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A critical comparison of migration policies: Entry fee versus quota

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Oded Stark, Lukasz Byra, Alessandra Casarico, and Silke Uebelmesser

A critical comparison of migration policies: Entry fee versus quota

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Abstract

We ask which migration policy a developed country will choose when its objective is to attain the optimal skill composition of the country’s workforce, and when the policy menu consists of an entry fee and a quota. We compare these two policies under the assumptions that individuals are heterogeneous in their skill level as well as in their skill type, and that individuals of one skill type, say “scientists,” confer a positive externality on overall productivity whereas individuals of the other skill type, say “managers,” do not confer such an externality. We find that a uniform entry fee encourages self-selection such that the migrants are only or mostly highly skilled managers. The (near) absence of migrant scientists has a negative effect on the productivity of the country’s workforce. Under a quota: the migrants are (a) only averagely skilled managers if the productivity externality generated by the scientists is weak, or (b) only averagely skilled scientists if the productivity externality generated by the scientists is strong. In (a), a uniform entry fee is preferable to a quota. In (b), a quota is preferable to a uniform entry fee. If, however, the entry fee for scientists is sufficiently below the entry fee for managers, then migrants will be only or mostly highly skilled scientists, rendering a differentiated entry fee preferable to a quota even when the productivity externality is strong. Instituting a differentiated fee comes, though, at a cost: the fee revenue is not as high as it will be when migrants are only or mostly managers. We conclude that if maximizing the revenue from the entry fee is not the primary objective of the developed country, then a differentiated entry fee is the preferred policy.

Keywords: International migration; A quota; A uniform entry fee; A differentiated entry fee; Heterogeneous human capital; Optimal skill composition of the developed country’s workforce; Total factor productivity

JEL classification: D62; F22; J24
1. Introduction

Countries that receive migrants regularly evaluate their policies, and assess and weigh the advantages and disadvantages of alternative rules and admission procedures. Take the case of the US. Ever since The Immigration Restriction Act of 1921, the US has controlled the inflow of migrants by means of quotas, selecting migrants by their characteristics. At the outset, quotas were based on nationality, yet with the enactment of the Immigration and Nationality Act of 1965, the focus shifted to migrants’ skills and family ties to US citizens. Several other migrant-receiving countries such as Canada, Australia, and New Zealand have had in place skill-based admission procedures.¹

If a receiving country seeks to admit skilled workers, and if it does that by means of a skill-selective quota, economics-based reasoning would suggest a seemingly simpler tool: selling the right to enter. The idea proposed by Freeman (2006) and Becker (2011), among others, is as follows. If the private returns from migration, as measured by a prospective migrant’s earnings, increase with the migrant’s skill level, then it would be more beneficial for high-skilled individuals to migrate than for low-skilled individuals. Consequently, the imposition of a high enough entry fee will discourage low-skilled individuals for whom the cost of entry will be higher than the gain from increased earnings. If the number of migrants decreases with the level of the entry fee, fine-tuning the fee will also control the number of migrants.

This seemingly attractive policy may not be as appealing as it might appear at first sight. It stands to reason that individuals differ not only in their skill level, but also in their skill type (Willis, 1986; Grogger and Eide, 1995; Iyigun and Owen, 1998, 1999; Krueger and Lindahl, 2001; Stark and Zakharenko, 2012), that different skill types generate different social returns, and that the skill types that generate high social returns (high production externalities) are not at the upper end of the pay distribution. Recent studies attest to this. For example, Peri et al. (2014, 2015) present evidence of the significant impact of STEM workers (Scientists, Technology professionals, Engineers, and Mathematicians) on total factor productivity in US cities. However, in 2015 the annual mean wage of a mathematician was 80 percent of the annual mean wage of a marketing manager, and 60 percent of the annual mean wage of a chief executive (BLS, 2015). In such a constellation, levying an entry fee may discourage migration by individuals with relatively low private returns but high social returns, who

¹ Kerr et al. (2016) discuss how the US, Canada, and Australia have used skill-based admission procedures.
would migrate under a selective quota based on skill type. The absence of such individuals among the migrants can have a negative effect on the overall productivity in the receiving country.

A more attractive admission policy could be based on a differentiated entry fee: individuals in an occupation that generates high production externalities but pays a relatively low wage such as science, will be charged a fee that is far enough below the fee charged to individuals in an occupation that generates little production externalities but pays a relatively high wage such as management. A careful calibration of the fees will benefit the receiving country by attracting workers of the desirable skill type.

There are few analyses of the implications of introducing an entry fee. Collie (2009) considers entry fee revenue as a means of compensating the native inhabitants for the lower terms of trade caused by the expansion of export industries following the arrival of migrants. Chao et al. (2013) suggest that entry fee revenue could be used to compensate the native inhabitants for the congestion in public services caused by migrants. Bianchi (2013) studies a setting in which migrants are heterogeneous in skill level, refers to fees or bureaucratic requirements that can be levied and imposed on the migrants by the receiving country, and assesses how such impositions affect the level of migration and the skill level of migrants. The desirable and undesirable effects of selective migration policies on the quality of migrants are studied by Bertoli et al. (2016). In this paper, we study the implications of introducing an entry fee from a different angle.

We develop an analytical framework that enables us to compare two admission procedures: a selective quota based on skill type, and an entry fee (either uniform or differentiated). Under these two admission procedures we first study the impact of “opening up” to migration on the skill composition of the workforce in the receiving country, and we then assess which policy is better from the perspective of the native inhabitants (workers), henceforth natives, in that country. As a baseline, we consider a setting with no migration. Workers in a developed country are characterized by their endowments and preferences. They differ in their exogenously given skill level (productivity) and in the value that they attach to working in a prestigious occupation (occupational prestige); and they derive utility from consumption and from occupational prestige. A single consumption good is produced by workers of two types: “scientists” and “managers.” By raising the economy’s total factor productivity (TFP), scientists generate externalities that boost the productivity of the entire workforce. Working as a scientist confers prestige, whereas working as a manager does not.
However, managers are compensated for the lack of occupational prestige by earnings that are higher than those of scientists. Given this setting, we let the developed country receive migrants from a developing country under the two alternative admission procedures mentioned above.

Our main findings are as follows. A uniform entry fee encourages self-selection such that most or all of the migrants are highly skilled managers. The (near) absence of migrant scientists has a negative effect on the productivity of the country’s workforce. Under a quota: the migrants are (a) only averagely skilled managers if the productivity externality generated by the scientists is weak, or (b) only averagely skilled scientists if the productivity externality generated by the scientists is strong. In (a), a uniform entry fee is preferable to a quota. In (b), a quota is preferable to a uniform entry fee. If, however, the entry fee for scientists is far enough below the entry fee for managers, then all or most migrants will be highly skilled scientists, rendering a differentiated entry fee preferable to a quota even when the productivity externality is strong. Instituting a differentiated fee comes, though, at a cost: the fee revenue is not as high as it will be when all or most migrants are managers. We conclude that if maximizing revenue from the entry fee is not the primary objective of the developed country, then a differentiated entry fee is the preferred policy.

The remainder of this paper is structured as follows. In Section 2 we present a benchmark model of a developed country with no migration. In Section 3 we let the country “open up” to migration under a selective quota or under a uniform entry fee, and we study the extent to which the developed country can control the skill composition of migration under these two policies. In Section 4 we calculate the optimal level and skill composition of migration under a selective quota and under a uniform entry fee, and we compare these two policies. In Section 5 we compare a selective quota with a differentiated entry fee. In Section 6 we bring entry fee revenue into the picture and study the extent to which the developed country can simultaneously maximize its fee revenue and attain the optimal size and skill composition of its workforce. Section 7 concludes.

2. A no-migration setting in a developed country

Consider a developed country populated by a continuous set of individuals (workers) of measure one. Individuals in this country work in an occupation of their choice, and derive utility from consumption and occupational prestige. There are two occupations to choose...
from: science, denoted by $S$, and management, denoted by $M$. Initially, individuals differ in their productivity in the labor market, and in their preference for occupational prestige. The utility function of an individual in occupation $j = S, M$ is

$$u_j = \ln c_j + \kappa(j) \varepsilon,$$  

(1)

where $c_j$ denotes consumption, $\kappa(j)$ is a function such that $\kappa(S) = 1$ and $\kappa(M) = 0$, and $\varepsilon$ is a random variable defined over the interval $[0, E]$, $E \in \mathbb{R}_+$, with a probability distribution function and a cumulative distribution function denoted, respectively, by $f(\cdot)$ and $F(\cdot)$, such that $f(z) = F'(z) > 0$ for all $z \in [0, E]$. The variable $\varepsilon$ measures the individual’s preference for working in a prestigious occupation, with both $\kappa(S) = 1$ and $\kappa(M) = 0$ implying that only science is considered prestigious.\footnote{According to a recent Harris Poll (Birth, 2016), working as a scientist in the US is ranked second in terms of occupational prestige, with 83 percent of the respondents considering that occupation prestigious, whereas the corresponding rank for a business executive is seventeen, with 59 percent of the respondents considering that occupation prestigious.}

\footnote{It could be argued that if the notion of prestige is expanded to include other non-pecuniary job characteristics (such as power or control), then some individuals might prefer management to science (when earnings in the two occupations are controlled for). The results obtained in this paper carry through qualitatively to a setting in which some individuals prefer management to science (for the same level of wages in both occupations) if the share of such individuals is sufficiently small. Such an assumption seems plausible: despite a prevailing wage differential, a great many bright college graduates choose science rather than management.}

That $\varepsilon$ varies across individuals reflects the observation that the value attached by individuals to working in a prestigious occupation depends on individual-specific characteristics such as personality, values, and family background.\footnote{The construction of our model is inspired by the structure of the model of Fan and Stark (2011). In particular, the formulation of the utility function, as well as the properties of the preference towards one occupation as opposed to another, as delineated below, are akin to those in Fan and Stark (2011), with the difference that whereas Fan and Stark (2011) consider occupational stigma, we consider occupational prestige.}

The consumption of an individual is equal to his earnings, which, in turn, are given by the individual’s skill level, or productivity in the labor market, $\theta$, times the wage per unit of productivity, $w_j$, namely $c_j = \theta w_j$. We assume that $\theta$, which is the same in both occupations, is a random variable over the interval $(0, T]$, $T \in \mathbb{R}_+$, with a probability distribution function and a cumulative distribution function denoted, respectively, by $g(\cdot)$ and $G(\cdot)$, such that $g(z) = G'(z) > 0$ for all $z \in (0, T]$. Mean productivity is $\overline{\theta} = \int_0^T \theta g(\theta) d\theta = 1$.\footnote{According to a recent Harris Poll (Birth, 2016), working as a scientist in the US is ranked second in terms of occupational prestige, with 83 percent of the respondents considering that occupation prestigious, whereas the corresponding rank for a business executive is seventeen, with 59 percent of the respondents considering that occupation prestigious.}

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We assume that productivity and preference for occupational prestige are distributed independently in the population, namely that $\text{cov}(\varepsilon, \theta) = 0$.

At the beginning of his life, each individual chooses his occupation by comparing utilities. Science will be preferred to management if

$$u_S = \ln(\theta_{wS}) + \varepsilon \geq \ln(\theta_{wM}) = u_M,$$

or, equivalently, if

$$\varepsilon \geq \ln w_M - \ln w_S.$$  \hfill (3)

Individuals for whom $\varepsilon < \ln w_M - \ln w_S$ will choose management. The supplies of scientists, $L'_S$, and of managers, $L'_M$, are, respectively,

$$L'_S = P(\varepsilon \geq \ln w_M - \ln w_S) = 1 - F(\ln w_M - \ln w_S) \quad \text{and} \quad L'_M = F(\ln w_M - \ln w_S),$$  \hfill (4)

where superscript $s$ indicates supply.

A large number of competitive firms employ scientists and managers to produce the economy’s consumption good, which is sold at a unit price. The production of firm $i$, $Y_i$, is

$$Y_i = A(l)\left(\bar{\theta}_SL_S\right)^\alpha \left(\bar{\theta}_ML_M\right)^{1-\alpha},$$  \hfill (5)

where $\bar{\theta}_j$ denotes the average productivity of workers of type $j = S, M$ employed by firm $i$, $L_j$ denotes the size of the workforce of type $j$ employed by firm $i$, $\bar{\theta}_j L_j$ are the effective units of work of type $j$ employed by firm $i$, and $\alpha$ and $1-\alpha$, $0 < \alpha < 1$, are the output elasticities of scientific work and of managerial work, respectively.\footnote{Even though productivity of an individual is the same in either occupation, the average productivity of workers employed by a single firm can vary between occupations.} $A(l)$, the economy’s total factor productivity (TFP) common to all the firms, depends on the effective units of scientific work in the economy’s workforce according to the function

$$A(l) = l^\eta = \left(\frac{\bar{\theta}_SL_S}{W - \bar{\theta}_SL_S}\right)^\eta,$$  \hfill (6)
where \( \bar{\bar{\delta}}_s = \frac{1}{L_s} \sum \bar{\bar{\delta}}_s L_s \) and \( L_s = \sum L_s \) are the average productivity and the aggregate size of the scientific workforce, respectively, \( W = \bar{\bar{\delta}} \) is the size of the effective workforce, and where \( \eta > 0 \), a measure of the strength of the externality generated by the scientists, is a constant such that \( \eta < 1 - \alpha \).

Because there are many firms in the economy, the employment decisions of any single firm cannot dent the ratio of scientists to managers in the economy’s workforce; a single firm is too small to affect the ratio. A profit maximizing firm will employ effective units of work of type \( j \) up to the point at which the marginal product of the effective unit of work of each type is equal to the market wage per unit of productivity, namely up until

\[
w_s = \alpha A(l) \left( \frac{\bar{\bar{\delta}}_m L_M}{\bar{\bar{\delta}}_s L_S} \right)^{1-\alpha} \text{ and } w_m = (1-\alpha) A(l) \left( \frac{\bar{\bar{\delta}}_s L_S}{\bar{\bar{\delta}}_m L_M} \right)^{\alpha}.
\]

Upon dividing \( w_m \) by \( w_s \) in (7) and rearranging, we obtain the relative demand for the effective work of firm \( i \),

\[
\frac{\bar{\bar{\delta}}_s L_S}{\bar{\bar{\delta}}_m L_M} = \frac{\alpha}{1-\alpha} \frac{w_m}{w_s}.
\]

Because firms are identical and face the same market wages, it follows from (8) that the ratio of effective units of scientific work to effective units of managerial work employed by each firm is the same, which implies that this is also the market ratio, namely \( \frac{\bar{\bar{\delta}}_s L_S}{\bar{\bar{\delta}}_m L_M} = \frac{\bar{\bar{\delta}}_s L_S}{\bar{\bar{\delta}}_m L_M} \), where \( \bar{\bar{\delta}}_M = \frac{1}{L_M} \sum \bar{\bar{\delta}}_m L_M \) is the average productivity of the managerial workforce, and \( L_M = \sum L_M \) is the aggregate size of the managerial workforce. Therefore, we can replace the ratio of effective units of scientific work to effective units of managerial work employed by a particular firm in (7) and (8) with the ratio of the aggregate scientific workforce to the aggregate managerial workforce to obtain the profit maximization conditions

\[
w_s = \alpha A(l) \left( \frac{\bar{\bar{\delta}}_m L_M}{\bar{\bar{\delta}}_s L_S} \right)^{1-\alpha} \text{ and } w_m = (1-\alpha) A(l) \left( \frac{\bar{\bar{\delta}}_s L_S}{\bar{\bar{\delta}}_m L_M} \right)^{\alpha},
\]

and the market relative demand for work
\[
\frac{\partial \bar{L}_S}{\partial \bar{L}_M} = \frac{\alpha}{1-\alpha} \frac{w_M}{w_S}.
\] (10)

Because an individual’s occupational choice depends on the wage per unit of productivity and on the individual’s preference for working in a prestigious occupation, but not on his productivity, the expected representation of individuals with different levels of productivity will be the same in the two occupations. We assume that the actual representation of the individuals in the two occupations is equal to the expected representation, which implies that \(\bar{s} = \bar{m} = \bar{\theta} = 1\). Upon utilizing this together with the \(L_s + L_m = 1\) constraint on the size of the workforce, we get that (10) yields the aggregate demand for scientists and for managers, respectively:

\[
L_s^d = \frac{\alpha w_M}{1-\alpha + \alpha \frac{w_M}{w_S}} \quad \text{and} \quad L_m^d = \frac{1-\alpha}{1-\alpha + \alpha \frac{w_M}{w_S}},
\] (11)

where superscript \(d\) indicates demand.

In equilibrium, \(L_j^i = L_j^d\) and, therefore, from equalization of the left-hand sides, or, equivalently, of the right-hand sides of (4) with (11), we get that in equilibrium

\[
F \left( \ln w_M - \ln w_S \right) = \frac{1-\alpha}{1-\alpha + \alpha \frac{w_M}{w_S}}.
\] (12)

We denote by \(w\) the wage ratio of managerial work to scientific work, \(w = \frac{w_M}{w_S}\). Utilizing this, (12) is rewritten as

\[
F \left( \ln w \right) = \frac{1-\alpha}{1-\alpha + \alpha w}.
\] (13)

And we denote by \(w^n\) the value of \(w\) that solves (13), where the superscript \(n\) indicates the equilibrium level of a variable in the no-migration setting. We now have the following proposition.

**Proposition 1.** (a) \(w^n\) exists, and is unique. (b) \(w^n > 1\).

**Proof.** The proof is in Appendix A.
Proposition 1 aligns with the principle of a “compensating wage differential,” which applies when there are non-pecuniary aspects of different occupations, in our case a prestige component in the individual’s utility function. Because \( w^n = \frac{w^n_M}{w^n_S} > 1 \), managers are compensated for not working in a prestigious occupation by means of wages that are higher than those of scientists.

Upon utilizing \( w^n \) in (4) and (11), we obtain

\[
L^n_S = 1 - F(\ln w^n) = \frac{\alpha w^n}{1 - \alpha + \alpha w^n} \quad \text{and} \quad L^n_M = F(\ln w^n) = \frac{1 - \alpha}{1 - \alpha + \alpha w^n},
\]

where the middle parts of each of the expressions in (14) are the equilibrium supplies of scientific and managerial work, and where the right-hand parts are the equilibrium demands for scientific and managerial work, respectively. Thereafter, by inserting the right-hand sides of (14) into \( w^n_S \) and \( w^n_M \) in (9), and into \( A(I) \) in (6), we obtain, respectively, the equilibrium values of the wages paid per unit of scientific work and per unit of managerial work

\[
w^n_S = \alpha^{\alpha+\eta} (1 - \alpha)^{1-\alpha-\eta} (w^n)^{\alpha+\eta-1} \quad \text{and} \quad w^n_M = \alpha^{\alpha+\eta} (1 - \alpha)^{1-\alpha-\eta} (w^n)^{\alpha+\eta}.
\]

3. Introducing migration

In this section, we let the developed country, referred to henceforth as the “receiving” country, accept migrants from a developing country, referred to henceforth as the “sending” country, under two alternative migration regimes: a selective quota based on skill type, henceforth a quota, and a uniform (flat) entry fee. At this stage, we do not “allow” the receiving country to set different fees for different skill types. The reason for that is that we seek to highlight the importance of accounting for skill type heterogeneity in policy formation.\(^6\)

Let the workforce in the sending country consist of workers of the same two types as in the receiving country. The sending country is assumed to be less developed than the receiving country, which is reflected in lower wages of scientists and managers per unit of

\(^6\) In a simple model with a single skill type we show that, unlike a quota, an entry fee can be used to attract the most productive migrants. The model is available on request.
productivity. To enable us to concentrate on essentials, we assume that the preference “premium” for working in a prestigious occupation, as well as the distribution of productivity in the labor market, are universal. The size of the migration inflow is expressed as a fraction of the native workforce (which, it will be recalled, is of measure one). We denote by $Q_s$ the stock of migrant scientists, and by $Q_m$ the stock of migrant managers admitted by the receiving country under a given migration admission policy. We assume that before the receiving country opens up to migration, the ratio of the wage (per unit of productivity) paid to managers in the sending country to the wage paid to scientists in that country is the same as the corresponding ratio of the wages in the receiving country, namely that

$$w^F_{m} / w^F_{s} = w^F = w^F > 1,$$  \hfill (16)

where $w^F_j$ is wage per unit of productivity paid to workers of type $j$ in the sending country, $F$.\footnote{This assumption is equivalent to assuming that both $\alpha$ and $F(\cdot)$ are universal.} From a rewrite of (16), and on recalling that $w^n_j > w^F_j$ for $j = S, M$, we get that

$$w^n_m - w^F_m > w^n_s - w^F_s,$$  \hfill (17)

namely absent migration, the wage difference between the two countries is higher for managers than for scientists.\footnote{It might be argued that scientific work is utilized more in production in a developed country than in a developing country ($\alpha > \alpha'$), or that working as a scientist in a developed country is associated with greater prestige than working as a scientist in a developing country ($\kappa(S) > \kappa'(S)$). In such cases, the balance of the returns from migration will tilt in favor of scientific work. However, managers will continue to benefit more from migration if the production technologies in the two countries are not too distinct (if $\alpha'$ does not exceed $\alpha$ by too much) or if the gain in prestige reaped by scientists upon migration is not too large (if $\kappa'(S)$ does not exceed $\kappa'(S)$ by too much).}

To further aid us focusing on essentials, we also assume that migration is small relative to the size of the workforce in the sending country, which implies that the wages of scientists and managers in that country are not affected by migration and can, thus, be considered exogenous to the model.\footnote{The assumption that migration will be small relative to the size of the workforce in the sending country is not crucial for this model; the results reported in this paper carry through qualitatively to the case with a relatively large flow of migrants. That wages in the sending country do not change on the departure of migrants, whereas the wages in the receiving country do change with the migrants’ arrival, is internally consistent if we assume that the workforce in the sending country is much larger than the workforce in the receiving country. In turn, it is...} Finally, we assume that the receiving country deciphers without cost...
the skill type of migrants, but not their productivity (a migrant’s productivity is his private information).

The purpose of this section is threefold. First, we study the composition of migration by skill type and by migrants’ productivity under two alternative migration policies set by the receiving country. Second, we investigate the impact of migration on the equilibrium in the labor market in that country under each migration policy. Third, we enlist results that will be used to study the repercussions of migration for the optimal skill composition of the workforce in the receiving country in Section 4.

3.1. Migration under a quota

The receiving country chooses the quota of migrant scientists, $Q_s$, and the quota of migrant managers, $Q_m$; then, aware of the announced migration policy, the natives make their occupational choices. Upon the arrival of $Q_s$ scientists and $Q_m$ managers, there will be $\tilde{L}_s = L_s + Q_s$ scientists and $\tilde{L}_m = L_m + Q_m$ managers in the receiving country. Because the receiving country cannot select migrants by their productivity, and because the distribution of productivity in the sending country is the same as in the receiving country, the average productivity of the migrants will be the same as that of the natives, $\bar{\theta}_m = \bar{\theta}_s = 1$, where superscript $m$ indicates a magnitude that pertains to the migrants. Therefore, $\tilde{L}_s = L_s + Q_s$ and $\tilde{L}_m = L_m + Q_m$ also denote effective units of scientific work and of managerial work, respectively. The TFP under a quota is given by $A(\tilde{L}) = \left( \frac{\tilde{L}_s}{\tilde{W} - L_s} \right)^{\gamma}$, where $\tilde{W} = 1 + Q_s + Q_m$ denotes the size of the effective workforce under a quota. As in the no-migration setting, firms employ effective units of scientific work and of managerial work up to the point where their marginal product equals their respective wages, and we assume that the firms are indifferent as to whether they employ a native or a migrant. By replicating the steps taken in the no-migration setting, and upon adding the constraint on the size of the workforce, $\tilde{L}_s + \tilde{L}_m = \tilde{W}$, we obtain the size of the scientific workforce and the size of the managerial workforce under a quota, namely

easy to justify this assumption if we treat the sending country as the rest of the world - a collection of countries that are less developed than the receiving country.
\[
\bar{L}_S^q = 1 - F\left(\ln w^q\right) + Q_S = \frac{\alpha w^q}{1 - \alpha + \alpha w^q} \bar{W} \quad \text{and} \quad \bar{L}_M^q = F\left(\ln w^q\right) + Q_M = \frac{1 - \alpha}{1 - \alpha + \alpha w^q} \bar{W}, \tag{18}
\]

and the wages paid per unit of productivity to the two types of workers,

\[
w_S^q = \alpha^{\eta + \eta} (1 - \alpha)^{1 - \eta} (w^q)^{\alpha + \eta} \quad \text{and} \quad w_M^q = \alpha^{\eta + \eta} (1 - \alpha)^{1 - \eta} (w^q)^{\alpha + \eta}, \tag{19}
\]

where \(w^q\) constitutes the value of \(w\) which equates the supply of workers of each type with the demand for workers of each type, and where, henceforth, superscript \(q\) denotes the equilibrium level of a variable under a quota. By following a similar procedure as in the proof of Proposition 1, it can be shown that \(w^q\) exists, that it is unique, and that \(w^q > 1.10,11\)

Having established the size of the scientific workforce and the size of the managerial workforce in equilibrium under a quota, we ask how they relate to their counterparts in the no-migration setting. We have the following proposition.

**Proposition 2.** Under a quota, as compared to the no-migration setting: (a) \(w^q = w^n\), if the composition of migration by skill type is the same as the composition of the native workforce, \(Q_S = L^n_S ; Q_M = L^n_M\); (b) \(w^q > w^n\), if the composition of migration by skill type is such that migrants are only or mostly scientists, \(Q_S > L^n_S ; Q_M > L^n_M\); (c) \(w^q < w^n\), if the composition of migration by skill type is such that migrants are only or mostly managers, \(Q_S < L^n_S ; Q_M < L^n_M\).

**Proof.** The proof is in Appendix A.

Proposition 2 reveals how the composition of migration by skill type affects the ratio of the (per unit of productivity) wage of managers to the wage of scientists, which, as exhibited in (18) and (19), uniquely determines the division of the native workers between skill types and the wages per unit of productivity of the two skill types. When we divide the right-hand

---

10 Unlike in the no-migration setting, under a quota it is possible that, in equilibrium, *all* the natives will choose the same occupation, in which case the other occupation will be manned entirely by migrants. Throughout we assume that not all the natives prefer the same occupation. We note that the reported results carry through qualitatively to the case in which all the natives choose the same occupation when migration becomes an option.

11 We note that \(w^n\) is a function of \(Q_S\) and \(Q_M\). When modeling in Section 4 the optimal choice of the level and composition of migration by skill type, we allow \(Q_S\) and \(Q_M\), and thereby \(w^q\), to vary. Thereafter, for the sake of brevity, we will still use the notation \(w^q\) rather than \(w^q(Q_S, Q_M)\).
side of the first formula in (18) by the right-hand side of the second formula in (18), we get that $w^q$ also determines the ratio of (the effective units of) scientific work to (the effective units of) managerial work. For example, for part (b) in Proposition 2, we have that fewer natives choose to become scientists, $L^q_s < L^n_s$; the wages of scientists decrease, and the wages of managers increase, $w^q_s < w^n_s$ and $w^q_M > w^n_M$, respectively; and the ratio of effective units of scientific work to managerial work increases, $\frac{L^q_s}{L^q_M} > \frac{L^n_s}{L^n_M}$. Thus, if the receiving country seeks to increase the share of (the effective units of work of) one skill type in its workforce, it should set a relatively large quota for that skill type, and a relatively small quota for the other skill type.

3.2. Migration under a uniform entry fee

Suppose now that the receiving country introduces a uniform entry fee: anyone who pays the fee, irrespective of the type of skill, can come. For a given entry fee, each worker in the sending country calculates his returns from migration net of the entry fee in order to determine whether migration pays off. Because scientists and managers experience the same level of occupational prestige in both countries, the reasons underlying the decision to migrate are purely pecuniary. An individual in the sending country will choose to migrate as long as the entry fee is lower than the gross gain in earnings upon migration, that is, as long as

$$\begin{cases} 
(w_s - w^F_s) \theta^m > x & \text{if he is a scientist} \\
(w_M - w^F_M) \theta^m > x & \text{if he is a manager},
\end{cases}$$

(20)

where $x$ is the entry fee.

We seek to find how the introduction of a uniform entry fee instead of a quota affects the composition of migration by skill type and the distribution of the migrants by their productivity, thus determining the equilibrium in the labor market of the receiving country. The timing of events is as follows. First, the receiving country sets the entry fee, bearing in mind that any individual will choose to migrate as long as his earnings net of the entry fee at destination are higher than his earnings at home. Then, the natives, aware of the level and composition by skill type of migration, make their occupational choices.

We first inquire what the composition of migrants by skill type and by skill level will be under the uniform entry fee. We have the following result.
**Proposition 3.** Under a uniform entry fee: (a) the composition of migration by skill type is such that migrants are all or mostly managers, \( \frac{Q_S}{Q_M} < \frac{L^n_S}{L^n_M} \); (b) for each fee-induced level of migration, the corresponding composition of migration by skill type is fixed; (c) migrants are of higher productivity than under a quota.

**Proof.** The proof is in Appendix A.

The logic underlying part (a) of Proposition 3 is as follows. Under a uniform entry fee, the receiving country cannot admit exclusively scientists because no level of the entry fee renders it beneficial for scientists, but not for managers, to pay the fee and migrate. It is also impossible to increase the ratio of scientific work to managerial work over the corresponding ratio in the no-migration setting because when both skill types face the same entry fee, any decrease of the fee aimed at encouraging more scientists to come will also encourage more managers to come. It is a direct implication of part (b) of Proposition 3 that a uniform entry fee imposes limitations on the receiving country with respect to the set of feasible choices of the composition of migration by skill type. When the same fee applies to both skill types, fine-tuning the fee creates simultaneously incentives or disincentives to migrate for both skill types. Consequently, for a given overall level of migration, the composition of migration by skill type is fixed. The mechanism behind part (c) of Proposition 3 follows from (20): under a given uniform entry fee, only some foreign managers or only some foreign scientists are willing to migrate, and these are those whose productivity is sufficiently high. Thus, unlike under a quota where the group of migrants is a random selection of the foreign workers, an entry fee encourages positive self-selection by the migrants.\(^{12}\)

We proceed with determining the equilibrium in the labor market. Under a uniform entry fee, because the fee leads to self-selection by the migrants such that migrants are from the upper end of the distribution of productivity, the average productivity of the migrants will be higher than that of the natives, \( \bar{\theta}_j^n > 1 \) for \( j = S, M \). Moreover, because the wage per unit of productivity is different between scientists and managers, it follows from (20) that for two individuals with the same productivity but who work in different occupations, the decision whether to migrate might be different. For this reason, the average productivity of the migrants will not be equal for the two skill types, namely \( \bar{\theta}_S^n \neq \bar{\theta}_M^n \). Therefore, upon the

\(^{12}\) How migrants self-select has recently been studied by Dequiedt and Zenou (2013).
arrival of $Q_S$ scientists and $Q_M$ managers, there will be $\hat{L}_S = L_S + \bar{\theta}_S^m Q_S$ effective units of scientific work, and $\hat{L}_M = L_M + \bar{\theta}_M^m Q_M$ effective units of managerial work in the receiving country. Because $\bar{\theta}_j^m > 1$ for $j = S, M$, then, for a given level and composition of migration by skill type, there are more effective units of each skill type in the receiving country under a uniform entry fee than under a quota. The TFP under a uniform entry fee is given by

$$A(i) = \left( \frac{\hat{L}_S}{\hat{W} - \hat{L}_S} \right) ^ \eta,
$$

where $\hat{W} = 1 + \bar{\theta}_S^m Q_S + \bar{\theta}_M^m Q_M$ denotes the size of the effective workforce under a uniform entry fee. By replicating the steps taken in the no-migration setting and under a quota, and upon adding the constraint on the size of the effective workforce, $\hat{L}_S + \hat{L}_M = \hat{W}$, we obtain the size of the effective workforce of scientists and the size of the effective workforce of managers under a uniform entry fee, namely

$$\hat{L}_S^{uef} = 1 - F(\ln w^{uef}) + \bar{\theta}_S^m Q_S \quad \hat{L}_M^{uef} = F(\ln w^{uef}) + \bar{\theta}_M^m Q_M,$$

and the wages paid per unit of productivity to the two types of workers,

$$w_S^{uef} = \alpha^{\alpha+\eta} (1 - \alpha)^{\lambda - \eta} (w^{uef})^{\alpha+\eta-1} \quad \text{and} \quad w_M^{uef} = \alpha^{\alpha+\eta} (1 - \alpha)^{\lambda - \eta} (w^{uef})^{\alpha+\eta},$$

where $w^{uef}$ constitutes the value of $w$ which equates the supplies of effective units of work of each type with their demands, and where, henceforth, superscript $uef$ denotes the equilibrium level of a variable under a uniform entry fee.

We now compare the repercussions of implementing a uniform entry fee and a quota for the equilibrium in the labor market of the receiving country. From Proposition 3 we know that if the receiving country seeks to increase the share of effective units of scientific work in its workforce, it cannot do that by means of a uniform entry fee. However, if it seeks to increase the share of effective units of managerial work in its workforce, it can achieve that by means of either a quota or a uniform entry fee. Therefore, a meaningful comparison to perform is between a uniform entry fee and a quota, controlling for the level and composition by skill type of migration (which can be the same under the two policies). Clearly, the productivity of the migrants will be different under the two policies. We then have the following proposition.
Proposition 4. \( w^\text{eff} < w^q < w^n \) if migration is of the same level and composition by skill type under a quota as under a uniform entry fee.

Proof. The proof is in Appendix A.

From Proposition 4 it follows that opening up to migration under a uniform entry fee brings about changes in the labor market that are akin to those resulting from opening up to migration under a quota, when most or all migrants are managers (cf. part (c) of Proposition 2). That \( w^\text{eff} < w^q \) is a direct result of the fact that the average productivity of the migrants is higher under a uniform entry fee than under a quota, which strengthens the impact of migration on the wages of both skill types in the receiving country as compared to a quota. When applied to (21) and (22), and compared, respectively, with (18) and (19), and with (14) and (15), the inequalities in \( w^\text{eff} < w^q < w^n \) establish that under a uniform entry fee, for migration of the same level and composition by skill type as under a quota, more natives choose to become scientists, \( L^\text{eff}_S > L^q_S > L^n_S \), the wage per unit of scientific work is higher, and the wage per unit of managerial work is lower, namely \( w^\text{eff}_S > w^q_S > w^n_S \) and \( w^\text{eff}_M < w^q_M < w^n_M \), respectively. Also, when we divide the right-hand sides of the first formulas in (21), (18), and (14) by the right-hand sides of the second formulas in (21), (18), and (14), respectively, and invoke \( w^\text{eff} < w^q < w^n \), we get that a uniform entry fee leads to the lowest ratio of effective units of scientific work to managerial work. Therefore, when the objective of the receiving country is to increase the share of (the effective units of) managerial work in its workforce, a uniform entry fee is more effective than a quota, because migration “delivers” more productive workers under the former policy than under the latter. However, if the receiving country seeks to increase the share of (the effective units of) scientific work in its workforce, it should not enact a uniform entry fee.

4. A quota vs. a uniform entry fee: The optimal policy

Which of the two policies considered in Section 3 should the receiving country adopt when its objective is to improve the welfare of the native population? To make this assessment, we introduce a measure of the welfare of the natives of the receiving country: a utilitarian social welfare function, SWF, defined as
\[ \text{SWF}^k(Q_S, Q_M) = \int_{\ln w^k}^{\ln w^k T} \int_{0}^{E} u^k_S g(\theta) f(\varepsilon) d\theta d\varepsilon + \int_{\ln w^k}^{T} \int_{0}^{E} u^k_M g(\theta) f(\varepsilon) d\theta d\varepsilon, \quad (23) \]

where superscript \( k = n, q, uef \) indicates the type of equilibrium; where the boundaries of the integrals are yielded by \( \theta \in (0, T) \) and \( \varepsilon \in [0, E] \), and upon recalling that the individuals for whom \( \varepsilon < \ln w^k_M - \ln w^k_S = \ln w^k \) will choose management, whereas the individuals for whom \( \varepsilon \geq \ln w^k \) will choose science.\(^{13,14}\)

In this section, we first search for the level and skill composition of migration that maximize the welfare of the natives of the receiving country under a quota and under a uniform entry fee, and we next ask which of the two policies delivers a higher maximum welfare level.

We assume that, combined, the migration of scientists and managers cannot exceed the limit \( Q \), namely that \( Q_S + Q_M \leq Q \).\(^{15}\) We consider only the impact of migration on the welfare of the natives via the labor market effects, referring to the SWF as displayed in (23), not taking into account the entry fee revenue; the revenue effect will be considered in Section 6.

\(^{13}\) Managing migration as a policy tool for enhancing welfare has been at the core of several papers that study the welfare of the population of the sending country (Stark and Wang, 2002; Fan and Stark, 2007a, 2007b; Bertoli and Brücker, 2011; Stark et al., 2012; Stark and Zakharenko, 2012; Byra, 2013), and that study the welfare of the population of both the receiving country and the sending country (Stark et al., 2009a., 2009b, 2012).

\(^{14}\) In (23), \( Q_S \) and \( Q_M \) are control (exogenous) variables not only under a quota, but also under a uniform entry fee. Formally, under a uniform entry fee the control variable (namely the instrument of migration policy controlled by the receiving country) is the fee, \( x \), and \( Q_S \) and \( Q_M \) are its functions. However, because \( Q_S \) and \( Q_M \) are monotonically decreasing in \( x \), there are only one value of \( Q_S \) and one value of \( Q_M \) that correspond to a given \( x \), and vice versa. Therefore, when calculating the optimal solution, we can just as well reverse the causality, namely treat \( Q_S \) and \( Q_M \) as control variables themselves, as long as they are interdependent as per part (b) of Proposition 3, and we can then determine the fee that corresponds to the chosen \( Q_S \) and \( Q_M \). We take this approach especially because it renders the results obtained in the quota setting and in the uniform entry fee setting comparable. And we adhere to this approach also in Section 5, where we introduce a differentiated entry fee.

\(^{15}\) The exogenous limit to the level of migration stems from the negative effects associated with a large inflow of migrants, that are not modeled-in. Those considerations might include, for example, increasing income inequality between natives and migrants, when the optimal migration policy mandates specialization by skill type of the migrants, which, as we further show, is the case. Other reasons might include the integration efforts of migrants, which are likely to decrease with the size of the migrant population. \( Q \) might be driven by a political-economy process where the natives have taken all these effects into account.
4.1. Optimal migration under a quota

Under a quota, the objective of the receiving country is to maximize (23), using $Q_s$ and $Q_M$ as choice variables. The outcome of the receiving country’s maximization problem is presented in the following proposition.

**Proposition 5.** Under a quota, the receiving country attains the optimal skill composition of its workforce (it maximizes $SWF$) when the level of migration is at the limit $Q_s + Q_M = Q$, and when the composition of migration by skill type is such that the migrants are

(a) all scientists, namely $Q_M = 0$, if $SWF^q(Q,0) > SWF^q(0,Q)$;

(b) all managers, namely $Q_s = 0$, if $SWF^q(Q,0) < SWF^q(0,Q)$.

**Proof.** The proof is in Appendix A.

Proposition 5 reveals that the welfare of the natives under a quota is strictly higher than in the no-migration setting because optimally, the receiving country will not elect to have no migrant scientists and no migrant managers. That the optimal skill composition of the workforce is attained under full specialization by skill type of the migrants up to the quota limit is quite intuitive. When migration is of level $Q$ and consists exclusively of scientists or exclusively of managers, then the decline in the wages of the native workers of the same skill type as that of the migrants, and the increase in the wages of the native workers of the other skill type, are both more substantial than under migration of any other level and composition by skill type. However, because migrants “push” natives from the occupation that suffers a decline in wages into the occupation that experiences an increase in wages, the proportion of those who sustain a loss on account of lower wages declines with the level of migration (and is the lowest under migration of level $Q$).\(^{16}\)

\(^{16}\) Corner solutions, such as the one reported in Proposition 5, are not uncommon in the received migration policy literature. For example, in a political economy setting, Benhabib (1996) shows that the population of the migrant receiving country will be polarized in terms of the preferred migration policy, with one section of the population opting for admitting migrants with as much capital as possible, and with the remainder section preferring to admit migrants with as little capital as possible. Which of these two policies ends up being implemented depends on the size of the two sections. In yet another political economy setting, Ortega (2010) shows that when the native workforce consists of skilled and unskilled workers, and when citizenship is not granted to the children of migrants who are born in the host country (so as to avoid them voting against the interests of the unskilled native workers), then the preference of the unskilled native workers is to admit only skilled migrants. This preference is formed when the wages of unskilled native workers increase with the size of the skilled workforce, and when income redistribution (in a welfare state) from skilled workers to unskilled native workers is increasing with these wages.
Proposition 5 narrows the set of potentially optimal realizations in the level and composition by skill type of migration to only two, yet it does not provide us with a means of selecting between the realizations (other than a comparison of the values of the SWF). In general, we cannot specify when it is better to admit exclusively scientists or exclusively managers, because the choice of whom to admit evolves from the interaction between the returns from science and the preference for occupational prestige among the natives. However, we can be specific when the limit to the level of migration is relatively small. Under such a constellation we show how the choice of the preferred skill type of migrants varies with the strength of the externality generated by the scientists.

We calculate the maximal level of migration for which we can identify the preferred skill type of migrants exclusively by referring to the strength of the externality. We denote by $Q_1$ the limit to the level of migration such that

$$
\begin{align*}
Q_1 &> 0 \quad \text{and} \quad SWF^q (Q_1, 0) = SWF^n, \quad \text{if} \quad \eta < L^n_S - \alpha \\
Q_1 &> 0 \quad \text{and} \quad SWF^q (0, Q_1) = SWF^n, \quad \text{if} \quad \eta > L^n_S - \alpha \\
Q_1 &= 0, \quad \text{if} \quad \eta = L^n_S - \alpha.
\end{align*}
$$

That is, $Q_1$ is a specific value of the limit to the level of migration such that if the externality is relatively weak (strong), and if migration is of level $Q_1$ with only scientists (managers) migrating, then the welfare of the natives, as represented by (23), is the same as in the no-migration setting. (When $\eta = L^n_S - \alpha$, there is no positive value of $Q_1$ for which the levels of the SWF in the two settings are equal.) We now have the following lemma and proposition.

**Lemma 1.** $Q_1$ exists, and is unique.

**Proof.** The proof is in Appendix A.

**Proposition 6.** Under a quota, when $Q \leq Q_1$, the receiving country attains the optimal skill composition of its workforce (it maximizes SWF) when the level of migration is at the limit $Q_S + Q_M = Q$, and when the composition of migration by skill type is such that the migrants are

(a) all scientists, namely $Q_M = 0$, if the externality generated by the scientists is sufficiently strong, that is, if $\eta > L^n_S - \alpha$;
(b) all managers, namely \( Q_S = 0 \), if the externality generated by the scientists is sufficiently weak, that is, if \( \eta < L^u_S - \alpha \).

**Proof.** The proof follows from the intersection of Proposition 5, (24), and Claim 3 (incorporated in the proof of Lemma 1). Q.E.D.

Proposition 6 underscores the role of the externality generated by the scientists in combination with full specialization by skill type of the group of migrants up to the quota limit in determining the optimal skill composition of the workforce in the receiving country. When this externality is weak, migration exclusively of managers results in an optimal skill composition of the workforce. When this externality is strong, it is migration exclusively of scientists that attains that goal.

To discern why the choice of the preferred type of skill of migrants depends on the strength of the externality generated by the scientists, we need to identify the positive and negative effects associated with the migration of scientists only, and likewise with the migration of managers only. As already noted, when migration is specialized by skill type, the wages paid to the same skill type as that of the migrants decrease, whereas the wages paid to the other skill type increase, thus “pushing” the natives from the occupation that suffers a decline in wages into the occupation that experiences an increase in wages. This “crowding out effect” is stronger when the natives are being driven into science rather than into management, due to the shape of the utility function: a utility increase of low-earning scientists in response to a marginal increase in their wage is higher than a utility increase of high-earning managers in response to the same stimulus. By admitting only managers, the receiving country ensures that the natives specialize in science and, thus, that they are the ones who experience a large increase in utility, benefiting from the “crowding out effect.” However, such migration entails a decrease in the share of scientists in the receiving country’s workforce, thereby reducing the country’s TFP, lowering the earnings of the natives and of the migrants alike. By admitting only scientists rather than only managers, the receiving country benefits from the “TFP effect,” at the cost of driving the natives into managerial occupations, who thereby forfeit the utility gain from the “crowding out effect.” Which of the two effects dominates depends on the strength of the externality generated by the scientists, \( \eta \), and on how much the wages of scientists and managers differ in equilibrium, as delineated by \( L^u_S - \alpha \), which measures the “crowding out effect” (noting that \( L^u_S - \alpha \) maps onto the ratio of the wage per unit of productivity of managerial work to scientific work through (10)).
Figure 1 illustrates how the optimal choice of the skill type of the migrants depends on the strength of the externality when \( Q \leq Q_s \). Lighter colors indicate higher values of the social welfare function. It is better to pursue migration of only managers under the specifications used to draw Figure 1(a), whereas it is better to pursue migration of only scientists under the specifications used to draw Figure 1(b).

(a) Weak externality  
(b) Strong externality

Figure 1. The values of the SWF under a quota as a function of the level of migration of scientists and of managers.

Note: Figure 1 is drawn for a uniform distribution of \( \varepsilon \) on the interval \([0, 1]\), and for values of the parameters \( \alpha = 0.5 \), and \( Q = 0.2 \). In drawing panel (a), we assume that \( \eta = 0.06 \); in drawing panel (b), we assume that \( \eta = 0.16 \). (For drawing this Figure, the distribution of \( \theta \) is immaterial.)

4.2. Optimal migration under a uniform entry fee

We now ask what level and composition of migration by skill type and by productivity achieve the optimal skill composition of the workforce under a uniform entry fee.\(^{17}\) Because migrants are more productive under a uniform fee than under a quota, then for a given level and composition of migration by skill type, the value of the SWF under the former policy will differ from its value under the latter policy. Consequently, the maximal level of migration for which we can identify the preferred skill type of migrants exclusively by referring to the

\(^{17}\) Recalling the clarification in footnote 14, our usage of \( Q_s \) and \( Q_M \) as control variables instead of usage of the fee, \( x \), leads to the same optimal solution as does usage of \( x \) as a control variable.
strength of the externality will differ as well. We denote by $Q_2$ the limit to the level of migration such that

$$
\begin{cases}
Q_2 > 0 \quad \text{and} \quad SWF^q(Q_2,0) = SWF^n, & \text{if } \eta < L^n_S - \alpha \\
Q_2 > 0 \quad \text{and} \quad SWF^{qef}(0,Q_2) = SWF^n, & \text{if } \eta > L^n_S - \alpha \\
Q_2 = 0, & \text{if } \eta = L^n_S - \alpha,
\end{cases}
$$

(25)

and by $x$ the level of the (uniform) entry fee below which scientists find it beneficial to migrate alongside managers. We now have the following lemma and proposition.

**Lemma 2.** (a) $Q_2$ exists, and is unique. (b) $Q_2 < Q_1$ if $\eta > L^n_S - \alpha$; $Q_2 = Q_1$ if $\eta \leq L^n_S - \alpha$.

**Proof.** The proof is in Appendix A.

**Proposition 7.** Under a uniform entry fee, when $Q \leq Q_2$, the receiving country attains the optimal skill composition of its workforce (it maximizes SWF) when

(a) the level of migration is zero, $Q_S = Q_M = 0$, if the externality generated by the scientists is sufficiently strong, that is, if $\eta > L^n_S - \alpha$;

(b) the level of migration is at $\min\{Q(x),Q_2\}$, if the externality generated by the scientists is sufficiently weak, that is, if $\eta < L^n_S - \alpha$.

**Proof.** The proof is in Appendix A.

Part (a) of Proposition 7 implies that by setting the fee so as to allow some migration, the receiving country will act against the goal of attaining the optimal skill composition of its workforce. This implication is due to the negative impact of migrant managers on TFP in the receiving country: when the externality generated by the scientists is strong, then the “TFP effect” is stronger than the “crowding out effect” and, thus, the migration of managers reduces the welfare of the natives. Part (b) of Proposition 7 indicates that when the externality generated by the scientists is weak, it is optimal to admit as many managers as possible and only managers. Once scientists too find it beneficial to migrate, additional migration will no longer bring about the desired “crowding out effect” and, thus, is not optimal.
4.3. Choosing the optimal migration policy

We now inquire which of the two migration policies fares better as a tool for attaining the optimal skill composition of the workforce in the receiving country. We have the following proposition.

**Proposition 8.** From the perspective of the receiving country, when \( Q \leq Q_2 \), the receiving country attains the optimal skill composition of its workforce (it maximizes SWF) under

\[
\begin{align*}
& \text{(a)} \quad \text{a quota, if the externality generated by the scientists is sufficiently strong, that is, if } \eta > L^n_S - \alpha, \text{ or if the externality generated by the scientists is sufficiently weak and the size of migration is sufficiently large, that is, if } \eta < L^n_S - \alpha \text{ and } Q > \bar{\theta}_m^{sw} Q(x); \\
& \text{(b)} \quad \text{a uniform entry fee, if the externality generated by the scientists is sufficiently weak and the size of migration is sufficiently small, that is, if } \eta < L^n_S - \alpha \text{ and } Q < \bar{\theta}_m^{sw} Q(x).
\end{align*}
\]

**Proof.** The proof follows from combining the proofs of Propositions 6 and 7. Q.E.D.

That a uniform entry fee is strictly preferable to a quota if the externality generated by the scientists is weak stems from the positive self-selection by the migrants under the former policy, which strengthens the “crowding out effect.” However, if that externality is strong, by implementing a uniform entry fee rather than a quota, the receiving country acts against the welfare interest of the natives, as it forfeits the TFP boost that it would have enjoyed under a quota. The latter policy will also be preferable to the uniform entry fee if under a quota the receiving country optimally admits more effective units of managerial work than under a uniform entry fee, namely if \( Q > \bar{\theta}_m^{sw} Q(x) \).

To gain further insight into which of the two policies is more likely to be preferable in attaining the optimal skill composition of the workforce, we present an illustrative calculation based on US data. From Proposition 8 we know that a quota should be chosen if the “TFP effect” is stronger than the “crowding out effect,” that is, if \( \eta > L^n_S - \alpha \). On the basis of empirical studies, we assessed numerically the two sides of this inequality (details are in Appendix B). With \( \alpha = 0.042 \) and with \( \bar{L}^n_S = 0.061 \), which we can use instead of \( L^n_S \) because
in the case of the US $\tilde{L}_{S} > L_{S}$, it follows that the US should aim at increasing the share of scientists among migrants if $\eta > 0.019$. The indirect methods of evaluating $\eta$ on the basis of empirical studies (Kerr and Lincoln, 2010; Peri et al., 2015) indicate that, for the US, $\eta$ exceeds 0.019. In the specific case of the US, imposition of a uniform entry fee instead of a quota would cause adjustments in the country’s labor market that are disadvantageous to the welfare of its natives.

In Table 1 we present evidence on the balance of foreign-born individuals in the workforces of selected countries, and among scientists and managers in the countries. In all the reported countries, the share of the foreign-born among scientists exceeds the share of the foreign-born in the overall workforce. In contrast, the share of the foreign-born among managers is about the same as or lower than the share of the foreign-born in the overall workforce. If the reported countries were to adopt a uniform entry fee, then the balance of the foreign-born between the two professions could be reversed.

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18 Hanson and Slaughter (2015) report that the share of foreign-born workers among STEM workers (Scientists, Technology professionals, Engineers, and Mathematicians) in the US is higher than their share in the overall workforce.
<table>
<thead>
<tr>
<th>Country</th>
<th>Foreign-born as a percent of workers</th>
<th>Foreign-born as a percent of scientists</th>
<th>Foreign-born as a percent of managers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Australia</td>
<td>27.73</td>
<td>41.46</td>
<td>28.79</td>
</tr>
<tr>
<td>Canada</td>
<td>22.77</td>
<td>28.82</td>
<td>23.45</td>
</tr>
<tr>
<td>Ireland</td>
<td>21.12</td>
<td>25.90</td>
<td>19.08</td>
</tr>
<tr>
<td>Norway</td>
<td>10.20</td>
<td>14.36</td>
<td>5.40</td>
</tr>
<tr>
<td>New Zealand</td>
<td>28.42</td>
<td>38.30</td>
<td>27.06</td>
</tr>
<tr>
<td>US</td>
<td>17.61</td>
<td>21.84</td>
<td>13.31</td>
</tr>
</tbody>
</table>

Table 1. Foreign-born as a percent of workers in selected countries.

Source: Database on Immigrants in OECD Countries, 2010-2011.

Notes:
1. Scientists are defined as follows:
   - codes 21 (Physical, Mathematical and Engineering Science Professionals) and 22 (Life Science and Health Professionals) for Australia and Ireland;
   - codes 21 (Science and Engineering Professionals) and 22 (Health Professionals) for Canada and Norway;
   - codes 23 (Design, Engineering, Science and Transport Professionals), 25 (Health Professionals), and 26 (Information and Communications Technology Professionals) for New Zealand;
   - codes 15 (Computer and Mathematical Occupations), 17 (Architecture and Engineering Occupations), and 19 (Life, Physical and Social Science Occupation) for the US.
2. Managers are defined as follows:
   - codes 12 (Corporate Managers) and 13 (General Managers) for Australia and Ireland;
   - codes 12 (Administrative and Commercial Managers), 13 (Production and Specialized Services Managers), and 14 (Hospitality, Retail and Other Services Managers) for Canada and Norway;
   - code 11 (Chief Executives, General Managers and Legislators) for New Zealand;
   - code 11 (Management Occupations) for the US.

5. Migration under a differentiated entry fee

In Section 3 we have shown that under a uniform entry fee, the receiving country faces limitations to the choice of the composition of migration by skill type; under such policy, it can encourage migration only or mostly of managers, which renders the policy unfit for the task of improving the skill composition of the workforce in the receiving country, if the externality generated by the scientists is sufficiently strong, which, as shown in Section 4.3, is a reasonable assumption to make. It stands to reason that by setting different fees for different skill types, the receiving country could overcome those limitations. Let then the receiving country introduce instead of a single uniform entry fee of $x$, two distinct fees for the two skill
types: $x_S$ for scientists, and $x_M$ for managers. In such a setting, an individual in the sending country will pay the fee and migrate as long as the fee is lower than his returns from migration, that is, as long as
\[
\begin{cases}
(w_s - w_s^F)\theta^m > x_S & \text{if he is a scientist} \\
(w_M - w_M^F)\theta^m > x_M & \text{if he is a manager.}
\end{cases}
\]
(26)

We first ask what composition of migration by skill type and by productivity will be brought about by a differentiated entry fee. We have the following proposition.

**Proposition 9.** Under a differentiated entry fee: (a) the receiving country can encourage the migration of any mix of scientists and managers; (b) the migrants are of higher productivity than under a quota.

**Proof.** (a) From (26) it follows straightforwardly that migration by each skill type depends on the entry fee set for the skill type. (b) Whereas under a quota the migrants constitute a random selection from the workforce of the sending country, under a differentiated entry fee the migrants’ skill level is higher than a certain threshold, as defined by (26). Q.E.D.

What follows from part (a) of Proposition 9 is that under a differentiated entry fee, the receiving country does not face limitations to the choice of the composition of migration by skill type that are present under a uniform entry fee, thus it can replicate any choice of composition by skill type of migration set under a quota. As far as the migrants’ productivity is concerned (part (b) of Proposition 9), it follows from (26) that just as under a uniform entry fee, in this setting too we have a positive self-selection by the migrants: migrants are from the upper end of the productivity distribution.

Under a differentiated entry fee there will be $\hat{L}_s = L_s + \bar{\theta}_s^mQ_s$ effective units of scientific work and $\hat{L}_M = L_M + \bar{\theta}_M^mQ_M$ effective units of managerial work in the receiving country, where $\bar{\theta}_s^m > 1$ and $\bar{\theta}_M^m > 1$. Again, all equilibrium values of the model’s endogenous variables are identified by the wage ratio of managerial work to scientific work, which we denote as $w^{def}$, with superscript $def$ indicating the equilibrium level of a variable under a differentiated entry fee. The equations describing the equilibrium in the labor market are the same as under a uniform entry fee, namely (21) and (22). The following proposition relates the equilibrium values of variables under a differentiated entry fee to the respective values
under a quota of the same level and composition of migration by skill type, and in the no-migration setting.\textsuperscript{19}

**Proposition 10.** Under a differentiated entry fee, as compared to a quota for which migration is of the same level, and to the no-migration setting: (a) \( w^\text{def} = w^d = w^n \), if \( \frac{Q_S}{Q_M} = \frac{L^n_S}{L^n_M} \); (b) \( w^\text{def} > w^d > w^n \), if all migrants are scientists; (c) \( w^\text{def} < w^d < w^n \), if all migrants are managers.

**Proof.** The proof is analogous to the proof of Proposition 4.

Just as under a uniform entry fee, under a differentiated entry fee the repercussions of opening up to migration are of a higher magnitude than when opening up to migration under a quota. The reason for this result is also the same, and follows from the positive self-selection by the migrants. We consider case (b) in Proposition 10. When applied to (21) and (22), and compared, respectively, with (18) and (19), and with (14) and (15), the inequalities in \( w^\text{def} > w^d > w^n \) establish that under a differentiated entry fee, for migration of only scientists of the same level as under a quota, we get that fewer natives choose to become scientists, \( L^n_S < L^n_S < L^n_S \), the wages (per unit of productivity) of scientists are lower and the wages of managers are higher, \( w_S^\text{def} < w_S^d < w_S^n \) and \( w_M^\text{def} > w_M^d > w_M^n \), respectively, and the ratio of effective units of scientific work to managerial work is higher. For case (c), we have the opposite results.

We now ask whether a differentiated entry fee fares better than a quota in securing the optimal skill composition of the workforce in the receiving country. We denote by \( Q_3 \) the limit to the level of migration such that

\[
\begin{align*}
Q_3 > 0 \quad & \text{and} \quad SWF^\text{def} (Q_3, 0) = SWF^n, \quad \text{if} \quad \eta < L^n_S - \alpha \\
Q_3 > 0 \quad & \text{and} \quad SWF^\text{def} (0, Q_3) = SWF^n, \quad \text{if} \quad \eta > L^n_S - \alpha \\
Q_3 = 0, \quad & \text{if} \quad \eta = L^n_S - \alpha.
\end{align*}
\]

We have the following lemma and propositions.

**Lemma 3.** (a) \( Q_3 \) exists, and is unique. (b) \( Q_3 < Q_1 \).

\textsuperscript{19} Clearly, because a uniform entry fee can be conceived as a special case of a differentiated entry fee, the uniform entry fee can at best be as good as the differentiated entry fee. Therefore, in what follows, we do not compare a differentiated entry fee with a uniform entry fee.
Proof. The proof is analogous to the proof of Lemma 2.

Proposition 11. Under a differentiated entry fee, when $Q \leq Q_s$, the receiving country attains the optimal skill composition of its workforce (it maximizes SWF) when the level of migration is at the limit $Q_s + Q_m = Q$, and when the composition of migration by skill type is such that the migrants are

(a) all scientists, namely $Q_m = 0$, if the externality generated by the scientists is sufficiently strong, that is, if $\eta > L^s - \alpha$;

(b) all managers, namely $Q_s = 0$, if the externality generated by the scientists is sufficiently weak, that is, if $\eta < L^s - \alpha$.

Proof. The proof is in Appendix A.

Proposition 12. From the perspective of the natives of the receiving country, an optimal differentiated entry fee is strictly preferable to an optimal quota.

Proof. The proof is straightforward.

Proposition 11 indicates that under a differentiated entry fee, the receiving country should set the fees so as to encourage the same level and composition of migration by skill type as is optimal under a quota. If it does so, then the resulting skill composition of the workforce in the receiving country will be more beneficial to the natives than that which obtains for an optimal quota (cf. Proposition 12). This is so because of the positive self-selection by the migrants, which strengthens the “TFP effect” when the externality generated by the scientists is strong, or the “crowding out effect” when that externality is weak.

The results of Proposition 11 regarding the optimal level of migration and of its composition by skill type can be expressed in terms of the corresponding entry fees. A summary of the optimal entry fees, conditional on the strength of the externality, is provided in Table 2, where $\theta^n_j$ stands for the skill level of an individual whose skill type is $j$ and who is indifferent between paying the fee and not migrating and where, to recall, $T$ is the migrant with the highest skill level. The entry fee for managers when the externality is strong, and the entry fee for scientists when the externality is weak, are given as the minimum fees needed to discourage workers of each skill type from migration, as indicated by the strict inequality signs in the respective optimal entry fees in the second and third columns of Table 2. The
entry fee for the scientists when the externality is strong, and the entry fee for the managers when the externality is weak, ensure that exactly $Q$ scientists or $Q$ managers will pay the fee.

<table>
<thead>
<tr>
<th>Strength of the externality</th>
<th>Scientists ($x_s$)</th>
<th>Managers ($x_M$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta &gt; L_\alpha^s - \alpha$</td>
<td>$x_s = \left(w_s(Q,0) - w^f_s\right)Q^m_s (Q,0)$</td>
<td>$x_M &gt; \left(w_M(Q,0) - w^f_M\right)T$</td>
</tr>
<tr>
<td>$\eta &lt; L_\alpha^s - \alpha$</td>
<td>$x_s &gt; \left(w_s(0,Q) - w^f_s\right)T$</td>
<td>$x_M = \left(w_M (0,Q) - w^f_M\right)Q^m_M (0,Q)$</td>
</tr>
</tbody>
</table>

Table 2. Optimal differentiated entry fees.

### 6. Attaining optimal skill composition of the workforce vs. maximizing the entry fee revenue

It might be tempting for the receiving country, when it pursues an entry fee policy, to set the fees so as to maximize revenue. In this section we ask whether under a differentiated entry fee maximization of the entry fee revenue aligns with maximization of the SWF. We have the following proposition.

**Proposition 13.** The highest revenue is attained when migrants are only or mostly managers, namely when $\frac{Q_s}{Q_M} < \frac{L^s_\alpha}{L^s_M}$.

**Proof.** The proof is in Appendix A.

Proposition 13 together with part (a) of Proposition 3 imply that a revenue-maximizing country will not want to introduce a differentiated entry fee, or that it will differentiate the fee only slightly. By attracting only or mostly managers, a revenue-maximizing country will attain optimal (or close to optimal) skill composition of its workforce only if the externality generated by the scientists is weak (cf. Proposition 11). If that externality is strong, however, then by setting the fees that yield the highest possible revenue, the receiving country will forfeit the optimal skill composition of its workforce, because such a composition will require migrants to be all scientists. We conclude that if the externality generated by the scientists is strong, then the revenue maximization comes at a cost of inducing unfavorable changes in the skill composition of the receiving country’s workforce. Seen from a different perspective,
attaining the optimal skill composition of the country’s workforce, which requires all migrants
to be scientists, comes at a cost of foregone revenue that could be obtained if the migrants
were all or mostly managers.

Under the objective of the maximization of revenue, the optimal entry fees depend on
the rate at which the entry fee for managers needs to be lowered to encourage a marginal
increase in the size of managerial migration, and on the rate at which the migration premium
for scientists increases in response to a decrease in the scientists-to-managers ratio brought
about by the increase of managerial migration. Both rates depend on the distributions of
prestige and of productivity. These dependences render it impossible to present a Table that is
analogous to Table 2. Still, because revenue maximization requires migrants to be all or
mostly managers, the optimal entry fees needed to maximize the entry fee revenue will be
either the same or close to the ones reported in Table 2 for the case of weak externality.

7. Conclusions

It can reasonably be expected that when a receiving country charges for the right to work
within its borders, it will attract the most productive individuals who will generate the highest
returns from the investment in the entry fee. We showed why this expectation is only a
fragment of the overall picture. We constructed a model which we used to assess the
implications of selling the right to enter a receiving country, as opposed to administering a
quota, under the assumption that migrants are heterogeneous not only in skill level but also in
skill type, and that one skill type, scientists, confers positive production externality, whereas
the other, managers, does not.

We found that under a quota, the receiving country will optimally control the level of
migration as little as possible and that it will admit only scientists or only managers,
depending on whether the production externality is strong or weak, respectively. The
disadvantage of a quota is that it does not encourage desirable self-selection by the migrants.
By contrast, when enacting a uniform entry fee, the receiving country can select migrants by
skill level, but not by skill type: it will attract only those highly skilled foreign workers who
generate the highest returns from incurring the entry fee; in our case these are only or mostly
managers. A comparison of a uniform entry fee with a quota yields the result that the former
is better when the production externality generated by the scientists is weak, whereas when
this externality is strong, the ranking reverses. Illustrative calculations suggest that in the case of the US, the externality generated by the scientists is strong.

By setting different fees for different skill types, the receiving country can overcome the limitations it faces under a uniform entry fee: it can select migrants by skill level and by skill type. This renders a differentiated fee strictly preferable to a quota if the aim of the receiving country is to attain the optimal skill composition of its workforce rather than to maximize its entry fee revenue. However, if the receiving country seeks to maximize its entry fee revenue and if the externality generated by the scientists is strong, then the pursuit of such a maximization does not deliver the optimal skill composition of its workforce.
Appendix A

For ease of reference, prior to providing proofs we replicate the propositions and lemmas presented in the body of the paper.

Proposition 1. (a) \( w^n \) exists, and is unique. (b) \( w^n > 1 \).

Proof. (a) Let \( b(w) = \frac{1-\alpha}{1-\alpha + \alpha w} \). Note that \( w^n \) is defined as a solution to

\[
F(\ln w) = b(w),
\]

in which case the left-hand side and the right-hand side of (A1) are the equilibrium supply of and the equilibrium demand for managerial work in the no-migration setting, respectively. To prove the existence of \( w^n \), we note that for \( w = e^0 = 1 \) we have that \( F(\ln 1) = 0 < 1-\alpha = b(1) \), whereas for \( w = e^E \) we have that \( F(\ln 1) = 1 > \frac{1-\alpha}{1-\alpha + \alpha e^E} = b(e^E) \). From the continuity of \( F(\cdot) \) and \( b(\cdot) \) it follows that there exists \( w^n \in (1,e^E) \) such that \( w^n \) is the solution to (A1). Furthermore, because

\[
\frac{\partial F(\ln w)}{\partial w} = \frac{F'(\ln w)}{w} = \frac{f(\ln w)}{w} > 0,
\]

and because \( b'(w) = -\frac{\alpha(1-\alpha)}{(1-\alpha + \alpha w)^2} < 0 \), the left-hand side and the right-hand side of (A1) cross exactly once, which guarantees uniqueness of the solution to (A1).

(b) Because \( w^n \in (1,e^E) \) where \( E \in \mathbb{R}_+ \), as shown in part (a) of this proof, it follows that \( w^n > 1 \). Q.E.D.

Proposition 2. Under a quota, as compared to the no-migration setting: (a) \( w^q = w^n \), if the composition of migration by skill type is the same as the composition of the native workforce,

\[
\frac{Q_s}{Q_M} = \frac{L_s^n}{L_M^n} ;
\]

(b) \( w^q > w^n \), if the composition of migration by skill type is such that migrants are only or mostly scientists,

\[
\frac{Q_s}{Q_M} > \frac{L_s^n}{L_M^n} ;
\]

(c) \( w^q < w^n \), if the composition of migration by skill type is such that migrants are only or mostly managers,

\[
\frac{Q_s}{Q_M} < \frac{L_s^n}{L_M^n} .
\]

Proof. We first present and prove a claim.
Claim 1. $\frac{\partial w^q}{\partial Q_s} > 0$ and $\frac{\partial w^q}{\partial Q_M} < 0$.

**Proof.** Recalling that $b(w) = \frac{1-\alpha}{1-\alpha + \alpha w}$, consider the function

$$B(w, Q_s, Q_m) = F(\ln w) + Q_m - b(w)(1+Q_m + Q_s).$$

(A2)

Because $w^q$ is defined as a solution to $B(w, Q_s, Q_m) = 0$ (cf. (18)), in which case the right-hand side of (A2) is the difference between the equilibrium supply of and the equilibrium demand for managerial work under a quota, it follows that $B(w^q, Q_s, Q_m) = 0$. Applying the implicit function theorem to $B(w^q, Q_s, Q_m)$ yields $\frac{\partial w^q}{\partial Q_s} = -\frac{B_{Q_s}}{B_{w^q}}$, where $B_{Q_s}$ and $B_{w^q}$ are the first derivatives of $B(w^q, Q_s, Q_m)$ with respect to $Q_s$ and $w^q$, respectively. Because

$$B_{Q_s} = -b(w^q) < 0,$$

(A3)

and because

$$B_{w^q} = \frac{1}{w^q} \left[ f(\ln w^q) + b(w^q)(1-b(w^q))(1+Q_m + Q_s) \right] > 0,$$

(A4)

it follows that

$$\frac{\partial w^q}{\partial Q_s} = \frac{b(w^q)w^q}{f(\ln w^q) + b(w^q)(1-b(w^q))(1+Q_m + Q_s)} > 0.$$

Applying the implicit function theorem to $B(w^q, Q_s, Q_m)$ once again yields

$$\frac{\partial w^q}{\partial Q_m} = -\frac{B_{Q_m}}{B_{w^q}},$$

where $B_{Q_m}$ is the first derivative of $B(w^q, Q_s, Q_m)$ with respect to $Q_m$.

Because

$$B_{Q_m} = 1-b(w^q) > 0,$$

(A5)

and recalling (A4), it follows that

$$\frac{\partial w^q}{\partial Q_M} = -\frac{(1-b(w^q))w^q}{f(\ln w^q) + b(w^q)(1-b(w^q))(1+Q_m + Q_s)} < 0.$$

Q.E.D.

We now proceed with the proof of Proposition 2.
(a) When \( \frac{Q_S}{Q_M} = \frac{L^n_S}{L^n_M} \), using the relationship \( L^n_S + L^n_M = 1 \) we can rewrite (A2) as

\[
B(w, Q_S, Q_M) = F(\ln w) + Q_M - b\left(w\right)\left(1 + \frac{Q_M}{b\left(w^\alpha\right)}\right).
\]  

(A6)

Recalling that \( L'_M = F(\ln w^\alpha) = \frac{1-\alpha}{1-\alpha + \alpha w^\alpha} \) (cf. (14)), utilizing \( b(w) = \frac{1-\alpha}{1-\alpha + \alpha w} \), and rearranging, we write (A6) as

\[
B(w, Q_S, Q_M) = F(\ln w) + Q_M - b\left(w\right)\left(1 + Q_M \frac{1}{b\left(w^\alpha\right)}\right).
\]  

(A7)

We know that \( w^\alpha \) is defined as a solution to \( B(w, Q_S, Q_M) = 0 \), namely we have that

\[
F(\ln w^\alpha) + Q_M = b\left(w^\alpha\right)\left(1 + Q_M \frac{1}{b\left(w^\alpha\right)}\right).
\]  

(A8)

For \( w^\alpha = 1 \), \( F(\ln 1) + Q_M = Q_M < 1-\alpha + Q_M \left(1-\alpha + \alpha w^\alpha\right) = b(1) + Q_M \frac{b(1)}{b\left(w^\alpha\right)} \), whereas for \( w^\alpha = e^E \), \( F(E) + Q_M = 1 + Q_M > \frac{1-\alpha}{1-\alpha + \alpha e^E} + Q_M \frac{1-\alpha + \alpha w^\alpha}{1-\alpha + \alpha e^E} = b\left(e^E\right) + Q_M \frac{b\left(e^E\right)}{b\left(w^\alpha\right)} \). From the continuity of \( F(\cdot) \) and \( b(\cdot) \) it follows that the left-hand side and the right-hand side of (A8) have to cross at least once, which ensures existence of a solution to (A8). Furthermore, because \( \frac{\partial F(\ln w)}{\partial w} > 0 \), and because \( b'(w) < 0 \), the left-hand side of (A8) and the right-hand side of (A8) cross exactly once, which guarantees uniqueness of the solution to (A8). Having that for \( w^\alpha = w^\alpha \) (A8) becomes \( F(\ln w^\alpha) = b\left(w^\alpha\right) \), which, as shown in the proof of Proposition 1, holds, then \( w^\alpha = w^\alpha \) has to be the unique solution to (A2) when \( \frac{Q_S}{Q_M} = \frac{L^n_S}{L^n_M} \).

(b) Because when \( \frac{Q_S}{Q_M} = \frac{L^n_S}{L^n_M} \), then \( w^\alpha = w^\alpha \), as shown in part (a) of the proof, we can divide \( \text{any pair } (Q'_S, Q'_M) \) where \( \frac{Q'_S}{Q'_M} > \frac{L^n_S}{L^n_M} \) into a preliminary choice \( (Q'_S, Q'_M) \) where \( \frac{Q'_S}{Q'_M} = \frac{L^n_S}{L^n_M} \),
and a residual choice \((Q'_S, 0)\), where \(Q'_S + Q'_M = Q_S\). For the preliminary choice \((Q'_S, Q'_M)\), there is no change in the equilibrium level of \(w\) in comparison with the no-migration setting (cf. part (a) of the proposition), namely \(w^q(Q'_S, Q'_M) = w^p\). For the residual choice \((Q''_S, 0)\), because \(\frac{\partial w^q}{\partial Q'_S} > 0\) (cf. Claim 1), \(w^q(Q'_S + Q''_S, Q'_M) > w^q(Q'_S, Q'_M)\). In combination, when \(\frac{Q''_S}{Q'_M} > \frac{L''_S}{L'_M}\), it follows that \(w^q = w^q(Q'_S + Q''_S, Q'_M) > w^q(Q'_S, Q'_M) = w^p\).

(c) To prove part (c) of the proposition, we follow a procedure similar to the one used to prove part (b). Q.E.D.

**Proposition 3.** Under a uniform entry fee: (a) the composition of migration by skill type is such that migrants are all or mostly managers, \(\frac{Q''_S}{Q'_M} < \frac{L''_S}{L'_M}\); (b) for each fee-induced level of migration, the corresponding composition of migration by skill type is fixed; (c) migrants are of higher productivity than under a quota.

**Proof.** (a) We first show that ensuring migration only of scientists is impossible under a uniform entry fee. If under such a fee only scientists were to migrate, then in equilibrium we would have \(w^q_{S_M} < w^n_{S_M}\) and \(w^q_{M_M} > w^n_{M_M}\) (the proof is analogous to the proof of part (b) of Proposition 2). In a setting without migration we have that \(w^q_{M_M} - w^F_{M_M} > w^q_{S_S} - w^F_{S_S}\) (cf. (17)), which, together with \(w^q_{S_S} < w^n_{S_M}\) and \(w^q_{M_M} > w^n_{M_M}\), implies that \(w^q_{M_M} - w^F_{M_M} > w^q_{S_S} - w^F_{S_S}\), or that under a uniform entry fee with only scientists migrating, the wage difference between the two countries will be higher for managers than for scientists. However, if \(w^q_{M_M} - w^F_{M_M} > w^q_{S_S} - w^F_{S_S}\) were to obtain, then managers too will find it beneficial to migrate and, thus, we reach a contradiction.

We next show that ensuring migration only of managers is possible under a uniform entry fee. By choosing the entry fee a little below the between-countries difference in the earnings of a manager with the highest skill level, \(x < \left(w^q_{M_M} - w^F_{M_M}\right)T\), managers will find it beneficial to migrate, but scientists will not (cf. (20) in conjunction with (17)). Because the wages of managers decrease as more managers enter the receiving country, the inflow of migrant managers will cease as soon as those wages drop to a level at which it is no longer profitable for them to migrate, which obtains when \(\left[w^q_{M_M}(Q_M(x)) - w^F_{M_M}\right]Q^q_{M}(Q_M(x)) = x\),
where $\theta_m^m$ stands for the skill level of a manager who is indifferent between paying the fee and not migrating. Because the wages of scientists increase as more managers enter the receiving country, a direct consequence of migration of only managers is convergence of the wages of the two skill types. As a result, migration will consist exclusively of managers when the entry fee is above a certain level, denoted by $x$, such that if the entry fee is lower than $x$, scientists find it beneficial to migrate alongside managers. Specifically, $x$ is determined by equalizing the returns from migration to the most skilled scientist (namely the first one to migrate) with the returns from migration to the manager who is indifferent so as to whether to pay the fee or not to migrate (namely the last one to migrate when migration is manned only by managers), that is, $x$ is such that
\[
\left( w^w_s (Q_m(x)) - w^w_s \right) \theta^m_m = \left( w^w_m (Q_m(x)) - w^w_m \right) \theta^m_m (Q_m(x)) = x.
\]

By setting the entry fee at a level below $x$, the receiving country will encourage migration of both scientists and managers. We next show that this migration cannot exceed the ratio $\frac{Q_s}{Q_M} = \frac{L_s^s}{L_M^m}$. Imagine differently, namely that $\frac{Q_s}{Q_M} \geq \frac{L_s^s}{L_M^m}$. If so, then the average skill level of the migrants, and the skill level of an individual who is indifferent as to whether to pay the fee or not to migrate, will be lower for scientists than for managers, that is, we will have $\theta^m_s < \theta^m_M$ and $\theta^w_s < \theta^w_M$, respectively. Because migration occurs up to the point at which
\[
\left( w^w_s (Q_M(x)) - w^w_s \right) \theta^m_m = \left( w^w_m (Q_M(x)) - w^w_m \right) \theta^m_m (Q_M(x)) = x,
\]
we have $w^w_M - w^w_s > w^w_M - w^w_s$, or, on rearrangement and upon recalling that $w^w_M / w^w_s = w^F = w^m > 1$, we will have $w^w_M - w^w_s < w^w_M - w^w_s$, which requires $\frac{w^w_M}{w^w_s} < \frac{w^w_M}{w^w_s}$. However, $\frac{w^w_M}{w^w_s} < \frac{w^w_M}{w^w_s}$ can only obtain if migration is only or mostly of managers; therefore, we have a contradiction.

(b) Imagine otherwise, namely that for a given overall level of migration corresponding to a given entry fee, there can be two compositions of migration by skill type: $Q'_s + Q'_M = Q$ and $Q^s_M + Q^m_M = Q$, where $Q'_s > Q^s_M$, which implies that $Q'_s < Q^s_M$. From $Q'_s > Q^s_M$ and $Q'_s < Q^s_M$ it follows that $w^w_M (Q'_s, Q'_M) - w^w_M > w^w_M (Q^s_M, Q^m_M) - w^w_M$ and that $w^w_s (Q'_s, Q'_M) - w^w_s < w^w_s (Q^s_M, Q^m_M) - w^w_s$, or that the returns from migration to managers (scientists) are higher (lower) under the migration of $Q'_s$ scientists and $Q'_M$ managers than the returns from...
migration to managers under the migration of $Q'_s$ scientists and $Q'_M$ managers; and that $\theta_M^m (Q'_S, Q'_M) > \theta_S^m (Q'_S, Q'_M)$ and $\theta_S^m (Q'_S, Q'_M) < \theta_M^m (Q'_S, Q'_M)$, or that the skill level of a manager (scientist) who is indifferent as to whether to pay the fee or not to migrate is higher (lower) under the migration of $Q'_S$ scientists and $Q'_M$ managers than under the migration of $Q''_S$ scientists and $Q''_M$ managers.

For there to be two combinations of the (same) overall level of migration corresponding to a given entry fee $x_1$, it has to be the case that in equilibrium

$$\left( w^m_M (Q'_S, Q'_M) - w^F_M \right) \theta_M^m (Q'_S, Q'_M) = \left( w^m_S (Q'_S, Q'_M) - w^F_S \right) \theta_S^m (Q'_S, Q'_M) = x_1$$

and

$$\left( w^m_M (Q''_S, Q''_M) - w^F_M \right) \theta_M^m (Q''_S, Q''_M) = \left( w^m_S (Q''_S, Q''_M) - w^F_S \right) \theta_S^m (Q''_S, Q''_M) = x_1,$$

or that the migration premium of a scientist and of a manager who are indifferent as to whether to pay the fee or not to migrate is equal to the entry fee. Suppose that, indeed, $\left( w^m_M (Q'_S, Q'_M) - w^F_M \right) \theta_M^m (Q'_S, Q'_M) = \left( w^m_S (Q'_S, Q'_M) - w^F_S \right) \theta_S^m (Q'_S, Q'_M) = x_1$ holds. Because $w^m_M (Q'_S, Q'_M) - w^F_M > w^m_S (Q'_S, Q'_M) - w^F_S$ and $w^m_S (Q'_S, Q'_M) - w^F_S < w^m_M (Q'_S, Q'_M) - w^F_M$, and because $\theta_M^m (Q'_S, Q'_M) > \theta_S^m (Q'_S, Q'_M)$ and $\theta_S^m (Q'_S, Q'_M) < \theta_M^m (Q'_S, Q'_M)$, it follows that $\left( w^m_M (Q'_S, Q'_M) - w^F_M \right) \theta_M^m (Q'_S, Q'_M) < x_1$ and that $\left( w^m_S (Q'_S, Q'_M) - w^F_S \right) \theta_S^m (Q'_S, Q'_M) > x_1$, or that fewer than $Q'_M$ managers and more than $Q'_S$ scientists will find it beneficial to migrate when required to pay the fee of $x_1$. Therefore, both $Q'_S + Q'_M = Q$ and $Q''_S + Q''_M = Q$ cannot obtain for the same level of the entry fee.

(c) Whereas under a quota the migrants constitute a random selection from the workforce of the sending country, under a uniform entry fee the migrants’ skill level is higher than a certain threshold, as defined by (20). Q.E.D.

**Proposition 4.** $w^m < w^q < w^r$ if migration is of the same level and composition by skill type under a quota as under a uniform entry fee.

**Proof.** We first present a claim.
Claim 2. \( \frac{\partial w^{\text{pref}}}{\partial \bar{\theta}^{\text{m}}_s Q_s} > 0 \) and \( \frac{\partial w^{\text{pref}}}{\partial \bar{\theta}^{\text{m}}_M Q_M} < 0 \).

Proof. By following an analogous procedure as that in the proof of Claim 1, on recalling that \( w^{\text{pref}} \) is the solution to

\[
B\left(w, \bar{\theta}^{\text{m}}_S Q_S, \bar{\theta}^{\text{m}}_M Q_M\right) = F\left(\ln w\right) + \bar{\theta}^{\text{m}}_M Q_M - b\left(w\right)\left(1 + \bar{\theta}^{\text{m}}_S Q_S + \bar{\theta}^{\text{m}}_M Q_M\right),
\]

we get that

\[
\frac{\partial w^{\text{pref}}}{\partial \bar{\theta}^{\text{m}}_S Q_S} = \frac{b\left(w^{\text{pref}}\right)w^{\text{pref}}}{f\left(\ln w^{\text{pref}}\right) + b\left(w^{\text{pref}}\right)\left(1 - b\left(w^{\text{pref}}\right)\right)\left(1 + \bar{\theta}^{\text{m}}_S Q_S + \bar{\theta}^{\text{m}}_M Q_M\right)} > 0,
\]

and

\[
\frac{\partial w^{\text{pref}}}{\partial \bar{\theta}^{\text{m}}_M Q_M} = -\frac{\left(1 - b\left(w^{\text{pref}}\right)\right)w^{\text{pref}}}{f\left(\ln w^{\text{pref}}\right) + b\left(w^{\text{pref}}\right)\left(1 - b\left(w^{\text{pref}}\right)\right)\left(1 + \bar{\theta}^{\text{m}}_S Q_S + \bar{\theta}^{\text{m}}_M Q_M\right)} < 0.
\]

Q.E.D.

We now proceed with the proof of Proposition 4. The proof that \( w^{\text{pref}} < w^a \) is analogous to the proof of part (c) of Proposition 2, with a reference to Claim 2 replacing the reference to Claim 1. Because the right-hand side of the inequality, namely \( w^a < w^a \), holds from part (c) of Proposition 2, we can focus on the left-hand side of the inequality, namely on \( w^{\text{pref}} < w^a \).

Under a uniform entry fee, migration that is relatively small in size has to consist only of managers, as shown in part (a) of Proposition 3. Because the equilibrium value of \( w \) is a decreasing function of \( Q_M \), and because \( \bar{\theta}^{\text{m}}_M > 1 \) the inflow of effective units of managerial work under an entry fee is larger than the inflow of managers, namely \( \bar{\theta}^{\text{m}}_M Q_M > Q_M \), then it has to be the case that \( w^{\text{pref}} < w^a \) when under a uniform entry fee and under a quota all the migrants are managers.

By reducing the entry fee below the level \( \bar{x} \), defined in the proof of part (a) of Proposition 3, scientists will migrate as well as managers. Any subsequent decrease of the entry fee aimed at inducing a larger inflow of migrants will attract relatively more scientists than managers because any decrease of the entry fee benefits relatively more the low-earning scientists than the high-earning managers. The relatively larger inflow of scientists than of
(additional) managers raises $w^\text{ref}$. Under a quota, such a relatively larger inflow of scientists than of (additional) managers will also increase $w^\theta$ (recalling that we compare $w^\text{ref}$ with $w^\theta$ for migration of the same level and composition by skill type under the two policies). However, under a uniform entry fee, scientists will be of higher skill level than the (additional) managers. Therefore, the inflow of effective units of scientific work relative to the inflow of (additional) effective units of managerial work will be higher under a uniform entry fee than under a quota for the same level of migration and composition by skill type. Consequently, $w^\text{ref}$ will increase with the level of migration (that is, with the lowering of the entry fee) at a higher rate than $w^\theta$. Equalization of $w^\text{ref}$ and $w^\theta$ will occur only in the limit, that is, in a hypothetical setting where the entry fee is set at zero, in which case all foreigners, scientists and managers alike, will find it beneficial to migrate and, thus, all the migrants will have the same average skill level. Therefore, $w^\text{ref} < w^\theta$ continues to hold under a joint migration of scientists and managers as long as $x > 0$. Q.E.D.

**Proposition 5.** Under a quota, the receiving country attains the optimal skill composition of its workforce (it maximizes SWF) when the level of migration is at the limit $Q_S + Q_M = Q$, and when the composition of migration by skill type is such that the migrants are

(a) all scientists, namely $Q_M = 0$, if $SWF^q(Q,0) > SWF^q(0,Q)$;

(b) all managers, namely $Q_S = 0$, if $SWF^q(Q,0) < SWF^q(0,Q)$.

**Proof.** Under a quota, the SWF is given by

$$SWF^q(Q_S, Q_M) = \int_{\ln w^\theta}^{\ln w^\theta} \int_0^T u_M^q(\theta) f(\varepsilon) d\theta d\varepsilon + \int_{\ln w^\theta}^{\ln w^\theta} \int_T^F u_S^q(\theta) f(\varepsilon) d\theta d\varepsilon.$$  \hspace{1cm} (A10)

Upon substitution for $u_M^q$ and $u_S^q$ from (1), and recalling that $c_j = \theta w_j$, and upon noting that

$$\int_0^T \ln{\theta} g(\theta) d\theta = K$$ is a constant, (A10) becomes

$$SWF^q(Q_S, Q_M) = \int_{\ln w^\theta}^{\ln w^\theta} \left( K + \ln w_M^q \right) f(\varepsilon) d\varepsilon + \int_{\ln w^\theta}^{\ln w^\theta} \left( K + \ln w_S^q + \varepsilon \right) f(\varepsilon) d\varepsilon.$$  \hspace{1cm} (A11)
Given that \( w_S^q \) and \( w_M^q \) do not depend on \( \varepsilon \), and that 
\[
\int_{\ln w^q}^E f(\varepsilon) d\varepsilon = 1 - F(\ln w^q) = L_M^q, \quad \text{and} \quad \int_{\ln w^q}^E f(\varepsilon) d\varepsilon = L_S^q,
\]
we can rewrite the objective function of the receiving country as
\[
SWF^q(Q, Q_M) = K + L_M^q \ln w_M^q + L_S^q \ln w_S^q + \int_{\ln w^q}^E \varepsilon f(\varepsilon) d\varepsilon. \quad \text{(A12)}
\]

Because (A12) depends on the behavior of individuals and firms, and because the receiving country first chooses the quota of migrant scientists, \( Q_S \), and the quota of migrant managers, \( Q_M \), and thereafter, aware of the declared migration policy, individuals make their occupational choices, we can incorporate the responses of individuals and firms to migration into the receiving country’s optimization problem. These reactions are exhibited by the expressions \( w_S^q \) and \( w_M^q \) in (19), by \( L_S^q = 1 - F(\ln w^q) \), and by \( L_M^q = F(\ln w^q) \). Upon substitution for \( w_S^q \), \( w_M^q \), \( L_S^q \), and \( L_M^q \) into (A12), the SWF becomes
\[
SWF^q(Q, Q_M) = D + \ln w^q \left[ F(\ln w^q) - 1 + \alpha + \eta \right] + \int_{\ln w^q}^E \varepsilon f(\varepsilon) d\varepsilon, \quad \text{(A13)}
\]

where \( D = (\alpha + \eta) \ln \alpha + (1 - \alpha - \eta) \ln (1 - \alpha) + K \).

The receiving country chooses a quota \( Q_S \) of migrant scientists and a quota \( Q_M \) of migrant managers, namely a pair \( (Q_S, Q_M) \), with the aim of maximizing (A13) subject to the non-negativity constraints on the choice variables, \( Q_S \geq 0 \) and \( Q_M \geq 0 \), and subject to the constraint on the level of migration, \( Q_S + Q_M \leq Q \). Because these three constraints are linear, the feasible region is a triangle given by the intersection of \( Q_S \geq 0 \), \( Q_M \geq 0 \), and \( Q_S + Q_M \leq Q \), with vertices at \((0,0)\), \((Q,0)\), and \((0,Q)\). The Lagrangian for the constrained optimization problem is
\[
V(Q_S, Q_M) = D + \ln \left[ w^q(Q_S, Q_M) \right] \left[ F(\ln \left[ w^q(Q_S, Q_M) \right]) - 1 + \alpha + \eta \right] + \int_{\ln w^q(Q, Q_M)}^E \varepsilon f(\varepsilon) d\varepsilon + \lambda (Q - Q_S - Q_M), \quad \text{(A14)}
\]

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where, for the sake of transparency, we emphasize that \( w^q \) is a function of \( Q_s \) and \( Q_m \). The first-order conditions for the SWF maximization problem are

\[
\frac{\partial V}{\partial Q_j} \leq 0, \quad Q_j \geq 0, \quad \text{and} \quad Q_j \frac{\partial V}{\partial Q_j} = 0, \quad (A15)
\]

and

\[
\frac{\partial V}{\partial \lambda} \geq 0, \quad \lambda \geq 0, \quad \text{and} \quad \lambda \frac{\partial V}{\partial \lambda} = 0. \quad (A16)
\]

That

\[
\frac{\partial V}{\partial Q_j} = \left[ F \left( \ln w^q \right) - 1 + \alpha + \eta \right] \frac{\partial \ln w^q}{\partial Q_j} + \ln w^q \frac{\partial F \left( \ln w^q \right)}{\partial Q_j} + \frac{\partial}{\partial Q_j} \int \varepsilon f \left( \varepsilon \right) d\varepsilon - \lambda
\]

\[
= \left[ F \left( \ln w^q \right) - 1 + \alpha + \eta \right] \frac{\partial \ln w^q}{\partial Q_j} + \left( \ln w^q \right) f \left( \ln w^q \right) \frac{\partial \ln w^q}{\partial Q_j} - \left( \ln w^q \right) f \left( \ln w^q \right) \frac{\partial \ln w^q}{\partial Q_j} - \lambda
\]

\[
= \left[ F \left( \ln w^q \right) - 1 + \alpha + \eta \right] \frac{\partial \ln w^q}{\partial Q_j} - \lambda,
\]

yields

\[
\frac{\partial V}{\partial Q_s} = \left[ F \left( \ln w^q \right) - 1 + \alpha + \eta \right] \frac{\partial \ln w^q}{\partial Q_s} - \lambda \quad (A17)
\]

and

\[
\frac{\partial V}{\partial Q_m} = \left[ F \left( \ln w^q \right) - 1 + \alpha + \eta \right] \frac{\partial \ln w^q}{\partial Q_m} - \lambda. \quad (A18)
\]

Finally,

\[
\frac{\partial V}{\partial \lambda} = Q - Q_s - Q_m. \quad (A19)
\]

We first show that the maximum of the SWF cannot occur under a migration of both scientists and managers, that is, it cannot obtain for the intersection of \( Q_s > 0 \) and \( Q_m > 0 \). It follows from the first-order conditions \( (A15) \) that if a maximum to the SWF were to obtain for \( \left( Q_s^*, Q_m^* \right) \) such that \( Q_s^* > 0 \) and \( Q_m^* > 0 \), then it would be required that

\[
\frac{\partial V \left( Q_s^*, Q_m^* \right)}{\partial Q_s} = \frac{\partial V \left( Q_s^*, Q_m^* \right)}{\partial Q_m} = 0. \quad (A20)
\]
Let \( h(\ln w^\theta) = F(\ln w^\theta) - 1 + \alpha + \eta \). On substitution from 
\[
\frac{\partial V}{\partial Q_s} = h(\ln w^\theta) \frac{\partial \ln w^\theta}{\partial Q_s} - \lambda
\]
and
\[
\frac{\partial V}{\partial Q_M} = h(\ln w^\theta) \frac{\partial \ln w^\theta}{\partial Q_M} - \lambda
\]
(cf. (A17) and (A18), respectively, upon incorporating the definition of \( h(\ln w^\theta) \)) in (A20), and on rearrangement, we get that (A20) obtains only if
\[
h(\ln \left[w^\theta(Q_s', Q_M')\right]) = 0,
\]
which in turn implies that \( \lambda = 0 \) (noting that \( \frac{\partial \ln w^\theta}{\partial Q_s} = \frac{1}{w^\theta} \frac{\partial w^\theta}{\partial Q_s} > 0 \)) whereas
\[
\frac{\partial \ln w^\theta}{\partial Q_M} = \frac{1}{w^\theta} \frac{\partial w^\theta}{\partial Q_M} < 0,
\]
(cf. Claim 1)). When \( \lambda = 0 \), any point for which (A20) holds is an ordinary stationary point which has to obey the second partial derivative test. The Hessian matrix for any stationary point is given by
\[
H = f(\ln w^\theta) \begin{bmatrix}
\left( \frac{\partial \ln w^\theta}{\partial Q_s} \right)^2 & \frac{\partial \ln w^\theta}{\partial Q_s} \frac{\partial \ln w^\theta}{\partial Q_M} \\
\frac{\partial \ln w^\theta}{\partial Q_s} \frac{\partial \ln w^\theta}{\partial Q_M} & \left( \frac{\partial \ln w^\theta}{\partial Q_M} \right)^2
\end{bmatrix}.
\]
We have that \( \left( \frac{\partial \ln w^\theta}{\partial Q_s} \right)^2 > 0 \) and that \( \det H = 0 \) and, thus, the second partial derivative test is inconclusive.\(^{20}\) However, because \( H \) has positive entries on the main diagonal, it cannot constitute a maximum of the SWF. Therefore, a maximum of the SWF can obtain only either when the migrants are all scientists, or when the migrants are all managers, or when there is no migration at all. We explore each of these possible cases in turn.

If migration exclusively of scientists were to maximize the SWF, that is, if a maximum of the SWF were to obtain for \( (Q_s', 0) \), where \( Q_s' > 0 \), then the first-order conditions given by

---

\(^{20}\) That \( \det H = 0 \) follows from the properties of the CRS Cobb-Douglas production function. When using such a production function for calculating the equilibrium levels of wages, the ratios of the two types of workers matter, not their numbers. For any initial ratio of scientists to managers, we can add several scientists and several managers in such a proportion that the ratio of scientists to managers remains unchanged. Such an addition will not affect the distribution of the individuals by skill types as well as by the wages paid to different skill types in equilibrium. Consequently, the equilibrium ratio of managerial work to scientific work will not change either and, similarly, the value of the SWF will not change either because it depends only on \( w^\theta(Q_s, Q_M) \). This is why the SWF does not strictly increase (or strictly decrease) locally in the neighborhood of any point of the feasible region, and why the second derivative test (and also higher-order derivative tests) is (are) inconclusive.
(A15) are \( \frac{\partial V(Q^*,0)}{\partial Q_S} = 0 \) and \( \frac{\partial V(Q^*,0)}{\partial Q_M} \leq 0 \). We consider two cases: \( Q^*_S < Q \) and \( Q^*_S = Q \). If \( Q^*_S < Q \), then it follows from the first-order condition (A16) that \( \lambda = 0 \). With \( \lambda = 0 \), and on recalling (A17) and (A18), we get that \( \frac{\partial V(Q^*,0)}{\partial Q_S} = 0 \) holds only if \( h(\ln w^0(Q^*,0)]\frac{\partial \ln w^0(Q,0]}{\partial Q_S} = \lambda \) which, in turn, and together with \( \lambda = 0 \), implies that \( \frac{\partial V(Q^*,0)}{\partial Q_M} = 0 \). Because a point \( \frac{\partial V}{\partial Q_S} = \frac{\partial V}{\partial Q_M} = 0 \) cannot constitute a maximum of the SWF (as shown in the preceding part of this proof), any point \( (Q^*,0) \) such that \( 0 < Q^*_S < Q \) does not maximize the SWF.

If \( Q^*_S = Q \), then \( \lambda \geq 0 \). With \( \lambda \geq 0 \), and recalling (A17) and (A18), we get that

\[
\frac{\partial V(Q,0)}{\partial Q_S} = 0 \quad \text{and} \quad \frac{\partial V(Q,0)}{\partial Q_M} \leq 0
\]

jointly hold if \( h(\ln w^0(Q,0)]\frac{\partial \ln w^0(Q,0]}{\partial Q_S} = \lambda \) and

\[
h(\ln w^0(Q,0)]\frac{\partial \ln w^0(Q,0]}{\partial Q_M} \leq \lambda .
\]

Substituting for \( \lambda \) from the preceding equation into the last inequality, we get that for a maximum to obtain at \( (Q,0) \), it is required that

\[
h(\ln w^0(Q,0)]\frac{\partial \ln w^0(Q,0]}{\partial Q_M} \leq h(\ln w^0(Q,0)]\frac{\partial \ln w^0(Q,0]}{\partial Q_S},
\]

which holds if \( h(\ln w^0(Q,0)] > 0 \). Because \( h(\ln w^0(Q,0)] > 0 \) can well be satisfied, \( (Q,0) \) can constitute a (local) maximum to the SWF.

For the case of migration consisting exclusively of managers, the proof tracks the same steps as those taken for the case of migration consisting exclusively of scientists. In this case, \( (0,Q) \) constitutes a (local) maximum of the SWF if \( h(\ln w^0(0,Q)] < 0 \).

For the no-migration state to constitute a maximum of the SWF, it is required that

\[
\frac{\partial V(0,0)}{\partial Q_S} \leq 0 \quad \text{and} \quad \frac{\partial V(0,0)}{\partial Q_M} \leq 0 \quad (\text{cf. (A15)})
\]

or, upon recalling (A17) and (A18), that

\[
h(\ln w^0)\frac{\partial \ln w^0}{\partial Q_S} \leq \lambda \quad \text{and} \quad h(\ln w^0)\frac{\partial \ln w^0}{\partial Q_M} \leq \lambda .
\]

Because \( \frac{\partial V(0,0)}{\partial \lambda} > 0 \), then from (A16) it follows that \( \lambda = 0 \) and, thus, a maximum of the SWF will be obtained for the no-migration state...
state if $h(\ln w^q) \frac{\partial \ln w^q}{\partial Q_s} \leq 0$ and $h(\ln w^q) \frac{\partial \ln w^q}{\partial Q_m} \leq 0$. However, because $\frac{\partial \ln w^q}{\partial Q_s} > 0$ whereas $\frac{\partial \ln w^q}{\partial Q_m} < 0$, both $h(\ln w^q) \frac{\partial \ln w^q}{\partial Q_s} \leq 0$ and $h(\ln w^q) \frac{\partial \ln w^q}{\partial Q_m} \leq 0$ cannot hold simultaneously and, thus, a maximum of the SWF cannot be obtained for the no-migration state.

Thus far we have shown that the only points that might constitute a maximum of the SWF are $(Q,0)$ (which locally maximizes SWF if $h(\ln \left[ w^q (Q,0) \right]) > 0$) and $(0,Q)$ (which locally maximizes SWF if $h(\ln \left[ w^q (0,Q) \right]) < 0$). We next show that at least one of these two points actually locally maximizes SWF. Because $\frac{\partial h(\ln w^q)}{\partial Q_s} = f(\ln w^q) \frac{\partial \ln w^q}{\partial Q_s} > 0$, and $\frac{\partial h(\ln w^q)}{\partial Q_m} = f(\ln w^q) \frac{\partial \ln w^q}{\partial Q_m} < 0$, as we increase the level of migration consisting exclusively of scientists (managers) from zero to a positive value, $h(\ln w^q)$ increases (decreases). If in the no-migration setting we have that $h(\ln w^q) \geq 0$, then it has to be that $h(\ln \left[ w^q (Q,0) \right]) > 0$, in which case $(Q,0)$ locally maximizes SWF. If, however, $h(\ln w^q) \leq 0$, then it has to be that $h(\ln \left[ w^q (0,Q) \right]) < 0$, in which case $(0,Q)$ locally maximizes SWF. Because either $h(\ln w^q) \geq 0$ or $h(\ln w^q) \leq 0$, then at least one of the two points will locally maximize SWF.

If only one of the two points $(Q,0)$ and $(0,Q)$ locally maximizes SWF, then that point maximizes SWF globally. If, however, both $(Q,0)$ and $(0,Q)$ locally maximize SWF, which occurs if $h(\ln \left[ w^q (Q,0) \right]) > 0$ and $h(\ln \left[ w^q (0,Q) \right]) < 0$, then $(Q,0)$ globally maximizes SWF if $SWF^q (Q,0) > SWF^q (0,Q)$. The inverse of the latter inequality yields $(0,Q)$ as a global maximum of the SWF. Q.E.D.

**Lemma 1.** $Q_1$ exists, and is unique.

**Proof.** $Q_1$ is defined as
\[ Q_i > 0 \quad \text{and} \quad SWF^\eta(Q_i, 0) = SWF^\eta, \quad \text{if} \quad \eta < L_S^n - \alpha \]
\[ Q_i > 0 \quad \text{and} \quad SWF^\eta(0, Q_i) = SWF^\eta, \quad \text{if} \quad \eta > L_S^n - \alpha \]
\[ Q_i = 0, \quad \text{if} \quad \eta = L_S^n - \alpha. \]

To show that \( Q_i \) exists and that it is unique, we address in turn the cases \( \eta < L_S^n - \alpha \), \( \eta > L_S^n - \alpha \), and \( \eta = L_S^n - \alpha \). We first present a claim.

**Claim 3.** Under a quota, when migrants are of the same skill type, as we increase the level of migration from zero to a positive value, the value of the SWF

(a) first decreases and then increases when migrants are all scientists, and continuously increases when migrants are all managers, if the externality generated by the scientists is sufficiently weak, that is, if \( \eta < L_S^n - \alpha = 1 - F(\ln w^\eta) - \alpha \);

(b) continuously increases when migrants are all scientists, and first decreases and then increases when migrants are all managers, if the externality generated by the scientists is sufficiently strong, that is, if \( \eta > L_S^n - \alpha \);

(c) continuously increases when migrants are all scientists and when migrants are all managers, if the externality generated by the scientists is neither strong nor weak, that is, if \( \eta = L_S^n - \alpha \).

**Proof.** The change in the value of the SWF brought about by a marginal increase in the level of migration of a given type is measured by

\[
\frac{\partial \text{SWF}^\eta(Q_S, Q_M)}{\partial Q_j} = h(\ln w^\eta) \frac{\partial \ln w^\eta}{\partial Q_j}, \quad j = S, M
\]

(recalling that \( h(\ln w^\eta) = F(\ln w^\eta) - 1 + \alpha + \eta \)). Because \( \frac{\partial \ln w^\eta}{\partial Q_S} = \frac{1}{w^\eta} \frac{\partial w^\eta}{\partial Q_S} > 0 \) and

\[
\frac{\partial \ln w^\eta}{\partial Q_M} = \frac{1}{w^\eta} \frac{\partial w^\eta}{\partial Q_M} < 0 \]

(cf. Claim 1), the sign of \( \frac{\partial \text{SWF}^\eta(Q_S, Q_M)}{\partial Q_j} \) at each point of the feasible region depends on the sign of \( h(\ln w^\eta) \). Upon opening up to migration, and because \( \ln[w^\eta(0, 0)] = \ln w^\eta \), the direction of the change in the value of the SWF brought about by a marginal increase in the level of migration of a given type from zero to a small positive value depends on whether \( h(\ln w^\eta) < 0 \), \( h(\ln w^\eta) > 0 \), or \( h(\ln w^\eta) = 0 \), or, upon recalling that
\[ h(\ln w^a) = F(\ln w^a) - 1 + \alpha + \eta = \alpha - L^a_n, \] it depends on whether \( \eta < L^a_n - \alpha \), \( \eta > L^a_n - \alpha \), or \( \eta = L^a_n - \alpha \), respectively. These three possibilities correspond to parts (a), (b), and (c) of this claim; we attend to the three parts in turn.

(a) When \( \eta < L^a_n - \alpha \), then \( \frac{\partial \text{SWF}^q(0,0)}{\partial Q_s} < 0 \) and \( \frac{\partial \text{SWF}^q(0,0)}{\partial Q_M} > 0 \), which indicate that an increase in the level of migration consisting exclusively of scientists (managers) from zero to a small positive value will decrease (increase) the value of the SWF. Because

\[
\frac{\partial \ln w^a}{\partial Q_M} = \frac{1}{w^a} \frac{\partial w^a}{\partial Q_M} < 0 \quad \text{and} \quad \frac{\partial h(\ln w^a)}{\partial Q_M} = f(\ln w^a) \frac{\partial \ln w^a}{\partial Q_M} < 0,
\]

it follows that \( \frac{\partial \text{SWF}^q(0,Q_M)}{\partial Q_M} > 0 \) for all \( Q_M \). Consequently, the value of the SWF continuously increases with the level of migration consisting exclusively of managers, thus \( \text{SWF}^q(0,Q_M) > \text{SWF}^q(0,0) = \text{SWF}^a \) for all \( Q_M > 0 \), given that \( \eta < L^a_n - \alpha \). When the level of migration consisting exclusively of scientists is small, \( h(\ln w^a) \approx h(\ln w^a) < 0 \) and, thus,

\[
\frac{\partial \text{SWF}^q(Q_s,0)}{\partial Q_s} < 0, \quad \text{whereas when it is large enough, it follows from} \quad \frac{\partial \ln w^a}{\partial Q_s} = \frac{1}{w^a} \frac{\partial w^a}{\partial Q_s} > 0, \quad \frac{\partial h(\ln w^a)}{\partial Q_s} = f(\ln w^a) \frac{\partial \ln w^a}{\partial Q_s} > 0, \quad \text{and} \quad h(\ln e^\alpha) = \alpha + \eta > 0, \quad \text{that} \quad h(\ln w^a) > 0 \quad \text{and, thus}, \]

\[
\frac{\partial \text{SWF}^q(Q_s,0)}{\partial Q_s} > 0. \quad \text{Consequently, holding} \quad Q_M = 0, \quad \text{the value of the SWF first decreases and then increases with the level of migration consisting exclusively of scientists.}
\]

(b) The proof of (b) tracks the same steps as those taken in the proof of (a).

(c) When \( \eta = L^a_n - \alpha \), then \( \frac{\partial \text{SWF}^q(0,0)}{\partial Q_s} = 0 \) and \( \frac{\partial \text{SWF}^q(0,0)}{\partial Q_M} = 0 \). Because \( \frac{\partial \ln w^a}{\partial Q_s} > 0 \) and

\[
\frac{\partial h(\ln w^a)}{\partial Q_s} = f(\ln w^a) \frac{\partial \ln w^a}{\partial Q_s} > 0 \quad \text{and} \quad \frac{\partial h(\ln w^a)}{\partial Q_M} = f(\ln w^a) \frac{\partial \ln w^a}{\partial Q_M} < 0, \quad \text{it follows that} \quad \frac{\partial \text{SWF}^q(Q_s,0)}{\partial Q_s} > 0 \quad \text{and} \quad \frac{\partial \text{SWF}^q(0,Q_M)}{\partial Q_M} > 0 \]

for any positive value of \( Q_s \) and \( Q_M \). Consequently, the SWF continuously increases with the
level of migration consisting exclusively of scientists or exclusively of managers, thus
\( SWF^q(Q_S,0) > SWF^q(0,0) = SWF^n \) and \( SWF^q(0,Q_M) > SWF^q(0,0) = SWF^n \) for all \( Q_S, Q_M > 0 \). Q.E.D.

We now return to the proof of Lemma 1. From Claim 3 we know that when \( \eta < L^n_S - \alpha \),
then upon increasing the level of migration that consists exclusively of scientists from zero to
a positive value, the value of the SWF first decreases and then increases. The remaining
question is whether the eventual increase is large enough to compensate for the initial
decrease, that is, whether for large enough migration consisting exclusively of scientists the
value of the SWF will be higher than the corresponding value in the no-migration setting.

Because the wages of scientists and managers are given by
\[ w_S = \alpha \left( \frac{\bar{L}_M}{\bar{L}_S} \right)^{1-\alpha-\eta} \]
and
\[ w_M = (1-\alpha) \left( \frac{\bar{L}_S}{\bar{L}_M} \right)^{\alpha/\eta} \], respectively, as we increase \( Q_S \) but not \( Q_M \), and thereby increase
\( \bar{L}_S \) relative to \( \bar{L}_M \) (cf. part (b) of Proposition 2), the wages of scientists go down and the
wages of managers go up, eventually leading to all the natives choosing management over
science, which occurs when \( \ln w_M^n > \ln w_S^n + E \). As \( Q_S \) increases further, the wages of
managers eventually become high enough for the individual with the highest occupational
prestige preferring management under a quota to science under no migration, namely
\( \ln w_M^n > \ln w_S^n + E \). At that point, all the natives are better off than in the no-migration setting,
thus clearly \( SWF^q(Q_S,0) > SWF^n \). Altogether, when \( \eta < L^n_S - \alpha \), \( SWF^q(0,Q_M) > SWF^n \) for
all \( Q_M \), whereas \( SWF^q(Q_S,0) < SWF^n \) for small \( Q_S \), and \( SWF^q(Q_S,0) > SWF^n \) for large
\( Q_S \). Because the sign of \( \frac{\partial SWF^q(Q_S,0)}{\partial Q_S} \) changes only once, there can be only one magnitude
of migration such that \( SWF^q(Q_S,0) = SWF^n \).

For the case when \( \eta > L^n_S - \alpha \), the proof follows steps that are akin to the ones taken in
the case \( \eta < L^n_S - \alpha \). For \( \eta = L^n_S - \alpha \), a proof is not needed. Q.E.D.

**Lemma 2.** (a) \( Q_2 \) exists, and is unique. (b) \( Q_2 < Q_1 \), if \( \eta > L^n_S - \alpha \); \( Q_2 = Q_1 \), if \( \eta \leq L^n_S - \alpha \).

**Proof.** (a) The proof is analogous to the proof of Lemma 1.
(b) That $Q_2 < Q_1$ if $\eta > L^s_s - \alpha$ follows because under a uniform entry fee, when all the migrants are managers, the SWF attains the same values as under a quota when migration is at a lower level than under a quota. This is so because migration of a given level is of more effective units of work under a uniform entry fee than under a quota, which is due to the positive self-selection by the migrants (cf. part (c) of Proposition 3). Thus, fewer managers are needed under a uniform entry fee than under a quota for the SWF to be of equal value to that in the no-migration setting. That $Q_2 = Q_1$ if $\eta < L^s_s - \alpha$ follows from a comparison of (24) and (25). Q.E.D.

**Proposition 7.** Under a uniform entry fee, when $Q \leq Q_2$, the receiving country attains the optimal skill composition of its workforce (it maximizes SWF) when

- (a) the level of migration is zero, $Q_s = Q_m = 0$, if the externality generated by the scientists is sufficiently strong, that is, if $\eta > L^s_s - \alpha$;
- (b) the level of migration is at $\min\{Q(x), Q\}$, if the externality generated by the scientists is sufficiently weak, that is, if $\eta < L^s_s - \alpha$.

**Proof.** We first present the following claim.

**Claim 4.** Under a uniform entry fee, as we increase the level of migration from zero to a positive value, the value of the SWF

- (a) continuously increases, until scientists too find it beneficial to migrate, if the externality generated by the scientists is sufficiently weak, that is, if $\eta \leq L^s_s - \alpha$;
- (b) first decreases and then increases when migrants are all managers, if the externality generated by the scientists is sufficiently strong, that is, if $\eta > L^s_s - \alpha$.

**Proof.** The proof is analogous to the proof of Claim 3.

We now return to the proof of Proposition 7. The proof follows from the intersection of Claim 4 and (25) for part (a), and when $\min\{Q(x), Q\} = Q$ for part (b). When $\min\{Q(x), Q\} = Q(x)$, then by setting the fee at $x < x$, the receiving country will encourage migration of both scientists and managers. Because the incoming scientists are of higher average skill level than the incoming managers, $\bar{\theta}_s^m > \bar{\theta}_m^s$, then by increasing the level of migration, the receiving country will admit relatively more units of effective scientific work.
than of effective (additional) managerial work, which reduces the desired “crowding out effect” (cf. the proof of Proposition 4). Therefore, it is not optimal for the receiving country to have an overall migration larger than $Q(x)$. Q.E.D.

**Proposition 11.** Under a differentiated entry fee, when $Q \leq Q_3$, the receiving country attains the optimal skill composition of its workforce (it maximizes SWF) when the level of migration is at the limit $Q_3 + Q_M = Q$, and when the composition of migration by skill type is such that the migrants are

(a) all scientists, namely $Q_M = 0$, if the externality generated by the scientists is sufficiently strong, that is, if $\eta > L_s^\alpha - \alpha$;

(b) all managers, namely $Q_S = 0$, if the externality generated by the scientists is sufficiently weak, that is, if $\eta < L_s^\alpha - \alpha$.

**Proof.** We first present two claims.

**Claim 5.** Under a differentiated entry fee, the welfare of the natives is maximized when the level of migration is at the limit $Q_S + Q_M = Q$, and when the composition of migration by skill type is such that the migrants are

(a) all scientists, namely $Q_M = 0$, if $\text{SWF}^{\text{def}} (Q,0) > \text{SWF}^{\text{def}} (0,Q)$;

(b) all managers, namely $Q_S = 0$, if $\text{SWF}^{\text{def}} (Q,0) < \text{SWF}^{\text{def}} (0,Q)$.

**Proof.** The proof is analogous to the proof of Proposition 5.

**Claim 6.** Under a differentiated entry fee, when migrants are of the same skill type, as we increase the level of migration from zero to a positive value, the value of the SWF

(a) first decreases and then increases when migrants are all scientists, and continuously increases when migrants are all managers, if the externality generated by the scientists is sufficiently weak, that is, if $\eta < L_s^\alpha - \alpha$;

(b) continuously increases when migrants are all scientists, and first decreases and then increases when migrants are all managers, if the externality generated by the scientists is sufficiently strong, that is, if $\eta > L_s^\alpha - \alpha$;
(c) continuously increases when migrants are all scientists and when migrants are all
managers, if the externality generated by the scientists is neither strong nor weak,
that is, if $\eta = L_s^\alpha - \alpha$.

**Proof.** The proof is analogous to the proof of Claim 3.

We now return to the proof of Proposition 11. The proof follows from the intersection of
Claim 5, Claim 6, and (27). Q.E.D.

**Proposition 13.** The highest revenue is attained when migrants are only or mostly managers,
namely when $\frac{Q_s}{Q_M} < \frac{L_s^\alpha}{L_m^\alpha}$.

**Proof.** We prove the proposition by contradiction, showing that migration only or mostly of
scientists cannot yield the highest possible revenue. This follows from the combination of two
observations. First, by marginally increasing the level of migration from zero to a positive
value, the revenue will be highest when the migrants are only managers because their wages,
and consequently the entry fee that can be charged to them, are higher than the wages of
scientists (cf. (17)). Second, for migration only or mostly of scientists, the wages of scientists
(per unit of productivity) decrease, whereas the wages of managers increase as compared to
the no-migration setting, because $w_{\text{def}}^s > w_m$ (the proof is analogous to the proof of
Proposition 2 with a reference to Claim 2 replacing the reference to Claim 1), thereby further
increasing the wage gap between the two skill types and, thus, the entry fee that can be
charged to them. We conclude that when migrants are only or mostly scientists, the entry fee
revenue will always be higher if several migrant scientists are replaced by migrant managers,
and that the solution to the revenue-maximization problem has to be migration only or mostly
of managers. Q.E.D.

**Appendix B: An illustrative calculation of the strength of the externality generated by
the scientists, based on US data**

We seek to find out whether $\eta > L_s^\alpha - \alpha$ for the US. To calculate $\alpha$, we use the equation for
$w_s$ as displayed in (9), which, upon rearrangement and upon recalling that
$A(l)\left(\frac{L_m}{L_s}\right)^{1-\alpha} = \frac{Y}{L_s}$, yields $\alpha = \frac{w_s L_s}{Y}$. Calculating $\alpha$ requires US data on the wages paid to
the scientists, their number, and the country’s GDP. For the purpose of this calculation we consider scientists to be STEM workers.\(^{21}\) According to the Bureau of Labor Statistics (BLS) database, in May 2015 there were 8.47 million STEM workers in the US, with a mean annual wage of $88,881. These data, together with the US GDP, which in the second quarter of 2015 was estimated at $17,998.3 billion, yield \(\alpha = 0.042\).\(^{22}\)

Data on the share of scientists in the US workforce in the no-migration setting, \(L^n_s\), are not available. However, \(\eta > L^n_s - \alpha\) will hold if instead of the no-migration share of scientists in the US workforce we use that share under a quota, \(\tilde{L}^q_s\), provided that \(\tilde{L}^q_s > L^n_s\). In turn, \(\tilde{L}^q_s > L^n_s\) will hold if foreigners among scientists constitute a larger share than foreigners in the US workforce, that is, if \(\frac{Q_s}{L_s} > Q\). In 2010, the share of foreigners among STEM workers in the US was about 21.8 percent, whereas the share of foreigners in the US workforce was only 17.6 percent (Table 1), which allows us to substitute \(L^n_s\) with \(\tilde{L}^q_s\).\(^{23}\) Because the share of STEM workers in the US is estimated at \(\tilde{L}^q_s = 0.061\) (our calculations based on the BLS data), the US should seek to increase the share of scientists amongst migrants if \(\eta > 0.019\).

Unfortunately, we cannot estimate \(\eta\) directly by applying the official US data to any of our model’s equations; for that we need to refer to the received literature. Moreover, whereas empirical studies that measure the social returns of higher education exist, the studies that measure externalities generated by specific skill types, science in particular, are scarce and, to the best of our knowledge, none measures the impact of STEM workers on TFP. Therefore, we calculate the value of \(\eta\) indirectly drawing on the available empirical studies. The results of two studies (Kerr and Lincoln, 2010; and Peri et al., 2015) can be used for such indirect calculation. The methods of obtaining \(\eta\) in these studies are similar but differ somewhat. Kerr and Lincoln find no effect of migration of scientists and engineers on the wages of native

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\(^{21}\) We take the list of STEM occupations from the US BLS, which can be found at www.bls.gov/oes/stem_list.xlsx.

\(^{22}\) A rather small estimated value for the output elasticity of scientific work, \(\alpha\), does not imply that the estimate for the output elasticity of managerial work is close to one, as would follow from the latter elasticity being defined as \(1 - \alpha\); when calculated directly, the estimate for output elasticity of managerial work is also small. The two elasticities add up to one only for a simple economy with two skill types as factors of production.

\(^{23}\) Peri et al. (2015), who use a different definition of STEM workers than the one used by the US Bureau of Labor Statistics, estimate the share of foreign-born among STEM workers in the US at 26 percent.
scientists and engineers in the US. This finding can formally be expressed as \( \frac{\partial \ln w_s}{\partial \ln Q_s} = 0 \).

Because in our model (cf. (9) in conjunction with log-differentiation) \( \frac{\partial \ln w_s}{\partial \ln Q_s} = \alpha + \eta - 1 \), then \( \frac{\partial \ln w_s}{\partial \ln Q_s} = 0 \) implies that \( \eta = 1 - \alpha = 0.958 \). Peri et al. estimate that a one percent increase in the supply of STEM workers will increase the wages of college-educated workers by 4 to 6 percent, and will have no effect on the wages of workers who are not college-educated. This finding can formally be expressed as \( 4 < \frac{\partial \ln w_{\text{EDU}}}{\partial \ln L_s} < 6 \) and \( \frac{\partial \ln w_{\text{NEDU}}}{\partial \ln L_s} = 0 \), where the subscript EDU stands for college-educated workers, and the subscript NEDU stands for not college-educated workers. The weighted average of these effects, where as weights we use the shares of college-educated workers and not college-educated workers in the US workforce, which are 0.39 and 0.61, respectively, is not larger than 1.56.\(^{24}\) In a model analogous to ours but with more than two skill types as inputs in the economy’s production function, the percentage change in the wage of each skill type other than science in response to a one percent increase in the size of the scientific workforce will be the same for each skill type and it will be equal to \( \alpha + \eta \) (just as in our model, cf. (9) in conjunction with log-differentiation, we have that \( \frac{\partial \ln w_s}{\partial \ln L_s} = \alpha + \eta \)). Because STEM workers constitute a small fraction of the US workforce, we use \( \alpha + \eta \) as an approximation of the effect of a one percent increase in the size of the STEM workforce on the wages of all, STEM and non-STEM, workers in the US, that is, \( 1.56 \approx \alpha + \eta \). Therefore, upon recalling that \( \alpha = 0.042 \), and upon rearrangement, we get that \( \eta \) is not smaller than 1.518. Both values of \( \eta \) that we calculated on the basis of received empirical literature are significantly higher than \( \tilde{L}_s^\eta - \alpha = 0.019 \), which suggests that in the US, the externality generated by STEM workers is strong.

These rudimentary calculations provide a rough measure of the interaction between the model’s parameters that determine the optimal composition of migration by skill type. Still, a large gap between the calculated “TFP effect,” \( \eta \), and the calculated “crowding out effect,” \( L_s^\alpha - \alpha \), implies that there is considerable room for the actual values of the relevant parameters to differ from the estimates that we have presented. Overall, the numerical

\(^{24}\) The shares were calculated using the US BLS data, which can be found at http://www.bls.gov/cps/cpsaat07.htm.
illustration points to a scientists-only migration as optimal for the receiving country when such a country can be characterized by parameters akin to the ones for the US.
References


