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Unionisation Structures and Innovation Incentives

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Unionisation Structures and Innovation Incentives*

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Abstract

This paper examines how different unionisation structures affect firms’ innovation incentives and industry employment. We distinguish three modes of unionisation with increasing degree of centralisation: (1) “Decentralisation” where wages are determined independently at the firm-level, (2) “coordination” where one industry union sets individual wages for all firms, and (3) “centralisation” where an industry union sets a uniform wage rate for all firms. While firms’ investment incentives are largest under “centralisation,” investment incentives are non-monotone in the degree of centralisation: “Decentralisation” carries higher investment incentives than “coordination.” Labour market policy can spur innovation by decentralising unionisation structures or through non-discrimination rules.

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How unions affect firms' performance, innovation and labour productivity is a highly controversial issue (for a survey see, e.g., Flanagan, 1999). On the one hand, unions are argued to hurt firms as unionisation may increase wage demands and, thereby, firms’ labour costs (see, e.g., Oswald, 1985; Farber, 1986; Hirsch, 1991). On the other hand, unions are regarded as part of a constructive labour market regime which smoothens industrial relations, thereby promoting labour productivity and lowering average costs (see Freeman and Medoff, 1984). In general though, it is not the mere existence of unions that is decisive for firms' performance, but rather the specific mode of labour market organisation (see, e.g., Calmfors and Driffill, 1988; Soskice, 1990; Layard et al., 1991).

Labour markets differ substantially between countries. A salient dimension that differentiates national unionisation structures is the degree of wage setting centralisation (Calmfors and Driffill, 1988; Moene and Wallerstein, 1997; Flanagan, 1999; Wallerstein, 1999). At the industry level, a decentralised wage setting structure is commonly contrasted with a completely centralised one. While in the former case, wages are set between a single employer and a firm-level union, in the latter case an industry union negotiates a standard wage for the entire industry. An important feature of centralised agreements is their egalitarian nature, as they usually do not account for differences between firms, so that firms have to pay the same wage rate for the same kind of labour. This feature is also mirrored in cross-country studies, which consistently report lower wage dispersion for countries with centralised wage bargaining, as in Scandinavia, and much higher wage dispersion for economies with decentralised wage setting systems, as e.g., in Japan and North America (see Freeman, 1988; Freeman and Schettkat, 2000).

The rigidities associated with centralised wage setting have recently come under attack in the policy debate over labour market organisation and economic performance. A commonly held view is that labour market rigidities are generally bad for overall economic performance (see, for example, Nickell, 1997, and Siebert, 1997), so that any move towards more decentralised and, hence, more flexible structures is good for employment and overall economic prosperity. Consistent with this view, the OECD Jobs Study (OECD, 1996, p. 15) recommends to “make wage and labour costs more flexible by removing restrictions that prevent wages from reflecting local conditions (...).”

\[1\] There exists a large literature about the possible key characteristics that are crucial for the relative performance of different modes of labour market organisation. For comparisons of countries' labour market institutions see, e.g., Nickell (1997), OECD (1997, c 3), Blau and Kahn (1999), and Wallerstein (1999).
Given this policy recommendation, tendencies to introduce more flexibility into centralised wage systems have given rise to intermediate structures, which allow for adjustments to local conditions at the firm-level even though wage setting remains under the auspices of an industry union. For example, in Germany collective wage agreements between industry unions and employer associations have started to contain so-called “opening clauses” according to which firms are allowed to pay wages below the collectively agreed rate under certain conditions (see Sachverständigenrat, 1998, pp. 117-27). While in Germany industry unions’ successfully protected their monopoly power at the industry level, trends towards less centralised wage setting in other countries, as, e.g. Denmark, Sweden, or New Zealand, have led to more decentralised wage setting at the firm-level and, thereby, substantially reduced unions’ monopoly power.\(^2\)

Motivated by the institutional diversity of industry unionisation and by recent trends towards more flexible wage setting regimes, this paper analyses how various unionisation structures that differ in the degree of wage centralisation affect firms’ incentives for implementing labour productivity enhancing technologies and, as a consequence, overall industry employment.

Most of the existing theoretical work on the relationship between unionisation and innovative activity has focused on the effects that union bargaining power and objectives have on firms’ incentives to invest and to cut labour costs. Following the standard incomplete contracts approach, the conventional wisdom has been that firms’ investment incentives are decreasing with union bargaining power, as unions have incentives to hold up innovative firms and demand higher wages once an investment is sunk so as to appropriate parts of the rent, which in turn leads firms to invest less (see Grout, 1984; Van der Ploeg, 1987; for a survey, Malcomson, 1997; and Van Reenen, 1986, for empirical evidence). More recent work by Tauman and Weiss (1987) and Ulph and Ulph (1994, 2001) has qualified this underinvestment result, however, by considering the strategic aspects of oligopolistic competition between firms (for a survey, see Ulph and Ulph, 1998). Overall these papers have shown that firms that face strong unions may enjoy a strategic advantage vis-à-vis those competitors that either pay competitive wages or that face weaker unions. In such an environment, unionisation can increase the concerned firm’s incentives

\(^2\)For a country-wise survey of recent trends towards more flexible wage setting see, e.g., Katz (1993). For Australia see in particular Wailes and Lansbury (1999). The breakdown of centralised wage bargaining in Denmark and Sweden is documented in Iversen (1996). The Swedish case has also been extensively studied by Hibbs and Locking (2000). For the period from 1950 to 1992, a more conservative view is expressed in Wallerstein et al. (1997).
to innovate, and hence, its overall competitiveness.

As this literature has focused exclusively on decentralised modes of wage setting, where wages are set by independent unions at the firm-level, the relative performance of more centralised wage setting systems remains an open issue, even though the degree of wage centralisation has been identified as a crucial feature of different unionisation structures. Our concern is, therefore, the relative performance of different unionisation structures as measured by their impact on firms’ innovation incentives and on overall industry employment.

For this purpose, we develop a framework that allows us to analyse how the degree of wage centralisation affects both the hold-up problem associated with unionisation and the strategic interaction between firms. The basic set-up follows Ulph and Ulph (1994) and analyses a unionised oligopoly model, where two firms are racing to be the first to introduce an exclusive new technology. Once the innovation has been implemented and the according investment is sunk, wages are determined within a given unionisation structure. Finally, firms compete in Cournot fashion in the product market.

To analyse how different unionisation structures affect innovation incentives three modes of unionisation are distinguished with an increasing degree of centralisation: (1) “Decentralisation” where wages are determined independently at the firm-level, (2) “coordination” where an industry union sets individual wages for all firms at the firm-level, and (3) “centralisation” where a uniform wage rate is set for the entire industry. As we will show, firms’ investment incentives are largest under “centralisation” and smallest under “coordination”. Hence, investment incentives are not monotone in the degree of centralisation, as “decentralisation” carries higher investment incentives than “coordination”. The intuition behind this result is that “coordination” allows the monopoly union to fully exploit its hold-up potential by setting discriminating wages for the two firms. In contrast, “centralisation” and “decentralisation” both constrain the unions’ power either through the uniformity rule or through competition between firm level unions. Overall, our analysis shows that the uniformity rule is more effective in constraining the union’s hold-up potential than the competitive threat resulting from decentralisation. Hence, the positive effect that centralisation can have on firms’ innovation incentives may possibly serve as an efficiency defence for labour market rigidities that confine a strong industry union to an “equal pay for equal work” policy.

Regarding employment, “decentralisation” yields the most favourable outcome, while either
“coordination” or “centralisation” perform worst in this regard. Given these results concerning innovation incentives and employment, our analysis has two potentially important policy implications: First, any institutional change from a centralised structure towards a less centralised system that does not constrain the union’s monopoly power at the industry level should be critically reviewed, especially if it opens the window for more wage discrimination, which should have negative effects on firms’ innovation incentives. Second, even if decentralisation is a feasible policy option, policy makers are still confronted with the trade-off between innovation incentives and employment. While “centralisation” is most conducive for innovation incentives, industry employment is largest when wages are determined at the firm level under a decentralised wage setting regime.

These results also relate to arguments that have been put forward in the Swedish debate over “solidaristic” bargaining (Rehn, 1952) and that have been recently formalised in Agell and Lommerud (1993) and Moene and Wallerstein (1997). According to this literature, nationwide wage settlements that are associated with a high degree of wage equality drive inefficient firms off the market and expedite structural change, thereby fostering growth. In contrast, our analysis focuses on the role that different unionisation structures can have in overcoming the hold-up problem associated with unionisation in oligopolistic industries. Moreover, we do not only analyse the polar cases of centralised and decentralised wage setting institutions, but also consider an intermediate degree of centralisation.

The rest of this paper is now organised as follows: In Section 1 we introduce the model’s structure and define the unionisation structures. We solve the model for the static case where firms’ productivity levels are given in Section 2. Section 3 solves for firms’ innovation incentives and compares our results to innovation incentives in perfectly competitive labour markets. In Section 4 we discuss implications for labour market policy before Section 5 finally concludes.

1 The Model and Unionisation Structures

Let us consider a homogeneous goods Cournot duopoly with two firms $i = 1, 2$. Both firms operate under constant returns to scale, with labour being the only factor of production. To produce a unit of the final good each firm $i$ requires $\alpha_i$ units of homogeneous labour so that firm $i$’s marginal cost is given by $\alpha_i w_i$, where $w_i$ denotes the wage rate that firm $i$ has to pay. Also let $q_i$ denote the quantity of the final good produced by firm $i$, and let $l_i$ be its labour demand.
Since firm $i$ requires $\alpha_i$ units of labour per unit of output, it follows that $l_i = q_i \alpha_i$. We assume a linear inverse demand function of the standard form $p = A - q_1 - q_2$, for $q_1 + q_2 \leq A$.

Suppose that initially the two firms are identical and have the same labour productivity, $1/\alpha_1 = 1/\alpha_2$, which we normalise to unity. Then both firms engage in a patent race where each firm has the same chance of being the first to find an innovation that reduces its labour requirement per unit of output by $\Delta$, ($\Delta > 0$). The cost of implementing the innovation is exogenous and will be denoted by $I(\Delta) > 0$. Once the innovation is implemented and the according cost sunk, the firm’s labour productivity increases instantaneously. The values of $\Delta$ and $I(\Delta)$ are commonly known before firms decide about participating in the tournament. As $I$ is the price that has to be paid in order to implement a productivity enhancing technology, it also measures how severe the hold-up problem is that the investing firm faces under unionisation. If an innovation does not involve any specific investment, $I$ is zero and, accordingly, the hold-up problem vanishes. Conversely, as $I$ becomes larger the hold-up problem becomes more severe, and the specific mode of unionisation becomes a critical determinant of firms’ investment incentives.

The opportunity cost of labour, given through the workers’ outside option such as their alternative income, is denoted by $w_0$, with $w_0 > 0$. It is assumed that the union maximises its members’ wage bill relative to the opportunity cost of labour, and we adopt the right-to-manage assumption: The union can set the wage while firms retains the right to choose their employment level.\(^3\)

We consider a three stage game with the following timing: In the first stage, both firms decide whether or not to participate in an R&D tournament, where the two firms have the same chance of obtaining the exclusive right to an innovation, $I(\Delta)$, through which the labour requirement per unit of output can be reduced by $\Delta$, if the firm implements the technology at a cost of $I$. In the second stage, wages are determined, and we distinguish three unionisation structures $\rho = D, C, U$ with the following properties:

1. **Decentralisation** ($\rho = D$): There are two firm-level unions which set firm-level wages $w_1$

\(^3\)In contrast to the right-to-manage assumption efficient bargaining models assume that unions and firms bargain over both wages and firms’ employment levels (Oswald and Turnbull, 1985; Layard *et al.*, 1991; and Booth, 1995). While unions usually neither have perfect monopoly power nor do they exclusively care about their members’ wage bill, these simplifying assumptions allow us to concentrate exclusively on the effects of wage setting rigidities.
and \( w_2 \) for their firms. The two unions choose their wage demands simultaneously and noncooperatively.

2. **Coordination** \((\rho = C)\): An industry union coordinates the wage demands \( w_1 \) and \( w_2 \) so as to maximise the industry wage bill.

3. **Centralisation** \((\rho = U)\): There is one industry-wide union which sets a *uniform* industry wage \( w \) for both firms so as to maximise the industry wage bill.

Finally, in the third stage of the game the two firms compete in quantities, taking productivity levels and wage rates as given.

This timing of the game is intended to reflect the planning horizon usually associated with the respective decisions. Investment decisions are mostly long-run while wage contracts are usually negotiated for a much shorter time horizon, and product market quantities can usually be adjusted on an even shorter basis.

The three unionisation structures differ with respect to the degree of wage centralisation in the following way: The \( D \)-regime can be viewed as the most decentralised system of collective wage setting, where firm-level unions do not cooperate and set firm-specific wages depending on the relative efficiency of their employer. In contrast, the \( U \)-regime stands for the most centralised wage setting system, as labour supply is perfectly monopolised and the industry union determines one uniform wage for all firms in the industry. This regime embodies the famous union-slogan “equal pay for equal work” and is, therefore, the most egalitarian one. The \( C \)-regime lies in between those polar cases. On the one hand labour supply is completely monopolised, as an industry union coordinates wage demands at the firm level. On the other hand firm-level wages are adjustable to the firms’ relative competitiveness. Consequently, different wages are likely to prevail in this case. We take the \( C \)-regime as reflecting recent trends in continental Europe towards flexible wage setting, while the union’s monopoly power remains largely intact (see also OECD, 1997, for a summary of recent trends in this regard).

At this point two remarks about the multidimensional character of wage centralisation are at hand: Firstly, our notion of centralisation requires either an industry union or intense cooperation among firm-level unions to assure maximisation of the industry wage bill. Secondly, a completely centralised regime further embodies an egalitarian wage rule by which “wage flexibility” between firms is suppressed.
Before we compare the different regimes, let us introduce the following assumption in order to exclude corner solutions in which the non-innovating firm is driven off the market:\footnote{Assumption 1 is derived in the Appendix. Similar restrictions are also employed in Ulph and Ulph (1994, 1998, 2001).}

**Assumption 1.** The labour productivity enhancing innovations under consideration lead to non-drastic productivity improvements in the sense that the union prefers the less efficient firm to remain active in the market even under the most centralised wage setting regime $U$; i.e.,

$$w_0 < \bar{w}_0 = \frac{(1 - 3\Delta)A}{1 - \Delta^2},$$

which implies that we restrict attention to productivity increases $\Delta < 1/3$.

Assumption 1 ensures that all optimisation problems in the second and third stage of the game stay globally concave. We maintain Assumption 1 throughout the rest of the paper.

## 2 The Static Case: Given Productivity Levels

Let us begin our analysis by solving for the subgame perfect equilibrium quantities and wages, taking firms’ productivity levels as given. We suppose that firm 1 is the innovating firm, so that its labour requirement per unit of output is reduced by $\Delta$. Firm 1’s profit function is then given by

$$\Pi_1 = (A - q_1 - q_2)q_1 - w_1(1 - \Delta)q_1.$$

Firm 2 is the non-innovating firm, and its profit function is

$$\Pi_2 = (A - q_1 - q_2)q_2 - w_2q_2.$$

Maximising firms’ profits for given wages $w_1$ and $w_2$, and solving the first order conditions, we obtain the firms’ optimal strategies

$$q_1(w_1, w_2, \Delta) = \frac{A - 2w_1(1 - \Delta) + w_2}{3},$$
$$q_2(w_1, w_2, \Delta) = \frac{A - 2w_1 + w_2(1 - \Delta)}{3}.$$

We now turn to the wage setting stage. Wage-bill maximisation implies that the union’s optimal wage setting strategy, $w_i^p$, regarding firm $i$ is defined as

$$w_i^p = \arg \max_{w_i \geq 0} U_i^p(w_i, w_j^p) \text{ for } i = 1, 2, i \neq j,$$
for regimes $\rho = U, C, D$, where $U^D_i = l_i(w_i - w_0)$, $U^C_i = \sum_{i=1}^2 l_i(w_i - w_0)$, and $U^U_i = \sum_{i=1}^2 l_i(w - w_0)$ stand for the wage bills under the respective regimes, while labour demands $l_i$ are derived from equations (2) and (3). Comparing the resulting equilibrium wages and the wage differentials, $d^\rho_w \equiv w_1^\rho - w_2^\rho$, between the two firms across the three different regimes, we obtain the following orderings:

**Proposition 1.** For all $\Delta > 0$, the orderings of the wages, $w_1^\rho$ and $w_2^\rho$, and the wage differential, $d^\rho_w$, under the different unionisation structures $\rho = U, C, D$ are as follows:

(i) Firm 1’s wages: $w_1^C > w^U > w_1^D$,
(ii) Firm 2’s wages: $w^U > w_2^C > w_2^D$,
(iii) Wage differentials: $d^D_w > d^C_w > d^U_w (= 0)$.

Proposition 1 shows how wage setting depends on the particular mode of unionisation. Decentralised wage setting ($\rho = D$) leads to the lowest wage levels compared to more centralised structures. Under coordinated wage setting ($\rho = C$) a positive wage differential results with the efficient firm paying the highest wage. However, the wage differential under regime $C$ is lower than under system $D$. The ordering of the wage differentials mirrors our notion of wage setting centralisation as discussed above. Wage setting under the decentralised regime $D$ is most responsive to firms’ characteristics, so that productivity differences between firms translate into the largest wage differentials. At the other end of the spectrum, centralised wage setting under regime $U$ completely suppresses the wage gap.\(^6\)

The ordering of wage differentials under the unionisation structures reflects the empirical finding that wage dispersion is negatively correlated with wage centralisation, which is documented in Rowthorn (1992), OECD (1997), Flanagan (1999), and Wallerstein (1999). Interestingly, even though the intermediate regime $C$ and the decentralised regime $D$ both allow for full wage flexibility at the firm-level, the wage profile is more compressed under regime $C$ than under $D$. This is because under decentralised wage setting the union of the less efficient firm is willing to accept a lower wage in order to restall its firm’s competitiveness in the product market. In contrast, an industry union fully internalises the negative “business stealing” externality of this

\(^5\)The resulting equilibrium wage profile $(w_1^\rho, w_2^\rho)$ and output levels $(q_1^\rho, q_2^\rho)$ are stated in Lemmas 1 and 2 in the Appendix.

\(^6\)It should be noted that centralised wage agreements often establish wage floors where firms may decide to pay higher wages. This may be explained by efficiency wage considerations or other frictions in labour market contracting that are beyond the scope of this paper.
policy. Hence, under a coordinated wage setting regime (C) the union’s incentive to adjust the non-innovating firm’s wage to a lower level in response to an increase in the innovating firm’s productivity is much weaker.\footnote{Accordingly, from Lemma 1 (Appendix) it follows that $\partial w_2^D/\partial \Delta < \partial w_2^C/\partial \Delta$.}

Proposition 1 also helps us to explore how the severity of the hold-up problem varies under the three unionisation structures. Noting that $\Pi_i = (q_i)^2$ holds in equilibrium and using equation (2), we can also write firm 1’s profits as

$$\Pi_1(w_1, d_w, \Delta) = \left[ A - w_1(1 - 2\Delta) - d_w \right]^2 / 9.$$  

This expression of the innovator’s profit allows us to identify two different hold-up mechanisms. Firstly, firm 1’s profit is reduced as its wage level, $w_1$, increases, and secondly, the profit decreases as the wage differential between the two firms widens. Hence, there are two kinds of hold-up:

1. **Wage-level hold-up**: An increase in the innovating firm’s wage level - while holding the wage differential constant - unambiguously reduces the gains from innovation; i.e. $\partial \Pi_1/\partial w_1 < 0$.

2. **Wage-differentiation hold-up**: An increase in the wage differential - while holding the innovating firm’s wage level constant - unambiguously reduces the gains from innovation; i.e. $\partial \Pi_1/\partial d_w < 0$.

While the first kind of hold-up has received some attention in the respective literature, the second way of rent extraction seems to be much less recognised. From Proposition 1 we see that the wage-level hold-up is largest under regime $C$ and lowest under regime $D$. This ordering may suggest that decentralised wage setting is the mode of labour market organisation that is most conducive to innovation. However, as part (iii) of Proposition 1 reveals, decentralised wage setting also involves the largest hold-up potential via wage-differentiation. This may counter the positive effects of lower wage levels on innovation incentives.

Comparison of regimes $C$ and $U$ shows that both the innovating firm’s wage rate and the wage differential are strictly smaller under regime $U$ than under $C$. Hence, the uniformity rule under centralised wage setting restricts the union’s hold-up potential and, therefore, should induce larger innovation incentives. Comparison with the decentralised wage setting regime, however, remains ambiguous so far. While under regime $D$ the wage level is the lowest, it also involves the largest scope for hold-up via wage differentiation.
It is also instructive to compare the wage outcomes under the unionisation structures with a perfectly competitive labour market outcome. Clearly, with perfect competition in the labour market, wages are forced down to workers’ reservation wage, \( w_0 \). Quite obviously, unionisation always implies higher wage levels. Similarly, union monopoly power also implies positive wage differentials except for the centralised case, where the uniformity rule suppresses any inter-firm wage differential as does a perfectly competitive labour market. While one might conclude from these observations that innovation incentives are highest under a perfectly competitive labour market, we will show below that such a conclusion is premature with respect to a centralised wage setting regime.

Before turning to firms’ innovation decisions, let us shortly examine the effects that the different unionisation structures have on employment at the industry level. These effects are summarised in the following proposition:

**Proposition 2.** For all \( \Delta > 0 \), the orderings of the employment levels at the industry level, \( L^\rho := l_1^\rho + l_2^\rho \), under the different unionisation structures \( \rho = U, C, D \) are \( L^C = L^U < L^D \).

**Proof.** See Appendix.

Proposition 2 broadly mirrors the wage ordering as stated in Proposition 1. As the two firms’ wages are lowest, overall employment is largest under regime \( D \). Interestingly enough, total employment is the same under regimes \( U \) and \( C \) for given productivity levels.

3 The Dynamic Case: Productivity Improvements

3.1 Innovation Incentives

In the first stage of the game, the two firms decide whether or not to participate in an R&D tournament, in which both firms have the same chance of winning the exclusive patent right to a labour productivity enhancing innovation. Both firms are certain about the productivity gain, indicated by the reduction in labour requirement per unit of output, \( \Delta \), and the cost, \( I(\Delta) \), which has to be sunk in order to implement the innovation. Accordingly, given that both firms participate in the tournament, the expected profit from participating in the tournament is \( (1/2) [\Pi_1(\Delta) + \Pi_2(\Delta) - I(\Delta)] \), where \( \Pi_1(\Delta) \) denotes the profit of the innovating firm (i.e., the patent right holder), while \( \Pi_2(\Delta) \) stands for the losing firm’s profit (as both firms are equally likely to win the race we can assign the innovation to firm 1 without loss of generality). Each
firm chooses to participate in the tournament as long as the expected profit is higher than the
certain profit from unilaterally abstaining, given by \( \Pi_2(\Delta) \), (note that in this case the rival firm
would obtain the patent for sure). As we abstract from any R&D costs that have to be spent
in the course of the patent race, both firms will participate in the race for the innovation if the
gain from participation, \( \Psi \), is larger than the implementation costs, \( I \); i.e.
\[
\Psi(\Delta) := \Pi_1(\Delta) - \Pi_2(\Delta) > I(\Delta).
\]
The value \( \Psi(\Delta) \) measures the maximum willingness to pay for implementing a labour productivity
enhancing innovation, \( \Delta \), and therefore, also each firm’s incentive to participate in the patent
tournament. If this value is smaller than the cost that has to be sunk in order to implement the
innovation, then neither firm has an incentive to participate in the R&D tournament.\(^8\),\(^9\)

The following proposition states the main result regarding firms’ innovation incentives:

**Proposition 3.** For any given \( \Delta > 0 \), the firms’ innovation incentives, \( \Psi(\Delta) \), are largest
under unionisation structure \( U \) and smallest under structure \( C \); i.e. the ordering \( \Psi^U > \Psi^D > \Psi^C \) holds.

**Proof.** See Appendix.

Proposition 3 shows that different unionisation structures have different effects on innovation incentives. Our ordering of unionisation structures along the dimension of wage centralisation shows that a completely centralised wage setting system carries the largest innovation incentives. Furthermore, the relationship between wage centralisation and innovative activity is non-monotone. Innovation incentives are lowest when centralisation is intermediate; i.e. if an industry union can differentiate wages across firms as under “coordination”. This means

\(^8\)Our specification of the R&D tournament gives rise to the same incentives to innovate as in Ulph and Ulph’s
(1994) model, where two firms bid for the exclusive right to an innovation, which is auctioned off by an innovator
with zero reservation price. However, as the innovation is already available, their set-up does not address the
hold-up problem, which is at the core of our analysis. While the innovation is always implemented in Ulph and
Ulph (1994), positive implementation costs may result in a non-innovation outcome in our model.

\(^9\)Alternatively, we may also assume that only one firm, say firm 1, has the opportunity to implement the
innovation. In this case, a firm’s incentive to innovate would be given by the profit differential \( \Pi_1(\Delta) - \Pi_1(0) \),
where \( \Pi_1(0) \) stands for firm 1’s profit when neither firm implements the innovation (see Bester and Petrakis, 1993,
for this approach). As we have shown in Haucap and Wey (2002), all our results are qualitatively robust in this
regard.
that while the wage-differentiation hold-up is less severe under intermediate centralisation than under decentralisation (as the wage structure is more compressed), the magnitude of the wage-level hold-up under intermediate centralisation outweighs this. Overall, the hold-up problem is therefore larger under intermediate centralisation, so that intermediate centralisation provides the smallest innovation incentives among the three regimes.\textsuperscript{10}

The first finding has important implications for empirical work on the relationship between unionisation and productivity or innovation. Starting with the seminal work of Brown and Medoff (1978) there is a large body of empirical literature studying the effects of unionisation on productivity and innovation (see, e.g., Freeman and Medoff, 1984; Connolly \textit{et al.}, 1986; Addison and Hirsch, 1989; Hirsch, 1991; Bronas and Deere, 1993; Addison and Wagner, 1994). While empirical studies from North America tend to report negative impacts of unions on R&D, European studies do not find these negative effects. All in all the empirical results are mixed: There is no unambiguous relation between union power (measured by union density or union coverage) and innovation activities (for surveys see Menezes-Filho \textit{et al.}, 1998, and Menezes-Filho and Van Reenen, 2003). This may possibly also be due to the fact that much of the empirical work has focused on union coverage and union density as measures of unionisation. As our results indicate, the degree of centralisation in the wage setting process can also significantly affect firms’ innovative behaviour.

Our second finding, namely that investment incentives are non-monotone with respect to centralisation, calls for a critical reassessment of recent trends towards more flexibility in industry-wide wage settlements. As these agreements often remain highly coordinated on the union’s side we find that flexibility can also adversely affect innovation incentives while the desired positive effects on employment may remain small or even negligible as long as labour supply remains monopolised. Before we elaborate on this issue in Section 4 let us now shortly examine the employment implication at the industry level when innovation is endogenous. Comparison of the industry level employment effects yields the following result:

\textsuperscript{10}As one referee suggested, the unionisation structures may also be ordered differently so that union strength is interpreted as being largest under regime C and smallest under regime D. While we agree that there is some arbitrariness in ranking the different unionisation structures, our ordering (according to wage differentials) is in line with the empirical work cited above. Moreover, ranking regimes U and D with respect to union power is difficult if the industry wage bill (namely, the ability to extract rents from firms) is used as a measure for union power, as the wage bill can be larger under regime D than under regime U.
Proposition 4. Consider any $I(\Delta)$ with $\Delta > 0$. Then, in the long run, when firms’ innovation decisions are endogenous, industry employment levels are ordered as follows:

(i) If $\Psi^U(\Delta) > I(\Delta) > \Psi^C(\Delta)$ holds, i.e. when the innovation is undertaken under the centralised regime ($\rho = U$), but not under the coordinated regime ($\rho = C$), then for all $\Delta < 1/7$ there exists a threshold value $w_{0}'' > 0$ such that the ordering $L^C < L^U < L^D$ holds if and only if $w_0 > w_{0}''$. Otherwise, the ordering $L^U < L^C < L^D$ holds. Moreover, the threshold value $w_{0}''$ is monotonically increasing in $\Delta$.

(ii) In all remaining cases the ordering of the industry employment levels is $L^C = L^U < L^D$.

Proof. See Appendix.

Proposition 4 clarifies that, while a centralised regime can lead to higher overall employment levels than a coordinated regime in the long run, the reverse may also hold. The latter is less likely, however, if the hold-up problem, as measured through the implementation cost $I(\Delta)$, is sufficiently large, given $w_0$. More precisely, the overall employment is higher under centralisation than under coordination if an investment project is only undertaken with a centralised regime, but labour productivity does not increase too much. However, since the reverse can also hold, the higher powered incentives to increase labour productivity under centralisation may eventually come at the cost of lower industry employment than under coordinated wage setting.

3.2 Comparison with Perfectly Competitive Labour Markets

Since in policy debates over labour market reform it is often argued that policy makers should take a more active role in deregulating labour markets, it is useful to relate our results to the benchmark case of perfectly competitive labour markets where unionisation is completely suppressed. Comparing our three regimes to a perfectly competitive labour market (under a product market duopoly) yields the following result (to proceed in a parsimonious way, we integrate the perfectly competitive labour market regime into the set of unionisation structures as $\rho = \ast$):

Proposition 5. Innovation incentives are strictly larger under a perfectly competitive labour market ($\rho = \ast$) than under coordinated ($\rho = C$) or decentralised wage setting ($\rho = D$). However, innovation incentives are larger under centralised wage setting ($\rho = U$) than under perfectly competitive labour markets ($\rho = \ast$) if $w_0 < A/3$. For all $A/3 < w_0 < A$ there exists a threshold
value $\Delta \in (0, 1/3)$ such that $\Psi^* > \Psi^U$ if $\Delta < \tilde{\Delta}$ and $\Psi^* < \Psi^U$ if $\Delta > \tilde{\Delta}$. Moreover, $\tilde{\Delta}$ is monotonically increasing in $w_0$.

**Proof.** See Appendix.

Proposition 5 shows that unionisation usually reduces innovation incentives when compared to a perfectly competitive labour market. The only exception is the centralised regime $U$, where the risk of a hold-up through wage differentiation is completely eliminated. As the union pushes the wage rate above the competitive wage, firms face a more elastic demand for their products, which can induce higher investment activity. If, however, the reservation wage is sufficiently large, this effect becomes smaller and a perfectly competitive market is likely to exhibit larger investment incentives. Interestingly though, the centralised regime can also lead to largest investment incentives for relatively high reservation wages if the innovation is large enough (i.e., $\Delta > \tilde{\Delta}$ holds) in which case the uniformity rule is most effective in restricting the union’s hold-up potential.

Finally, Proposition 5 sheds some light on an important empirical regularity regarding unions and innovation reported by Menezes-Filho and Van Reenen (2003). As the authors conclude in their survey, empirical studies from North America consistently find strong and negative impacts of unions on R&D. In contrast, European studies do generally not find negative effects of unionisation on R&D. As union formation in the US mainly occurs at the firm-level and, therefore, gives rise to a decentralised unionisation structure, while unionisation is generally more centralised in Europe, our results may help to explain those different empirical findings. According to Proposition 5, decentralisation should unambiguously decrease R&D investment when compared to a competitive labour market, while the higher degrees of centralisation in Europe may increase or decrease innovation incentives depending on the degree of wage setting centralisation. Hence, the reported ambiguity is not only consistent with our model, but may potentially also be explained if the degree of wage setting centralisation is taken into account as an explanatory variable.

4 Labour Market Policy Implications

What are the implications our model has for labour market policy? At the latest since Calmfors and Drifill (1988) the question of the optimal degree of wage setting centralisation has been
most contentious and subject to a vigorous debate. The central questions are how labour market organisation affects unemployment on the one hand and productivity on the other, and relatedly, whether a change in labour market policy can induce more favourable outcomes. While quite a number of economists argue that labour market rigidities and centralised wage setting institutions are at the root of the unemployment problem and also responsible for the poor economic performance of many European countries (see, e.g., Siebert, 1997), others point at the positive dynamic efficiency effects as firms have stronger incentives to increase their labour productivity when labour markets are less flexible (see, e.g., Kleinknecht, 1998). While the first line of reasoning is regularly put forward by economic experts such as the council of economic advisers in Germany (see, e.g., Sachverständigenrat, 1998, pp. 117-27), union representatives usually concur with the second argument and claim that wage differentiation opens the window for wage dumping (Schmutzkonkurrenz), which reduces firms’ incentives to increase their labour productivity (see, e.g., Flassbeck and Scheremet, 1995; Soltwedel, 1997). Similar arguments have also been put forward by Swedish trade union economists (see Rehn, 1952, and Agell, 1999).

As we have demonstrated in our model, there may be some truth in both lines of reasoning, depending on the severity of the hold-up problem, the nature of innovation, and other factors such as workers’ reservation wage. Therefore, and since policy makers usually care about both employment effects and investment/productivity, it is useful to summarise our results for policy purposes as follows:

**Proposition 6.** Decentralised wage setting (D) provides both larger innovation incentives and higher employment levels than coordinated wage setting (C). Centralised wage setting (U) provides the largest innovation incentives but results in lower employment than decentralised wage setting (D).

In light of Proposition 6, an extension of antitrust rules to labour markets, as called for by some economists (see, e.g., Simons, 1944; Baird, 2000; Haucap et al., 2001), may not be unwarranted. A strict application of antitrust rules would mean that the formation of industry-wide unions and collective wage agreements should not be allowed due to their monopolisation effects. While such a prohibition may imply lower productivity, our model predicts that employment would increase. If, however, the creation of monopoly unions is allowed for some reason, another antitrust rule may come into force, namely non-discrimination rules. The requirement
not to discriminate between firms would unambiguously increase investment incentives while employment, however, may increase or decrease as we have shown in Proposition 4.

Overall, policy makers face a trade-off between more employment and innovative activity. Interestingly though, allowing for an industry union and wage flexibility at the firm-level appears to be the least preferable regime for policy makers. While innovation incentives are lowest under coordinated wage setting, employment may only exceed the centralised outcome at the cost of forgone labour productivity improvements, which is particularly problematic in the face of intensified international competition. Hence, in the light of our model labour market policy may be well advised either to restrict union formation altogether or to impose non-discrimination rules on collective wage agreements. Based on these accounts, we are left with the uncomfortable finding that labour markets are nevertheless exempted from antitrust law.\footnote{For the European Union and, e.g., Germany there is no dispute that the labour market is completely exempted from antitrust regulations (see, e.g., Rittner, 1999). While in the United States the Pennington case has proved that antitrust laws can be imposed on agreements between unions and employers, the overall picture is similar as in Europe (for an assessment of the US law see, e.g., Sullivan and Grimes, 2000, pp. 716-27).}

Let us finally also briefly comment on recent trends in continental Europe, and in particular Germany, to augment centralised tariff agreements by opening clauses, which allow for wage adjustments at the firm-level. In Germany, a deregulation policy towards decentralised union structures, where independent unions compete against each other within the same industry, is not a feasible policy option, as the German constitution and the labour law explicitly protect the monopoly power of industry unions (for an analysis of the cartelisation effects of the German labour market institutions, see Haucap \textit{et al.}, 2003). Given this institutional constraint, our model suggests that, even though reforms which allow for differentiated wage profiles in highly centralised labour markets may result in higher industry employment than a regime with uniform wages, this tends to come at the cost of curbing innovative activity.

\section{Conclusion}

In unionised industries firms’ incentives to innovate are determined by their own wage as well as their competitors’ wage and the according adjustments following an innovation. In this paper we have examined how these wage adjustments are determined under different modes of unionisation structures which we have differentiated according to the degree of wage setting centralisation.
As we have shown, firms’ innovation incentives are non-monotone in this regard. If coordinated wage setting is combined with strict uniform wage rules innovation incentives are largest, while coordinated wage setting alone performs worst in terms of innovative activity. This may possibly also help to explain why the empirical literature is generally not conclusive on the relationship between various measures of labour market rigidity and productivity growth. The OECD, for example, has recently concluded: “While higher unionisation and more co-ordinated bargaining lead to less earnings inequality, it is more difficult to find consistent and clear relationships between those key characteristics of collective bargaining systems and aggregate employment, unemployment, or economic growth” (OECD, 1996, p. 2). Our results suggest to distinguish coordinated wage regimes along the lines of wage flexibility. For this purpose, it should prove useful that the degree of centralisation is monotone in the interfirm wage differential, which suggests that it should be used as an explanatory variable in a reduced form approach.

While it is conventional wisdom that rigidities in European labour markets are the main cause for the high unemployment in Europe, we would also point to the commitment value that these rigidities provide, as they help to reduce the hold-up problem associated with unionism. Since conventional arguments for labour market deregulation are often based on a static framework without innovation, they fail to capture the commitment aspects associated with different forms of labour market organisation. In contrast, our paper has analysed the strategic incentives to innovate under different modes of labour market organisation, and we have argued that “equal pay for equal work” rules may be beneficial as they can encourage innovation. In this case, policy makers face a trade-off between high employment and productivity when designing labour market regulations and labour market policy more generally.

While we do not wish to over-emphasise this point, we believe that understanding the institutional complementarities of labour market organisation and innovation is crucial for discussing the effects of labour market deregulation. The costs and benefits of labour market regulation are likely to be less clear-cut than is sometimes argued (see, for example, Siebert, 1997). While decentralisation leads to higher employment levels in our framework, it also reduces innovation incentives when compared with a centralised wage setting regime. An intermediate degree of centralisation with only some (in)flexibility appears to be especially undesirable in the light of our analysis. Some empirical support for this finding is provided by Bassanini and Ernst (2002) who find in a cross-country study that in countries with coordinated wage setting sys-
tems there is a negative relationship between R&D intensity and labour market flexibility, at least for high-technology industries.

For our model, we have used the simplifying assumption that firms are initially symmetric. If, however, we assume instead that firms are already asymmetric when they decide about any innovation (they may for instance operate in regionally differentiated labour markets with different reservation wages prevailing), the natural question arises how wage setting systems affect the evolution of oligopoly markets. While we have to leave a definite answer to further research, we conjecture that centralised wage setting under a uniformity rule is likely to increase asymmetries between firms, and consequently concentration, while a decentralised system may give rise to offsetting effects. Other areas for further research may be to fully endogenise the choice of labour market institutions and to analyse innovation incentives under different degrees of centralisation with different bargaining patterns or union preferences.
Appendix

Derivation of Condition (1) in Assumption 1.

We first derive condition (1), which is a sufficient condition that ensures that all firms produce strictly positive output levels. Then we show that Assumption 1 guarantees interior equilibrium outcomes such that the less efficient firm 2 has a strictly positive production quantity under all three unionisation structures.

Assume that we face regime U, and, for the moment, also suppose that firm 2 is not active. Then, firm 1’s optimal choice in the third stage is given by

\[ q_1(w, \Delta) = \left[ A - w(1 - \Delta) \right]/2. \]

Accordingly, the union sets \( w \) to maximise its wage bill \( U = (1 - \Delta)q_1(w, w_0) \) which yields the optimal wage \( w(\Delta) = [A + w_0(1 - \Delta)]/[2(1 - \Delta)] \). This, however, cannot constitute an equilibrium outcome as long as firm 2’s labour demand remains strictly positive. Firm 2’s best response function in the third stage of the game is given by

\[ q_2(q_1, w) = \max \{ A - q_1 - w \}, 0 \}

and by substituting \( q_1(w, \Delta) \) and \( w(\Delta) \) we obtain that \( q_2(q_1, w) > 0 \) if condition (1) holds.

Hence, condition (1) is sufficient to exclude corner solutions under regime U where only the efficient firm stays in the market.

We next show that condition (1) guarantees interior solutions under all three unionisation structures. Under the different structures, firm 2’s equilibrium output levels (which are stated in Lemma 2 below) are strictly positive for

\[ w_0 < \bar{w}_0^D = 5A/(5 + 2\Delta), \quad w_0 < \bar{w}_0^C = A/(1 + \Delta), \quad \text{and} \quad w_0 < \bar{w}_0^U = (2 + 5\Delta - 5\Delta)A/[2(1 + \Delta)(1 - \Delta + \Delta^2)] \]

for \( D, C \), and \( U \), respectively. It is straightforward to check that \( \bar{w}_0 < \bar{w}_0^D < \bar{w}_0^C < \bar{w}_0^U \). As condition (1) is the most restrictive one, it ensures that firm 2 has strictly positive output levels under all unionisation structures.

Equilibrium Wages and Outputs under the Different Unionisation Structures.

Lemma 1 and 2 summarise the equilibrium wages and quantities for given labour productivity enhancing innovations, \( \Delta \):

**Lemma 1.** For the different unionisation structures \( \rho = D, C, U \) the equilibrium wages are as follows:

(i) Decentralisation (D): \( w_1^D = \frac{5A + 2w_0(5 - 4\Delta)}{15(1 - \Delta)} \) and \( w_2^D = \frac{5A + 2w_0(5 - \Delta)}{15} \),

(ii) Coordination (C): \( w_1^C = \frac{A + w_0(1 - \Delta)}{2(1 - \Delta)} \) and \( w_2^C = \frac{A + w_0}{2} \),

(iii) Centralisation (U): \( w_i^U = \frac{(2 - \Delta)A + 2w_0(1 - \Delta + \Delta^2)}{4(1 - \Delta + \Delta^2)} \) for \( i = 1, 2 \).

Substitution of the equilibrium wages into equations (2) and (3) gives the firms’ equilibrium
output levels.

**Lemma 2.** Under the different unionisation structures $\rho = D, C, U$ the equilibrium production quantities for firms 1 and 2 are as follows:

(i) Decentralisation ($D$): $q_1^D = \frac{10A - 2w_0(5 - 7\Delta)}{45}$ and $q_2^D = \frac{10A - 2w_0(5 + 2\Delta)}{45}$.

(ii) Coordination ($C$): $q_1^C = \frac{A - w_0(1 - 2\Delta)}{6}$ and $q_2^C = \frac{A - w_0(1 + \Delta)}{6}$.

(iii) Centralisation ($U$): $q_1^U = q_2^C + \frac{A\Delta}{4(1 - \Delta + \Delta^2)}$ and $q_2^U = q_2^C - \frac{A\Delta(1 - \Delta)}{4(1 - \Delta + \Delta^2)}$.

**Proof of Proposition 2.**

By definition, $l_1^\rho = (1 - \Delta)q_1^\rho$, $l_2^\rho = q_2^\rho$, and $L^\rho = (1 - \Delta)q_1^\rho + q_2^\rho$, so that employment levels follow immediately from Lemma 2. The calculations to prove part (ii), (iii), and the ordering of $l_1^\rho$ in part (i) are straightforward. It remains to derive the ordering of $l_1^U$ and $l_1^D$ in part (i) of the proposition. Comparing $l_1^U$ and $l_1^D$, we obtain

$$l_1^U - l_1^D = \frac{(1 - \Delta)2w_0(3 - 3\Delta + 3\Delta^2 + 2\Delta^3) - 5A(2 - 11\Delta + 2\Delta^2)}{1 - \Delta + \Delta^2},$$

which is positive if and only if

$$w_0 > w_0^* = \frac{5A}{2} \frac{2 - 11\Delta + 2\Delta^2}{5 - 3\Delta + 3\Delta^2 + 2\Delta^3}.$$

We have to check that $w_0' < w_0^*$ holds, which is true for the considered parameter domain $\Delta \in (0, 1/3)$.

**Proof of Proposition 3.**

We have to compare $\Psi^\rho := \Pi_1^\rho(\Delta) - \Pi_2^\rho(\Delta)$ for $\rho = D, C, U$. Let us first compare $\Psi^D$ and $\Psi^C$. We obtain $\Psi^D = 4w_0\Delta\left[2(A - w_0) + w_0\Delta\right]/45$, and $\Psi^C = w_0\Delta\left[2(A - w_0) + w_0\Delta\right]/12$, so that $\Psi^D > \Psi^C$ if $w_0 < 2A/(2 - \Delta)$, which holds by Assumption 1.

Now let us turn to the comparison of $\Psi^U$ and $\Psi^D$. Define $a \equiv (A - w_0)/3$, $b \equiv 2\Delta w_0/3 + A\Delta/[2(1 - \Delta + \Delta^2)]$, $c \equiv 7\Delta w_0/15$, $d \equiv 2\Delta w_0/15$ and $e \equiv \Delta w_0/3 + A\Delta(1 - \Delta)/[2(1 - \Delta + \Delta^2)]$. We can write the innovation incentives under regimes $U$ and $D$ as follows: $\Psi^U = (2ab + b^2 + 2ae - e^2)/4$ and $\Psi^D = 4(2ac + c^2 + 2ad - d^2)/9$. It follows that $\Psi^U > \Psi^D$ if and only if

$$a\left[2(b + e) - 32(c + d)/9\right] + b^2 - e^2 > 16(e^2 - d^2)/9.$$

For this condition to be satisfied, it is sufficient to show that the following two conditions are jointly fulfilled:

$$2(b + e) > 32(c + d)/9;$$

$$b^2 - e^2 > 16(e^2 - d^2)/9.$$  

(5)

(6)
Given that $b + e = \Delta w_0 + \frac{A\Delta(2 - \Delta)}{2(1 - \Delta + \Delta^2)}$ and $c + d = 3\Delta w_0 / 5$, (5) holds for $w_0 < \frac{15A(2 - \Delta)}{2(1 - \Delta + \Delta^2)}$, which is implied by Assumption 1. Turning now to requirement (6) note that

\begin{align*}
\Delta^2 w_0 \left[ w_0 + A(1 + \Delta)/(1 - \Delta + \Delta^2) \right] / 3 &> 16(\Delta w_0)^2 / 45,
\end{align*}

which reduces to $w_0 < \frac{5A(1 + \Delta)}{2(1 - \Delta + \Delta^2)}$, which again holds by Assumption 1. Hence, $\Psi^U > \Psi^D$ follows, and therefore, $\Psi^U > \Psi^D > \Psi^C$.

**Proof of Proposition 4.**

We only derive the threshold value $w''_0$ as the remaining comparisons are obtained by straightforward calculations. Suppose that $\Psi^C < I(\Delta) < \Psi^U$, which implies that the innovation is implemented under regime $\rho = U$ but not under regime $\rho = C$. Comparison of total employment $L^U(\Delta)$ and $L^C(0)$ gives

\[ L^U(\Delta) - L^C(0) = -\frac{\Delta}{6} [A - 2w_0(1 - \Delta)]. \]

Hence, $L^U(\Delta) > L^C(0)$ holds for $w_0 > w''_0 \equiv \frac{A}{2(1 - \Delta)}$. As $w''_0 < \bar{w}_0$ is only true for $\Delta < 1/7$, we obtain the result stated in the proposition.

**Proof of Proposition 5.**

We first derive the second-best innovation incentives $\Psi^*$ with perfectly competitive labour markets and Cournot-duopoly competition in the product market. Then we prove the second part of the assertion which compares $\Psi^*$ and $\Psi^U$. Then we prove the first part of the proposition.

If the labour market is perfectly competitive, then the prevailing wage rate is $w^* = w_0$. Hence, the innovative firm’s equilibrium profits are $\Pi_1(\Delta) = [A - w_0(1 - 2\Delta)]^2 / 9$. Accordingly, second-best innovation incentives are defined by $\Psi^* = \Pi_1(\Delta) - \Pi_2(\Delta)$ for which we obtain $\Psi^* = w_0 \Delta [2(A - w_0) + w_0 \Delta] / 3$. Comparing $\Psi^*$ and $\Psi^U$ we obtain

\begin{align*}
\Psi^U - \Psi^* & = \frac{\Delta\psi_1(\Delta, w_0)}{48(1 - \Delta + \Delta^2)^2}, \quad \text{with} \quad \psi_1(\Delta, w_0) = A^2(8 - 12\Delta + 18\Delta^2 - 7\Delta^3) - 4Aw_0(8 - 16\Delta + 21\Delta^2 - 13\Delta^3 + 5\Delta^4) \\
& \quad + 4w_0^2(6 - 15\Delta + 24\Delta^2 - 21\Delta^3 + 12\Delta^4 - 3\Delta^5). \quad (7)
\end{align*}
As the denominator is strictly positive the sign of $\Psi^U - \Psi^*$ is positive if

$$\psi_1(\Delta, w_0) > 0. \quad (8)$$

Condition (8) is quadratic in $w_0$, which suggests an indirect way to prove the existence of a unique threshold $\tilde{\Delta}(w_0) > 0$ such that $\psi_1(\tilde{\Delta}(w_0), w_0) = 0$. Note that this also implies $\psi_1(\Delta, w_0) > 0$ for $\Delta > \tilde{\Delta}$ and $\psi_1(\Delta, w_0) < 0$ for $\Delta < \tilde{\Delta}$. Solving the quadratic form we obtain two critical values

$$w_{0,1} = \frac{A}{6} \left( \frac{4 - 4\Delta + 7\Delta^2}{2 - 3\Delta + 3\Delta^2 - \Delta^3} \right), \quad (9)$$

$$w_{0,2} = \frac{2}{2} \left( \frac{2 - \Delta}{1 - \Delta + \Delta^2} \right),$$

such that (8) holds if $w_0 < w_{0,1}$ or $w_0 > w_{0,2}$. First note that $w_{0,2} > w_0(\Delta)$ for all $\Delta \geq 0$, so that the second inequality never holds. We next show that (9) is monotonically increasing in $\Delta$. Calculating its derivative reveals that the sign of the derivative is determined by the expression

$$7\Delta^4 - 8\Delta^3 + 3\Delta^2 + 4\Delta + 4. \quad (10)$$

Note that expression (10) is strictly positive for all $0 \leq \Delta < 1/3$, so that the threshold value $w_{0,1}$ is monotonically increasing in $\Delta$. Moreover, for $\Delta = 0$ we obtain $w_{0,1} = A/3$ and for $\Delta = 1/3$ the value $w_{0,1} = 31(A/70)$. Combining the values for $w_{0,1}$ at the boundaries with the monotonicity of $w_{0,1}$ in $\Delta$ proves the existence of the unique threshold value $0 < \tilde{\Delta}(w_0) < 1/3$ and its monotonicity in $w_0$ for all $A/3 < w_0 < A$ as asserted in the proposition.

We finally show $\Psi^* > \Psi^D$ holds for all $0 < \Delta < 1/3$. First note that the difference $\Psi^* - \Psi^D$ is increasing in $\Delta$, which follows from

$$\frac{\partial(\Psi^* - \Psi^D)}{\partial \Delta} = \frac{22w_0}{45} [A - w_0(1 - \Delta)] > 0.$$ 

For $\Delta = 0$ we get $\Psi^* - \Psi^D = 0$, so that $\Psi^* > \Psi^D$ holds for all $0 < \Delta < 1/3$. By Proposition 3 it also follows that $\Psi^* > \Psi^C$.

References


