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# Strategic Renegotiation in Repeated Games

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## Abstract

Cooperative approaches to study renegotiation in repeated games have assumed that Pareto-ranked equilibria could not coexist within the same renegotiation-proof set. With strategic renegotiation, however, a proposal to move to a Pareto-superior equilibrium can be deterred by a different continuation equilibrium which harms the proposer and rewards the rejector. This paper studies strategic renegotiation in repeated games, defining stable social norms and renegotiation-proof outcomes in terms of a simple equilibrium refinement. We provide distinct necessary and sufficient conditions for renegotiation-proofness which converge to each other as renegotiation frictions become negligible. Renegotiation-proof outcomes always exist and can be all included within a single, most permissive social norm that is straightforward to characterize graphically. The analysis suggests a novel mechanism explaining inefficient equilibria, such as mis-coordination, inertia, and failure to upset an oppressive status quo even when information is complete, communication is frictionless, and players can credibly agree on efficient outcomes.

## 1 Introduction

The punishment equilibria used to sustain cooperation in repeated games are often Pareto inefficient. This puts into question their viability and, hence, the implementability of cooperative outcomes based on such punishments when players are free to renegotiate any continuation of the game. Incorporating renegotiation satisfactorily in repeated games has been a longstanding challenge.

To address this question, economists have introduced various concepts of renegotiation-proofness based on the following idea: roughly speaking, an equilibrium is *not* renegotiation-proof if it entails a continuation play that is Pareto dominated by some “credible” equilibrium (Pearce (1987), Bernheim and Ray (1989), Farrell and Maskin (1989), Abreu and Pearce (1991), and Asheim (1991)).<sup>1</sup>

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<sup>1</sup>The first discussion along these lines is due to Farrell (1983), which is subsumed by Farrell and Maskin (1989). Other approaches to renegotiation include DeMarzo (1988), Benoît and Krishna (1993), and Bergin and MacLeod (1993). All these papers follow axiomatic approaches.

These concepts mainly differ regarding what “credible” means and yield contrasted results: while Pearce (1987) argued,<sup>2</sup> as in the first paragraph, that maximal cooperation may not be sustained due to the lack of a credible and severe enough punishment, Farrell and Maskin (1989) found that most renegotiation-proof outcomes, as players become arbitrarily patient, *had* to be on the Pareto frontier of the feasible set.<sup>3</sup>

Owing to their cooperative (i.e., non-strategic) nature, these concepts have left unexplored an aspect of renegotiation which arises naturally when one considers an explicit protocol of renegotiation: what happens when a player rejects another player’s proposal? Suppose that during the punishment phase of a two-player repeated game, the continuation payoffs are  $(X_1, X_2)$  and player 1 proposes a Pareto-improving equilibrium with payoffs  $(Y_1, Y_2)$ . Clearly, such a Pareto-improvement need not be accepted if, by rejecting 1’s proposal, player 2 gets rewarded by a higher continuation payoff  $Z_2 > Y_2$ . Moreover, if 1’s continuation payoff  $Z_1$  after 2 has rejected his offer is less than  $X_1$ , then it is suboptimal for 1 to propose the Pareto improvement in the first place. With strategic renegotiation, a Pareto dominated equilibrium may thus withstand renegotiation as long as any off-path proposal may be deterred in this fashion. Punishing a player who deviates (here, in proposals) and rewarding other players is standard in repeated game analysis. It also seems plausible: for example, if an agent tries to bribe another one to obtain some advantage (a Pareto improving scheme for these agents!), the agent who rejects and exposes the bribe may be rewarded and the corruptor punished as a result.

This paper considers strategic renegotiation in repeated games by appending a simple stage at the end of each period: after actions and payoffs have been chosen and observed in period  $t$ , one of the players may be selected, with a fixed probability, to propose a continuation *plan*. A plan for period  $t + 1$  is more easily described recursively: it prescribes players’ actions, proposals, and acceptance decisions in period  $t + 1$ , as well as the continuation plan for period  $t + 2$  as a function of the actions, proposals, and acceptance decisions observed in period  $t + 1$ . In repeated game with renegotiation a set  $\mathcal{N}$  of equilibria is called a *norm* if, roughly speaking, all continuations of these equilibria—except, possibly, accepted off-path proposals—belong to  $\mathcal{N}$ . This definition aims to capture the idea of a social norm, known to all players, which describes all “usual” equilibria. A set  $\mathcal{N}$  is a norm if for any equilibrium in the norm, any off-path proposal is rejected and followed by an equilibrium which stays in the norm. Players can propose new plans in each period, with a

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<sup>2</sup>See also Abreu and Pearce (1991) and Abreu, Pearce and Stacchetti (1993).

<sup>3</sup>Farrell and Maskin, like Bernheim and Ray, introduce weak and strong concepts of renegotiation-proofness. The strong notion is arguably the more satisfactory one as it allows external comparisons (for example, the repetition of any static Nash equilibrium forms a weakly renegotiation-proof equilibrium of the repeated game, but it can be challenged by other equilibria according to the strong concept). The strong concept is well-behaved (existence, uniqueness) when players are arbitrarily patient, although the set of strongly-renegotiation proof equilibria may be very small due to the lack of punishments outside of a line that goes through the Pareto frontier.

fixed probability.<sup>4</sup>

To assess the stability of a norm, we introduce a strategic<sup>5</sup> equilibrium refinement. A norm  $\mathcal{N}$  is *stable*, when for any equilibrium in the norm, if a player makes a proposal which is accepted by other players, then this proposal is played. In particular, the refinement rules out babbling equilibria in which all proposals are ignored and any equilibrium of the underlying repeated game can be implemented. In fact, we find that as players become arbitrarily patient, the folk theorem need not hold any more when renegotiation is allowed.

Our analysis first covers the case of two players. We consider the set of all *renegotiation-proof* equilibrium payoffs, which are the payoffs that belong to the payoff set of some stable norm. We characterize this set when players are patient and renegotiation frictions—modeled as the probability that no one gets to make a proposal within any fixed time window—become negligible. This set is well behaved: it is always non empty and is straightforward to describe. In fact, its shape depends on only three points in the set of feasible payoffs: the minmax payoff vector  $V$  and the payoff vectors  $P_1$  and  $P_2$  that delimit the Pareto frontier. The set of renegotiation-proof payoffs is the intersection of two positive orthants with the feasible set: the orthant with vertex  $V$  (as in the Folk Theorem) and the orthant whose boundaries go through  $P_1$  and  $P_2$ . The sets characterizing necessary and sufficient conditions are distinct at all friction levels of renegotiation, and converge to each other as renegotiation frictions become negligible.

All renegotiation-proof payoffs can be implemented within the same stable norm. Therefore, there is no issue of competition or indeterminacy between multiple norms, at least not for the most permissive norm.

Our construction implies *path dependence* for the set of proposals considered acceptable within the norm. For example, the cooperative proposal  $(Y_1, Y_2)$  mentioned above may be acceptable at the beginning of the game, but not after a deviation. The relevance of this path dependence has been emphasized earlier (Abreu and Pearce (1991) and Asheim (1991)), and arises naturally when renegotiation is considered to be part of the equilibrium of a larger game, rather than a restriction on the set of equilibria of the underlying repeated game. It also implies that Farrell and Maskin’s and Bernheim and Ray’s identical notions of weak renegotiation proofness and internal consistency *may* in fact rule out stable norms. These reduced-form concepts, often viewed as a minimal restriction for any renegotiation-proof norm, require that no Pareto ranked equilibria coexist within such a norm. As noted, however, Pareto-dominated equilibria may withstand renegotiation as long as the social norm specifies clear punishment and reward for proposers and rejectors of Pareto improving

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<sup>4</sup>Our results do not hinge on allowing players to propose arbitrary innovations: the sets of payoffs characterizing necessary and sufficient conditions are identical if players are restricted to propose “credible” innovations, as explained in Section 5.

<sup>5</sup>Section 5 introduces a set-theoretic notion which captures this strategic approach and is payoff-equivalent.

equilibria.

The analysis provides a new explanation for the existence of inefficient equilibria, such as coordination failures and the difficulty to oppose oppressive regimes even when agents are freely able to communicate and credibly agree on efficient equilibria. With strategic negotiation, a norm must prescribe the continuation play not only after off-path actions but also after off-path proposals, such as suggestions to move to a Pareto-improving equilibrium. In the context of an oppressive regime, for instance, the norm may specify that any “subversive” (regime-threatening) proposal trigger a punishment for the proposer and reward other players who reject the proposal.

With three or more players, new conceptual issues emerge. In particular, what happens if only a subset of players accept the proposal made by a given player?<sup>6</sup> We explore several specifications and show that, depending on the assumptions, our predictions range from the Folk Theorem to Pareto efficiency. The simplest specification assumes that the continuation equilibrium when a proposal fails to receive unanimous approval is independent of the identity of the rejectors. Under this condition, our necessary and sufficient conditions are both characterized by payoff sets which take the form of upper-orthants, a useful qualitative property to model renegotiation in repeated games. In all our specifications, the sets characterizing necessary and sufficient conditions become arbitrarily close to each other as renegotiation frictions vanish. The analysis of this more general environment is contained in Section 7.

As noted, the stable norms introduced here differ conceptually from earlier notions and yield different predictions. Compared to weakly renegotiation-proof or internally consistent sets, they are more demanding by allowing their elements to be challenged by outside proposals. On the other hand, the stable norms are more permissive by allowing Pareto-ranked equilibria to coexist within a given norm and acceptable proposals and equilibria to be path dependent. For these latter reasons, the stable norms of this paper are more inclusive than the strongly renegotiation-proof sets of Farrell and Maskin (1989).

Several papers have studied strategic negotiation in which players engage in several rounds of cheap talk before choosing their actions in a one-shot game, and asked whether this pre-play communication could help select efficient equilibrium in the one-shot game. Farrell (1987) considers an entry game in which firms simultaneously announce their intention of whether to enter the market. With pre-play communication, firms achieve a higher payoff than they do in the symmetric one-shot equilibrium, but do not achieve perfect coordination. In Rabin (1994), players simultaneously propose Nash equilibria of the one-shot game and an equilibrium is played if both players propose it. With sufficiently many rounds of communication, each player is guaranteed to get at least her

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<sup>6</sup>A related issue is to understand what happens if a player makes a proposal to a subset of players. Although such a proposal may be understood as a global proposal which requires only the approval of a subset of players, it has a specific structure which we do not investigate in this paper and hope to explore in future work.

worst payoff in the “Pareto meet,” which is the set of Pareto-efficient Nash equilibria in the one-shot game. Players need not achieve a Pareto-efficient outcome however. In these papers, inefficiency can arise because players keep proposing their preferred outcome with high probability and may fail to reach an agreement.

The papers closest to ours are Santos (2000) and Miller and Watson (2013). In the alternative-offer model studied by Santos (2000), players bargain over Nash equilibria of a one-shot game, and play whichever equilibrium is agreed upon. In that paper, each player is guaranteed to get a payoff in the Pareto meet, but players may still end up playing a Pareto-inefficient equilibrium. More recently, Miller and Watson (2013) study equilibrium selection in a repeated game with an explicit bargaining protocol and transfers. Their goals and analysis are quite different from this paper’s. In particular, they are interested in understanding how axiomatic restrictions on disagreements affect bargaining outcomes. This, together with the presence of a transfer stage separate from the action stage, distinguishes their analysis and results from ours. The relation between these papers and ours is explained in detail in Sections 4.3 and 6.

## 2 Setting

We start the analysis by considering a repeated game between two players indexed by  $i \in \{1, 2\}$ . Player  $i$ ’s stage-game action,  $a_i$ , lies in a finite set denoted  $\mathcal{A}_i$ . The vector  $\mathbf{a} = (a_1, a_2)$  of actions determines the players’ payoffs for the current period,  $\mathbf{u}(\mathbf{a}) = (u_1(\mathbf{a}), u_2(\mathbf{a}))$ . A distribution  $\alpha_i$  over  $\mathcal{A}_i$  is a *mixed action* for  $i$ , and  $\alpha = (\alpha_1, \alpha_2)$  denotes a vector of mixed actions for both players. Players have a common discount factor  $\delta \in (0, 1)$ , and we will often find it convenient to work with the current-period weight  $\varepsilon = 1 - \delta$ .

Each period consists of the following stages:

- 1) Players observe the realization  $z$  of a public randomization device taking values in  $[0, 1]$ ;
- 2) They simultaneously choose mixed actions  $\alpha_i \in \Delta(\mathcal{A}_i)$ ,  $i \in \{1, 2\}$ . Mixing probabilities are not observable. Conditional on the realization  $z$  of the public randomization device, players choose their mixed actions independently from each other;
- 3) The vector  $\mathbf{a}$  of actions is observed and the period’s payoffs are realized;<sup>7</sup>

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<sup>7</sup>In accordance to the accepted standard in the modern analysis of repeated games, we allow players to use a public randomization device and private mixed strategies. This feature distinguishes our analysis from some of the earlier work on renegotiation. For example, Farrell and Maskin (1989) assume that players can observe each other’s mixing strategies, rather than merely observing the action outcomes of the randomization. It is possible to construct examples in which this distinction is important, as shown in Appendix H. Intuitively, when players observe mixing, there is without loss a single continuation payoff vector, conditional on players’ mixing strategies. When mixtures are unobservable, however, there must be a continuation vector for every possible outcome of the mixture—chosen so as to make each player indifferent across all actions in the support of her mixing strategy—and all of these vectors

4) With probability  $p < 1$ , one of the players is chosen to propose a new *plan* describing the continuation of the game. Each player has the same probability of  $\frac{p}{2}$  being chosen.<sup>8</sup> The chosen player may, however, conceal his proposal opportunity and remain silent instead, or mix between making a proposal or staying silent. The object of the proposal is an infinite-horizon plan  $m$  from the set  $\mathcal{M}$  of all possible plans, and will be described shortly;

5) If  $i$  made a proposal, player  $-i$  decides whether to accept it, possibly mixing between acceptance and rejection. The resulting decision,  $D_{-i}$ , is set to 1 if  $-i$  accepts the proposal and 0 if he rejects it;

The *public* history for the stage consists of the realisation  $z$  of the randomization device, the action vector  $\mathbf{a}$ , the proposal (which we will later denote as  $\mu_i$ ) or absence thereof and, if applicable, the acceptance decision  $D_{-i}$ . In addition, each player privately observes the mixing probability used for each of her decisions.

A *plan* at period  $t$  describes players' strategy for the infinite repetition of the stage-game described above, *from period  $t + 1$  onwards*. Those decisions (actions, proposals, and acceptance mixtures) are history-dependent. Because the setting is time invariant, the set  $\mathcal{M}$  of plans can be more conveniently defined recursively.

Specifically, a plan  $m \in \mathcal{M}$  at any period  $t$  is characterized by the following elements:

a) For each realization  $z$  of the public randomization device, a pair  $\alpha = \alpha[m](z)$  of mixed actions that players should play in period  $t + 1$ ;

b1) For each player  $i$ , a distribution  $\bar{\mu}_i = \bar{\mu}_i[m](z, \mathbf{a}) \in \Delta(\mathcal{M} \cup \emptyset)$  over proposals, where the outcome  $\emptyset$  means that  $i$  abstains from making a proposal (unbeknownst to player  $-i$ ). We assume that distributions have a finite support over plans.<sup>9</sup> The proposer's choice of a proposal distribution is conditional on the realization  $z$  of the public randomization device and on the pair  $\mathbf{a}$  of observed actions. Because  $p < 1$ , not observing any proposal from either player is always consistent with "on-path" behavior. The realized proposal is denoted  $\mu_i$ ;

b2) A probability  $q_{-i} = q_{-i}[m](z, \mathbf{a}, \mu_i)$  that  $-i$  accepts  $i$ 's proposal (whenever  $\mu_i \neq \emptyset$ ), conditional on  $z$ ,  $\mathbf{a}$ , and  $\mu_i$ ;

b3) If no one made a proposal, the acceptance stage is skipped. To economize on notation, we assume that some player  $i$  is, even in that case, conventionally selected (randomly or deter-

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must belong to the renegotiation-proof set. This is problematic because some of these continuations may have Pareto-ranked payoffs, violating weak renegotiation-proofness. Bernheim and Ray (1989) rule out mixing altogether, focusing the analysis on pure-strategy equilibria.

<sup>8</sup>Our results extend to the case of asymmetric probabilities. The sufficient conditions are unchanged, but necessary conditions entail a payoff lower bound on each player, which increases with that player's proposal probability, consistent with the intuition that a higher proposal probability means an increased bargaining power. The extension is discussed in Appendix B.

<sup>9</sup>We will in fact impose a uniform upper bound on this support as explained below.

ministically) as the proposer and let  $\mu_i = \emptyset$  and  $D_{-i} = 0$ . (So,  $-i$ 's conventional response is to systematically “reject”  $i$ 's non proposal.)

c) A continuation plan  $m_{+1} = m_{+1}[m](z, \mathbf{a}, i, \mu_i, D_{-i}) \in \mathcal{M}$  for period  $t + 2$  onwards, as a function of  $z, \mathbf{a}, i, \mu_i, D_{-i}$ , where  $i$  indicates the identity of the last proposer. Obviously, this plan must be independent of  $i$  whenever  $\mu_i = \emptyset$ , so that the convention chosen for the proposer in the absence of any actual proposal is indeed irrelevant. This restriction is applied throughout.

This protocol allows plans in which any proposal is ignored (babbling). In the next section, we introduce a stability refinement requiring some accepted proposals to be played. The protocol also allows plans for which any rejected proposal results in the same “default” continuation, which is the “No-Fault Disagreement” Axiom studied by Santos (2000) and Miller and Watson (2013) and, in a simultaneous-offer setting, by Farrell (1987), Rabin (1994), Arvan, Cabral, Santos (1999).<sup>10</sup>

While the above definition seems natural, it turns out to be too permissive for the set of plans to be well-defined: there does not exist a set of plans so large as to contain all the possible continuation prescriptions allowed above. For example, notice that the above construction must specify an acceptance decision for each possible proposal. This means that, in general, each plan  $m$  must specify—among other things—a function which maps each element of  $\mathcal{M}$  (the proposal) to a binary decision (acceptance). This implies that the set  $\mathcal{M}$  of plans must contain, in order to include all possible prescriptions, its power set  $2^{\mathcal{M}}$ . Such a set does not exist, since any set has a strictly lower cardinality than its power set, by Cantor’s Power Set Theorem (see, e.g., Mendelson (1997)).

In the Appendix, we provide restrictions on plan prescriptions which guarantee that the set of plans is well-defined, but flexible enough to include all the plans discussed in this paper. These restrictions are of three kinds: first, we assume that the cardinalities of support of  $\bar{\mu}_i[m](z, \mathbf{a})$  (the proposal distribution) and the set  $\mathcal{M}^{+1}[m]$  of possible continuation plans are uniformly bounded over  $m, z$ , and  $\mathbf{a}$ . Second, a receiver is prescribed to accept on-path proposals and reject off-path ones. Third, the continuation plan is chosen in  $\mathcal{M}^{+1}[m]$  according to a choice rule which depends only on the following information: i) whether the proposal was on-path (i.e., in the support of the proposal distribution prescribed by  $m$  for the proposer), ii) whether it was accepted, and iii) for each player and continuation plan in  $\mathcal{M}^{+1}[m]$ , the pairwise ranking of the proposal’s payoff relative that continuation plan’s payoff. The set of plans resulting from these restrictions can be chosen so as to have cardinality  $\beth_2$ —the cardinality of the set of all real-valued functions over  $\mathbb{R}$ . To avoid cluttering the analysis, we defer the details to Appendix A.

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<sup>10</sup>The protocol also allows more counter-intuitive plans for which an accepted proposal is followed by a continuation plan which has nothing to do with the initial proposal. Appendix B explains why one could without loss restrict attention to plans which are “truthful” (i.e., *on-path* proposals which are accepted are played). However, we delay this discussion to avoid cluttering the analysis.



### 3 Concepts

The previous section has introduced an infinite-horizon game which we call “negotiated game”. Any strategy profile of this game can be identified with a plan. Indeed, a plan defines—explicitly or recursively—an arbitrary history-dependent mixture of actions at each stage of the game.<sup>11</sup> Accordingly, the subgame perfect equilibria (SPEs) of the negotiated game can be identified as a subset  $\mathcal{S}$  of  $\mathcal{M}$ . Unless stated otherwise, in this paper “SPE” refers to an equilibrium of the negotiated game (not to be confused with the subgame perfect equilibria of the underlying repeated game without renegotiation).

**DEFINITION 1** *A subset  $\mathcal{N}$  of  $\mathcal{S}$  is a **norm** if for any  $m \in \mathcal{N}$  such that  $\mu_i \in \bar{\mu}_i[m](z, \mathbf{a})$  or  $D_{-i} = 0$ ,  $m_{+1}[m](z, \mathbf{a}, i, \mu_i, D_{-i}) \in \mathcal{N}$ ;*

We interpret  $\mathcal{N}$  as a social norm: it describes the set of all continuation plays which players consider possible under “business as usual”. A social norm alone does not determine everyday interaction, since it allows for multiple possibilities; players have to keep in mind which continuation in the norm is being currently played. According to this norm, players may be punished if they deviate from the equilibrium path, but they are always punished *within* the norm, regardless of the history. However, players may in principle agree to switch to an equilibrium outside of the norm. This happens if a player makes a deviation in proposal (hence creating an “innovation”), which the other player accepts.

Until now, negotiation has not imposed any restriction on the set of equilibria which may be played. For example, it does not rule out babbling equilibria. The key concept is the following notion of stability.

**DEFINITION 2** *A norm  $\mathcal{N}$  is **stable** if for any SPE of  $\mathcal{N}$ , whenever  $i$  proposes an equilibrium  $\mu \in \mathcal{S}$  and  $-i$  accepts it,  $\mu$  is implemented.*

To understand the concept, notice that a stable norm must be able to withstand any proposal, including proposals which lie outside the norm (and are thus off-path, by definition of a norm)<sup>12</sup>. For if such a proposal were accepted, it would be taken seriously by the players and be implemented. Stability thus requires that no player has ever an incentive to make proposals outside of the norm. As noted, stability can be intuitively achieved by rewarding the receiver if he rejects such a proposal,

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<sup>11</sup>In the actual game, the absence of a proposal triggers the next period. Therefore, a plan’s independence from the conventionally chosen proposer in the absence of an actual proposal is not restriction on the set of strategy profiles being considered.

<sup>12</sup>A union of two stable norms  $\mathcal{N}_1$  and  $\mathcal{N}_2$  is another stable norm  $\mathcal{N}_1 \cup \mathcal{N}_2$ , with even larger set of possible punishments for any deviation in proposal. This shows the existence of the largest stable norm.

and punishing the proposer by a worse continuation than if he had stayed on path. Of course, this deterrence continuation has to lie within the norm.

This definition of stability may seem particularly demanding: why should players treat all proposals as “credible”? Section 5 shows that a much more restrictive notion of credibility yields the same necessary and sufficient conditions that we obtain when we consider all proposals.<sup>13</sup>

Conceptually, it should be emphasized that all proposals are equilibria of the repeated game with renegotiation. For instance, if player 1 says to player 2: “let’s move to this other equilibrium of the underlying repeated game and treat all future proposals as babbling,” and 2 accepts this, then it is indeed an equilibrium for them to treat all ulterior proposals as babbling.

Another potential concern about the definition of stability is that credible proposals are not required to obey a similar stability refinement. However, the refinement considered by definition 2 is imposed on off-path proposals.<sup>14</sup> Players may accept, once, to leave the norm and implement an innovative proposal. Any further proposal which is off-path, relative to the current one, should not be played even if accepted. Otherwise the first proposal would not be credible and should be ignored when being made initially.

Stability thus amounts to a simple equilibrium refinement which rules out pure cheap talk, giving some bite to renegotiation. This means that any SPE of the stable norm  $\mathcal{N}$  is able to withstand arbitrary proposals. Recalling that on-path continuations must all belong to the norm (by definition of a norm), stability implicitly requires that any proposal that is *not* in the norm is rejected. We allow considerable leeway in proposals, though it does not affect either our necessary or sufficient conditions, stated in the next section. Both the conditions are rigorously identical if one restricts proposals to a much smaller subset of “credible” proposals, which are roughly speaking equilibria such that any deviation triggers a reversal to the norm. Since such a restriction is not needed for the results, we postpone the analysis of credible innovations to Section 5.

Before defining renegotiation-proofness, we need to introduce notation to distinguish players’ payoffs at different stages of the game. Given a subset  $\mathcal{L}$  of SPEs, the set  $\mathcal{U}(\mathcal{L})$  (or just  $\mathcal{U}$ , when there is no confusion) denotes the set of expected payoffs for the players, across all possible SPEs in  $\mathcal{L}$ , computed *before* public randomization.  $\mathcal{V}$  is defined identically but computed *after* the realization of the randomization device  $z$ . In particular,  $\mathcal{U}$  is included in the convex hull of  $\mathcal{V}$ . Finally,  $\mathcal{W}$  consists of continuations payoffs *after* actions and payoffs are observed and incurred in the current period, but *before* the proposal stage. Any payoff vector of  $\mathcal{W}$  is a mixture of three payoff vectors of  $\mathcal{U}$ , seen as continuation payoffs for the next period: these payoffs correspond to the three possible

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<sup>13</sup>In that section, a proposal is credible relative to a norm if any ulterior deviation from this proposal triggers a reversal to the norm.

<sup>14</sup>In fact, one can easily show that any norm is equivalent to another one in which all on-path proposals are accepted and implemented, as shown in Appendix B.

cases for the proposal stage: player 1 gets to make a proposal, player 2 does, or no one does. Because continuation payoffs in  $\mathcal{W}$  do not include the payoff from the current period, in order to make them commensurate with payoffs in  $\mathcal{U}$ , they are computed in terms of the next period (i.e., ignoring the discount factor between the two periods). With this convention, payoffs in  $\mathcal{W}$  are indeed mixtures of elements in  $\mathcal{U}$ .

Elements of  $\mathcal{U}$ ,  $\mathcal{V}$ , and  $\mathcal{W}$  are points of two-dimensional sets. For any point  $U$  of such a set, we let  $\pi_i(U)$  denote the  $i^{\text{th}}$  component of  $U$ , i.e.,  $i$ 's continuation payoff at corresponding stage of the game.

**DEFINITION 3** *A point  $A$  is  **$q$ -renegotiation-proof** if there exists  $\bar{\varepsilon} \in (0, \frac{1}{q})$  such that for all  $\varepsilon \leq \bar{\varepsilon}$  and  $p = q\varepsilon$ , there exists a stable norm  $\mathcal{N}$  such that  $A \in \mathcal{U}(\mathcal{N})$ . Moreover,  $A$  is **renegotiation-proof** if it is  $q$ -renegotiation-proof for all  $q$ 's large enough.*

The coefficient  $q$  is inversely related to the amount of renegotiation frictions in the game:<sup>15</sup> when  $q = 0$ , players never get a chance to renegotiate and the game reduces to a standard repeated game. As with the standard Folk theorem, any point of the feasible IR set (in the usual sense of repeated games, absent any renegotiation) is 0-sustainable. Our main objective is to characterize the set of sustainable payoffs. To do so, we first study the set of  $q$ -sustainable payoffs for any fixed  $q$ , and then let the renegotiation frictions go to zero (i.e.,  $q$  go to  $\infty$ ).

In the definition above,  $A$  needs only belong to  $\mathcal{U}$  which, unlike  $\mathcal{V}$ , includes the initial use of a public randomization device. As it turns out however, this distinction is unimportant for our results.

## 4 Main Result

### 4.1 Statement

We let  $v_i$  denote  $i$ 's minmax payoff in the stage game of the repeated game (absent any renegotiation).<sup>16</sup> The set of all feasible stage-game payoffs is a convex polygon. Similarly, let  $P_i$  denote the feasible payoff vector that gives  $i$  his maximal payoff in the stage game.<sup>17</sup> The 'weak' Pareto frontier (consisting of all points which are not *strictly* Pareto dominated) is a piecewise linear curve joining  $P_1$  and  $P_2$ .

$$\text{Let } v_1 = \max\{v_1; \pi_1(P_2)\} \text{ and } v_2 = \max\{v_2; \pi_2(P_1)\}.$$

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<sup>15</sup>Players could be considered to take actions in continuous time, with  $\varepsilon$  being the time interval between the subsequent actions. For small  $\varepsilon$ , the coefficient  $q$  would then be equal to the expected number of possibilities for some player to make a proposal, within a time interval of 1.

<sup>16</sup>As usual, player  $-i$  is allowed to mix across actions to minmax  $i$ .

<sup>17</sup>If several such points exist, we choose the point among those with the lowest payoff for  $-i$ .

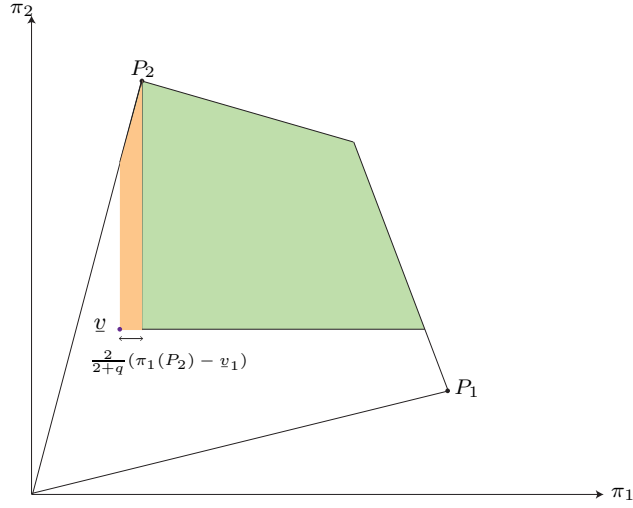


Figure 1: Necessary and sufficient conditions for fixed  $q$

**THEOREM 1 (RENEGOTIATION-PROOF SET)** *Suppose that  $P_1 \neq P_2$ . Then, the following holds:*

**Sufficiency** *If*

$$\pi_i(A) > v_i \quad \text{for } i \in \{1, 2\}, \quad (1)$$

*then the point  $A$  is  $q$ -renegotiation-proof for all  $q \in \mathbb{R}_+$  and hence renegotiation-proof.*

**Necessity** *If  $A$  is  $q$ -renegotiation-proof, then*

$$\pi_i(A) \geq v_i + \max \left\{ 0; \frac{q}{2+q} (\pi_i(P_{-i}) - v_i) \right\} \quad (2)$$

*for  $i \in \{1, 2\}$ . If  $A$  is renegotiation-proof, inequalities in (1) must hold for both players as weak inequalities.*

If  $P_1 = P_2$ , the only renegotiation-proof point is  $P_1$ , which is played forever. In this case, players have perfectly aligned interests as they both want to implement  $P_1$ . The necessary conditions in this case are the same as in (2), selecting point  $P_1$  as the only possible outcome, as renegotiation frictions become negligible.<sup>18</sup>

The statement of Theorem 1 can be visualized on Figure 1 for a fixed friction level of renegotiation. The green domain represents the set of points which are known to be renegotiation-proof (i.e., part of a stable norm), while the orange domain represents the additional points which *may* be renegotiation-proof. When  $q = 0$  (no renegotiation), the orange domain extends all the way back to the minmax point  $v$ , and we obtain the Folk Theorem. As the renegotiation frictions vanish ( $q \rightarrow +\infty$ ), the orange domain disappears: necessary and sufficient conditions become identical (up to the boundary).

<sup>18</sup>In the non-generic case of several Pareto-efficient points, each giving the same payoff to one player, the best Pareto point is renegotiation-proof. Other Pareto points may or may not be renegotiation-proof.

It is straightforward to characterize renegotiation-proof points when frictions vanish. This may be done graphically, and Figure 2 represents the corresponding sets for various configuration. In configuration (a), renegotiation constrains the set of implementable payoffs because the deterrence points  $P_1$  and  $P_2$  are too close to each other (relative to the vector of minmax payoffs). Configuration (b) represents a perfectly cooperative game. The only renegotiation-proof outcome is the Pareto efficient point. In configuration (c), the punishment/reward points are sufficiently far apart, and the Folk Theorem holds despite the presence of renegotiation.

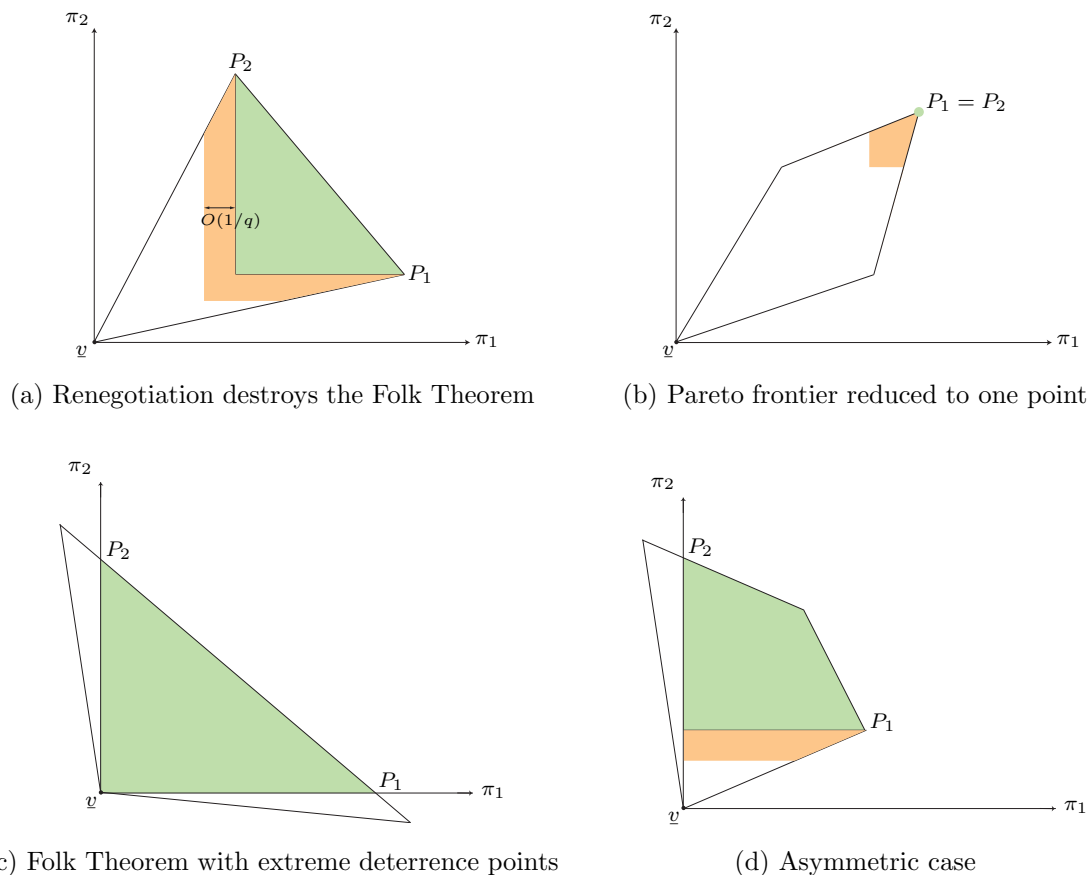


Figure 2: Necessary and sufficient conditions for various configurations

### Scope for Cooperation and Renegotiation-proof Outcomes

As Figure 2 illustrates, the impact of renegotiation hinges on the structure of the stage game. As the game becomes less cooperative (moving from (b) to (a) to (c), on the figure), there is more scope for disagreement among the players, which can be used to implement a larger set of feasible payoffs. In our framework, renegotiation does not destroy the implementability of individually-

rational, Pareto efficient payoffs. However, when players have diverging interests, one can depart more and more from this.

Seeing this result from the perspective of a mechanism designer, it may be that what is good for these two players is actually bad for a larger group. Getting close to the Pareto point may mean that both players are slacking off, or polluting, which negatively affects some unmodeled entity, such as the environment or other society members not explicitly included in the game. Thus, a certain scope of disagreement (actions which benefit only one player but not the other) may keep players working or making effort (even if this is, for them, a bad Nash equilibrium). The construction explained below shows how such equilibrium can be self-implemented by using dynamic norms among the two players. The only role of the mechanism designer is simply to specify a norm of how players should interpret deviations in actions and proposals, i.e., what continuation equilibria they should expect as a result of these deviations. Once this common understanding is reached at the beginning of the game, the mechanism designer has no role to play and can completely withdraw from the game: the two players can enforce the norms themselves and punish each other if one of them ever deviates.

### **Examples illustrating the limits of renegotiation to achieve efficiency**

Perhaps the most intriguing aspect of the present analysis is the fact that players self-enforce a norm that prevents them from moving to Pareto efficient payoffs. One may interpret any deviating proposal as a rupture in the social norm between players. After all, when being in a norm, there is a common understanding between players of the sequence of actions that should take place and of off-path sequences in case of deviations. This common understanding can be modified or shattered by various proposals, resulting in a new equilibrium. This new equilibrium may not necessarily follow a proposal, as illustrated by the following examples.

**Cournot competition.** Consider two symmetric firms which, in equilibrium, produce a higher total output than the monopolistic output. These firms could achieve a higher profit if they each produced half of monopolistic output. However, even when they can renegotiate this Pareto inefficient equilibrium, the business norm in these firms' industry may prevent them from implementing a coordinated output reduction, by treating any such proposal as "corrupt" behavior. The other firm rejects the agreement and both firms understand (as part of a business norm) that the new equilibrium prescribes the rejector to produce the Stackelberg leader's output in each period, while the proposer chooses the Stackelberg follower's output. These new outputs constitute an equilibrium. The proposer has a lower payoff compared to initial equilibrium, and the opponent has a higher payoff than under the split monopoly proposal. The punishment for the proposer, i.e., the

Stackelberg equilibrium, is not Pareto-efficient. However, it achieves the objective of deterring a Pareto-improving proposal.

This example illustrates the point, mentioned earlier, that the designer of the norm may be a regulator outside of the game whose payoff (or welfare objective) depends on the actions of the players in the game. However, once the norm is set in place, no outside involvement is needed to sustain the Pareto-dominated equilibrium.

It may be tempting for one of the firms to reach out to the other firm anyway and argue that they really ought to forget all about the present norm and move to a Pareto improving production. However, such a message is exactly the kind of proposal that this model aims to capture. If it is clear to all firms that reaching out in this way constitutes a proposal punishable by the norm, then it is definitely in the receiver's interest to reject the proposal, which deters the first firm from making a proposal. Indeed, a perfect norm should prescribe reactions to any possible message. This opens the possibility of studying "weak" norms, which are not immune to convincing neologisms, perhaps because the first player's message cannot be mapped unambiguously into a proposal for which the norm has a clear prescription.

One can push further the idea that the environment includes not only "active" players of the game, but also a third party—the norm designer. The third party is not directly involved in the game, and it does not want the players to cooperate (or, depending on applications, to collude). For example, it could be a manager who faces high costs to monitor his employees. Designing a norm may provide a cheap, effective way of preventing collusion.

The logic of the argument may be illustrated by the a dictatorship facing the possibility of a revolution. Under the dictatorship, citizens must pay high taxes, as a large share of their income is captured by the dictator. Each citizen may start persuading others to start a revolution (which is not a part of the current norm). If everyone agrees, the dictator is thrown out, and every citizen gets a better payoff. However, the current norm addresses this threat by rewarding anyone who reveals such a plot, so that the revolution may be prevented by punishing the deviator. Importantly, the reward and the punishment are performed by the citizens, without the dictator needing to get involved. The logic of the argument relates to a symmetric setting with many players, as described in Section 7. It provides a novel completely endogenous explanation for the stability of dictatorships, which allows coordination but exposes the limits of attempts to coordinate when the norm in place anticipates such attempts.

These examples also hint at the dynamic nature of social norms, understood in our model as the interpretation of proposals: for example, starting from a Pareto dominated equilibrium, a proposal to move to a cooperative, Pareto-improving equilibrium may be welcome and accepted. However, the accepted proposal triggers a new continuation in the norm, in which if a player subsequently

deviates from the cooperative play, the Pareto-dominated equilibrium is played forever. Any new proposal to move to a Pareto improving plan is punished.

An equilibrium of the repeated game with renegotiation defines its own changes as one moves along the game. It is as if players have a “reputation” within the current equilibrium—which is not about an intrinsic type (such type doesn’t exist in our model) but about how players understand to be treated in the continuation of the game. If a player makes a proposal to change the equilibrium (cooperation, revolution), she risks losing her reputation if the other player rejects the proposal. Following a rejected proposal, the initial plan is still implemented, but with changed players’ reputation and payoffs.

## 4.2 Sufficient conditions: construction of a stable norm

We now present the construction of a stable norm which permits to establish the sufficient conditions. The necessary conditions are derived in Appendix C

**Outline.** We construct, for any payoff vector  $A$  satisfying (1), a norm  $\mathcal{N}$  containing an equilibrium with expected payoff  $A$  and which is stable for any  $q \geq 0$ . To avoid confusion, for any payoff vector  $X$  which is achieved at some SPE in the norm, we denote that SPE as  $X^{\mathcal{N}}$ . The construction starts by choosing two points  $A_1, A_2$  such that  $A_i$  defines  $i$ ’s worst possible payoff according to the norm.<sup>19</sup> Given any continuation payoff far away from  $A_i$ , it is always in  $i$ ’s interest to follow the prescribed play in action, since any deviation provides a gain of order  $\varepsilon$  and can be punished by moving to  $A_i^{\mathcal{N}}$ . The key is to choose  $A_i$  so that  $i$  is adequately incentivized near  $A_i$  and to complete the norm with enough equilibria to guarantee that the norm is stable. That last part is achieved by including some Pareto-optimal points  $D_1, D_2$  in the norm so that for any proposal that  $-i$  may deviate to,  $i$  can always be rewarded, and  $-i$  punished, by rejecting  $-i$ ’s proposal and have  $D_i^{\mathcal{N}}$  implemented.

For each player  $i$ , there are two cases to consider, depending on whether  $i$ ’s minmax payoff  $v_i$  lies above or below  $\pi_i(P_{-i})$ . We treat the former case first.

**Case 1:**  $v_1 > \pi_1(P_2)$  and  $v_2 > \pi_2(P_1)$

Consider any point  $A$  satisfying (1). For  $\varepsilon$  small enough, the points  $A_1$  and  $A_2$  with coordinates

$$\pi_1(A_1) = v_1 + \varepsilon^{\frac{1}{2}}; \quad \pi_2(A_1) = \pi_2(A)$$

and

$$\pi_1(A_2) = \pi_1(A); \quad \pi_2(A_2) = v_2 + \varepsilon^{\frac{1}{2}}$$

are individually rational and such that  $\pi_1(A_1) < \pi_1(A)$  and  $\pi_2(A_2) < \pi_2(A)$ .

The element  $A_1^{\mathcal{N}}$  is implemented as follows ( $A_2^{\mathcal{N}}$  has a similar implementation):

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<sup>19</sup>Unless stated otherwise, payoffs are elements of the set  $\mathcal{U}$ , i.e., at the beginning of a period.



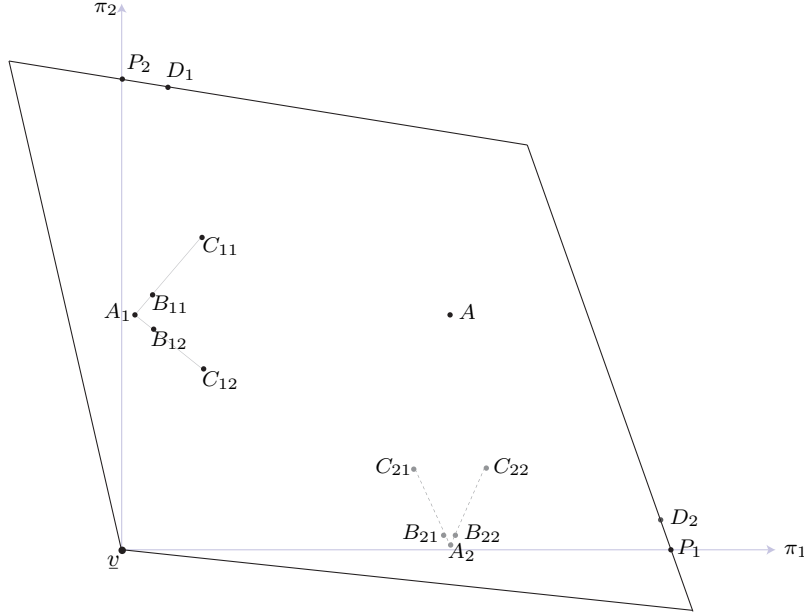


Figure 3: Stable norm construction: outline

1) Action stage: player 2 minmaxes player 1, possibly mixing between several actions  $a_{2j}$  and 1 best responds by a pure action  $a_{1,minmax}$  achieving his minmax payoff.

1a) If no deviation in action is observed, the continuation payoff vector  $B_{1j} \in \mathcal{W}$  is a function of 2's realized action,  $a_{2j}$ , chosen so as to satisfy the promise-keeping condition. The payoffs  $B_{1j}$  are chosen so as to make 2 indifferent between the actions  $a_{2j}$  used to minmax 1 and all give 1 the same continuation payoff. This implies that

$$\pi_1(A_1) = \varepsilon v_1 + (1 - \varepsilon)\pi_1(B_{1j}) \quad (3)$$

Note that all  $B_{1j}$ 's are within an  $\varepsilon$ -proportional distance of  $A_1$ .

1b) If 2 deviates in action (i.e., chooses an action outside of the mixture used to minmax 1), the continuation payoffs jump to the point  $A_2$ , mentioned above, which gives her the lowest possible payoff in the norm.<sup>20</sup> This punishment is clearly enough to incentivize 2, because any gain is of order  $\varepsilon$ , whereas  $\pi_2(A_2)$  is arbitrarily close to 2's minmax payoff and thus at an  $\varepsilon$ -independent distance from  $\pi_2(A_1)$  (and, hence,  $\pi_2(B_{1j})$ 's)

1c) If 1 deviates in action, disregard this. Such a deviation is obviously suboptimal, since 1 was prescribed to best respond to being minmaxed by 2.

2) Proposal stage: the element  $B_{1j}^N$  is implemented as follows: if either 2 gets a chance to make a proposal, or no player does, the continuation payoffs return to  $A_1^N$ . (2 is prescribed to remain

<sup>20</sup>More precisely, it jumps to the point  $B_{21}$ , which is the analogue of the point  $B_{11}$ , following the implementation of  $A_2$ .

silent.) If 1 gets a chance to make a proposal, he proposes an element  $C_{1j}^{\mathcal{N}}$ , which payoff  $C_{1j}$  lies on the line going through  $A_1$  and  $B_{1j}$ , and is chosen so as to satisfy the promise-keeping condition:

$$\pi_1(B_{1j}) = \left(1 - \frac{p}{2}\right) \pi_1(A_1) + \frac{p}{2} \pi_1(C_{1j}) \quad (4)$$

Player 2 is prescribed to accept proposal  $C_{1j}^{\mathcal{N}}$ . The points  $\{C_{1j}\}_j$  give the same payoff to 1, independently of  $j$ . Their implementation is described in 3) below.

2a) If 1 proposes any plan other than  $C_{1j}^{\mathcal{N}}$ , he is punished by the implementation of an element  $D_1^{\mathcal{N}}$  chosen so that i)  $\pi_1(D_1) < \pi_1(C_{1j})$  and ii) 2 prefers  $D_1^{\mathcal{N}}$  to 1's proposal, Pareto dominating  $C_{1j}^{\mathcal{N}}$ . Precisely,  $D_1^{\mathcal{N}}$  is defined as the point of the Pareto frontier that gives 1 a payoff of:

$$\frac{\pi_1(A_1) + \pi_1(C_{1j})}{2} \quad (5)$$

2b) If 2 deviates by making a proposal or rejecting 1's offer to move to  $C_{1j}$ , she is punished by the player-2 analogue of point  $D_1^{\mathcal{N}}$ .

3) Next periods: the element  $C_{1j}^{\mathcal{N}}$  is easily implemented, because it gives 1 a payoff of the order of  $\sqrt{\varepsilon}$  above what  $A_1^{\mathcal{N}}$  and, hence,  $B_{1j}^{\mathcal{N}}$ 's give him, as explained shortly. A deviation in action by 1 brings a gain of order  $\varepsilon$  and is punished by a drop of order  $\sqrt{\varepsilon}$  in 1's continuation payoff, and is thus suboptimal, for  $\varepsilon$  small enough. More precisely, the element  $C_{1j}^{\mathcal{N}}$  can be implemented by a deterministic sequence of actions keeping players' continuation payoffs within a distance  $K\varepsilon$  from  $C_{1j}$ . The rules implementing that sequence are simple: play a deterministic action profile keeping continuation payoffs  $\varepsilon$ -close to  $C_{1j}$  and do not allow any proposal. If 1 deviates in actions, move to one of the points  $B_{1j}^{\mathcal{N}}$ ; if he deviates in proposals, move to  $D_1^{\mathcal{N}}$ . A similar rule is applied for player 2, who has even more to lose from deviating.

4) Finally, the payoff  $D_1$  lies at a distance of order  $\sqrt{\varepsilon}$  from  $A_1$ , and  $D_1^{\mathcal{N}}$  can therefore be implemented similarly to  $C_{1j}^{\mathcal{N}}$  by a deterministic sequence of actions which keep the continuation payoff within a distance  $K\varepsilon$  from  $D_1$ . Any proposal is ignored.

This construction is represented on Figure 3, and the relative distances between continuation payoffs are represented on Figure 4.

We verify the claim that all  $C_{1j}$ 's lie at a  $\sqrt{\varepsilon}$ -proportional distance to the right of  $A_1$ . From (3) and (4), we get

$$\pi_1(A_1) = \varepsilon v_1 + (1 - \varepsilon) \pi_1(B_j) = \varepsilon v_1 + (1 - \varepsilon) \left[ \left(1 - \frac{q\varepsilon}{2}\right) \pi_1(A_1) + \frac{q\varepsilon}{2} \pi_1(C_{1j}) \right]$$

Ignoring terms of order  $\varepsilon^2$  and higher, this implies that

$$\pi_1(A_1) = \varepsilon v_1 + \left(1 - \left(1 + \frac{q}{2}\right) \varepsilon\right) \pi_1(A_1) + \frac{q\varepsilon}{2} \pi_1(C_{1j}).$$

Subtracting  $\pi_1(A_1)$  from both sides and dividing by  $\varepsilon$  yields

$$\varepsilon^{\frac{1}{2}} = \pi_1(A_1) - v_1 = \frac{q}{2} (\pi_1(C_{1j}) - \pi_1(A_1)), \quad (6)$$

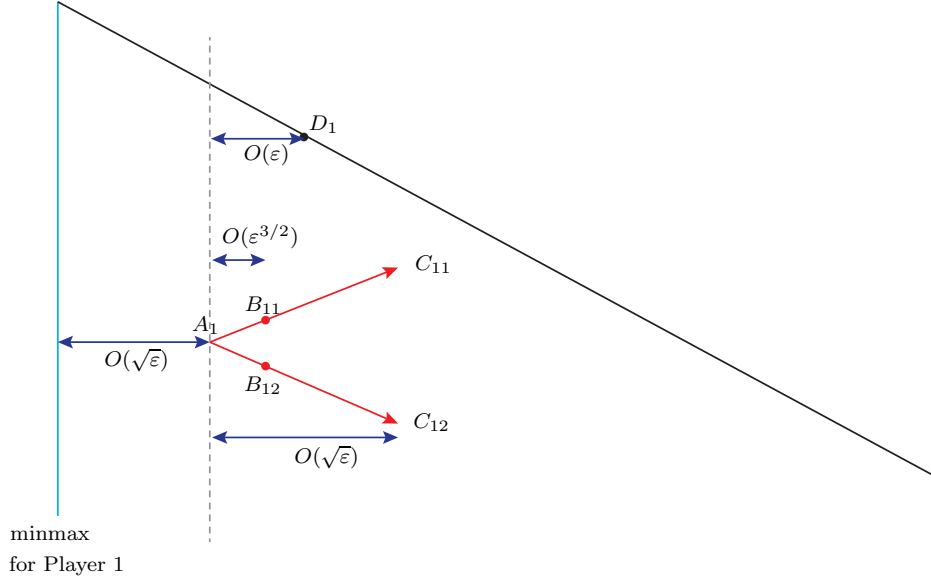


Figure 4: Construction details

which shows the claim.

The direction of each vector  $\overrightarrow{A_1 C_{1j}}$ , which is also  $\overrightarrow{A_1 B_{1j}}$ 's direction, depends only on 2's action,  $a_{2j}$ ; it does not change when  $\varepsilon$  goes to 0. This shows that, for  $\varepsilon$  small enough,  $C_{1j}$  is a feasible payoff and  $\pi_2(C_{1j})$  exceeds  $\pi_2(A_2)$  by an  $\varepsilon$ -independent value.

As noted, the system of actions and proposals implementing  $A_i^N$ 's,  $B_{ij}^N$ 's and  $C_{ij}^N$ 's and  $D_i^N$ 's is incentive compatible in actions and in proposals. To conclude the construction, we observe that  $A$  gives each player  $i$  a payoff higher than  $A_i$ , by an amount that is bounded below away from zero and thus independent of  $\varepsilon$  as  $\varepsilon$  goes to zero. One may therefore implement  $A$  by a deterministic sequence of actions, chosen so that the continuation payoffs stay within a distance  $K\varepsilon$  of  $A$ .<sup>21</sup> Deviations in actions are punished by moving to  $B_{11}^N$  or  $B_{21}^N$ , depending on which of the players, 1 or 2, has deviated. Deviations in proposals are similarly punished by moving to  $D_1^N$  or  $D_2^N$ .

To verify that the norm is stable, notice that whenever 1 gets to make a proposal, his payoff is at least  $\pi_1(D_1)$ . Since  $D_1$  is on the Pareto frontier, any proposal giving 1 strictly more than  $\pi_1(D_1)$  must give 2 less than  $\pi_2(D_1)$ . This means that  $D_1^N$  can serve as a punishment in case 1 makes such a proposal.

**Remaining cases:**  $v_1 \leq \pi_1(P_2)$  and/or  $v_2 \leq \pi_2(P_1)$

The construction of the norm is almost identical to the previous case. The only difficulty is that the difference  $\pi_1(A_1) - v_1$  is now bounded below away from zero, whereas it was previously

<sup>21</sup>One can modify the implementation of payoffs  $A$ ,  $A_i$  in a way that the continuation payoff will eventually converge to a Pareto-efficient point. Thus, if players switch to a Pareto-inefficient element in the stable norm after a deviation, they will eventually forgive and forget, rather than being stuck at an inefficient element forever.

of order  $\sqrt{\varepsilon}$ . This may lead to situations in which the points  $C_{1j}$  constructed above are no longer feasible and/or may give 2 a payoff lower than  $\pi_2(A_2)$ . This difficulty is easily addressed by adding, for each  $j$ , a new point  $E_{1j}$  lying on the segment  $[A_1B_{1j}]$ —and thus also on the line  $(A_1C_{1j})$ —such that if player 2 gets a chance to make a proposal, or if nobody does, continuation payoffs jump to the point  $E_{1j}$ . The promise keeping condition (4) becomes

$$\pi_1(B_{1j}) = \left(1 - \frac{p}{2}\right) \pi_1(E_{1j}) + \frac{p}{2} \pi_1(C_{1j}) \quad (7)$$

By choosing  $E_{1j}$  close enough to  $B_{1j}$ , one can make the point  $C_{1j}$  within a distance  $\sqrt{\varepsilon}$  of  $B_{1j}$  and, hence, of  $A_1$ . This guarantees that  $C_{1j}$  is feasible and does not drop below  $\pi_2(A_2)$ , so that the rest of the argument for the first case can be applied. Finally, whenever the element  $E_{1j}^{\mathcal{N}}$  must be implemented in the next period, we use the public randomization device to implement it as a probabilistic mixture between  $A_1^{\mathcal{N}}$  and  $C_{1j}^{\mathcal{N}}$ .

### 4.3 Relation to the existing literature

When renegotiation is viewed as a strategic interaction, stable norms may contain Pareto-ranked equilibria. This happens when the norm dissuades Pareto improving proposals by punishing the proposer and rewarding the rejector above and beyond the higher payoff offered to him by the proposer. This idea also underlies the results of Santos (2000), who considers players bargaining over which equilibrium to play in a one-shot game, as well as Miller and Watson’s (2013) Theorem 1, which shows that renegotiation has no restrictive power when it must only obey their “Internal Agreement Consistency” Axiom. To understand the difference between that theorem and ours, a key observation is that their argument requires unbounded transfers: to punish a proposer, say player 1, one requires him to make a very high transfer to 2 in the next period. If the weight of a single period is  $\varepsilon$ , the transfer must be of order  $\frac{1}{\varepsilon}$  (hence, the necessity of assuming unbounded transfers, as  $\varepsilon$  goes to zero). These large transfers permit 1’s continuation value to immediately jump discretely from some punishment payoff  $v_1^0$  to a higher continuation value  $v_1$ , which is easy to implement. Moreover, the transfer stage takes place, in each period, before the action stage (and, in particular, is distinct from it), if 1 deviates by making a lower or no transfer, it suffices to have him minmaxed by the other player and reset the continuation value to  $v_1^0$  for the next period to punish this deviation.

When stage-game payoffs are bounded, as in our setting, the continuation value of a player cannot jump by an  $\varepsilon$ -independent amount. The equilibrium construction must thus keep track of continuation values and make sure that these continuation values are implementable at all stages and following all deviations. In the absence of a separate transfer stage, moreover, if player 1 deviates in action when implementing  $v_1^0$ , his continuation value must fall below  $v_1^0$ . Implementing

this lower value may be difficult or even impossible. In fact, it is this impossibility which creates new restrictions on the set of renegotiation-proof payoffs and destroys the folk theorem obtained in Miller and Watson’s Theorem 1.

Both Santos (2000) and Miller and Watson (2013) consider a further restriction, which is that the continuation of the game, in case of a disagreement, be independent of the identity of the proposer and of the nature of the proposals. This restriction guarantees a higher level of efficiency. The consequences for our model of such a refinement are studied in Section 6.

#### 4.4 Comparative statics: bargaining frictions and discounting

In standard repeated games with public randomization, it is well known that the set of implementable payoffs increases with the discount factor. However, the same property does not hold with renegotiation. For example, let the stage game have an inefficient Nash equilibrium, lying outside of the renegotiation-proof set predicted by Theorem 1, for a fixed value of  $q = \frac{1}{2}$ . When  $\varepsilon$  is small, Theorem 1 implies that this Nash equilibrium payoff, along with an open neighborhood around it, is not renegotiation-proof. However as  $\varepsilon$  goes to 1, there is an equilibrium in which players follow this Nash equilibrium in the first period (before possibly renegotiating to a Pareto superior continuation). Since the current-period weight is arbitrarily close to 1, players’ payoffs are arbitrarily close to the inefficient Nash equilibrium’s payoffs, which was impossible with a small enough value of  $\varepsilon$ .

Although discount-factor monotonicity is violated in the presence of renegotiation, a different kind of monotonicity arises here, with respect to negotiation frictions: the more opportunities players have to renegotiate their continuation equilibrium, the smaller the renegotiation-proof set. This result holds at all discount factor levels, as proved in Appendix F.

PROPOSITION 1 *For any fixed  $\varepsilon \in (0, 1)$ , the set of renegotiation-proof payoffs is decreasing in  $q$ .*

## 5 Equivalent notions of stability

Previous sections derived conditions characterizing renegotiation-proof payoffs based on a particular equilibrium refinement. When the renegotiation frictions become negligible, these two conditions converge to an orthant with coordinates given by either minmax or Pareto points  $P_1, P_2$ . In this section, we show that the set of renegotiation-proof payoffs is the same if we consider alternative concepts of renegotiation. We start by restricting the set of players’ possible deviations from the norm: it is possible only if they propose an innovation which is “credible” with respect to the norm. Second, we also consider a set-theoretic definition of the renegotiation-proof set, and show that it is payoff-equivalent to the strategic notion analyzed in earlier sections.

## 5.1 Credible innovations

Our definition of stability allowed players to implement any Pareto-improving continuation equilibrium that is proposed and accepted at some point of the game. When players are used to a given norm  $\mathcal{N}$ , however, one may question why they would take such a proposal seriously. As it turns, both the necessary and the sufficient conditions of Theorem 1 remain identical if one restricts proposals to a much smaller subset.

*DEFINITION 4* Given a norm  $\mathcal{N}$ , an  $\mathcal{N}$ -credible (or just “credible”, when there is no confusion) proposal is an SPE such that any off-equilibrium play (action, proposal, or acceptance decision) is followed by a continuation in  $\mathcal{N}$  at the next stage;

A credible proposal is thus an SPE which can be supported under the assumption that any deviation will be followed by a reversal to the norm. For example, if a norm includes a harsh punishment equilibrium for both players, then it supports many credible equilibria, any deviation of which triggers a reversal to the norm and, more precisely, to the punishment equilibrium.

*DEFINITION 5* A norm  $\mathcal{N}$  is stable with respect to credible innovations if it satisfies the refinement of Definition 2 for all  $\mathcal{N}$ -credible proposals.

Definition 5 is clearly more permissive than Definition 2, because it imposes the refinement over a smaller set of proposals. However, we get the following result.

*THEOREM 2* All the conclusions of Theorem 1 continue to hold if the norms sustaining renegotiation-proof payoffs are only required to be stable with respect to credible innovations.

The proof is straightforward. Because this second definition of stability is more permissive, our construction for the sufficiency condition also works in this case. Moreover, it is easy to check that the proposals considered to derive the necessary conditions are credible.

## 5.2 Set-theoretic definition

The norms that we defined earlier were **open** in the sense that they allowed players to depart from the norm in case an off-equilibrium proposal is made and accepted. This openness is necessary for stability since we want to treat such proposals seriously. It is possible however to bring our work closer to the set-theoretic approach that was studied in the late eighties and early nineties. In fact, we show in this section that our earlier analysis can be entirely reexpressed in terms of set-theoretic definitions, yielding exactly the same characterization.

In this section, players do not actually take proposals outside of the norm seriously. They consider their norm as the only possible outcomes. We thus begin by “closing” our definition of a norm:

**DEFINITION 6** *A subset  $\mathcal{N}$  of  $\mathcal{S}$  is a **closed norm** if for any  $m \in \mathcal{N}$ ,  $m_{+1}[m](z, \mathbf{a}, i, \mu_i, D_{-i}) \in \mathcal{N}$ .*

The only difference with the earlier definition of a norm is that continuations belong to the norm, including when an off-equilibrium proposal is made and accepted. Next, our earlier definition of stability is translated into set-theoretic terms. To keep in line with the previous section, we state the definition for credible proposals. It should be noted however that the same definition dropping “credible” yields the same set.

**DEFINITION 7** *A closed norm  $\mathcal{N}$  is **stable** if the following holds: consider any SPE of  $\mathcal{N}$  and history at which  $i$  gets a chance to make a proposal and let  $\hat{U}_i$  denote  $i$ 's continuation payoff. Then, for any credible proposal with payoff vector  $U$  which gives  $i$  a payoff  $\pi_i(U) > \hat{U}_i$ , there exists a payoff vector  $U'$  in the norm such that  $\pi_{-i}(U') \geq \pi_{-i}(U)$  and  $\pi_i(U') \leq \hat{U}_i$ .*

**THEOREM 3**

1. *For any closed norm  $\mathcal{N}^c$ , there exists an open norm  $\mathcal{N}^o$  which has the same payoff set, and vice versa.*
2. *For any stable closed norm  $\mathcal{N}^c$ , there exists a stable open norm  $\mathcal{N}^o$  which has the same payoff set, and vice versa.*

## 6 Renegotiation-proof equilibria in the absence of proposer-specific punishments

The analysis thus far has explored a negotiation environment in which deviating proposals may trigger continuations that incentivize receivers to reject the proposals and dissuade proposers from making them in the first place. As we showed, this mechanism confers credibility to punishment phases which cannot be easily renegotiated away. This feature of our model arises as a natural consequence of describing explicitly the strategic aspect of the negotiation process between players. One may, however, wonder what equilibria may be sustained when proposers cannot be punished. Such a scenario occurs when one restricts the continuation play after a rejection to be independent of the identity of the proposer and the nature of the proposal. This restriction has been modeled elsewhere<sup>22</sup> as a “No-Fault Disagreement” (NFD) axiom, which requires the continuation after

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<sup>22</sup>See Santos (2000) and Miller-Watson (2013). A similar idea appears in Farrell (1987), Rabin (1994), and Arvan, Cabral, Santos (1999) for the case of simultaneous announcements.

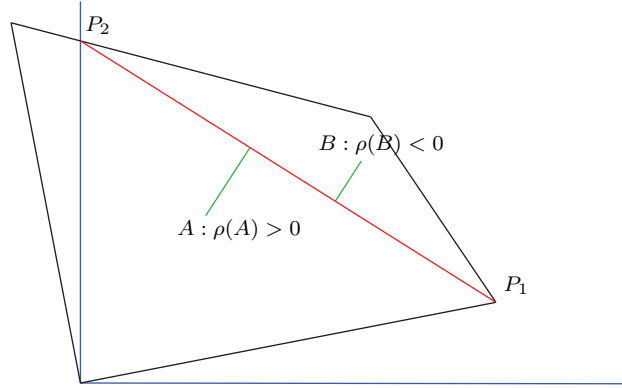


Figure 5: Signed distance from  $(P_1P_2)$

rejection to coincide with the default continuation in case no proposal is made. For completeness, we describe how the results are modified in this case.

DEFINITION 8 *A stable norm  $\mathcal{N}$  is forgiving if for any SPE  $m$  in  $\mathcal{N}$ , for any  $i$  and  $\mu_i$ ,  $m_{+1}[m](z, \mathbf{a}, i, \mu_i, 0) = m_{+1}[m](z, \mathbf{a}, i, \emptyset, 0)$ .*

Our concepts of renegotiation-proofness are modified as follows. A payoff vector  $A$  is said to be *forgivingly  $q$ -renegotiation-proof* if for all  $\varepsilon$  small enough, there is a forgiving stable norm containing an equilibrium which expected payoff is equal to  $A$ .  $A$  is *forgivingly renegotiation-proof* if it is *forgivingly  $q$ -renegotiation-proof* for all  $q$ 's large enough.

The main result in this case is given by the novel necessary conditions, which are much more restrictive than before: the continuation payoffs must lie within a distance  $O(\frac{1}{q})$  of the convex hull of the (individually-rational) Pareto frontier. More precisely, for each feasible payoff vector  $A$ , let  $\rho(A)$  denote the signed distance from the line  $(P_1P_2)$ , counted positively if  $A$  lies below  $(P_1P_2)$ , and negatively otherwise, as indicated by Figure 5).

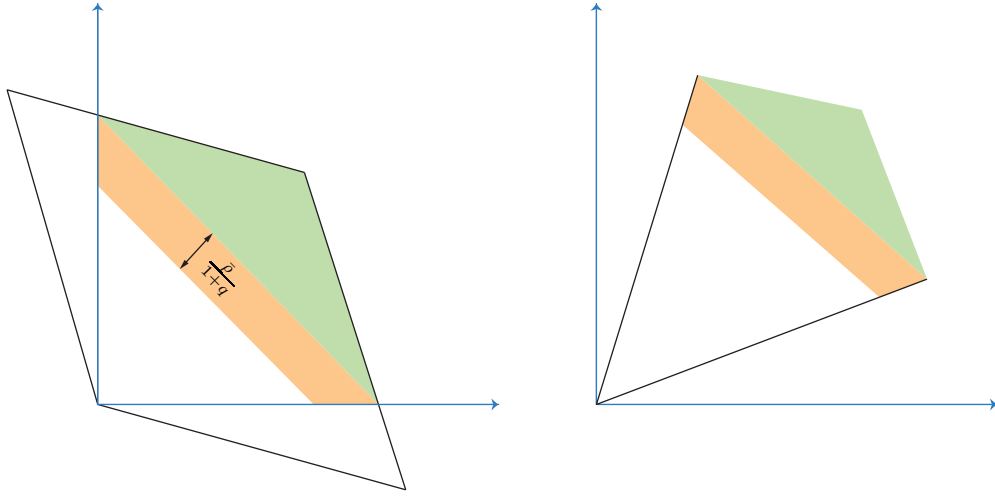
Let  $\bar{\rho}$  denote the maximum value of  $\rho$  among all feasible payoff vectors.

THEOREM 4 *If  $A$  is forgivingly  $q$ -renegotiation-proof, then  $\rho(A) \leq \frac{\bar{\rho}}{1+q}$ .*

One may also wonder whether all the feasible payoffs lying above  $(P_1, P_2)$  can be achieved in this case. The next result provides a positive answer which is independent of negotiation frictions. To establish this result, we slightly modify the definition of stability, as follows: deviating proposal which is accepted needs to be implemented only if it improves the proposer's payoff by more than a constant  $\eta > 0$ , arbitrarily small but fixed, over his equilibrium payoff without the deviation.<sup>23</sup>

<sup>23</sup>Using the refinement in Theorem 4 affects the corresponding bound by a factor  $\eta$ .





DEFINITION 9 A norm  $\mathcal{N}$  is  $\eta$ -stable if the following holds: consider any SPE of  $\mathcal{N}$  and history at which  $i$  gets a chance to make a proposal and let  $\hat{U}_i$  denote  $i$ 's continuation payoff. Then, whenever  $i$  proposes a plan  $\mu \in \mathcal{S}$  giving him at least  $\hat{U}_i + \eta$ , and  $-i$  accepts it,  $\mu$  is implemented.

THEOREM 5 Assuming  $\eta$ -stability, any payoff vector  $A$  strictly above the segment  $(P_1, P_2)$  is forgivingly renegotiation-proof.

The role of  $\eta$ , which is to prevent off-equilibrium proposals near the most extreme continuation payoffs in the norm, is pointed out in the proof.

## 7 Three or more players

The analysis so far has focused on two players, which is a common restriction in the study of renegotiation in repeated games (e.g., Farrell and Maskin (1989), Benoît and Krishna (1993), and Santos (2000)).<sup>24</sup> Extending the analysis to more players raises new conceptual issues. Can proposals be targeted toward a subset of individuals? What happens if only a subset of the players accepts the proposal?

This section explores some of these issues, allowing for an arbitrary number,  $n \geq 3$ , of players. After a player has made a proposal, other players vote on accepting the proposal. The continuation payoff may *a priori* depend on the identity of the players who voted for the proposal. We consider several dependence structures, which vary in their flexibility. Our analysis is mainly focused on environments in which players vote simultaneously over the acceptance decision. We briefly discuss the case of sequential acceptance decisions afterwards.

<sup>24</sup>Abreu et al. (1993) focus instead on symmetric equilibria.

The setting is identical to the two-player case, except as noted below. At the proposal stage, each player  $i$  has the same probability  $\frac{p}{n}$  ( $p < 1$ ) of being chosen to propose a new plan. This player may choose to conceal his proposal opportunity. If  $i$  makes a proposal, other players decide on whether to accept it, resulting in a vector of acceptance votes  $D_{-n} \in \{0, 1\}^{n-1}$ .<sup>25</sup> Given a plan  $m$  at the beginning of a period, the continuation plan at the next period,  $m_{+1} = m_{+1}[m](z, \mathbf{a}, i, \mu_i, D_{-i}) \in \mathcal{M}$  is a function of  $z$ ,  $\mathbf{a}$ ,  $i$ ,  $\mu_i$ , and  $D_{-i}$ , where  $i$  indicates the identity of the last proposer.

Norm stability is extended as follows.

**DEFINITION 10** *A subset  $\mathcal{N}$  of  $\mathcal{S}$  is a **norm** if, for any  $m \in \mathcal{N}$  such that  $\mu_i \in \bar{\mu}_i[m](z, \mathbf{a})$  or  $D_{-i} \neq \{1\}^{n-1}$ ,  $m_{+1}[m](z, \mathbf{a}, i, \mu_i, D_{-i}) \in \mathcal{N}$ ;*

**DEFINITION 11** *A norm  $\mathcal{N}$  is **stable** if for any SPE of  $\mathcal{N}$ , whenever  $i$  proposes an equilibrium  $\mu \in \mathcal{S}$  and all other players accept it,  $\mu$  is implemented.*

Stability specifies the continuation payoff when *everyone* accepts the proposal. One must also define players' continuation payoffs following the rejection of the proposal. We consider three specifications. The most flexible one allows continuation payoffs to depend arbitrarily on the identity of the players who accepted and rejected the proposal. The other two specifications entail two continuation payoffs: the first one arises if all players agree on a proposal; the second one arises if at least one player rejects the proposal. These specifications differ with regard to the continuation payoff in case of a rejection: can the proposer be punished if his proposal is rejected, or does the game proceed as if no proposal had taken place, as already prescribed by the No-Fault Disagreement axiom studied in Section 6?

These specifications are captured by the following concepts.

**DEFINITION 12** *A stable norm  $\mathcal{N}$  is **simple** if, for any  $m \in \mathcal{N}$  and  $D_{-i}, D'_{-i} \neq \{1\}^{n-1}$ ,*

$$m_{+1}[m](z, \mathbf{a}, i, \mu_i, D_{-i}) = m_{+1}[m](z, \mathbf{a}, i, \mu_i, D'_{-i}).$$

**DEFINITION 13** *A simple norm  $\mathcal{N}$  is **forgiving** if, for any SPE  $m \in \mathcal{N}$ ,  $m_{+1}[m](z, \mathbf{a}, i, \mu_i, D_{-i} \neq \{1\}^{n-1}) = m_{+1}[m](z, \mathbf{a}, i, \emptyset, \{0\}^{n-1})$ , for any  $i, \mu_i$ .*

The definitions of (forgivingly)  $q$ -renegotiation-proof payoffs and (forgivingly) renegotiation-proof payoffs are identical to those of the two-player case.

Throughout the analysis, we assume that the individually-rational payoff set has a full dimension as in Fudenberg and Maskin (1986).

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<sup>25</sup>Similar to two-player case, if no proposal is made, the identity of a proposer is arbitrarily chosen, and the null proposal is rejected by everyone else.

## 7.1 Voter-dependent continuations

Suppose, first, that continuation payoffs can depend arbitrarily on the voting decision of each player—except if everyone agrees on a proposal, in which case stability dictates that the proposal is implemented. With this high degree of flexibility, norms may be constructed so that all negotiation proposals are dissuaded and the Folk Theorem obtains.

**THEOREM 6** *For any feasible payoff vector  $\pi$  with  $\pi_i > \underline{v}_i$  for all  $i$ ,  $\pi$  is renegotiation-proof.*

To understand this result, we recall that in the underlying repeated game without negotiation, any strictly individually-rational payoff vector can be implemented for  $\varepsilon$  small enough by minmaxing any player  $i$  who deviates in actions, and switch to minmaxing any player  $j \neq i$  who deviates when minmaxing player  $i$ . The same idea can be applied when negotiation is possible, by deterring it as follows: if a player,  $i$ , proposes another continuation, everyone else is prescribed to reject the proposal and to start minmaxing player  $i$ . If another player,  $j$ , deviates from the prescribed rejection by accepting  $i$ 's proposal, and all other players reject it, then players are prescribed to minmax  $j$  instead of  $i$ . If two or more players accept  $i$ 's proposal, it is implemented, which guarantees that the norm satisfies our stability refinement. This prescription guarantees that it is always suboptimal for a player to unilaterally accept a proposal and, consequently, that it is also suboptimal to make any proposal. Unless some additional restrictions are imposed on the continuation payoffs, allowing for the possibility of renegotiation with three or more players thus has no more predictive power on the set of equilibria and payoffs than the standard Folk Theorem.

## 7.2 Simple Norms

Suppose now that only two continuations may follow each proposal, depending on whether all players have agreed to it. As usual with voting games, we eliminate equilibria involving weakly dominated strategies.

**ASSUMPTION 1** *A player votes in favor of the proposal if it gives him a strictly higher payoff than its continuation payoff in case of a rejection.*

Let  $\mathcal{P}$  denote the Pareto frontier of the feasible payoffs in the stage game and, for each  $i$ ,  $P_{-i}$  denote any individually-rational payoff vector of  $\mathcal{P}$  which minimizes  $i$ 's payoff.

The key question, for characterizing stable norms, is this: if player  $i$  makes an unprescribed proposal, what is the worst “credible” punishment for him? Suppose that player  $i$  proposes an SPE of the renegotiated game, with corresponding payoff vector  $C$ , and let  $\mathcal{V}$  denote the set of achievable payoff vectors in our candidate norm,  $\mathcal{N}$ . If  $\mathcal{N}$  is stable,  $C$  will be implemented if all players accept  $i$ 's proposal. If anyone rejects the proposal, norm simplicity implies that there is a

single payoff vector *in the norm*,  $D(C)$ , which will be realized. If  $D(C)$  gives  $\pi_j(C)$  or more to at least one player  $j \neq i$ , that player will refuse the implementation of  $C$ , and  $D(C)^N$  will be played.

Following any proposal with a payoff  $C$  by player  $i$ , the worst punishment in  $\mathcal{V}$  for player  $i$  minimizes  $i$ 's utility over the set:<sup>26</sup>

$$\mathcal{D}(C, \mathcal{V}) = \{D(C) \in \mathcal{V} : \exists j \neq i : \pi_j(D(C)) \geq \pi_j(C)\}.$$

Let  $\underline{\pi}_i(C, \mathcal{V})$  denote  $i$ 's utility under this worst punishment.

Viewing  $\underline{\pi}_i(C, \mathcal{V})$  as a function of  $C$ , one can then find the proposal with a continuation  $C(\mathcal{V})$  which maximizes  $i$ 's payoff at the worst punishment:  $C(\mathcal{V}) = \arg \max_C \{\underline{\pi}_i(C, \mathcal{V})\}$ , and the corresponding payoff,  $\underline{\pi}_i(\mathcal{V})$ , for  $i$ .

One should think of the payoff  $C(\mathcal{V})$  as follows. The most efficient way to prevent player  $i$  from making a non-prescribed proposal is by implementing his worst punishment. Anticipating this, if player  $i$  deviates from his prescription, he may as well choose the optimal proposal, which gives the payoff  $C(\mathcal{V})$ . In the appendix, we show that  $C(\mathcal{V})$  can without loss of generality be taken to lie on the relative weak Pareto frontier of the set  $\mathcal{V}$ , which we denote  $\mathcal{P}(\mathcal{V})$ .

LEMMA 1  $C(\mathcal{V}) \in \mathcal{P}(\mathcal{V})$

These observations lead to the following sequential construction. We start from the set  $\mathcal{F}$  of strictly individually-rational payoffs in the stage game, i.e., what would be implementable without renegotiation. We consider the minimal payoffs  $\underline{\pi}_i(\mathcal{F})$ ,  $i \in \{1, \dots, n\}$  which any player  $i$  could guarantee himself if having a chance to make a proposal and the payoffs sustained by the norm were in  $\mathcal{F}$ . We will build *two* decreasing sequences of sets, starting from  $\mathcal{F}$ , which will generate separate necessary and sufficient conditions for a payoff to be renegotiation-proof.

To derive sufficient conditions, the  $k^{\text{th}}$  set in the sequence,  $\mathcal{F}_s^k$ , is reduced by removing all the payoffs below  $\underline{\pi}_i(\mathcal{F}_s^k)$ , to form the  $k+1$ -th set in the sequence, starting with  $\mathcal{F}_s^0 = \mathcal{F}$ . We will show that this process converges to a stable set which defines sufficient conditions.

To derive necessary conditions, let  $\pi_{\min,i}(\mathcal{F}_n^k)$  denote the lowest expected payoff for player  $i$  at the beginning of a period, among all payoff vectors in  $\mathcal{F}_n^k$ . This value is lower than the continuation payoff  $\underline{\pi}_i(\mathcal{F}_n^k)$  that  $i$  can guarantee himself when he gets a chance to make a proposal. We have

$$\pi_{\min,i}(\mathcal{F}_n^k) \geq \varepsilon \underline{v}_i + (1 - \varepsilon)[q\varepsilon \underline{\pi}_i(\mathcal{F}_n^k) + (1 - q\varepsilon)\pi_{\min,i}(\mathcal{F}_n^k)]$$

Indeed, as in the two-player case,  $i$  gets at least  $\underline{v}_i$  as his current payoff, and can guarantee himself  $\underline{\pi}_i(\mathcal{F}_n^k)$  if he has a chance to make a proposal. As  $\varepsilon$  goes to 0, one can express the value  $\pi_{\min,i}(\mathcal{F}_n^k)$  as:

$$\pi_{\min,i}(\mathcal{F}_n^k) \geq \frac{\underline{v}_i + q\underline{\pi}_i(\mathcal{F}_n^k)}{1 + q}. \quad (8)$$

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<sup>26</sup>For the existence of a *worst* punishment, the set  $\mathcal{V}$  needs to be closed. Our construction will satisfy this condition.

At each step the set  $\mathcal{F}_n^k$  is being reduced by removing the payoffs below (8). Iterations of this procedure converge to a steady set, as we show in the Appendix.

**PROPOSITION 2** *Both procedures converge to steady sets.*

We denote the limiting sets by  $\mathcal{V}_s$  and  $\mathcal{V}_n$ . They both are positive orthants, whose vertices give lower bounds on players' payoffs (calculated at the beginning of period) under both procedures, and are denoted  $\pi_{min,i}(\mathcal{V}_s)$  and  $\pi_{min,i}(\mathcal{V}_n)$ , for any player  $i$ . By construction, expression (8) holds as an equality for  $\mathcal{V}_n$ :

$$\pi_{min,i}(\mathcal{V}_n) = \frac{v_i + q\underline{\pi}_i(\mathcal{V}_n)}{1 + q} \quad (9)$$

Similarly, we have  $\pi_{min,i}(\mathcal{V}_s) = \underline{\pi}_i(\mathcal{V}_s)$ .

We can now state the main result of this section. Let  $\mathcal{R}$  denote the open positive orthant whose vertex is the vector  $(\pi_i(P_{-i}))_{i=1}^n$ . In the two-player case, this set characterized the sufficient conditions for renegotiation-proofness. With  $n > 2$  players, we show that  $\mathcal{R}$  still consists of renegotiation-proof vectors, though it might not include all of them. The theorem is formulated for the case where Pareto frontier supports for each player a non-zero range of payoffs.

**THEOREM 7** *Any renegotiation-proof payoff lies in  $\mathcal{V}_n$ , and generically any payoff in the interior of  $\mathcal{V}_s$  is renegotiation-proof. Moreover, any payoff in the interior of  $\mathcal{R}$  is renegotiation-proof.*

In case of feasible set having the unique Pareto-efficient point, it is the only renegotiation-proof payoff.<sup>27</sup>

In the two-player case, the necessary and sufficient conditions became arbitrary tight as renegotiation frictions vanished. The same is true in this more general environment, as shown in Appendix G.5.

**PROPOSITION 3** *The sets  $\mathcal{V}_s$  and  $\mathcal{V}_n$  converge to each other as  $q$  goes to infinity.*

If the players are making responses to proposals sequentially, then one has the same result<sup>28</sup>. Each proposal has only two continuations. If the continuation in case of rejection benefits at least one responder, he rejects the proposal in any extended game. Otherwise, from backward induction, each player votes for the proposal if it gives higher payoff than rejection - same as in simultaneous voting.

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<sup>27</sup>In the non-generic case of several Pareto points, each giving the same payoff to one of players, in case of at least two other players having different payoffs on Pareto frontier, the renegotiation-proof payoffs always form a non-empty full-dimensional orthant. In case Pareto frontier gives the same payoff to all but one player, there is a best Pareto point, which is renegotiation-proof.

<sup>28</sup>We actually got the equivalence between sequential and simultaneous votes when assuming that player votes for the proposal if it gives him higher payoff compared to rejection.

### 7.2.1 Sequential voting in case of no restrictions

Sequential voting permits more than two continuation payoffs, depending on the sequence of acceptance decisions of the players. The resulting stable norm is qualitatively similar to the earlier analysis with only two continuations, and it is more permissive.

PROPOSITION 4 *Suppose that each proposal is decided by sequential voting. Then, analogous constructions to the two-continuation case yield sufficient and necessary conditions characterized by upper orthants. Moreover, each of these sets is larger than the corresponding set obtained with only two continuations.*

Sequential voting with many continuations thus provides more predictive power than simultaneous voting, but less predictive power than the simultaneous-voting specification with only two continuations.

### 7.3 Three or more players without proposer-specific punishment

Finally, consider the most restrictive case of a simple norm that is also forgiving, as defined in the two-player case. The necessary conditions resemble the two-player case. Consider the set of individually-rational Pareto-efficient payoffs  $\mathcal{P}'$ , and consider the convex hull of this set,  $Co(\mathcal{P}')$ . Then one has:

PROPOSITION 5 *If  $A$  is forgivingly  $q$ -renegotiation-proof, the distance from  $A$  to  $Co(\mathcal{P}')$  is bounded above by a decreasing function of  $q$ , which converges to 0 as  $q$  becomes arbitrarily large.*

The proof closely mirrors the argument used for the two-player case and is only sketched here. Suppose that  $A$  is the point of the norm which has the largest distance from  $Co(\mathcal{P}')$  and that  $A$  lies “too far” down away from  $Co(\mathcal{P}')$ . Whenever a player gets to make a proposal—which happens with probability proportional to  $q$ —he proposes a Pareto point (or close to it). Moreover, the continuation payoff  $A'$  which follows if the proposal is rejected cannot lie farther away from  $Co(\mathcal{P}')$  than  $A$  does. Combining this puts a bound on  $A$ 's distance to  $Co(\mathcal{P}')$ , which vanishes as  $q$  gets large.

We conclude this section with sufficient conditions, whose derivation is more involved and described in Appendix G.7.

THEOREM 8 *Assuming  $\eta$ -stability, any point  $A$  in the set  $Co(\mathcal{P}')$  lying strictly above the minmax is forgivingly renegotiation-proof.*

## 8 Discussion

Understanding and tractably modeling renegotiation in repeated games has been a longstanding challenge. Nevertheless, the protocol and concepts studied in the paper lead to a particularly simple characterization of stable norms and renegotiation-proof equilibria. We hope that our characterization can serve as a useful benchmark for applied economists who need to incorporate renegotiation in their models.

Renegotiation in repeated games raises two challenges: defining “credible” proposals and prescribing what happens when a proposal is rejected. Cooperative concepts have focused on the first aspect, implicitly assuming that a rejection would be bad for both players, and faced important conceptual difficulties. Internal notions of credibility (consider renegotiation only within the norm) were too weak, ignoring natural proposals—such as proposals which Pareto dominate all equilibria in a given “stable” norm—while external notions proved overly restrictive.

A contribution of this paper is to show that this first challenge becomes moot when the second one is taken into account: the stability concept is essentially independent of any reasonable notion of credibility, at least for the case of two players. This independence results for two players is already important in itself because many games concern only two players (and, in fact, many existing works on renegotiation in repeated games have focused on two players).

Even with more than two players, the present analysis suggests that under natural specifications the set of renegotiation-proof payoffs has useful analytical (convergence of necessary and sufficient conditions) and geometric properties (upper orthant characterization). Although the present paper makes progress even in this more complicated case, some important issues need to be explored. In particular, what happens if a player makes a proposal to a subset of the players? How does this impact the equilibrium played by all players? This question arises naturally when players can be divided into relatively homogeneous groups, which should intuitively be more easily able to coordinate their actions. Understanding how renegotiation shapes equilibrium outcomes in environments with segmented groups seems particularly helpful for applied work in political economy and other areas.

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## A Restricting plans

We refine steps b1), b2), and c) as follows. There exist constants  $k$  and  $k'$  with  $k' > 3k > 0$ <sup>29</sup> such that

- b1) for any  $z$ ,  $\mathbf{a}$ , and  $i$ , the support of  $\mu_i[m](z, \mathbf{a})$  contains at most  $k$  proposals;
- b2)  $-i$  is prescribed to surely accept (reject) any on-path (off-path) proposal;
- c) for any  $z$  and  $\mathbf{a}$ , the set  $\mathcal{M}^{+1}[m](z, \mathbf{a}, \mu)$  of possible continuation plans has at most  $k'$  elements and contains, in this order: the current proposal,  $\mu$ ; a default option (used if no proposal was made); the set of possible on-path proposals for each player (at most  $2k$  elements); a finite number of alternative continuation plans, pre-specified by  $m$  as a function of  $z$  and  $\mathbf{a}$ , which may be used as punishments against deviating proposers.

The condition  $k' > 3k$  guarantees that the list of continuation plans can indeed be included in  $\mathcal{M}^{+1}[m](z, \mathbf{a}, \mu)$  as long as there are no more than  $k' - 2k - 2 > 0$  alternative continuations plans.

To specify the continuation plan selected for the next period, we partition the proposal stage according to i) who (if any) got to make a proposal, ii) in the event that a proposal was made, whether the proposal was prescribed (out of at most  $k$  possibilities) or off-path, iii) whether the proposal was accepted or rejected. As is readily checked, this partition consists of at most  $\hat{k} = 1 + 2 \times (k + 1) \times 2$  elements. In addition, we also compare the current proposal,  $\mu$  (or the default option in case of no proposal), to each of the (at most)  $k' - 1$  other feasible continuation plans. Specifically, we consider, for each player and alternative continuation, which of the current proposal or the alternative continuation plan gives the higher payoff to that player. Allowing three comparison outcomes ( $\{=, <, >\}$ ) for each player yields  $3^{2(k'-1)}$  combinations.

Let  $\mathcal{E}$  denote the set of possible *events* at the proposal stage, and  $\bar{k} = \hat{k} \times 3^{2(k'-1)}$  denote its cardinality. A *choice rule*  $g$  determines, for each event, the continuation plan—an element of  $\mathcal{M}^{+1}[m](z, \mathbf{a}, \mu)$ —for the next period. The choice rule  $g[m](z, \mathbf{a})$  is pre-specified by  $m$  as a function of  $z$  and  $\mathbf{a}$ ; and it is an element of a finite set  $\mathcal{G}$  with a cardinality  $(k')^{\bar{k}}$ .

In summary, each plan prescribes, for each realization of  $z$  and  $\mathbf{a}$ , a successor (a plan for the following period) as a function of the choice rule (an element from  $\mathcal{G}$ ) and of the event which occurred. Letting  $\mathcal{A} = \mathcal{A}_1 \times \mathcal{A}_2$  denote the (finite) set of action profiles, this stage adds  $\mathcal{M}_{prop} = (k')^{\bar{k}|\mathcal{G}|}[0,1]^{\times \mathcal{A}}$  elements to the prescription, which has the cardinality of the power set of the continuum.<sup>30</sup>

For any set  $M$ , let  $T(M)$  denote the set of plans obtained from the recursive construction above, choosing proposals from  $M$  and continuation plans from  $M$ .  $T(M)$  is the Cartesian product of the prescriptions obtained at each step of the construction. We now show that there exist sets  $M$ 's for which  $T(M)$  has the same cardinality as  $M$ . These sets have cardinality  $\beth_2 = |2^{\mathbb{C}}|$ , i.e., the cardinality of the power set of the continuum, which is also the cardinality of  $\mathbb{R}^{\mathbb{R}}$ —the set of all real-valued functions over  $\mathbb{R}$ —see, e.g., Forster (1995).<sup>31</sup>

**PROPOSITION 6** *If  $M$  has cardinality  $|2^{\mathbb{C}}|$ , then so does  $T(M)$ .*

This implies that any set  $M$  with cardinality  $|2^{\mathbb{C}}|$  is in bijection with  $T(M)$ .<sup>32</sup> The set  $\mathcal{M}$  of plans is then structured as follows: let  $\phi$  denote the bijection between  $2^{\mathbb{C}}$  and  $T(2^{\mathbb{C}})$ . To any  $m \in 2^{\mathbb{C}}$ , we can associate the plan, defined recursively through  $\phi(m)$ , which specifies mixed strategies for each realization  $z$  of the

<sup>29</sup>The plan for  $n > 2$  players is described similarly, with increased constants  $k$  and  $k' > (n + 1)k > 0$ .

<sup>30</sup>Since  $[0, 1]$  has the cardinality of the continuum,  $\mathcal{M}_{prop}$  has the same cardinality as the set of functions which maps real numbers into a finite set, which is the same as the cardinality of  $2^{\mathbb{C}}$ , the power set of the continuum.

<sup>31</sup>Perhaps a simple way to see this is the chain  $|\mathbb{R}^{\mathbb{R}}| = |(2^{\aleph_0})^{2^{\aleph_0}}| = |2^{\aleph_0 2^{\aleph_0}}| = |2^{2^{\aleph_0}}| = |2^{\mathbb{R}}|$ , where the third inequality holds because  $\aleph \times \aleph$  is no larger than  $\aleph$ .

<sup>32</sup>By definition, two sets have the same cardinality if there exists a bijection between them. See, e.g., Kuratowski and Mostowski (1968).

randomization device, proposals as a function of  $z$  and of the observed action profile  $\mathbf{a}$ , and continuation plans which are elements of  $2^c$ . Thus, each element of  $\mathcal{M} = 2^c$  specifies a plan, which is defined recursively. *Proof.* Given the cardinality of  $\mathcal{M}$ , for each part from a) to c) in the prescription of a plan, one must find the cardinality added to the plan choice from that part. Since these parts are related by a Cartesian product and we are dealing with infinite sets, the cardinality of the Cartesian product coincides with the cardinality of the largest component of the product.<sup>33</sup> Part a) maps real-line outcomes (randomization device) into mixed strategies over  $\mathcal{A}$  for both players, which is a subset of  $\mathbb{R}^{|\mathcal{A}|}$ . Since  $\mathbb{R}^{\mathbb{R}}$  has the cardinality of  $2^c$ , this has the same cardinality as  $\mathcal{M}$ . Part b1) maps any outcome  $(z, \mathbf{a})$  and proposer into  $k$  possible plans in  $\mathcal{M}$  and a distribution over these plans, and thus has  $\mathcal{M}^{k \times [0,1] \times \mathcal{A}} \times \Delta_k(\mathbb{R})^{[0,1] \times \mathcal{A}}$  elements where  $\Delta_k(\mathbb{R})$  is the probability simplex in  $\mathbb{R}^k$ . Again, this set has the cardinality of  $\beth_2$  since  $\mathcal{M}^{\mathbb{R}}$  is equivalent to  $\mathbb{R}^{\mathbb{R}}$ . Part b2) does not add any cardinality since the prescription is to accept only prescribed proposals. Similarly, part b3) does not add any cardinality. Part c) adds the choice rule specification with a cardinality  $\mathcal{G}^{[0,1] \times \mathcal{A}} = 2^c$  and the set  $\mathcal{M}^{+1}$  of continuations with the cardinality of  $\mathcal{M}^{k'}$ , which is the same as  $\mathcal{M}$ 's, concluding the proof. ■

### Special cases used in the analysis

- 1) Babbling equilibria. These are the SPEs of the underlying repeated game, ignoring any renegotiation of continuation play. Babbling equilibria are captured by the choice rule which imposes the default continuation (the second element of  $\mathcal{M}^{+1}[m](z, a, \mu)$ ), no matter what happens during the proposal stage.
- 2) Stable norms. We can use choice rules such that if a proposal is accepted it is used as the continuation plan
- 3) Conditionally stable norms. Similarly, we can also define a choice rule such that if a proposal is accepted *and* gives everyone at least as much as the expected proposal of some player, or some default option, then it becomes the continuation plan.
- 4) Forgiving norms. The choice rule specifies that if a proposal is rejected, the default continuation is played regardless of the proposal and the identity of the proposer.

## B Refinement and Extensions

### Payoff equivalence to more permissive plans

The current description of a plan imposes restriction on the number of proposals for each player and the number of possible continuations; plus only the on-path proposals are prescribed to be accepted. That is, any equilibrium is *truthful*: any on-path proposal is always accepted and implemented. Those restrictions are necessary to guarantee the existence of a set of plans. However, even if one could find more permissive plans and a related stable norm  $\mathcal{N}$ , there would be a payoff-equivalent stable norm, using the same concept of stability, and built with the restrictive plans used in this paper. Indeed, let's consider any stable norm  $\mathcal{N}$  which uses plans with no restrictions. When player  $i$  gets a chance to propose, he can make any number of proposals in equilibrium. Given that player  $i$  got a chance to propose, one can find the related expected continuation payoff  $C$  and alter the equilibrium by prescribing player  $i$  to make only one proposal with payoff  $C$ . The altered equilibrium would prescribe every other player to accept the proposal, and to implement  $C$  regardless of the acceptance decision. The payoff  $C$  can be implemented using public randomization.

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<sup>33</sup>In particular, for  $\beth_0 = \mathbb{N}$ , we have  $|\mathbb{N} \times \mathbb{N}| = |\mathbb{N}|$ . The same is true for higher beth numbers, such as  $\beth_1 = \mathbb{R}$  and  $\beth_2 = 2^c$ .

If player  $i$  deviates and proposes a Pareto improvement over  $C$ , then everyone is prescribed to reject it. In case of  $n > 2$  rejecting the off-path proposal is an equilibrium. In case of  $n = 2$  players or original norm  $\mathcal{N}$  being simple there are only two continuations in  $\mathcal{N}$  following the off-path proposal. The new norm would prescribe to have the same rejection continuation as in the original norm,  $\mathcal{N}$ . Since the incentives to accept the proposal did not change, everyone is incentivized to reject an off-path proposal.

Finally, if one has enough continuations to incentivize the players to reject any off-path proposal in a simple norm, it is possible to use only  $n - 1$  of them. If one considers all possible off-path proposals of player  $i$ , for each such a proposal there is at least one player  $j \neq i$  who rejects it. This means that for any off-path proposal rejected by player  $j$  there is an element  $C_j^{\mathcal{N}}$  in the norm which gives  $j$  at least as high payoff as the off-path proposal. Any norm can be expanded to have all the payoffs in its closure by properly designing the prescriptions. Thus, one can consider the element  $C_j^{\mathcal{N}}$  with the highest payoff for  $j$  and use it as a punishment; and have at most  $n - 1$  continuations in total for the simple norm to be stable.

The equivalence to more permissive plans also holds if one does not impose the stability requirement. Player  $i$  is prescribed to propose  $C^{\mathcal{N}}$  and the continuation is prescribed to be  $C^{\mathcal{N}}$  regardless of acceptance decision. If player  $i$  makes another proposal, a default option is always played. This makes accepting  $C^{\mathcal{N}}$  and rejecting any other proposal an equilibrium.

### Asymmetric proposing probabilities and bargaining power

It is easy to extend the analysis to a protocol in which one of the players has a higher probability factor  $q_i$  of proposal than the other player. The sufficient conditions are unchanged in this setting, but the necessary conditions become tighter for the player whose proposal probability is higher, which translates into a higher minimal guaranteed payoff for that player, across all renegotiation-proof equilibria. To see this starkly suppose that  $v_1 < \pi_1(P_2)$  and  $v_2 < \pi_2(P_1)$  (configuration (a) in Figure 2), so that renegotiation potentially benefits both players, compared to the minmax payoffs, and consider the case in which 1 can make frequent proposals while 2 never gets a chance to make a proposal (i.e.,  $q_1$  is arbitrarily large while  $q_2 = 0$ ). Then, 2's minimal guaranteed renegotiation-proof payoff collapses to her minmax payoff, while 1 is guaranteed to get a payoff of at least  $\pi_1(P_2)$ . More generally, player  $i$ 's minimal payoff, given by (2), is calculated using the probability  $q_i$  that he gets an opportunity to make a proposal, and is independent of the other player's probability of getting that opportunity. As  $q_i$  increases, player  $i$ 's guaranteed continuation payoff increases as well, and vice versa.

## C Proof of Theorem 1 (Necessary Conditions)

The interesting case is when  $v_i < \pi_i(P_{-i})$ —otherwise, Theorem 1 predicts only that  $i$ 's payoff must be individually rational. We derive the necessary condition for player 1. The proof for player 2 is identical.

Let us thus assume that  $\pi_1(P_2) > v_1$  and suppose, by contradiction, that there is a point  $A$  such that  $\pi_1(A) < v_1 = v_1 + \frac{q}{2+q}(\pi_1(P_2) - v_1)$ , which is  $q$ -renegotiation-proof. This means that one can construct, for any  $\varepsilon$  small enough, a stable norm  $\mathcal{N}$  that contains  $A$  as one of its continuation payoffs, that is, an element  $A^{\mathcal{N}}$ .

In this case, we first build a specific equilibrium which yields the payoff vector  $P_2$ . The point  $P_2$  is easily shown to be the payoff vector of an equilibrium of the game, and hence has to be considered in the Definition 2 to check that  $A$  belongs to a stable norm.<sup>34</sup> However, we argue later in the paper (Section 5) that our necessary and sufficient conditions are unchanged if one restricts attention, in Definition 2, to *credible* proposals instead of arbitrary ones. A proposal is credible with respect to some norm  $\mathcal{N}$  if it is an SPE

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<sup>34</sup>By the Folk Theorem,  $P_2$  can be implemented by an SPE of the repeated game without renegotiation. By treating all proposals as cheap talk,  $P_2$  can then also be implemented as an SPE of the game with renegotiation.

such that any deviation (in action or proposal) leads to reversal to an equilibrium of the norm  $\mathcal{N}$ .<sup>35</sup> To this end, we now show that  $P_2$  is the payoff of a credible proposal. The SPE implementing  $P_2$  is constructed as follows: players are prescribed to play, in all periods, the pure action profile with payoff  $P_2$ , and to abstain from making any proposal. Any deviation, whether in action or in proposal, triggers the implementation of element  $A^{\mathcal{N}}$ . Clearly, player 2 cannot benefit from deviating as she is getting her highest possible payoff in the game. Moreover, the difference  $\pi_1(P_2) - \pi_1(A)$  is by assumption bounded below by  $\frac{2}{2+q}(\pi_1(P_2) - \underline{v}_1)$ , which is  $\varepsilon$ -independent. Therefore, 1 cannot benefit from deviating either: a deviation in action may create an immediate gain of order  $\varepsilon$ , but triggers a drop in continuation payoffs that is  $\varepsilon$ -independent and dominates the gain. A deviation in proposal triggers  $A^{\mathcal{N}}$ , which again is detrimental to 1. We thus have a constructed an equilibrium of the game with payoff  $P_2$ .

Let  $C_1$  denote 1's infimum payoff in  $\mathcal{N}$  when it is his turn to make a proposal. Since  $P_2$  is a possible proposal payoff, and since it Pareto dominates all payoffs with  $\pi_1 < \pi_1(P_2)$ ,  $A$  is a payoff of  $\mathcal{N}$  only if

$$\pi_1(P_2) \leq C_1$$

We now show that it is impossible. Let  $A_1 = \inf_{V \in \mathcal{V}(\mathcal{N})} \pi_1(V)$ ,  $B_1 = \inf_{W \in \mathcal{W}(\mathcal{N})} \pi_1(W)$ , and  $D_1 = \inf_{U \in \mathcal{U}(\mathcal{N})} \pi_1(U)$ , and consider a sequence  $\{V_m\} \in \mathcal{V}(\mathcal{N})$  such that  $\pi_1(V_m) \rightarrow_{m \rightarrow +\infty} A_1$ . For any  $V_m$  there is an action that implements it in the first period of the corresponding SPE. However, if player 1 deviates, he can guarantee himself an immediate payoff of at least  $\underline{v}_1$ , and the worst punishment for him after deviation gives him at least  $B_1$ . Therefore,  $\pi_1(V_m) \geq \varepsilon \underline{v}_1 + (1 - \varepsilon)B_1$ . Since this inequality holds for all  $V_m$  we obtain, taking the limit:

$$A_1 \geq \varepsilon \underline{v}_1 + (1 - \varepsilon)B_1 \tag{10}$$

Because any element of  $\mathcal{U}(\mathcal{N})$  lies in the convex hull of  $\mathcal{V}(\mathcal{N})$ , and  $C_1$  is a mixture of points in  $\mathcal{U}(\mathcal{N})$ ,<sup>36</sup> we have

$$C_1 \geq D_1 \geq A_1$$

Consider now a sequence  $\{W_m\} \in \mathcal{W}(\mathcal{N})$  such that  $\pi_1(W_m) \rightarrow B_1$ . Any element  $W_m$  is a weighted average of an expected payoff vector  $EU_m^1$  whenever 1 gets a chance to make a proposal, an expected payoff vector  $EU_m^2$  when it is 2's turn to make a proposal, and a payoff vector  $U_m^0$  in case no one gets to make a proposal:

$$W_m = \frac{p}{2}(EU_m^1) + \frac{p}{2}(EU_m^2) + (1 - p)(U_m^0) \tag{11}$$

We note that  $EU_m^1$  is a mixture of elements of  $\mathcal{U}$  resulting from 1's mixture over proposals and 2's mixture over her acceptance decision. Similarly,  $EU_m^2$  is a mixture of elements of  $\mathcal{U}$ .

Since all elements  $U_m$ 's belong to  $\mathcal{U}(\mathcal{N})$ , we have  $\pi_1(EU_m^2) \geq A_1$  and  $\pi_1(U_m^0) \geq A_1$ . Equation (11) thus implies that

$$\pi_1(W_m) \geq (1 - \frac{p}{2})A_1 + \frac{p}{2}\pi_1(EU_m^1).$$

Recalling that  $C_1$  denotes 1's infimum payoff when he gets to make a proposal, we get

$$\pi_1(W_m) \geq (1 - \frac{p}{2})A_1 + \frac{p}{2}C_1.$$

Taking limits,

$$B_1 \geq (1 - \frac{p}{2})A_1 + \frac{p}{2}C_1$$

or

$$B_1 \geq (1 - \frac{q\varepsilon}{2})A_1 + \frac{q\varepsilon}{2}C_1. \tag{12}$$

<sup>35</sup>See Definition 4 for a precise definition.

<sup>36</sup>It is a mixture over the payoffs obtained following  $-i$ 's acceptance/rejection decision to  $i$ 's proposal.

Combining (10) and (12), we conclude that

$$A_1 \geq \varepsilon v_1 + (1 - \varepsilon) \left[ \left(1 - \frac{q\varepsilon}{2}\right) A_1 + \frac{q\varepsilon}{2} C_1 \right]$$

or, ignoring terms of order  $\varepsilon^2$  in right-hand side,

$$A_1 \geq \varepsilon v_1 + (1 - [1 + \frac{q}{2}]\varepsilon) A_1 + \frac{q\varepsilon}{2} C_1.$$

Subtracting  $A_1$  on both sides of the last equation and dividing by  $\varepsilon$ , we obtain

$$0 \geq v_1 - [1 + \frac{q}{2}] A_1 + \frac{q}{2} C_1 \tag{13}$$

From  $A_1 \leq \pi_1(A)$ ,  $C_1 \geq \pi_1(P_2)$ , and  $\pi_1(A) < v_1 = v_1 + \frac{q}{2+q}(\pi_1(P_2) - v_1)$ , we get

$$0 < v_1 - [1 + \frac{q}{2}] A_1 + \frac{q}{2} C_1$$

which contradicts (13). This shows the necessary condition for player 1. An identical reasoning for player 2 shows the second necessary condition. This proves the result. When  $P_1 = P_2$ , a similar reasoning implies the result.

## D Concept equivalence

1. Any closed norm  $\mathcal{N}^c$  is an open norm as well, so the first statement is trivially true. Now consider any open norm  $\mathcal{N}^o$ . To construct a payoff-equivalent closed norm  $\mathcal{N}^c$ , we modify each plan/equilibrium  $m$  of  $\mathcal{N}^o$  as follows:  $m$ 's rules on and off the equilibrium path are kept unchanged except when a player, say  $i$ , makes a proposal  $\mu_i$  which is off the equilibrium path. In this case, because  $\mathcal{N}^o$  is an open norm, the continuation equilibrium if  $-i$  accepts the proposal need not lie in  $\mathcal{N}^o$ . Following such a proposal, players are instead prescribed to behave as if  $i$  had remained silent. The new rules define an equilibrium: when playing the original equilibrium  $m$ ,  $i$  was not making the proposal  $\mu_i$  anyway, so removing this option does not affect *equilibrium* behavior and payoffs. By construction, the set of modified equilibria form a closed norm  $\mathcal{N}^c$ , and because each equilibrium of  $\mathcal{N}^o$  has been modified into a single payoff-equivalent equilibrium of  $\mathcal{N}^c$ , the norms are payoff equivalent.

2. We start with the observation that if two norms  $\mathcal{N}^c$  and  $\mathcal{N}^o$  have the same payoff sets, then any proposal that is credible according to either norm is credible according to the other norm.

We now consider any stable open norm  $\mathcal{N}^o$  and construct the corresponding closed norm  $\mathcal{N}^c$  as in Part 1. To show that  $\mathcal{N}^c$  is stable, consider any SPE  $m$  of  $\mathcal{N}^c$ , history at which player  $i$  gets to propose, and credible proposal  $U$  such that  $\pi_i(U)$  is strictly greater than  $i$ 's continuation payoff  $\hat{U}_i$ . From the above observation,  $U$  is also credible for  $\mathcal{N}^o$ . If the proposal  $U$  gives player  $-i$  lower payoff compared to  $\hat{U}$ , then the game proceeds as if there is no proposal. If the proposal  $U$  Pareto dominates  $\hat{U}$ , then for the equilibrium  $\tilde{m}$  of  $\mathcal{N}^o$  corresponding to  $m$ , and the same history,  $-i$  must reject  $U$  with positive probability (for otherwise  $\pi_i(U)$  would coincide with  $\hat{U}_i$ ). Let  $U'$  denote the continuation payoff if  $-i$  rejects  $U$ . By stability of  $\mathcal{N}^o$ ,  $-i$  knows that if he accepts  $U$  it will be implemented. Since it is weakly optimal for  $-i$  to reject  $U$ , it must therefore be the case that  $\pi_{-i}(U') \geq \pi_{-i}(U)$ . Moreover, it must also be the case that  $\pi_i(U') \leq \hat{U}_i$ , for otherwise it would be strictly optimal for  $i$  to deviate by proposing  $U$ , and  $\tilde{m}$  would not be an equilibrium. Using this  $U'$  in Definition 7, this implies that  $\mathcal{N}^c$  is stable.

Next, consider any stable closed norm  $\mathcal{N}^c$ . To construct a payoff-equivalent stable open norm  $\mathcal{N}^o$ , we simultaneously modify all SPE's of  $\mathcal{N}^c$ . The modification proceeds in two steps, and is based on the recursive definition a plan. Recall that a plan is a prescription of actions, proposals and acceptance decisions for the

next period (each depending on what happened in earlier stages), along with a continuation plan resulting from these stages to be applied in the period after next. In Step 1, we modify the prescriptions for time  $t + 1$ , and still use plans of  $\mathcal{N}^c$  as continuation plans. The purpose of this step is to make a prescription compatible with the requirement that if a Pareto-improving, credible proposal is made and accepted, then it has to be played. In Step 2, we replace these continuation plans of  $\mathcal{N}^c$  by those built in Step 1, to guarantee that the rule applies at all periods, ensuring that credible proposals which are accepted are implemented, so that Definition 5 holds at all periods.

Consider any SPE  $m$  of  $\mathcal{N}^c$ . We modify  $m$  as follows. For the modified SPE  $\tilde{m}$ , the action stage and on-path proposals are prescribed exactly as in  $m$ .<sup>37</sup> Now consider a history at which  $i$  makes any proposal  $U$  which is not prescribed by  $m$  but which is  $\mathcal{N}^c$ -credible. If  $-i$  accepts the proposal, we construct  $\tilde{m}$  by prescribing that players implement this proposal.<sup>38</sup> If the proposal gives  $i$  a strictly higher payoff than his equilibrium continuation payoff  $\hat{U}_i$ , then by stability of  $\mathcal{N}^c$ , there must exist a payoff vector  $U'$  corresponding to some equilibrium  $m'$  of  $\mathcal{N}^c$ , which gives player  $-i$  at least as much as  $U$ , and which gives player  $i$  at most  $\hat{U}_i$ . We prescribe playing the equilibrium corresponding to  $U'$  in case player  $-i$  rejects the proposal. If  $U$  does not improve upon  $i$ 's equilibrium continuation payoff, we prescribe playing the continuation equilibrium corresponding to any of  $i$ 's equilibrium proposals in case  $-i$  rejects  $U$ . Finally, if  $i$  makes a non-credible proposal, the proposal is ignored as if  $i$  had stayed silent.

We now verify that  $\tilde{m}$  is an SPE that yields the same payoff as  $m$ . Since  $\tilde{m}$  prescribes the same actions as  $m$ , players are incentivized to follow the prescription. If  $i$  gets a chance to make a proposal, any proposal prescribed by  $m$  (and hence  $\tilde{m}$ ) yields the same continuation payoff as in  $m$ . If player  $i$  makes a credible, off-equilibrium proposal that improves upon his equilibrium payoff, then player  $-i$  is incentivized to reject it, and  $i$ 's continuation payoff is weakly lower than his equilibrium payoff. It is never optimal for  $i$  to make a credible proposal that is lower than his equilibrium payoff, regardless of  $-i$ 's acceptance decision. Finally, we replace all continuation plans by their modified versions.

There remains to verify that the set consisting of all modified equilibria forms a stable open norm, denoted  $\mathcal{N}^o$ , which is payoff equivalent to  $\mathcal{N}^c$ . First, we notice that continuation equilibria outside of  $\mathcal{N}^o$  may only arise when a player makes an off equilibrium proposal (which, by construction, also has to be credible) which is accepted by the other player. Thus,  $\mathcal{N}^o$  is an open norm. By construction, each element of  $\mathcal{N}^o$  corresponds to exactly one element of  $\mathcal{N}^c$ , which yields the same expected payoff. Therefore, the norms are payoff equivalent. As observed earlier, this implies that they have the same set of credible proposals. This, in turn, implies that any Pareto-improving, credible proposal of  $\mathcal{N}^o$  that is accepted is played and, hence, that  $\mathcal{N}^o$  is stable.

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<sup>37</sup>We need to make another small modification to  $m$  issue whenever  $i$  is proposing a continuation  $\mu$  outside of the norm  $\mathcal{N}^c$ , which  $-i$  is supposed to accept, and which is followed by a continuation  $\mu'$  in the norm  $\mathcal{N}^c$  (as it should, since the norm is closed). In that case, we can replace this play by  $i$  proposing instead  $\mu'$  and have it accepted by  $-i$ . That this modification can be done while preserving the equilibrium is straightforward to check. In fact, any SPE of the game can be turned into a payoff equivalent “truthful” SPE of the game, i.e., one in which any proposal that is made and accepted *on the equilibrium path* is implemented. See Appendix B.

<sup>38</sup>At this point, we do not know yet that the proposal is  $\mathcal{N}^o$ -credible. We only know that it is  $\mathcal{N}^c$ -credible. However, the norm  $\mathcal{N}^o$  that we are constructing will be payoff equivalent to  $\mathcal{N}^c$  and hence have the same credible proposals.

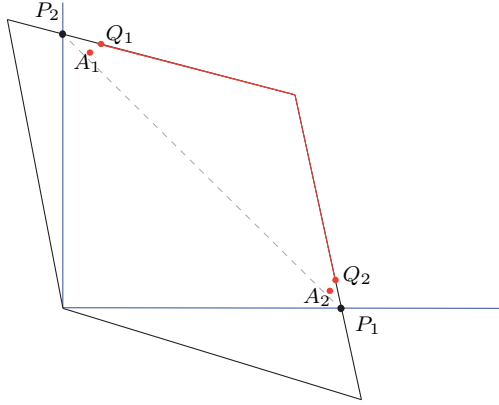


Figure 6: Norm construction with NFD (payoffs)

## E Proofs of Section 6

### E.1 Proof of Theorem 5 (Sufficient Conditions)

Consider two feasible Pareto points,  $Q_1$  and  $Q_2$ , lying at an arbitrarily small but strictly positive distance from  $P_2$  and  $P_1$ , respectively, and illustrated by Figure E.1. It suffices enough to show that for any  $\varepsilon$  small enough, there exists a forgiving stable norm  $\mathcal{N}$  which includes  $Q_1$  and  $Q_2$  as equilibrium payoffs, that is, norm has elements  $Q_1^{\mathcal{N}}, Q_2^{\mathcal{N}}$ . By public randomization, this will imply that this norm can also be made to contain all payoffs above the segment  $[Q_1, Q_2]$ . The argument below focuses on the case in which  $P_2$  and  $P_1$  are determined by the minmax payoffs, which is the harder one.<sup>39</sup>

We construct a norm which continuation payoffs just after the public randomization stage (before the action stage) consist of the Pareto frontier contained between  $Q_1$  and  $Q_2$  and of two additional points,  $A_1$  and  $A_2$ , respectively lying within  $\varepsilon$ -proportional distance from  $Q_1$  and  $Q_2$ , as indicated on Figure E.1. We describe the implementation of  $A_1^{\mathcal{N}}$  and  $Q_1^{\mathcal{N}}$ ;  $A_2^{\mathcal{N}}$  and  $Q_2^{\mathcal{N}}$  have a symmetric implementation.

While  $Q_1$  is taken as given, the location of  $A_1$  depends on  $\varepsilon$ , and is determined by the following conditions

$$\begin{aligned}\pi_1(A_1) &= \pi_1(Q_1) - K\varepsilon \\ \pi_2(A_1) &= \pi_2(Q_1) - L\varepsilon,\end{aligned}\tag{14}$$

for constants  $K$  and  $L$  which will be determined ulteriorly.

To implement  $A_1^{\mathcal{N}}$ , players are prescribed to minmax each other. The continuation payoff  $B$  after the action stage is a function of the players' realized actions,  $a_1$  and  $a_2$ :  $B = B(a_1, a_2)$ . The implementation is illustrated by Figure 7. For any action  $a_i$  of player  $i$  the continuation payoff  $\pi_i(B(a_i, a_j))$  does not depend on  $a_j$ .

Given that player 2 has minmaxed player 1, let  $Eu_1(a_1)$  denote 1's expected payoff for the period, as a function of his chosen action,  $a_1$ . 1's continuation payoff,  $\pi_1(B(a_1, a_2))$ , satisfies the promise-keeping condition

$$\pi_1(A_1) = \varepsilon Eu_1(a_1) + (1 - \varepsilon)\pi_1(B(a_1, a_2)).$$

A similar relation holds for 2's continuation payoff. By appropriately choosing players' continuation payoffs  $B(a_1, a_2)_{(a_1, a_2) \in \mathcal{A}}$ , the construction can make players indifferent between taking *any* action in the game.

<sup>39</sup>If, say,  $\pi_1(P_2) > \underline{v}_1$ , it suffices to set  $Q_1 = P_2$  in our construction and use it as as the best proposal for player 2.



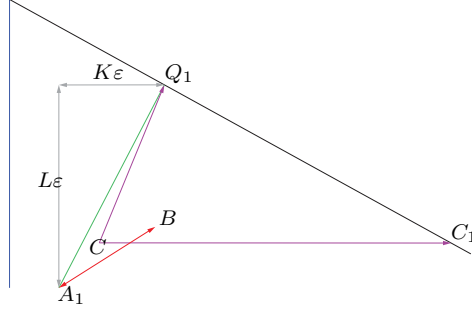


Figure 7: Norm construction with NFD (implementation)

Moreover, if the constant  $K$  appearing in (14) is large enough, then for any action profile  $(a_1, a_2)$ , one necessarily has  $\pi_1(B(a_1, a_2)) < \pi_1(Q_1)$ .<sup>40</sup>

Consider any of the continuation payoffs  $B(a_1, a_2)_{(a_1, a_2) \in \mathcal{A}}$  after the action stage—henceforth referred to as ‘ $B$ ’ for simplicity.  $B$  is a weighted average of three continuation payoffs corresponding to the following events: player 1 makes a proposal, player 2 makes a proposal, no one makes a proposal. Let  $C$  denote the continuation payoff in case no one makes a proposal (this payoff is computed before the public randomization taking place in the following period).

For the norm to be forgiving, any rejected proposal results in payoff  $C$ . This implies that if player 1 gets to make a proposal, in equilibrium he proposes the element with a Pareto-efficient payoff  $C_1$  which gives 2 her default value  $\pi_2(C)$ , making player 2 to accept the proposal in equilibrium.

The situation is different if player 2 gets to make a proposal.  $B^N$  gives player 1 a lower payoff than  $Q_1^N$ , and player 2 is prescribed to propose an element  $Q_1^N$ , which achieves her highest payoff in the norm and also gives player 1 a higher payoff than  $C^N$  does.

As shown on Figure 7, at element  $B^N$  if player 1 gets a chance to make a proposal, he proposes  $C_1^N$ , if 2 gets a chance to make a proposal, she proposes  $Q_1^N$ .  $B$  is thus a weighted average of  $C$ ,  $C_1$  and  $Q_1$ . Given any point  $B$ , one can find a default option  $C$  such that  $B$  is indeed the right weighted average, given the probabilities of proposal for each player.

We will verify at the end of this proof that the constants  $K$  and  $L$  from (14) may be chosen so that  $C$  lies to the right of the line  $(A_1, Q_1)$ . If this is true,  $C^N$  may be implemented, before public randomization, as a weighted average of  $A_1^N$ ,  $Q_1^N$ , and  $Q_2^N$ .

The remaining element of interest,  $Q_1^N$ , is implemented as follows: players are prescribed to choose the pure-strategy Pareto-efficient payoff northwest of  $Q_1$ . If 1 deviates in action, the continuation payoff jumps to  $B$ ; if 2 deviates, it jumps to the analog of  $B$  near  $Q_2$ . Players are incentivized to play as prescribed as long as  $\frac{\pi_1(Q_1) - \pi_1(B)}{\varepsilon}$  is large enough. This is achieved by judiciously choosing the constants  $K$  and  $L$  arising in (14), as explained next.

### Determination of the constants $K$ and $L$

First, we observe that for  $K$  large enough, the threat of jumping to continuation  $B^N$  is enough to incentivize player 1 to play as prescribed in the implementation of  $Q_1^N$ . We fix such a  $K$ —this choice is independent of  $\varepsilon$ . We now show that for  $L$  big enough, for any realization of  $B$  (which depends on which actions players choose while implementing  $A^N$ ), the point  $C$  will lie to the right of line  $A_1Q_1$ , as mentioned earlier.

<sup>40</sup>Indeed, the distance between  $A$  and  $B(a_1, a_2)$  is proportional to  $\varepsilon$ , with a coefficient bounded above by the highest absolute value of the payoff of the stage game.

Since a player's probability of proposal and the distance from  $B$  to the Pareto line are both proportional  $\varepsilon$ , the distance between  $B$  and  $C$  must be proportional to  $\varepsilon^2$ . Therefore, if we can show that each continuation point  $B(a_1, a_2)$  lies to the right of the line  $A_1Q_1$ , at a strictly positive  $\varepsilon$ -proportional distance, so does the point  $C$ , for sufficiently small  $\varepsilon$ .

The points  $B(a_1, a_2)$  are constructed by promise-keeping conditions. Let  $B^*$  denote the continuation payoff, out of all continuations  $B(a_1, a_2)$ , which gives the lowest payoff to player 1 and the highest payoff to player 2.  $B^*$  corresponds to the highest value  $Eu_1(a_1)$  out of all actions  $a_1$  and to the lowest value  $Eu_1(a_2)$  out of all actions  $a_2$ . It suffices to show that  $B^*$  lies to the right of  $A_1Q_1$ . We recall the promise-keeping conditions

$$\begin{aligned}\pi_1(A_1) &= \varepsilon Eu_1(a_1) + (1 - \varepsilon)\pi_1(B^*) \\ \pi_2(A_1) &= \varepsilon Eu_2(a_2) + (1 - \varepsilon)\pi_2(B^*)\end{aligned}$$

or, equivalently,

$$\begin{aligned}[\pi_1(A_1) - \pi_1(B^*)] &= \varepsilon[Eu_1(a_1) - \pi_1(B^*)] \\ [\pi_2(A_1) - \pi_2(B^*)] &= \varepsilon[Eu_2(a_2) - \pi_2(B^*)].\end{aligned}$$

The ratio of the absolute values of the right-hand sides in the two equations above,  $|\frac{Eu_2(a_2) - \pi_2(B^*)}{Eu_1(a_1) - \pi_1(B^*)}|$ , determines the tangent of the angle of the vector  $A_1B^*$  above the horizontal. Since  $B^*$  is at an  $\varepsilon$ -distance from  $Q_1$ , this ratio simplifies to  $|\frac{Eu_2(a_2) - \pi_2(Q_1)}{Eu_1(a_1) - \pi_1(Q_2)}|$ , plus  $\varepsilon$ -terms which can be ignored.

Player 1 cannot obtain a higher payoff than his minmax  $\underline{v}_1$  (as player 2 is minmaxing him), and player 2 cannot obtain a lower payoff than her lowest possible payoff in the game, which we denote as  $\underline{v}$ . Therefore, the angle of the vector  $A_1B^*$  above the horizontal is no higher than  $|\frac{\underline{v} - \pi_2(Q_1)}{\underline{v}_1 - \pi_1(Q_2)}|$ , a finite value independent of  $L$  and  $\varepsilon$ .

The tangent of the angle of the line  $(A_1Q_1)$  above the horizontal is equal to  $\frac{L}{K}$ . By choosing  $L$  high enough, this ratio exceeds twice the ratio  $|\frac{\underline{v} - \pi_2(Q_1)}{\underline{v}_1 - \pi_1(Q_2)}|$ . This guarantees that the vector  $A_1B^*$  lies strictly to the right of the line  $(A_1Q_1)$ , as desired.

There remains to check that the norm satisfies all the conditions of Theorem 5. First, both players are incentivized to propose as prescribed: player 1 proposes the best available option for him, given the default option  $C$ . If player 2 wants to improve upon  $Q_1$ , she has to propose a continuation which gives her at least  $\eta$  more than her on-path continuation payoff. For  $\varepsilon$  small enough, however, the only proposals that would achieve this would have to give player 1 less than  $\pi_1(C)$ , and would therefore be rejected. Second, the continuation payoff,  $C$ , is the same when a proposal is rejected, regardless of the identity of the proposer and the nature of the proposal. The norm is thus forgiving. Finally, the point  $Q_1$  is a continuation of the norm both after and before the public randomization, as desired.

## E.2 Proof of Theorem 4 (Necessary Conditions)

Consider a forgiving stable norm  $\mathcal{N}$ . For simplicity, we assume that at each stage of the game—before the action stage, before the proposal stage, and before the public randomization stage—there exist equilibria in the norm with respective payoff vectors  $A$ ,  $B$ , and  $C$ , that yield the maximal value of  $\rho$  at the corresponding stage.<sup>41</sup> Let  $\alpha$  denote the (possibly mixed) action profile corresponding to the first-period play implementing element  $A^{\mathcal{N}}$ —the continuation before the action stage, and let  $v(\alpha)$  denote the expected current payoff resulting from  $\alpha$ . Since  $\rho(v(\alpha)) \leq \bar{\rho}$ , we necessarily have

$$\rho(A) \leq \varepsilon \bar{\rho} + (1 - \varepsilon)\rho(B)$$

Point  $B$ , which is a continuation payoff before the proposal stage, is the weighted average of the continuation payoffs following accepted proposals, and of the default option. When a player—player 1, say—gets a chance

<sup>41</sup>If the supremum values are not achieved, the proof can be easily adjusted by taking appropriate limits.

to make a proposal, the expected continuation payoff must lie within at most an  $\sqrt{\varepsilon}$ -distance from the Pareto line. Otherwise, player 1 could propose a Pareto point which increases both players' payoffs by a value proportional to  $\sqrt{\varepsilon}$ , and is an equilibrium lying above the minmax.<sup>42</sup> This proposal would then be accepted by player 2 and would be a profitable deviation for player 1. Therefore, if a player gets a chance to make a proposal, which happens with probability  $q\varepsilon$ , the resulting continuation cannot have a positive value of  $\rho$  that exceeds  $\sqrt{\varepsilon}$ . When no one makes a proposal, the continuation payoff is dictated by the default continuation, whose value of  $\rho$  is at most  $\rho(C)$ . This implies that

$$\rho(B) \leq q\varepsilon \times \sqrt{\varepsilon} + (1 - q\varepsilon)\rho(C).$$

Finally, since  $C$  is a convex combination of payoffs, obtained by public randomization, of equilibrium payoffs before the action stage whose maximal  $\rho$ -value is achieved by  $A$ ,

$$\rho(C) \leq \rho(A).$$

Combining the above inequalities and getting rid of second-order  $\varepsilon$  terms shows Theorem 4.

## F Comparative statics

Consider any  $q > q'$  and any norm  $\mathcal{N}$  that is stable for  $q$ . We will show the existence of a norm  $\mathcal{N}'$ , stable for  $q'$  and payoff-equivalent to  $\mathcal{N}$ , which implies that all payoffs implemented by  $\mathcal{N}$  are  $q'$ -renegotiation-proof for the lower value of  $q'$ .

In the new norm  $\mathcal{N}'$ , any payoff  $A$  achieved by  $\mathcal{N}$  before the action stage is implemented using the same mixed actions and the same subsequent continuations as prescribed by  $\mathcal{N}$ . Consider now any vector payoff  $B$ , calculated before the proposal stage, implemented by some equilibrium  $B^{\mathcal{N}}$  of  $\mathcal{N}$ .  $B^{\mathcal{N}}$  is a mixture of three continuation equilibria:  $C_1^{\mathcal{N}}$ , which arises when 1 gets a chance to make a proposal and is calculated after the proposal stage;  $C_2^{\mathcal{N}}$  which arises if 2 gets to make a proposal; and  $C^{\mathcal{N}}$ , which arises if no one gets to make a proposal.

With the new negotiation factor  $q'$ ,  $B$  is implemented as follows: players are prescribed to make exactly the same proposals (with the same prescribed punishments if someone made an off-path proposal). For  $B$  to still to be the weighted average of the continuations occurring after the three proposal events, we change the continuation payoff in case no proposal is made: the new continuation payoff in this case,  $C'$ , has to lie on the line between  $B$  and  $C$ . The new continuation  $C'^{\mathcal{N}}$  can be implemented by using a public randomization, as it lies in the triangle  $(C, C_1, C_2)$ . This, essentially, yields the new implementation.

There might be a problem, however, with this candidate implementation. One needs to make sure that players are correctly incentivized to make a proposal, when they get an opportunity to do so, rather than to conceal this opportunity. This is the case if  $\pi_1(C_1) \geq \pi_1(C)$  and  $\pi_2(C_2) \geq \pi_2(C)$ , i.e., if each player gets at least as high a payoff when he makes a proposal as when he remains silent. When one moves point  $C$  to  $C'$ , these incentives might get violated, and the construction above must be adjusted as follows.

The new continuation payoff when no proposal is made,  $C'$ , lies in between  $C$  and  $B$ . Suppose that it violates 1's incentives to make his prescribed proposal:  $\pi_1(C') > \pi_1(C_1)$ . Since, in the old norm, we had  $\pi_1(C_1) \geq \pi_1(C)$ , such a violation is possible only if  $\pi_1(C_2) > \pi_1(C_1)$ . In this case, we modify the prescribed proposal for player 1 by moving point  $C_1$  towards  $C_2$ . As this happens, the value of  $\pi_1(C_1)$  increases and the value  $\pi_1(C')$  decreases (to keep  $B$  the weighted average). When these values become equal, the incentives for

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<sup>42</sup>With the more permissive concept of an  $\eta$ -stable norm, the continuation payoff has to lie within a distance of  $\sqrt{\varepsilon} + \eta$  from the Pareto line. Otherwise player 1 could make a proposal which gives him  $\eta$  more, and gives player 2  $\sqrt{\varepsilon}$  more than the continuation payoff.

player 1 to make a proposal start holding again. With the new continuation payoff  $C'_1$  for player 1's proposal and renewed continuation payoff in case of no proposal  $C''$ , player 1 is incentivized to make the prescribed proposal. One then can check that both new points can be implemented: the payoff  $C'_1$  lies between  $C_1$  and  $C_2$  and therefore can be implemented using public randomization, while point  $C''$  lies within the triangle  $(C, C_1, C_2)$  and can therefore also be implemented.

The same procedure can be done for player 2. The modified continuation payoffs can be implemented using public randomization device. The new norm  $\mathcal{N}'$  therefore has the same set of payoffs as the old norm  $\mathcal{N}$  at any stage; and it is stable given the new value of  $q'$ .

## G Proofs of Section 7 (Three or More Players)

### G.1 Proof of Theorem 6

Since  $v \in \mathcal{F}$ , the usual Folk Theorem implies that for  $\varepsilon$  small enough  $v$  can be achieved by an SPE of the underlying repeated game. This SPE can be embedded into an equilibrium of the repeated game with renegotiation. In this equilibrium, no proposals are ever prescribed at any stage of the game. If any player  $i$  makes a proposal, other players are all prescribed to reject it, and the continuation payoff is player  $i$ 's punishment, as in the underlying SPE. If only one player  $j \neq i$  accepts  $i$ 's proposal, the continuation is the punishment equilibrium for  $j$ . If at least two players accept the proposal, it is implemented. These prescriptions guarantee that any unilateral deviation in action, proposal, or acceptance decision is suboptimal.

### G.2 Proof of Lemma 1

If  $i$  makes a non-prescribed proposal with payoff  $C$ , which is not Pareto-optimal, there must exist another point  $C'$ , which is the payoff of another non-prescribed proposal for player  $i$  which Pareto dominates  $C$ , given that the number of prescribed proposals is finite and given our maintained full-dimensionality assumption.

The set  $\mathcal{D}(C, \mathcal{V})$  of possible punishment payoffs<sup>43</sup> is strictly larger than the set  $\mathcal{D}(C', \mathcal{V})$ , since the latter set gives every player  $j \neq i$  a higher lower-bound on his payoff. This implies that the proposal  $C'$  gives player  $i$  a worst punishment payoff  $\underline{\pi}_i(C', \mathcal{V})$  at least as high as the proposal associated with payoff  $C$ . Therefore, player  $i$  can without loss always choose a point on Pareto frontier  $\mathcal{P}(\mathcal{V})$ .

### G.3 Proof of Proposition 2

We fix one of the two procurers and let  $\mathcal{F}_k$  denote the set corresponding to the  $k$ -th step in the sequential reduction of the set  $\mathcal{F}$  under this procedure. We first show that points on the relative Pareto frontier  $\mathcal{P}(\mathcal{F}_k)$  of  $\mathcal{F}_k$  are never removed by the procedure. Suppose, contrary to the claim, that some point  $A \in \mathcal{P}(\mathcal{F}_k)$  was removed by the procedure. Then there would be a player  $i$  such that  $\pi_i(A) < \underline{\pi}_i(\mathcal{F}_k)$ . If  $A$  was prescribed as a punishment payoff for any proposal of player  $i$ , then for  $i$ 's optimal proposal with payoff  $C \in \mathcal{F}_k$ , the punishment payoff  $A$  would not be credible as it is removed at the  $k$ -th step. That is, any  $j \neq i$   $\pi_j(A) < \pi_j(C)$ . Since  $A$  lies on the Pareto frontier of  $\mathcal{F}_k$ , this means that  $\pi_i(C) < \pi_i(A)$ :  $C$  gives  $i$  a lower payoff than  $\underline{\pi}_i(\mathcal{F}_k)$ , which contradicts  $C$ 's assumed optimality. One could simply prescribe both continuations to have  $\bar{C}$  as their payoff vector, and this would give  $i$  a lower payoff than  $\underline{\pi}_i(\mathcal{F}_k)$ .

Since no point on the relative Pareto frontier of  $\mathcal{F}$  is removed, the set of optimal proposals for any player  $i$  remains the same along the sequence. However, the set of possible punishments keeps decreasing at each step, which weakly increases, as a result, the minimal value  $\underline{\pi}_i(\mathcal{F}_k)$  with  $k$ . (Recall that  $\underline{\pi}_i(\mathcal{F}_k)$  is  $i$ 's minimal

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<sup>43</sup>In Appendix B we show that it's enough to have only a finite number of punishment continuations, satisfying the description of a plan.

payoff if he gets a chance to make a proposal). At each step, the set  $\mathcal{F}_k$  is characterized by the  $n$  lower bounds of the players' payoffs  $\{\pi_{\min,i}(\mathcal{F}_k)\}_{i \in \{1, \dots, n\}}$ . These lower bounds are weakly increasing at each step, which implies that the procedure converges to a stable point.

## G.4 Proof of Theorem 7

### Necessity

Suppose that  $A$  lies outside of  $\mathcal{V}_n$  and, for any small enough  $\varepsilon > 0$ , there exists a stable norm  $\mathcal{N}(\varepsilon)$  such that  $A \in \mathcal{N}(\varepsilon)$ . A norm  $\mathcal{N}(\varepsilon)$  has to satisfy the inequality 8 (if being used as an argument instead of  $\mathcal{F}_n^k$ ), up to an  $\varepsilon$ -term. Let's limit  $\varepsilon$  to 0 and consider a sequence of norms  $\mathcal{N}(\varepsilon)$  which payoff sets converge. This limit payoff set contains  $A$  and satisfies the inequality 8, which in turn means that  $A$  should have not been removed from any of the sets  $\mathcal{F}_n^k$ . However, that would make  $A$  an element in  $\mathcal{V}_n$ , a contradiction.

### Sufficiency: $\mathcal{R}$

We first prove that any point in  $\mathcal{R}$  is renegotiation-proof. Consider any point  $A$  with  $\pi_i > \pi_i(P_{-i})$  for any  $i$ . As in the two-player case, one can find  $n$  points  $A_i$  such that for  $j \neq i$   $\pi_j(A_i) = \pi_j(A)$  and  $\pi_i(A_i) = \pi_i(P_{-i}) + \sqrt{\varepsilon}$ . Each point  $A_i$  will give the lowest payoff for player  $i$  in the constructed norm  $\mathcal{N}$ . In the equilibrium of the norm  $A_i^{\mathcal{N}}$  associated with payoff vector  $A_i$ , player  $i$  is being minmaxed. Since players other than  $i$  may have to use mixed strategies, this generates a set  $\mathcal{B}$  of continuation payoffs, following the action stage, which depend on the realization of actions of players other than  $i$ . Any continuation  $B \in \mathcal{B}$  is implemented as follows: if player  $i$  can make a proposal, he is prescribed to propose some continuation with payoff  $C$ ; other players are prescribed to remain silent; in the absence of any proposal, the continuation returns to  $A_i^{\mathcal{N}}$ . As in the two-player case, one can guarantee (possibly using the public randomization), that the distance  $A_i C$  is of order  $\sqrt{\varepsilon}$ .

Since the Pareto frontier is connected, so is its truncation to points for which  $i$ 's payoff lies above  $\pi_i(P_{-i})$ . One can therefore find a connected subset  $S_\varepsilon$  of the frontier consisting of all points giving, for each  $i$ , a payoff greater than or equal to  $\pi_i(A_i) + K\varepsilon$ , where  $K$  is a constant chosen large enough that players are incentivized not to deviate in actions.

Continuation equilibria with payoffs in  $S_\varepsilon$  are implemented in such a way that each player  $i$  gets at least  $\pi_i(A_i) + K\varepsilon$  in all continuations. Players are prescribed to stay silent. Since each point of  $S_\varepsilon$  is Pareto-efficient, there are no unanimously improving proposals anyway. Moreover, using  $A_i^{\mathcal{N}}$  as a punishment if  $i$  deviates in actions guarantees that such deviation would be suboptimal.

When implementing  $A_i^{\mathcal{N}}$ , players are already incentivized to follow the prescribed actions. If  $i$  wants to make a non-prescribed proposal, then by construction of  $S_\varepsilon$  there exists a continuation with a payoff  $Q_i$  in set  $S_\varepsilon$  which gives player  $i$  a lower payoff than  $C$ . Indeed, the lower bound for  $\pi_i$  at the set  $S_\varepsilon$  is  $\pi_i(A_i) + K\varepsilon$ , while  $\pi_i(C) - \pi_i(A_i)$  is of order  $\sqrt{\varepsilon}$ .

### Sufficiency: General Conditions

The proof is similar to the two-player case. For any point  $A \in \mathcal{V}_s$  with  $\pi_i > \pi_{\min,i}(\mathcal{V}_s)$ , consider the set of points  $A_i \in \mathcal{V}_s$  such that for any  $i$   $\pi_i(A_i) = \pi_{\min,i}(\mathcal{V}_s) + \sqrt{\varepsilon}$  and  $\pi_{-i}(A_i) = \pi_{-i}(A)$ . The points  $A_i$  have a smaller  $i$ -th coordinate than  $A$  provided that  $\varepsilon$  is small enough. In addition, we have  $\pi_i(A_j) - \pi_i(A_i) \gg \sqrt{\varepsilon}$  for any  $j \neq i$  without loss of generality.

We build a stable norm  $\mathcal{N}$  such that  $A_i$  gives the lowest payoff to player  $i$  in the norm (calculated at the start of the period). At  $A_i^{\mathcal{N}}$ , player  $i$  is minmaxed. Since players other than  $i$  may have to mix their actions, we construct a set of continuations with payoffs  $B \in \mathcal{B}$ , corresponding to the observed actions of players  $-i$ . For any continuation equilibrium  $B^{\mathcal{N}}$  associated with some payoff  $B \in \mathcal{B}$ ,  $i$  is prescribed to make a proposal with some payoff vector  $C$ , and all other players are prescribed to remain silent. As with the two-player case,  $C$  can be assumed to lie at a distance of order  $\sqrt{\varepsilon}$  from  $A_i$ . When implementing the equilibrium  $C^{\mathcal{N}}$  associated with  $C$ , players are prescribed to follow a deterministic sequence of actions such

that the continuation payoff remains within an  $\varepsilon$ -distance from  $C$ . Players are prescribed not to make any proposals.

The initial point  $A$  is also implemented by deterministic actions and no proposals. Moreover, each point in the orthant with lower bounds  $\pi_i(A_i) + K\varepsilon$  is included in the norm  $\mathcal{N}$  and implemented in such a way that  $\pi_i > \pi_i(A_i) + K\varepsilon$ :  $A_i^{\mathcal{N}}$  is severe enough a punishment for  $i$  that it makes it suboptimal for him to deviate in action.

This norm can be shown to be generically stable. The only new issue concerns  $i$ 's incentives to deviate in proposal. We have reduced (increased the lower bounds on payoffs) the initial set  $\mathcal{V}_s$  by an order of  $\sqrt{\varepsilon}$ . The whole orthant defined by  $\pi_i > \pi_i(A_i) + K\varepsilon$  for all  $i$  is part of the norm, but some points below are removed from the original set  $\mathcal{V}_s$ . As a result, the value  $\underline{\pi}_i(\cdot)$ , which  $i$  can guarantee if having a chance to propose, can now be larger. Our goal is to show that, nevertheless, generically the value of  $\underline{\pi}_i(\cdot)$  is smaller than  $\pi_i(C)$ , and therefore player  $i$  is incentivized to propose  $C^{\mathcal{N}}$ .

When building a set  $\mathcal{V}_s$  by sequentially removing payoffs with  $\pi_{min,i}(\cdot) < \underline{\pi}_i(\cdot)$ , the initial set of individually-rational payoffs gets reduced. If for player  $i$  the final value of  $\pi_{min,i}(\mathcal{V}_s)$  is strictly larger than his minmax payoff  $\underline{v}_i$ , then the value of  $\pi_i(A_i) - \underline{v}_i$  is of order  $\varepsilon^0$ . This means that the distance  $A_i C$  can be made of  $\varepsilon^{\frac{1}{4}}$ -order. At the same time, the set  $\mathcal{V}_s$  (and, respectively, the value  $\underline{\pi}_i(\cdot)$ ) were changed by an order of  $\sqrt{\varepsilon}$ , guaranteeing that  $\underline{\pi}_i(\cdot) < \pi_i(C)$ .

If player  $i$ 's payoff  $\pi_{min,i}(\mathcal{V}_s)$  equals to minmax  $\underline{v}_i$ , this means that  $i$ 's payoff was not increased when building set  $\mathcal{V}_s$ . Put it differently, one can consider a hyperplane of the set  $\mathcal{V}_s$  with  $\pi_i = \underline{v}_i$ , and find the maximum payoffs of other players  $\bar{\pi}_j$ ,  $j \neq i$  on that hyperplane. The  $n - 1$ -dimensional payoff vector  $\{\bar{\pi}_j\}_{j \neq i}$  cannot lie within an interior of  $\mathcal{V}_s$  (otherwise, player  $i$  could make a proposal dominating  $\{\bar{\pi}_j\}_{j \neq i}$  and thus guaranteeing himself a payoff higher than  $\underline{v}_i$ ). When the set  $\mathcal{V}_s$  is reduced by (an arbitrarily small)  $\sqrt{\varepsilon}$ -order, player  $i$  can gain incentives to make an off-path proposal, only if the vector  $\{\bar{\pi}_j\}_{j \neq i}$  lies exactly on the Pareto frontier of  $\mathcal{V}_s$ . However, this possibility is not generic.

## G.5 Proof of Proposition 3

**Intuition.** The sets  $\mathcal{V}_s$  and  $\mathcal{V}_n^q$ —necessary conditions depend on  $q$ , hence the superscript—are both obtained from  $\mathcal{F}$  by sequentially increasing the lower bounds on each player's payoff when he gets a chance to make a proposal.  $\mathcal{V}_s$  is obtained by removing payoffs below  $\underline{\pi}_i(\cdot)$  at each step, while  $\mathcal{V}_n^q$  is obtained by removing payoffs below  $\frac{\underline{v}_i + q\underline{\pi}_i(\cdot)}{1+q}$ . When  $q$  goes to infinity, the sets of payoffs removed at each step of these procedures converge to each other. As we show below, this implies that  $\mathcal{V}_n^q$  converges to the set  $\mathcal{V}_s$  as  $q$  goes to infinity.

The set of sufficient conditions,  $\mathcal{V}_s$ , can be characterized by two sets of lower bounds for each player  $i$ :  $\underline{\pi}_i(\mathcal{V}_s)$  is the lower bound on  $i$ 's payoff when he gets a chance to make a proposal and  $\pi_{min,i}(\mathcal{V}_s)$  is the lower bound for his payoff at the beginning of a period.  $\mathcal{V}_s$  was constructed in such a way that  $\underline{\pi}_i(\mathcal{V}_s) \leq \pi_{min,i}(\mathcal{V}_s)$ .

To capture the above intuition, we first show by induction that  $\mathcal{V}_s$  is the largest set  $\mathcal{S}$  of individually rational payoffs whose Pareto frontier is equal to  $\mathcal{P}(\mathcal{V})$  and such that  $\underline{\pi}_i(\mathcal{S}) \leq \pi_{min,i}(\mathcal{S})$  for any  $i$ . Consider such a set  $\mathcal{S}$ . The sequence of sets  $\mathcal{F}_s^k$  converging to  $\mathcal{V}_s$  starts with  $\mathcal{F}_s^0 = \mathcal{F}$ , the set of all individually rational points. This implies that  $\underline{\pi}_i(\mathcal{S}) \geq \underline{\pi}_i(\mathcal{F}_s^0)$ , since  $\mathcal{F}_s^0$  contains  $\mathcal{S}$  and, hence, the set of punishments if  $i$  makes an unprescribed proposal is higher with  $\mathcal{F}_s^0$  than with  $\mathcal{S}$ , resulting in a lower bound  $\underline{\pi}_i$ . We now show the induction hypothesis: if  $\underline{\pi}_i(\mathcal{S}) \geq \underline{\pi}_i(\mathcal{F}_s^k)$ , then the same condition holds for  $k + 1$ . Due to the way the payoffs are cut at step  $k$ , one has for each  $i$ ,  $\pi_{min,i}(\mathcal{F}_s^{k+1}) = \max\{\pi_{min,i}(\mathcal{F}_s^k), \underline{\pi}_i(\mathcal{F}_s^k)\} \leq \underline{\pi}_i(\mathcal{F}_s^k)$ , which does not exceed  $\underline{\pi}_i(\mathcal{S}) \leq \pi_{min,i}(\mathcal{S})$ . Since the lower bound  $\pi_{min,i}(\mathcal{F}_s^{k+1})$  is lower than  $\pi_{min,i}(\mathcal{S})$ , the set  $\mathcal{F}_s^{k+1}$  contains  $\mathcal{S}$ , and one has that  $\underline{\pi}_i(\mathcal{S}) \geq \underline{\pi}_i(\mathcal{F}_s^{k+1})$ . By induction, the limit set  $\mathcal{V}_s$  contains  $\mathcal{S}$ .

Let  $\mathcal{V}_n$  denote the limit of  $\mathcal{V}_n^q$  as  $q$  goes to infinity. We wish to show that  $\mathcal{V}_n = \mathcal{V}_s$ . Consider the sequences  $\{\mathcal{F}_n^{k,q}\}_{k=0}^{+\infty}$  resulting from the procedure applied, for any fixed  $q$ , to derive necessary conditions for this value of  $q$ . Due to the way points are removed at each step, it is easy to check that  $\mathcal{F}_n^{k,q'} \subset \mathcal{F}_n^{k,q}$  whenever  $q' > q$ ; by the same logic, it is straightforward to check that  $\mathcal{V}_s$  is contained in  $\mathcal{V}_n$ . To prove the

reverse inclusion, note for each  $q$  and  $i$ , we have  $\pi_{\min,i}(\mathcal{V}_n^q) \geq \frac{v_i + q\pi_i(\mathcal{V}_n^q)}{1+q}$ , as this inequality holds at each step  $k$  of the procedure. Taking the limit as  $q$  goes to infinity, the limiting set  $\mathcal{V}_n$  must satisfy for each  $i$   $\pi_i(\mathcal{V}_n) \leq \pi_{\min,i}(\mathcal{V}_n)$ . From the previous paragraph, this implies that  $\mathcal{V}_s$  contains  $\mathcal{V}_n$ , which concludes the proof.

## G.6 Proof of Proposition 4 (Sketch)

Consider for simplicity the case of three players: player 1 makes a proposal and player 2 responds first, followed by player 3. Depending on responding players' votes, there are four possible continuations, one of which is equal to 1's proposal and arises when 2 and 3 accept the proposal.

The ability to punish 2 for accepting player 1's proposal is constrained by the following issue: if 2 accepts the proposal, 3 will reject it only if the punishment for player 2 gives him at least the same payoff as 1's proposal, which will be implemented if he accepts it. This puts a lower bound on 2's punishment payoff, which is higher than the minmax  $v_2$ .

As a result, 1's punishment for making an off-path proposal is also limited. Since fewer punishments are available, fewer equilibria are renegotiation-proof: sequential voting has more predictive power than simultaneous voting.

By nature of the arguments used to derive necessary and sufficient conditions, these conditions are characterized by upper orthants, even if players randomize their acceptance decision.

Since allowing only two continuations—as simple norms do with simultaneous voting—is a special case of the more numerous continuations allowed by sequential voting, it follows that simple stable norms have more predictive power than the stable norms obtained with sequential voting.

## G.7 Proof of Theorem 8 (Sketch)

We construct a forgiving  $\eta$ -stable norm  $\mathcal{N}$  as follows. The norm  $\mathcal{N}$  includes all Pareto-efficient payoffs which lie at some arbitrary small, but  $\varepsilon$ -independent distance from the minmax values. The norm  $\mathcal{N}$  also includes, for each player, a set of Pareto-inefficient elements used to build a punishment equilibrium for that player, all elements in each set lie within a distance of order  $\varepsilon$  from the Pareto-efficient elements of the norm. For each player  $i$ , there is a Pareto-inefficient payoff vector  $A_i$  which gives  $i$  his worst payoff in  $\mathcal{N}$ . The equilibrium  $A_i^{\mathcal{N}}$  which achieves payoff  $A_i$ , together with its continuations, form the punishment set for player  $i$ , as described below.

If players were unable to make any proposal, one could implement payoff  $A_i$  as follows. Player  $i$  is being minmaxed, which may require other players to use mixed strategies. As described in earlier proofs, this results in a set  $\mathcal{B}^1$  of continuation payoffs, (potentially) one for each observed action profile (these various continuations are needed to incentivize the minmaxing strategy). Each continuation payoff  $B^1 \in \mathcal{B}^1$  is implemented by minmaxing player  $i$ , which again generates several continuation payoffs in the next period, with generic element denoted as  $B^2$ . Player  $i$  is minmaxed in this way for several periods. In each period  $i$ 's continuation payoff,  $\pi_i$ , increases by an amount of order  $\varepsilon$ . One can compute the number  $m$  of periods needed to minmax player  $i$ , so that  $\pi_i$  exceeds  $\pi_i(A_i)$  by a sufficiently high amount that  $i$  can be incentivized to play any action by the threat of returning to  $A_i$ . After these  $m$  periods, each continuation payoff  $B^m$  can be implemented by playing a deterministic sequence of actions so that the continuation payoff always lies within some  $\varepsilon$ -proportional distance from  $B^m$ . This implementation is an equilibrium, since the payoff  $A_i$  prevents any deviation from player  $i$ , and any deviation by another player leads to an even larger drop in the continuation payoff of the deviator.

When proposals are re-introduced in the game, there will be changes in the implementation of  $A_i^{\mathcal{N}}$ , but these changes will be insignificant. After the first round of minmaxing player  $i$ , the resulting continuation payoff  $B^1$  is calculated taken into account the possibility of proposals. That is,  $B^{1\mathcal{N}}$  is the convex combination

of some default option,  $C^{1\mathcal{N}}$ , if no one makes a proposal, and of proposals payoffs  $C_i^{\mathcal{N}}$  for each player, which are chosen to be Pareto efficient elements of the norm  $\mathcal{N}$ . The distance between the payoffs  $B^1$  and  $C^1$  is of order  $\varepsilon^2$ —as explained the similar proofs seen earlier. In the next period, the continuation payoff before the actions will be  $C^1$  (instead of  $B^1$ , in the previous paragraph). Therefore, if one repeats minmaxing player  $i$  for  $m$  periods, the resulting continuation payoff compared to the case with no proposals, will differ by an amount of order  $\varepsilon^2$ , which is negligible as  $\varepsilon$  becomes arbitrarily small. As players become arbitrarily patient, the modified implementation of  $A_i$ , based on minmaxing player  $i$  for  $m$  periods and then choosing a deterministic sequence of actions, will thus be an equilibrium even with the possibility of proposals.

Finally, the payoff  $A_i$  (and, therefore, all the default continuation payoffs  $C$ 's) can be chosen so as to lie within some distance  $K\varepsilon$ -distance from the Pareto line. With  $\varepsilon$  small enough, no player can make an off-equilibrium proposal that would give him a payoff of at least  $\eta$  more than the equilibrium proposal, while keeping all other players at least as well as off as with the default payoff  $C$ . Therefore, the constructed norm is  $\eta$ -stable. Using initial public randomization, one can then include in the norm any point in the convex hull  $Co(\mathcal{P}')$ , which concludes the proof.

## H Observability of mixing strategies

We have assumed throughout the paper that when a player randomizes across several actions or proposals, only the outcome of this randomization is observed by the other player. In particular, players' continuation values cannot directly depend on their choice of mixed strategy. Our results do not change if instead we assume that mixed strategies are observable. For sufficient conditions, this fact is straightforward because our construction is clearly compatible with players observing more information. For necessary conditions, payoff lower bounds were computed using only that any player can guarantee himself at least his minmax payoff during the action stage and at least some particular payoff during the proposal stage which satisfies the responder. These lower bounds do not change when mixing is observable.

The observability of mixed strategies does affect, however, the set of weakly renegotiation-proof (WRP) equilibria defined by Farrell and Maskin (1989), defined as follows. An SPE  $\sigma$  is *weakly-renegotiation proof* if there do not exist continuation equilibria  $\sigma^1, \sigma^2$  of  $\sigma$  such that  $\sigma^1$  strictly Pareto dominates  $\sigma^2$ . If a payoff vector arises as players' continuation payoff following some history of a WRP equilibrium, we will also say that these payoffs are WRP.

Assuming that mixing probabilities are observable, Farrell and Maskin obtained a sufficient condition for any feasible payoff to be WRP in the context of two-player repeated games. To formulate this condition, they define  $c_i(\alpha) = \max_{a'_i} \pi_i(a'_i, \alpha_{-i})$  as the *cheating* payoff of player  $i$  when he chooses a best response to the (mixed) action  $\alpha_{-i}$ , and establish the following result.

**PROPOSITION 7** *Let  $\pi = (\pi_1, \pi_2)$  denote a feasible payoff. If there exist (mixed) action pairs  $\alpha^i = (\alpha_1^i, \alpha_2^i)$  (for  $i = 1, 2$ ) such that  $c_i(\alpha^i) < \pi_i$ , and  $\pi_{-i}(\alpha^i) \geq \pi_{-i}$ , then the payoff  $\pi$  is WRP if players are sufficiently patient.*

We now present an example with a Pareto-efficient payoff that satisfies the requirement of the above proposition, but cannot be WRP if mixing probabilities are unobserved.<sup>44</sup> The stage game is as follows:

9,-4	-2,-4
-2,-4	9,-4
0,8	8,0

<sup>44</sup>The definition of WRP is the same as before. The only difference is that equilibrium strategies now depend only on the history of realized actions rather than on the history that included mixed strategies.



The payoffs  $(0, 8)$  and  $(8, 0)$  are Pareto efficient and the minmax values of players are  $\underline{v}_1 = \frac{72}{19} = 4 - \frac{4}{19}$  and  $\underline{v}_2 = -4$ , as is easily checked (for  $v_1$ , 2 mixes so as to make 1 indifferent between the first and last rows). The Pareto-efficient, individually-rational point  $A$  with payoffs  $\pi_1 = 4 + \frac{1}{100}$ ,  $\pi_2 = 4 - \frac{1}{100}$  satisfies the premise of Proposition 7 with  $\alpha^1$  defined by player 1 choosing the last row and player 2 mixing equally between two columns and  $\alpha^2$  defined by player 1 choosing the first row and player 2 choosing the first column. We will nevertheless show that  $A$  cannot be WRP for low enough  $\varepsilon$  (arbitrarily patient players), even if players have access to a public randomization device.

COUNTER-EXAMPLE 1 *With unobservable mixed strategies,  $A$  is not WRP.*

Suppose, by way of contradiction, that  $A$  is the continuation payoff of some WRP equilibrium  $\sigma$ , and consider the payoff vector  $A'$  corresponding to player 1's lowest payoff and, hence, player 2's highest payoff among all continuations payoff of  $\sigma$  before public randomization.<sup>45</sup> When implementing  $A'$ , depending on the outcome of public randomization, player 2 plays a pure strategy with some probability  $\beta$  and mixes with the complement probability  $(1 - \beta)$ .

Since  $A'$  gives 1 his lowest possible payoff, when implementing  $A'$  player 1 cannot get a period payoff higher than  $\pi_1(A')$ , even if he always plays a stage-game best response. Otherwise, the promise-keeping constraint would have to prescribe a continuation giving 1 a payoff lower than  $\pi_1(A')$ . If player 2 chooses a pure strategy, player 1 can guarantee himself a payoff of at least 9. If player 2 chooses a mixed strategy, player 1 can guarantee himself his minmax payoff of  $\frac{72}{19}$ . This puts an upper bound on the probability  $\beta$  of player 2 choosing pure strategy:

$$\pi_1(A') \geq 9\beta + \frac{72}{19}(1 - \beta) \quad (15)$$

The continuation payoff  $\pi_2(A')$  of player 2 is a mixture between continuation payoffs  $\pi_{2,pure}$  and  $\pi_{2,mixed}$  conditional on her playing a pure and a mixed strategy:

$$\pi_2(A') = \pi_{2,pure}\beta + \pi_{2,mixed}(1 - \beta) \quad (16)$$

If 2 mixes between the two columns, by indifference any choice has to give her the same payoff  $\pi_{2,mixed}$ . Player 2 cannot get more than 0 when choosing the right column, and the continuation payoff cannot exceed the maximal value  $\pi_2(A')$  in WRP equilibrium. This puts an upper bound on 2's continuation payoff:

$$\pi_{2,mixed} \leq 0 \times \varepsilon + (1 - \varepsilon)\pi_2(A')$$

Similarly, since 2 cannot get a payoff higher than 8, we have

$$\pi_{2,pure} \leq 8 \times \varepsilon + (1 - \varepsilon)\pi_2(A')$$

Combining these inequalities with (16) yields

$$\pi_2(A') \leq 8 \times \varepsilon\beta + (1 - \varepsilon)\pi_2(A').$$

Rearranging, we get the following lower bound for  $\beta$ :

$$\pi_2(A') \leq 8\beta \quad (17)$$

Combining (15) and (17) yields  $\pi_2(A') \leq \frac{8}{99}(19\pi_1(A') - 72)$ . Since  $\pi_2(A) \leq \pi_2(A')$  and  $\pi_1(A) \geq \pi_1(A')$  this implies that

$$4 - \frac{1}{100} \leq \frac{8}{99} \left( 19 \left( 4 + \frac{1}{100} \right) - 72 \right),$$

which is false (the right-hand side is approximately equal to 0.34) and yields the desired contradiction.

<sup>45</sup>Since  $\sigma$  is WRP, 1's lowest continuation payoff is achieved for 2's highest continuation payoff. The proof can be easily adjusted if  $\sigma$ 's payoff extrema are not achieved.