZLB to bind, the central bank becomes unable to prevent a strong deflationary effect of the shock by lowering the interest rate and stimulating demand. The expectations of further deflation raises the real interest rate and stalls consumption demand. The central banks inability to react to the shock, amplifies the negative effect of inflation and output. The asymmetry shown in the left panels of the graph reflect the far stronger response than in the case of a positive demand shock. In period 6 after the shock the zero lower bound is not binding anymore and responses become attenuated, and are now proportional to the positive shock. Note that at every point the algorithm perfectly captured and anticipated the future equilibrium path.

We now turn to the productivity shock. At period 0, the shock hits the economy and lowers the price level, and immediately drags inflation in the region where the ZLB is effective. Since the central bank can not further decrease the nominal interest rate, the real rate is very high due to low expectations on inflation, which leads to a drop in consumption as well. As the shock slowly fades out, inflation increases, which goes hand in hand with a decrease in the real interest rate. Now agents raise their demand above the steady state level. Perfectly anticipating that the central bank will raise the interest rate from period 8 onwards, the negative effect of the ZLB smoothes out and output transitions to the normal rational expectations path.

Now lets turn to the case, in which the monetary authority conducts forward guidance of its nominal interest rate path. By doing so, it affects the expectations of agents

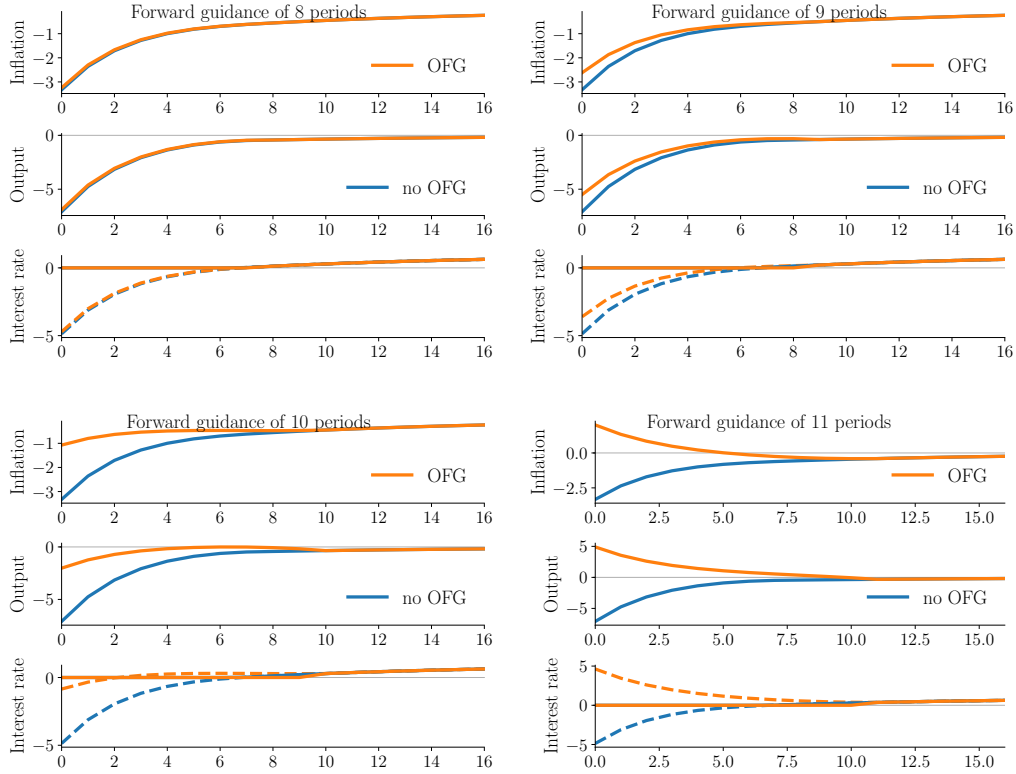


Figure 2: Forward Guidance over 8 periods (upper left) to 11 periods (lower right) whereas the Taylor rule without OFG would exit the ZLB after 6 periods. The dashed lines represent the interest rate as implied by the Taylor rule. The ZLB is caused by a strong negative demand shock.

regarding the duration of the ZLB episode. Let us have a look at Figure 2, which, given the state vector, shows how the trajectory evolves, if the central bank ex-ante commits to setting the interest rate to zero for the next  $\bar{k}$  periods. Deviations of inflation and output are lower than at the rational expectations path with  $k = k^*$ . The Figures also reveal why any  $\bar{k} > k^*$ , in the absence of any central bank commitment, can not be a rational rational expectations equilibrium. Let us call the interest rate that is implied by strictly following the Taylor rule to be the *sunny rate*. For  $\bar{k} = 9$  the exit date implied the sunny rate is roughly the same as without OFG. Given that the conditions from the above Theorem are satisfied, still  $k^* = 7$  is the only true and self-fulfilling value for  $k$  and hence any higher value can not be a rational equilibrium.

For our policy analysis this does also mean that, if central bank credibility would not matter, at this point the central bank has an incentive to deviate from the commitment and start raising interest rates. This effect is even stronger for  $\bar{k} = 10$ . Here the anticipation of low future interest rates merely bisects the initial response of output and inflation, and likewise the exit date implied by the sunny rate would be even earlier than  $k^*$ . This introduces another dimension to the problem of time inconsistency: switching

from the OFG-path back to the Taylor rule would set back the interest rate to zero and be accompanied with drops in output and inflation.

Finally, for  $\bar{k} = 11$ , in the lack of any other state variables the economy would jump upwards upon announcement with a sunny rate that is consistently not only above the ZLB but also above equilibrium level.

Furthermore, there exists a channel that can worsen the impact of a negative demand shock. This is the effect of uncertainty, which is not covered by our method due to its deterministic nature. The tail distribution of future demand shocks can possibly further prolong the ZLB period and promises further hazard. Without explicitly modelling this channel it should be clear that OFG can help to mitigate this uncertainty on when to exit the ZLB period.

	no ZLB	ZBL, no OFG	8	9	10 periods OFG
with hand-to-mouth	.80	1.63	1.93	3.44	7.03
w/o hand-to-mouth	.73	1.13	1.40	2.39	4.54

Table 2: Government spending multiplier for different durations of OFG. The spending shock is normalized to one percent of SS government consumption. For the case without hand-to-mouth agents we use a shock of smaller size, i.e.  $\sigma_v = .8$  to make results comparable and obtain similar baseline impacts on  $y$  and a  $k^* = 6$ .

Lastly, let us now turn to the effects of government spending at the ZLB, and to the role of OFG for the multiplier. In Table 2 we show the government spending multiplier together with the initial responses of output and inflation to a negative demand shock. The multiplier is small and less than one if the ZLB is not binding, a finding that is independent of the share of hand-to-mouth households. The multiplier is larger than one if the ZLB binds and increasing considerably when government spending is accompanied with OFG, whereas longer announced periods of low interest rates increase the multiplier further. This can be explained by the absence of a crowding-out effect, i.e. the Ricardian equivalence does not hold at the ZLB. While qualitatively this result is robust to the share of hand-to-mouth agents, a larger share of the latter also leads to higher multipliers for the similar reason that these are non-Ricardian agents that do not smooth consumption over time.

## 5 Conclusion

In this work we contrast Odyssean Forward Guidance to a monetary policy that follows the Taylor rule as soon, as the economic recovery allows for an exit from the zero lower bound. We find that, when facing strong negative demand shocks, the central bank can improve economic conditions by setting interest rates low for longer. Policy makers however face two problems. First, committing to a path of low interest rates for too long might lead to overshooting and create unnecessarily high inflation and overheat the economy.

Second, the central bank faces a problem of time inconsistency. While promising to keep rates low for long boosts the economy, this boost might provide the central bank with the option to switch back to a Taylor rule policy and to increase the interest rate earlier than announced. We show that while in some scenarios, this switch can push the economy right back to the zero lower bound, in others, the switch reinstates the

Taylor rule policy as in normal times and can potentially prevent an overshooting of the economy. In the latter case, the central bank has a rationale to renege on its earlier promise to keep interest rates low for long and the time-inconsistency problem emerges.

For our analyzes we present an analytical solution method for occasionally binding constraints in linearized difference equations. Due to the nature of analytical methods, this procedure does not rely on a grid and finds the exact solution for each point in the state space. We show that the method is numerically robust. We also show that the rational expectations equilibrium in our benchmark case is unique.

Time inconsistency is of particular relevance because deviating from a once promised path of interest rates impacts on the central bank credibility. This credibility is paramount for the functioning of the central bank, and especially so in the time after a severe crisis, which is the focus of our analysis. In the face of future and potentially large negative shocks, a credible commitment to forward guidance, untainted by strong incentive to forego the commitment can be a helpful tool in the toolbox of monetary policy.

## Appendix A Proofs

### Appendix A.1 Proof of Lemma 1

Using Equation 24 the function

$$f(\mathbf{y}) = \mathbf{N}^j \mathbf{y} + \mathbf{b} \sum_{0 \leq i < j} \mathbf{N}^i \quad (\text{A.1})$$

traces back the trajectory of the  $k^*$  equilibrium. For any  $k^*$  to be the *exact* number of periods at the constraint it is necessary that

- i) the trajectory was never in the unconstrained region before  $k = k^*$  (Condition 26)
- ii) any other  $k'$  for which  $L(k', \mathbf{v}_t) \geq \bar{x}$  is not an equilibrium because their trajectory already hits the unconstrained region earlier than in period  $k'$  (Condition 27).

### Appendix A.2 Proof of Theorem 1

Let us write out the conditions in Lemma 1 in more detail. The equilibrium of  $k^*$  is unique **iff**

$$\mathbf{b}L(k^*) \geq \bar{x} \quad \wedge \quad \mathbf{b}L_{k < k^*}(k^*) > \bar{x} \quad (\text{A.2})$$

$$\wedge \quad \mathbf{b}L(k^* + 1) < \bar{x} \quad \vee \quad \mathbf{b}L_{k^*}(k^* + 1) > \bar{x} \quad (\text{A.3})$$

$$\wedge \quad \mathbf{b}L(k^* + 2) < \bar{x} \quad \vee \quad \mathbf{b}L_{k^*}(k^* + 2) > \bar{x} \quad \vee \quad \mathbf{b}L_{k^*+1}(k^* + 2) > \bar{x} \quad (\text{A.4})$$

$$\wedge \quad \mathbf{b}L(k^* + 3) < \bar{x} \quad \vee \quad \mathbf{b}L_{k^*}(k^* + 3) > \bar{x} \quad \vee \quad \mathbf{b}L_{k^*+1}(k^* + 3) > \bar{x} \quad \vee \quad \dots \quad (\text{A.5})$$

From here it is clear that the equilibrium is unique if

- i) the system is unconstrained after  $k$  periods, then it is also unconstrained after  $k$  periods if it is expected to be unconstrained in  $k + 1$  periods,

$$\mathbf{b}L(k) \geq \bar{x} \implies \mathbf{b}L_k(k + 1) > \bar{x} \quad (\text{A.6})$$

- ii) the system is unconstrained in the period it is expected to be unconstrained when it is already unconstrained one period before

$$\mathbf{b}L_{k-1}(k) \geq \bar{x} \implies \mathbf{b}L(k) > \bar{x}. \quad (\text{A.7})$$

Compare the difference between the two systems for  $k = 1$ . Writing out (9) yields

$$\mathbf{M}_1 \mathbf{x}_t^u + \mathbf{M}_2 \mathbf{v}_t = \mathbf{P}_1 E_t \mathbf{x}_{t+1}^u + \mathbf{c}_x (\mathbf{b}_x \mathbf{x}_t^u + \mathbf{b}_v \mathbf{v}_t) \quad (\text{A.8})$$

$$\mathbf{M}_x \mathbf{x}_t^u + \mathbf{M}_v \mathbf{v}_t = \mathbf{v}_{t+1}^u + \mathbf{c}_v (\mathbf{b}_x \mathbf{x}_t^u + \mathbf{b}_v \mathbf{v}_t), \quad (\text{A.9})$$

for the unconstrained  $\mathbf{x}_t^u$  and setting  $\mathbf{x}_{t+1}^u = \mathbf{\Omega} \mathbf{v}_{t+1}^u$  enables to rewrite

$$(\mathbf{M}_1 - \mathbf{P}_1 \mathbf{\Omega} \mathbf{M}_x) \mathbf{x}_t^u + (\mathbf{M}_2 - \mathbf{P}_1 \mathbf{\Omega} \mathbf{M}_v) \mathbf{v}_t = (\mathbf{c}_x - \mathbf{P}_1 \mathbf{\Omega} \mathbf{c}_v) (\mathbf{b}_x \mathbf{x}_t^u + \mathbf{b}_v \mathbf{v}_t). \quad (\text{A.10})$$

Repeat the same under the assumption that  $\mathbf{y}_t$  is constrained. and again set  $\mathbf{x}_{t+1}^c = \mathbf{\Omega} \mathbf{v}_{t+1}^c$  in order to get

$$(\mathbf{M}_1 - \mathbf{P}_1 \mathbf{\Omega} \mathbf{M}_x) \mathbf{x}_t^c + (\mathbf{M}_1 - \mathbf{P}_1 \mathbf{\Omega} \mathbf{M}_v) \mathbf{v}_t = (\mathbf{c}_x - \mathbf{P}_1 \mathbf{\Omega} \mathbf{c}_v) \bar{x}. \quad (\text{A.11})$$

Combine Equations A.10 and A.11 and pre-multiply by  $\mathbf{b}$  to obtain

$$\mathbf{b}_x \mathbf{x}_t^c = \mathbf{b}_x \mathbf{x}_t^u + \underbrace{\psi (\mathbf{b}_x \mathbf{x}_t^u + \mathbf{b}_v \mathbf{v}_t - \bar{x})}_{>0 \text{ iff } \mathbf{y}_t \text{ unconstrained}} \quad (\text{A.12})$$

with  $\psi = \mathbf{b}_x (\mathbf{M}_1 - \mathbf{P}_1 \mathbf{\Omega} \mathbf{M}_x)^{-1} (\mathbf{c}_x - \mathbf{P}_1 \mathbf{\Omega} \mathbf{c}_v)$  being a scalar. If  $\psi > 0$  as stated in the theorem, then condition i. holds. Since the result implies as well that the constrained system is also moving further away from the constraint than the unconstrained system, condition ii. also holds. Additionally this ensures the existence of a solution.

## Appendix B Implementation of method

---

```
def S(k, v):          # implement solution here
    return (N**k)[:m,:m].dot(...)

def L(k, v):          # define the function L(k)v + l(k)
    ...

def F(v):              # the actual algorithm
    k = 0
    while b.dot(L(k, v)) - x_bar < 0:
        k +=1
    print 'Periods until unconstrained: ', k
    return S(k, v)

N, b, x_bar, ... = ... # give values to vectors and matrices
v = ...               # define the vector of state variables
x = F(v)
print 'Solution vector of endogenous variables: ', x
```

---

Figure B.3: Python-like code for the main solution method.

## Appendix C Writing the model in matrix form

With  $\mathbf{x}_t = \begin{bmatrix} \pi_t \\ y_t \end{bmatrix}$  and  $\mathbf{v}_t = \begin{bmatrix} v_t^\pi \\ v_t^y \end{bmatrix}$  we can transfer this nicely into matrix notation by

$$\mathbf{M}_1 \mathbf{x}_t = \mathbf{P} E_t \mathbf{x}_{t-1} + \mathbf{v}_t, \quad (\text{C.1})$$

$$\mathbf{R} \mathbf{v}_t = \mathbf{v}_{t+1} + \varepsilon_t, \quad (\text{C.2})$$



and then the system matrices are given by

$$\mathbf{M} = \begin{bmatrix} \mathbf{M}_1 & \mathbf{I} \\ \mathbf{0} & \mathbf{R} \end{bmatrix}, \quad \mathbf{N} = \begin{bmatrix} \mathbf{M}_1 & \mathbf{I} \\ \mathbf{0} & \mathbf{R} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{P} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix}, \quad \text{and} \quad \mathbf{c} = \begin{bmatrix} \mathbf{c}_x \\ \mathbf{0} \end{bmatrix}.^5 \quad (\text{C.3})$$

The constraint for being in the system  $\begin{bmatrix} \mathbf{x}_t \\ \mathbf{v}_t \end{bmatrix} = \mathbf{N}E_t \begin{bmatrix} \mathbf{x}_{t+1} \\ \mathbf{v}_{t+1} \end{bmatrix}$  is that  $\mathbf{b} \begin{bmatrix} \mathbf{x}_t \\ \mathbf{v}_t \end{bmatrix} - \bar{x} < 0$  where  $\mathbf{b} = [1 \ 0 \ 0 \ 0]$  and  $\bar{x} = \bar{i}$ . Convince yourself that the condition in the theorem is satisfied.

## Appendix D Derivation of the models equation

### Appendix D.1 Households

As stated in the main text, both types of households maximize the following life-time utility function

$$U_t = E_0 \sum_{t=0}^{\infty} \beta^t e^{V_t} \left( \log(C_t) - \frac{N_t^{1+\psi}}{1+\psi} \right)$$

We assume that  $C^o = C^r = C$  and  $N^o = N^r = N$  hold in steady state. The budget constraint of optimizing households is

$$P_t C_t^o + B_{t+1}^o = B_t^o(1 + R_t) + W_t N_t^o + T_t.$$

The linearized first order conditions of the optimizing household are the Euler Equation for its consumption, and the optimal labor supply.

$$\hat{C}_t^o = E_t[\hat{C}_{t+1}^o] - [(R_{t+1} - R) - \pi_{t+1} + V_{t+1} - V_t] \quad (\text{D.1})$$

$$\hat{W}_t = \hat{C}_t^o + \psi \hat{N}_t^o. \quad (\text{D.2})$$

The budget constraint of hand-to-mouth households is

$$P_t C_t^r = W_t N_t^r,$$

and the first order conditions of the hand-to-mouth household are

$$\hat{C}_t^r = \hat{W}_t + \hat{N}_t^r, \quad (\text{D.3})$$

$$\hat{W}_t = \hat{C}_t^r + \psi \hat{N}_t^r. \quad (\text{D.4})$$

A direct implications of the latter equations is, that the labor supply of hand-to-mouth consumers is constant (i.e.  $\hat{N}_t^r$ ), and it holds that  $\hat{C}_t^r = \hat{W}_t$ . Aggregating consumption and labor hours of the two household type yields

$$\hat{C} = \lambda \hat{C}_t^r + (1 - \lambda) \hat{C}_t^o, \quad (\text{D.5})$$

$$\hat{N} = \lambda \hat{N}_t^r + (1 - \lambda) \hat{N}_t^o = (1 - \lambda) \hat{N}_t^o. \quad (\text{D.6})$$

---

<sup>5</sup>For completeness,  $\mathbf{M}_1 = \begin{bmatrix} 1 & -\kappa \\ 0 & 1 \end{bmatrix}$ ,  $\mathbf{P} = \begin{bmatrix} \beta & 0 \\ 1 & 1 \end{bmatrix}$ ,  $\mathbf{R} = \begin{bmatrix} 1/\rho_\pi & 0 \\ 0 & 1/\rho_y \end{bmatrix}$  and  $\mathbf{c}_x = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$ .

The labor supply equations of both household types share can be aggregated to yield

$$\hat{W}_t = \hat{C}_t + \psi \hat{N}_t. \quad (\text{D.7})$$

#### Appendix D.2 Firms

Firms are monopolistic competitors. The production function of firm  $i$  is simply:  $Y_t(i) = N_t(i)$ . As firms face nominal rigidities as developed by (Calvo, 1983b), the known New Keynesian Phillips Curve (NKPC) emerges

$$\hat{\pi}_t = \beta E_t[\hat{\pi}_{t+1}] + \kappa \hat{M}C_t. \quad (\text{D.8})$$

Given the simple production function and the labor supply of households, one can rewrite the NKPC as:

$$\hat{\pi}_t = \beta E_t[\hat{\pi}_{t+1}] + \kappa \left( \hat{C}_t + \psi \hat{N}_t \right). \quad (\text{D.9})$$

The linearized good market clearing condition is

$$\hat{Y}_t = (1 - g)\hat{C}_t + g\hat{G}_t \quad (\text{D.10})$$

implies that one can write the NKPC in the following way

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \left( \left( \frac{1}{1 - g} + \psi \right) \hat{Y}_t - \frac{g}{1 - g} \hat{G}_t \right). \quad (\text{D.11})$$

#### Appendix D.3 Further equations and simplifications

So far our full set of equilibrium equations is:

$$\hat{C}_t^o = E_t[\hat{C}_{t+1}^o] - [(R_{t+1} - R) - \pi_{t+1} + V_{t+1} - V_t], \quad (\text{D.12})$$

$$\hat{W}_t = \hat{C}_t + \psi \hat{N}_t \quad (\text{D.13})$$

$$\hat{C}_t^r = \hat{W}_t, \quad (\text{D.14})$$

$$\hat{C} = \lambda \hat{C}_t^r + (1 - \lambda) \hat{C}_t^o, \quad (\text{D.15})$$

$$\hat{N} = (1 - \lambda) \hat{N}_t^o \quad (\text{D.16})$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \left( \left( \frac{1}{1 - g} + \psi \right) \hat{Y}_t - \frac{g}{1 - g} \hat{G}_t \right), \quad (\text{D.17})$$

$$\hat{Y}_t = \hat{N}_t \quad (\text{D.18})$$

$$\hat{Y}_t = (1 - g)\hat{C}_t + g\hat{G}_t \quad (\text{D.19})$$

$$R_{t+1} = \max \left\{ 0, \rho_R R_t + (1 - \rho_R) \left( R + \frac{\phi_1}{\beta} \pi_t + \frac{\phi_2}{\beta} \hat{Y}_t \right) \right\} \quad (\text{D.20})$$

$$\hat{G}_t = \rho_g \hat{G}_{t-1} + \epsilon_t^g \quad (\text{D.21})$$

$$\hat{V}_t = \rho_g \hat{V}_{t-1} + \epsilon_t^v \quad (\text{D.22})$$

Replacing  $\hat{N}_t^o$  with  $\frac{1}{1-\lambda}\hat{N}_t$ ,  $\hat{N}_t$  with  $\hat{Y}_t$ , and  $\hat{C}_t^r$  with  $W_t$  yields:

$$\hat{C}_t^o = E_t[\hat{C}_{t+1}^o] - [(R_{t+1} - R) - \pi_{t+1} + V_{t+1} - V_t], \quad (\text{D.23})$$

$$\hat{W}_t = \hat{C}_t + \psi \hat{Y}_t \quad (\text{D.24})$$

$$\hat{C}_t = \lambda \hat{W}_t + (1 - \lambda) \hat{C}_t^o, \quad (\text{D.25})$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \left( \left( \frac{1}{1-g} + \psi \right) \hat{Y}_t - \frac{g}{1-g} \hat{G}_t \right), \quad (\text{D.26})$$

$$\hat{Y}_t = (1-g) \hat{C}_t + g \hat{G}_t \quad (\text{D.27})$$

$$R_{t+1} = \max \left\{ 0, \rho_R R_t + (1 - \rho_R) \left( R + \frac{\phi_1}{\beta} \pi_t + \frac{\phi_2}{\beta} \hat{Y}_t \right) \right\} \quad (\text{D.28})$$

$$\hat{G}_t = \rho_g \hat{G}_{t-1} + \epsilon_t^g \quad (\text{D.29})$$

$$\hat{V}_t = \rho_g \hat{V}_{t-1} + \epsilon_t^v \quad (\text{D.30})$$

Plugging (24) in (25) yields

$$\hat{C}_t = \lambda \left( \hat{C}_t + \psi \hat{Y}_t \right) + (1 - \lambda) \hat{C}_t^o \quad (\text{D.31})$$

or

$$\hat{C}_t^o = \hat{C}_t - \frac{\lambda \psi}{(1 - \lambda)} \hat{Y}_t \quad (\text{D.32})$$

Thus (31) can be written as:

$$\left( \hat{C}_t - \frac{\lambda \psi}{(1 - \lambda)} \hat{Y}_t \right) = E_t[\hat{C}_{t+1} - \frac{\lambda \psi}{(1 - \lambda)} \hat{Y}_{t+1}] - [(R_{t+1} - R) - \pi_{t+1} + V_{t+1} - V_t], \quad (\text{D.33})$$

$$(\text{D.34})$$

Lastly, one plug the aggregate good market clearing condition into the Euler Equation for consumption and multiply both sides of the equation with  $(1 - g)$  to obtain

$$(\hat{Y}_t - g \hat{G}_t - \frac{\lambda \psi (1 - g)}{(1 - \lambda)} \hat{Y}_t) = E_t[\hat{Y}_{t+1} - g \hat{G}_{t+1} - \frac{\lambda \psi (1 - g)}{(1 - \lambda)} \hat{Y}_{t+1}] - (1 - g)[(R_{t+1} - R) - \pi_{t+1} + V_{t+1} - V_t], \quad (\text{D.35})$$

$$(\text{D.36})$$

Rearranging the equation yields.

$$\Theta \hat{Y}_t - g \hat{G}_t = E_t[\Theta \hat{Y}_{t+1} - g \hat{G}_{t+1}] - (1 - g)[(R_{t+1} - R) - \pi_{t+1} + V_{t+1} - V_t] \quad (\text{D.37})$$

with  $\Theta = \left(1 - \frac{\lambda\psi(1-g)}{(1-\lambda)}\right)$ . Alternatively

$$\hat{Y}_t - \frac{g}{\Theta} \hat{G}_t = E_t[\hat{Y}_{t+1} - \frac{g}{\Theta} \hat{G}_{t+1}] - \frac{(1-g)}{\Theta} [(R_{t+1} - R) - \pi_{t+1} + V_{t+1} - V_t] \quad (\text{D.38})$$

Thus, our final set of equation in the Variables  $(Y_t, R_t, \pi_t, \hat{G}_t, \hat{V}_t)$  reads:

$$\hat{Y}_t - \frac{g}{\Theta} \hat{G}_t = E_t[\hat{Y}_{t+1} - \frac{g}{\Theta} \hat{G}_{t+1}] - \frac{(1-g)}{\Theta} [(R_{t+1} - R) - \pi_{t+1} + V_{t+1} - V_t] \quad (\text{D.39})$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \left( \left( \frac{1}{1-g} + \psi \right) \hat{Y}_t - \frac{g}{1-g} \hat{G}_t \right), \quad (\text{D.40})$$

$$R_{t+1} = \max \left\{ 0, \rho_R R_t + (1 - \rho_R) \left( R + \frac{\phi_1}{\beta} \pi_t + \frac{\phi_2}{\beta} \hat{Y}_t \right) \right\} \quad (\text{D.41})$$

$$\hat{G}_t = \rho_g \hat{G}_{t-1} + \epsilon_t^g \quad (\text{D.42})$$

$$\hat{V}_t = \rho_g \hat{V}_{t-1} + \epsilon_t^v \quad (\text{D.43})$$

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