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Tenure Choice, Portfolio Structure and Long-term Care - Optimal Risk Management in Retirement

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Tenure Choice, Portfolio Structure and Long-term Care - Optimal Risk Management in Retirement*

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Abstract

Our study analyzes the savings behavior of elderly and highlights the interplay between tenure decisions, stock market investment and long-term care risk. Housing equity serves a dual purpose as a consumption good and as an asset, consequently it is important for the optimal risk structure of the financial portfolio. In addition, recent contributions also point out its implicit insurance provision to buffer long-term care shocks. Our stylized life cycle model captures these links and indicates that in Germany long-term care risks may be an important driver for homeownership. In our preferred set-up housing equity is a rather low-risky investment that even encourages stock market participation among elderly homeowners.

JEL Classifications: C61, D15, G11, H55

Keywords: Homeownership, Life-cycle models, Stock market participation, Long-term care insurance provision

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1 Introduction

The importance of housing equity in retirement wealth is documented since a long time. With the notably exception of Germany and the Netherlands, typically more than 70 percent of households in OECD countries own a home before retirement, see Chiuri and Jappelli (2010, 649). Housing equity is by far the most valuable asset in most developed countries, dwarfing retirement accounts and other financial and nonfinancial assets. Recent evidence from the Eurosystem Household Finance and Consumption Survey (HFCS) indicates that in Europe residential property accounts for roughly 50% of wealth of households, see Mathä et al. (2017).

Two specific aspects of homeownership have received specific attention: its role as a key determinant of the financial portfolio and – more recently – its role as a "precautionary savings device" that buffers health and especially long-term care shocks in old age.

The interplay between homeownership and stock market participation has a long history in the literature, but the findings are rather inconclusive. Owning a house introduces asset price risk and may even lead to higher liquidity risk (due to the indivisible and illiquid nature of a house). Consequently, homeownership seems to reduce the demand for risky financial assets, see Hochguertel and van Soest (2001) for the Netherlands or Yamashita (2003) for the U.S. On the other hand, homeowners are perfectly hedged against rent fluctuations (Sinai and Souleles, 2005) and inflation (Wu and Pandey, 2012) which may in turn encourage their risk taking in financial assets. Not surprisingly, Beaubrun-Diant and Maury (2016) present evidence from the Panel Study of Income Dynamics (PSID) in the U.S. that the fraction of stockholders is significantly higher among homeowners than among renters while Michielsen et al. (2016) find no effect at all of home equity and mortgage debt on the risky asset share of Dutch households. Chetty et al. (2018) argue that home equity and mortgage debt have a different impact on stockholding. While rising residential prices reduce stockholding, increases in home equity wealth raise stockholding.

The seemingly contradictory empirical evidence is also evident in dynamic life-cycle models that highlight the interaction of tenure and portfolio choice. The seminal study in this direction was Cocco (2005), who still abstracts from tenure choice and models housing as a durable consumption good and a risky investment. Owning a house generates returns in the form of consumption services from which the investor derives utility. However, house prices are uncertain and homeowners may incur large losses when they sell their house later in life. As a consequence, homeownership crowds out stock market investment and therefore helps to explain the empirically observed low participation rates on stock markets. Previous studies (without housing) required extremely high fix cost in order to generate realistic stock market investment levels. The combination of house price risk, minimum house size and high transaction cost generates observed patterns of housing consumption over the life cycle and empirically plausi-
ble correlations in leverage and stock holdings. Yao and Zhang (2005) extend this approach by incorporating the choice between renting and owning a house. They find that investors choose substantially different portfolio allocations when owning a house versus when renting housing services. Homeowners substitute home equity for risky stocks, but hold a higher equity proportion in their liquid financial portfolio. They also quantify the welfare cost from suboptimal allocations of housing wealth (i.e. no choice, only renting or only owning) over the life cycle. Hu (2005) also considers the tenure decision, but in his approach owning a house generates a higher utility than renting it. Again he documents the negative impact of homeownership on stock market participation. Exactly the opposite result is derived by Vestman (2017) in a model with preference heterogeneity where households with a large savings motive choose to become homeowners and stock market participants, while those with small savings motives become renters and participate much less in the stock market. These results are confirmed by U.S. and Swedish savings data.

More recently, a second aspect of homeownership has gained increasing attention. Already Venti and Wise (2004), Painter and Lee (2009) or Poterba et al. (2011) point out that relatively few U.S. retirees tap into their housing equity to finance ordinary consumption needs. Instead it is rather used to buffer shocks to the family status (i.e. divorce, widowhood) or health (i.e. entry to a nursing home). Very similar findings are confirmed for Germany (Keese, 2012) and Great Britain (Banks et al., 2012), while Angelini et al. (2014) provide evidence for 13 European countries using life history data from the Survey of Health, Ageing and Retirement in Europe (SHARELIFE). Davidoff (2009, 2010) was among the first who analyzed in a life-cycle model the role of home equity as an insurance against long-term care (LTC) risk in old age. As already discussed above, home equity is a large fraction of the wealth for the elderly and tends to be liquidated only late in life or when in long-term care. When hit by LTC shocks, homeowners may convert their equity into a reverse mortgage. The latter reduces the demand for LTC insurance especially for illiquid homeowners who hold annuities to eliminate longevity risk. Consequently, illiquid homeowners who hold annuities in the form of pension products will hardly demand LTC insurance products on private markets, see also Laferre (2012). Nakajima and Telyukova (2013) estimate a structural model of saving and housing decisions for retired households who face benefits from homeownership, longevity risk, uncertain medical expenses and borrowing constraints. In this model set-up, homeowners dissave slower than renters since they prefer to stay in their house as long as possible. Nakajima and Telyukova (2013) quantify the role of the different forces to understand their interactions with household savings. Their simulations indicate that without considering homeownership retirees’ net worth would be 28-53 % lower in the U.S. Using the same model, Nakajima and Telyukova (2017) analyze the welfare consequences of reverse mortgage programs. On average the expected welfare gain of reverse mortgages is equivalent to just $ 252 or 0.84% of annual after-tax income. However, welfare gains are much larger for lower income and for older households.
We will combine both strands of the literature in order to analyze the interplay between savings behavior, tenure choice and long-term care risk in Germany. For various reasons, Germany seems to be an especially interesting country to study these issues. On the one side, the German homeownership rate is among the lowest in Europe since many years. Currently only 44% of German households are homeowners compared to 60% in the whole Euro area, see Matthä et al. (2017). Overall, the real estate market in Germany is quite different than in Anglo-Saxon and other European countries. The existence of a sophisticated rental market as well as the German insistence on prudential lending arrangements dampen homeownership and bank default risk. Since homes strongly reflect individual tastes and ownership is considered as a long-term investment which reduces regional mobility, turnover rates are very low and residential price dynamics are very modest. As a consequence, the German housing market was one of the few that avoided a slump in the wake of the 2008-2009 global financial crisis, see Voigtlander (2014), Matthä et al. (2017) as well as Kholodilin et al. (2017). On the other side, Germany was one of the first countries that introduced a compulsory long-term care insurance (LTCI) in 1995. Individuals are eligible to claim benefits from the LTCI if they are in need of care because of an illness or disability. Since the most recent reform in 2017 the condition "in need of care" is derived in five grades which determine the benefit in cash and in kind. At the highest grade the maximum benefit for inpatient care reaches more than 2000 € per month. However, as argued by the Verband der Privaten Krankenversicherer (2016), the LTCI benefits only cover a share of roughly 40-50% of total care cost at any grade level. The remaining difference between costs of care and LTCI benefits has to be borne by the person in need of care. If the care taker has no further resources, children or near relatives have to pay before ultimately social assistance steps in. Consequently, despite the LTCI, long-term care risk still plays an important role for retired households in Germany especially when they care for their children.

In the following we will present empirical facts reflecting saving and homeownership patterns in Germany. Afterwards we develop a life-cycle simulation model that captures the interaction between tenure choice, portfolio choice and LTC risk in retirement. In our set-up, homeowners will sell their property when hit by a LTC shock, even without reverse mortgage arrangements. As we will show, this feature will have a significant positive impact on homeownership already before retirement.

2 Homeownership and portfolio choices in Germany

In this section we analyze the second wave of the Eurosystem Household Finance and Consumption Survey (HFCS) which was collected in Germany between beginning 2013 and 2015 and released in December 2016. The HFCS collects household level data on earnings, wealth

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1 A detailed description of the survey methodology of the HFCS can be found in ECB (2016).
and consumption in twenty European Union countries which is comparable. Similar as in the U.S. Survey of Consumer Finances (SCF), wealthy households are oversampled but the provision of specific household weights allows to cover the total number of households in each country. The German sample has a size of 4,461 households which represent more than 40 mio. households. In order to make the sample more compatible with our model, we eliminated from the total sample those households who were younger than 30 years and had already a net wealth of more than 500,000 €. In addition, we eliminated homeowners where the mortgage was higher than the house value and renters who had other real estate. These adjustments reduced the sample size to 4,122 households which represent more than 36 mio. households in Germany. Table 1 gives some summary statistics about annual labor income and assets in our (slightly adjusted) data set.

<table>
<thead>
<tr>
<th></th>
<th>Homeowner</th>
<th>Renter</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Annual labor income</td>
<td>36,260</td>
<td>21,530</td>
<td>28,330</td>
</tr>
<tr>
<td>Net wealth</td>
<td>427,470</td>
<td>29,880</td>
<td>213,310</td>
</tr>
<tr>
<td>Financial assets</td>
<td>83,290</td>
<td>25,370</td>
<td>52,095</td>
</tr>
<tr>
<td>Deposits/Bonds</td>
<td>40,860</td>
<td>14,400</td>
<td>26,610</td>
</tr>
<tr>
<td>Stocks</td>
<td>16,995</td>
<td>3,885</td>
<td>9,935</td>
</tr>
<tr>
<td>Pensions/Life insurance</td>
<td>20,180</td>
<td>5,850</td>
<td>12,460</td>
</tr>
<tr>
<td>Other financial assets</td>
<td>5,255</td>
<td>1,235</td>
<td>3,090</td>
</tr>
<tr>
<td>Real assets</td>
<td>390,140</td>
<td>7,355</td>
<td>183,955</td>
</tr>
<tr>
<td>Real estate (HMR)</td>
<td>233,060</td>
<td>–</td>
<td>107,520</td>
</tr>
<tr>
<td>Real estate (other)</td>
<td>85,335</td>
<td>–</td>
<td>39,370</td>
</tr>
<tr>
<td>Other real assets</td>
<td>71,745</td>
<td>7,355</td>
<td>37,065</td>
</tr>
<tr>
<td>Debt</td>
<td>45,960</td>
<td>2,845</td>
<td>22,740</td>
</tr>
<tr>
<td>Mortgage (HMR)</td>
<td>31,880</td>
<td>–</td>
<td>14,710</td>
</tr>
<tr>
<td>Mortgage (other)</td>
<td>10,780</td>
<td>–</td>
<td>4,975</td>
</tr>
<tr>
<td>Other debt</td>
<td>3,300</td>
<td>2,845</td>
<td>3,055</td>
</tr>
<tr>
<td>Observations (unweighted)</td>
<td>2,555</td>
<td>1567</td>
<td>4,122</td>
</tr>
<tr>
<td>Observations (weighted)</td>
<td>16993,457</td>
<td>19839,929</td>
<td>36833,387</td>
</tr>
<tr>
<td>Fractions (in %)</td>
<td>46</td>
<td>54</td>
<td>100</td>
</tr>
<tr>
<td>Stock market participation rates (in %)</td>
<td>34</td>
<td>18</td>
<td>25</td>
</tr>
</tbody>
</table>

Source: Own calculations based on second wave of HFCS.

Homeowners are defined by having declared real estate as their home main residence (HMR). Table 1 reveals that about 46% of our sample households are then homeowners. They have a significantly higher annual labor income than renters and participate much more than renters in the stock market. On average, their net wealth is roughly 427,500 € and therefore almost fifteen times higher than that of renters. As one would expect, real estate dominates their net wealth while financial assets are mainly held as deposits in checking accounts. Mortgages only amount to roughly 13% of the total real estate held.
Next we group our observations into 13 data cells of 5-year age groups ranging from 25-29 until 85-89. Then we split each data cell into net wealth quintiles and compute the ownership shares for the whole data cell as well as its top and bottom wealth quintile. The left part of Figure 1 shows the homeownership rates over the life cycle for the three considered groups. Not surprisingly, homeownership rises at younger ages and reaches an average share of roughly 60 per cent in the age groups 55-75. Afterwards it falls again slightly. Figure 1 also documents a striking difference between the top and bottom wealth quintile. While the lowest quintile consists of mainly renters, households in the top income quintile are typically homeowners after age 45.

The right part of Figure 1 reports for the same groups the average share of mortgages as a percentage of house values over the life cycle. Of course, depending on the amount of own savings when buying the house, mortgage shares will vary quite substantially across households. In addition, especially when buying a house at younger ages typically bequests reduce the necessary amount of mortgages. Consequently, when households start to buy a house at younger ages mortgage shares rise until they reach for the lowest wealth quintile a level of about 70 per cent of the house value at age 35. This fraction then falls in the following years. Of course, for the top wealth quintile the mortgage share decreases considerable and reaches only about 20 per cent at age 40. The profile of average mortgage shares lies in between. After age 65 less than 10 per cent of the house value is covered by a mortgage.

Next we compare the net wealth profiles (relative to average income) along the life cycle for both owners and renters in the left part of Figure 2. Already at young ages owners have a net wealth of 200,000 € (probably due to bequests) which is roughly seven times average income. After age 35 their net wealth rises steeply up to roughly 600,000 € (or about twenty times average income) around age 62 when they retire. Afterwards net wealth decreases again as one would expect. In contrast, the net wealth profile of renters is fairly flat around 50,000 €.

**Figure 1: Homeownership and Mortgage shares**

Source: Own calculations.
Finally, we compare participation rates in the stock market for homeowners and renters in the right part of Figure 2. Both participation rates seem to be (slightly) higher during retirement than during working years. In almost all years, homeowners have higher participation rates than renters.

Consequently, our results for Germany are in line with the results from Vestman (2017) or Chetty et al. (2018) and cast some doubt that homeowners invest less in risky assets than renters. Although we do not document the shares of risky assets explicitly, the data at least indicates that homeowners do not have a lower share of risky assets as predicted by the older literature cited before. In our opinion this is at least partly due to the fairly stable housing market in Germany. This specific feature will be therefore incorporated in the simulation model which we develop in the next section.

3 The Model

Our model is a discrete time life-cycle model where each period corresponds to one year. Households face income, health and lifespan uncertainty and need to decide about consumption, savings, homeownership and stock market participation. Our benchmark calibration abstracts from correlations between risky stocks and labor income shocks. During retirement, households may experience an irreversible health shock (think of long-term care), which reduces their resources, their life expectancy but also the utility from living in their own house. The latter establishes a link between homeownership and long-term care insurance despite the omitted reverse mortgage market. As long as they are healthy, households have a preference for owning a house instead of renting it. Tenure choice is further influenced by the regulations regarding minimum housing investment and down payment requirements.
3.1 Demographic structure and household preferences

More formally we assume that our household’s life starts at age 1 and may finally end at some maximum age \(J\), where \(j = 1, \ldots, J\) denotes the actual age. At the beginning of life, individuals are assigned a skill level \(\theta\) which affects their labor income until forced retirement at age \(j_r\). Survival from one period to the next is stochastic where \(\psi_j(s_{j-1})\) denotes the conditional survival probability from age \(j - 1\) to age \(j\), which depends on the health status \(s_{j-1} \in (0, 1)\) of the previous age \(j - 1\) with \(\psi_j(0) > \psi_j(1)\) and \(\psi_{J+1}(s_J) = 0\).

Households receive utility from ordinary consumption \(c\) and housing services \(c_h\). They may also value leaving housing and asset equities to their heirs. The momentary utility function is

\[
u(c, c_h) = c^\alpha (\chi c_h)^{1-\alpha}\]

where \(\alpha\) denotes the Cobb-Douglas expenditure share of ordinary consumption. Consumption of housing services may be derived from ownership or from rental housing and also depends on the health status. Similar as Hu (2005) or Nakajima and Telyukova (2013, 2017) we assume a preference for ownership for healthy households while this preference disappears for households who are in need of long-term care. Homeowners consume their housing equity \(h > h_{\text{min}}\) as housing services (i.e. \(c_h = h\)), while renters (where \(h = 0\)) have to purchase housing services on the rental market. The taste parameter is set to \(\chi < 1\) whenever the household is either a renter or in bad health (i.e. \(s = 1\)), in the other case (homeowner in good health) we set \(\chi = 1\). This so-called “pride of ownership” term in the utility function is meant to capture insurance properties or tax advantages of homeownership or the full flexibility in house adjustment. The utility cost from long-term care may reflect the fact that in case of LTC need specific investments in the own house are no longer important.\(^2\)

3.2 Income process over life-cycle

Agents start their working life at age 1 and, conditional on surviving, retire at age \(j_r\). In each period during the working years, an agent receives an endowment of labor productivity, which she can supply to the market at the wage rate \(w\). Labor supply is inelastic and equal to unity, so that changes in labor productivity translate one-to-one into changes in labor income \(y_j\). Labor productivity is a function of a deterministic, skill-specific age-profile of earnings \(e_j(\theta)\) and two transitory components \(\eta_j(\theta)\) and \(\xi_j\). While the fixed productivity \(\theta\) is drawn at the beginning of the life cycle, the second component \(\eta_j\) has an AR(1) autoregressive structure so that

\[
\eta_j = \rho \eta_{j-1} + \varepsilon_j \quad \text{with} \quad \varepsilon_j \sim N(0, \sigma^2) \quad \text{and} \quad \eta_1 = 0.
\]

\(^2\) However, Rouwendahl and Thomese (2013) find that in the Netherlands homeowners move later in nursing homes than renters. Therefore one may also argue that the utility from the own house even rises following a LTC shock.
The idiosyncratic innovation term $\epsilon_j$ is also normally distributed with mean zero and variance $\sigma^2_\epsilon$. Both, the autoregressive correlation term $\rho$ as well as the variance of the innovation term $\sigma^2_\epsilon$ are assumed to be skill-specific, see Fehr et al. (2013). In contrast, the white noise component $\zeta_j$ is skill-independent and normally distributed with mean zero and variance $\sigma^2_\zeta$. Household’s labor income is therefore

$$y_j = \begin{cases} \omega e_j(\theta) \cdot \exp(\eta_j(\theta) + \zeta_j) & \text{for } j < j_r \\ 0 & \text{otherwise} \end{cases}$$

At age $j_r$ the household retires and $\epsilon_j$ and therefore labor productivity drops to zero. We assume that throughout retirement we have $\eta_{j+1} = \eta_j$. In this phase the agent receives pension income $pen$ which is assumed to be a constant fraction of the last working period’s permanent income, i.e.

$$pen = \begin{cases} \kappa_1 \cdot we_{j-1}(\theta) \cdot \exp(\eta_{j-1}(\theta)) & \text{for } j \geq j_r \\ 0 & \text{otherwise} \end{cases}$$

During the retirement phase the household may experience a long-term care shock. In this case, health changes to the state $s = 1$, survival probabilities fall and additional cost $m = \kappa_2 \cdot \bar{g}$ will arise for the remaining lifetime. For simplicity we assume that the latter are a constant fraction $\kappa_2$ of average labor income.

### 3.3 House transaction cost and stock market participation cost

In each period, households have to decide how much to consume and how much to save and whether to become a renter or owner in the next period. Households who want to buy a house have to select a specific house size $h^+$ and the amount of financial assets $a_f^+$ they want to save. The difference between financial assets and maximum mortgage $\xi h^+$ the household could take out against the house determines either the actual mortgage debt or liquid assets which could be invested on the financial market. The maximum mortgage is fixed by the maximum loan-to-value ratio $\xi$ which is specified exogenously. As a consequence, households split up their aggregate savings $a^+$ into the selected house size $h^+$, the resulting transaction cost $tr(h, h^+)$ of changing the house size and either mortgage debt or liquid assets, i.e. $a^+ = h^+ + tr(h, h^+) + (a_f^+ - \xi h^+)$. In order to select a specific house size $h^+$, the household chooses $\omega_{h^+}$ as a fraction of his total assets which are required for minimum down payment, i.e. $h^+ = \frac{\omega_{h^+} a^+}{1 - \xi}$. Financial assets are then determined as $a_f^+ = (1 - \omega_{h^+}) a^+ - tr(h, h^+)$. Households who become renter (i.e. $\omega_{h^+} = h^+ = 0$) can invest all their savings net of transaction cost on the financial market,
i.e. $a_f^+ = a^+ - tr(h, 0)$. Transaction cost $tr(h, h^+)$ only apply to homeowners when they change their house value by more than $\varphi$ per cent:

$$
tr(h, h^+) = \begin{cases} 
0 & \text{if } h^+ \in [(1 - \varphi)h; (1 + \varphi)h], \\
\phi_1 ph + \phi_2 p^+ h^+ & \text{otherwise.}
\end{cases}
$$

Following Yang (2009) homeowners may change their housing consumption by undertaking housing renovation or by allowing depreciation up to a fraction of $\varphi$ their house size. If the size is reduced more or appreciated more than this fraction then it is assumed that the house has been sold. In the latter case the household has to pay transaction cost which are a fraction $\phi_1$ of its selling value and a fraction $\phi_2$ of its buying value.

Financial investors who hold liquid wealth ($a_f^+ - \zeta h^+ > 0$) may split it up into a fraction $\omega^+$ of stocks and a fraction $1 - \omega^+$ of bonds. In case they enter the stock market, households must pay a one-time entry cost which is modeled as a fraction $\kappa_3$ of average labor income $\bar{y}$. The binary state variable $I_j$ keeps track of whether entry has occurred up until age $j$. We therefore define

$$
I_j = \begin{cases} 
1 & \text{if } I_{j-1} = 1 \text{ or } \omega > 0 \\
0 & \text{otherwise}
\end{cases}
$$

with $I_{j+1} = 0$.

### 3.4 Financial market and relative prices

The return on households stock holdings $r(\vartheta)$ evolves stochastically where we assume the following relation between $r$ with the risk free rate $r_f$:

$$
r(\vartheta) - r_f = \mu_r + \vartheta.
$$

The excess return of the risky asset over the risk free rate is defined by the risk premium $\mu_r$ and the interest rate shock $\vartheta$. The latter is independent over time and may be correlated with the white noise shock $\zeta$, so that we have

$$
\left[ \begin{array}{c} \zeta \\ \vartheta \end{array} \right] \sim N \left( \left[ \begin{array}{c} 0 \\ 0 \end{array} \right], \left[ \begin{array}{cc} \sigma_\zeta^2 & \sigma_\zeta \sigma_\vartheta \\
\sigma_\zeta \sigma_\vartheta & \sigma_\vartheta^2 \end{array} \right] \right).
$$

Similarly, the price of housing consumption $p$ to non-housing consumption may also evolve stochastically as

$$
p^+ = \exp(v)
$$
where \( \nu \) defines the i.i.d. normally distributed price shock with mean zero and variance \( \sigma_\nu^2 \). Again, the latter may be correlated with the interest rate shock:

\[
\begin{bmatrix}
\nu \\
\theta
\end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\
0 
\end{bmatrix}, \begin{bmatrix}
\sigma_\nu^2 & \rho_2 \sigma_\nu \sigma_\theta \\
\rho_2 \sigma_\nu \sigma_\theta & \sigma_\theta^2
\end{bmatrix}\right).
\]

Consequently, the total value of financial savings in the next period can then be written as

\[
R_p(\omega^+, \theta^+) (a_j^+ - \xi h^+),
\]

where

\[
R_p(\omega^+, \theta^+) = \begin{cases}
1 + r_f + \omega^+ \cdot (\mu_r + \theta^+) & \text{if } a_j^+ > \xi h^+ \\
1 + r_f + r_p & \text{otherwise}
\end{cases}
\]

The household therefore first pays back the mortgage debt (i.e. when \( a_j^+ < \xi h^+ \)) where the mortgage interest rate \( r_f + r_p \) might be above the risk free rate. He only invests in the stock market when the mortgage is fully eliminated.

### 3.5 Household’s optimization problem

Given the specific structure of the model we can define the problem with cash-on-hand \( X \), so that the state vector is then defined by \( z = (j, X, s, \theta, \eta, I, h) \) where \( h = 0 \) for a renter. In what follows, we solve the problem recursively defining the set of control variables by \( \Omega = (c, a^+, \omega_h^+, \omega^+, I^+) \) where \( I^+ \) is a binary variable that changes from zero to one as soon as the investor makes his first investment in the stock market. Afterwards it remains at one forever.

Following Vestman (2017) we assume that every household has Epstein-Zin (1991) preferences over momentary utility and bequest so that the optimization problem of the household then reads

\[
V(z) = \max_{\Omega} \left\{ u(c, c_h)^{1-\frac{1}{\gamma}} + \beta \left[ \psi_{j+1}(s) \left( 1 + a_j^{\omega_h^+} E \left[ V(z^+) \right]^{\frac{1}{1-v}} \right) + 1 \right] E \left[ V(z^+) \right]^{\frac{1}{1-v}} \right\}
\]

where \( \beta \) denotes the discount rate, \( \gamma \) is the the intertemporal elasticity of substitution (IES) between momentary utility and expected future utility, \( E \) is a conditional expectations operator and \( \nu \) is the coefficient of relative risk aversion (RRA). The parameters \( b \) and \( q \) in the bequest function capture the strength of the bequest motive and the luxury good property, see De Nardi (2004).
Current homeowners (i.e. where $h > h_{\text{min}}$) are subject to the constraints

\[
I^+ = \begin{cases} 
1 & \text{if } I = 1 \text{ or } I = 0 \text{ and } \omega^+ > 0 \\
0 & \text{otherwise.}
\end{cases}
\]

\[
a^+ = X - c, \quad 0 \leq \omega^+, \omega_h^+ \leq 1, \quad a^+ \geq tr(h, h^+), \quad h^+ = \frac{\omega_h^+ a^+}{1 - \xi} \geq h_{\text{min}},
\]

\[
a_f^+ = (1 - \omega_h^+) a^+ - tr(h, h^+)
\]

\[
X^+ = R_p(\omega^+, \vartheta^+) (a_f^+ - \xi h^+) + y^+ + pen^+ + (1 - \delta_h) p^+ h^+ - 1_{s^+ = 1} m^+ - (I^+ - I) \kappa \tilde{y}
\]

with $1_{k=x}$ denoting an indicator function that returns 1 if $k = x$ and 0 if $k \neq x$

The expectation operators $E$ in equation (1) are with respect to the stochastic processes of productivity $\eta$ and health $s$. Current resources are split between consumption and aggregate assets. Households who buy a house are restricted to a maximum loan-to-value ratio $\xi$ and a minimum house size $h_{\text{min}}$. In case of borrowing, households face a fixed interest rate of $r_f + r_p$. Future resources $X^+$ are derived from financial assets (including interest), gross labor income, pensions, housing investments net of depreciation $\delta_h$, LTC and stock market participation cost.

The objective function of a current renter is quite similar except that we have $h = 0$ so that the current constraint changes to

\[
X = c + r_h c_h + a^+.
\]

Renters have to pay rent $r_h$ per housing unit $c_h$. Of course, they face the same restrictions on future resources, future house size and borrowing as current owners. The rental price $r_h$ is linked to the return of financial assets through

\[
r_h = r_f + \delta_h + \mu_h
\]

which makes sure that renters implicitly bear the maintenance cost of the house plus some additional cost $\mu_h$. Consequently, tenure choice and portfolio allocation decisions over a households life cycle are closely interlinked and provide an insurance and consumption value.

When looking at the optimization problem (1), we recognize that it has a particular structure that allows us to solve it in the following five steps:

1. **Equity exposure in liquid wealth:** Given a current state $\bar{z} = (j, s, \theta, \eta, h, I)$ as well as future financial assets $a_f^+$ and house size $h^+$, we can solve the household’s portfolio optimization problem at age $j$ which yields $\omega^+ = \omega(\bar{z}, a_f^+, h^+)$. Of course, in case of a future renter we only need to derive $\omega(\bar{z}, a_f^+)$.

2. **Stock market participation decision:** Given a current state $\bar{z} = (j, s, \theta, \eta, h, I = 0)$ we have to derive the participation decision by comparing the respective value functions without
participation (i.e. where \( I^+ = 0 \) and \( \omega^+ = 0 \)) and with participation (i.e. where \( I^+ = 1 \) and \( \omega^+ = \omega(\tilde{z}, a^+_h, h^+) \)). Of course, in case the household has already participated in the past (i.e. \( I = 0 \)), no further decision is required at this stage.

3. **Wealth exposure in real estate**: Given a current state \( \tilde{z} \) as well as the optimal equity exposure \( \omega^+ \) and participation \( I^+ \), we need to split total savings \( a^+ \) between financial and housing assets which yields \( \omega^+_h = \omega_h(\tilde{z}, a^+) \). In case of a future renter household we simply set \( \omega^+_h = \omega_h(\tilde{z}, a^+) = 0 \).

4. **The consumption-savings decision**: Given a current state \( z \) (i.e. including resources \( X \)) and the optimal split between financial and housing assets \( \omega^+_h \), we can solve the consumption savings decision depending on future homeownership status \( o^+ \) in order to get \( c(z, o^+) \) and \( a^+(z, o^+) \) for a (current) homeowner and in addition \( c_h(z, o^+) \) for a (current) renter.

5. **The tenant decision**: Finally, given optimal consumption and savings for both ownership options \( o^+ = O \) and \( o^+ = R \), we can determine the respective value functions and select the optimal future homeownership \( o^+(z) \).

In order to save space the optimization procedure is explained in the appendix in more detail. The following section describes the parameter choice for the benchmark model before we present some simulation results.

### 4 Calibration of the benchmark model

The models time period is one year, agents therefore start life at age 20 (\( j = 1 \)), are forced to retire at age 65 (\( j_r = 45 \)) and can live at maximum up to age 100 (\( J = 80 \)). Survival probabilities for households are taken from the 2012/14 Life Tables for Germany reported in Statistisches Bundesamt (2016). They are slightly increased for those households in good health \( \psi_j(0) \) and reduced by about 15 percent for those households who have received a LTC shock. The probability to become a LTC patient in a certain year is rising from 0.5% at age 65 up to 15% at age 100, so that we roughly match the fractions of LTC patients among the older cohorts as reported in Table 2. We also try to match at age 65 the expected years without care and with care reported in Unger et al. (2011). Based on German LTC data from 2004-2008 this study finds that on average someone in good health at age 65 can expect to life 17 more years if they are in good health and 2.1 years if subject to long-term care. Table 2 shows that our model roughly matches these numbers. The models’ reported overall life expectancy at age 65 of 19.4 years is exactly equal to the respective life expectancy reported in Statistisches Bundesamt (2016).

The remaining baseline parameters are provided in Table 3. Our choices for preference parameters are selected in order to (at least roughly) match the build up of financial assets during the
life cycle as well as the overall homeownership rates and mortgage fraction in the data. Consequently, the consumption share parameter is specified at $\alpha = 0.6$ while the time discount factor is $\beta = 0.96$. The shift parameter for housing utility after a long-term care shock or the move into rental housing is set at $\chi = 0.6$ similar to Nakajima and Telyukova (2013). Hu (2005) assumes a value of 0.8 when renting is compared to ownership. The literature typically assumes a $\gamma$ value between 0.1 and 0.5, we therefore set the intertemporal elasticity of substitution (IES) to $\gamma = 0.2$. Since our benchmark calibration should reflect the regular CES utility function, the relative risk aversion (RRA) parameter is set to $\nu = 5$. This combination will be analyzed in the sensitivity analysis. Finally, similar to De Nardi (2004) the benchmark model specifies a bequest parameter $b = -5$ and a bequest utility shift parameter $q = 5.0$ in order to have a realistic old-age wealth level.

With respect to the labor productivity parameters we follow Fehr et al. (2013) and distinguish three skill levels that reflect high-, middle- and low-skilled households in Germany. The latter study provides the skill-specific deterministic age-profiles, as well as the correlation and variance terms. The skill-independent white noise component $\sigma^2_\zeta$ as well as the interest rate shocks are taken from Cocco et al. (2005). The same study also provides the risk-free interest rate of $r_f$ and an equity premium of $\mu_r$. As already discussed before, house price risk in Germany is much lower than in Anglo-saxon countries. Consequently, our standard deviation of 0.015 is much lower than the respective 0.062 in Cocco (2005). Finally, our benchmark simulation abstracts from mortgage premiums and any correlations between labor productivity, stock market returns and house prices.

When households want to become owners, they need to invest at least 5.25 times the average annual income. This figure is higher than in Hu (2005) or Yang (2009), but reflects the German situation where houses are typically more expensive due to specific safety and environmental regulations. Reasonable values for down payment in Germany are between 20 and 30 percent of the value of the house. However, Voigtländer (2016) expects a likely increase so that we fix the loan-to-value ratio $\xi = 0.7$. Voigtländer (2016) also computes the taxation and registry cost for housing purchases in Germany to $4.6 - 8\%$ property values, which makes us set $\phi_2 = 0.08$. Transaction cost when selling the house are considerably smaller so that we specify $\phi_1 = 0.05$.
Table 3: Baseline parameter values

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>Ordinary consumption share</td>
<td>0.65</td>
</tr>
<tr>
<td>$\chi$</td>
<td>Owner preference</td>
<td>0.60</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Time discount factor</td>
<td>0.96</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Intertemporal elasticity of substitution (IES)</td>
<td>0.20</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Relative risk aversion (RRA)</td>
<td>5.00</td>
</tr>
<tr>
<td>$b$</td>
<td>Utility weight on bequest</td>
<td>-5.00</td>
</tr>
<tr>
<td>$q$</td>
<td>Shifter of bequest utility</td>
<td>5.00</td>
</tr>
<tr>
<td>$e_j(\theta)$</td>
<td>Productivity of agent at age $j$</td>
<td>[1]</td>
</tr>
<tr>
<td>$\rho$</td>
<td>AR(1) correlation</td>
<td></td>
</tr>
<tr>
<td></td>
<td>low-skilled</td>
<td>0.95666</td>
</tr>
<tr>
<td></td>
<td>middle-skilled</td>
<td>0.95687</td>
</tr>
<tr>
<td></td>
<td>high-skilled</td>
<td>0.95828</td>
</tr>
<tr>
<td>$\sigma^2_e$</td>
<td>Transitory variance</td>
<td></td>
</tr>
<tr>
<td></td>
<td>low-skilled</td>
<td>0.02321</td>
</tr>
<tr>
<td></td>
<td>middle-skilled</td>
<td>0.02812</td>
</tr>
<tr>
<td></td>
<td>high-skilled</td>
<td>0.03538</td>
</tr>
<tr>
<td>$\sigma^2_\zeta$</td>
<td>White noise variance</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.0738</td>
</tr>
<tr>
<td>$\sigma^2_v$</td>
<td>House price variance</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.015²</td>
</tr>
<tr>
<td>$\sigma^2_\rho$</td>
<td>Stock market variance</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.157²</td>
</tr>
<tr>
<td>$\kappa_3$</td>
<td>Participation cost</td>
<td></td>
</tr>
<tr>
<td>$r_f$</td>
<td>Risk free interest rate</td>
<td>0.02</td>
</tr>
<tr>
<td>$\mu_r$</td>
<td>Risk premium</td>
<td>0.04</td>
</tr>
<tr>
<td>$\theta_1$</td>
<td>... stock returns and labor income</td>
<td>0.00</td>
</tr>
<tr>
<td>$\theta_2$</td>
<td>... stock returns and house prices</td>
<td>0.00</td>
</tr>
<tr>
<td>$r_p$</td>
<td>Mortgage premium</td>
<td>0.00</td>
</tr>
<tr>
<td>$h_{min} / \bar{g}$</td>
<td>Minimum house size</td>
<td>5.25</td>
</tr>
<tr>
<td>$\delta_h$</td>
<td>Depreciation rate</td>
<td>0.015</td>
</tr>
<tr>
<td>$\xi$</td>
<td>Loan-to-value ratio</td>
<td>0.70</td>
</tr>
<tr>
<td>$\mu_h$</td>
<td>Rental premium</td>
<td>0.0075</td>
</tr>
<tr>
<td>$\phi_1$</td>
<td>... of selling price</td>
<td>0.04</td>
</tr>
<tr>
<td>$\phi_2$</td>
<td>... of buying price</td>
<td>0.08</td>
</tr>
<tr>
<td>$\phi$</td>
<td>... free fraction</td>
<td>0.20</td>
</tr>
<tr>
<td>$\kappa_1$</td>
<td>Pension replacement rate</td>
<td>0.50</td>
</tr>
<tr>
<td>$\kappa_2$</td>
<td>Private long-term care cost</td>
<td>0.20</td>
</tr>
</tbody>
</table>

[¹] Taken from Fehr et al. (2013).

which is roughly in line with the literature. We also assume that transaction cost only arise when the house size changes by more than 20 per cent, see Yang (2009) who specifies a slightly
lower fraction. Follow Cocco (2005), we set the depreciation rate of houses at \( \delta_h = 0.015 \).

Finally, the replacement rate of the pension system is set at \( \kappa_1 = 0.5 \). As already mentioned above, average private cost for long-term care (i.e. after the provisions of the statutory LTC insurance) may rise up to a level of an average pension. However, in the benchmark model we assume a rather conservative value for \( \kappa_2 \), so that long-term care cost amount to the roughly the half of average pension benefits.

Table 1 reports that roughly 46 percent of German households live in their own house. The resulting real estate wealth is about 75 percent of their total net wealth and about 11 times their annual income while about 13 percent of their own house is financed by a mortgage. Finally, roughly 34 percent of homeowners are participating in the stock market while only 18 percent of renters own stocks. Table 4 shows that the resulting aggregate indicators from the model match at least the wealth structure quite well. Only stock market participation rates of homeowners in the model are significantly higher than in the data.

| Table 4: Aggregate key indicators in the data and the model (in %) |
|---------------------------------|----------------|----------------|-------------|-------------|----------------|
|                                 | Homeowner-   | Real Estate  | Mortgage     | Stock market | Stock market |
|                                 | ship rate    | relative to  | fraction     | participation | participation |
| Model                           | 47           | 78           | 968          | 9            | 54            |
| Data (Table 1)                  | 46           | 75           | 1100         | 13           | 34            |

Our model also does a good job in matching the life cycle profiles. Figure 3 shows the path for homeownership and the house value (relative to average income). In Figure 1 above, the average homeownership rate increased to 70% at age 55, then remained fairly stable for 20 years before it slightly decreased afterwards. While homeownership reached almost 100% in the top wealth quintile after age 55, much less owned a home in the bottom quintile. The left part of Figure 3 documents that our model roughly matches these figures. Households save during early ages in order to buy a house after age 30. Then the average homeownership rate rises up to roughly 80 percent at age 65 when they retire. Afterwards it declines again especially for those who have a long-term care shock. Healthy households try to keep their home until death. This distinct behavior of healthy and non-healthy individuals already indicates the insurance property of home equity. Almost all households in the top wealth quintile own a home and keep it as long as they are healthy. On the other hand, households in the bottom quintile build up some home equity around retirement, but much less than the average population. The right panel of Figure 3 shows the house value (relative to average income) for different wealth quintiles of homeowners. While the bottom quintile only buys slightly above the minimum house size, households in the top wealth quintile own a house that has a value of up to twenty or
thirty times average income. This rise in house values in old age is mainly due to composition effects when less wealthy households become renters. These composition effects dominate the value reductions from downsizing the house.

Figure 4 reveals that in our setup households typically accumulate assets until retirement. On average, wealth at retirement age in the model is about fourteen times average income which is also shown in Figure 2 where we report the wealth profiles of homeowners and renters separately. After retirement wealth declines, but does not disappear completely due to the bequest motive. Note that on the right side of Figure 4 the homeowner and renter wealth still increases slightly during retirement. Again, this is mainly due to composition effects. When less wealthy homeowners become renters, average assets of both (remaining) homeowners and renters increase.

The left panel of Figure 5 shows the mortgage fractions over the life cycle for different wealth quintiles of homeowners. The lowest quintile is clearly bounded by the maximum loan-to-value ratio and also pays back the mortgage almost until age 65. Wealthier households need much lower fractions of their house value as a mortgage and repay much faster. In the top quintile repayments are already finished at age 45. The right panel shows the participation rates in
the stock market for homeowners and renters. Clearly, homeowners have higher participation rates than renters but overall they are much higher than in the data. The sharp increase of participation rates after retirement is again due to a composition effect when former homeowners become renters.

Finally, Figure 6 shows the shares of risky assets over the life cycle in the aggregate and separated for homeowners and renters. Because of participation cost, renters do not invest their assets into equity at young ages. After age 30 the risky fraction rises but then falls again sharply until reaching retirement since they need to reduce their risk exposure in later life. Homeowners on the other hand first need to repay and therefore hold no financial assets. When they start investing in equity after age 40 their equity share increases sharply far beyond the respective share of renters. Of course, this reflects the fact that homeownership is concentrated in the higher wealth classes who also invest more risky.
5 Homeownership and social security

The idea of the paper is to isolate the insurance properties of homeownership and analyze the special relation with government provided social security. In order to quantify the impact of social security on homeownership and risk management we eliminate in this section the long-term care risk and the pay-as-you-go financed pension system successively. With respect to the long-term care risk we eliminate successively the cost, then the impact on longevity and finally the impact on the homeownership utility.

In the first simulation we assume that the government covers all cost if households are hit by a long-term care shock. Consequently, elderly now only face the risk of the reduced longevity and an ownership preference shock. The left part of Figure 7 shows that homeownership especially at older ages now decreases significantly. On the one side households will save less because they now face no LTC cost in retirement. Higher consumption at younger ages might even explain the slight increase in homeownership. On the other hand they don’t need to keep a house as an insurance against LTC shocks and therefore homeownership drops especially in retirement. The right part of Figure 7 shows the impact on risky financial investment. At younger ages there is now a significant drop in risky financial investment while at older ages households increase their financial risk exposure. The latter is simply due to the fact that the government has eliminated financial risk in old age. Consequently, our model clearly indicates the substitution between LTC insurance provision and homeownership. Countries which have higher (lower) homeownership rates need less (more) public and private LTC insurance coverage. The general connection was already highlighted by Davidoff (2009, 2010), but our model does not require reverse mortgages by pointing out the link due to the reduced consumption utility of a house.

Next we assume that a LTC shock would as well no longer affect longevity, so that people who are subject to LTC only derive lower utility from owner-occupied home equity. Of course, this
will make the share of households in old age who are in long-term care increase significantly. These households do not want to own a house and therefore the homeownership rate decreases especially at older ages. Note that in the right panel of Figure 8 the risky investment fraction increases. Households now face a longer lifespan and hence build up more savings again, which makes them willing to take more risk in their investment.

Figure 8: Homeownership and portfolio choice: Preference shock only

Finally, we completely eliminate LTC risk in the model. Not surprisingly, the left panel of Figure 9 shows now a strong increase of homeownership during retirement due to the elimination of the preference shock for homeowners. Since homeowners are also prefer more risk in their financial portfolio the right panel shows a slight increase in the risky investment share in older ages.

Figure 9: Homeownership and portfolio choice: No LTC risk

Next we analyze the impact of the pension system. Chen (2010) argues that the key mechanism of the elimination of social security is the fall in interest rates which boost housing demand. In our partial equilibrium set up bond and stock returns are not affected by the pension reform. Nevertheless, the left panel of Figure 10 shows a significant increase of homeownership at middle ages and a drop at old ages when pensions are eliminated. The right panel shows a dramatic increase in the risk exposure during younger ages and a lower financial risk share at
older ages. The mechanism here is therefore much different to Chen (2010). When households

increase their private savings to substitute for paygo pension benefits in old age, they build up
more risky investments at younger ages. In order to balance the risk exposure they buy houses
which are less risky. At later ages they sell their houses again and invest the proceeds in less
risky bonds in order to finance retirement. Consequently, the pension system therefore affects
both homeownership rates and portfolio choice dramatically in our model.3

6 Sensitivity analysis

Finally, we analyze the impact of specific parameters on homeownership and risky investment.
Table 5 repeats in the first line the initial indices which were already discussed above. Then we
also report the aggregate indices of the four social security simulations which we also discussed
in the last section. In aggregate the impact of LTC risk is fairly small, which might explain why
it has been hardly considered in such models before. However, as shown above LTC risk has a
strong impact during years of retirement and therefore should be discussed within a lifecycle
model. The lower part of Table 5 reports the results of some parameter variations.

When household risk aversion is increased from 5.0 to 10.0 then investment decisions tilt to-
wards more savings, more housing and more bonds. Consequently, homeownership increases
as well as the share of housing wealth relative to annual income. Due to higher savings the
mortgage fraction decreases as well as the participation rates in the stock market for both own-
ers and renters. If we increase the volatility of house prices then homeownership becomes less
attractive. Homeowners reduce their housing investment and substitute towards stock market
investments. Renters are only indirectly affected by the increase in the number of renters. As
one would expect, the ownership rate increases if renting becomes less attractive because of

3 Note, however, that we do not consider the impact of the elimination of paygo contributions during working
time. This should further boost savings and homeownership.
Table 5: Sensitivity of aggregate values (in %)

<table>
<thead>
<tr>
<th>Homeownership rate</th>
<th>Real Estate relative to net wealth</th>
<th>Mortgage fraction</th>
<th>Stock market participation</th>
<th>Renters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td>47</td>
<td>78</td>
<td>968</td>
<td>9</td>
</tr>
<tr>
<td>Policy simulations</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\kappa_2 = 0.0$</td>
<td>45</td>
<td>81</td>
<td>967</td>
<td>11</td>
</tr>
<tr>
<td>$+\psi_j(s) = \psi_j(2)$</td>
<td>44</td>
<td>78</td>
<td>982</td>
<td>10</td>
</tr>
<tr>
<td>no LTC, $\kappa_1 = 0.0$</td>
<td>46</td>
<td>79</td>
<td>973</td>
<td>9</td>
</tr>
<tr>
<td>Sensitivity simulations</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\nu = 10.0$</td>
<td>54</td>
<td>73</td>
<td>1047</td>
<td>3</td>
</tr>
<tr>
<td>$\sigma^2_\nu = 0.062^2$</td>
<td>40</td>
<td>65</td>
<td>1000</td>
<td>2</td>
</tr>
<tr>
<td>$\mu_h = 0.02$</td>
<td>51</td>
<td>79</td>
<td>950</td>
<td>9</td>
</tr>
<tr>
<td>$\vartheta = 0.25$</td>
<td>45</td>
<td>76</td>
<td>915</td>
<td>10</td>
</tr>
</tbody>
</table>

rising rental rates. Since homeowners are now less wealthy, they hold more mortgages and invest less in risky assets. Finally, if risk during employment increases due to a positive correlation between labor and capital income, the portfolio structure will be tilt towards bonds in order to reduce the risk exposure in savings. Consequently, households save less in real estate (although the ownership rate hardly declines) and in stocks while the mortgage rates increase significantly.

7 Extensions and future research

The main idea of this study is to analyze the interplay between homeownership, health risk and portfolio choice. For this reason we build a life-cycle model which features risky investment opportunities and tenure choice as well as interlinks the long-term care cost with the utility from housing consumption. We are able to roughly replicate a realistic life cycle ownership pattern for Germany. In our set-up, tenure choice is affected by long term care risk. The risk exposure of households during working periods turns out to be of minor importance. The simulated changes in homeownership rates are economically intuitive.

A quite natural extension of the model is to allow for some specific reverse mortgage (RM) options and analyze the resulting welfare consequences. There is currently a growing interest in RM products and their efficiency in financing retirement consumption. For example, Hanewald et al. (2015) develop a stylized three period model for a retired homeowner who faces longevity, long-term care, house price and interest rate risk. They compare reverse mortgages and home reversion plans with respect to the optimal choice for the release of home equity either with or without government provided LTC insurance. While the availability of both products in-
crease individual welfare, they find that the RM product offers a higher benefit. The already mentioned study by Nakajima and Telyukova (2017) extends the approach of Nakajima and Telyukova (2013) by considering an RM option. Their rich structural model allows to identify the determinants of RM demand, to explain why the RM market is currently so thin and how current RM contracts could be modified in order to make them more attractive. Their study also finds significant welfare gains for homeowners when they are given access to fair priced RM contracts.

A second interesting line of research would be to model the long-term care risk in more detail and include a choice between home care and nursing home care. Homeowners might have better options to receive home care than renters which would explain why the move less frequently to a nursing home, see Rowendahl and Thomese (2013). Kopecky and Koreshkova (2014) have recently modeled the nursing home choice in the U.S. but they did not consider the preceding tenure choice problem. In Germany the choice between home care and nursing home care is even more distorted because households receive transfers from the long-term care insurance if they stay at home.

A final important extension of our model concerns the consequences of the differentiated tax treatment of owner occupied and rented houses. Up to now we abstracted from this important issue mainly because we wanted to analyze housing investment from a portfolio choice selection perspective. In our partial equilibrium model homeownership reduces the risk exposure of households in equity markets. The analysis of taxation issues, however, requires a general equilibrium approach with overlapping generations. However, such a set-up makes it extremely difficult to handle aggregate investment risk and therefore only idiosyncratic labor income risk is typically considered. Recent contributions in this direction by Chambers et al. (2009) or Floetotto et al. (2016) have already studied the impact of the U.S. tax system on individual tenure choice and welfare. Other papers by Chen (2010) and Chun (2015) analyze the interplay between tenure choice and the pension systems in the U.S. and Australia. Recently, Kaas et al. (2017) implemented such a general equilibrium approach also for Germany. They analyzed the role of social housing, transfer taxes and no tax deductions for mortgage interest in explaining the low ownership rates.
References


25
Appendix

At any state \( z = (j, X, s, \theta, \eta, I, h) \), the household has to decide how to split up resources \( X \) into ordinary consumption \( c(z) \) and total wealth \( a^+(z) \) and total wealth into financial assets \( a^+_j = (1 - \omega^+_h) a^+ - tr(h, h^+) \) and future real estate \( h^+ = \frac{\omega^+_h a^+}{1 - \omega^+_h} \) and whether to participate on the stock market or not and (in case of participation) how to split positive financial wealth into stocks \( \omega^+(a^+_f - \xi h^+) \) and bonds \( (1 - \omega^+)(a^+_f - \xi h^+) \). Note that the current renter is identified whenever \( h = 0 \) while \( h^+ = 0 \) for the future renter. The corresponding optimization problem can be solved in five steps:

1. **Equity exposure in liquid wealth:** Given a specific state \( \tilde{z} = (j, s, \theta, h, I) \), and a combination of future financial assets \( a^+_f \) and future house \( h^+ \), future owners (i.e. where \( h^+ \geq h_{\text{min}} \)) and future renters (i.e. where \( h^+ = 0 \)) solve

   \[
   Q(\tilde{z}, a^+_f, h^+) = \max_{0 \leq \omega^+ \leq 1} \left[ \psi_{j+1}(s) E \left[ V(z^+)^{1-\nu} \right] \frac{1}{1 - \psi_{j+1}(s)} + (1 - \psi_{j+1}(s)) b \left( 1 + \frac{\hat{a^+} - \hat{\xi}}{q} \right)^{1-\frac{1}{\gamma}} \right]^{1-\frac{1}{\gamma}}
   \]

   subject to

   \[
   \begin{align*}
   I^+ &= \begin{cases} 
   1 & \text{if } I = 1 \text{ or } I = 0 \text{ and } \omega^+ > 0 \\
   0 & \text{otherwise}.
   \end{cases} \\
   X^+ &= R_p(\omega^+, \theta^+)(a^+_f - \xi h^+) + y^+ + pen^+ + (1 - \delta_h) p^+ h^+ - 1_{s=1} \cdot m^+ - (I^+ - I) \kappa_3 \tilde{g} \\
   \hat{a}^+ &= R_p(\omega^+, \theta^+)(a^+_f - \xi h^+) + (1 - \delta_h) p^+ h^+ - (I^+ - I) \kappa_3 \tilde{g}
   \end{align*}
   \]

   where

   \[
   R_p(\omega^+, \theta^+) = \begin{cases} 
   1 + r_f + \omega(\tilde{z}, a^+_f, h^+) (\mu_r + \theta^+) & \text{for } a^+_f - \xi h^+ > 0 \\
   1 + r_f + r_p & \text{otherwise}.
   \end{cases}
   \]

   The solution to this problem is \( \omega^+ = \omega(\tilde{z}, a^+_f, h^+) \).

2. **Stock market participation decision:** In case the household has not participated in the stock market before (i.e. \( I = 0 \)) and now decided \( \omega^+ > 0 \) so that \( I^+ = 1 \) we need to decide about participation by comparing

   \[
   Q(\tilde{z}, a^+_f, h^+) = \max[Q(\tilde{z}, a^+_f, h^+, I^+ = 0); Q(\tilde{z}, a^+_f, h^+, I^+ = 1)]
   \]

   In the case the household has already participated before (i.e. \( I = 1 \)) no further participation cost are due and the optimal decision from the first step suffices.

3. **Wealth exposure in real estate:** The household who wants to become a renter simply has to sell his house (in case he/she is a owner) and pay the resulting transaction cost. Given that we
already know the optimal equity exposure $\omega^+$ as well as the value function $Q(\bar{z}, a^+_f, 0)$ from step 2, we can define

$$S(\bar{z}, a^+, o^+ = R) = Q(\bar{z}, a^+_f, 0)$$

where we need to make sure that

$$a^+_f = a^+ - tr(h, 0) \geq 0.$$

with $tr(h, 0)$ defining the transaction cost of moving to a rented home. Note that all combinations where $a^+ < tr(h, 0)$ are not feasible!

The situation when the household wants to become (or remain) a homeowner is more complicated since we need to split total savings $a^+$ between future financial assets and housing and make sure that the minimum house requirement is fulfilled and again that the transaction cost are taken into account. The sub-optimization problem is now

$$S(\bar{z}, a^+, o^+ = O) = \max_{\omega(a^+) \leq \omega^+_h \leq 1} Q(\bar{z}, a^+_f, h^+)$$

s.t.

$$a^+_f = (1 - \omega^+_h) a^+ - tr(h, h^+) \geq 0.$$

Note that we can define an implicit minimum housing share $\omega_h(a^+) = \frac{(1 - \bar{z}) h_{\min}}{a^+}$. The solution to this problem gives us $\omega^+_h = \omega_h(\bar{z}, a^+)$. 

4. The consumption-savings decision: Finally knowing how much wealth to allocate to real estate and how much liquid assets to hold, it is possible to set up the consumption savings problem for current homeowners and renters separately. The former own a positive housing stock $h \geq h_{\min}$ that is consumed. The maximization problem then reads

$$W(z, o^+) = \max_{c, a^+} \left\{ u(c, c_h)^{1 - \frac{1}{\gamma}} + \beta S(\bar{z}, a^+, o^+)^{1 - \frac{1}{\gamma}} \right\}^{\frac{1}{1 - \frac{1}{\gamma}}}$$

s.t. $X = c + a^+.$

Current renters (i.e. where $h = 0$) have to decide how to split their resources between ordinary consumption, housing consumption and savings. They therefore maximize

$$W(z, o^+) = \max_{c, r_h c_h, a^+} \left\{ u(c, c_h)^{1 - \frac{1}{\gamma}} + \beta S(\bar{z}, a^+, o^+)^{1 - \frac{1}{\gamma}} \right\}^{\frac{1}{1 - \frac{1}{\gamma}}}$$

s.t. $X = c + r_h c_h + a^+$

Substituting the first order conditions

$$\frac{c}{c_h} = \frac{\alpha}{1 - \alpha} r_h$$
into the above budget constraint we derive the optimal housing choice

\[ c_h = \frac{(1 - \alpha)(X - a^+)}{r_h}. \]

The solution yields the consumption and rental housing demand as well as the savings functions \( c(z, o^+), c_h(z, o^+) \) and \( a^+(z, o^+) \), respectively.

4. **The tenant decision:** Finally substituting the consumption and housing demand \( c(z, o^+), c_h(z, o^+) \) and savings functions \( a^+(z, o^+) \) from the last step into the respective objective functions we can derive the values \( W(z, o^+ = R) \) and \( W(z, o^+ = O) \) for future renters and owners, respectively. The final value function is then simply derived from

\[ V(z) = \max \{ W(z, o^+ = R), W(z, o^+ = O) \}. \]