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A New Theory of Demand-Restricted Growth

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A New Theory of Demand-Restricted Growth

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Mainstream growth theory is dominated by variations of the neoclassical approach. Growth is explained fully by elements of the supply side. In this paper we examine the general mechanism of technology growth and capital accumulation. However, instead of following a fully supply-side driven neoclassical approach we suggest a hybrid approach that allows for growth restrictions induced by demand-side elements. We obtain a demand-restricted growth by suggesting an unconventional equilibrium concept in a stochastic environment. We define macroeconomic equilibrium as stationary no-expectation-error equilibrium. This equilibrium concept relates to the Nash idea of individual stationary behavior as long as expectations prove to be realized. No rigidities are introduced. Even if potential growth is generated by technical change and capital accumulation, the level of the path and the growth rate are restricted by effective earnings. Both can be stable below the neoclassical potential growth. The growth rate in the demand restricted regime is semi-endogenous and determined by entrepreneurial conditions and activities. However, the growth process mutates to the neoclassical process if effective earnings are sufficient and the earnings ratio turns to one. As a result, the demand side cannot generate growth, but the demand side can permanently restrict growth which could have been generated by the supply side. Our hybrid model could help to bridge a gap between Keynesian and neoclassical ideas of economic growth.

JEL classifications: E12, E13, O40, E60

Keywords: Demand restricted growth, no-expectation-error equilibrium,
neoclassical growth theory

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1 Introduction

In mainstream neoclassical growth theory, the supply side determines the growth process. Starting with the growth mechanics described in Solow (1956), neoclassical growth theory developed a number of extensions and upgrades. For example, in the 1960s contributions by Phelps, (1961), Cass (1965) or Koopmans (1965) introduced concepts of optimal growth. An endogenous choice of the growth rate and additional input factors such as human capital (see Mankiv et al. (1992)) were introduced in the so called "New Growth Theory" of the late 1980s and 1990s. This phase started with Romer (1986, 1990) or Lucas (1988), and includes contributions such as Grossman and Helpman (1991), Aghion and Howitt (1992), Mankiv (1995), or Barro and Sala-I-Martin (1995). Excellent reviews of this approach are provided by Aghion and Howitt (1998) or Jones (1999). However, all these contributions are characterized by the notion of resource- and/or technology-restricted growth: Savings restrict investments and capital accumulation, resources like labor or human capital restrict the growth of technologies, and the speed of technical progress restricts productivity growth. This notion not only holds for theory building, it also dominates most empirical growth research. Many empirical papers, e.g. Benhabib and Spiegel (1994) or Quah (1997), apply supply-side-oriented growth accounting methods for global growth processes, and this direction of research continues right up to a very recent analysis by Bloom et al. (2017), who look at the level of scale economies in the R&D sector. Demand-side elements do not matter, neither in the theory of aggregate growth, nor in empirical research on aggregate growth determinants. Due to this dominance in neoclassical thinking, demand-restricted growth models and the related empirical research are only marginally represented. Neglecting the demand side is all the more astounding since the experiences of the 2008 economic crisis as well as a long-term decline in growth rates in advanced economies (sometimes referred to as secular stagnation (Summers, 2014) or Great Recession) suggest that we should allow for unconventional thinking and alternative approaches. Effective demand as a persistent restriction of macroeconomic growth seems unthinkable to many growth economists. Therefore, in this paper we scrutinize the current dominance of supply-side economics and offer an extended demand-restricted growth model, which we refer to as the Keynesian growth process. However, we do not neglect supply-side elements. Instead, we synthesize

two sides of the same coin in our model. Technical change and capital accumulation remain the engine of potential growth on the one hand side, on the other hand side, potential growth can be systematically demand-restricted.

Looking at the literature that relates to Keynesian growth we see a number of heterogeneous models. This kind of discussion started with the pioneering work of Harrod (1939) right up to the more recent work of Palley (2014). In an acknowledgment of the profound discussions of the various literature strands by, e.g., Eichner (1978) or Lavoie (2007), we only need to briefly revisit some elements of this compound literature, which relate to our work. Even if considered Keynesian, one strand suggests that growth is limited by the availability of resources used for accumulation, with Kaldor (1957) a well-known contributor. A second strand claims that growth is indeed constrained by aggregate demand. There is a sustainable production below full capacity with idle resources as a recurring element, e.g., in the work of Kalecki (1971). However, in both strands we find several typical elements. Often, two types of income are generally considered, with workers distinct from capitalists. It is assumed that labor income is entirely consumed and not saved, i.e., economy-inherent savings stem completely from capitalists. Wage determination is often not explicitly modeled, as in the work of Sawyer (2011) or assumed to be exogenously given, as in Dutt (1984). Capitalists' income from savings and investment depends on profits, which are in turn determined by the rate of profit and again by capacity utilization. Thus, savings and investments and hence capital accumulation represent the first growth-generating mechanism in these models; the second important mechanism is technical change. In this respect, Keynesian growth theories identify the same growth force as neoclassical theories, however they are implemented in a rather different way.

In our general model potential growth is driven by neoclassical mechanisms of technical change and capital accumulation, simultaneously we draw on Keynesian ideas of effective demand such as Dutt (2006) or Palley (1996, 1997, 2014). Our distinctive contribution is that we suggest the concept of a stationary no-expectation-error equilibrium at individual and aggregate level. Without assuming market rigidities, our concept allows for a stationary under-utilization of potential production. We obtain a stationary effective earning below potential capacity level. Even if potential growth is generated by technical change and capital accumulation, this approach gives the

demand side a role to play in determining the degree of under-utilization and the level of the steady-state growth path. In the special case of earning at full capacity level, the model mutates to the well-known neoclassical mechanism. As a result, the demand side cannot generate growth, but the demand side can permanently restrict growth which otherwise could have been generated by the supply side. Therefore, our hybrid model could help to bridge a gap between Keynesian and neoclassical notion of economic growth.

2 The Basic Model

Our hybrid model uses the idea of a demand-restricted growth process. This idea is basically and essentially Keynesian. However, in order to draw comparisons with neoclassical fundamental mechanisms we relate many elements of our model to the Solow (1956) model as the neoclassical benchmark. This allows us to identify the substantial differences between the two approaches. In other words, in this hybrid model of a demand-restricted growth theory, our primary concern is ease of comparison. Very often we (over)simplify in many directions, as we do not primarily intend to provide convincing micro-foundation of consumption, savings or investment behavior, elegance, endogeneity of the growth rate, and others. That said, we nevertheless hope that this approach contributes to a broader discussion of growth processes, that include elements of the demand side.

2.1 Representative firm production

Potential production of representative firm i is generated by a standard (Cobb-Douglas) production function with labor in efficiency units Λ_i , capital K_i , and capital intensity $k_i = K_i/\Lambda_i$. Labor in efficiency units, however, is defined as physical labor L_i times the index describing the current level of technology A_i , $\Lambda_i = L_i A_i$. The current level of technology is determined by the set of implemented technologies determining the total factor productivity. Each implemented technology j stands for a differentiated good and contributes one unit to total factor productivity $a(j) = 1$. Hence, the level of the current technology index is equal to the total number of implemented technologies $\int_0^{A_i} a(j) dj = A_i a = A_i$. Thus *potential production* is given

by

$$Y_i^p(t) = k_i(t)^{1-\alpha} L_i \int_0^{A_i(t)} a(j) dj \quad (1)$$

From individual firm i 's perspective all technologies $A_i = \int_0^{A_i} a(j) dj$ stand for the firm's differentiated goods. Each of these goods earns the price of one unit of output $P_i = 1$. However, due to random reasons not all of these goods may be currently successful in the market. Only the number A_i^e of implemented technologies is currently successful and effectively generates income. This share of currently effective earning technologies defines the ratio of *earnings* to *potential* production and is referred to as *earnings ratio*

$$\lambda_i(t) = \frac{A_i^e(t)}{A_i(t)}. \quad (2)$$

In other words, λ_i stands for the share of effectively earning production capacity, and thus describes a kind of capacity utilization.¹ Thus, fraction $1 - \lambda_i$, is *idle*. To simplify the modeling, we assume that idle technologies will currently not generate net earnings, such that this idle production is fully depreciated.² Thus, the number of effectively earning projects A_i^e directly relates to the *effective production* and *income* and is

$$A_i^e(t) = k_i(t)^{-(1-\alpha)} \frac{Y_i^e(t)}{L_i} \quad (3)$$

Therefore, in this economy we distinguish between *potential production value* Y_i^p and *effective production* Y_i^e . While potential production is all production that potentially generates earnings and income, effective production is the production that indeed generates income because it is the value that can be sold in the market. The difference between potential and effective production leads to the third category of production which we refer to as *unused/idle production* Y_i^u . Idle production is defined as produced goods that currently cannot be sold in the market (later we explain how and why this may happen). These terms relate as

$$Y_i^p(t) \geq Y_i^e(t) + Y_i^u(t). \quad (4)$$

¹Note, while the term capacity utilization normally refers to production capacity that is producing versus not producing; here we use this term with respect to capacity that is earning versus not earning. In this sense the earning ratio is the ratio of capacity utilization.

²Even if this assumption is made to simplify the analysis, in section we give some more economics reasoning.

2.2 Income and Expenditure

Labor income and consumption: In competitive goods and labor markets, wages are paid according to marginal productivity rules. However, firms can only pay wages out of the earnings they effectively realize. Hence, the effective (realized) wage rate paid by the representative firm is³

$$w_i^e(t) = \frac{\partial Y_i^e}{\partial L_i}(t) = \lambda_i(t) w_i^p(t), \quad w_i^p(t) = \alpha k_i(t)^{1-\alpha} A_i(t). \quad (5)$$

Using (3) and (5), total effective labor income on aggregate is $w^e L = \alpha k^{1-\alpha} L A^e = \alpha Y^e$. Further, we import a concept often used in Keynesian growth models (inter alia by Dutt, 1984, Kaldor and Mirrlees, 1962, and Kalecki, 1968) and assume that all wage income is consumed and effective consumption is

$$C^e(t) = w^e(t) L = \alpha Y^e(t) = c Y^e(t). \quad (6)$$

With this assumption, wage income is the fraction α of total income and the economy's consumption rate c is equal to α .

Capital income and savings: In a competitive goods and capital market, capital returns are paid according to marginal productivity rules. However, firms can only pay returns out of the earnings they effectively realize. Hence, the effective interest rate paid by the representative firm is

$$r_i^e(t) = \frac{\partial Y_i^e}{\partial K_i}(t) = \lambda_i(t) r_i^p(t), \quad r_i^p(t) = (1 - \alpha) k_i(t)^{-\alpha}. \quad (7)$$

Using (3) and (7), total effective capital income on aggregate becomes $r^e K = (1 - \alpha) k^{-\alpha} K \lambda = (1 - \alpha) Y^e$. Again, for in the tradition of Keynesian models (inter alia Kaldor, 1961) we assume that all capital income is saved and effective saving is

$$S^e(t) = r^e(t) K(t) = (1 - \alpha) Y^e(t) = s Y^e(t), \quad (8)$$

and the savings rate of the economy s is equal to $(1 - \alpha)$.

³Note that in case λ equals one, total production capacity is successfully sold in the market and the effective wage is the potential wage.

Investments: Unlike in the benchmark Solow model, in which investments are determined by savings (Say’s Law), we suggest a Keynesian mechanism, in which investments and innovation determine the income-expenditure equilibrium. Further, as the discussion of an alternative growth mechanism is the focus of this paper, we introduce a particularly simple investment hypothesis independent of savings, which is in line with Keynesian traditions, such as the work of Kaldor (1957) and Bhaduri and Marglin (1990).

Innovations and technologies: We suggest that the purpose of an investment is to place new goods and technologies on the market. We link investments to new technologies and consider innovative goods. As entrepreneurs try to place new technologies on the market, we also refer to these new technologies as new innovative entrepreneurial projects successfully entering the market \dot{A} . The number of these innovative projects introduced in the market is a share μ of the total number of potential new technologies and ideas $\dot{\theta}$. Economically, the value μ indicates the entrepreneurial activities associated with the market entry of new products

$$\dot{A}(t) = \mu \dot{\theta}(t) \quad \mu \leq 1. \quad (9)$$

As in non-endogenous growth theory (Solow model, and others), we assume the availability of such new ideas $\dot{\theta}$ is exogenous. For simplicity, we also regard μ as given.

Investment expenditure: Placing innovative products in the market and implementing the respective production capacity requires investment. The idea that technical change relates to investments also links-up with the Keynesian tradition (starting with Kaldor, 1957). As a first simplifying step, we assume that there is a given investment amount η that has to be spent to build production capacity for each innovative product. Hence, the number of innovations placed on the market (9) and the investment per new project η translate into the respective total amount of innovation-related investment

$$I_A(t) = \eta \dot{A}(t). \quad (10)$$

A second, again very simple modeling of investments is directly related to effective demand. If effective demand is high, investors are inclined to invest correspondingly and expand their capacities. For simplicity, we assume that investments are simply

proportional to effective demand and income:

$$I_Y(t) = \varepsilon Y^e(t). \quad (11)$$

Total investments are the sum of both capacity- and innovation-related investments

$$I(t) = I_Y(t) + I_A(t). \quad (12)$$

2.3 The Keynesian Income-Expenditure Equilibrium

The Keynesian income-expenditure equilibrium suggests that aggregate demand Y^D determines the aggregate value of income and hence effective income Y^e . This effective income Y^e is consistent with spending decisions determining aggregate demand. Therefore, in income-expenditure equilibrium, aggregate effective demand equals effective income

$$Y^e(t) \stackrel{!}{=} Y^D(t) \equiv cY^e(t) + \varepsilon Y^e(t) + I_A(t).$$

As a result, effective expenditure - which is consistent with the income-expenditure equilibrium - is

$$Y^e(t) = \frac{1}{1 - c - \varepsilon} I_A(t) = \frac{1}{1 - c - \varepsilon} \eta \dot{A}(t) \quad (13)$$

which is well known from undergraduate macroeconomics.

2.4 Accumulation Dynamics and Capital Intensity

Assuming away labor force growth ($L = 1, \dot{L} = 0$) and supposing that capital, once installed, can be used for producing all kinds of products in the aggregate production process (no depreciation of product-specific installed capital), capital intensity $k(t) = K(t)/A(t)L$ changes by

$$\dot{k}(t) = \frac{\dot{K}(t)}{A(t)L} - k(t)g_A. \quad (14)$$

g_A denotes the growth rate of new technologies

$$g_A = \frac{\dot{A}(t)}{A(t)} \quad (15)$$

In *neoclassical models*, all new ideas add to the technology index $\dot{A}(t) = g_A A(t)$, and g_A is the given technology growth rate. Capital accumulation \dot{K} is full determined by savings, $\dot{K} = I = S = sY$.

In this new *demand-restricted growth model*, technology growth is modeled similar to the neoclassical approach. The growth rate of potential new technologies g_θ is given. However, in this model only a fraction μ of potential new technologies enters the market via entrepreneurial activities (9). As we relate all changes in technologies to existing and installed technologies ($A = \theta$), for the growth rate of projects through market entry we obtain

$$g_A = \frac{\dot{A}(t)}{A(t)} = \mu \frac{\dot{\theta}(t)}{\theta(t)} = \mu g_\theta. \quad (16)$$

Further, capital accumulates through investment $I(t)$ independent of savings; and as investment has two components (12) we obtain

$$\dot{K}(t) = I(t) = I_Y(t) + I_A(t). \quad (17)$$

Having identified the value of investments and capital change, we can now determine the accumulation dynamics with respect to k . Using (10), (11), (16) and (17) the motion of $k(t)$ over time can be described as

$$\dot{k}(t) = \frac{\varepsilon Y^e(t) + \eta \dot{A}(t)}{A(t)} - k(t) g_A \quad (18)$$

2.5 Defining the no-expectation-error equilibrium

Broadly speaking, in this new demand-restricted growth model we depart from the neoclassical approach and substitute the “allocation equilibrium concept of Say’s Law” (no excess-supply in the aggregate market, production finds the corresponding demand) through the concept of a stationary “no-expectation-error equilibrium”. While Say’s law is related to aggregate market clearing as an equilibrium concept, the suggested “stationary no-expectation-error equilibrium” is related to the Nash-idea of individual stationary behavior.

Specifically, the suggested equilibrium concept is based on the perception of individuals in a stochastic environment. According to this concept, each economic agent has his individual interpretation of the outcome of the stochastic market process. Each worker or capital owner has an individual perspective, so they behave in an optimal manner according to their individual expectations.⁴ In particular, if a manager of

⁴For simplicity, we assume risk neutrality.

firm i manages firm i 's projects (product variations) he knows that in the current period, some projects can randomly succeed while others randomly fail. If a project is randomly successful it will fully earn; if it is randomly unsuccessful, the product goes on sale and earns less, may be even less than what it cost to produce. To illustrate, let a fashion producer design a collection of shoes, shirts and jackets, each in a particular style and launch it on the market. The example in table 2.5 shows the sales revenues.

Table 2.5: % of potential revenue					
period	1	2	3	4	total
jackets	100%	100%	20%	100%	80%
pants	20%	100%	100%	100%	80%
shirts	100%	100%	100%	20%	80%
shoes	100%	20%	100%	100%	80%
total	80%	80%	80%	80%	80%

In the first year jackets are fully sold at the standard price of one unit of output. However, the pink pants completely failed to match the fashion trend color, which was grey, and flopped. The sale led to a massive price reduction such that the firm had to depreciate 80% of the potential value of pants in this period. As result, the four projects (shoes, jackets, shirts and pants) did not fully earn on average. Overall 80% of potential earnings could be realized. In period two, pants could be fully sold; the design of the shoes fails to match the taste of the buying public, and again could not be sold. Again, on average (with a randomly changed structure of successful sales) the total production of all items can realize around 80% of potential earnings. What would the manager of this fashion label do if he does his very best and continuously observes this kind of market results? He would expect that - due to random results of markets - offered products of the various projects only earn an average fraction (around 80% in this example) of potential earnings; and he would include this expectation in his business calculations. He would expect an effective earning ratio of 80% of potential earnings, 20% of potential earnings must be systematically depreciated, they could not be realized in the markets. For simplicity, in this model we assume

that a failing project cannot be sold at all and generates no value at all.⁵ Therefore, in our simplifying formal analysis projects either fully earn, these being part of firm i 's successful effective projects A_i^e , or they currently do not earn and hence describe a production that is unused or idle and are fully depreciated. For each existing project of firm i the probability of earning and finding demand is $\pi_i(t)$. As we assume symmetry among projects, all project have identical probabilities of success and the total portfolio of projects can be expected to earn accordingly (in the above example 80%). However, after this general discussion we can now state the definition of the equilibrium concept at individual agent level.

Definition 1 *Individual no-expectation-error equilibrium (n-ee):*

A) (i) If a firm $i \in F$ effectively earns what it expects to earn with its projects, firm i is in "no-expectation-error equilibrium". (ii) If the effective return on capital for each capital owner of firm i is equal to the expected returns, each capital owner of firm i is in "no-expectation-error equilibrium". (iii) If the effective wage of each worker of firm i is equal to the expected wage, each worker of firm i is in "no-expectation-error equilibrium". Formally,

$$\begin{aligned} EY_i(t) &= \pi_i(t)Y_i^P(t) = Y_i^e(t), \\ Er_i(t) &= \pi(t)_i r^p(t) = r_i^e(t), \quad Ew_i(t) = \pi(t)_i w^p(t) = w_i^e(t). \end{aligned} \quad (19)$$

B) *Individual adjustment dynamics:*

In a stochastic environment, if an economic agent i (project manager, capital owner or worker) observes a realized value that is unexpected, the expectation error is construed as an error in their individual expectation. Thus they adjust their individual expectation towards the observe realization at speed ϑ . Formally,

$$E\dot{Y}_j = -\vartheta (EY_j - Y_j^e), \quad E\dot{r}_i = -\vartheta (Er_i - r_i^e), \quad E\dot{w}_i = -\vartheta (Ew_i - w_i^e). \quad (20)$$

⁵Even if this assumption is made to simplify the mathematical model, we may give some economic reasoning. What can a firm do if the pink pants were obviously not successful. First, they have extra storage costs. If these extra costs are high net earnings quickly turn to zero even if they sell try to at lower prices. Thus sometimes firms rather quickly fully depreciate their failing projects than waiting longer for potentially consumers to buy. A recent examples for this strategy is H&M that according to Bloomer TV (2017) destroyed a large amount of clothes which did not match costumers taste. Therefore, our simplifying assumption of not earning is not completely unrealistic.

This definition implies that in a stochastic environment, if an economically rational, optimally acting agent of a firm i effectively realizes all the values he expects to realize (no-expectation-error), he would not change his choices. In fact this equilibrium idea relates to the Nash-equilibrium idea of stationary choices in a game. In *no-expectation-error equilibrium* individuals have no reason to change their behavior and hence would not change their decisions. With this definition of a stationary choice equilibrium at individual level, the model is complete and can be solved.

3 Determining the No-Expectation-Error Equilibrium

3.1 Firms:

Individual firm's probability of success: In the above examples we suggest that a project j may be a random success or a random failure. For simplicity we assume that projects do not systematically differ in their probability of failure. Further, as we assume that probabilities of success are stochastically independent (i.i.d), we can make the following statement.

Lemma 2 (*probability of success of individual firm i*): If we assume that the probability of success of a project j of firm i , $\pi_i(j)$, is identical for all projects j of firm i , and $\pi(j)$ are i.i.d., $\pi_i(j)$ is also the probability of success of the entire firm, $\pi_i(j) = \pi(t)_i$ and is determined by the fraction of successful projects of firm i

$$\pi_i(t) = \frac{A_i^e(t)}{A_i(t)}. \quad (21)$$

For a proof see appendix A.1.

So far, we have only looked at the individual micro level. However, in macroeconomic growth theory the market level should be included as well. A failure of project j of firm i may be due to idiosyncratic conditions or it may have systematic market reasons. In the above example, an idiosyncratic reason may be that a given product does not fully match the taste of potential consumers; a systematic market reason may be a downturn in the business cycle. However, for simplicity and as we want to focus on macro-mechanisms, we assume away idiosyncratic reasons. In this model, only systematic market reasons determine the probability of success. However, firms

do not have this information – they just experience failure. Therefore, each firm simply adjusts to the fact that projects may fail in their market; from their individual perspective, market conditions are taken as an exogenously given fact. Hence we can state.

Lemma 3 (*probability of market success for all firms*): *If the probability of success of an individual firm i is fully determined by systematic market factors and the individual probability of success of projects is i.i.d., at time t the probability of success for each firm $\pi(t)_i$ is identical and determined by the fraction of successful projects in the overall market $\pi(t)$,*

$$\pi(t)_i = \pi(t) = \frac{A^e(t)}{A(t)}, \quad \forall \text{ firms } i. \quad (22)$$

For a proof see appendix A.1. Implication (ii) is important. As $\pi(t)$ holds for any firm or economic agent, it is a market signal that goes out to all market participants simultaneously. $\pi(t)$ coordinates expectations. All participants use this identical message from the market to calculate their own probability of success. As this signal is the only available information to agents, it also coordinates the expectations of economic agents in a consistent way.

Firm's no-expectation-error equilibrium: A manager of firm i estimates expected earnings of all projects of the firm. Then, they compare their expectations with what is realized in the market. In no-expectation-error equilibrium (n-e-ee) the firm would not need to correct its expectations and actions.

Proposition 4 (*no-expectation-error equilibrium*): *Part I (n-e-ee of (manager of) firm i) If (a) (22) holds, (b) firm i has no information about the true reasons why projects fail, and (c) all markets function such that no systematic arbitrage profits can be expected in all goods and factor markets, the no-expectation-error equilibrium (Definition 1) holds for firm i with respect to earnings and the expected rate of success $\pi(t)$ is equal to the realized earnings ratio $\lambda(t)$*

$$EY_i(t) = Y_i^e(t), \quad \pi(t)_i = \lambda(t)_i \quad \forall i \in F,$$

Part II (n-e-ee in the aggregate market)

If i is an element of the set F of all firms, and if all firms $i \in F$ are symmetric,

n-e-ee holds for all firms simultaneously. At aggregate level we obtain

$$EY(t) = Y^e(t), \quad \pi(t) = \lambda(t). \quad (23)$$

For a proof see appendix A.1.

3.2 Capital owners:

Once firms have formed their expectations, they give promises to capital owners (elements of the set of capital owners J) or make contracts with workers (elements of the set of workers N).

Firms promise a return on capital to capital owners according to the best of their knowledge and hence promise a return on capital according to the expected marginal productivity of capital. As capital owners trust the firms, they also expect the promised return. In no-expectation-error equilibrium capital owners would expect what firms have promised and these promised returns are indeed the effective returns realized.

Lemma 5 (*n-e-ee for capital owner of firm i*): *If a capital owner $j \in J$ of firm i expects what the firm promises to pay as return on capital, and if this firm i gives promises according to its own expectations, the expected return for the capital owner will be identical for all capital owners of firm i*

$$Er_{ji}(t) = Er_i(t), \quad \forall \quad j \in J \text{ of firm } i \in F.$$

At aggregate level if firms are in n-e-ee (proposition 4) each capital owner j is in no-expectation-error equilibrium

$$\begin{aligned} Er_{ji}(t) &= Er_i(t) = r_i^e(t), \quad \forall \quad j \in J \text{ of firm } i, \\ Er(t) &= r^e(t). \end{aligned} \quad (24)$$

For a proof see appendix A.1. All capital owners expect the same return on capital which they indeed realize. Hence there is no reason to change decisions.

3.3 Workers:

Firms have flexible wage contracts with their workers. That is, although firms offer a given wage to their workers, the contracts are still flexible and allow firms to cut the agreed wage if their earnings are insufficient. However, they make their wage offer according to the best of their knowledge and hence according to their expected marginal productivity of labor. If worker h agrees to the contract, they will expect to obtain the offered wage. In no-expectation-error equilibrium workers would expect that wage agreed in the contract to indeed be the effective wage they earn.

Lemma 6 (*n-e-ee for worker h of firm i*): If a worker $h \in N$ of firm i expects the wage $Ew_{hi}(t)$ according to their (flexible) wage contract, and if firm i 's wage contracts are determined according to expected marginal labor productivity, the expected wage of each worker $Ew_{hi}(t)$ of firm i will be identical

$$Ew_{hi}(t) = Ew_i(t), \quad \forall \quad h \in N, \text{ employed by firm } i \in F.$$

At aggregate level if firms are in n-e-ee (proposition 4) each worker h is in no-expectation-error equilibrium

$$\begin{aligned} Ew_{hi}(t) &= Ew_i(t) = w_i^e(t), \quad \forall \quad h \in N, \text{ employed by firm } i, \\ Ew(t) &= w^e(t). \end{aligned} \tag{25}$$

For a proof see appendix A.1.

3.4 Probability of success and income-expenditure equilibrium

According to proposition 4 no-expectation-error equilibrium requires that effective earnings are expected. From (22) we know that the probability of success is determined by the share of successfully earning projects $\pi = \frac{A^e}{A}$; and from (2) we know that the realized earnings ratio is $\lambda = \frac{A^e}{A}$. Further, from (3) we obtain that $\frac{A^e}{A} = \frac{Y^e}{Y^p}$. Thus, we need to understand what determines the ratio of effective and potential earnings. In this Keynesian approach the market conditions are driven by the well known income-expenditure equilibrium process. Effective earnings are accordingly

the result of the income multiplier (13). Potential production is defined by the neo-classical production function (1). Combining these elements, proposition 7 identifies both the determinants of the probability of success and of the realized earnings ratio.

Proposition 7 (*probability of success and income-expenditure equilibrium*): *In each period, the probability of success of projects $\pi(t)$ is simultaneously determined by the Keynesian income-expenditure multiplier and by production capacity. Further, in no-expectation error equilibrium, the probability of success of the projects equals the realized earnings ratio $\lambda(t)$*

$$\pi(t) = \frac{A^e(t)}{A(t)} = \frac{\eta \dot{A}(t)}{(1-c-\varepsilon) k(t)^{1-\alpha} A(t)} = \lambda(t). \quad (26)$$

For calculations see appendix A.1. As we observe from (26) the realized *earnings ratio* still depends on the motion of capital intensity k . However, as soon as a stationary capital intensity is reached, we also obtain a stationary probability of success and earnings ratio.

4 Growth process

4.1 General motion of economy

Motion of capital intensity: Departing from (14) we can solve for the motion of capital intensity. $\dot{k}(t)$ is determined by investments (17), the Keynesian income-expenditure multiplier (13), and technology growth (16) and gives⁶

$$\frac{\dot{k}}{k}(t) = \left(\left(\varepsilon \frac{\eta}{1-c-\varepsilon} + \eta \right) k(t)^{-1} - 1 \right) \mu g_\theta, \quad \frac{d\frac{\dot{k}}{k}}{dk} < 0. \quad (27)$$

For a simple calculation, see appendix A.2.1. Graphically, the motion of capital intensity is illustrated by the $\frac{\dot{k}}{k}$ -curve in figure 1. The $\frac{\dot{k}}{k}$ -curve is monotone with a negative slope. If we can show that the $\frac{\dot{k}}{k}$ -curve intersects the k -axis at some $\tilde{k} > 0$ (see the steady-state analysis in section 4.2) we have a stable dynamic adjustment towards \tilde{k} (arrows in figure 1).

⁶Note that this dynamic system has two simultaneous motions, one of k and one of π , respectively λ . Therefore, we would expect an interdependent two dimensional dynamic system. Fortunately, as shown in appendix A.2.1 the motion of λ depends recursively on the motion of k , such that a dynamic analysis of k is sufficient to determine both motions.

[insert figure 1 here]

Motion of earnings ratio: The motion of the realized earnings ratio $\lambda(t)$ is directly determined by the motion of the income-expenditure equilibrium (13) and production capacity growth (4). Over time, the demand determined income-expenditure equilibrium must consistently move with the growing supply side and production capacity. From the equilibrium motion of the income-expenditure multiplier $\dot{Y}^e(t) = \frac{\dot{I}_A(t)}{1-c-\varepsilon}$ and the change of the production side $\dot{Y}^e(t) = \frac{dY^e(t)}{dt} = \frac{d(k(t)^{(1-\alpha)\lambda(t)A(t)})}{dt}$ we obtain the equilibrium *growth of innovation-driven investments*

$$\frac{\dot{I}_A}{I_A} = \frac{\eta\ddot{A}}{\eta\dot{A}} = g_A, \quad (28)$$

With this investment growth we can determine the motion of the income-expenditure equilibrium and the *motion of the earnings ratio*⁷

$$\frac{\dot{\lambda}}{\lambda}(t) = -(1-\alpha) \frac{\dot{k}}{k}(t); \quad (29)$$

and accordingly we obtain the general *motion of effective income*

$$\frac{\dot{Y}^e}{Y^e}(t) = (1-\alpha) \frac{\dot{k}}{k}(t) + \frac{\dot{\lambda}}{\lambda}(t) + \frac{\dot{A}}{A}(t) = g_A. \quad (30)$$

Figure 2a illustrates that the *earnings ratio* declines as long as the capital intensity is growing. The economic interpretation in this Keynesian model is straight-forward. The expansion of potential production capacity due to additional capital accumulation does not meet with sufficient demand. The earnings ratio declines and effective earnings cannot grow as much as they could owing to the expansion of potential capacity. *Income growth is demand-restricted.*

[insert figure 2 here]

In figure 2b we draw the growth path of such a *demand-restricted growth* process for both, the adjustment phase, and the steady-state phase (steady-state is discussed in section 4.2). During the adjustment phase, potential production (solid line in

⁷For calculations see appendix A.2.1.

figure 2b) has two engines of growth, growth in technologies and growth in capital intensity. The demand-side (dashed line in figure 2b) however, has only one engine of growth, the rate of technology growth. Specifically, the rate of technology growth determines the growth rate of investments while investment growth drives the growth in aggregate demand. As result the economy develops an *earnings gap* which is the difference between potential production and effective income (figure 2b). This increasing earnings gap points towards a *built-in* mechanism of the demand restricted expansion or even *stagnating growth*.

4.2 Steady-state with (exogenous) growth

Steady-state capital intensity \tilde{k} can be determined using (27) at $\frac{\dot{k}}{k} = 0$. In steady-state not only capital intensity, but also the realized earnings ratio λ and effective income in efficiency units y^e as well as production capacity in efficiency units reach a stationary level. We obtain the following proposition:

Proposition 8 (*steady-state of capital intensity and earnings ratio*): *If $(1 - c)(1 - c - \varepsilon)\eta > k(0)$, we can determine a stationary capital intensity \tilde{k} which is restricted by elements of the demand side, c, ε and η*

$$\tilde{k} = \left(\frac{\varepsilon}{1 - c - \varepsilon} + 1 \right) \eta, \text{ with } \frac{d\tilde{k}}{dc} > 0, \quad \frac{d\tilde{k}}{d\eta} > 0. \quad (31)$$

For the earnings ratio we find as steady-state value

$$\tilde{\lambda} = \frac{A^e(t)}{A(t)} = \frac{\eta}{(1 - c - \varepsilon)} \frac{g_A}{\tilde{k}^{1-\alpha}}, \quad \frac{d\tilde{\lambda}}{dc} > 0. \quad (32)$$

Using (4), (31) and (32) we obtain for the steady-state income per labor in efficiency units \tilde{y}^e and the effective income path $\tilde{Y}^e(t)$

$$\tilde{Y}^e(t) = \tilde{y}^e A(0) e^{\mu g_\theta t}, \quad \tilde{y}^e = \frac{\eta}{(1 - c - \varepsilon)} \mu g_\theta \quad (33)$$

For a simple calculation, see appendix A.2.2. Before we discuss the implications, we also need to look at the steady-state growth rate.

Proposition 9 (*effective income growth in steady-state*): *In no-expectations-error equilibrium the stationary state effective income growth is determined purely by the growth rate of potential innovation g_θ and the entrepreneurial activity rate μ ,*

$$g_{Y^e} = \frac{\dot{Y}^e}{Y^e} = \frac{\dot{A}}{A} = \mu g_\theta \quad (34)$$

For a simple calculation see appendix A.2.2.

4.3 Steady state with semi-endogenous growth

So far the implications of this modeling seem to affect the steady state level only. The steady state path may be some small fraction below the Solow-path, and hence this result may not be too exciting. However, in this section we show that also the growth rate is affected.

In the above model, the number of new technologies brought to the market and thus effectively installed is $\dot{A} = \mu \dot{\theta}$ (see 9). While in neoclassical economics the discussion is focusing on the rate of technical innovations, in this approach the economically interesting parameter is μ . μ indicates the successful market entry of new technologies. In this model the implementation of new technologies not only contributes to the production and supply side, it is also a driving force of investment expenditure on the demand side. Further, μ sheds some more light on the important role of entrepreneurs. While μ so far was just a parameter it will now be endogenized in a straight forward manner.

We assume μ stands for the efficiency of an aggregate entrepreneurial matching process of bringing innovations into the market. For simplicity we assume that the market match of a new innovation becomes more likely if market opportunities are large. If we assume that the probability of success for a new product is related to the probability of success for existing products, π becomes relevant for innovations and their respective market success. We assume that successful market entry μ is a function of π times the absolute number of goods already in the economy πA , and the number of new technical innovations $\dot{\theta}$ ready to be introduced to the market. For the matching technology we assume constant economies to scale, such that the total number of new products in the market is

$$\dot{A} = \mu = (\pi A)^\gamma \dot{\theta}^{1-\gamma} \quad (35)$$

From (34) we know that the rate of market entry of innovations determines investments and aggregate demand. Thus, we have an additional effect on the effective demand. The following proposition is summarizing these results and shows that the growth rate is also affected by demand restrictions.

Proposition 10 *If successful market entry of new products relates to the success rate π , the growth rate of implemented technologies \tilde{g}_A (and the overall growth rate) becomes semi-endogenous and demand restricted*

$$\tilde{g}_A = \left(\frac{\eta}{(1-c-\varepsilon)} \tilde{k}^{-(1-\alpha)} \right)^{\frac{\gamma}{1-\gamma}} g_\theta,$$

$$\frac{d\tilde{g}_A}{d\eta} > 0, \quad \frac{d\tilde{g}_A}{dc} > 0,$$

For a proof see appendix A.3.

4.4 Implications and comparative statics

With proposition 8 and 10 the results of this demand restricted growth model are in sharp contrast to neoclassical results and is thus of particular interest.

(i) according to proposition 8 steady-state levels of all variable \tilde{k} , $\tilde{\lambda}$, and \tilde{y}^e are determined by the Keynesian income-expenditure multiplier mechanism $\frac{\eta}{(1-c-\varepsilon)}$. The higher this demand-side multiplier, the higher the steady-state values. In figure 2b we already saw that the Keynesian growth processes - during the adjustment and steady-state phase - is clearly below the potential growth path which is basically the neoclassical path (Solow model). In this demand-restricted growth process, potential production is not fully absorbed; in other words, potential savings are not fully used for investments $I < S^p$.

(ii) Further the steady state growth rate will be restrained by demand limitation. The demand side is responsible for the success of innovations in the market and to what extend innovative goods can eventually survive during market entry. The growth rate is affected by demand-restrictions. The demand side is a determining factor of successful market entry of new technologies.

[insert figure 3 here]

What are the economic implications? As an example we take-up the discussion on growth and inequality. What does this model predict with respect to the growth process if the share of labor income and capital income changes. According to Karabarbounis & Neiman (2014) or Lawrence (2015) we observe a reduction in the share of labor income during the last decades. In this model a reduction of the income share of labor is identical to a reduction in the consumption rate. When looking at comparative statics in this model and the example of a change in the consumption rate and savings rate, we observe that the resulting effects in this model are in sharp direct contrast to the results of the standard neoclassical model. From neoclassical models we know that an increase in the savings rate or a decrease in the consumption rate $dc < 0$ is positive for capital accumulation, and leads to a higher steady-state capital intensity \tilde{k} , as well as a higher level of the income path (figure 3b). In this demand-restricted growth process (figure 3a) we see the opposite. With an increase in savings rate and a decrease in consumption rate, steady-state capital intensity \tilde{k} and earnings ratio $\tilde{\lambda}$ decreases such that \tilde{y}^e also declines, and effective income can grow only on a lower path (figure 3a). Further more with the semi-endogenous growth mechanism we also see a decline in the steady state growth rate \tilde{g}_A . This directly contradicts standard neoclassical results.

More generally in this demand-restricted growth process the economy can grow at a stationary path of under-utilization of earning potentials. A demand expansion can improve the earnings ratio and bring the economy closer to the path of potential earnings. However, once all capacities earn the earnings ratio becomes $\lambda = 1$ and we are back to the Solow mechanism.

5 Conclusion

Mainstream growth theory is dominated by the neoclassical notion of long term full-employment equilibrium growth. Hence, growth is explained fully by elements of the supply side, namely technology growth and capital accumulation. Say's law and the notion of 'savings determine investments and capital accumulation' is the fundamental ingredient of all these models.

In this new demand-restricted growth model, however, we depart from this neoclassical notion by substituting the "allocation equilibrium concept of Say's Law" (no

excess-supply in the aggregate market) by the concept of a stationary “no-expectation-error equilibrium”. While Say’s law is related to aggregate market clearing as an equilibrium concept, the suggested “stationary no-expectation-error equilibrium” is related to the Nash-idea of individual stationary behavior.

Without assuming market rigidities, the economy may remain stationary on a path of under-utilization of potential production. As a result, effective income can remain consistently below potential income and the stationary earnings ratio is less than one. This gives the demand side a role to play in the overall growth mechanics. The level of this steady-state earnings ratio is determined by the well known income-expenditure multiplier and other demand-side elements. Consequently, policy implications of this long-term demand-restricted growth mechanism contrast with neoclassical implications. For example, in this model a reduction in the share of labor income and a respective reduction in the consumption rate reduces the economy’s steady state position and growth path. In contrast, in a neoclassical model, a respective increase in the share of capital income and the savings rate improves steady state and the level of the growth path. However, in the special case of full earning at capacity level, the model mutates to the well-known neoclassical mechanism. In summary, the demand side cannot generate growth, but the demand side can permanently restrict growth which could have been generated by the supply side! Therefore, our hybrid model could help to bridge a gap between Keynesian and neoclassical ideas of economic growth.

References

- [1] Aghion, P. and Howitt, P. 1992. A Model of Growth through Creative Destruction, *Econometrica*, vol. 60, no. 2, 323-351.
- [2] Aghion, P. and Howitt, P. 1998. *Endogenous Growth Theory*, Cambridge, MIT Press MA.
- [3] Barro, R. J. and Sala-i-Martin, X. 1995. *Economic Growth*, New York, McGraw-Hill.
- [4] Basu, D. and Vasudevan, R. 2013. Technology, distribution and the rate of profit in the US economy: understanding the current crisis, *Cambridge Journal of Economics*, vol. 37, 57–89.

- [5] Benhabib, J. and Spiegel, M. M., 1994. The role of human capital in economic development evidence from aggregate cross-country data, *Journal of Monetary Economics*, vol. 34, no. 2, 143-173.
- [6] Bhaduri, A. and Marglin, S. 1990. Unemployment and the real wage: the economic basis for contesting ideologies, *Cambridge Journal of Economics*, vol. 14, no. 4, 375-393.
- [7] Bloom, N., Jones C.I., van Renen, J., and Webb M. 2017. Are ideas getting harder to find? *Centre for Economic Performance, Discussion Paper*, no. 1496, Sept. 2017.
- [8] Bloomberg TV 2017. A Power Plant Is Burning H&M Clothes Instead of Coal, By Jesper Starn 24. November 2017, 01:01 MEZ Updated on 24. November 2017, 09:21 MEZ
- [9] Cass, D. 1965. Optimum Growth in an Aggregative Model of Capital Accumulation, *The Review of Economic Studies*, vol. 32, no. 3, 233-240.
- [10] Dutt, A. K. 1984. Stagnation, income distribution and monopoly power, *Cambridge Journal of Economics*, vol. 8, no. 1, 25-40.
- [11] Dutt, A. K. 2006. Aggregate Demand, Aggregate Supply and Economic Growth, *International Review of Applied Economics*, vol. 20, no. 3, 319-336.
- [12] Eichner, A. S. 1979. *A Guide to Post-Keynesian Economics*, New York, M.E. Sharpe Inc.
- [13] Grossman, G. M. and Helpman, E. 1991. Trade, knowledge spillovers, and growth, *European Economic Review*, vol. 35, no. 2, 517-526.
- [14] Harrod, R. F. 1939. An Essay in Dynamic Theory, *The Economic Journal*, vol. 49, no. 193, 14-33.
- [15] Jones, C. 1995. R&D-Based Models of Economic Growth, *Journal of Political Economy*, vol. 103, no. 4, 759-784.
- [16] Jones, C. 1999. Growth: With or Without Scale Effects? *American Economic Review*, vol. 89, no. 2, 139-144.
- [17] Kaldor, N. 1957. A Model of Economic Growth, *The Economic Journal*, vol. 67, no. 268, 591-624.
- [18] Kaldor, N. 1961. Capital Accumulation and Economic Growth, in F. A. Lutz and D. C. Hague (Eds.), *The Theory of Capital*, 177-222, London, Palgrave

Macmillan.

- [19] Kaldor, N. and Mirrlees, J. A. 1962. A New Model of Economic Growth, *The Review of Economic Studies*, vol. 29, no. 3, 174-192.
- [20] Kalecki, M. 1971. Trend and the Business Cycle, in *Selected Essays on the Dynamics of a Capitalist Economy 1933-1971*, Cambridge, Cambridge University Press.
- [21] Karabarbounis L. and Neiman, B. 2014. The global decline of the labor share, *Quarterly Journal of Economics*, vol. 129, no. 1, 61–103.
- [22] Keynes, J. M. 1936. *The General Theory of Employment, Interest, and Money*, New York, Macmillan.
- [23] Koopmans, T., C. 1965. *On the concept of optimal economic growth. The econometric approach to development planning*, Chicago.
- [24] Lavoie, M. 2007. *Introduction to Post-Keynesian Economics*, New York, Palgrave Macmillan.
- [25] Lawrence R.Z. 2015. Recent declines in labor’s share in US income: a preliminary neoclassical account, *NBER Working Paper*, no. 21296.
- [26] Lucas, R. E. Jr. 1988. On the Mechanics of Development Planning, *Journal of Monetary Economics*, vol. 22, no. 1, 3-42.
- [27] Mankiw, N. G. 1995. The Growth of Nations, *Brookings Papers on Economic Activity*, vol. 1, 275-326.
- [28] Mankiw, N. G., Romer, D., and Weil, D. 1992. A Contribution to the Empirics of Economic Growth, *Quarterly Journal of Economics*, vol. 107, 407-37.
- [29] Molero-Simarro, R. 2017. Growth and inequality revisited: the role of primary distribution of income. A new approach for understanding today’s economic and social crises, *Cambridge Journal of Economics*, 41, 367–390
- [30] Palley, T. I. 1996. Growth theory in a Keynesian mode: some Keynesian foundations for new growth theory, *Journal of Post Keynesian Economics*, vol. 19, no. 1, 113-135.
- [31] Palley, T. I. 1997. Aggregate Demand and Endogenous Growth. A Generalized Keynes-Kaldor Model of Economic Growth, *Metroeconomica*, vol. 48, no. 2, 161–176.
- [32] Palley, T. I. 2012. From Financial Crisis to Stagnation: The Destruction of

Shared Prosperity and the Role of Economics, Cambridge, UK, Cambridge University Press.

- [33] Palley, T. I. 2014. A neo-Kaleckian-Goodwin model of capitalist economic growth: monopoly power, managerial pay and labour market conflict, *Cambridge Journal of Economics*, vol. 38, no. 6, 1355–1372.
- [34] Phelps, E. 1961. The Golden Rule of Accumulation: A Fable for Growthmen, *American Economic Review*, vol. 51, no. 4, 638-643.
- [35] Quah, D. T., 1997. Empirics for growth and distribution: stratification, polarization, and convergence clubs, *Journal of economic growth*, vol. 2, no. 1, 27-59.
- [36] Romer, P. M. 1986. Increasing Returns and Long-Run Growth, *Journal of Political Economy*, vol. 94, no. 5, 1002-1037.
- [37] Romer, P. M. 1990. Endogenous Technological Change, *Journal of Political Economy*, vol. 98, no. 5, pt. II, 71–102.
- [38] Sawyer, M. 2011. The Kaleckian Analysis of Demand-Led Growth, *Metroeconomica*, vol. 63, no. 1, 7-28.
- [39] Solow, R. M. 1956. A contribution to the theory of economic growth, *The Quarterly Journal of Economics*, vol. 70, no.1, 65-94.
- [40] Summers, L. H. 2014. U.S. Economic Prospects: Secular Stagnation, Hysteresis, and the Zero Lower Bound, *Business Economics*, vol. 49, no. 2, 65-73.
- [41] Togati, T.D. 2016. How can we explain the persistence of the Great Recession? A balanced stability approach, *Cambridge Journal of Economics* 2016, 40, 1077–1101.

A The First Appendix

A.1 Proofs of no-expectation-error-equilibrium

Proof. [Lemma 2] $\pi(j)$ is the probability of success for each project j . We assume that all projects j are i.i.d. such that the conditional probability simplifies to $\pi(j | h) = \pi(j)$ and each project j and project h has the same probability of success, $\pi(j) = \pi(h)$. Further, at firm level, we can determine the expected number of effectively earning projects of firm i by $E[A_i^e] = \int_0^{A_i} \pi(j)_i a(j)_i dj$. As $\pi(j)$ is i.i.d., and using the assumption $a(j)_i = 1$, we obtain that $E[A_i^e] = \pi(j)_i \int_0^{A_i} dj = \pi(j)_i A_i$. If A_i^e is the number of effectively earning projects of firm i in the markets and A_i is the number of all projects of firm i , $\pi_i = \frac{A_i^e}{A_i}$ is the probability of success of the complete product portfolio of firm i such that $\pi_i = \frac{A_i^e}{A_i}$. Thus, $\pi_i = \frac{A_i^e}{A_i} = \frac{EA_i^e}{A_i} = \pi(j)_i$, that is, the probability of success of the entire firm equals the probability of success of each single project. ■

Proof. [Lemma 3] According to (21), $\pi(t)_i = \frac{A_i^e(t)}{A_i(t)}$. As the probability of success is driven only by market reasons (by assumption no idiosyncratic reasons) all firms are affected by the same market conditions and have on average a constant share in the market. Hence, at each time t

$$\pi(t)_i = \pi(t) \quad \text{and} \quad \pi(t) = \frac{A^e(t)}{A(t)}.$$

■

Proof. [Proposition 4:] $EY_i(t) = \pi(t)_i Y_i(t) + (1 - \pi(t)_i) 0$ and using (21), (1), and (3) gives

$$EY_i(t) = \pi(t)_i Y_i(t) = \frac{A_i^e(t)}{A_i(t)} Y_i(t) = Y_i^e(t).$$

Further, as firms have no information about the true reason of project failure and the true reason is by assumption only due to market conditions, market conditions signal identical probabilities of success to all firms, $\pi(t)_i = \pi(t), \forall i \in F$ (see 22). Eventually, as all firms i have identical potentials, $Y_i^P(t) = Y_j^P(t), \forall i, j \in F$, firms

form identical earning expectations

$$\begin{aligned} EY_i(t) &= \pi(t)_i Y_i^P(t) \quad \forall \quad i \in F \\ EY_i(t) &= \pi(t) Y_i^P(t) \quad \forall \quad i \in F \\ EY(t) &= \pi(t) Y^P(t) \end{aligned}$$

Therefore, expected earnings equalize for all firms. Further, using (2) and as effective income has the same value as the expected income of each firm all firms are in no-expectation-error equilibrium

$$EY(t) = Y^e(t) = \frac{A^e(t)}{A(t)} Y^P(t)$$

■

Proof. [Lemma 5] As each capital owner $j \in J$ of firm i trust the expected return promised by firm i , $Er_i(t) = Er_{ji}(t) = Er_{hi}(t) \quad \forall \quad h, j \in J$. Further, as $Er_{ji}(t) = \pi(t)_i r_{ji}^p(t) + (1 - \pi(t)_i) 0$ the expected return of the firm and each single capital owner is determined by the firm's probability of success and the potential return (r_i^p) of firm i ,

$$Er_{ji}(t) = Er_i(t) = \pi(t)_i r_i^p(t), \quad \forall \quad j \in J.$$

Using (21), (1), (2), and (3) gives for all capital owner of firm i

$$Er_{ji}(t) = \frac{A_i^e(t)}{A_i(t)} r_i^p(t) = r_i^e(t) \quad \forall \quad j \in J.$$

Further, applying Proposition 4 all capital owners will be in no-expectation-error equilibrium

$$\begin{aligned} Er_{ji}(t) &= \pi(t) (1 - \alpha) Y_i^P(t) \quad \forall i \in F \wedge \forall j \in J \\ Er(t) &= \pi(t) r^p(t) \\ Er(t) &= r^e(t) = \frac{A^e(t)}{A(t)} (1 - \alpha) Y^P(t) \end{aligned}$$

■

Proof. [Lemma 6] As each worker $h \in N$ of firm i expects wages of their wage contract offered by firm i are according to the firms expectations of marginal labor productivity, wages for each worker are identical $Ew_i(t) = Ew_{hi}(t) = Ew_{ji}(t) \quad \forall \quad h, j \in J$.

Further, as $Ew_{hi}(t) = \pi(t)_i w_{hi}^p(t) + (1 - \pi(t)_i) 0$ the expected marginal labor productivity of firm i and the expected wage of each single worker is determined by the firm's probability of success and the potential marginal labor productivity (potential wage, w_i^p) of firm i ,

$$Ew_{hi}(t) = Ew_i(t) = \pi(t)_i w_i^p(t), \quad \forall \quad h \in N.$$

Using (21), (1), (2), and (3) gives for all capital owner of firm i

$$Ew_{hi}(t) = \frac{A_i^e(t)}{A_i(t)} w_i^p(t) = w_i^e(t) \quad \forall \quad h \in N.$$

Further, applying Proposition 4 all workers will be in no-expectation-error equilibrium

$$\begin{aligned} Ew_{hi}(t) &= \pi(t) \alpha Y_i^p(t) \quad \forall i \in F \wedge \forall h \in N \\ Ew(t) &= \pi(t) w^p(t) \\ Ew(t) &= w^e(t) = \frac{A^e(t)}{A(t)} \alpha Y^p(t) \end{aligned}$$

■

Proof. [Proposition 7 (probability probability of success and income-expenditure equilibrium: $\pi(t) = \lambda(t)$)] :

Thus, departing from implication (ii) and using (1), (2), (13), (10+11) and (22), we obtain

$$\begin{aligned} \pi(t) &= \frac{A^e(t)}{A(t)} = \frac{k^{-(1-\alpha)} Y^e}{k^{-(1-\alpha)} Y^p} \\ \pi(t) &= \frac{\eta \mu \dot{\theta}}{(1 - c - \varepsilon) k^{1-\alpha} A} = \lambda(t) \\ \pi(t) &= \frac{\eta \mu g_\theta}{(1 - c - \varepsilon) k^{1-\alpha}} = \lambda(t) \end{aligned}$$

■

A.2 Solving the dynamic model

A.2.1 General Dynamics

Proof. $\frac{\dot{k}}{k}$, Determining differential equation for capital intensity growth: \dot{k}

collecting from (18) $\dot{k} = \frac{\dot{K}}{AL} - k g_A$, (17) $\dot{K} = I = \varepsilon Y^e + \eta \dot{A}$, (16) $g_A = \frac{\dot{A}}{A} = \mu g_\theta$, (13)

$Y^e = \frac{I_A}{1-c-\varepsilon}$, and (10) $I_A = \eta\mu\dot{\theta}$ we can solve for the motion of capital intensity

$$\begin{aligned}\dot{k}(t) &= \frac{\varepsilon Y^e(t) + \eta\mu\dot{\theta}(t)}{A(t)L} - k(t)\mu g_\theta, & L &= 1 \\ &= \frac{\varepsilon Y^e(t) + \eta\mu\dot{\theta}(t)}{A(t)} - k(t)\mu g_\theta, & \lambda(t) &= \pi(t) \\ &= \left(\varepsilon \frac{\eta}{1-c-\varepsilon} + \eta - k(t) \right) \mu g_\theta \\ \frac{\dot{k}}{k}(t) &= \left(\left(\varepsilon \frac{\eta}{1-c-\varepsilon} + \eta \right) k(t)^{-1} - 1 \right) \mu g_\theta\end{aligned}$$

slope of differential equation

$$\frac{d\frac{\dot{k}}{k}}{dk} = - \left(\varepsilon \frac{\eta}{1-c-\varepsilon} + \eta \right) k^{-2} \mu g_\theta < 0$$

$$\lim_{k \rightarrow \infty} \frac{d\frac{\dot{k}}{k}}{dk} = -0$$

■

Proof. Motion of income-expenditure equilibrium and $\frac{\dot{\lambda}}{\lambda}$:

The demand determined equilibrium must consistently move over time. The static income-expenditure equilibrium $Y^e = \frac{I_A}{1-c-\varepsilon} = \frac{\eta\mu\dot{\theta}}{1-c-\varepsilon}$ is not sufficient. Four elements are needed to determine the dynamics of the income expenditure equilibrium. First, the change in income-expenditure equilibrium is $\dot{Y}^e = \frac{\dot{I}_A}{1-c-\varepsilon}$. Second, the growth rate of innovation related investments is $\frac{\dot{I}_A}{I_A} = \frac{\eta\mu\ddot{\theta}}{\eta\mu\dot{\theta}} = g_\theta$ (note that $\frac{\ddot{\theta}}{\dot{\theta}} = g_\theta$ for exponential growth ($\theta(t) = e^{g_\theta t}$)). Third, combining the time derivative of effective production $Y^e = k^{(1-\alpha)}\lambda A$ and (10) we obtain $\dot{Y}^e = (1-\alpha)k^{(-\alpha)}\lambda A\dot{k} + k^{(1-\alpha)}\lambda\dot{A} + k^{(1-\alpha)}\dot{\lambda}A$. Fourth, (16) gives $\dot{A} = \mu\dot{\theta}$ and $I_A = \eta\mu\dot{\theta}$, Fifth, production dynamics must equal income-expenditure dynamics and gives $\dot{Y}^e(\text{expenditure}) = \dot{Y}^e(\text{production})$. Now, all these elements are used to determine the dynamics of λ .

$$\begin{aligned}Y^e &= k^{(1-\alpha)}A^e = k^{(1-\alpha)}\lambda A, & I_A &= \eta\mu\dot{\theta} = \eta\dot{A} \\ \dot{Y}^e &= (1-\alpha)k^{(-\alpha)}A^e\dot{k} + k^{(1-\alpha)}\lambda\dot{A} + k^{(1-\alpha)}\dot{\lambda}A, & \frac{I_A}{\eta} &= \dot{A} \\ &= (1-\alpha)k^{(-\alpha)}\lambda A\dot{k} + k^{(1-\alpha)}\lambda\frac{I_A}{\eta} + k^{(1-\alpha)}\dot{\lambda}A\end{aligned}$$

$$\begin{aligned}
\frac{\dot{I}_A}{1-c-\varepsilon} &= (1-\alpha) k^{(-\alpha)} \lambda A \dot{k} + k^{(1-\alpha)} \lambda \frac{I_A}{\eta} + k^{(1-\alpha)} \dot{\lambda} A \\
\frac{\dot{I}_A}{1-c-\varepsilon} &= \left((1-\alpha) k^{(-\alpha)} \frac{A}{I_A} \eta \dot{k} + k^{(1-\alpha)} + \eta k^{(1-\alpha)} \frac{\dot{\lambda}}{\lambda} \frac{A}{I_A} \right) \lambda \frac{I_A}{\eta} \\
\frac{\dot{I}_A}{I_A} &= \left((1-\alpha) k^{(-\alpha)} \frac{A}{\eta \mu \theta} \dot{k} \eta + k^{(1-\alpha)} + \eta k^{(1-\alpha)} \frac{\dot{\lambda}}{\lambda} \frac{A}{\eta \mu \theta} \right) \lambda \frac{1-c-\varepsilon}{\eta},
\end{aligned}$$

the ratio of $\frac{\ddot{A}}{\dot{A}} = g_A$ for an exponential growth process $e^{g_A t}$ with constant g_A .

$$\begin{aligned}
\frac{\eta \ddot{A}}{\eta \dot{A}} &= g_A = \mu g_\theta = \left((1-\alpha) k^{(1-\alpha)} \frac{A}{\mu \dot{\theta}} \frac{\dot{k}}{k} + k^{(1-\alpha)} + k^{(1-\alpha)} \frac{\dot{\lambda}}{\lambda} \frac{A}{\mu \dot{\theta}} \right) \lambda \frac{1-c-\varepsilon}{\eta} \\
\mu g_\theta &= \left((1-\alpha) \frac{\dot{k}}{k} + \mu g_\theta + \frac{\dot{\lambda}}{\lambda} \right) \frac{1}{\mu g_\theta} k^{(1-\alpha)} \lambda \frac{(1-c-\varepsilon)}{\eta}
\end{aligned}$$

$$\mu^2 g_\theta^2 k^{-(1-\alpha)} \frac{\eta}{\lambda (1-c-\varepsilon)} = \left((1-\alpha) \frac{\dot{k}}{k} + \mu g_\theta + \frac{\dot{\lambda}}{\lambda} \right)$$

$$\begin{aligned}
\frac{\dot{\lambda}}{\lambda} &= \mu^2 g_\theta^2 k^{-(1-\alpha)} \frac{\eta}{\lambda (1-c-\varepsilon)} - \mu g_\theta - (1-\alpha) \frac{\dot{k}}{k} \\
&= \left(\frac{\eta \mu g_\theta}{(1-c-\varepsilon)} - k^{(1-\alpha)} \lambda \right) \frac{\mu g_\theta}{k^{(1-\alpha)} \lambda} - (1-\alpha) \frac{\dot{k}}{k}
\end{aligned}$$

$$\begin{aligned}
&= \left(\overbrace{\frac{\overbrace{\eta g_A}^{y^e(\text{expenditure})}}{(1-c-\varepsilon)} - \frac{\overbrace{k^{(1-\alpha)} \lambda}^{y^e(\text{production})}}{k^{(1-\alpha)} \lambda}}^{=0} \right) \frac{g_A}{\lambda \eta} k^{-(1-\alpha)} - (1-\alpha) \frac{\dot{k}}{k} \\
&\quad \frac{\dot{\lambda}}{\lambda} = - (1-\alpha) \frac{\dot{k}}{k}
\end{aligned}$$

■

A.2.2 steady-state growth process

Proof. [Proposition 8] stationary capital intensity \tilde{k} :

In order to determine the stationary capital intensity \tilde{k} we rearrange (27) to obtain

a function $\tilde{k} = \tilde{k}(\dots)$

$$\frac{\dot{k}}{k}(t) = \left(\left(\varepsilon \frac{\eta}{1-c-\varepsilon} + \eta \right) k(t)^{-1} - 1 \right) \mu g_\theta$$

proof: total differential

$$0 = \left(\varepsilon \frac{\eta}{1-c-\varepsilon} + \eta \right) \frac{1}{k} - 1$$

$$\tilde{k} = \varepsilon \frac{\eta}{1-c-\varepsilon} + \eta$$

Proof. [Proposition 8] stationary earning ratio $\tilde{\lambda}$:

We can also determine the stationary $\tilde{\lambda}$ by rearranging (29) in steady-state $\frac{\dot{\lambda}}{\lambda} = \frac{\dot{k}}{k} = 0$, thus

$$\frac{\dot{\lambda}}{\lambda} = \mu^2 g_\theta^2 k^{-(1-\alpha)} \frac{\eta}{\lambda(1-c-\varepsilon)} - \mu g_\theta - (1-\alpha) \frac{\dot{k}}{k}$$

$$0 = \mu g_\theta k^{-(1-\alpha)} \frac{\eta}{\lambda(1-c-\varepsilon)} - 1$$

$$\tilde{\lambda} = \frac{\eta \mu g_\theta}{(1-c-\varepsilon)} \tilde{k}^{-(1-\alpha)}$$

$$\tilde{\lambda} = \frac{\eta \mu g_\theta}{(1-c-\varepsilon)} \left(\overbrace{\varepsilon \frac{\eta}{1-c-\varepsilon} + \eta}^{\tilde{k}} \right)^{-(1-\alpha)}, \quad \frac{d\tilde{\lambda}}{dc} > 0$$

$$\begin{aligned} \frac{d\tilde{\lambda}}{dc} &= \eta g_A (1-c-\varepsilon)^{-2} (\varepsilon \eta (1-c-\varepsilon)^{-1} + \eta)^{-(1-\alpha)} + \\ &\quad - (1-\alpha) \eta g_A (1-c-\varepsilon)^{-1} (\varepsilon \eta (1-c-\varepsilon)^{-1} + \eta)^{-(2-\alpha)} \varepsilon \eta (1-c-\varepsilon)^{-2} \\ &= \eta g_A (1-c-\varepsilon)^{-2} (\varepsilon \eta (1-c-\varepsilon)^{-1} + \eta)^{-(1-\alpha)} \\ &\quad \left[1 - (1-\alpha) (\varepsilon \eta (1-c-\varepsilon)^{-1} + \eta)^{-1} \varepsilon \eta (1-c-\varepsilon)^{-1} \right], \end{aligned}$$

thus $\left[1 - (1-\alpha) (\varepsilon \eta (1-c-\varepsilon)^{-1} + \eta)^{-1} \varepsilon \eta (1-c-\varepsilon)^{-1} \right]$ need to be positive:

$$1 - (1-\alpha) (\varepsilon \eta (1-c-\varepsilon)^{-1} + \eta)^{-1} \varepsilon \eta (1-c-\varepsilon)^{-1} > 0$$

$$(1-c-\varepsilon) (\varepsilon \eta (1-c-\varepsilon)^{-1} + \eta) - (1-\alpha) \varepsilon \eta > 0$$

$$\varepsilon \eta + (1-c-\varepsilon) \eta - (1-\alpha) \varepsilon \eta > 0$$

$$\varepsilon + 1 - c - \varepsilon > (1-\alpha) \varepsilon$$

$$1 - c - \varepsilon > -\alpha \varepsilon$$

■

Proof. [Proposition 8] Effective income per efficiency unit \tilde{y}^e :

$$\begin{aligned}
 Y^e &= \tilde{k}^{1-\alpha} \tilde{\lambda} A \\
 Y^e &= \left(\varepsilon \frac{\eta}{1-c-\varepsilon} + \eta \right)^{1-\alpha} \frac{\eta \mu g_\theta}{(1-c-\varepsilon)} \left(\overbrace{\varepsilon \frac{\eta}{1-c-\varepsilon} + \eta}^k \right)^{-(1-\alpha)} A \\
 \tilde{y}^e &= \frac{Y^e}{A} = \frac{\eta \mu g_\theta}{(1-c-\varepsilon)}
 \end{aligned}$$

■

Proof. [Proposition 9]: Effective income growth rate g_{Y^e} :

$$\begin{aligned}
 \dot{Y}^e &= (1-\alpha) k^{(-\alpha)} \lambda A \dot{k} + k^{(1-\alpha)} \lambda \dot{A} + k^{(1-\alpha)} \dot{\lambda} A \\
 \frac{\dot{Y}^e}{Y^e} &= (1-\alpha) \frac{k^{-\alpha} \lambda A}{k^{1-\alpha} \lambda A} \dot{k} + \frac{k^{1-\alpha} \lambda}{k^{1-\alpha} \lambda A} \dot{A} + \frac{k^{(1-\alpha)}}{k^{1-\alpha} \lambda A} \dot{\lambda} A \\
 g_{Y^e} = \frac{\dot{Y}^e}{Y^e} &= (1-\alpha) \frac{\dot{k}}{k} + \frac{\dot{A}}{A} + \frac{\dot{\lambda}}{\lambda}, \quad \frac{\dot{\lambda}}{\lambda} = -(1-\alpha) \frac{\dot{k}}{k} \\
 g_{Y^e} = \frac{\dot{Y}^e}{Y^e} &= \frac{\dot{A}}{A} = \mu g_\theta
 \end{aligned}$$

■

A.3 Solving for the semi-endogenous growth rate

Proof. [Proposition 10]: semi-endogenous growth rate: \tilde{g}_A

departing from $\dot{A} = \mu = (\pi A)^\gamma \dot{\theta}^{1-\gamma}$ and using proposition 32 gives

$$\begin{aligned}
 \dot{A} &= \left(\frac{\eta}{(1-c-\varepsilon)} \tilde{k}^{-(1-\alpha)} g_A A \right)^\gamma \dot{\theta}^{1-\gamma} \\
 \dot{A} g_A^{-\gamma} A^{-\gamma} &= \left(\frac{\eta}{(1-c-\varepsilon)} \tilde{k}^{-(1-\alpha)} \right)^\gamma \left(\theta \frac{\dot{\theta}}{\theta} \right)^{1-\gamma} \\
 \frac{\dot{A}}{A} g_A^{-\gamma} A^{1-\gamma} &= \left(\frac{\eta}{(1-c-\varepsilon)} \tilde{k}^{-(1-\alpha)} \right)^\gamma (\theta g_\theta)^{1-\gamma} \\
 g_A^{1-\gamma} A^{1-\gamma} &= \left(\frac{\eta}{(1-c-\varepsilon)} \tilde{k}^{-(1-\alpha)} \right)^\gamma (\theta g_\theta)^{1-\gamma} \\
 g_A A &= \left(\frac{\eta}{(1-c-\varepsilon)} \tilde{k}^{-(1-\alpha)} \right)^{\frac{\gamma}{1-\gamma}} (\theta g_\theta) \\
 \tilde{g}_A &= \left(\frac{\eta}{(1-c-\varepsilon)} \tilde{k}^{-(1-\alpha)} \right)^{\frac{\gamma}{1-\gamma}} g_\theta
 \end{aligned}$$

■

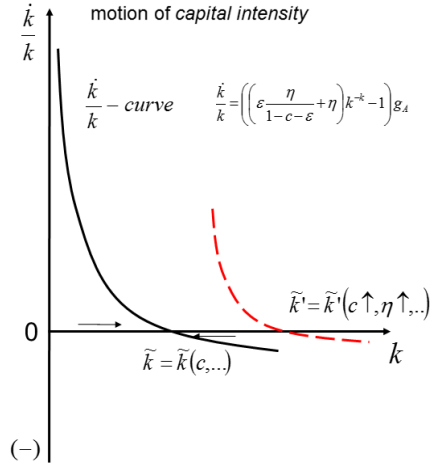


Figure 1: Motion of *capital intensity*

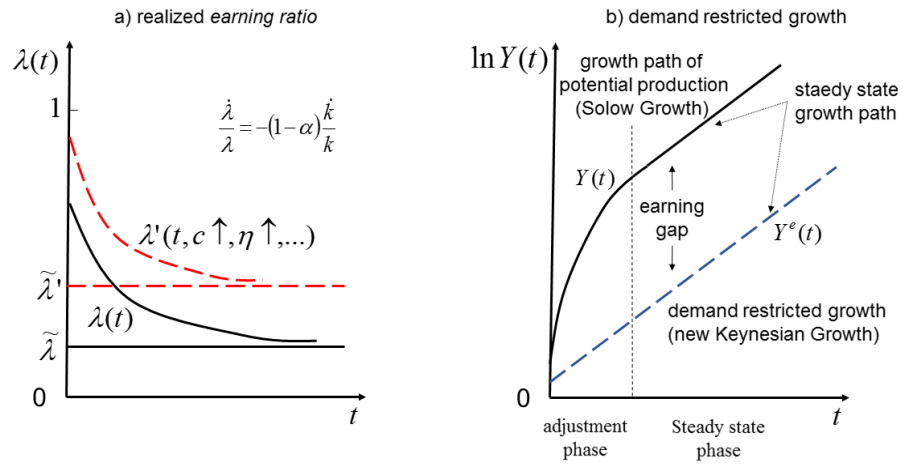


Figure 2: Path of a *demand-restricted growth process*

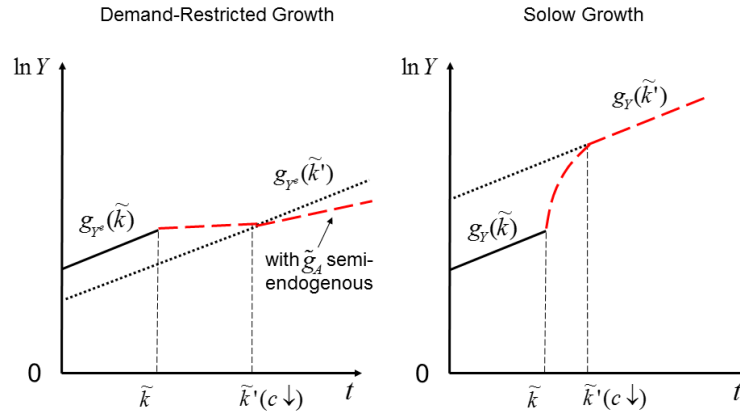


Figure 3: Comparative statics, new Keynesian growth versus neoclassical growth