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## Unemployment and Growth: Putting Unemployment into Post Keynesian Growth Theory

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# FMM WORKING PAPER

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## UNEMPLOYMENT AND GROWTH: PUTTING UNEMPLOYMENT INTO POST KEYNESIAN GROWTH THEORY

Thomas Palley\*

### ABSTRACT

Post Keynesian (PK) growth models typically fail to model unemployment. That shows up in the absence of any equilibrium condition requiring the growth of employment equal effective labor supply growth. Consequently, the models can have an imploding or exploding unemployment rate. The underlying analytical problem is failure to resolve the Harrod (1939) knife edge problem. This paper shows how the knife-edge problem can be resolved via a Kaldor – Hicks technological progress function. The paper applies the concept to several different PK growth models. In the Harrod, super-multiplier, Cambridge, and neo-Kaleckian models the warranted rate rules the roost and natural rate forces have no impact on the equilibrium growth rate. However, in a modified neo-Kaleckian model with labor market distribution conflict both warranted rate and natural rate forces impact steady state growth.

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# **Unemployment and Growth: Putting Unemployment into Post Keynesian Growth Theory**

## **Abstract**

Post Keynesian (PK) growth models typically fail to model unemployment. That shows up in the absence of any equilibrium condition requiring the growth of employment equal effective labor supply growth. Consequently, the models can have an imploding or exploding unemployment rate. The underlying analytical problem is failure to resolve the Harrod (1939) knife edge problem. This paper shows how the knife-edge problem can be resolved via a Kaldor – Hicks technological progress function. The paper applies the concept to several different PK growth models. In the Harrod, super-multiplier, Cambridge, and neo-Kaleckian models the warranted rate rules the roost and natural rate forces have no impact on the equilibrium growth rate. However, in a modified neo-Kaleckian model with labor market distribution conflict both warranted rate and natural rate forces impact steady state growth.

*Keywords:* Growth, unemployment, Harrod Knife-edge, endogenous technical progress, Hicks, Kaldor.

JEL ref.: O4, O41, O33, E12.

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## **1. Introduction: the need to model unemployment**

Post Keynesian (PK) growth models typically fail to model the labor market. That failure shows up in the absence of any equilibrium condition requiring the growth of employment equal effective growth of the labor supply. Consequently, the models can have an imploding or exploding unemployment rate. Additionally, the failure to model the labor market means unemployment and the state of the labor market are precluded from playing a role in determining distributional outcomes.<sup>1</sup>

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<sup>1</sup> Goodwin's (1967) cyclical growth model is an exception. However, though of interest to Post Keynesians, the Goodwin model is not Keynesian. Instead, it is better described as classical Marxist. Classical because saving drives investment: Marxist because labor market conflict determines the functional distribution of income.

The standard PK defense is to invoke a dichotomous Harroddian world of perfectly elastic labor supply and full employment. However, this assumption seems implausible for long run growth models describing industrialized economies as those economies can swing between high unemployment and full employment over the course of a single business cycle.

The underlying analytical problem is failure of PK growth theory to resolve the Harrod (1939) knife edge problem. Reflecting their belief in the primacy of AD, Post Keynesians have therefore tended to ignore the supply-side of the economy and the implications of the natural rate of growth. Instead, they have focused almost exclusively on the demand side and the determinants of the warranted rate of growth, and assumed industrialized economies grow at the warranted rate.

This paper shows how the Harrod knife-edge problem can be resolved using the concept of a Kaldor – Hicks technological progress function (Palley, 2012, 2014). The attribution to Kaldor (1957) reflects the fact that technological progress is endogenous and impacted by the rate of capital accumulation. The attribution to Hicks (1932) reflects the fact that the state of the labor market impacts technological progress, with firms having a greater incentive to innovate when labor markets are tight and unemployment is low.<sup>2</sup>

The paper applies the Kaldor – Hicks technological progress function to several different core PK growth models. Doing so is revealing about the character of the models. In the Harrod, super-multiplier, Cambridge, and standard neo-Kaleckian models the warranted rate rules the roost and natural rate forces have no impact on the equilibrium

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<sup>2</sup> Dutt (2006) has also recognized the absence of unemployment in Post Keynesian growth models. His resolution of the problem makes technological progress a function of the rate of change of the unemployment rate. That microeconomic set-up means the steady state unemployment rate depends on initial conditions and any unemployment rate is feasible in principle. The Kaldor – Hicks technological progress function has a different microeconomic logic, which generates different macroeconomic outcomes.

growth rate. However, in a modified neo-Kaleckian model with labor market conflict both warranted rate and natural rate forces impact the equilibrium growth rate.

## 2. The general model

The starting point is a general framework that can encapsulate different Keynesian models of growth. It is given by equations (1) – (7) below. These equations constitute the basic structure, and each Keynesian approach is completed by adding different behavioral equations and different assumptions about what are the endogenous and exogenous variables. It is those assumptions that distinguish the different theories, but all use the basic structure. The core equations are:

$$(1) Y = \text{Min}[K/v, L/b]$$

$$(2) U = 1 - L/N$$

$$(3) g_Y = g_K = g_L$$

$$(4) g_K = I/K$$

$$(5) g_L = g_L + g_a$$

$$(6) g_{LS} = g_N + g_a$$

$$(7) g_U = g_N - g_L$$

$$(8) I/K = S/K$$

$Y$  = output,  $K$  = capital,  $v$  = capital - output ratio,  $L$  = effective labor input,  $b$  = effective labor - output ratio,  $I$  = investment,  $S$  = saving,  $U$  = unemployment rate,  $L$  = employment,  $N$  = population,  $g_Y$  = output growth,  $g_K$  = growth of the capital stock,  $g_L$  = employment growth,  $g_{LS}$  = effective labor supply growth,  $g_N$  = labor force growth,  $g_a$  = rate of labor augmenting technical progress,  $g_U$  = rate of change of the unemployment rate.

Equation (1) – (7) constitute the supply side. Equation (1) is the aggregate

production function which is Leontieff. Labor input equals effective labor, reflecting the assumption that technical progress is labor augmenting.<sup>3</sup> Equation (2) is the definition of the unemployment rate. The unemployment rate depends on actual labor employed rather than effective labor employed. Equation (3) has output growth equal to the rate of capital accumulation, which in turn must equal the rate of growth of effective labor input growth given Leontieff technology. Equation (4) defines the growth of the capital stock which equals the rate of capital accumulation. Equation (5) defines the rate of effective labor input growth, which is equal to employment growth plus the rate of labor augmenting technical progress. Equation (6) defines the rate of effective labor supply growth, which is equal to labor force growth plus the rate of labor augmenting technical progress. Equation (7) determines is the rate of change of the unemployment rate, which is equal to labor force growth minus actual employment growth.

Equation (8) is the goods market clearing condition which requires that investment per unit of capital (i.e. the rate of capital accumulation) equal saving per unit of capital. This last condition holds in all PK growth models, and differences across models frequently concern the specification of investment and saving behavior.

In medium run analyses, employment growth can differ from labor supply growth so that the unemployment rate changes. Likewise, labor supply growth can differ from population growth owing to change in the labor force participation rate. In long run analyses labor supply growth must equal employment growth ( $g_N = g_L$ ) so that the unemployment rate is constant ( $g_U = 0$ ). Additionally, labor supply growth must equal

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<sup>3</sup> The capital-effective labor ratio is obtained from equation (1) and is given by  $k = K/L = v/b$ . Unlike neoclassical growth theory, the capital-effective labor ratio is of no significance. In neoclassical growth theory it is essential as it determines labor productivity and the real wage.

population growth so that labor force participation rate is constant.

### 3. The Kaldor – Hicks technical progress function: the critical element

The critical building block is the Kaldor – Hicks technological progress function introduced by Palley (2012, 2014). It makes the rate of labor augmenting technical progress a negative function of the unemployment rate.<sup>4</sup> The technical progress function serves as the mechanism that equilibrates and stabilizes the labor market. When the unemployment rate is high, the rate of technical progress falls, thereby lowering the rate of effective labor supply growth and equalizing it with labor demand growth.

The technical progress function is given by:

$$(9) \quad g_a = a(g_K, U) \quad a_{gK} > 0, a_U < 0$$

Substituting equations (3), (5), and (9) into equation (7) yields

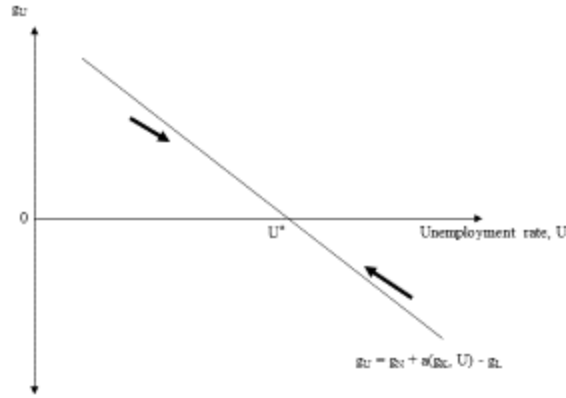
$$(10) \quad g_U = g_N + a(g_K, U) - g_K$$

Equation (10) is the fundamental differential equation governing the evolution of the unemployment rate. The necessary condition for stability is that  $dg_U/dU < 0$ . As the unemployment rate increases, the rate of increase decreases so that the unemployment rate eventually stabilizes. Figure 1 shows the case of a stable unemployment rate adjustment process. The necessary assumption is that  $a_U < 0$ .

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<sup>4</sup> An equivalent specification is to make the rate of technical progress a positive function of the employment rate (E) since  $U = 1 - E$ . The only impact is to change the sign of the partial derivative of the technical progress function with respect labor market conditions. In log linear models there is an advantage to using E as the argument.

Figure 1. The stable unemployment rate adjustment mechanism.



For purposes of clarity, the rest of the paper assumes a linear endogenous technical progress function given by

$$(11) \quad g_a = \alpha_0 + \alpha_1 g_K - \alpha_2 U \quad \alpha_0 > 0, \alpha_1 > 0, \alpha_2 > 0$$

This linear form helps identify the economic factors that matter.

Substituting equation (11) in equation (10) then yields a specific form fundamental differential equation given by

$$(12) \quad g_U = g_N + \alpha_0 - \alpha_2 U - [1 - \alpha_1]g_K$$

The long run equilibrium solution is obtained by setting  $g_U = 0$  and solving, which yields

$$(13) \quad U^* = \{g_N + \alpha_0 - [1 - \alpha_1]g_K\} / \alpha_2$$

The steady state unemployment rate is a positive function of labor force growth and a negative function of the rate of capital accumulation.<sup>5</sup>

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<sup>5</sup> Not only must the fundamental differential equation be stable, it must also yield a solution  $0 < U^* < 1$ . That implies the condition  $0 < g_N + \alpha_0 - [1 - \alpha_1]g_K < 1$ . In the rest of the paper it is assumed this condition is satisfied.



#### 4. The Harrod model

The Harrod model (Harrod, 1939) is the oldest model of modern growth theory and provides the starting point for understanding the problem of unemployment in PK growth models. The two fundamental equations of the model are

$$(14) g_K = su$$

$$(15) g_K = g_L$$

$$(16) g_L = g_L + g_a$$

$$(17) g_{LS} = g_N + g_a$$

$$(18) g_U = g_N - g_L$$

$s$  = propensity to save,  $u$  = output – capital ratio. The output - capital ratio can be viewed as the exogenously given capacity utilization rate. Equation (14) is the goods market clearing condition, which requires the rate of capital accumulation equal the saving rate. An important feature of the model is that saving drives capital accumulation, so that the Harrod model is classical as regards the accumulation process, and not Keynesian.

The fundamental challenge is how to balance the warranted growth rate determined by the goods market (equation (14)) with the natural growth rate determined by the supply-side (equation (17)). If  $g_N > g_L$  the unemployment rate will steadily increase, producing an explosive deflationary gap. If  $g_N < g_L$  the unemployment rate will steadily fall, producing an implosive inflationary gap. It is the possibility of these unemployment rate dynamics that give the model its Keynesian patina.

Combining equations (12) and (14) yields the fundamental differential equation for the unemployment rate in the Harrod model

$$(19) g_U = g_N + \alpha_0 - \alpha_2 U - [1 - \alpha_1]su$$

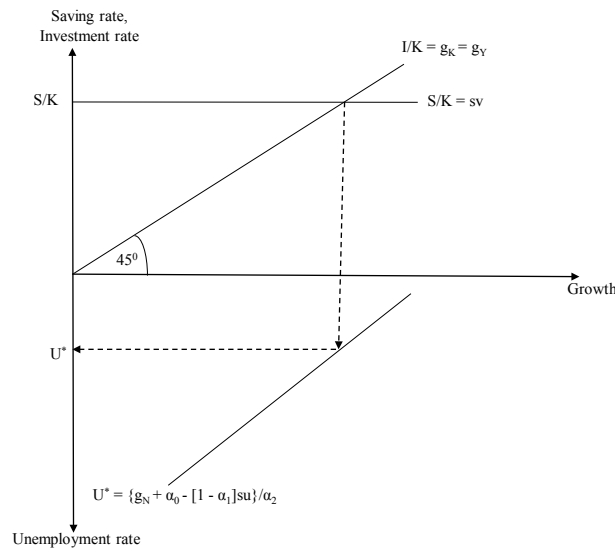
The stability is satisfied since  $dg_U/dU = -\alpha_2 U < 0$ . Setting  $g_U = 0$  then enables solution for the steady state unemployment rate which is given by

$$(20) U^* = \{g_N + \alpha_0 - [1 - \alpha_1]su\}/\alpha_2$$

To be feasible, the unemployment must also satisfy  $0 \leq U^* \leq 1$ . That imposes the parameter restrictions  $g_N + \alpha_0 - [1 - \alpha_1]su > 0$  and  $g_N + \alpha_0 - [1 - \alpha_1]su < 1$ . Capital accumulation cannot be too low or the unemployment rate will explode and the economy will be driven to an immiserized state. Conversely, capital accumulation cannot be too fast or the economy will hit the full employment boundary.

The Harrod model is illustrated in Figure 2. The upper panel show the goods market and the determination of the warranted growth rate. The lower panel traces the relationship between the growth rate and the equilibrium unemployment rate. There are several features to note.

Figure 2. The Harrod model with Kaldor – Hicks endogenous technological progress.



First, the model has a classical accumulation process whereby the saving rate

determines the rate of capital accumulation. A higher saving rate increases capital accumulation and growth, and lowers the steady state unemployment rate. A higher capital – output rate (lower normal capacity utilization) lowers growth and increases the unemployment rate. The logic is a saving rate results in slower capital accumulation and slower employment growth, which lowers growth and increases the unemployment rate.

Second, there is a negative relationship between growth and unemployment, but the causality runs from the goods market to the labor market. The natural rate of growth adjusts to the warranted rate via adjustment of the rate of labor augmenting technical progress, as governed by the Kaldor – Hicks technical progress function.

Third, in the Harrod model with a Kaldor - Hicks technical progress the warranted rate rules the roost, and labor force growth has no effect on steady state growth. Fourth, faster labor force growth ( $g_N$ ) increases the unemployment rate.

Fifth, increases in the parameters  $\alpha_0$  and  $\alpha_1$  increase the unemployment rate. These parameters increase the rate of productivity growth, which raises the rate of effective labor supply growth and crowds out jobs for actual workers. That causes technological progress unemployment. The reverse holds for an increase in the parameter  $\alpha_2$ . In this case, higher unemployment slows technological progress more strongly, which helps lower the unemployment effect of technological progress.<sup>6</sup>

## **5. The super-multiplier model**

A second model with a long history is the super-multiplier model and it has recently been subject to a Post Keynesian revival (Serrano, 1995; Serrano and Freitas, 2017). The logic of the simplest super-multiplier model is very similar to the Harrod model, except that it

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<sup>6</sup> Technological progress unemployment is different from technological displacement unemployment. The former is a rate effect which impacts the rate of future job creation. The latter is a level effect and displaces existing workers.

imposes a Keynesian accumulation process in place of a classical one. Investment determines saving instead of saving determining investment. The rate of accumulation is given by

$$(21) I/K = g^{ED}$$

$g^{ED}$  = firms' expected rate of demand growth. The assumptions are that firms' expectations of demand growth equal actual demand growth, and the goods market clears at all time. Consequently, firms must grow their capital stock at the rate of expected demand growth in order to be able to meet demand.

Using equations (21) and (13), the solutions to the super-multiplier model are given by

$$(22) g_Y = I/K = g^{ED}$$

$$(23) U^* = \{g_N + \alpha_0 - [1 - \alpha_1]g^{ED}\}/\alpha_2$$

The model is shown in Figure 3. It is near identical to the Harrod model, but the economics of accumulation are very different.<sup>7</sup>

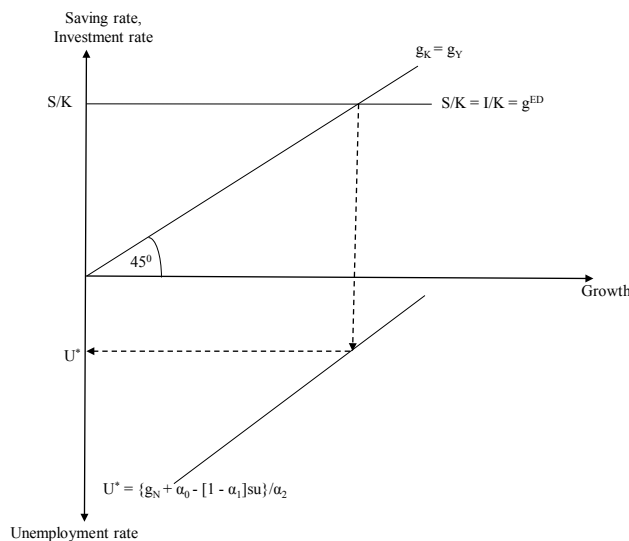
A higher rate of expected demand growth raises growth by raising the rate of capital accumulation. It also lowers unemployment by increasing employment growth, but causality runs from the goods market (the warranted rate) to the labor market. The mechanism is the Kaldor – Hicks technical progress function, whereby labor augmenting technical progress increases to accommodate faster effective labor demand growth caused by faster capital accumulation. Once again, the warranted rate rules the roost and labor

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<sup>7</sup> Fazzari et al. (2018) use the Kaldor-Hicks technical progress function into their super-multiplier model. However, they do not dig into the labor market to show how the technical progress function equilibrates labor supply and demand growth. Instead, their focus is showing how aggregate supply accommodates faster aggregate demand growth. The mechanism is the Kaldor-Hicks function. Faster demand growth lowers unemployment which stimulates labor augmenting technical progress, which raises aggregate supply growth. Implicitly, below the surface is the labor market adjustment process described herein.

force growth has no impact on growth. However, faster labor force growth again increases steady state unemployment. Saving has no impact on growth, and is assumed to accommodate capital accumulation in a manner akin to forced saving.<sup>8</sup>

Figure 3. The simple super-multiplier model with Kaldor – Hicks endogenous technological progress.



## 6. The Cambridge model

The third model to consider is the Cambridge model which was developed in the late 1950s and early 1960s. The Cambridge model incorporates income distribution using Kaldor's (1955-56) theory of income distribution, according to which functional income distribution adjusts to equalize output and aggregate demand.<sup>9</sup>

The equations of the Cambridge model are as follows

$$(24) \quad I/K = \gamma_0 + \gamma_1 \pi \quad \gamma_0 > 0, \gamma_1 > 0$$

<sup>8</sup> In more complicated super-multiplier models saving is not passive, and that requires a goods market clearing mechanism (Palley, 2018b).

<sup>9</sup> This section is based on Palley (2012) who presents a Cambridge model with a Kaldor - Hicks technical progress function. However, that model is significantly more complicated as the saving structure is affected by Goodwin-styled labor market conflict that affects the distribution of the wage bill across workers and managers.

$$(25) S/K = \beta_0 + \beta_1 \pi$$

$$\beta_0 > 0, 0 < \gamma_1 < \beta_1 < 1$$

$$(26) s_\pi = \Pi/Y$$

$$(27) \pi = [\Pi/Y] \cdot [Y/K] = s_\pi u$$

$$(28) I/K = S/K$$

$\Pi$  = profits,  $s_\pi$  = profit share,  $\pi$  = profit rate,  $u$  = capacity utilization. Equation (24) has the rate of capital accumulation being a positive function of the profit rate, while equation (25) has saving being a positive function of the profit share. The rate of capacity utilization is exogenously fixed at its long run normal rate.

Equations (24) – (28) enable solution for the profit share, profit rate, rate of accumulation and growth. The equilibrium profit share is given by

$$(29) s_\pi^* = [\gamma_0 - \beta_0]/[\beta_1 - \gamma_1]u > 0$$

It is assumed that  $\gamma_0 - \beta_0 > 0$  so that net autonomous demand is positive. Additionally, to ensure Keynesian multiplier stability, it is assumed that  $\beta_1 > \gamma_1$  so that an increase in the profit share raises saving by more than it raises investment. The logic of determination of the profit share is that increases in investment raise AD, necessitating a higher profit share so that saving can adjust to equalize investment and clear the goods market. Increases in saving have the reverse effect.

Substituting (29) into equation (24) then yields

$$(30) I/K^* = \gamma_0 + \gamma_1[\gamma_0 - \beta_0]/[\beta_1 - \gamma_1] > 0$$

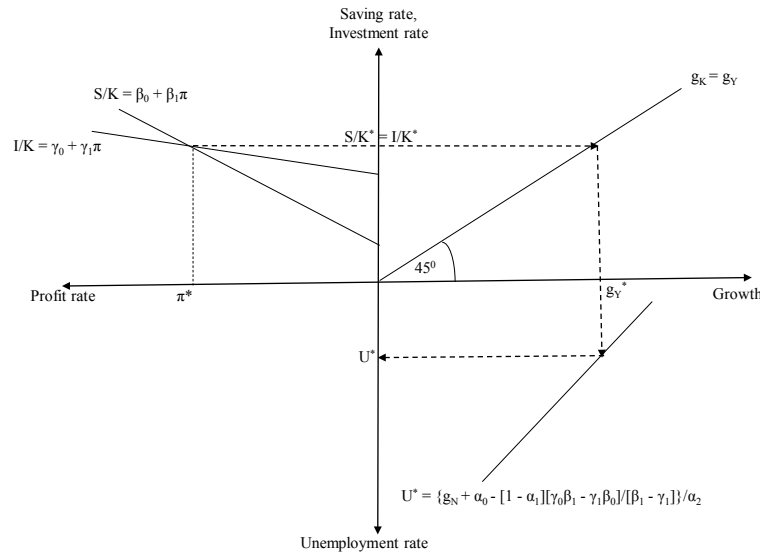
Lastly, substituting equation (30) into equation (13) yields the expression for equilibrium unemployment, which is given by:

$$(31) U^* = \{g_N + \alpha_0 - [1 - \alpha_1]\{\gamma_0 + \gamma_1[\gamma_0 - \beta_0]/[\beta_1 - \gamma_1]\}\}/\alpha_2$$

The model is illustrated in Figure 4. Many of the features remain the same, particularly as

regards the warranted ruling the roost. Saving and investment behavior determine the growth rate, and they also impact the unemployment rate. Labor force growth affects the unemployment rate, but has no impact on the growth rate. Once again, the warranted and natural growth rates are equalized by adjustment of the rate of labor augmenting technical progress via the Kaldor – Hicks endogenous technological progress function.

Figure 4. The Cambridge model with Kaldor – Hicks endogenous technological progress.



Demand again affects the growth rate and unemployment rate. An increase in the rate of accumulation raises the profit rate and growth rate, and lowers the unemployment rate. Conversely, an increase in the saving rate lowers the profit rate and rate of growth, and increases the unemployment rate. Faster population growth and faster technical progress both increase the unemployment rate, so that there can be technical progress unemployment and demographically induced unemployment. However, neither affects the growth rate.

## 7. The neo-Kaleckian model

The fourth model to consider is the neo-Kaleckian model which emerged in the 1980s as an alternative to the Cambridge model.<sup>10</sup> The key distinction between the two models is that the neo-Kaleckian model treats capacity utilization as endogenously variable, and capacity utilization influences capital accumulation and growth. That opens a channel for a higher wage share to increase growth – so-called “wage-led” growth – because a higher wage share can increase AD and raise capacity utilization, thereby increasing capital accumulation and growth.

The equations of the model are given by:

$$(32) I/K = I = \gamma_0 + \gamma_1 u + \gamma_2 \pi + \gamma_3 s_\Pi \quad \gamma_0 > 0, \gamma_1 > 0, \gamma_2 > 0, \gamma_3 > 0$$

$$(33) \pi = s_\Pi u$$

$$(34) s_W = 1 - s_\Pi \quad 0 < s_\Pi < 1, 0 < s_W < 1$$

$$(35) S/K = S = \beta_\Pi s_\Pi u + \beta_W s_W u \quad 1 > \beta_\Pi > \beta_W > 0$$

$$(36) I = S$$

$$(37) g = I/K$$

$\pi$  = profit rate,  $\beta_\Pi$  = propensity to save out of profits,  $\beta_W$  = propensity to save out of wages.

The endogenous variables are  $I$ ,  $S$ ,  $u$ ,  $\pi$ ,  $s_W$ , and  $g$ .

Equation (32) determines the rate of capital accumulation which is a positive function of capacity utilization, the profit share and the profit rate. Equation (33) determines the profit rate which equals the product of the profit share and the rate of capacity utilization. Equation (34) determines the wage share based on the income share adding up constraint. Equation (35) determines the aggregate saving rate. Equation (36) is

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<sup>10</sup> This section is based on Palley (2014) who presents a neo-Kaleckian model with a Hicks-Kaldor technical progress function. However, that model is more complicated because the saving structure is affected by Goodwin-styled labor market conflict that affects the distribution of the wage bill across workers and managers.



the dynamic IS condition that ensures investment - saving balance. Equation (37) determines the growth rate which is equal to the rate of capital accumulation.

The neo-Kaleckian growth model has three regimes (Bhaduri and Marglin, 1990): wage-led, conflictive, and profit-led. These regimes refer to the impact of an exogenous change in the profit share. In a wage-led regime, a higher profit share lowers both capacity utilization and growth. In a conflictive regime, a higher profit share lowers capacity utilization but increases growth. In a profit-led regime, a higher profit share raises both capacity utilization and growth.

The different regimes can be characterized by reference to the slope of the IS schedule in  $[u, s_\pi]$  space. A wage-led regime is characterized by a relatively flat negatively sloped IS; a conflictive regime is characterized by a relatively steep negatively sloped IS; and a profit-led regime is characterized by a positively sloped IS.

The IS schedule is given by

$$(38) \gamma_0 + \gamma_1 u + \gamma_2 s_\pi u + \gamma_3 s_\pi = \beta_\pi s_\pi u + \beta_{ws} w u$$

Its slope is  $ds_\pi/du = [S_u - I_u]/[I_{s_\pi} - S_{s_\pi}]$

$$= \{\beta_\pi s_\pi + \beta_{ws} - \alpha_1 - \alpha_2 s_\pi\} / \{\alpha_2 u + \alpha_3 + \beta_w u - \beta_\pi u\}$$

The slope depends on the distributional parameter  $s_\pi$ . If the Keynesian multiplier stability condition ( $S_u - I_u > 0$ ) holds, the numerator is positive and the sign of the slope depends exclusively on the sign of the denominator. The economy is wage-led or conflictive if the denominator is negative (i.e. the profit share has a larger absolute impact on consumption than investment), and profit-led if it is positive.

Solving equation (38) yields the solution for equilibrium capacity utilization which is given by:

$$(39) u^* = [\gamma_0 + \gamma_3 s_{\Pi}] / \{\beta_W + [\beta_{\Pi} - \beta_W] s_{\Pi} - \gamma_1 - \gamma_2 s_{\Pi}\}$$

Substituting equations (33) and (39) into equation (32) yields the solution for equilibrium growth which is given by:

$$(40) g_K^* = \gamma_0 + [\gamma_1 + \gamma_2 s_{\Pi}] [\gamma_0 + \gamma_3 s_{\Pi}] / [\beta_{\Pi} s_{\Pi} + \beta_W s_W - \gamma_1 - \gamma_2 s_{\Pi}] + \gamma_3 s_{\Pi}$$

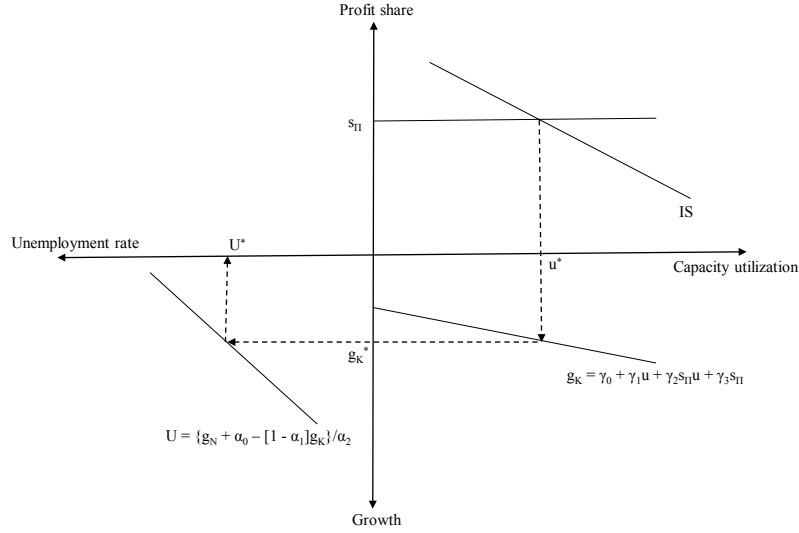
Substituting equation (40) into equation (12) yields the equilibrium unemployment rate which is given by:

$$(41) U^* = \{g_N + \alpha_0 - [1 - \alpha_1] g_K^*\} / \alpha_2$$

As is standard in the neo-Kaleckian model, the properties of the model depend on the nature of the regime. However, for all regimes the warranted rate again dominates the natural rate. The goods market determines capacity utilization and growth, and induced Kaldor – Hicks technological progress then brings the labor demand and supply growth into equilibrium. Natural rate and supply-side forces do not affect the equilibrium growth rate.

Figure 5 illustrates the model for a wage-led regime: hence, the negatively sloped IS schedule. There is a positive relation between growth and capacity utilization, and a negative relation between growth and the unemployment rate. An increase in firms' animal spirits shifts the IS right, raising capacity utilization and growth, which lowers the equilibrium unemployment rate. An increase in the profit share shifts up the  $s_{\Pi}$  line, which lowers capacity utilization. It also shifts the growth function south and rotates it. The capacity utilization effect dominates so that growth falls and the equilibrium unemployment rate rises.

Figure 5. The wage-led neo-Kaleckian model with Kaldor – Hicks endogenous technological progress.



In a conflictive regime the IS remains negatively sloped, but it has a greater absolute slope. The increase in the profit share still lowers the capacity utilization rate, but the growth function shifts south sharply as well as rotating clockwise. Now, the profit share effect dominates so that growth increases and the unemployment rate falls.

In a profit-led regime the IS is positively sloped. Increases in the profit share raise capacity utilization. That augments the positive profit share effect on the growth function, so that growth increases and the unemployment rate falls.

Lastly, the direction of co-movement of capacity utilization and the unemployment rate depends on the regime and the nature of the shock. A positive profit share shock produces negative co-movement in the wage-led regime (lower  $u$ , higher  $U$ ), negative co-movement in the profit led regime (higher  $u$ , lower  $U$ ), and positive co-movement in the conflictive regime (lower  $u$ , lower  $U$ ).

## 8. Extending the neo-Kaleckian model to include labor market conflict

Palley (1998) presents a short-run Kaleckian macro model with Goodwin (1967)-like labor market conflict over income distribution. The logic of that macroeconomic model can be incorporated within the neo-Kaleckian growth model by making the profit share a positive function of the unemployment rate, as follows:

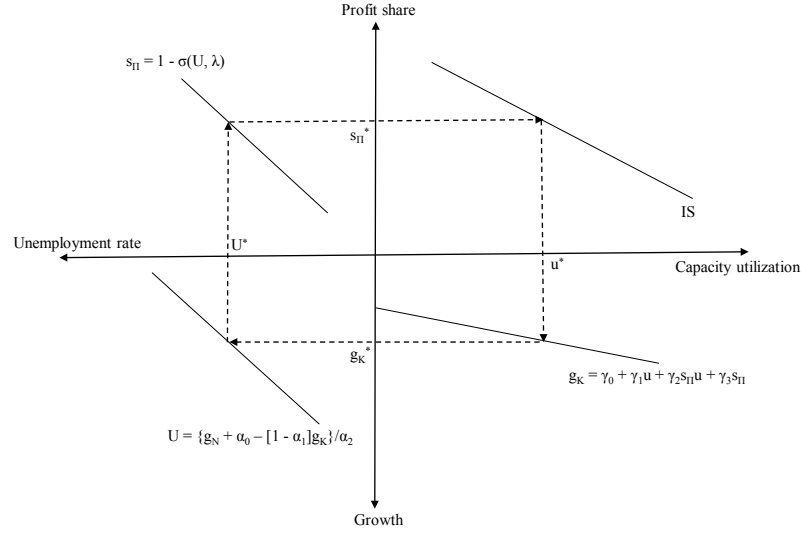
$$(42) s_{\Pi} = \sigma(U, \lambda) \quad \sigma_U > 0, \sigma_{\lambda} < 0$$

$\lambda$  is a worker bargaining power variable, and increases in  $\lambda$  lower the profit share. Given the adding up constraint on income shares, that implies the wage share is a negative function of the unemployment rate given by

$$(43) s_W = 1 - \sigma(U, \lambda) = \Sigma(U, \lambda) \quad \Sigma_U < 0, \Sigma_{\lambda} > 0$$

The extended model for a wage-led regime is shown in Figure 6. Now, the northwest quadrant contains an additional functional relation between the unemployment rate and the profit share. In equilibrium the goods market (the northeast quadrant) and the labor market (southeast quadrant) must clear simultaneously. In terms of Figure 6, the points of equilibrium must map back into themselves via the other quadrants. The warranted rate no longer rules the roost because there is interaction between the two markets. The natural rate affects income distribution in the labor market, which then impacts the warranted rate as determined in the goods market.

Figure 6. The extended wage-led neo-Kaleckian model with Kaldor – Hicks endogenous technological progress.



Mathematically, the simple neo-Kaleckian model was block recursive with the goods market determining capacity utilization, capacity utilization determining the warranted growth rate, and the warranted growth rate determining the unemployment rate. Now, there is simultaneous interaction so that the natural rate affects the unemployment rate, which affects the warranted rate and vice-versa.

The model can be expressed as a two equation simultaneous system in capacity utilization – unemployment rate space. Equilibrium capacity utilization is obtained by substituting equation (42) into equation (39) to yield

$$(44) \quad u = u(U, \gamma_0, \lambda, \gamma_1, \gamma_2, \gamma_3, \beta_W, \beta_{\Pi})$$

The slope of the relation depends on the nature of the regime. In a wage-led or conflictive regime it is negatively sloped in  $[u, U]$  space because a higher profit share lowers capacity utilization. In a profit-led regime it is positively sloped.

The equilibrium rate of capital accumulation is obtained by substituting equations

(35) and (44) into equation (32), yielding a reduced form for the rate of capital accumulation given by

$$(45) \quad g_K = g(u, U, \gamma_0, \lambda, \gamma_1, \gamma_2, \gamma_3) \quad g_u > 0, g_U < 0$$

Growth responds positively to higher capacity utilization. The response to the unemployment rate depends on the regime. The response is negative in a wage-led regime, and positive in a conflictive and profit-led regime.

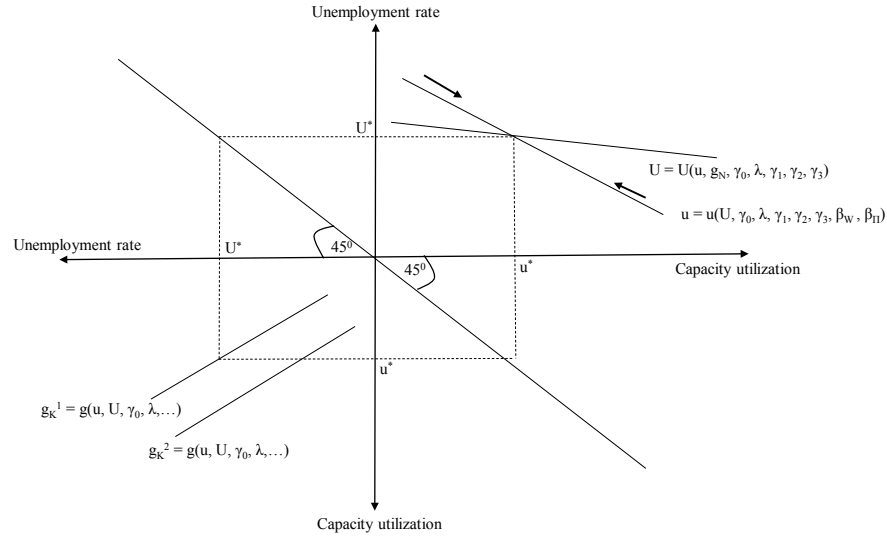
The equilibrium unemployment rate is obtained by substituting equation (45) into equation (13), which yields a reduced form given by

$$(46) \quad U = U(u, g_N, \gamma_0, \lambda, \gamma_1, \gamma_2, \gamma_3) \quad U_u < 0$$

Higher capacity utilization lowers the unemployment rate because it increases capital accumulation and job growth.

As is standard in the neo-Kaleckian model, there are three regimes each of which is of interest. Figure 7 shows the wage-led regime. The northwest quadrant shows the equilibrium capacity utilization and unemployment rate functions. The unemployment rate function is always negatively sloped in  $[u, U]$  space. The capacity utilization function is negatively sloped in a wage-led regime as lower unemployment increases the wage share, which raises AD and capacity utilization. The intersection of the two functions determines the general equilibrium.

Figure 7. Determination of equilibrium in the extended wage-led neo-Kaleckian model with Kaldor – Hicks endogenous technological progress ( $g_K^2 > g_K^1$ ).



The capacity utilization function must be absolutely steeper than the unemployment rate function if the labor market is stable.<sup>11</sup> Capacity utilization is a jump variable. The unemployment rate is a state variable. Consequently, the move to general equilibrium occurs by smoothly sliding down the capacity utilization function.<sup>12</sup>

The southeast quadrant shows a family of growth iso-contours, the slope of which is given by  $dU/du|_g = -g_u/g_U > 0$ . Iso-contours lying in a southeasterly direction correspond to higher levels of growth. *Ceteris paribus*, a lower unemployment rate raises the wage share, which raises growth because the economy is wage-led. *Ceteris paribus*, higher capacity utilization also raises growth.

There are four interesting comparative static experiments affecting both the

<sup>11</sup> Increased excess labor supply causes a fall in labor augmenting technical progress, which lowers effective labor supply growth and brings down the actual unemployment rate. The reverse happens with excess labor demand. Inspection of Figure 7 shows the dynamics are unstable if the unemployment rate function has greater absolute slope.

<sup>12</sup> The jump – state distinction is not widely recognized in the neo-Kaleckian growth discourse because the labor market is absent. Once the labor market is introduced and employment is treated as a state variable, capacity and utilization must jump via increased use of another “labor power” input, such as hours (Palley, 2014).

demand and supply sides of the economy. An increase in actual labor force growth ( $dg_N > 0$ ) shifts up the unemployment function in the northwest quadrant. The unemployment rate rises, and capacity utilization falls as the higher unemployment rate lowers the labor share and the economy is wage led. In the southwest quadrant, the economy moves to a lower growth iso-contour so growth falls.

An increase in worker bargaining power ( $d\lambda > 0$ ) increases the wage share. In the northwest quadrant, that shifts the capacity utilization function right. It also shifts the unemployment rate function up slightly since there is a slightly negative effect from  $\lambda$  on capital accumulation and job growth. However, because the economy is wage-led, the net effect is for capacity utilization to rise and the unemployment rate to fall. In the southwest quadrant, that corresponds to jumping to a higher iso-growth contour. However, the increase in  $\lambda$  also causes the iso-contours to be rebased with slightly lower growth. However, once again, growth is higher because the economy is wage-led.

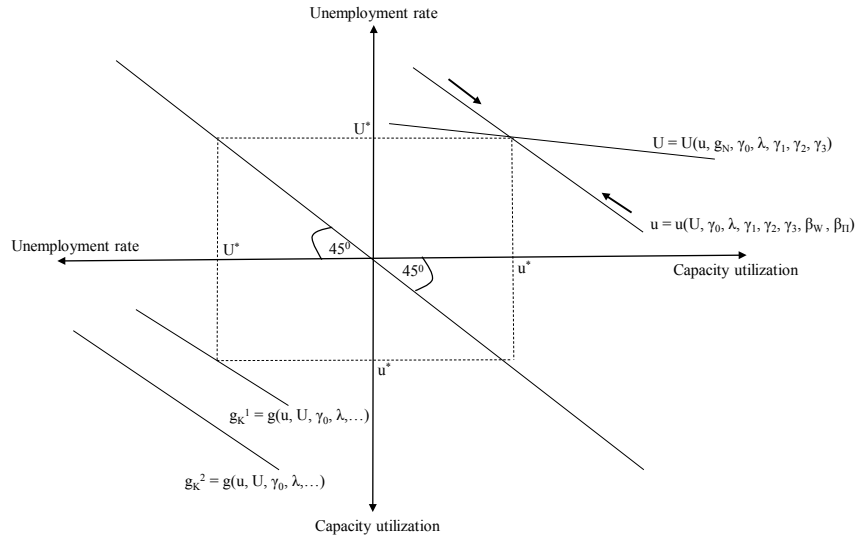
A third experiment is a decrease in the propensity to save out of wages ( $d\beta_w < 0$ ) or profits ( $d\beta_\pi < 0$ ). That shifts the capacity utilization function right, raising capacity utilization and lowering the unemployment rate. In the southwest quadrant, the economy moves to a higher iso-growth contour and growth increases.

Lastly, an increase in autonomous investment spending ( $d\gamma_0 > 0$ ) shifts the capacity utilization function right and the unemployment rate function down, so that capacity utilization increases and the unemployment rate falls. That moves the economy to a higher iso-growth contour with faster growth. Additionally, there is a further direct shot to growth as the increase in autonomous investment causes an upward re-basing of the iso-growth contours.



Figure 8 illustrates the determination of equilibrium in the conflictive version of the extended neo-Kaleckian model. Now, the iso-growth contours are negatively sloped, and growth increases in the southwesterly direction. Recall, the slope of the iso-growth contours is  $dU/du|_g = -g_u/g_U$ . In the conflictive regime, increased unemployment raises the profit share, which raises growth so that  $g_U > 0$ . Since  $g_u > 0$ , that implies  $dU/du|_g = -g_u/g_U < 0$ .

Figure 8. Determination of equilibrium in the extended conflictive neo-Kaleckian model with Kaldor – Hicks endogenous technological progress ( $g_K^2 > g_K^1$ ).



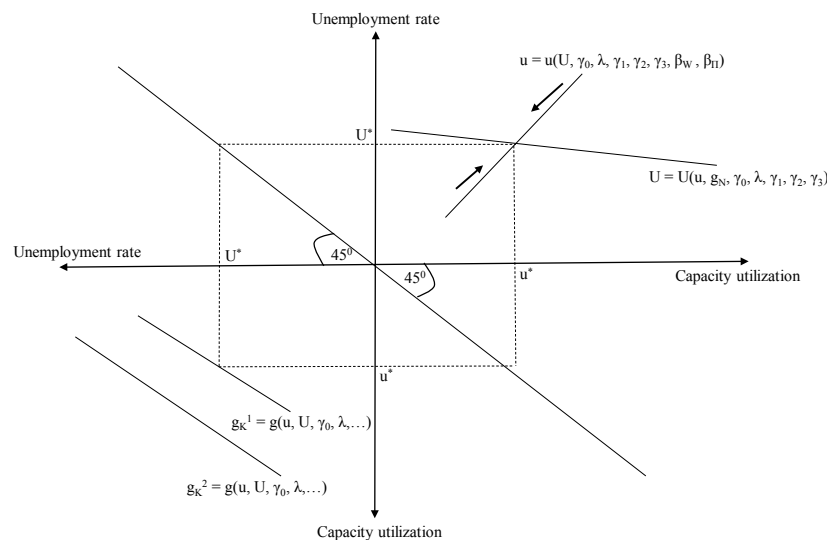
The comparative statics on the demand side ( $d\beta_w < 0$ ,  $d\beta_\pi < 0$ ,  $d\gamma_0 > 0$ ) are the same as the wage-led regimes, but those on the supply side are different. An increase in labor force growth ( $dg_N > 0$ ) shifts the unemployment rate function up, which raises the unemployment rate and lowers capacity utilization. However, growth increases because the higher unemployment rate increases the profit share, which strongly increases capital accumulation.

An increase in worker bargaining power ( $d\lambda > 0$ ) increases the wage share. That

shifts the utilization function up and lowers the unemployment function. Capacity utilization rises and the unemployment rate falls, which further lowers the profit share. Growth therefore falls because the profit share effect dominates in a conflictive regime.

Figure 9 illustrates the determination of equilibrium in the profit-led version of the extended neo-Kaleckian model. Now, the utilization function is positively sloped in the northwest quadrant. A higher unemployment rate raises the profit share, which increases utilization because the economy is profit-led. The iso-growth contours are negatively sloped, as in the conflictive regime.

Figure 9. Determination of equilibrium in the extended profit-led neo-Kaleckian model with Kaldor – Hicks endogenous technological progress ( $g_K^2 > g_K^1$ ).



The comparative statics on the demand side with respect to saving ( $d\beta_W < 0$ ,  $d\beta_\Pi < 0$ ) are very different. Lower saving shifts the utilization function right, increasing utilization and lowering unemployment. The effect on growth is ambiguous. On one hand, lower unemployment lowers growth because it lowers the profit share. On the other hand, higher utilization increases growth because it increases capital accumulation. If the profit

share is very sensitive to unemployment and the accumulation function is very sensitive to the profit share, growth is more likely to fall.

Increases in autonomous investment ( $d\gamma_0 > 0$ ) shift the utilization function right and shifts the unemployment rate function down. The unemployment rate falls. Utilization could also fall if there is a strong adverse impact on the profit share and the economy is very profit sensitive. That possibility makes the impact on growth formally ambiguous. The increase in autonomous capital accumulation raises growth, but it could be overwhelmed by extreme adverse profit share developments combined with extreme investment sensitivity to the profit share.

An increase in labor force growth ( $dg_N > 0$ ) shifts the unemployment rate function up, which raises the unemployment rate and increases capacity utilization. Capacity utilization increases because the higher unemployment rate increases the profit share which strongly increases capital accumulation. Growth also increases.

Lastly, an increase in worker bargaining power ( $d\lambda > 0$ ) increases the wage share. That shifts the utilization function left and the unemployment rate function up, because the lower profit share reduces capital accumulation. Capacity utilization falls, the unemployment rate increases, and growth therefore falls. The increase in the unemployment rate mitigates some of increased bargaining power effect on the profit share.

In sum, making the profit share endogenous with respect to the unemployment rate has significant implications for the neo-Kaleckian model. First, it means that the warranted rate no longer rules the roost in determining growth. Instead the warranted rate is influenced by the natural rate and supply side factor.

Second, the effects of an endogenous profit share depend importantly on the relationship between the unemployment rate and the profit share. The above analysis assumed a negative relationship, which means lower unemployment rates discourage growth in profit-led and conflictive regimes via the negative impact on the profit share. Conversely, it encourages growth in wage-led regimes. Those discouragement and encouragements effects would be reversed if the unemployment rate – profit share relationship was positive.

There are good theoretical and empirical arguments for believing the relationship may be a non-linear concave function, with the profit share increasing as the economy moves away from high unemployment rates and then falling as the economy moves toward full employment and tight labor market effects kick-in. In that case, the relationship between growth and unemployment will depend where the economy is on the concave profit share function. The big point is that not only do the predictions of the neo-Kaleckian model depend on type of regime, they also depend on the nature of the unemployment rate - profit share relationship.

Third, the pattern of co-movement between capacity utilization and the unemployment rate will depend on type of regime, the nature of the unemployment rate - profit share relationship, and the type of disturbance. That means there can be no assumed monotonic correspondence between capacity utilization and the unemployment rate.

## **9. Conclusions**

This paper has presented a general framework for incorporating labor markets and unemployment into PK growth models. The framework consists of a core structure of definitions and growth accounting relations plus the Kaldor – Hicks technological progress

function. That function is compatible with all Keynesian growth models and provides a mechanism for equalizing the warranted and natural rates of growth. It thereby overcomes the fundamental problem that has afflicted Keynesian growth theory since Harrod's (1939) inception of modern growth theory.

Historically, PK models have failed to address the labor market and incorporate labor market effects. Adding a Kaldor - Hicks technological progress function enables them to do so. Income distribution plays a central role in PK growth theory. If income distribution is determined independently of the unemployment rate, the warranted growth rate rules the roost in determining steady state growth. However, once income distribution is affected by the unemployment rate, that provides a channel for natural rate forces to influence AD and the goods market. In that case, steady state growth is determined by both natural rate (supply-side) and warranted rate (demand-side) forces.

## References

- Bhaduri, A. and Marglin S.A. (1990), "Unemployment and the real wage: the economic basis for contesting political ideologies," *Cambridge Journal of Economics*, 14, 375 – 93.
- Dutt, A.K. (2006): "Aggregate demand, aggregate supply and economic growth," *International Review of Applied Economics*, 20 (July), 319 – 36.
- Fazzari, S., Ferri, P. and Variato, A.M. (2018), "Demand-led growth and accommodating supply," FMM Working Papers, IMK Macroeconomic Policy Institute, Dusseldorf, Germany, 15, February.
- Goodwin, R.M. (1967), "A growth cycle," in Feinstein, C.H. (ed.), *Capitalism and Economic Growth*, Cambridge: Cambridge University Press, 54 – 58.
- Harrod, R. F. (1939), "An essay in dynamic theory," *Economic Journal*, 49, 14-33.
- Hicks, J.R. (1932), *The Theory of Wages*, London: Macmillan.
- Kaldor, N. (1956), "Alternative Theories of Distribution," *Review of Economic Studies*, 23, 83 - 100.
- (1957), "A model of economic growth," *Economic Journal*, 67, 591 – 624.
- Palley, T.I. (1998), "Macroeconomics with Conflict and Income Distribution," *Review of Political Economy*, 10, 329 - 342.
- (2012), "Growth, Unemployment and Endogenous Technical Progress: A Hicksian Resolution of Harrod's Knife-Edge," *Metroeconomica*, 63 (3), 512 – 541.
- (2014), "A neo-Kaleckian - Goodwin model of capitalist economic growth: Monopoly power, managerial pay, and labor market conflict," *Cambridge Journal of Economics*, 38 (6) (November), 1355 – 1372.
- Serrano, F. (1995), "Long period effective demand and the Sraffian supermultiplier," *Contributions to Political Economy*, 14 (1), 67-90.
- Serrano, F. and Freitas, F. (2017), "The Sraffian supermultiplier as an alternative closure for heterodox growth theory," *European Journal of Economics and Economic Policy: Intervention*, 14(1), 71 – 90.

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