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Friedrich Breyer and
Mathias Kifmann

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The German Retirement Benefit Formula: Drawbacks and Alternatives

Die Rentenformel in der gesetzlichen Rentenversicherung:
Nachteile und Alternativen

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Abstract

In this paper we identify a number of objectionable features of the German retirement benefit formula. We show that groups of insureds with higher than average life expectancy, in particular high-income groups, are subsidized by the rest of the membership because the formula neglects differences in group-specific life expectancy. Furthermore, the current formula leads to undesirable long-run effects if the earnings ceilings is raised, mandatory membership is extended, life expectancy rises or the rate of population growth declines. We present two alternative formulas which take group-specific life expectancy into account. In particular, a return-rate formula which rewards each Euro of contributions with the internal rate of return of the pay-as-you-go pension system proves to be superior to the current formula.

JEL-classification: H55, H22, J18.

Keywords: pay-as-you-go pension systems, implicit taxation, intra- and intergenerational equity

Zusammenfassung

Wir diskutieren eine Reihe von Problemen der Rentenformel in der deutschen gesetzlichen Rentenversicherung (GRV). Wir zeigen, dass Bevölkerungsgruppen mit überdurchschnittlicher Lebenserwartung, insbesondere die besser Verdienenden, von den übrigen Versicherten subventioniert werden, da die Rentenformel Unterschiede in den gruppenspezifischen Lebenserwartungen nicht berücksichtigt. Außerdem führt die Formel zu unerwünschten langfristigen Wirkungen, wenn die Beitragsbemessungsgrenze erhöht oder die Pflichtmitgliedschaft ausgeweitet wird, wenn die Lebenserwartung steigt oder das Bevölkerungswachstum abnimmt. Wir schlagen zwei Alternativen vor, die die gruppenspezifische Lebenserwartung berücksichtigen. Wir zeigen, dass insbesondere die Interne-Rendite-Formel, die jeden Euro an Beiträgen mit der internen Rendite des Umlageverfahrens belohnt, gegenüber der gegenwärtigen Formel überlegen ist.

Schlagwörter: Rentenversicherung, implizite Besteuerung, intra- und intergenerative Gerechtigkeit

1. Introduction

A well-known result from the theory of public pensions states that Pay-as-you-go (PAYG) pension systems can be regarded as a zero-sum game between generations if the economy is dynamically efficient.¹ In this case, the transfers to previous generations are exactly equivalent to the implicit tax burden of the public pension system in this case (see Sinn (2000)).² How this burden is shared between individuals of one generation and among generations depends on the specific design of the pension system. In this paper, we examine the case of the German Old-Age Insurance system (GRV). In particular, we concentrate on the following four characteristics of the German retirement benefit formula:

1. The formula establishes a proportional relationship between the monthly benefits and the contributed income over the working life.³
2. Labor income is only liable to contributions up to an earnings ceiling.
3. Not all population groups are included in the public pension system. In particular, high-income groups such as the self-employed and certain professions such as medical doctors and lawyers are not members.
4. The retirement benefits are linked to the average earnings of current employees and not to the total wage bill.

In this paper, we show that these four properties give rise to a number of problems. First of all, the linkage between individual contributions and benefits (“Teilhabe-Äquivalenz”), commonly interpreted as a guarantee both of fairness and long-run stability of the German system, neglects that life expectancy is positively correlated with income.⁴ For example, Reil-Held (2002, p.163), finds that among men,

¹This was first shown by Breyer (1989) who assumes that contributions have a lump-sum character. Fenge (1995) extended his result to contributions financed by a distortionary wage tax.

²The implicit taxes due to the participation in the PAYG system are defined as the difference between the present value of PAYG contributions and the present value of PAYG benefits. This measure was first used by Lüdeke (1988) and Homburg and Richter (1990).

³For an analysis of the way contributed income in different periods is aggregated in the pension formula and the resulting incentives to supply labor over the life cycle, see Beckmann (2000) and Fenge, Übelmesser, and Werding (2002).

⁴See also Breyer (1997).

members of the top income quartile can expect to live 10 years longer than members of the bottom quartile. As a consequence, groups of insureds with higher than average life expectancy, in particular high-income groups, are subsidized by the rest of the membership.

Furthermore, we demonstrate that the independence of the monthly benefit from life expectancy has two further peculiar consequences. On the one hand, an increase in the income ceiling for contributions lowers the contribution rate only in the short run but raises it in the long run. If the income ceiling is raised to share the implicit tax burden more even within a generation, future generations are therefore negatively affected. On the other hand, expanding mandatory membership to additional population groups raises the contribution rate in the long run if these groups have a higher-than-average life expectancy.

This last point is particularly critical in view of the vigorous discussion on the justification of exemptions from mandatory membership in a social insurance system which is built on the principle of solidarity. In particular, the German council of economic experts (Sachverständigenrat zur Begutachtung der gesamtwirtschaftlichen Entwicklung - SVR) has opposed expanding the mandatory membership precisely because the new members are likely to have a higher life expectancy (see e.g. SVR (2001) paragraphs 254-259). Kifmann (2001), however, has demonstrated that including further members can in principle lower the long-run contribution rate if the generations who pay lower contributions due to the expansion receive a corresponding cut in retirement benefits.⁵ His analysis, however, did not take into account that new members could be expected to live longer.

In addition to these effects, we also examine the consequences of an increase in life expectancy and a decrease in the rate of population growth. We find that both demographic changes increase the implicit tax burden for future generations under the current formula, in the latter case due to property 4 above. We therefore conclude that the current formula suffers from a number of drawbacks and turn to two alternative formulas. Firstly, we examine a slightly modified formula which adjusts pension benefits for life expectancy. Secondly, we analyze a formula under which the rate-of-return of individual contributions is set equal to the rate-of-return of the PAYG system as a whole. For the three options we determine how

⁵See Buslei (2002) for a general equilibrium analysis of this proposal.

the contribution rate, the implicit tax rate and the total size of revenues per worker of the public pension scheme react to the exogenous changes. We find that both alternative formulas fare better than the current formula in a number of respects.

The paper is organized in the following way: In Section 2, a very simple overlapping generations model is presented as a basis for the following discussion. In Section 3, we analyze the present benefit formula, Sections 4 and 5 are devoted to the two alternative formulas. Section 6 compares the three formulas. In Section 7 we discuss how the alternative formulas can be implemented. Section 8 concludes.

2. The Model

The model is a simple discrete-time two-period overlapping generations model. Everybody works when young and receives a gross wage in line with his or her productivity. There is no technical progress and workers are divided in two productivity groups with wages w^H and w^L , respectively ($w^H > w^L$). Population grows at a constant rate m and wages at rate g . The interest rate r is exogenous. We assume $1 + r > (1 + m)(1 + g)$, i.e. the rate of return of the PAYG system $((1 + m)(1 + g) - 1 \approx m + g)$ is below the interest rate. In period t , there are N_t workers. A fraction $\gamma_t < 1$ are members of the mandatory old-age pension system. Of these, the share λ_t belongs to the high-income group. The number of workers covered by the pension system may be raised. We assume that all low-income workers are already completely covered. Hence, expanding the PAYG system raises both γ_t and λ_t .

Of all workers of group $i = H, L$ and cohort t , the share π_t^i will live a second period, will stop working and thus collect retirement benefits. The survival probability and hence life expectancy differs across groups: $\pi_t^H > \pi_t^L$.

Retirement benefits are financed on a pay-as-you-go basis by levying a proportional contribution at a rate b_t on all earnings up to a ceiling G_t with $w_t^L < G_t < w_t^H$, so that total contributions of a representative member of either of the two groups are given by $B_t^L = b_t w^L$ and $B_t^H = b_t G_t$. For the ceiling G_t we assume $G_t = \alpha_t w_t^H$, $\alpha_t < 1$. Thus, the ceiling is raised in line with wages unless the parameter α is varied. α corresponds to the level of the earnings ceiling G .

We do not specify the retirement benefit formula at this point, because this will be done in the subsequent sections. But given any such formula, the following questions will be answered: How do (i) the contribution rate, (ii) the implicit tax and (iii) the total size of revenues of the pension system per worker react to the following exogenous and policy changes:

- an increase in life expectancy, i.e. an increase in π^L or π^H ,
- a decline in the rate of population growth m ,
- an increase in the earnings ceiling above its time trend by raising α ,
- an expansion of mandatory membership to additional high-income workers (i.e. a simultaneous increase in γ and λ).

3. The Present Point Formula

According to the existing formula, a worker of a cohort t achieves “earnings points” P_t^i corresponding to the ratio of his contributable income to the mean contributable income of all workers.⁶ Thus, an individual of group $i = L, H$ collects the points

$$P_t^L = \frac{w^L}{\lambda_t G_t + (1 - \lambda_t) w_t^L} \quad (1)$$

$$P_t^H = \frac{G_t}{\lambda_t G_t + (1 - \lambda_t) w_t^L}. \quad (2)$$

The retirement benefit is obtained by multiplying the number of earnings points P_t^i with the so-called “current pension value” (“aktueller Rentenwert”). The latter is the product of the benefit level n and the average contributable income, diminished by the contributions to the pension scheme. In practice, changes in average income lead to an adaptation of retirement benefits with a lag of one year. For the long-run dynamics of the pension system, however, this lagged adaptation is irrelevant. In

⁶See § 70 Sozialgesetzbuch VI.

our model, in which a period comprises a working life, we therefore assume that pensions are instantaneously adapted.⁷ The benefit in period t , x_t^i , is then given by

$$x_t^i = n[\lambda_t G_t + (1 - \lambda_t)w_t^L](1 - b_t)P_{t-1}^i. \quad (3)$$

By inserting (1), (2) and (3) in the budget equation of the pension fund,

$$b_t(\lambda_t G_t + (1 - \lambda_t)w_t^L)\gamma_t N_t = [\lambda_{t-1}\pi_{t-1}^H x_t^H + (1 - \lambda_{t-1})\pi_{t-1}^L x_t^L]\gamma_{t-1} N_{t-1}, \quad (4)$$

we can solve for the break-even contribution rate b_t :

$$b_t = n(1 - b_t) \frac{\gamma_{t-1} N_{t-1}}{\gamma_t N_t} \frac{\lambda_{t-1} \pi_{t-1}^H G_{t-1} + (1 - \lambda_{t-1}) \pi_{t-1}^L w_{t-1}^L}{\lambda_{t-1} G_{t-1} + (1 - \lambda_{t-1}) w_{t-1}^L}. \quad (5)$$

With $G_{t-1} = \alpha_{t-1} w_{t-1}^H$ and $w_{t-1}^i = (1 + g)^{t-1} w_0^i$, we have

$$b_t = n(1 - b_t) \frac{\gamma_{t-1} N_{t-1}}{\gamma_t N_t} \frac{\lambda_{t-1} \pi_{t-1}^H \alpha_{t-1} w_0^H + (1 - \lambda_{t-1}) \pi_{t-1}^L w_0^L}{\lambda_{t-1} \alpha_{t-1} w_0^H + (1 - \lambda_{t-1}) w_0^L}. \quad (6)$$

Assuming $N_t = (1 + m)N_{t-1}$, $\lambda = \lambda_{t-1}$, $\gamma = \gamma_t = \gamma_{t-1}$, $\alpha = \alpha_{t-1}$ and $\pi^i = \pi_{t-1}^i$ and solving for the steady state contribution rate $b^* = b_t$ we obtain

$$b^* = \frac{nf(\alpha, \lambda, \pi^H, \pi^L)}{1 + m + nf(\alpha, \lambda, \pi^H, \pi^L)} \quad (7)$$

with

$$f(\alpha, \lambda, \pi^H, \pi^L) = \frac{\lambda \pi^H \alpha w_0^H + (1 - \lambda) \pi^L w_0^L}{\lambda \alpha w_0^H + (1 - \lambda) w_0^L}. \quad (8)$$

Straightforward calculations show that this function has the following properties:

$$\begin{aligned} \frac{\partial f}{\partial \alpha} &= \frac{(\pi^H - \pi^L)\lambda(1 - \lambda)w_0^H w_0^L}{[\lambda w_0^H + (1 - \lambda)w_0^L]^2} > 0 \\ \frac{\partial f}{\partial \lambda} &= \frac{(\pi^H - \pi^L)\lambda \alpha w_0^H w_0^L}{[\lambda \alpha w_0^H + (1 - \lambda)w_0^L]^2} > 0 \\ \frac{\partial f}{\partial \pi^H} &= \frac{\lambda \alpha w_0^H}{\lambda \alpha w_0^H + (1 - \lambda)w_0^L} > 0 \\ \frac{\partial f}{\partial \pi^L} &= \frac{(1 - \lambda)w_0^L}{\lambda \alpha w_0^H + (1 - \lambda)w_0^L} > 0. \end{aligned} \quad (9)$$

⁷We also do not consider that increases in the maximum contributions to a voluntary “second pillar” are treated as increases of the contribution rate (see § 68 SGB VI). This policy is equivalent to an exogenous reduction of the benefit level.

Observe that

$$\frac{\partial b^*}{\partial f} = \frac{n(1+m)}{[1+m+nf(\alpha, \lambda, \pi^H, \pi^L)]^2} > 0.$$

Using (7) and (9) we get

$$\frac{\partial b^*}{\partial \alpha} > 0, \quad \frac{\partial b^*}{\partial \lambda} > 0, \quad \frac{\partial b^*}{\partial \pi^H} > 0, \quad \frac{\partial b^*}{\partial \pi^L} > 0, \quad \frac{\partial b^*}{\partial m} < 0. \quad (10)$$

Thus, the steady-state contribution rate must not only increase when life expectancy rises and the rate of population growth falls but also when the level of the earnings ceiling is raised or when the share of high-wage workers is increased by expanding mandatory membership to additional high-income groups. As can be seen from the derivatives of $f(\cdot)$, these last two results are only due to the assumption that life expectancy is positively correlated with income: expanding membership to more high-income individuals increases the contribution rate because the average life expectancy of pensioners rises. For the earnings ceiling, the effect can be explained by the changes in the points collected by each group. From equations (1) and (2) we have

$$\frac{\partial P_t^L}{\partial G_t} < 0 \quad \text{and} \quad \frac{\partial P_t^H}{\partial G_t} > 0,$$

i.e. an increase in G_t via α leads to a spread in the points. Hence, high-income individuals receive a larger share of retirement benefits which increases total pension payments due to their higher life expectancy.

As a next step, we calculate the implicit tax rate τ in a steady state.⁸ It is defined by the difference between per-capita contributions and the discounted value of per-capita retirement benefits both divided by the contributable earnings of the respective worker.⁹

⁸See e.g. Thum and von Weizsäcker (2000) and Kifmann and Schindler (2000) for further studies which use this concept.

⁹We thus ignore the regressive effect of the earnings ceiling.

For a low-wage worker we derive:

$$\begin{aligned}
\tau_t^L &= b_t - \pi_t^L \frac{x_{t+1}^{L*}}{(1+r)w_t^L} \\
&= b_t - \pi_t^L \frac{n(1-b_{t+1})[\lambda_{t+1}G_{t+1} + (1-\lambda_{t+1})w_{t+1}^L]P_t^L}{(1+r)w_t^L} \\
&= b_t - \pi_t^L \frac{n(1-b_{t+1})[\lambda_{t+1}G_{t+1} + (1-\lambda_{t+1})w_{t+1}^L]}{(1+r)\lambda_t G_t + (1-\lambda_t)w_t^L}
\end{aligned} \tag{11}$$

In a steady state, we have $\lambda = \lambda_{t+1} = \lambda_t$, $G_{t+1} = (1+g)G_t$ and $w_{t+1}^L = (1+g)w_t^L$. With $\pi^L = \pi_t^L$, the steady state implicit tax rate therefore equals

$$\tau^{L*} = \tilde{b}^* - \pi^L b(1-b^*) \frac{1+g}{1+r}. \tag{12}$$

Substituting for b^* from equation (7) and collecting terms yields

$$\tau^{L*} = \frac{n[f(\cdot)(1+r) - \pi^L(1+m)(1+g)]}{(1+r)(1+m+nf(\cdot))}. \tag{13}$$

Analogously, we get for high-wage workers

$$\tau^{H*} = \frac{n[f(\cdot)(1+r) - \pi^H(1+m)(1+g)]}{(1+r)(1+m+nf(\cdot))} \tag{14}$$

where $\pi^H > \pi^L$ implies $\tau^{L*} > \tau^{H*}$.

The mean implicit tax rate in a steady state is given by

$$\tau^{M*} = \frac{\tau^{H*}\lambda_t G_t + \tau^{L*}(1-\lambda_t)w_t^L}{\lambda_t G_t + (1-\lambda_t)w_t^L}. \tag{15}$$

Inserting (13) and (14) yields

$$\tau^{M*} = \frac{nf(\cdot)(1+r)}{(1+r)(1+m+nf(\cdot))} - \frac{n(1+m)(1+g)[\lambda_t \pi^L G_t + (1-\lambda_t)\pi^H w_t^L]}{(1+r)(1+m+nf(\cdot))(\lambda_t G_t + (1-\lambda_t)w_t^L)}. \tag{16}$$

By using $G_t = \alpha_t w_t^H$, $w_t^i = (1+g)^t w_0^i$ and the definition of $f(\cdot)$ (equation (8)), we obtain

$$\tau^{M*} = \frac{nf(\cdot)[(1+r) - (1+m)(1+g)]}{(1+r)(1+m+nf(\cdot))}. \tag{17}$$

Since we assume that $1 + r > (1 + m)(1 + g)$, the mean implicit tax rate must therefore be positive. Equation (17) implies that the steady state implicit tax rate depends in the same qualitative way as the steady-state contribution rate on α , λ and π^i . Thus, the steady state implicit tax rate reacts positively to (i) an increase in the level of the earnings ceiling α , (ii) an increase in λ by expanding membership to more high-income people, (iii) an increase in life expectancy and (iv) a decrease in the rate of population growth.

Finally, we examine the volume v_t per worker of the pension system, i.e. total expenditures of the pension fund V_t divided by the number of all workers N_t . This indicator shows how much on average is invested in the PAYG system and therefore serves as a measure of the total implicit tax burden imposed by the PAYG system. Since in such a system each period's expenditures must equal current revenues, we obtain for the steady state total expenditures in period t :

$$V_t^* = b^*(\lambda G_t + (1 - \lambda)w_t^L)\gamma N_t.$$

Thus,

$$v_t^* = b^*(\lambda G_t + (1 - \lambda)w_t^L)\gamma. \quad (18)$$

Clearly, this measures increases as well if α is raised, life expectancy increases or the rate of population growth falls. In addition, v_t^* must increase if additional high-income groups are included since this raises both λ and γ .

In summary, we conclude that the Present Point Formula has a number of weaknesses. In particular, it can be argued that it is inequitable that low income individuals face a higher implicit tax rate than high income individuals. Furthermore, policy measures which in principle could spread the implicit tax burden of the pay-as-you-go system in a more equitable way among the population such as raising the earnings ceiling or including additional high-income groups have negative long-run effects. For this reason, the SVR opposes an expansion of mandatory membership. Finally, the Present Point Formula does not cope well with the demographic changes. On the one hand, an increase in life expectancy has to be borne by future generations. On the other hand, a fall in the rate of population growth does not only decrease the rate of return of the PAYG pension system but also raises the contribution rate, and hence the amount invested in the pension system increases.

4. A Life-Expectancy Adjusted Point Formula

A key problem of the Present Point Formula is that it fails to take group-specific life expectancy into account. The straightforward way to avoid this problem is to adjust the earnings points in the following way:

$$\tilde{P}_t^i = \frac{P_t^i}{\pi_t^i}. \quad (19)$$

In this case retirement benefits in period t are given by

$$\tilde{x}_t^i = \tilde{n}[\lambda_t G_t + (1 - \lambda_t)w_t^L](1 - b_t)\tilde{P}_{t-1}^i. \quad (20)$$

Here \tilde{n} is a measure for the benefit level.¹⁰ From the budget constraint (4) and equation (19) and (20) we obtain for the contribution rate

$$\tilde{b}_t = \tilde{n}(1 - \tilde{b}_t) \frac{\gamma_{t-1}N_{t-1}}{\gamma_t N_t}. \quad (21)$$

Assuming, as above, $N_t = (1 + m)N_{t-1}$ and $\gamma = \gamma_t = \gamma_{t-1}$ yields the steady state contribution rate

$$\tilde{b}^* = \tilde{b}_t = \frac{\tilde{n}}{1 + m + \tilde{n}}. \quad (22)$$

Thus, using the Adjusted Point Formula makes the steady state contribution rate independent not only of the life expectancy itself but also of the level α of the earnings ceiling and the share of high-income members, λ . However, a fall in the rate of population growth m continues to raise the steady-state contribution rate.

For the implicit tax rate for low-wage workers we get

$$\begin{aligned} \tilde{\tau}_t^L &= \tilde{b}_t - \pi_t^L \frac{x_{t+1}^{L*}}{(1+r)w_t^L} \\ &= \tilde{b}_t - \pi_t^L \frac{\tilde{n}(1 - \tilde{b}^*)[\lambda_{t+1}G_{t+1} + (1 - \lambda_{t+1})w_{t+1}^L]\tilde{P}_t^L}{(1+r)w_t^L} \\ &= \tilde{b}_t - \frac{\tilde{n}(1 - \tilde{b}^*)[\lambda_{t+1}G_{t+1} + (1 - \lambda_{t+1})w_{t+1}^L]}{(1+r)\lambda_t G_t + (1 - \lambda_t)w_t^L}. \end{aligned} \quad (23)$$

¹⁰Since the Adjusted Point Formula (19) increases the point value, \tilde{n} needs to be lower than n to yield the same retirement benefit as in the Present Point Formula.

In a steady state, we have $\lambda_{t+1} = \lambda_t$, $G_{t+1} = (1+g)G_t$ and $w_{t+1}^L = (1+g)w_t^L$. Thus, the steady state implicit tax rate equals

$$\tilde{\tau}^{L*} = \tilde{b}^* - \tilde{n}(1 - \tilde{b}^*) \frac{1+g}{1+r}. \quad (24)$$

Substituting for \tilde{b}^* from (22) and simplifying yields

$$\tilde{\tau}^{L*} = \frac{[(1+r) - (1+m)(1+g)]\tilde{n}}{(1+r)(1+m+\tilde{n})}. \quad (25)$$

The same result is obtained for $\tilde{\tau}^{H*}$, i.e. the Adjusted Point Formula yields the same implicit tax rate for both income groups. Again, the implicit tax rate is positive if $(1+r) > (1+m)(1+g)$ and increases if m falls.

Note that neither \tilde{b}^* nor $\tilde{\tau}^{i*}$ depend upon α , λ or π^i and are therefore not affected by an increase in the level of the earnings ceiling, the inclusion of the self-employed or an increase in life expectancy. However, the expenditures per worker of the pension system are increased by each of the first two measures. In a steady state this is given by

$$\tilde{V}_t^* = \tilde{b}^*(\lambda G_t + (1-\lambda)w_t^L)\gamma N_t.$$

Thus, the expenditures per worker are

$$\tilde{v}_t^* = \tilde{b}^*(\lambda G_t + (1-\lambda)w_t^L)\gamma. \quad (26)$$

Including new groups of people into the mandatory system as such increases (via γ) the revenue size. This effect is strengthened if at the same time λ increases. Similarly, raising the income ceiling increases the size of the old-age pension system and therefore total implicit taxes. Finally, a decline in the rate of population growth raises \tilde{v}_t^* through \tilde{b}^* .

We conclude that adjusting the earnings points for life expectancy only partially solves the problems of the Present Point Formula. Although the long-run contribution and implicit tax rate are no longer affected by an increase in the earnings ceiling, the inclusion of new groups or an increase in life expectancy, the total implicit tax burden continues to rise upon these changes. Furthermore, a fall in the rate of population growth still increases the contribution rate and therefore the investment in the PAYG pension system.

5. The Return-Rate Formula

The third retirement benefit formula is based on the idea that all contributions to the pension system should yield the same gross return, which is defined by the internal return rate of any pay-as-you-go system, namely the growth rate of total earnings $(1+m)(1+g) - 1$.¹¹ Taking life expectancy and thus the expected duration of retirement into account, the per-period retirement benefit of a member of cohort t is then given by

$$\hat{x}_t^L = \frac{(1+m)(1+g)\hat{b}_{t-1}w_{t-1}^L}{\pi_{t-1}^L} \quad (27)$$

$$\hat{x}_t^H = \frac{(1+m)(1+g)\hat{b}_{t-1}G_{t-1}}{\pi_{t-1}^H}. \quad (28)$$

Solving the budget equation of the pension fund,

$$\begin{aligned} \hat{b}_t[\lambda_t G_t + (1-\lambda_t)w_t^L]\gamma_t N_t &= [\lambda_{t-1}\pi_{t-1}^H \hat{x}_t^H + (1-\lambda_{t-1})\pi_{t-1}^L \hat{x}_t^L]N_{t-1} \\ &= (1+m)(1+g)\hat{b}_{t-1}[\lambda_{t-1}G_{t-1} + (1-\lambda_{t-1})w_{t-1}^L]\gamma_{t-1}N_{t-1} \end{aligned} \quad (29)$$

for the contribution rate \hat{b}_t yields

$$\hat{b}_t = \hat{b}_{t-1}(1+g) \frac{\gamma_{t-1} \lambda_{t-1} G_{t-1} + (1-\lambda_{t-1})w_{t-1}^L}{\lambda_t G_t + (1-\lambda_t)w_t^L}. \quad (30)$$

As long as G is growing at rate g , λ is held constant and $\gamma_t = \gamma_{t-1}$, we have $\hat{b}_t = \hat{b}_{t-1}$, i.e. the contribution rate remains stable. From equation (30) we obtain

$$\begin{aligned} \frac{\partial \hat{b}_t}{\partial \gamma_t} &= -\frac{\hat{b}_t}{\gamma_t} < 0 \\ \frac{\partial \hat{b}_t}{\partial \lambda_t} &= -\frac{\hat{b}_t(G_t - w_t^L)}{\lambda_t G_t + (1-\lambda_t)w_t^L} < 0 \\ \frac{\partial \hat{b}_t}{\partial \alpha_t} &= \frac{\partial \hat{b}_t}{\partial G_t} \frac{\partial G_t}{\partial \alpha_t} = -\frac{\hat{b}_t \lambda_t w_t^H}{\lambda_t G_t + (1-\lambda_t)w_t^L} < 0. \end{aligned} \quad (31)$$

¹¹The same idea underlies the concept of the “notional defined contribution (NDC)” pensions, which have recently gained growing interest in the literature (see, e.g. Williamson (2001)). Also Kifmann (2001) examines a similar formula where the implicit tax rate on earnings is exogenously given. His formula is equivalent to the Return-Rate Formula if all individuals have the same life expectancy and the rate of population growth is constant. In particular, the expansion of mandatory membership lowers the contribution and implicit tax rate and keeps the expenditures of the pension system constant.

The last result is due to $G_t = \alpha_t w_t^H$. Thus, increasing the earnings ceiling in period t , and expanding mandatory membership permanently *decreases* the contribution rate. Changes in life expectancy or in the rate of population growth have no effect on the contribution rate.

The long-run effects therefore differ strongly from the ones under both point formulas. This is the case because the Return-Rate Formula does not automatically lead to an adjustment of retirement benefits if average income changes. Individual contributions always earn a rate of return $(1+m)(1+g) - 1$. In contrast, the point formulas lead to an increase in retirement benefits one period after average income has risen because the retirement benefits are linked to the average earnings of current employees and not to the total wage bill (see equations (3) and (20)). Thus, an increase in average income due to changes in α, λ or γ raises the retirement benefits for some generations even though the rate of return of the PAYG system has not changed. Furthermore, a decrease of the rate of population growth does not lead to a cut in benefits despite the lower rate of return of the pension system. These factors cause the contribution rate to rise in the long run under the point formulas. The Return-Rate Formula avoids these effects.

For the implicit tax rate of a low-wage worker we get

$$\begin{aligned}\hat{\tau}_t^L &= \hat{b}_t - \pi^L \frac{\hat{x}_t^L}{(1+r)w^L} \\ &= \hat{b}_t - \frac{1+r}{1+m} \hat{b}_t \\ &= \hat{b}_t \frac{r-m}{1+r}\end{aligned}\tag{32}$$

The same result is obtained for $\hat{\tau}_t^H$. Thus, the implicit tax rate is independent of π^i , whereas, via \hat{b}_t , it reacts to changes of α, λ and γ . In particular, the implicit tax rate falls when either of these variables increases.

The expenditures of the pension system per worker are given by

$$\hat{v}_t = \hat{b}_t (\lambda_t G_t + (1 - \lambda_t) w^L) \gamma_t.\tag{33}$$

Using equation (30) we obtain

$$\hat{v}_t = \hat{b}_{t-1} (\lambda_{t-1} G_{t-1} + (1 - \lambda_{t-1}) w^L) \gamma_{t-1} = \hat{v}_{t-1},\tag{34}$$

	increase in α	increase in λ and γ	increase in π^i	decrease in m
Present Point F.	+ / + / +	+ / + / +	+ / + / +	+ / + / +
Adjusted Point F.	0 / 0 / +	0 / 0 / +	0 / 0 / 0	+ / + / +
Return-Rate F.	- / - / 0	- / - / 0	0 / 0 / 0	0 / + / 0

Table 1: Long-run effects on the contribution rate / (average) implicit tax rate / expenditures per worker

i.e. the expenditures per worker are independent of G_t, λ_t and γ_t , i.e. changing α, λ and γ in period t has no effect on the implicit tax per worker! Furthermore, the rate of population growth does not change \hat{v}_t .

As with the Adjusted Point Formula, the Return-Rate Formula therefore yields equal implicit tax rates for both income groups. In contrast to the former, however, the latter is always size-neutral: By raising the earnings ceiling or expanding mandatory membership an intragenerational redistribution is effected, which implies a lower implicit tax rate for the initial membership, while a shift of the burden to later generations is avoided. Furthermore, a decline in the rate of population growth does not affect the expenditures and hence the implicit tax burden per worker due to the PAYG system.

6. Comparing the formulas

Table 1 summarizes and compares the effects of the various exogenous changes on three different measures of the size of the pension system (contribution rate, average implicit tax rate and expenditures per worker) under the three benefit formulas analyzed in this paper.

We observe from row 1 that under the Present Point Formula all three measures of the size of the pension system go up when either the membership or the basis of contributions is expanded or demographic ageing occurs. Row 2 demonstrates that these results are mainly due to the failure to adjust retirement benefits for life ex-

pectancy, because under the Adjusted Point Formula the contribution rate and the implicit tax rate are not affected by any of these exogenous changes except for a decline in fertility. Finally row 3 shows that only the Return-Rate Formula makes the contribution rate and the total size of the pension system immune against a decline in fertility, whereas a rise in the implicit tax burden is impossible to achieve in this case.

Of course, which of the three formulas is preferable depends upon the norms that a public pension system is expected to meet: If horizontal and vertical equity within a generation are regarded as important, then the Adjusted Point Formula and the Return-Rate Formula are superior to the Present Point Formula. They equalize the implicit tax rate and avoid negative effects on this rate when either new groups are forced to join or the contribution as a share of income is made more equitable by raising the earnings ceiling. If it is furthermore considered a valuable goal to avoid shifting the implicit tax burden to future generation, then the Return-Rate Formula is the best choice. Finally, intergenerational risk sharing can be an objective for the public pension system. Here, the Return-Rate Formula provides insurance against income risk (see Thøgersen (1998)) but not against changes in fertility. If, however, fertility is seen not as an aggregate risk, but rather as a choice variable, this effect may even be judged as desirable.

Our analysis has also shown that the objections of the German council of economic experts against an expansion of mandatory membership is only justified under the Present Point Formula. Once retirement benefits are adjusted for life expectancy, expanding membership has no more adverse effects on the long-run contribution and implicit tax rate. Furthermore, the result by Kifmann (2001) is shown to hold for the Return-Rate Formula as well. If the pension formula takes the higher life expectancy of high-income groups into account, then both the contribution and the implicit tax rate can be permanently lowered.

7. Implementation

What are the institutional and practical preconditions that have to be fulfilled to implement either the Adjusted Benefit Formula or the Return-Rate Formula?

To start with the former rule, it has to be decided how income is used as a predictor for life expectancy:

- a) in a limited number of income classes or as a continuous variable,
- b) as period income or on a lifetime income basis?

Income classes are easier to understand for the public and are more appropriate if the underlying relationship between income and life expectancy is non-linear. On the other hand, they invite attempts in particular by persons near the class boundaries to be classified in a lower class in order to get a better return on their contributions. Moreover, income classes are extremely difficult to form when the income variable to be used is lifetime income.

On the other hand, a continuous variable “period income” makes it easy to use a point schedule and to simply sum the points earned over the life cycle. There are two problems with this approach: First, low income as a result of part-time work will be misinterpreted as an indicator of short life expectancy. However, this is a typical phenomenon in tax systems which are based on observed income rather than (unobservable) potential income and thus should not be overemphasized. Secondly, to correctly reflect the increasing life expectancy, the point schedule must be concave in period income, which implies that people with more heavily fluctuating income get a lower return on their contributions than others with the same lifetime income. This may not be unjust after all if the maximum income earned over the life cycle is a good indicator of capabilities and if the latter are at the root of differences in life expectancy.

Another possible predictor for life expectancy is “education” which could be used as a discrete variable with a limited number of possible realizations, e.g. 1) no completed school career, 2) basic school diploma, 3) middle school diploma, 4) high school diploma, 5) university degree. This variable has several advantages over “income”. First, education is normally completed early in life so that in-

centives to manipulate this variable to receive a more favorable treatment in the public pension system are not important. Secondly, the underlying "true" relationship between education and life expectancy is probably even stronger than the one between income and life expectancy,¹² and finally the above mentioned problem of misclassification of part-time workers vanishes.

In addition to these problems, the Return-Rate Formula requires that the internal rate of return must be officially measured. But this task is very similar to the one in the existing practice of annual adjustments of the pension level. The only difference is that the reference is changed from the growth rate of average earnings per employed worker to the growth rate of the total wage bill.¹³ The statistical problem of defining and measuring the appropriate concepts remains virtually the same.

8. Concluding Remarks

We have shown in this paper that a number of undesirable features of the present German pension system can be avoided by changing the formula which links retirement benefits to the contributions of the respective member. In particular, an alternative to the current formula is a formula which takes differences in life expectancy into account and rewards each Euro of contributions with the internal rate of return of the PAYG pension system. Apart from equalizing the implicit tax rate, this Return-Rate Formula lowers the implicit tax rate when either new groups are forced to join or the contribution as a share of income is made more equitable by raising the earnings ceiling. Furthermore, it avoids shifts of the implicit tax burden to future generation if the the income base of the PAYG system is extended or when the rate of population growths declines.

In future research, the properties of the Return-Rate formula could be explored in more depth. In particular, possible incentive effects of this formula on individual behavior towards labor supply, savings and retirement could be analyzed in a

¹²For the United States, there are several studies which confirm the strong positive association between educational achievement and life expectancy. See, e.g. Liebman (2002).

¹³Note that the growth rate of the wage bill of members of the pension system cannot be used because this measure is affected by changes in the earnings ceiling and membership.

general equilibrium analysis. In an overlapping generations model in which individuals work for more than one period, different elasticities of labor supply over the life-cycle could be taken into account as in Fenge, Übelmesser, and Werding (2002). Finally, the issue of intergenerational risk sharing could be studied in a stochastic framework. By considering labor market imperfections, this would also allow to analyze whether current pensioners should share the unemployment risk of current workers.

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