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The Performance of Lot Sizing Heuristics in the Case of Sparse Demand Patterns

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G. Knolmayer, Kiel, August 1986

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The Performance of Lot Sizing Heuristics in the Case of Sparse Demand Patterns

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1. Introduction

During the last years a large number of lot sizing heuristics have been proposed for varying demand rates which are of special importance in Material Requirements Planning (MRP)-systems. The literature concentrates on procedures entitled Least-Unit-Cost, Least-Total-Cost (resp. Part-Period-Method) and Silver/Meal-heuristic. Less attention has been paid to several other proposals.

A few simulation studies identified heuristics which on the average produce significantly better results than their competitors \{4;8;9;12;17\}. Although the comparisons led to rather similar results, an important difference in ranking the best procedures exists: In the study by Wemmerlöv \{17\} Groff's heuristic performs significantly better than the Silver/Meal-procedure, whereas the results obtained by Knolmayer \{8\} show no statistically significant difference between the two procedures. One reason for this difference may be that Wemmerlöv considers a demand situation with intermittent demand in which zero demands occur in one third of the problems whereas Knolmayer regards dense demand patterns with a positive demand in each period.

Zero demands are particularly important in MRP-systems because lot sizing, e.g. for final products, may result in a situation where no dependent demand for intermediate parts exists in many periods.
Furthermore, some MRP-applications are moving to shorter planning periods (e.g. one day instead of one week as time unit). In this case, zero demands become more important on account of the more detailed planning procedure. 

Silver/Miltenburg (14) consider zero demands to be important enough to design a special heuristic for sparse demand patterns.

The goal of the present study is to determine whether demand patterns which are differently structured with respect to zero demands influence the performance of the heuristics as well in the degree of the error committed as in their ranking. The study aims at giving lot sizing recommendations for the important, but usually neglected sparse demand patterns. With the exception of zero demands, the same factors of the experiment were used as in the earlier comparison of lot sizing heuristics with dense demands. The experiment regards 1500 randomly generated demand patterns for 15 problem classes distinguished by mean positive demand and by the range of deviation of single to mean positive demand. Each demand pattern represents $T=200$ discrete demands. About 50 % of the demands equal zero; positive demands are uniformly distributed. The cost parameters used are $F=1000$ and $h=1$. All significance-judgements are based on an experimentwise error factor $\alpha = .01$ for a multiple-comparison sign test described by Miller (11).
2. Description and comparison of relevant heuristics

2.1. Description

This section describes only those heuristics which perform favourably in the case of dense demand patterns or which are often implemented in MRP-systems. The description serves as basis for discussing some modifications of the heuristics in the case of sparse demand patterns. A more complete survey of the different heuristics is given in [8].

The discrete lot-sizing-procedures assume that each lot covers the demand of several consecutive periods. For the lot arriving in period \( r \) one must determine the last period \( s \) the demand of which should be satisfied from this lot; the accompanying lot size is

\[
x_r = \sum_{j=r}^{s} d_j
\]  

(1)

The earliest heuristic for the case of varying demand rates, the Least-Unit-Cost (LUC)-procedure, is still implemented in many MRP-systems. It determines the lot size by minimizing

\[
\frac{t}{(F + h \sum_{j=r}^{t} d_j (j-r)) / (\sum_{j=r}^{t} d_j)}
\]  

(2)

for different values of \( t \); the minimizing index is assigned to \( s \).

In 1968 the Least-Total-Cost (LTC)-procedure was introduced as an alternative to LUC. From the very beginning, the idea of balancing order and inventory cost resulted in two different versions: DeMattheis/Mendoza (3;10) proposed a procedure in which the lot size is always rounded off and which therefore never results in higher relevant inventory cost than order cost:
\[
\begin{align*}
\sum_{j=r}^{s} (j-r) d_j & < F/h < \sum_{j=r}^{s+1} (j-r) d_j \\
\end{align*}
\]

The Part-Period-Method (PPM) differs from this procedure only by a Look-Back-Look-Ahead-refinement \{3\}.

On the contrary, Gorham \{5\} proposed to determine the lot size in such a way that the absolute difference between inventory and order cost is minimized per cycle:

\[
\begin{align*}
|F/h - \sum_{j=r}^{t} (j-r) d_j| = \text{Min} |(F/h - \sum_{j=r}^{t} (j-r) d_j)| \\
\end{align*}
\]

The approach chosen by Gorham should be better suited than the version implemented by DeMattheis/Mendoza if the cost balancing principle is valid. However, experiments show that the DeMattheis/Mendoza-version outperforms Gorham's proposal. The reason for this result is that the LTC-principle possesses several deficiencies which, however, are almost perfectly compensated by the DeMattheis/Mendoza-procedure. Both variants of the cost-balancing-principle have been used by several other authors just as other authors suggested further variants which, however, did not improve the performance of the DeMattheis/Mendoza-heuristic (for a survey see \{9\}).

The Silver-Meal (SM)-heuristic \{13\} minimizes the relevant cost per period

\[
\begin{align*}
\frac{(F + h \cdot \sum_{j=r}^{t} (j-r) d_j)}{(t-r+1)} \\
\end{align*}
\]
For increasing t criterion (5) may result in several local minima as shown in Fig. 1 for the data given in Table 1. As originally proposed, SM usually stops after having determined the first minimum. This minimum would possibly appear only as a local one if additional values t were examined. Therefore, one may also select the global minimum. Silver/Meal assume that these two variants lead to very similar results and that a distinction is unnecessary.

Groff {6} proposed that a new lot should arrive in period s if

\[ h \cdot d_{s+1} \cdot (s-r+1)(s-r+2) \geq 2F \]  

holds. This criterion is derived from an estimation of the marginal cost effects connected with adding the demand of one additional period. Groff's heuristic is remarkable because it possesses a property which one would expect from a decision theory point of view: If one decided to fulfil the demand in period t from the lot ordered in period r, the sunk cost of holding the demands \( d_r, \ldots, d_t \) are not relevant for deciding whether demand \( d_{t+1} \) should be included in the lot \( x_r \) or not.

Wemmerlöv {16} proposed three slightly different modifications of Groff's heuristic, all of which take past demands into account. However, these modifications do not improve the results obtained with Groff's heuristic. Therefore only Wemmerlöv's Rule 1 is described which determines the lot size by

\[ s \cdot \left( \sum_{j=r}^{s} d_j + d_{s+1} \cdot (s-r+1)^2 \right) \geq 2F \]  

and for which the comparatively best results are obtained.

Axsäter {2} and Karni {7} independently proposed a heuristic sometimes entitled Maximum-Part-Period-Gain (MPG). They start with a lot-for-lot solution and sequentially improve the solution by
combining two adjacent lots in one lot. The iterative process of combining lots is determined by computing the maximum cost saving. The computational effort of this heuristic is far larger than of the other heuristics, especially for large T. If the decision maker accepts this comparatively large computational effort to be worthwhile he may even consider applying the Wagner/Whitin (WW)-algorithm {15}.

Although the WW-algorithm provides an optimal solution to the lot sizing problem described, it is usually not applied in MRP-planning. Basically the WW-algorithm is a dynamic programming procedure which may need non-neglectable computing time when applied to thousands of items. Furthermore, the WW-algorithm provides optimal solutions only if $d_j=0$ holds for $j>T$ or if an independent planning horizon exists. Finally, the WW-algorithm is more difficult to understand than the heuristics described above. The WW-algorithm provides reference solutions for comparing the solutions obtained by different heuristics.

2.2. Comparison

Previous experimental results with these and further heuristics for dense demand patterns are summarized in Table 2. All heuristics are evaluated on the basis of additional relevant holding and setup costs compared to the WW-algorithm. Thus far, good results were obtained especially for the heuristics proposed by Silver/Meal and Groff. Table 2 mentions also some heuristics which will be explained later in this paper; the associated results are given in Table 2 for a comparison with their results in Table 5 for sparse demand patterns. Ineffective heuristics (e.g. McLaren's Order Moment Method and procedures immediately based on the continuous lot size model as EOQ-D and POQ and theoretically unsound procedures as Freeland/Colley's or Gaither's proposals and the ineffective LTC-variants) are not subject of this study. For a short description and results see e.g. {8;9;17}. 
3. Adaptation of heuristics for sparse demand patterns

Most of the previous literature does not make provisions for taking zero demands into account. An obvious property of reasonable solutions should be that each lot arrives in a period with positive demand. However, without special modification, several heuristics may violate this property and propose an inefficient solution. One example is LUC: If a zero demand follows the first minimum, two equal cost minima exist in (2). It depends on the usually neglected question whether > or ≤ is used in the implementation to select the lot size associated with the first or the second minimum. However, if the first minimum s is used, most implementations will assume that a lot arrives in period s+1 and will therefore compute avoidable holding costs in the next cycle. A similar problem may occur with the Periodic-Order-Quantity-Rule, some of Wemmerlöv's marginal rules and some LTC-variants (e.g. Gorham's).

For a fair comparison, all heuristics were adjusted in such a way that each lot arrives in a period with positive demand. The fact that adjustments of the heuristics described in literature are necessary in the case of sparse demand leads to presume that also some MRP-implementations can be improved by paying special attention to sparse demand patterns.
4. An improved Silver/Meal-heuristic for phase 1

In general, heuristics often consist of two different phases: In the first phase a preliminary solution is constructed. This solution may be changed e.g. by applying cost comparisons to differently modified solutions in phase 2.

While distinguishing between local and global minima in SM is not very important for dense demand patterns, sparseness results in a more sensitive situation. It may happen that the lot size connected with the global minimum is far larger than the one determined by the first local minimum. Therefore, in their modification 1 Silver/Miltenburg (14) restrict the number of demands to be covered by one lot by a rather lengthy procedure. An easier way of limiting the time between orders is to restrict the number of periods regarded after determining the first local minimum of (5). Let u be the first local minimum found in SM. Furthermore, assume that the search for a better minimum is prolonged until period u+v. In the literature, only v=1 and v=T-u have been regarded thus far. Table 3 and Fig. 2 show the average performance for values v=1,...,10 in 1500 cases with sparse demand patterns; on the average v=4 achieves the best results. Thus on the average neither the myopic selection of the first local minimum nor the search for the global minimum leads to best results over the total planning horizon. As Table 3 also shows, the best value v depends on the mean demand per period; in the 15 problem classes with sparse demand patterns regarded, values 2≤v≤8 are optimal.

It is remarkable that for dense demand patterns v=1 leads to best results in 12 of 15 problem classes. The differences between the performances obtained for different values of v are far smaller for dense demand patterns; values v>4 lead to identical results in all 1500 cases. Therefore it may be worthwhile to study the effect of the parameter v in one's individual demand patterns and determine an appropriate individual value. A global value as v=4 in phase 1 results in a conservative estimate for the solution quality obtainable with simple heuristics. There is a small, insignificant advantage of the Phase-one-heuristic of Silver/Miltenburg compared to the computationally less expensive heuristic operating with v=4.
5. Improving Phase 2

Aucamp/Fogarty (AF) \{1\} try to improve the solution obtained in phase 1 by testing whether costs can be reduced if a lot's arrival moment is slightly shifted backward or forward. In the case of sparse demand patterns, only periods with positive demand are candidates for modified arrival moments.

Silver/Miltenburg (SIMIL) \{14\} use a preliminary solution obtained by a (modified) global search \((v=T-u)\) for a minimum in (5) and test in phase 2 whether combining two adjacent lots in one common lot reduces cost.

The performance of these two suggestions for phase 2 cannot be compared with implementations of the original descriptions of the heuristics because of the different solutions obtained in phase 1 of the two heuristics. For a fair comparison, both phase-two-procedures were applied to solutions obtained with SM and \(v=4\) which is a well suited phase-one-procedure.

The two basic ideas for modifying the preliminary lot sizes may also be combined. In the first improvement step marginal modifications of arrival moments are evaluated as in the AF-proposal. In the second step the combination of two adjacent lots as in phase 2 of SIMIL is evaluated. These two improvement steps are executed successively until no modifications have occurred in the last cycle. Usually only a small number of cycles is necessary.

An example for the combined improvement phase is given in Table 4. The SM-heuristic with \(v=4\) provides a far better solution than SM with \(v=1\). Cost savings can be obtained from three successively applied AF-modifications in which the fourth, the third and again the fourth lot arrive earlier than in the previous solution. Finally, the SIMIL-procedure is able to combine the second and the third lot. For the solution obtained at the end of phase one, such a combination would not have been evaluated as cost-effective. The solution obtained by combined application of the improvement steps described above is optimal. Therefore a second improvement cycle cannot achieve a better solution.
6. Experimental results

In general, in the case of sparse demand patterns most heuristics provide far worse results than in case of exclusively positive demands (cf. Table 2 vs. Table 5). Therefore, executing a second phase is far more attractive for sparse demand patterns owing to the higher potential for improvement.

The myopic SM with \( v = 1 \) clearly outperforms the LTC versions of DeMattheis/Mendoza and of Gorham for dense demand patterns; for sparse demand patterns the myopic SM performs better than Gorham's heuristic but worse than DeMattheis/Mendoza's proposal. The myopic SM performs very similar to Groff's heuristic for dense demand but operates significantly poorer for sparse demand patterns. In the comparisons done by Wemmerlöv, the SM-procedure is probably implemented with \( v = 1 \). For sparse demand this may result in comparatively bad results and may be the reason why Groff's heuristic performs significantly better than the SM-heuristic in Wemmerlöv's experiment. In summary, for sparse demand patterns the myopic CM-principle looses its widely assumed efficiency.

For sparse demand patterns, a parameter \( v = T - u \) results in significantly better performance than \( v = 1 \). Therefore, the SIMIL-procedure starts from a significantly better preliminary solution than the AF-proposal applying SM with \( v = 1 \) in phase 1; this difference makes the SIMIL-solutions significantly better than the results obtained from the AF-procedure.

The best average result for SM in sparse demand patterns is obtained with \( v = 4 \). With this parameter value the disadvantage compared to WW-solutions is only about a third of the error obtained for \( v = 1 \) and about one half of the error resulting from \( v = T - u \) (cf. Table 3).

If the solution obtained with \( v = 4 \) is used as starting point for different improvement heuristics, the results show that the phase 2 suggested by AF is more powerful than the SIMIL-suggestion. The combined application of both improvement possibilities provides an even better solution. Its error is only about one tenth of the
error obtained from the myopic SM-heuristic with v=1. The SIMIL-modification is more extensive than the more locally oriented AF-proposal. The experiment showed significantly better results for starting with AF-modifications and considering SIMIL-changes only after no further AF-modification is possible than for applying these two proposals in the different order.

Whereas MPG does not perform better than the heuristics of Silver/Meal and Groff in the case of dense demand patterns, it is a good procedure for the more difficult case of sparse demand. However, the new procedure provides average results which are even (insignificantly) better than those for MPG whereas the computational effort is far smaller than for MPG.

7. Recommendations

The results of the experiment lead to the following recommendations for MRP-planners:

If sparse demand patterns are relevant in a MRP-environment, the usually employed heuristics may perform comparatively bad. Rankings of the heuristics obtained for dense demand patterns do not hold for sparse demand patterns.

For sparse demand a two-phase-procedure may be worthwhile. A good preliminary solution can be obtained by a slight modification of the Silver/Meal-principle which compromises on the number of additional periods regarded after finding the first local minimum. In the improvement phase, a combination of slightly shifting backward or forward the moment when the lot arrives and of combining adjacent lots is recommended. This can be done recursively until no further improvement is possible. The proposed heuristic reduces the average increase of relevant cost to less than 1 % even for the difficult case of sparse demand patterns and is computationally far less expensive than the Wagner-Whitin-algorithm.
8. **Symbols**

AF ... Aucamp/Fogarty
d<sub>j</sub> ... Demand in period j
F ... Fixed setup cost
h ... Holding cost per unit per unit time
j ... Period j, j=1,...,T
LTC ... Least-total-cost
LUC ... Least-unit-cost
MPG ... Maximum-part-period-gain
PPM ... Part-period-method
r ... First period of replenishment cycle
s ... Last period of replenishment cycle
SIMIL ... Silver/Miltenburg
SM ... Silver/Meal
t ... Period t
T ... Planning horizon
u ... Index of period in which the first local minimum is found with the Silver-Meal-heuristic
v ... Parameter of the Silver-Meal-heuristic
WW ... Wagner/Whitin
x<sub>r</sub> ... Lot size in period r

**Acknowledgement**

The author is indebted to Bodo Schnoor for helpful discussions and support in performing the experiments.
Silver—Meal—Heuristic

Several local minima:

Table 1: Example for illustrating several local minima in Silver/Meal criterion (5) for data given in Table 1.

<table>
<thead>
<tr>
<th>Period</th>
<th>Demand</th>
<th>Relevant cost per period</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>15</td>
<td>105.00</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>305.00</td>
</tr>
<tr>
<td>3</td>
<td>20</td>
<td>325.00</td>
</tr>
<tr>
<td>4</td>
<td>50</td>
<td>305.00</td>
</tr>
<tr>
<td>5</td>
<td>60</td>
<td>305.00</td>
</tr>
<tr>
<td>6</td>
<td>50</td>
<td>325.00</td>
</tr>
<tr>
<td>7</td>
<td>45</td>
<td>345.00</td>
</tr>
<tr>
<td>8</td>
<td>40</td>
<td>307.50</td>
</tr>
<tr>
<td>9</td>
<td>20</td>
<td>347.50</td>
</tr>
<tr>
<td>10</td>
<td>30</td>
<td>387.50</td>
</tr>
</tbody>
</table>

Figure 1: Several local minima in Silver/Meal criterion (5) for data given in Table 1.

Table 2: Relative relevant cost for selected lot size procedures in dense demand patterns.
Table 3: Influence of the parameter $v$ on relative relevant cost in the Silver/Meal-heuristic for sparse demand patterns.

Table 4: Lot sizes for combined application of Accord/Fogarty- and Silver/Millersburg-proposals.

Figure 2: Influence of the parameter $v$ for data given in Table 3.
Results for sparse demand patterns

Table 5: Relative relevant cost in percent for selected lot size procedures in different problem classes of sparse demand patterns.

<table>
<thead>
<tr>
<th>Problem class</th>
<th>Mean positive demand</th>
<th>Mean demand</th>
<th>Mean deviation from mean pos. demand inflow/outs</th>
<th>Procedure</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>Wagner/Miller</td>
<td>100.00</td>
<td>100.00</td>
<td>100.00</td>
<td>100.00</td>
</tr>
<tr>
<td>Silver/Meisel</td>
<td>100.00</td>
<td>100.00</td>
<td>100.00</td>
<td>100.00</td>
</tr>
<tr>
<td>Asaii/Ejiri (IPM)</td>
<td>101.00</td>
<td>101.00</td>
<td>101.00</td>
<td>101.00</td>
</tr>
<tr>
<td>Silver/Meisel</td>
<td>101.00</td>
<td>101.00</td>
<td>101.00</td>
<td>101.00</td>
</tr>
<tr>
<td>Silver/Meisel (only Phase 1)</td>
<td>101.00</td>
<td>101.00</td>
<td>101.00</td>
<td>101.00</td>
</tr>
<tr>
<td>Silver/Meisel</td>
<td>101.00</td>
<td>101.00</td>
<td>101.00</td>
<td>101.00</td>
</tr>
<tr>
<td>Silver/Meisel (old)</td>
<td>101.00</td>
<td>101.00</td>
<td>101.00</td>
<td>101.00</td>
</tr>
<tr>
<td>Silver/Meisel</td>
<td>101.00</td>
<td>101.00</td>
<td>101.00</td>
<td>101.00</td>
</tr>
<tr>
<td>Silver/Meisel (old)</td>
<td>101.00</td>
<td>101.00</td>
<td>101.00</td>
<td>101.00</td>
</tr>
<tr>
<td>Wepner/Meisel &amp; Ralston/Neumann (ILC)</td>
<td>101.00</td>
<td>101.00</td>
<td>101.00</td>
<td>101.00</td>
</tr>
<tr>
<td>Silver/Meisel</td>
<td>101.00</td>
<td>101.00</td>
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<tr>
<td>Silver/Meisel (old)</td>
<td>101.00</td>
<td>101.00</td>
<td>101.00</td>
<td>101.00</td>
</tr>
</tbody>
</table>

Figure 3: Relative relevant cost for lot size procedures selected from Table 5.
References


{ 5 } Gorham, T., Dynamic Order Quantities. Production & Inventory Management 9 (1968), Nr. 1, 75 – 81.

{ 6 } Groff, G.K., A Lot Sizing Rule for Time-Phased Component Demand. Production & Inventory Management 20 (1979), Nr. 1, 47 – 53.


