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Working Paper
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DIW Discussion Papers, No. 314

Provided in Cooperation with:
German Institute for Economic Research (DIW Berlin)

Suggested Citation: Fritsche, Ulrich; Kouzine, Vladimir (2002) : Do leading indicators help to predict business cycle turning points in Germany?, DIW Discussion Papers, No. 314, Deutsches Institut für Wirtschaftsforschung (DIW), Berlin

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Ulrich Fritsche and Vladimir Kouzine

Do Leading Indicators Help to Predict Business Cycle Turning Points in Germany?

Berlin, November 2002
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Do Leading Indicators Help to Predict Business Cycle Turning Points in Germany?*

by Ulrich Fritsche** and Vladimir Kouzine***

Abstract:
Using a binary reference series based on the dating procedure of Artis, Kontolemis and Osborn (1997) different procedures for predicting turning points of the German business cycles were tested. Specifically, a probit model as proposed by Estrella and Mishkin (1997) as well as Markov-switching models were taken into consideration. The overall results indicate that the interest rate spread, the long-term interest rate as well as some monetary indicators and some survey indicators can help predicting turning points of the business cycle.

JEL Classification: F.32, C.22, C25

Key words: Business cycle, leading indicators, probit model, McFadden's R², Markov switching models

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* The authors would like to thank Jörg Döpke, Jan Gottschalk, Gustav A. Horn, Sabine Stephan, Jürgen Wolters, and Kirsten Lommatzsch for helpful comments.

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1 Introduction

Leading indicators and their properties are of great practical relevance for business cycle research and forecast. In a companion paper\(^1\) business cycles' leading indicators for Germany were assessed according to specific requirements.

According to these requirements a reliable leading indicator should possess the following properties: (1) movements in the indicator series should resemble those in the business cycle reference series; (2) the relationship between the reference series and the indicator should be statistically significant and stable over time; (3) the inclusion of the indicator in out-of-sample forecasting procedures should improve the predictive power (compared to a "naïve" autoregressive prognosis).

The companion paper did, however, not answer an important and unmentioned fourth question: How well do leading indicators perform in forecasting turning points of the business cycle? This is of great practical interest since, in most cases, forecasters fail to forecast recessions. There are two main reasons for this: First, most collections of "stylised facts" of the business cycle mention that recessions happen suddenly and unexpectedly.\(^2\) Therefore recessions are by nature quite difficult to predict. Second, it is possible that some forecasters may hesitate to forecast recessions because they do not want to be blamed for creating a "self-fulfilling prophecy". The ambiguous quality of forecasting by German research institutes can partly be explained by having missed the turning points of the business cycle.\(^3\)

Figure 1 clearly shows that missing the turning point (the researches forecasts t-1 as a turning point instead of t, which is the "true" turning point) leads to a completely different average growth rate (scenario 1 and 2 respectively): This fact is well known by professional forecasters but regularly misunderstood in the public discussion. This misunderstanding is of great practical relevance since users of professional forecasts often only take note of the annual growth rate of GDP (which in fact is a result of the business cycle movement). Missing the turning point disturbs the forecast and has negative consequences for the reputation of the forecasting institution.

Therefore the question of this paper is: Are there information in leading indicators which can help forecasting the turning points of the business cycle?

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3 Cf. Döpke/Langfeldt (1994).
Traditional approaches that are used to investigate the properties of leading indicators focus on their behaviour over the whole cycle. To analyse the usefulness of indicators in forecasting turning points, however, binary or qualitative approaches have to be used. During the last couple of years, probit models have therefore attracted attention.

What is the research program of our paper? This paper is about assessing the behaviour of leading indicators at business cycle turning points and their ability to forecast the turning point. First, a binary time series for recession/boom periods had to be constructed (section 2). Because there is some degree of freedom in doing this, we decided to use the well-known and established procedure proposed by Artis/Kontolemis/Osborn (1997). Second, the properties of indicator variables to forecast a turning point had to be assessed. In this paper two completely different methods were tested: a probit model and a Markov switching model. In the probit model (section 3.1) indicator variables were regressed on the binary time series at a varying lag structure and a measure that is comparable to the well-known R² was calculated for each lag. In this paper a version of McFadden's R² as proposed by Estrella (1998) was used. The local maximum of the R² was interpreted as the lag with the highest probability of forecasting a turning point. For instance a local maximum at lag 8 should be interpreted as the (highest probable) "lead" of the indicator with respect to the business cycle turning point.

The probit approach was inspired by the paper of Estrella and Mishkin (1997). These authors, however, did not take into account the information inherent in the binary time series. As Dueker (1997) and Döpke (1999) pointed out, significant autocorrelation in the binary time series can disturb the results. Therefore we also tested a second version of the probit model in this paper checking whether lagged indicator series contributes to the explanation of the binary series in addition to an autoregressive process.

During the last couple of years Markov switching models became more and more popular. By construction, these models seem to be perfectly suited for the analysis of our problem (section 3.2). The Markov switching model is a "regime dependent" approach, whereby the probability of the regimes is modelled as a so-called Markov chain (see the detailed explanation in section 3.2). The regimes are unobservable and hidden in the data but their probability can be extracted using specific estimation techniques.

We assume a two-regime Markov process (which can be interpreted as a business cycle framework with boom and recession periods) and estimated univariate Markov switching models for each indicator. We asked if there is some information about the probability of a change in the

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5 We exploit a two regime business cycle approach (boom-recession-approach), cf. Artis/Kontolemis/Osborn (1997). There are, however, good reasons to think about a multiple-regime approach, cf. Heilemann/Muench (1999).
regime of the economy (from a recession to a boom phase and vice versa), which can be detected in the leading indicator series with a "lead" compared to the binary reference series. The time series of the recession probabilities derived from each indicator series were therefore also converted into a binary series and compared to the binary reference series at varying lags. The idea behind this approach is the following: If it is possible to detect the state of the regime in the leading indicator series "before" the business cycle passes a turning point (as measured by our binary reference series), this indicator seems to be a good leading indicator for predicting the turning points.

By using different approaches we were able to compare the results to identify "reliable" indicators (section 4). This serves as a robustness check. To guarantee the comparability with the companion paper, we have used the same data set here.

2 Determination of the Reference Series

Dating recessions is not invariant with regard to the method that is applied. The often-used detrending procedures have major theoretical and practical weaknesses. And there are different views of the business cycle as such. We decided to use a dating procedure developed by Artis/Kontolemis/Osborn (1997) to specify the recession and boom periods. This procedure has its drawbacks as well, but several advantages: The method was used for other studies for G-7 countries and the results are therefore easily comparable, the results can easily be reproduced and the results come close to definitions of the cycle which are used by practitioners. The idea behind the procedure of Artis/Kontolemis/Osborn (1997) goes back to the NBER approach of dating business cycles. The reference series is Germany's industrial production as it was in our companion paper. This time series will be analysed in original values and in a seven-month moving average representation. First outliers are identified and eliminated. Possible turning points (local maxima or minima that are in a range 12 months forward or backward) have to show up in both series, the original one and the moving-average representation. To be qualified as a turning point, some further conditions regarding the strength of the decline in output with


9 From a methodological point of view, detrending procedures are based on strong assumptions about the data-generating process and the kind of association between trend and fluctuations; from a practical point of view the generated trends and business cycle components often miss some "stylised facts" such as the often-cited business cycle asymmetry. Cf. Canova (1998a,b); Tichy (1994).

10 Cf. Tichy (1994), who distincts the Europaean approach (cyclical movements are deviations from a potential/trend) from the North American approach (booms and recessions are periods where a variety of predefined time series move in the same direction).


12 For instance the widely known rule of thumb that a recession is defined by two consecutive quarters of declining output.

respect to the period preceding the turning point have to be met. The result of this procedure applied to German industrial production is displayed in Figure 2 (shaded areas indicate recessions).

By visual inspection, the dating procedure of Artis/Kontolemis/Osborn (1997) seems to fit downswings in the reference series quite well and was therefore used as a base to construct the binary time series. For further analysis this binary time series serves as the reference series.

3 Methodology

3.1 Probit model

Following Estrella and Mishkin (1997), we used binary time series where the value one stands for recession and the value zero for non-recession periods. In our paper this binary series is based on the dating procedure proposed by Artis/Kontolemis/Osborn (1997). Estrella and Mishkin (1997) had been in the favourable situation that for the U.S. economy there is an official Business Cycle Dating Committee at NBER, which regularly publishes a schedule of booms and recession which can be used as a base for the construction of a respective binary time series.

We estimated a probit equation explaining the probability that a recession occurs \(R_t = 1\) by using lagged indicator time series [model I]:

\[
\text{Prob}(R_t = 1) = \Phi(\beta_0 + \beta_1 I_{t-k})
\]

In other words, we asked for the ability of the indicator to explain a recession period. Estrella (1998) proposed a modified McFadden's Pseudo-R^2 to test how good and at which lag an indicator series can predict recessions. This measure computes a Log-Likelihood ratio of the model under investigation compared to a model, which does not take the information of the more general model into account. In our case we compare the Log-Likelihood of model I, the model including the indicator, to the Log-Likelihood of a model where the binary series is only regressed on a constant (= unconstrained model):

\[
\text{Pseudo-R}^2 = 1 - \left[ \frac{L_u}{L_c} \right]^{(2)}
\]

where

- \(L_u\)...unconstrained Log-Likelihood (of the model)
- \(L_c\)...constrained Log-Likelihood (\(\beta_1 = 0\))
- \(n\)...number of observations

---


15 The original McFaddens R^2 is defined as \(1 - \frac{L_u}{L_c}\). The version proposed in Estrella (1998) furthermore adjusts for the number of regressors.
The higher the Log-Likelihood of model I in comparison to the unconstrained model becomes, the lower is the Log-Likelihood ratio and the closer is the (Pseudo)-$R^2$ to the value of 1.\footnote{The measure is called Pseudo-$R^2$ because it is a different concept compared with the well-known $R^2$ and in fact it only can come close to 1 but not equal to zero.}

The local maximum of the modified McFadden's $R^2$ – the point where the inclusion of the indicator mostly improves the forecasting quality – is interpreted as the "lead" of the indicator.

The main shortcoming of this approach – as mentioned by Dueker (1997) and Döpke (1999) – is the fact that the traditional probit estimation can be mis-specified if there is information content in the binary time series which is not taken into consideration. Or, as Dueker (1997: 45) described it: "...(I)t is implausible to assume that the conditional mean of $u_t$ is zero without reference to whether the economy has actually been in recession in recent periods." In traditional time series approaches we solve this problem by taking into account an autoregressive moving average filter. Here we use a similar technique.

Therefore we expanded the approach and specified the equation as model II:

$\text{Prob}(R_t = 1) = \Phi(\beta_0 + \beta_1 I_{t-k} + \beta_2 R_{t-k})$

We control, if the economy is still in a recession at time $t-k$ we measure the signal of the indicator. The Pseudo $R^2$ is now calculated in the same manner as explained above. The model II now yields the Log-Likelihood $L_u$. The restricted model with $\beta_1 = 0$ yields $L_c$. So here we tested for the information content that the indicator contributes to explain a recession additionally to those information already contained in the autoregressive structure of the binary time series. The estimation results at different lag lengths are shown in Figures 3 to 6.

**Insert Figures 3 to 6 about here**

### 3.2 Markov switching models

The crucial point of modeling of business cycles using Markov switching models is the decomposition of any observable economic time series into two parts: an unobservable discrete state and the remaining short-run autoregressive dynamics. The unobserved state variable is assumed to represent the fluctuations of the business cycle, which are unobservable in practice, too. The broadly accepted view of the business cycle as a series of contractions and expansions implies the discrete nature of the state variable.

A simple way to approximate the business cycle dynamics is given by a Markov chain with two possible states. The parameters of such a simple Markov chain are probabilities, which govern the transitional dynamics between two regimes. Figure 7 is an attempt to describe the model in an intuitive way:

**Insert Figure 7 about here**
The conditional probability \( \Pr\{B|B\} \), for example, is the probability to stay in a boom conditional on the fact, that the economy is actually booming. Obviously, all probabilities, conditional on the same regimes, are summing up to one. All probabilities are conditional only on the last state; therefore such a Markov chain is called a first order Markov chain. If the values of the probabilities \( \Pr\{B|B\} \) and \( \Pr\{R|R\} \) are close to one this in turn leads to a high persistence of the regimes.

The information content of Figure 3 can easily be represented in matrix form. The matrix of the transition probabilities is called the transition matrix \( P \)

\[
P = \begin{bmatrix} \Pr\{B|B\} & \Pr\{R|B\} \\ \Pr\{B|R\} & \Pr\{R|R\} \end{bmatrix},
\]

where \( \Pr\{B|B\} + \Pr\{R|B\} = \Pr\{B|R\} + \Pr\{R|R\} = 1 \). The Markov chain described above is a quite abstract stochastic process. It needs not to have some real valued realizations; only a set of possible regimes has to be defined. However, the Markov switching technique allows the real valued quantification of economic variables. Therefore, the mapping of the space of regimes into a parameter space of the data-generating process is necessary. In other words, some parameters of the data-generating process are assumed to be a continuous function of the discrete Markov chain. For the purpose of business cycle modeling it is straightforward to allow the intercept (or mean) of the estimated process to be dependent from some discrete Markov chain with two possible states. The following part of the subsection gives some analytical aspects of the methodology described above.

The Markov switching model is a special case of the generalized state-space model\(^{17}\). Let \( S_t \) be a discrete unobserved state variable following an ergodic first-order Markov chain with \( N \) states \( s_t \in \{1,2,\ldots,N\} \) and a transition matrix

\[
P = \begin{pmatrix} p_{11} & p_{12} & \cdots & p_{1N} \\ p_{21} & p_{22} & \cdots & p_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ p_{N1} & p_{N2} & \cdots & p_{NN} \end{pmatrix},
\]

where \( p_{ij} = \Pr\{s_{t+1} = j|s_t = i\}, \sum_{i=1}^{N} p_{ij} = 1 \quad \forall i,j \in \{1,2,\ldots,N\} \).

Let an observable leading indicator series \( x_t \) follow an autoregressive process of order \( p \)

\[
x_t - \mu(s_t) = \alpha_1 (x_{t-1} - \mu(s_{t-1})) + \cdots + \alpha_p (x_{t-p} - \mu(s_{t-p})) + u_t
\]

or an alternative specification

\[
x_t = \nu(s_t) + \alpha_1 x_{t-1} + \cdots + \alpha_{p-1} x_{t-p} + u_t
\]

\(^{17}\) Cf. Krolzig (1997).
where \( u_t \sim \text{NID}(0, \sigma) \) and the mean \( \mu(s_t) \) or the intercept \( \nu(s_t) \) are functions of the unobserved state variable \( S_t \). These specifications are denoted by MSM(N)-AR(p) and MSI(N)-AR(p) or Markov switching mean and Markov switching intercept. The states of the Markov chain \( S_t \) are not directly observable, therefore the statistical inference about any state \( j, j \in \{1, 2, \ldots, N\} \) is necessary. The subject of interest is the estimated probability \( \Pr(s_t = j | x_t; \Theta) \) for the state \( j \) in \( t \), conditional an all observations of \( x_t \) obtained through date \( t \) and the vector of all known parameters \( \Theta \). Under assumption of known parameters the rule of Bayes leads to the following non-linear recursive algorithm\(^{18}\):

\[
\Pr(s_t = j | x_t; \Theta) = \frac{f(x_t | s_t = j, x_{t-1}; \Theta) \Pr(s_t = j | x_{t-1}; \Theta)}{\sum_j f(x_t | s_t = i, x_{t-1}; \Theta) \Pr(s_t = i | x_{t-1}; \Theta)}
\]

or in vector form

\[
\hat{\xi}_{t+t} = \left( \hat{\xi}_{t+t-1} \otimes \eta_t \right) 1' \left( \hat{\xi}_{t+t-1} \otimes \eta_t \right)
\]

where \( \hat{\xi}_{t+t} \) and \( \eta_t \) are the vectors of \( \Pr(s_t = j | x_t; \Theta) \) and \( f(x_t | s_t = j, x_t; \Theta), j \in \{1, 2, \ldots, N\} \), \( \hat{\xi}_{t+t-1} = P \hat{\xi}_{t+t} \). \( \otimes \) denotes the element wise multiplication of vectors.

The likelihood function \( L(\Theta) \) for the observed indicator \( x_t \) evaluated at the value of \( \Theta \) that was used to perform the iterations can be calculated as a by-product of the recursive algorithm:

\[
L(\Theta) = \sum_{t=1}^{T} \log f(x_t | x_{t-1}; \Theta),
\]

where \( f(x_t | x_{t-1}; \Theta) = \sum_j f(x_t | s_t = j, x_{t-1}; \Theta) \Pr(s_t = j | x_{t-1}; \Theta) \). To obtain the estimates \( \hat{\Theta} \), the Expectation-Maximization (EM) algorithm can be used\(^{19}\). The EM algorithm is an iterative ML estimation technique designed for the general class of models, where the observed time series depends on some unobservable stochastic variables.

For the purpose of business cycle research, contractions and expansions can be modelled as realisations of the discrete Markov chain \( S_t \) with 2 states (\( N=2 \)). To get the inference about the states of the Markov chain, however, a Markov switching process has to be estimated\(^{20}\). The best-fitted model was selected.\(^{21}\) For most of the indicator series MSI(2)-AR(1)/-AR(2) and MSM(2)-AR(1)/-AR(2) yield reasonable results.

Insert Figures 8 to 9 about here

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\(^{20}\) A wide class of Markov switching models can be estimated using MSVAR for Ox 2.10 written by Hans-Martin Krolzig.

\(^{21}\) According to standard information criteria.
The obtained time series of the estimated recession probabilities \( \Pr_{x,t} = 1 \mid X_t ; \hat{\Theta} \) can be used to make conclusions about the current state of the business cycle. The time series of the recession probabilities are converted into binary series of 0 and 1 denoted by \( R^1_t \) according to the 50%-rule as follows:

\[
R^1_t = \begin{cases} 
0 & \text{if } \Pr_{x,t} = 1 \mid X_t ; \hat{\Theta} > 0.5 \\
1 & \text{if } \Pr_{x,t} = 1 \mid X_t ; \hat{\Theta} < 0.5
\end{cases}
\]

Thus \( R^1_t \) series are compared with the reference binary series \( R_t \). The share of correctly classified months can be calculated as a function of lead \( k \) from

\[
\text{Share}(k) = \frac{1}{n} \sum_{i=k}^{n} |R^1_{i+k} + R_i - 1|
\]

where \( n \) is the number of observations in the sample. If the local maximum of the share lies in the lead area \( (k>0) \), then the indicator series \( x_t \) is considered as a leading indicator. The sample of computation is 1979:1-1997:12\(^{22}\)

The function \( \text{Share}(k) \) is, of course, a quite descriptive measure of the indicator's predicting power, but at least it should be possible to distinguish the series in two subgroups: leading indicators and time series which have no indicator properties. Moreover, the graphs of \( \text{Share}(k) \) can be compared with the time series of the estimated recession probabilities to prove the plausibility of results.

**Insert Figures 10 to 11 about here**

### 4 Results

The results of the probit model exercise are shown in the Figures 3 to 6; the results of model I are represented by the thick line, those of model II by a dotted line. The \( R^2 \) for model II captures only the information content, which goes beyond the information inherent in the binary time series. However, to save space we plot both graphs together.

The results of the Markov switching models are plotted in the Figures 8 to 11, the Figures 8 and 9 show the best fitted model for each indicator; the reference business cycle dating according to Artis/Kontolemis/Osborn (1997) is given by the shadings. The results of the \( \text{Share}(k) \) computations are shown thereafter (Figures 10 to 11).

Frankly, the results are not at all satisfactory. Only some indicators showed a strong local maximum in the probit models – indicating a stable lead of this indicator with respect to turning points. This is true for \( ifo \) business expectations of producers of intermediate input (lead: three months), for the long-term nominal interest rate (lead: eleven months), for the interest rate

\(^{22}\) The computation sample 1979:01-1997:12 was selected shorter than the previous sample 1978:1-1998:12 due to the method of dating recessions and therefore possible problems at the ends of the sample.
spread (lead: four months) as well as for the money base M2 (lead: eleven months). The result for the interest rate spread is in line with the often-cited literature on forecasting quality based on this measure. Both probit models generally confirm the results. The value of the modified McFadden's-R² is generally lower for the second model. This is not surprising because we tested for additional information, which is not included in the binary time series itself. In the case of the interest rate spread as well as the long-term interest rate, the lead changed significantly – it is longer for the restricted model (in the case of the *ifo* business expectations of producers of intermediate input, the second model shows a lead of seven months instead of three months, in case of the interest rate spread, the second model shows a lead of seven months instead of four months).

The results of the Markov switching model estimates are more or less in line with those of the probit models. The comparison of the binary series according to the coefficient of correlation can of course only give a rough idea about the quality of the indicator series. However, it seems to be possible to identify two groups of indicator series: there are some indicators where the maximum of the coefficient of correlation lies definitely outside the "lead" area, whereas a second group seems to have leading indicator properties. The best leading indicators according to that measure seem to be: the interest rate spread, real and nominal money base M1 and the long-term interest rate. The *ifo* business expectation for intermediate input as well as for manufacturing industry seem to have leading indicator properties as well. So, there is a little group of possible leading indicators according to the methods in use here.

It is interesting to note, that 13 of 16 indicators gave a "false alarm" in 1989/1990 according to the Markov switching models – indicating that a recession was on the way at this time but was circumvented by the reunification boom.

The overall results for *ifo* indicators could indicate that they perform better as coincident indicators – which are in line with some tests in the companion paper. There is however another interesting result when the results of this paper are compared with the investigations in the companion paper. Whereas in the first paper (Fritsche/Stephan, 2002) the question was "Can indicators help in forecasting the annual growth rate of a reference series?" the question now became "Can indicators be useful in forecasting the turning points of the cycle?" The question is "yes" for both questions, but for different indicators. The indicators, which performed quite well in the first paper, were mainly order inflows and *ifo* (expectation-based) indicators. These indicators however performed badly if the question is the signalling of turning points (with the notable exception of the *ifo* business expectations of producers of intermediate input). In contrast to that finding, the interest rate spread, the long-term interest rate as well as the money base M2 performed bad in the first investigation but they are perhaps useful tools for the timely detection of turning points.

In general, we find no clear evidence that any of the indicators under investigation can be solely used for the purpose of identifying turning points. How can the results be interpreted? There is
some evidence that especially monetary indicators are useful to forecast turning points. The interest rate spread, long-term interest rate as well as the money supply M2 seem to be a good indicator for that specific purpose. On the other hand, our results could indicate that recessions in Germany were to a large extent caused by endogenous forces. Especially the "false signal" of a recession around 1989, indicated by most of the Markov switching models, which did not turn the economy into recession (because of the reunification boom), can be interpreted in this way. The recession, however, was not avoided but occurred two years later (after the influence of the "exogenous" special factor re-unification disappeared). This contradicts the Real Business Cycle school paradigm that exogenous shocks are the only driving force of business cycle movements. Further research should concentrate on the mechanisms driving endogenous cycles.
5 References


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Figure 1

Turning Points and Annual Growth Rates

Figure 2

Recession Phases in Germany
Figure 3

McFadden's $R^2$ in Probit Models to Predict German Recessions\(^1\)

Index of New Orders
Producers of Investment Goods

Real Effective Exchange Rate

Manufacturing Industry

Spread (Government Bonds and private Bonds)

Producers of Intermediate Input

 Consumer-Sentiment-Indicator

\(^1\) McFadden's Pseudo-R-Squared = 1 - (L_0/L_c)^{K(2/n)L_c}.\)
Figure 4

McFadden's R² in Probit Models to Predict German Recessions

1) McFadden's Pseudo-R-Squared = 1 - (L₀/L₀) - (2/n)L₀.

1) McFadden's Pseudo-R-Squared = 1 - (L₀/L₀) - (2/n)L₀.
McFadden's $R^2$ in Probit Models to Predict German Recessions

1) McFadden's Pseudo-R-Squared = 1 - $(L_0/L_D)^{(-2/n) L_0}$.
McFadden’s $R^2$ in Probit Models to Predict German Recessions

1) McFadden’s Pseudo-R-Squared = $1 - (L_u/L_c)^{1/(2/n)L_c}$. 
Figure 7

Recession

Boom

$Pr\{B \mid B\}$

$Pr\{B \mid R\}$

$Pr\{R \mid B\}$

$Pr\{R \mid R\}$
Figure 8

Filtered Probabilities

Ifo Business Expectations
Producer of Investment Goods
MSI(2)-AR(1)

Ifo Business Climate
Producer of Investment Goods
MSI(2)-AR(2)

Ifo Business Expectations
Manufacturing Industry
MSI(2)-AR(3)

Ifo Business Climate
Manufacturing Industry
MSI(2)-AR(2)

Ifo Business Expectations
Producers of Intermediate Input
MSI(2)-AR(1)

Ifo Business Climate
Producers of Intermediate Input
MSI(2)-AR(1)

Index of New Order
Producers of Investment Goods
MSM(2)-AR(1)

Index of New Order
Manufacturing Industry
MSM(2)-AR(1)
Figure 11

Index of New Order
Producers of Intermediate Input
MS(2)-AR(1)

Consumer Sentiment Indicator
MS(2)-AR(1)

Months Lead
Share

Interest Rate Spread
MS(2)-AR(2)

Real Money Supply/M1
MS(2)-AR(1)

Months Lead
Share

Nominal Money Supply/M1
MS(2)-AR(1)

Long-Term Interest Rate (Umlaufrendite)
MS(2)-AR(2)

Months Lead
Share

Spread between Government and Private
MS(2)-AR(1)

Real Credit Supply
MS(2)-AR(1)

Months Lead
Share