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Economics: The Open-Access, Open-Assessment E-Journal

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Asymmetry and Spillover Effects in the North American Equity Markets

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Abstract:
In this paper we examine the issue of asymmetry in the return and volatility spillover effects from the US equity market into the Canadian and Mexican equity markets. We model the conditional volatility of the returns in each of the three markets using the asymmetric power model of Ding, Granger and Engle (1993). The empirical findings indicate that the US market has a significant impact on the returns in the Canadian and Mexican markets. However, the findings for Canada vary considerably from those for Mexico. In particular, the empirical results indicate that volatility spillover effects, but not return spillover effects, exhibit an asymmetric behavior, with negative shocks from the US equity market impacting on the conditional volatility of the Canadian and Mexican equity markets more deeply than positive shocks. Moreover, while the impact of positive shocks from the US equity market is not much different between the two markets, this is not the case with negative shocks, which affect the volatility of the Mexican market more intensely than the volatility of the Canadian market.

JEL: C32, C53, F31, G15
Keywords: APARCH, Asymmetric Spillovers, North American Stock Markets

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1. Introduction

There is ample evidence that national equity markets have become more interdependent in recent years. The developments in the liberalization of capital movements and financial reforms, coupled with advances in computer technology and information processing, have reduced the isolation of national equity markets and increased their ability to react promptly to news and shocks originating from the rest of the world. Evidence of increased linkages between national equity markets has also been found following the October 1987 market crash, and the Asian and Russian financial crises. In general, most of the research has documented four stylized facts: 1) correlations across stock markets are time-varying; 2) returns in major markets tend to be more correlated when volatility is high; 3) all major episodes of high volatility are associated with market drops; and, finally, 4) correlations in volatility and returns appear to be causal from the US market.

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1 There is a substantial literature investigating the mechanisms through which returns and volatilities in one market are transmitted to other markets. See, e.g., Ng, Chang and Chow (1991), Lin, Engle and Ito (1994), Karolyi (1995), Kim and Rogers (1995), and Booth, Martikainen and Tse (1997).

2 These developments in information technology also indicate that the use of low frequency data is exceedingly restrictive and that it is essential to use high frequency data to examine the issue of spillover effects in any meaningful manner.

3 These studies include the important contributions by Eun & Shim (1989), Von Furstenberg and Jeon (1989), King and Wadhwani (1990), Schwert (1990), Hamao, Masulis and Ng (1990), King, Sentana and Wadhwani (1994), Arshanapalli and Doukas (1993), and Longin & Solnik (1995).
There also exists widespread evidence that national equity markets returns show strong asymmetries in conditional volatilities. Yet, there is little evidence in the literature documenting that the international transmission of stock returns and volatility also exhibits asymmetric behaviors. Further, while international transmission of stock returns and volatility has been widely detected in European and Asian countries, sparse attention has been devoted to transmission of stock return and volatility in North America markets.

The aim of this paper is to examine the return and volatility spillovers effects from the US equity market into the Canadian and Mexican equity markets. The extent in which the Canadian and Mexican stock markets depend upon the US stock market impacts directly on the issue of market integration of the North American continent, and, specifically, on the emergence of more integrated North American capital markets following the enactment of the North American Free Trade Agreement (NAFTA). NAFTA, which took effect on January 1, 1994, has greatly reduced or eliminated tariffs and other trade barriers between the three North American nations. In addition, NAFTA has promoted capital movements across borders by relaxing restrictions on cross-country

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4 See, e.g., Bae and Karolyi (1994), Ding, Granger and Engle (1993), Hentschel (1995) and Booth, Martikainen and Tse (1997). What is the main determinant of asymmetric volatility remains an open question. Black (1976) and Christie (1982) explain the asymmetric volatility property based on leverage. A drop in the value of the stock increases financial leverage, which makes the stock riskier and increases its volatility. Pindyck (1984), French, Schwert and Stambaugh (1987) and Campbell and Hentschel (1992), on the other hand, argue that the asymmetric nature of the volatility response to shock returns reflects the existence of time varying risk premiums. An increase in volatility raises the required return on equity, leading to an immediate stock decline. See Bekaert and Wu (2000) for a detailed analysis.

investing and ownership of foreign stocks. Trends toward greater economic interdependence and increased financial integration in the North American continent dilute the long-term diversification benefits available to market participants. Hedging strategies depend on shocks to stock markets being relatively isolated, idiosyncratic events, and if shocks to returns and volatilities in the US travel quickly across the Canadian and Mexican borders, the benefits of diversification may be undermined.

Apart from the focus on North America, this paper differs from previous research on spillover effects on two grounds. First, a key concern in the paper is the manner in which spillover effects from the US are transmitted across the North American markets. Specifically, our interest is in documenting (a) whether stock returns in an advanced, mature market (Canada) react differently from stock returns in an emerging market (Mexico) to return and volatility shocks from the US stock market, and (b) whether these spillover effects display nonlinear and asymmetric characteristics. The latter concern is motivated by the common observation that rising and declining patterns of a process frequently display nonlinear, asymmetric characteristics. This generality in the modeling of spillover effects has thus far been scant in the literature studying dependencies in national stock markets. Clearly, failure to properly account for these asymmetries, if they are present in the data, is likely to lead to incorrect inferences concerning the nature of the US spillover transmission across the North American markets. Second, whereas previous research on spillover effects has, with few exceptions, such as Bae and Karolyi (1994), used the standard Bollerslev’s GARCH (1, 1) specification, we model the volatility of the equity returns of the three North American markets using the asymmetric power APARCH model proposed by Ding, Granger and Engle (1993). The main
advantage of the APARCH specification is its functional flexibility. The APARCH model does not impose a common and uniform structure on the conditional volatility of the three North American equity market returns; rather, it uses a Box-Cox power transformation of the conditional standard deviation process and the asymmetric absolute residuals. This permits a virtually infinite range of transformations inclusive of any positive value. In addition, it accommodates asymmetries in the volatility of the idiosyncratic shocks in equity returns.

The empirical analysis proceeds through a two-step approach. First, we estimate an AR (1)-APARCH (1, 1) model for the US equity market returns in order to identify the volatility shocks from the US equity market. Second, we estimate an augmented AR (1)-APARCH (1, 1) model for the returns of the Canadian and Mexican equity markets that incorporates an asymmetric specification of the return and volatility spillover effects from the US market. Return spillovers occur when returns of the US market enter significantly in the anticipated part of the Canadian and Mexican returns. Volatility spillovers, on the other hand, occur when innovations in the US market have a significant effect on the unanticipated component of the Canadian and Mexican returns. It should be mentioned that the two-step approach is not without some conceptual limitations, as it excludes the possibility of reverse spillover effects from the Canadian and Mexican markets. Still, the focus on the US equity market as the reference, and hence as the center

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6 Ding, Granger and Engle (1993) show that the functional form of the APARCH (p, q) model encompasses seven other GARCH extensions as special cases, including, in addition to the standard class of Engle’s ARCH models (Engle 1982) and Bollerslev’s GARCH models (Bollerslev, 1986), the Taylor (1986) and Schwert (1990) GARCH in standard deviation model, the log-ARCH of Geweke (1986) and Pentula (1986), the threshold ARCH (TARCH) model of Zakoian (1994), the GJR-GARCH model of Glosten, Jagannathan and Runkle (Glosten et al., 1993), and the nonlinear ARCH (NARCH) model of Higgins and Bera (1992). See Ding, Granger and Engle (1993) for details.
of the international transmission of stock returns and volatility, is plausible based on the existing empirical evidence.\(^7\) Using this approach, we find strong statistical evidence that indicate that volatility spillover effects, but not in return spillover effects, exhibit an asymmetric behavior, with negative shocks from the US equity market impacting on the conditional volatility of the Canadian and Mexican equity markets more deeply than positive shocks.

The rest of the paper is organized as follows. Section 2 outlines the specification of the model. The data and their statistical properties are described in Section 3. In Section 4, we present the empirical results. A summary of the findings and some concluding remarks follow in Section 5.

2. A Spillover Model with Asymmetric Effects

Let \( R_{k,t} \) represent the return on the national equity index of country \( k \) (\( k = \) Canada, Mexico), and let \( R_{US,t} \) denote the return on the US equity index. The dynamics of the return on the US equity index \( R_{US,t} \) is specified in equations (1) through (5).

Following Bekaert and Harvey (1997), Ng (2000) and Baele (2005), we assume that US equity index returns consists of a predictable part, \( \mu_{US,t-1} \) and an unpredictable part, \( \epsilon_{US,t} \)

\[
R_{US,t} = \mu_{US,t-1} + \epsilon_{US,t} \tag{1}
\]

We model the predictable part of the US equity index returns, \( \mu_{US,t-1} \) as a first order autoregressive process:

\[
\mu_{US,t-1} = c_{0US} + c_{1US} R_{US,t-1} \tag{2}
\]

---

\(^7\) See, e.g., Masih and Masih (2001) and Eun and Shim (1989).
This is consistent with the partial adjustment model of stock returns of Amihud and Mendelson (1987) as well as the nonsynchronous trading hypothesis of Lo and MacKinlay (1990). The unpredictable part of the US equity index returns, $\varepsilon_{US,t}$ on the other hand, is defined simply in terms of a pure idiosyncratic shock, $e_{US,t}$

$$
(3) \quad \varepsilon_{US,t} = e_{US,t}
$$

which is assumed conditionally normal distributed

$$
(4) \quad e_{US,t}|I_{t-1} \sim N(0, \sigma^2_{US,t})
$$

and whose conditional standard deviation $\sigma_{US,t}$ is a time varying, positive and measurable function of the information set $I_{t-1}$ at time $t-1$, and governed by a GARCH process represented by the asymmetric power autoregressive conditional heteroskedasticity APARCH $(1, 1)$ model of Ding Granger and Engle (1993):

$$
(5) \quad \sigma^2_{US,t} = E(e^2_{US,t}|I_{t-1}) = \omega_{US} + \alpha_{US}(|e_{US,t}| - \gamma_{US} e_{US,t})^\delta + \beta_{US} \sigma^2_{US,t-1}
$$

where $\omega_{US} > 0$, $\delta \geq 0$, $\alpha_{US} \geq 0$, $\beta_{US} \geq 0$ and $-1 < \gamma_{US} < 1$. In equation (5) the standard ARCH and GARCH effects are estimated by the coefficients $\alpha_{US}$ and $\beta_{US}$, respectively, while the coefficient $\gamma_{US}$ accounts for the asymmetric response of volatility to unexpected shocks. The leverage effect, which reflects the greater impact of negative shocks than positive shocks, is present if the estimate of $\gamma_{US}$ is positive and significant. The magnitude of shocks is captured by $e_{US,t} - \gamma_{US} e_{US,t}$, and the sign effect of the shock is controlled by $-\gamma_{US} e_{US,t}$. Thus a negative shock, $e_{US,t} < 0$, with a positive $\gamma_{US}$ tends to strengthen the size of shocks while a positive shock, $e_{US,t} > 0$, tends to defuse it. The power term is denoted by $\delta$ in equation (5) and can be given by any positive values. In
particular, Ding, Granger and Engle (1993) conclude that when $\delta = 1$ the long memory property of stock returns is the strongest compared to other values of $\delta$.

The equity index return $R_{k,t}$ in country $k$, ($k = \text{Canada, Mexico}$) is specified in equations (6) through (10). As in the case of the US equity return, the return $R_{k,t}$ is also decomposed into a predictable part, $\mu_{k,t-1}$, and an unpredictable part, $\epsilon_{k,t}$.

$R_{k,t} = \mu_{k,t-1} + \epsilon_{k,t}$

However, as shown in equations (7) and (8), both $\mu_{k,t-1}$ and $\epsilon_{k,t}$ are extended to capture the asymmetric spillover effects from the US stock market. First, the AR (1) specification of the predictable part of the Mexican and Canadian stock returns is augmented to capture the asymmetric return spillover effects originating from US stock market:

$\mu_{k,t-1} = c_{0,k} + c_{1,k} R_{k,t-1} + \theta_{k}^{+} R_{US,t-1}^{+} + \theta_{k}^{-} R_{US,t-1}^{-}$

where $R_{US,t-1}^{+}$ and $R_{US,t-1}^{-}$ are time series defined by the rule $R_{US,t-1}^{+} = \max (0, R_{US,t-1})$, $R_{US,t-1}^{-} = \min (R_{US,t-1}, 0)$. The parameters $\theta_{k}^{+}$ and $\theta_{k}^{-}$ measure the return spillover intensities of positive and negative US returns at time $t-1$, respectively, on the returns of Canada and Mexico, respectively, at time $t$. Obviously, by definition, $R_{US,t-1} = R_{US,t-1}^{+} + R_{US,t-1}^{-}$ for every $t$. Note that the specification of the asymmetric return spillover process utilizes two different filters, one for positive return spillover effects, $R_{US,t-1}^{+}$ and one for negative return spillover effects $R_{US,t-1}^{+}$. It follows that when $\theta_{k}^{+} = \theta_{k}^{-} = \theta_{k}$ the return spillover process is symmetric, and the sequence $\mu_{k,t-1}$ reduces to:

$\mu_{k,t-1} = c_{0,k} + c_{1,k} R_{k,t-1} + \theta_{k} R_{US,t-1}$
Conversely, when $\theta_k^+ \neq \theta_k^-$, the impact of the return of the US market elicits a differential response in the returns of the Canadian and Mexican equity markets. Specifically, if $\theta_k^+ < \theta_k^-$ then the Canadian and Mexican equity markets respond more to the previous day negative returns in the US than to the previous day positive returns, while if $\theta_k^+ > \theta_k^-$ then the Canadian and Mexican equity markets respond more to the previous day positive returns than to the previous day negative returns. Second, the unpredictable part of the Mexican and Canadian stock returns is augmented by the volatility spillover effects from the US stock market:

\begin{equation}
\epsilon_{k,t} = \varphi^+_k e^+_{US,t} + \varphi^-_k e^-_{US,t} + e_{k,t}
\end{equation}

where $e^+_{US,t}$ and $e^-_{US,t}$ are time series defined by the rule $e^+_{US,t} = \max \{0, e_{US,t}\}$, $e^-_{US,t} = \min \{ e_{US,t}, 0 \}$, and $e_{US,t} = e^+_{US,t} + e^-_{US,t}$ for every $t$. The conditional means and variances of $e^+_{US,t}$ and $e^-_{US,t}$ are derived in the Appendix. The parameters $\varphi^+_k$ and $\varphi^-_k$ measure the volatility spillover intensities of positive and negative US shocks at time $t$, respectively, on the returns of Canada and Mexico, respectively, at time $t$. In the empirical analysis $e_{US,t}$ are the residuals from the AR (1)-APARCH (1, 1) estimation of the US return. As in the case of return spillover effects, the specification of the asymmetric volatility spillover process utilizes two different filters, one for positive volatility spillover effects, $e^+_{US,t}$, and one for negative volatility spillover effects, $e^-_{US,t}$. Obviously, when $\varphi^+_k = \varphi^-_k = \varphi_k$, the volatility spillover process is symmetric, and the sequence $\epsilon_{k,t}$ is defined by:

\begin{equation}
\epsilon_{k,t} = \varphi_k e_{US,t} + e_{k,t}
\end{equation}

Conversely, when $\varphi^+_k \neq \varphi^-_k$, the impact of the idiosyncratic shocks originating from US market elicits a differential response in the unexpected return of the Canadian and Mexican equity markets. Specifically, when $\varphi^+_k < \varphi^-_k$ then the Canadian and Mexican
equity markets respond more to negative shocks in the US than to positive shocks, while when $\phi_{k}^+ > \phi_{k}^-$ then the equity markets respond more to positive shocks than to negative shocks. Then, combining the expressions in equations (8) and (9) yields the conditional first moment of the APARCH(1, 1) model of the equity index return of Canada and Mexico, defined as:

$$R_{k,t} = c_{0,k} + c_{1,k} R_{k,t-1} + \theta_{k}^+ R_{US,t-1}^+ + \theta_{k}^- R_{US,t-1}^- + \phi_{k}^+ e_{US,t}^+ + \phi_{k}^- e_{US,t}^- + e_{k,t}$$

In equation (11) the returns of country $k$ are determined by local past information and past information arriving from the US market (the predictable or expected component of the returns) as well as by contemporary idiosyncratic shocks, initiated both locally and in the US market (the unpredictable or unexpected component of the returns). We further assume that the idiosyncratic shocks $e_{k,t}$ are conditionally normal distributed and are mutually uncorrelated, as well as uncorrelated with the US idiosyncratic shock, $e_{US,t}$, and have time-varying conditional standard deviations that are described by an APARCH (1, 1) process, i.e.,

$$e_{k,t} \mid I_{t-1} \sim N(0, \sigma_{k,t}^2)$$

$$E(e_{k,t} e_{j,t}) = 0, \text{ for all } k \neq j,$$

$$E(e_{US,t} e_{k,t}) = 0, \text{ for all } k$$

and

$$\sigma_{k,t}^\delta = E(e_{k,t}^\delta \mid I_{t-1}) = \omega_k + \alpha_k (e_{k,t-1} - \gamma_k e_{k,t}) + \beta_k \sigma_{k,t-1}^\delta$$

where $\omega_k > 0$, $\delta \geq 0$, $\alpha_k \geq 0$, $\beta_k \geq 0$, and $-1 < \theta_k < 1$. Note that the model specified in equations (1) through (15) accommodates three kinds of asymmetries. The first is defined through the idiosyncratic shocks of each of the three markets and is captured by the
leverage term in the APARCH (1, 1) model. The second is the asymmetry in the return spillover effects from the US market on the stock market of Canada and Mexico. This type of asymmetry is provided by the differential specification of the effects of the previous day positive and negative returns from the US stock market on the stock market of Canada and Mexico. Finally, the third kind of asymmetry refers to the asymmetry of the volatility spillover effects. This is accommodated by the differential impact of the contemporaneous positive and negative idiosyncratic shocks from the US stock market on the stock market of Canada and Mexico. The model implies that whenever shocks from the US market influence the unanticipated component of the Canadian and Mexican returns, they also contribute to their correlations. The implied time varying conditional variance, $h_{k,t}$, of the unpredictable part of the return of Canada and Mexico, based on information available at time $t-1$, given by equation (16), depends only on the volatility of the unpredictable part of the US return and their own idiosyncratic volatility:

$$h_{k,t} = Var((\varphi^+_k e^+_US,t + \varphi^-_k e^-US,t + e_{k,t})| I_{t-1})$$

$$= \sigma^2_{US,t} \left[ (\varphi^+_k)^2 \left( \frac{\pi - 1}{2\pi} \right) + (\varphi^-_k)^2 \left( \frac{\pi - 1}{2\pi} \right) + \varphi^+_k \varphi^-_k \left( \frac{1}{\pi} \right) \right] + \sigma^2_{t,t}$$

(See theorem A5 in the Appendix).

Note that the expression $\sigma^2_{US,t} \left[ (\varphi^+_k)^2 \left( \frac{\pi - 1}{2\pi} \right) + (\varphi^-_k)^2 \left( \frac{\pi - 1}{2\pi} \right) + \varphi^+_k \varphi^-_k \left( \frac{1}{\pi} \right) \right]$ in equation (16) denotes the variability of the returns of Canada and Mexico explained by the variability of the US returns. Thus, the conditional variance of the unexpected return for Canada and Mexico depends not only on the US and its own idiosyncratic volatility, but also on whether that volatility is driven by positive or negative US shocks. Similarly, the implied time varying conditional covariance between the unexpected return of Canada and
Mexico and the US unexpected return depends only on the US idiosyncratic volatility and the volatility spillovers intensities:

\[(17)\] \[h_{US,k,t} = \text{Cov}(e_{k,t}, e_{US,t} | I_{t-1})\]

\[= \text{Cov}(\varphi^+_{k} e_{US,t}^+ + \varphi^-_{k} e_{US,t}^-, e_{k,t}), (e_{US,t}^+ + e_{US,t}^-) | I_{t-1})\]

\[= \frac{\sigma^2_{US,t}}{2} (\varphi^+_{k} + \varphi^-_{k})\]

(See theorem A7 in the Appendix).

Note that equation (17) suggests that the larger the volatility of the US idiosyncratic shock, and the larger the volatility spillover intensities, the higher is the conditional covariance. Finally, the implied time varying conditional correlation between the unexpected returns of Canada and Mexico and the unexpected return in the US equity market depends on the covariance between the unexpected return of Canada and Mexico and the US unexpected return, their respective conditional variances of the unpredictable part of the returns and the US volatility:

\[(18)\] \[\rho_{US,k,t} = \frac{h_{US,k,t}}{\sqrt{h_{k,t} h_{US,t}}}\]

\[= \frac{\left(\varphi^+_{k} + \varphi^-_{k}\right)\sigma_{US,t}}{\sqrt{\sigma^2_{US,t} \left[\left(\frac{\pi - 1}{2\pi}\right)^2 + \left(\frac{\pi - 1}{2\pi}\right)^2 + \varphi^+_{k} \varphi^+_{k} \left(\frac{1}{\pi}\right)\right] + \sigma^2_{k,t}}} .\]

Equation (18) follows directly from equations (16) and (17). We present in the Appendix the formal details of the derivation of equations (16) and (17). It is straightforward to verify that under the hypothesis that the volatility spillover effects have a symmetric
impact on the unpredictable part of the returns of Canada and Mexico, equations (16)-(18) reduce to:

\[
\begin{align*}
&h_{k,t} = \phi_k^2 \sigma_{US,t}^2 + \sigma_{k,t}^2 \\
&h_{US,k,t} = \phi_k \sigma_{US,t}^2 \\
&\rho_{US,k,t} = \frac{\phi_k \sigma_{US,t}^2}{\sqrt{\phi_k^2 \sigma_{US,t}^2 + \sigma_{k,t}^2}}
\end{align*}
\]

From equation (16), it follows that the variance ratio \(VR_{k,t}^{US}\), i.e., the proportion of the variance of the unexpected returns of Canada and Mexico that is driven by US volatility is given by

\[
VR_{k,t}^{US} = \frac{h_{US,k,t}}{h_{k,t}} 
\]

\[
= \frac{\sigma_{US,t}^2 \left[ \left( \phi_k^+ \right)^2 \left( \frac{\pi - 1}{2\pi} \right) + \left( \phi_k^- \right)^2 \left( \frac{\pi - 1}{2\pi} \right) + \phi_k^+ \phi_k^- \left( \frac{1}{\pi} \right) \right]}{\sigma_{US,t}^2 \left[ \phi_k^+ \left( \frac{\pi - 1}{2\pi} \right) + \phi_k^- \left( \frac{\pi - 1}{2\pi} \right) + \phi_k^+ \phi_k^- \left( \frac{1}{\pi} \right) \right] + \sigma_{k,t}^2} \in [0,1]
\]

which reduces to

\[
VR_{k,t}^{US} = \frac{\phi_k^2 \sigma_{US,t}^2}{h_{k,t}} = \frac{\phi_k^2 \sigma_{US,t}^2}{\phi_k^2 \sigma_{US,t}^2 + \sigma_{k,t}^2} \in [0,1]
\]

under the assumption that volatility spillovers originating from the US equity market have symmetric effects on the volatility of the Canadian and Mexican returns, i.e., \(\phi_k^+ = \phi_k^- = \phi_k\). Similarly, the proportion of the variance of the unexpected returns of Canada and Mexico that is explained by their own local idiosyncratic shocks is \(VR_{k,t}^k = 1 - VR_{k,t}^{US}\), given by
Equation (24) reduces to

\[
VR_{k,t} = \frac{\sigma_{k,t}^2}{\sigma_{US,t}^2 \left( \varphi_k^+ \left( \frac{\pi - 1}{2\pi} \right) + \varphi_k^- \left( \frac{\pi - 1}{2\pi} \right) + \varphi_k^+ \varphi_k^- \left( \frac{1}{\pi} \right) \right) + \sigma_{k,t}^2} \in [0,1]
\]

if \( \varphi_k^+ \) is not significantly different from \( \varphi_k^- \).

3. The Data

The equity market data consist of daily observations on adjusted closing values of for the stock price indices of Canada, Mexico, and the US. The stock indices used are the Standard & Poor’s 500 Composite Index (S&P 500) for the US, the Standard & Poor’s/Toronto Stock Exchange Composite Index (S&P/TSX) for Canada, and the Índice de Precios y Cotizaciones (IPC) for Mexico. The S&P 500 index is the best-known US stock index and is considered the benchmark for US equity performance since it represents 70% of all US publicly traded companies. The index, as the name implies, is composed of 500 stocks traded on the New York Stock Exchange (NYSE), American Stock Exchange (AMEX) and the NASDAQ National Market System. The S&P/TSX Composite Index, which is widely considered as the benchmark for Canadian equities, can be considered as the Canadian counterpart of the S&P 500 index. The index, which includes 223 Canadian stocks which have the highest turnover volume and market capitalization among the stocks traded in the Toronto Stock Exchange (TSE), is a revision of the now defunct TSE 300 that became effective on May 1, 2002, but its
historical data remain completely comparable as its constituents and the index calculation method are exactly the same as before. The IPC is the leading indicator of performance of the Bolsa Mexicana de Valores (BMV) as a whole. The index, which in its current form was introduced in 1978, is composed of 34 stocks that participate in different sectors of the economy. All three indices are provided by Commodity Systems, Inc. and obtained from the Yahoo! Finance portal (http://finance.yahoo.com/intlindices). The web site reports for each day that the market is open the price at the close of the day and the adjusted close price (adj. close), which is the price at the close adjusted for dividends and splits. In the paper we used the latter, although there is no difference between the two.

For each of the three indices the data starts at the beginning of January 1992 and terminates at the end of December 2003. Since the US and Canadian markets are within the same time zone, and the Mexican market is located in the adjacent time zone, the trading time for the three markets are about the same, which minimizes the problems of time differences in trading. The econometric methodology, however, requires matched observations between the three markets. Consequently, the US returns are matched with the Canadian and Mexican return series. This results in the exclusions of returns for all three markets when one of the markets is closed for exchange-specific holidays or other reasons. This procedure yields a sample size of 2885 daily returns observations for each of the three markets.

The return, \( R_{i,t} \), of each index is expressed in continuously compounded terms as

\[
R_{i,t} = \ln P_{i,t} - \ln P_{i,t-1},
\]

where \( P_{i,t} \) represents the value of the equity index of country \( i \) for day \( t \), and, to detach the analysis from exchange rate volatility effects and incorporate hedging activities of investors against foreign exchange risk (Koutmos, 1996; De Santis
and Imrohoroglu, 1997), is denominated in local currency. The behavior of the return series for each of the three markets during the sample period January 1, 1992 to December 31, 2003 is illustrated in Figure 1. Visual inspection suggests that all three return series appear to be stationary in the mean, although not in the variance.

Figure 1. Equity Index Returns
January 1, 1992-December 31, 2003

a) US

b) Canada
As Figure 1 shows, the returns exhibit volatility clustering: large (small) shocks, of either sign, tend to follow large (small) shocks, a characteristic associated with a time varying conditional variance. There is also some visual evidence that periods of high volatility tend to be common across the three North-American countries. In addition, it appears that the volatility of the returns in the Mexican stock market has increased after the ratification of NAFTA and the peso crisis of December 1994. Summary statistics relating to the distributional and time series properties of the return series are presented in Table 1. The results indicate that the mean of the returns is higher for Mexico, as is the standard deviation, compared to Canada and the US.
Table 1. Descriptive statistics

<table>
<thead>
<tr>
<th></th>
<th>US</th>
<th>Canada</th>
<th>Mexico</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>January 1, 1992 to December 31, 2003</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No. of Obs.</td>
<td>2855</td>
<td>2855</td>
<td>2855</td>
</tr>
<tr>
<td>Mean</td>
<td>3.45E-04</td>
<td>2.99E-04</td>
<td>6.36E-04</td>
</tr>
<tr>
<td>Median</td>
<td>4.16E-04</td>
<td>5.97E-04</td>
<td>1.55E-04</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>0.011</td>
<td>0.009</td>
<td>0.018</td>
</tr>
<tr>
<td>Skewness</td>
<td>-0.131</td>
<td>-0.745</td>
<td>-0.059</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>6.722</td>
<td>10.353</td>
<td>8.122</td>
</tr>
<tr>
<td>Jarque-Bera</td>
<td>1656.652</td>
<td>6695.35</td>
<td>3123.69</td>
</tr>
<tr>
<td>Q(6)</td>
<td>16.504</td>
<td>35.111</td>
<td>29.988</td>
</tr>
<tr>
<td>Q^2(6)</td>
<td>510.290</td>
<td>354.69</td>
<td>392.470</td>
</tr>
<tr>
<td><strong>January 1, 1992 to September 30, 1997</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No. of Obs.</td>
<td>1379</td>
<td>1379</td>
<td>1379</td>
</tr>
<tr>
<td>Mean</td>
<td>5.98E-04</td>
<td>5.06E-04</td>
<td>9.52E-04</td>
</tr>
<tr>
<td>Median</td>
<td>5.32E-04</td>
<td>7.16E-04</td>
<td>3.95E-04</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>0.007</td>
<td>0.006</td>
<td>0.016</td>
</tr>
<tr>
<td>Skewness</td>
<td>-0.299</td>
<td>-0.702</td>
<td>-0.048</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>5.610</td>
<td>7.337</td>
<td>6.168</td>
</tr>
<tr>
<td>Jarque-Bera</td>
<td>412.12</td>
<td>1194.258</td>
<td>577.565</td>
</tr>
<tr>
<td>Q(6)</td>
<td>11.167</td>
<td>71.614</td>
<td>37.377</td>
</tr>
<tr>
<td>Q^2(6)</td>
<td>27.626</td>
<td>27.275</td>
<td>226.580</td>
</tr>
</tbody>
</table>
### Summary Statistics

![Summary Statistics Table]

- **No. of Obs.** 1454
- **Mean** 1.34E-04, 1.26E-04, 4.39E-04
- **Median** 2.73E-04, 3.75E-04, -1.68E-05
- **Standard Deviation** 0.014, 0.012, 0.018
- **Skewness** 0.003, -0.536, 0.121
- **Kurtosis** 4.621, 7.285, 6.847
- **Jarque-Bera** 159.271, 1182.612, 900.220
- **Q(6)** 9.367, 13.572, 12.155
- **$Q^2(6)$** 160.490, 131.040, 263.290

**Notes.** In a normal distribution skewness is zero and kurtosis is 3. The Jarque-Bera statistics is distributed as $\chi^2(2)$. The critical value at the 5% level is 5.99. $Q(6)$ and $Q^2(6)$ are the Ljung-Box statistics based on the returns and the squared returns respectively up to the 6th order. Both statistics are asymptotically distributed as $\chi^2(6)$. The critical value at the 5% level is 12.59.

All three return series exhibit negative skewness in returns, although only for Canada is skewness significantly different from zero, and are consistently leptokurtic, which is a typical characteristic of financial time series. For all three series, we can conclude that the respective distributions underlying the data have fatter tails and are unlikely to have been drawn from a normally distributed sample. As a confirmation, the Jarque and Bera (1987) test for normality rejects the null of normality in the distribution of these series. Table 1 also reports tests for serial correlation, using the Ljung and Box (1978) Q statistic up to and including the 6th lag. Also reported are tests for ARCH effects based on the methodology of Bollerslev (1986), which uses the Ljung and Box Q statistic applied to the squared returns. The well-known facts that stock returns are leptokurtic,
heteroskedastic, and not normally distributed are therefore confirmed by our findings. However, it is worth emphasizing that there are significant changes in the non-normal distributions of the returns over sub-periods. In particular, kurtosis is substantially less in each of the two sub-periods compared to the whole sample period, and negative skewness is present only for Canada. Further, the hypothesis that the US returns are not serially correlated can be rejected, at the 5 per cent level, for the whole period but not for the two sub-periods. Table 2 presents the cross-market correlation coefficients. Similarly, the hypothesis that the returns in the Mexican market are not serially correlated can be rejected, at the 5 per cent level, for the whole period and the first sub-period only. The coefficients are consistently positive, implying that the North American markets tend to move in the same direction.

**Table 2. Unconditional Cross-Market Correlations of Returns**

<table>
<thead>
<tr>
<th></th>
<th>US</th>
<th>Canada</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>January 1, 1992 to December 31, 2003</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>US</td>
<td>1.000</td>
<td></td>
</tr>
<tr>
<td>Canada</td>
<td>0.697</td>
<td>1.000</td>
</tr>
<tr>
<td>Mexico</td>
<td>0.506</td>
<td>0.452</td>
</tr>
<tr>
<td><strong>January 1, 1992 to September 30, 1997</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>US</td>
<td>1.000</td>
<td></td>
</tr>
<tr>
<td>Canada</td>
<td>0.583</td>
<td>1.000</td>
</tr>
<tr>
<td>Mexico</td>
<td>0.293</td>
<td>0.256</td>
</tr>
</tbody>
</table>
November 1, 1997 to December 31, 2003

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>US</td>
<td>1.000</td>
<td></td>
</tr>
<tr>
<td>Canada</td>
<td>0.717</td>
<td>1.000</td>
</tr>
<tr>
<td>Mexico</td>
<td>0.603</td>
<td>0.543</td>
</tr>
</tbody>
</table>

October 1, 1997 to October 31, 1997

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>US</td>
<td>1.000</td>
<td></td>
</tr>
<tr>
<td>Canada</td>
<td>0.910</td>
<td>1.000</td>
</tr>
<tr>
<td>Mexico</td>
<td>0.955</td>
<td>0.887</td>
</tr>
</tbody>
</table>

Notes. All cross-market correlation coefficients are significant at the 1% level.

The highest correlation coefficient, 0.6969, is found between the US and Canada. This is not surprising, since the US and Canadian economies are more related to each other. The lowest correlation coefficient, 0.4522, is between Mexico and Canada. It is obvious, however, that these values have restrictions, since they reveal only the average correlation during the period in question. If the variances change over time, these cross-market correlations are no longer valid. Table 2 shows that the cross-market correlations are substantially different before and after the US stock market crash of October 1997. The most isolated market in the North American region before October 1997 appears to be Mexico, which shows consistently lower correlations with the other two markets. As shown in Table 2, the cross-market correlations during the month of October 1997 are extremely high. This increase in cross-market correlations is consistent with the findings of King and Wadhwani (1990) and Lee and Kim (1993) and provides evidence of contagion (Forbes and Rigobon, 2001). Though cross-market correlations after the crash
decrease, they still remain higher than the correlations before the crash. The markets of the North America continent thus appear to follow the same changes in correlation patterns similar to those found in emerging markets. See, e.g., Bekaert and Harvey (1997), Solnik, Bourelle and Le Fur (1996), Bekaert and Urias (1999) and Meric et al. (2001).

4. Empirical Results

The results of estimating the AR (1)-APARCH (1, 1) model for the US, Canada and Mexico, under the assumption of no spillover effects are presented in Table 3. The AR (1)-APARCH (1, 1) model for the US represents the first-step regression from which the residuals extracted to be used as exogenous shocks in the spillover model for Canada and Mexico. The results of the AR (1)-APARCH (1, 1) model for Canada and Mexico are also reported for purposes of comparison. The model is estimated by numerical maximum likelihood procedures, using the algorithm developed by Berndt, Hall, Hall and Hausman (1974), but all test statistics and $t$-values are computed using the quasi-maximum likelihood methods (QML) described by Bollerslev and Wooldridge (1992), which are robust to distributional non-normalities.
Table 3. AR(1)-APARCH(1,1) Estimation of the Returns without Spillover Effects

January 1, 1992 - December 31, 2003

<table>
<thead>
<tr>
<th></th>
<th>US</th>
<th>Canada</th>
<th>Mexico</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_0$</td>
<td>2.42E-4</td>
<td>2.94E-4</td>
<td>5.83E-4</td>
</tr>
<tr>
<td></td>
<td>(1.538)</td>
<td>(2.2581)</td>
<td>(2.136)</td>
</tr>
<tr>
<td>$c_1$</td>
<td>0.022</td>
<td>0.156</td>
<td>0.1529</td>
</tr>
<tr>
<td></td>
<td>(1.212)</td>
<td>(8.446)</td>
<td>(7.729)</td>
</tr>
<tr>
<td>$\omega$</td>
<td>1.11E-4</td>
<td>5.49E-4</td>
<td>6.02E-5</td>
</tr>
<tr>
<td></td>
<td>(0.826)</td>
<td>(0.717)</td>
<td>(0.932)</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.067</td>
<td>0.084</td>
<td>0.117</td>
</tr>
<tr>
<td></td>
<td>(4.551)</td>
<td>(5.185)</td>
<td>(5.511)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.820</td>
<td>0.472</td>
<td>0.479</td>
</tr>
<tr>
<td></td>
<td>(5.192)</td>
<td>(3.434)</td>
<td>(4.249)</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.930</td>
<td>0.922</td>
<td>0.855</td>
</tr>
<tr>
<td></td>
<td>(78.671)</td>
<td>(58.120)</td>
<td>(35.651)</td>
</tr>
<tr>
<td>$\delta$</td>
<td>1.090</td>
<td>0.694</td>
<td>1.586</td>
</tr>
<tr>
<td></td>
<td>(4.692)</td>
<td>(2.602)</td>
<td>(5.928)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>-1.1E-3</td>
<td>7.2E-3</td>
<td>6.8E-2</td>
</tr>
<tr>
<td>Log Likelihood</td>
<td>9294.680</td>
<td>9783.424</td>
<td>7807.264</td>
</tr>
<tr>
<td>ARCH(1)</td>
<td>2.385</td>
<td>1.381</td>
<td>0.054</td>
</tr>
<tr>
<td>Q(6)</td>
<td>11.256</td>
<td>2.931</td>
<td>6.331</td>
</tr>
</tbody>
</table>
Q^{2}(6) & 5.067 & 3.813 & 6.931 \\
Jarque-Bera & 432.862 & 1394.417 & 278.346 \\
Skewness & -0.379 & -0.578 & 0.055 \\
Kurtosis & 4.750 & 6.223 & 4.525 \\

Notes. In parenthesis are the robust t-statistics based on Bollerslev and Wooldridge (1992). ARCH(1) is the value of the ARCH-LM test of order 1. The statistics is asymptotically distributed as $\chi^{2}(1)$. The critical value at the 5% level is 3.84. Q(6) and $Q^{2}(6)$ are the Ljung-Box statistics based on the standardized residuals and the squared standardized residuals respectively up to the 6th order. Both statistics are asymptotically distributed as $\chi^{2}(6)$. The critical value at the 5% level is 12.59. The Jarque-Bera statistics is distributed as $\chi^{2}(2)$. The critical value at the 5% level is 5.99.

The estimates of the conditional mean indicate that the autoregressive parameter is significant at the 1 per cent level or better only for Canada and Mexico. For the US, instead, the estimate is not significantly different from zero. Conversely, the estimates of the conditional variance are all significant at the 1% level or better. The estimate of $\alpha$, the ARCH coefficient, is the highest for Mexico and the lowest for the US, suggesting that idiosyncratic shocks tend to linger around longer in the Mexican stock market than in the US and Canadian markets. This may be an indication that the Mexican stock market is less efficient than the US and Canadian markets since the effects of idiosyncratic shocks take longer to dissipate. Interestingly, the estimate of $\beta$, the GARCH coefficient, is also not the same in the three markets. Rather, it is the highest for the US and the lowest for Mexico. The estimate of $\gamma$, the asymmetry coefficient, is also the highest for the US, while for Canada and Mexico is approximately the same, and about half of that found for the US. The leverage effect does exist in the returns of all three markets, but has a different impact among the three markets. In each market a negative shock has a
greater impact than a positive shock, but a negative shock in the US market has an even
greater effect on the volatility of the US than a negative shock of equal magnitude in the
Canadian and Mexican markets has on the volatility for the US and Mexico. The
estimates of $\delta$, the power coefficient, range in value from 0.69 for Canada to 1.58 for
Mexico. The $t$-statistics corresponding to the null hypothesis that $\delta = 1$ are 0.39, -1.08
and 2.19 for the US, Canada and Mexico, respectively. Similarly, the $t$-statistics
corresponding to the null hypothesis that $\delta = 2$ are -3.92, -4.79 and -1.55, respectively.
Thus, for both the US and Canada, the estimated $\delta$ is significantly different from 2
(Bollerslev GARCH), but not from 1 (Taylor/Schwert model), while for Mexico the
estimated $\delta$ is significantly different from 1, but not from 2. As indicated in Ding,
Granger and Engle (1993), a series of likelihood ratio tests can be constructed in which
the restricted case is either the Bollerslev’s GARCH ($\delta = 2, \gamma = 0$) or the
Taylor/Schwert model ($\delta = 1, \gamma = 0$). Let $l_0$ be the log-likelihood value under the null
hypothesis that the true model is the Bollerslev’s GARCH and $l$ the log-likelihood value
under the alternative that the true model is APARCH (1, 1). Then the likelihood ratio test,
$2(l - l_0)$, has a chi-squared distribution with 2 degrees of freedom when the null
hypothesis is true. The outcome of the likelihood ratio tests provides a clear rejection of
both the Bollerslev and the Taylor/Schwert models against the APARCH (1, 1) model.
When the restricted case is the Bollerslev’s GARCH, the likelihood ratio test yields a chi-
squared value of 100.94 for the US, 84.98 for Canada, and 108.16 for Mexico. Similarly,
when the restricted case is the Taylor/Schwert model, the likelihood ratio test yields a
chi-square value of 95.09 for the US, 51.34 for Canada and 126.80 for Mexico. The
Ljung-Box statistics for the standardized and the squared standardized residuals up to and
including the 6th lag show that the APARCH (1, 1) model is appropriate to describe the linear and non-linear dependencies in the return series, and none of the LM statistics is significant under the 10% level, implying that no other ARCH effects are left with the APARCH (1, 1) model. Table 3 also reports the estimated third (skewness) and fourth (kurtosis) moments for the standardized residuals. The critical value at the 5 percent significance level for the test of zero skewness against non-zero skewness is approximately ±0.09. The null of zero skewness is rejected for Canada and the US, but not for Mexico. The critical value at the 5 percent significance level for the test of zero excess kurtosis (the estimated fourth moment minus three) against non-zero excess kurtosis is approximately ±0.18. The null can therefore be rejected for all three series.

Although the Jarque-Bera test for normality of the standardized residuals fails to accept normality, and there is some residual negative skewness and positive excess kurtosis, in comparison to the statistics for the returns in Table 1, the model appears to capture much of the non-normality of the data.

In Tables 4 and 5 we present the estimation results for Canada and Mexico, respectively, of the AR (1)-APARCH (1, 1) model extended to incorporate the return and volatility spillovers from the US. In each Table we present the results of four alternative estimations, corresponding to models 1 through 4. Model 1 reports the estimates of the model under the restrictive assumption of symmetry in both return and volatility spillover effects (θk+ = θk− and φk+ = φk−). Model 2 allows for asymmetry in volatility spillover effects, while maintaining the assumption of symmetry in return spillover effects (θk+ = θk− but φk+ ≠ φk−). Similarly, model 3 allows for asymmetry in return spillover effects, but not in volatility spillover effects (φk+ = φk− but θk+ ≠ θk−). Finally, model 4
allows for asymmetry in both return and volatility spillover effects \((\theta_+^k \neq \theta_-^k \text{ and } \phi_+^k \neq \phi_-^k)\). Inferences can be then made on how positive and negative spillovers explain returns and variabilities. In particular, the symmetry of positive and negative spillovers can be investigated. If \(\theta_+^k \text{ and } \theta_-^k\), as well \(\phi_+^k \text{ and } \phi_-^k\) are statistically equal to each other, then the decomposition offers no advantage over the approach used by Bekaert and Harvey (1997), Baele (2005) and Ng (2000) in that the effects of return and volatility spillovers are symmetric.
### Table 4. AR(1)-APARCH(1,1) Estimation of the Returns with Spillover Effects:

**Canada**

January 1, 1992- December 31, 2003

<table>
<thead>
<tr>
<th></th>
<th>Model 1</th>
<th>Model 2</th>
<th>Model 3</th>
<th>Model 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_0$</td>
<td>1.93E-04</td>
<td>7.04E-04</td>
<td>2.86E-04</td>
<td>6.94E-04</td>
</tr>
<tr>
<td></td>
<td>(1.888)</td>
<td>(3.903)</td>
<td>(2.047)</td>
<td>(3.449)</td>
</tr>
<tr>
<td>$c_1$</td>
<td>0.095</td>
<td>0.091</td>
<td>0.093</td>
<td>0.091</td>
</tr>
<tr>
<td></td>
<td>(4.506)</td>
<td>(4.516)</td>
<td>(4.435)</td>
<td>(4.518)</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.08</td>
<td>0.073</td>
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<td></td>
<td>(5.506)</td>
<td>(4.927)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\theta^+$</td>
<td></td>
<td>0.062</td>
<td>0.075</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(2.871)</td>
<td>(3.592)</td>
<td></td>
</tr>
<tr>
<td>$\theta^-$</td>
<td></td>
<td>0.092</td>
<td>0.072</td>
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</tr>
<tr>
<td></td>
<td></td>
<td>(4.202)</td>
<td>(3.222)</td>
<td></td>
</tr>
<tr>
<td>$\varphi$</td>
<td>0.536</td>
<td>0.537</td>
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</tr>
<tr>
<td></td>
<td>(31.021)</td>
<td>(30.916)</td>
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</tr>
<tr>
<td>$\varphi^+$</td>
<td></td>
<td>0.456</td>
<td>0.456</td>
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</tr>
<tr>
<td></td>
<td></td>
<td>(20.831)</td>
<td>(20.734)</td>
<td></td>
</tr>
<tr>
<td>$\varphi^-$</td>
<td></td>
<td>0.609</td>
<td>0.610</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(16.409)</td>
<td>(16.433)</td>
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</tr>
<tr>
<td>$\omega$</td>
<td>2.81E-05</td>
<td>2.40E-05</td>
<td>2.71E-05</td>
<td>2.41E-05</td>
</tr>
<tr>
<td></td>
<td>(0.476)</td>
<td>(0.499)</td>
<td>(0.470)</td>
<td>(0.499)</td>
</tr>
<tr>
<td>-----</td>
<td>---------</td>
<td>---------</td>
<td>---------</td>
<td>---------</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.061</td>
<td>0.064</td>
<td>0.061</td>
<td>0.064</td>
</tr>
<tr>
<td></td>
<td>(3.830)</td>
<td>(4.021)</td>
<td>(3.789)</td>
<td>(4.011)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.138</td>
<td>0.141</td>
<td>0.137</td>
<td>0.141</td>
</tr>
<tr>
<td></td>
<td>(1.094)</td>
<td>(1.178)</td>
<td>(1.091)</td>
<td>(1.180)</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.943</td>
<td>0.941</td>
<td>0.943</td>
<td>0.941</td>
</tr>
<tr>
<td></td>
<td>(56.521)</td>
<td>(57.476)</td>
<td>(56.315)</td>
<td>(57.112)</td>
</tr>
<tr>
<td>$\delta$</td>
<td>1.145</td>
<td>1.162</td>
<td>1.147</td>
<td>1.162</td>
</tr>
<tr>
<td></td>
<td>(3.486)</td>
<td>(3.597)</td>
<td>(3.453)</td>
<td>(3.601)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.494</td>
<td>0.495</td>
<td>0.493</td>
<td>0.495</td>
</tr>
<tr>
<td>Log Likelihood</td>
<td>10587.00</td>
<td>10600.05</td>
<td>10587.44</td>
<td>10600.06</td>
</tr>
<tr>
<td>ARCH(1)</td>
<td>0.444</td>
<td>1.701</td>
<td>1.083</td>
<td>1.685</td>
</tr>
<tr>
<td>Q(6)</td>
<td>5.121</td>
<td>4.649</td>
<td>5.207</td>
<td>4.651</td>
</tr>
<tr>
<td>$Q^2(6)$</td>
<td>5.834</td>
<td>7.123</td>
<td>6.115</td>
<td>7.103</td>
</tr>
<tr>
<td>Jarque-Bera test</td>
<td>1083.575</td>
<td>683.143</td>
<td>1066.565</td>
<td>682.188</td>
</tr>
<tr>
<td>Skewness</td>
<td>-0.462</td>
<td>-0.354</td>
<td>-0.457</td>
<td>-0.354</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>5.872</td>
<td>5.289</td>
<td>5.851</td>
<td>5.287</td>
</tr>
</tbody>
</table>

Notes. In parenthesis are the robust t-statistics based on Bollerslev and Wooldridge (1992). ARCH(1) is the value of the ARCH-LM test of order 1. The statistics is asymptotically distributed as $\chi^2(1)$. The critical value at the 5% level is 3.84. Q(6) and $Q^2(6)$ are the Ljung-Box statistics based on the standardized residuals and the squared standardized residuals respectively up to the 6th order. Both statistics are asymptotically distributed as $\chi^2(6)$. The critical value at the 5% level is 12.59. The Jarque-Bera statistics is distributed as $\chi^2(2)$. The critical value at the 5% level is 5.99.
Table 5. AR(1)-APARCH(1,1) Estimation of the Returns with Spillover Effects:

Mexico

January 1, 1992- December 31, 2003

<table>
<thead>
<tr>
<th></th>
<th>Model 1</th>
<th>Model 2</th>
<th>Model 3</th>
<th>Model 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c_0 )</td>
<td>5.41E-04</td>
<td>1.55E-03</td>
<td>9.53E-04</td>
<td>1.74E-03</td>
</tr>
<tr>
<td></td>
<td>(2.276)</td>
<td>(4.140)</td>
<td>(2.751)</td>
<td>(4.174)</td>
</tr>
<tr>
<td>( c_1 )</td>
<td>0.132</td>
<td>0.129</td>
<td>0.131</td>
<td>0.128</td>
</tr>
<tr>
<td>( \theta )</td>
<td>0.031</td>
<td>0.022</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1.176)</td>
<td>(0.831)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \theta^+ )</td>
<td></td>
<td>-0.020</td>
<td>-0.006</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.486)</td>
<td>(-0.144)</td>
<td></td>
</tr>
<tr>
<td>( \theta^- )</td>
<td>0.085</td>
<td>0.053</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2.017)</td>
<td>(1.244)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \varphi )</td>
<td>0.657</td>
<td>0.656</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(29.066)</td>
<td>(29.255)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \varphi^+ )</td>
<td></td>
<td>0.527</td>
<td>0.532</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(12.448)</td>
<td>(12.355)</td>
<td></td>
</tr>
<tr>
<td>( \varphi^- )</td>
<td>0.782</td>
<td>0.775</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(16.536)</td>
<td>(16.195)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \omega )</td>
<td>2.74E-05</td>
<td>1.84E-05</td>
<td>2.63E-05</td>
<td>1.80E-05</td>
</tr>
<tr>
<td></td>
<td>(0.760)</td>
<td>(0.714)</td>
<td>(0.749)</td>
<td>(0.711)</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>α</td>
<td>0.104</td>
<td>0.102</td>
<td>0.106</td>
<td>0.104</td>
</tr>
<tr>
<td></td>
<td>(5.726)</td>
<td>(5.847)</td>
<td>(5.837)</td>
<td>(5.904)</td>
</tr>
<tr>
<td>γ</td>
<td>0.328</td>
<td>0.321</td>
<td>0.331</td>
<td>0.321</td>
</tr>
<tr>
<td></td>
<td>(4.018)</td>
<td>(4.092)</td>
<td>(4.087)</td>
<td>(4.136)</td>
</tr>
<tr>
<td>β</td>
<td>0.877</td>
<td>0.877</td>
<td>0.875</td>
<td>0.875</td>
</tr>
<tr>
<td></td>
<td>(40.302)</td>
<td>(42.155)</td>
<td>(39.985)</td>
<td>(41.776)</td>
</tr>
<tr>
<td>δ</td>
<td>1.658</td>
<td>1.738</td>
<td>1.667</td>
<td>1.744</td>
</tr>
<tr>
<td></td>
<td>(5.129)</td>
<td>(5.086)</td>
<td>(5.094)</td>
<td>(5.084)</td>
</tr>
<tr>
<td>R²</td>
<td>0.255</td>
<td>0.257</td>
<td>0.254</td>
<td>0.256</td>
</tr>
<tr>
<td>Log Likelihood</td>
<td>8171.074</td>
<td>8179.756</td>
<td>8172.483</td>
<td>8180.192</td>
</tr>
<tr>
<td>ARCH(1)</td>
<td>1.972</td>
<td>2.312</td>
<td>2.170</td>
<td>2.355</td>
</tr>
<tr>
<td>Q(6)</td>
<td>6.687</td>
<td>6.073</td>
<td>6.593</td>
<td>6.341</td>
</tr>
<tr>
<td>Q²(6)</td>
<td>9.637</td>
<td>10.401</td>
<td>9.281</td>
<td>10.042</td>
</tr>
<tr>
<td>Jarque-Bera</td>
<td>315.036</td>
<td>247.946</td>
<td>313.971</td>
<td>250.209</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.121</td>
<td>0.14</td>
<td>0.124</td>
<td>0.151</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>4.609</td>
<td>4.41</td>
<td>4.605</td>
<td>4.418</td>
</tr>
</tbody>
</table>

Notes. In parenthesis are the robust t-statistics based on Bollerslev and Wooldridge (1992). ARCH(1) is the value of the ARCH-LM test of order 1. The statistics is asymptotically distributed as $\chi^2(1)$. The critical value at the 5% level is 3.84. Q(6) and Q²(6) are the Ljung-Box statistics based on the standardized residuals and the squared standardized residuals respectively up to the 6th order. Both statistics are asymptotically distributed as $\chi^2(6)$. The critical value at the 5% level is 12.59. The Jarque-Bera statistics is distributed as $\chi^2(2)$. The critical value at the 5% level is 5.99.
In both Table 4 and Table 5, and for all models, the specification tests in terms of the $Q(12)$ and $Q^2(12)$ and ARCH(1) statistics indicate that the series are adequately modeled without any remaining serial correlation or residual ARCH effect. The autocorrelations of the standardized residuals and squared standardized residuals all lie within the asymptotic bounds of $2/\sqrt{N}$; however, the structure of the standardized residuals still reflects a significant amount of kurtosis, although significantly decreased in the case of Canada. As expected, the Jarque-Bera test rejects the normality of standardized residuals. As a further specification test, we checked the correlations of the idiosyncratic shocks. A correct specification of the model requires that the idiosyncratic shocks $e_{k,t}$ are mutually uncorrelated, as well as uncorrelated with the US idiosyncratic shock, $e_{US,t}$, i.e. $E(e_{k,t}e_{j,t} | I_{t-1}) = 0$ for all $k \neq j$, and $E(e_{US,t}e_{k,t} | I_{t-1}) = 0$ for all $k$. The correlation results confirm that the idiosyncratic shocks in each of the three markets are indeed orthogonal to each of the other two. Based on the estimates of model 2, the correlation between the US and Canada is 0.070, between the US and Mexico is 0.019, and between Mexico and Canada is 0.089. These correlations are substantially lower than the correlations of the returns, implying that indeed the model has substantial explanatory power.

Comparison of the parameter estimates for Canada and Mexico presented in Table 4 indicates few substantial differences from extending the model to include US spillover effects. For all four models, the estimate of the power coefficient for Canada increases, but still remains significantly different from 2 but not from 1, and the estimate of the power coefficient for Mexico increases, but still remains significantly different from 1 but not from 2. Thus, the estimate of the power coefficient is invariant to the specification
of the model. Similarly, the estimate of the autoregressive parameter in the conditional mean is slightly reduced, but remains highly significant. The remaining coefficients estimates are also approximately the same, with one exception. As a result of the inclusion of the US spillovers, the estimate of the asymmetry coefficient for Canada becomes insignificant. For Mexico, the coefficient estimates of the US return spillover effects are also statistically insignificant, regardless of the maintained hypothesis. Conversely, the return from the US market has a significant and positive effect on the Canadian market, implying that an episode of weakness in the US market today leads to a drop in the Canadian stock market tomorrow. However, the Wald test fails to reject the hypothesis that $\theta_c^+ = \theta_c^-$ in models 2 and 4, which indicates that returns spillovers do not have asymmetric effects. For Canada, the Wald test yields a chi-squared statistic of 0.844 ($p$-value = 0.358) and 0.007 ($p$-value = 0.931) for model 2 and model 4 respectively. For Mexico, the corresponding chi squared statistics are 2.600 ($p$-value = 0.107) and 0.776 ($p$-value = 0.3783). Regarding the magnitude of the impact, a 10 percent change in the US stock market is estimated to cause the Canadian stock market to change by approximately 7 percent in the next day. Unlike return spillover effects, volatility spillover effects are statistically significant in both Canada and Mexico. The estimated coefficients of the volatility spillover effects are positive, indicating that shocks, of any sign, in the US market increase the variance of the unexpected returns in Canada and Mexico. The effect, however, is larger for Mexico than Canada. Furthermore, the hypothesis of symmetry of volatility spillover effects, $\varphi_k^+ = \varphi_k^-$, is rejected by the Wald test in both cases, suggesting that negative shocks from the US equity market affect the volatility of the unexpected returns for Canada and Mexico more than positive shocks.
For Canada, the Wald test yields a chi-squared statistic of 8.481 ($p$-value = 0.036) and 8.562 ($p$-value = 0.034) for model 3 and model 4 respectively. For Mexico, the corresponding chi-squared statistics are 10.734 ($p$-value = 0.001) and 9.412 ($p$-value = 0.002). In the case of Canada, negative shocks increase volatility by approximately 60 percent, while positive shocks increase volatility by about 40 percent. In the case of Mexico, however, positive shocks increase volatility by about 50 percent, while negative shock increases it by about 70 percent.

To assess the importance of the US volatility spillover effects on the variance of the unexpected return of Canada and Mexico, we construct the time series of the variance ratios using the estimates of model 2. They are presented in Figure 2. Over the whole sample period, the variance ratio series for Canada has a mean of approximately 42% and a standard deviation of 14%.

Figure 2. Variance Ratios Computed Using the Estimates from Model 2
January 1, 1992-December 31, 2003

a) Canada
The US volatility spillover effects make up approximately between 6% and 80% of the conditional variance of the unexpected returns of Canada. The mean of the variance ratio for Mexico lower than the mean for Canada, and is approximately 20%, but the standard deviation is about the same. Over the whole sample period the US volatility spillover effects make up between 0.4% and 70% of the conditional variance of the unexpected return of Mexico. There are two features of the variance ratio series in Figure 2 that stand out. The first is that in the Canadian equity market the variance ratios series appears to be more stable. The means of the variance ratio in the period preceding the October 1997 crisis are approximately 38% and 12% respectively for Canada and Mexico. However, in the period following the October 1997 crisis, the mean of the variance ratios for Mexico increases substantially, to approximately 45%, while the mean for Canada slightly subsides, to 28%, although, during the period of the month of October 1997, the mean for Canada is about 64%, which is almost three times that for Mexico. A second interesting feature is that in the case of Mexico, starting in the period following the October 1997 crisis, the variance ratios series shows an upward tendency, reflecting the increasing
impact of the US market. This is further highlighted by the conditional correlations between the unexpected return of the US and that of Canada and Mexico. These are shown in Figure 3. We can see that the conditional correlation between the two largest and more mature markets, the US and Canada, is the most stable. The correlations between the US and Canadian returns fluctuate within a narrow band around 0.6. Conversely, the correlations between the US and the Mexican returns fluctuate around 0.3 in the period prior to the October 1997, while in the following period they begin to increase substantially, peaking at more than 0.8 in 2001.

*Figure 3. Correlations between Unexpected Returns Computed Using the Estimates from Model 2. January 1, 1992-December 31, 2003*

a) US and Canada
Our findings indicate that the US stock market helps explain the evolution of returns in Canada and Mexico, suggesting reduced long-term diversification benefits available to market participants. However, the findings for Canada vary considerably from those for Mexico, documenting that stock returns in an advanced, mature and developed capital market react differently from stock returns in an emerging capital market to shocks from the US equity markets. These findings hold true for both return and volatility spillover effects. We find strong evidence that returns from the US stock market are significant carriers of information for Canada, whereas they have an insignificant influence on the Mexican stock market. However the hypothesis of symmetry of return spillover effects cannot be rejected for the Canadian stock market. In contrast, we find strong evidence that volatility spillover effects not only impact significantly both on the Canadian and the Mexican stock markets, but also exhibit a significant asymmetric behavior, with negative shocks from the US equity market
impacting on the conditional volatility of the Canadian and Mexican equity markets more deeply than positive shocks. Moreover, while the impact of positive shocks is not much different between the two markets, this is not the case with negative shocks, which impact on the volatility of the Mexican stock market more intensely than on the volatility of the Canadian stock market. This finding is generally consistent with the results of Bekaert and Harvey (1995), Karolyi and Stulz (1996) and Bekaert and Harvey (1997) and has implications for the debate on financial liberalization in emerging markets. The Mexican equity market is the only emerging market in North America. This market has undergone substantial transformations following the liberalization reforms of May 1989 and January 1992, which allowed foreign investors to purchase and trade shares in its domestic market. For Mexico, as for most emerging markets, liberalization is an essential policy tool that attracts much needed foreign capital. On the other hand, our findings reaffirm that liberalization makes emerging equity markets more vulnerable to shocks originated in advanced and mature markets.

APPENDIX

Assume that conditional on $I_{t-1}$, $e_{US,t} \sim N(0, \sigma^2_{US,t})$, $e_{k,t} \sim N(0, \sigma^2_{k,t})$, and $e_{US,t}$ and $e_{k,t}$ are mutually uncorrelated, i.e., $E(e_{US,t} e_{k,t}) = 0$ for all $t$ and $k$. Let $e^+_{US,t}$ and $e^-_{US,t}$ define respectively, the left and right censored at 0 random variables obtained from the decomposition of $e_{US,t}$: $e^+_{US,t} = \max(0, e_{US,t})$ and $e^-_{US,t} = \min(0, e_{US,t})$. Then,

(A1) $e_{US,t} = e^+_{US,t} + e^-_{US,t}$.
The unexpected return of country $k$, $\varepsilon_{k,t}$ is

(A2) \[ \varepsilon_{k,t} = \varphi_k^+ \epsilon_{US,t}^+ + \varphi_k^- \epsilon_{US,t}^- + \varepsilon_{k,t} \]

Theorem A1. \( E(\epsilon_{US,t}^+) = \frac{\sigma_{US,t}}{\sqrt{2\pi}} \) and \( E(\epsilon_{US,t}^-) = -\frac{\sigma_{US,t}}{\sqrt{2\pi}} \)

Proof. Using the formulas for the moments of the truncated normal distribution (see, e.g., Maddala, 1983, p. 365) and applying the law of iterated expectations yields

(A3) \[ E(\epsilon_{US,t}^+) = E(\epsilon_{US,t}^+, | \epsilon_{US,t} \leq 0) \ P(\epsilon_{US,t} \leq 0) + E(\epsilon_{US,t}^+, | \epsilon_{US,t} \geq 0) \ P(\epsilon_{US,t} > 0) \]

\[ = E(\epsilon_{US,t}^+, | \epsilon_{US,t} \geq 0) \ P(\epsilon_{US,t} > 0) \]

\[ = \sigma_{US,t} \left[ \frac{\phi(0)}{1 - \Phi(0)} \right] (1 - \Phi(0)) \]

\[ = \sigma_{US,t} \phi(0) \]

\[ = \sigma_{US,t} \frac{1}{\sqrt{2\pi}} \]

since \( E(\epsilon_{US,t}^+, | \epsilon_{US,t} \leq 0) \ P(\epsilon_{US,t} \leq 0) = 0 \) and \( \phi(0) = \frac{1}{\sqrt{2\pi}}, \Phi(0) = \frac{1}{2} \), where \( \phi(0) \) and \( \Phi(0) \) denote, respectively, the probability density function and the cumulative distribution function of a \( N(0,1) \) random variable evaluated at \( k = 0 \). Similarly, from the law of iterated expectations,

(A4) \[ E(\epsilon_{US,t}^-) = E(\epsilon_{US,t}^-, | \epsilon_{US,t} \leq 0) \ P(\epsilon_{US,t} \leq 0) + E(\epsilon_{US,t}^-, | \epsilon_{US,t} \geq 0) \ P(\epsilon_{US,t} > 0) \]

\[ = E(\epsilon_{US,t}^-, | \epsilon_{US,t} \leq 0) \ P(\epsilon_{US,t} \leq 0) \]

\[ = -\sigma_{US,t} \left[ \frac{\phi(0)}{1 - \Phi(0)} \right] (1 - \Phi(0)) \]

\[ = -\sigma_{US,t} \phi(0) \]
\[ = -\frac{\sigma_{US,t}}{\sqrt{2\pi}}, \text{ since } E(e_{US,t}^+ | e_{US,t} \geq 0) P(e_{US,t} > 0) = 0. \]

**Theorem A2.** \( \text{Var}(e_{US,t}^+) = \text{Var}(e_{US,t}^-) = \sigma_{US,t}^2 \left( \frac{\pi - 1}{2\pi} \right). \)

**Proof.** \( \text{Var}(e_{US,t}^+) = \text{Var} \left[ E(e_{US,t}^+ | e_{US,t} > 0) \right] + E \left[ \text{Var}(e_{US,t}^+ | e_{US,t} > 0) \right] \) can be written as

\[
\text{Var}(e_{US,t}^+) = \text{Var} \left( \frac{\sigma_{US,t} \phi(0)}{1 - \Phi(0)} \cdot I_{e_{US,t} > 0} \right) + E \left[ \sigma_{US,t}^2 \left( 1 - \left( \frac{\phi(0)}{1 - \Phi(0)} \right)^2 \right) \cdot I_{e_{US,t} > 0} \right]
\]

where \( I_{e_{US,t} > 0} \) is an indicator function with value 1 if \( e_{US,t} > 0 \) and 0 otherwise, and \( E(I_{e_{US,t} > 0}) = \frac{1}{2} - \Phi(0) = \frac{1}{2} \) and \( \text{Var}(I_{e_{US,t} > 0}) = \Phi(0)(1 - \Phi(0)) = \frac{1}{4} \) (since \( I_{e_{US,t} > 0} \) is a Bernoulli random variable). Thus,

\[
(A5) \quad \text{Var}(e_{US,t}^+) = \sigma_{US,t}^2 \left[ \frac{\phi(0)}{1 - \Phi(0)} \right]^2 \Phi(0)(1 - \Phi(0)) + \sigma_{US,t}^2 \left[ 1 - \left( \frac{\phi(0)}{1 - \Phi(0)} \right)^2 \right] (1 - \Phi(0))
\]

\[
= \sigma_{US,t}^2 \left( \frac{1}{2\pi} + \frac{\pi}{2\pi} - \frac{2}{2\pi} \right)
\]

\[
= \sigma_{US,t}^2 \left( \frac{\pi - 1}{2\pi} \right).
\]

Similarly,

\[
(A6) \quad \text{Var}(e_{US,t}^-) = \text{Var} \left[ E(e_{US,t}^+ | e_{US,t} < 0) \right] + E \left[ \text{Var}(e_{US,t}^+ | e_{US,t} < 0) \right]
\]

\[
= \text{Var} \left( \frac{\sigma_{US,t} \phi(0)}{1 - \Phi(0)} \cdot I_{e_{US,t} < 0} \right) + E \left[ \sigma_{US,t}^2 \left( 1 - \left( \frac{\phi(0)}{1 - \Phi(0)} \right)^2 \right) \cdot I_{e_{US,t} < 0} \right]
\]

\[
= \sigma_{US,t}^2 \left[ \frac{\phi(0)}{1 - \Phi(0)} \right]^2 \Phi(0)(1 - \Phi(0)) + \sigma_{US,t}^2 \left[ 1 - \left( \frac{\phi(0)}{1 - \Phi(0)} \right)^2 \right] (1 - \Phi(0))
\]
\[= \sigma^2_{US,t} \left( \frac{\pi - 1}{2\pi} \right).\]

Theorem A3. \( E(e^+_{US,t} e^-_{US,t}) = 0 \)

Proof. \( E(e^+_{US,t} e^-_{US,t}) = E(e^+_{US,t} e^-_{US,t} \mid e_{US,t} \geq 0) P(e_{US,t} > 0) + E(e^+_{US,t} e^-_{US,t} \mid e_{US,t} \leq 0) P(e_{US,t} \leq 0) \)

\[= 0 \cdot [1 - \Phi(0)] + 0 \cdot \Phi(0) = 0 \]

Theorem A4. \( \text{Cov}(e^+_{US,t}, e^-_{US,t}) = \frac{\sigma^2_{US,t}}{2\pi} \)

Proof. By definition, \( \text{Cov}(e^+_{US,t}, e^-_{US,t}) = E(e^+_{US,t} e^-_{US,t}) - E(e^+_{US,t}) E(e^-_{US,t}) \), which reduces to \(-E(e^+_{US,t}) E(e^-_{US,t})\) since, from Theorem A3, \( E(e^+_{US,t} e^-_{US,t}) = 0 \). Hence, using the expressions in (A3) and (A4), we obtain

\[ (A7) \quad \text{Cov}(e^+_{US,t}, e^-_{US,t}) = -\left( \frac{\sigma_{US,t}}{\sqrt{2\pi}} \right) \left( \frac{\sigma_{US,t}}{\sqrt{2\pi}} \right) \]

\[= \frac{\sigma^2_{US,t}}{2\pi} \]

Theorem A5. \( \text{Var}(e_{k,t} \mid I_{t-1}) = \sigma^2_{US,t} \left[ (\varphi_k^+)^2 \left( \frac{\pi - 1}{2\pi} \right) + (\varphi_k^-)^2 \left( \frac{\pi - 1}{2\pi} \right) + \varphi_k^+ \varphi_k^- \left( \frac{1}{\pi} \right) \right] + \sigma^2_{k,t} \)

Proof. By definition, \( \text{Var}(e_{k,t} \mid I_{t-1}) = \text{Var}( (\varphi_k^+ e^+_{US,t} + \varphi_k^- e^-_{US,t} + e_{k,t}) \mid I_{t-1}) \).

Since \( \text{Cov}(e^+_{US,t}, e_{k,t}) = 0 \), and \( \text{Cov}(e^-_{US,t}, e_{k,t}) = 0 \), \( \text{Var}(e_{k,t} \mid I_{t-1}) \) reduces to

\[ \text{Var}(e_{k,t} \mid I_{t-1}) = (\varphi_k^+)^2 \text{Var}(e^+_{US,t}) + (\varphi_k^-)^2 \text{Var}(e^-_{US,t}) + 2 \varphi_k^+ \varphi_k^- \text{Cov}(e^+_{US,t}, e^-_{US,t}) + \text{Var}(e_{k,t}) \]

Then, using Theorems A2 and A4, \( \text{Var}(e_{k,t} \mid I_{t-1}) \)
(A9) \[ \text{Var}(\varepsilon_{k,t} | I_{t-1}) = \]
\[ \sigma_{US,j}^2 \left[ (\varphi_k^+) \left( \frac{\pi - 1}{2\pi} \right) + (\varphi_k^-) \left( \frac{\pi - 1}{2\pi} \right) \right] + \sigma_{US,j}^2 \varphi_k^+ \varphi_k^- \left[ 0 - \frac{\sigma_{US,j}^2}{2\pi} \right] + \sigma_{k,t}^2 \]

\[ = \sigma_{US,j}^2 \left[ (\varphi_k^+) \left( \frac{\pi - 1}{2\pi} \right) + (\varphi_k^-) \left( \frac{\pi - 1}{2\pi} \right) + \varphi_k^+ \varphi_k^- \left( \frac{1}{\pi} \right) \right] + \sigma_{k,t}^2 \]

Theorem A6. \[ E(e_{US,j}^+)^2 = \frac{\sigma_{US,j}^2}{2} \]

Proof. By definition, \[ E(e_{US,j}^+)^2 = \text{Var}(e_{US,j}^+) + \left[ E(e_{US,j}^+) \right]^2 \]. Then, using the results of Theorems A1 and A2,

\[ (A10) \quad E(e_{US,j}^+)^2 = \sigma_{US,j}^2 \left( \frac{\pi - 1}{2\pi} \right) + \frac{\sigma_{US,j}^2}{2\pi} \]

\[ = \frac{\sigma_{US,j}^2}{2} \] .

Similarly, \[ E(e_{US,j}^-)^2 = \text{Var}(e_{US,j}^-) + \left[ E(e_{US,j}^-) \right]^2 \]. Then, using the results of Theorems A1 and A2

\[ (A11) \quad E(e_{US,j}^-)^2 = \text{Var}(e_{US,j}^-) + \left[ E(e_{US,j}^-) \right]^2 \]

\[ = \sigma_{US,j}^2 \left( \frac{\pi - 1}{2\pi} \right) + \frac{\sigma_{US,j}^2}{2\pi} \]

\[ = \frac{\sigma_{US,j}^2}{2} . \]

Theorem A7. \[ h_{US,k,t} = \frac{(\varphi_k^+ + \varphi_k^-)\sigma_{US,j}^2}{2} \]

Proof. By definition,

\[ (A12) \quad h_{US,k,t} = \text{Cov}(\varepsilon_{k,t}, \varepsilon_{US,j}) = E(\varepsilon_{k,t} \varepsilon_{US,j} | I_{t-1}) - E(\varepsilon_{k,t})E(\varepsilon_{US,j}) \]
\[ E(e_{k,t} | I_{t-1}) \quad \text{(since} \quad E(e_{US,t}) = 0) \]

\[ E \left[ \left( \psi_k^+ e_{US,t}^+ + \psi_k^- e_{US,t}^- + e_{k,t} \right) \left( e_{US,t}^+ + e_{US,t}^- \right) \right] | I_{t-1} \]  

(from equations A1 and A2)

\[ E \left[ \psi_k^+ \left( e_{US,t}^+ \right)^2 + e_{US,t}^+ e_{US,t}^- + \psi_k^- \left( e_{US,t}^- \right)^2 + e_{US,t}^+ e_{US,t}^- + e_{k,t} e_{US,t}^+ + e_{k,t} e_{US,t}^- \right] | I_{t-1} \]

Since \( E(e_{k,t} e_{US,t} | I_{t-1}) = 0 \), we have \( E(e_{k,t} e_{US,t}^+ | I_{t-1}) = E(e_{k,t} e_{US,t}^- | I_{t-1}) = 0 \). Using theorems A3 and A6, \( h_{US,k,t} \) simplifies to

(A13) \[ h_{US,k,t} = \psi_k^+ E \left( \left( e_{US,t}^+ \right)^2 \right) | I_{t-1} \]  

\[ + \psi_k^- E \left( \left( e_{US,t}^- \right)^2 \right) | I_{t-1} \]

\[ + \psi_k^+ E \left( e_{US,t}^+ e_{US,t}^- \right) | I_{t-1} \]

\[ = \frac{\psi_k^+ \sigma_{US,t}^2}{2} + \frac{\psi_k^- \sigma_{US,t}^2}{2} = \frac{\left( \psi_k^+ + \psi_k^- \right) \sigma_{US,t}^2}{2}. \]
References


*Further information in IDEAS/RePEc*


