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Evaluating Inflation Targeting
Using a Macroeconometric Model

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Abstract:
This paper uses a structurally estimated macroeconometric model, denoted the MC model, to evaluate inflation targeting in the United States. Various interest rate rules are tried with differing weights on inflation and output, and various optimal control problems are solved using differing weights on inflation and output targets. Price-level targeting is also considered. The results show that 1) there are output costs to inflation targeting, especially for price shocks, 2) price-level targeting is dominated by inflation targeting, 3) the estimated interest rate rule of the Fed (in Table 4) is consistent with the Fed placing equal weights on inflation and unemployment in a loss function, 4) the estimated interest rate rule does a fairly good job at lowering variability, and 5) considerable economic variability is left after the Fed has done its best. Overall, the results suggest that the Fed should continue to behave as it has in the past.

JEL: E52
Keywords: inflation targeting, interest rate rules, optimal control

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1 Introduction

There has been much discussion in the recent literature on whether inflation targeting (IT) by a monetary authority is a good idea. One approach is to look at performances of IT versus non IT countries. Using this approach, Ball and Sheridan (2005) find no evidence that IT improves a country’s performance. There are, however, a number of endogeneity problems associated with this approach, as the authors are aware, and it is not clear what to make of the results. In comparisons across countries it is hard to hold other things constant, especially initial conditions. In addition, a country that is not formally an IT country may behave roughly like one, and a country that is formally an IT country may behave somewhat more flexibly.

An alternative approach is to examine how an IT policy performs relative to other types of monetary policies in an economic model. Different rules can be examined, or formal optimal control problems can be solved. This is the approach taken in this paper. The model used is a version of the multicountry (MC) macroeconometric model in Fair (2004). The MC model is quite different from the macro model that is primarily used in the current literature, namely the “New Keynesian” (NK) model, and some justification is needed for using a different model. The NK and MC models are briefly compared in Section 2. Section 3 then examines inflation targeting using the MC model. Various interest rate rules are tried with differing weights on inflation and output, and various optimal control problems are solved using differing weights on inflation and output targets. Price-level targeting is also considered.
2 The NK and MC Models

2.1 NK Model

Goodfriend and King (1997) lay out what they call the “New Neoclassical Synthesis,” which is represented by the NK model. The four features of this synthesis are: 1) intertemporal optimization, 2) rational expectations, 3) imperfect competition, and 4) costly price adjustment. The NK model plays a prominent role in Clarida, Galí, and Gertler (1999) in their review of recent research in monetary policy, as it does in Woodford (2003). Virtually all the papers in Taylor (1999a) use some version of this model. Ireland (2004c, p. 923) states that “The development of the forward-looking, microfounded New Keynesian model stands, in the eyes of many observers, as one of the past decade’s most exciting and significant achievements in macroeconomics.” Woodford (2006, p. 17) suggests that NK models have “sufficient claim to quantitative realism to be of interest to policy-making institutions.”

In the NK model an infinitely lived, representative household maximizes the discounted value of expected future utility. An intertemporal optimality condition relates current consumption to expected future consumption and the real in-

est rate. Equating consumption to output yields an aggregate demand equation in which current output depends on expected future output and the real interest rate. The price equation, which has come to be called the “new-Keynesian Phillips curve,” is a forward-looking Phillips curve in which current inflation depends on expected future inflation and an output gap. It is derived from the optimizing behavior of monopolistically competitive firms, where firms change prices randomly as discussed in Calvo (1983) or face some kind of adjustment costs.\(^2\) An interest rate rule is then sometimes added as a third equation in which the nominal interest rate depends on inflation and the output gap.

Data on output (usually real GDP), inflation (usually the percentage change in the GDP deflator), and the federal funds rate or the three-month Treasury bill rate are typically used for the model. Sometimes data on a few other variables are used, depending on the setup. In particular, the labor income share is sometimes used in the price equation in place of the output gap, as in Galí and Gertler (1999) and Sbordone (2002), if the price equation is being analyzed separately. Sometimes all the parameters are calibrated and sometimes some parameters are calibrated and some are estimated. Estimation includes maximum likelihood and matching the model’s impulse responses to those of an estimated VAR. This work is all done under the assumption of rational expectations. The parameters that are calibrated or estimated are usually the structural parameters of the theoretical model, and so

\(^2\)Recent studies dealing with the New Keynesian Phillips curve but not the entire NK model are Batini, Jackson, and Nickell (2005), Galí, Gertler, and López-Salido (2005), which is a defense of the earlier widely cited Galí and Gertler (1999) paper, Kurmann (2005), Mankiw and Reis (2002), who propose an alternative price equation, Mavroeidis (2005), Nessen and Vestin (2005), Rudd and Whelan (2005), Sahuc (2005), and Sbordone (2005), which is a defense of the earlier widely cited Sbordone (2002) paper.
this analysis is not subject to the Lucas (1976) critique.

2.2 MC Model

The theoretical model upon which the MC model is based was first presented in Fair (1974a). An easier-to-read presentation is in Fair (1984). The following is a brief outline of the model. It has two of the four features of the New Neoclassical Synthesis, namely intertemporal optimization and imperfect competition. Households maximize expected future utility and firms maximize expected future after-tax cash flow. The horizons for the maximization problems are finite. The choice variables for a household are consumption, leisure, and money holdings. The main choice variables for a firm are its price, wage rate, production, and investment. Expectations of future values by households and firms are based on current and past values; they are not assumed to be rational. Disequilibrium is allowed for, and it takes the form of firms telling households the maximum amount of labor they will hire in the period and of actual sales differing from expected sales.

A household takes as given its initial values of money and bonds and the current values of the price, wage rate, interest rate, personal income tax rate, transfer payments, and the labor constraint from firms. It forms expectations of the future values of these variables and solves it optimization problem given a terminal condition on the value of its money plus bonds.

A firm faces a putty-clay technology. Adjustment costs are postulated for changes in labor and the capital stock. Firms set prices and wages in a monopolistic competitive setting. The demand for a firm’s product depends on its price relative
to the prices of the other firms. A firm expects that other firms’ prices are affected by the price that it sets. In other words, a firm expects that other firms will raise (lower) their prices if the firm raises (lowers) its own price. Similarly, the supply of labor to a firm depends on its wage rate relative to the wage rates of the other firms, and a firm expects that other firms’ wage rates are affected by the wage rate that it sets.\(^3\)

A firm takes as given all the initial values, including the initial values of other firms’ prices and wage rates and the current values of the interest rate and the profit tax rate. It forms expectations of the relevant future values, where again its expectations of other firms’ prices and wage rates depend on its own behavior, and solves its optimization problem. It chooses its price, wage rate, amount of each type of machine to purchase, and production. Given its price and wage rate decisions, a firm has an expectation of its sales and of the amount of labor that will be supplied to it. If actual sales turn out to be different from expected, this results in an unexpected change in inventories. If actual labor supply exceeds expected labor supply, the firm is assumed to hire only the expected amount. In fact, the model is set up so that firms communicate to households the amount of labor they are willing to hire (namely, the firms’ expected amounts), and households optimize under this constraint, as noted above.

Regarding the expectations of households and firms in the theoretical model, for a number of variables equations are postulated specifying how the expectations are

\(^3\)No adjustment costs are postulated for price changes and wage rate changes, and all firms can change their prices and wage rates each period. This is contrary to the fourth feature of the New Neoclassical Synthesis mentioned above, namely costly price adjustment. This assumption of costly price adjustment is, of course, controversial, and it is not necessarily a desirable feature of the synthesis. Bils and Klenow (2004) is a recent study casting doubt on the sticky price assumption.
formed. For the overall model in Fair (1974a) it is also specified that households and firms estimate the parameters of these equations based on past data. In this sense the expectations are sophisticated. The key point about expectations, however, is that they are not specified to be rational or converge to being rational. Because expectations are not rational, disequilibrium can occur, which drives many of the properties of the model. Households and firms never learn the true model; they grope around in a complex world, never quite understanding everything.

Government fiscal policy decisions are exogenous. The government chooses the two tax rates, transfer payments, the amount of goods to purchase, and the amount of labor to hire. On the monetary policy side, an interest rate rule is postulated in which the interest rate depends on inflation and unemployment. Unemployment in the model is the difference between the labor that households would supply if the labor constraint were not binding and the amount they actually supply taking into account the labor constraint in their optimizing problem.

All flows of funds and balance sheet constraints are accounted for in the model. One sector’s saving is some other sector’s dissaving. One sector’s financial liability is some other sector’s financial asset.

The model in Fair (1974a) was a closed-economy model, but a two-country model was introduced in Fair (1984). Again, all flows of funds and balance sheet constraints among the sectors of the countries are accounted for. The choice of a household now includes how much to purchase of the foreign good, which is affected by the price of the foreign good relative to the price of the home good. The exchange rate is determined by a reaction function of one of the country’s monetary authorities.
The model is solved by numerical techniques, given chosen parameter values and initial conditions. In a model in which disequilibrium is possible, the order of transactions matters, and the order chosen is 1) the government, 2) firms, and then 3) households. Transactions take place after households have optimized. Because firms don’t have complete knowledge of the model, their price and wage setting behavior may result in sales differing from expected sales and labor demand differing from the unconstrained labor supply. The numerical work consists of running various experiments for the individual optimization problems and then running experiments using the entire model. The experiments are designed to explore the properties of the theoretical model.

Returning to the expectational assumptions used in the model, Mankiw and Reis (2002, 2006) in recent work have modified the standard NK model by adding the assumption of “sticky information.” Households and firms are inattentive and base their decisions on outdated information sets. This work is essentially incorporating ideas from behavioral economics into the NK model. This assumption of sticky information is to some extent in the spirit of the expectational assumptions described above. Agents do not know the true model and therefore do not form rational expectations. They have limited information. Contrary to the case in the present model, however, in the Mankiw and Reis (2006) model, when agents update, they do know everything. For example, if there is no sticky information, the Mankiw and Reis model is just a standard classical flexible-price model. In the model above, on the other hand, agents never know everything. But there are similarities, and in general the above expectational assumptions are in the spirit of the assumptions of behavioral economics in that there is a lot that agents don’t
know.

The main differences so far between the theoretical work behind the MC model and that behind the NK model are that the MC work considers more decisions (is more general), does not assume price stickiness, and does not assume rational expectations. The lack of rational expectations leads to possible disequilibrium since firms may not set market clearing prices and wage rates. There can be unintended inventory investment and unemployment (as defined above).

Another major difference concerns estimation. The theoretical work behind the MC model is used to guide the specification of a model to be estimated (the MC model). Essentially, the theoretical work is used to guide the choice of left hand side and right hand side variables. The empirical equations that are specified are meant to be approximations to the decision equations of the households and firms. The left hand side variables are the decision variables and the right hand side variables are those that the agents take as given in the optimization process. Moving from theoretical work to empirical specifications is a messy business, and extra theorizing is usually involved in this process, especially regarding lags and assumptions about unobserved variables.

Although the estimated decision equations are only approximations, they do not suffer from the Lucas (1976) critique if expectations are not rational. More specifically, agents are assumed to form future expectations on the basis of past values, where the parameters multiplying these values are constant. Expectations are backward looking in this sense. The parameters in the expectation equations

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4Evans and Ramey (2006) have shown that in some cases the Lucas critique is a problem even if expectations are not rational. These cases are specific to the Evans and Ramey framework, and it is unclear how much they can be generalized.
are assumed not to depend on the parameters in the model: expectations not model consistent (rational). In the specification of a decision equation to estimate, if expected future values influence the current decision (which is usually the case), these values are substituted out by replacing them with the lagged values upon which they are assumed to depend. The decision equation is then estimated with these values included. If the parameters in the expectation equations are constant, then this substitution does not introduce non constant parameters in the decision equation. It is usually not the case that one can back out from the estimated decision equation the parameters of the expectations equations, but there is usually no need to do so. Under the above assumptions, expectations have been properly accounted for in the decision equation.

This treatment of expectations does not mean that policy changes have no effect on behavior. Say that the Fed announces a new policy regime, one in which it is going to weight inflation more than it has done in the past. If expectations are rational, this announcement will immediately affect them and thus immediately affect current decisions. Current decisions can be affected even before the Fed has actually changed the interest rate. In the treatment here expectations and thus decisions will be affected only after the interest rate has been changed. Decisions respond to policy changes, but only in response to actual changes in the policy variables. Announcements of new policy rules and the like have no effect on decisions because agents don't know the model and thus don't use it to form their expectations. If expectations were rational, the parameters would change as regimes change, with the Lucas critique then being relevant. In the current treatment the parameters of the estimated decision equations are constant across
policy regimes, although the decisions obviously change as the policy variables change.

The equations of the MC model are estimated by two-stage least squares, and the model has been heavily tested. The latest test results are presented in Fair (2004), and these results will not be discussed here. In general the model does well in the tests. The current version of the MC model consists of 328 estimated equations, with 1,502 coefficients estimated, plus 1,220 estimated trade share equations. None of the coefficients are chosen by calibration. There are 59 countries in the model, where for 21 countries only trade share equations are estimated. In the United States part of the model there are 31 estimated equations and about 100 identities. Many of the identities are needed to account for all the flows of funds and balance sheet constraints.

To summarize, then, the parameters of the theoretical model that is behind the MC model are never estimated, unlike the parameters of the NK model. In the DSGE approach, the theoretical model is the one brought directly to the data, not some approximation of it. If the NK model is well specified, the DSGE approach has the advantage that deep parameters are being estimated. If, on the other hand, the model is not well specified, the estimated model may be a poor approximation.

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5The estimation periods begin in 1954 for the United States and as soon after 1960 as data permit for the other countries. They generally end between 2004 and 2006. The estimation accounts for possible serial correlation of the error terms. The variables used for first stage regressors for a country are the main predetermined variables in the model for the country.

6The latest description of the MC model is in Fair (2004). The model can be analyzed on line or downloaded from the website listed in the introductory footnote. The list of first stage regressors for each equation is also available from the website. Data sources and definitions for all the variables used in the next section are listed in Fair (2004) and on the website.
2.3 Critique of the Basic NK Model

The following critique pertains to the basic NK model in the literature. There has been much work modifying and expanding the basic model, and some of the following criticisms do not pertain to some versions of the model. It may be that the following criticisms become moot as the basic NK model continues to be improved. The main argument here is that at the present time NK models are not likely to be good enough approximations of the economy to be trustworthy for evaluating inflation targeting and that the MC model is a better choice.

There are a number of reasons to think that the basic NK model is not a good approximation of the economy. First, the government and foreign sectors are ignored, both of which are important parts of the macroeconomy. Second, the aggregate demand equation seems much too simple. It does not take into account the different determinants of consumption and investment demand (as well as of import and export demand). In the MC model, for example, consumption is disaggregated into services, nondurables, and durables, and investment is disaggregated into residential, nonresidential fixed, and inventory. The estimated equations for these six categories are quite different. For example, stock effects are different. The initial stock of durable goods affects durable spending; the initial stock of housing affects housing investment; the initial stock of capital affects nonresidential fixed investment; and the initial stock of inventories affects inventory investment. Also, there are important initial wealth effects (driven mostly by stock market fluctuations) on consumption and housing investment. Other key explanatory variables in the consumption and housing investment equations are after-tax real income and
interest rates. There are thus many important variables are missing from the right hand side of the NK aggregate demand equation. Third, the price equation of the NK model ignores wages.\(^7\) In the MC model prices affect wages and vice versa,\(^8\) and this specification has been found to fit the data better than the specification of a single price equation with no right hand side wage variable.\(^9\)

Regarding the use of the NK model to analyze monetary policy, one of its key properties seems wrong.\(^{10}\) In the NK model a positive price shock with the nominal interest rate held constant is explosive (or in some cases indeterminate): inflation increases from the price equation, demand increases from the aggregate demand equation because the real interest rate falls, inflation increases more from the price equation, and so on. In order for the model to be stable, the nominal interest rate must be increased more than the rate of inflation, and so the coefficient on the inflation rate in the nominal interest rate rule must be greater than one. In the MC model, on the other hand, not only is a positive price shock with the nominal interest rate held constant not explosive, it is in fact contractionary. First, real wealth falls, which negatively affects consumption demand. Second, wages lag prices (a property of the estimated price and wage equations) and so real income falls, which also negatively affects consumption demand. Finally, the empirical results suggest that except for nonresidential fixed investment, nominal interest

\(^7\)A recent exception to leaving wages out of the model is Christiano, Eichenbaum, and Evans (2005), where both staggered wage and price contracts are postulated.

\(^8\)This result is compatible with the theoretical model outlined above in that initial values of other firms’ prices and wages affects the firm’s price and wage decisions.

\(^9\)Also, the results in Fair (2000) suggest that the long run dynamics of NAIRU style equations, like the New Keynesian Phillips curve, are not right given their focus on inflation rates rather than price levels. For present purposes, however, the more important criticism of the New Keynesian Phillips curve is that it ignores price and wage interactions.

\(^{10}\)A more extensive discussion of the following points is in Fair (2002).
rates matter rather than real interest rates, and so there is no positive effect on
demand from a lower real interest rate except for nonresidential fixed investment.
The net effect from a positive price shock with the nominal interest rate constant
is contractionary in the MC model. So not only does the Fed not have to raise the
nominal interest rate more than the inflation rate to prevent an explosive reaction,
it does not have to increase the nominal interest rate at all! If this property of the
MC model is in fact right, it suggests that the NK model is likely to lead a monetary
authority to overreact to an positive inflation shock since the contractionary effects
of the shock are not taken into account.

Another way of evaluating the NK model is to see how well it explains the actual
data, in this case the data on output and inflation. A useful procedure for comparing
models is to compute and compare outside-sample (i.e., outside the estimation
period) root mean squared errors (RMSEs). Ireland (2004b) computes outside
sample RMSEs for a RBC model; Del Negro, Schorfheide, Smets, and Wouters
(2006) do the same for a NK model; and outside sample RMSEs are computed in
Fair (2004) for the United States part of the MC model. The prediction periods
used in these three cases are close enough to allow at least a rough comparison
across models to be made. The RMSEs are presented in Table 1.

The “US” model uses actual values of the exogenous variables, and the “US+”
model uses forecasted values of the exogenous variables. Ireland considers two
versions of the RBC model, a “hybrid” version and a “diagonal” version. He does
not compute eight-quarter-ahead predictions, and the model does not include a
price variable. The prediction periods and table references are presented at the
bottom of Table 1. There are 76 four-quarter-ahead observations for the US and
Table 1
Outside Sample RMSEs
(percentage points)

<table>
<thead>
<tr>
<th>Model</th>
<th>Real GDP Qtrs ahead</th>
<th>GDP Deflator Qtrs ahead</th>
<th>No. Obs. Qtrs ahead</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>4</td>
<td>8</td>
<td>4</td>
</tr>
<tr>
<td>1. US</td>
<td>1.02</td>
<td>1.46</td>
<td>0.78</td>
</tr>
<tr>
<td>2. US+</td>
<td>1.33</td>
<td>1.84</td>
<td>0.87</td>
</tr>
<tr>
<td>3. Hybrid RBC</td>
<td>3.45</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. Diagonal RBC</td>
<td>2.16</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5. NK</td>
<td>2.62</td>
<td>6.05</td>
<td>0.88</td>
</tr>
</tbody>
</table>

- Rows 1 and 2 rows from Fair (2004), Table 14.1, p. 166.
- Rows 3 and 4 from Ireland (2004b), Table 5, p. 1218.
- Row 5 computed from Del Negro et al. (2006), Table 2, p. 36.

US+ models, 70 for the RBC models, and 55 for the NK model.

Table 1 shows that the NK model does poorly regarding real GDP. The four-quarter-ahead RMSE is about twice as large as those for the US and US+ models, and the eight-quarter-ahead RMSE is over three times as large. For the four-quarter-ahead results, the NK model is better than the hybrid RBC model, but worse than the diagonal RBC model. The NK model is much closer to the US and US+ models for the GDP deflator. These results thus suggest that the NK aggregate demand equation is not well specified, a point argued above. In light of these results the quote from Woodford (2006) at the beginning of this section seems premature.

Another way of testing the NK model is to test the assumption of rational expectations, which play a large role in the model. Although it is hard to test
this assumption, results have generally not been supportive—see, for example, Fair (2004), Fuhrer and Rudebusch (2004), and Rudd and Whelan (2006). The results in Rudd and Whelan (2006) are particularly strong against the assumption of rational expectations in the new-Keynesian Phillips curve. Given the results to date, a useful working hypothesis would appear to be that expectations are not rational rather than rational.

Returning to methodology, early examples of the estimation of equations that are meant to approximate the decision rules of economic agents are Tinbergen (1939) and Klein (1950). There is considerable economic theory involved in this work, and in fact nearly half of Klein’s book is devoted to intertemporal optimizing models of households and firms.11 But none of this early empirical work directly estimated the parameters of the theoretical models. Theory was only used to guide the choice of left hand side and right hand side variables. This approach dominated macro model building through the 1960s. The Lucas (1976) critique in the early 1970s changed the macro research landscape, and it eventually led to the DSGE approach that is currently popular. Whether this was a positive change for macro is an open question. Given the heterogeneity of agents, the complexity of the actual decision making processes, the complexity of the interactions among agents, and the quality of the macro data, it may be too much to expect that a good approximation of the economy can be obtained by directly estimating the parameters of a representative-agent theoretical model like that of the NK model. It may be better to settle for estimated approximations to decision rules. And if expectations are not rational, the Lucas critique is not likely to be a problem. The

11For an interesting discussion of this, see Solow (1991).
basic NK model does not appear trustworthy for analyzing monetary policy issues, including inflation targeting. Models more tied to the data are needed, and the MC model is one alternative. It is used for the work in the next section. Table 2 summarizes the comparison of the basic NK and the MC model discussed in this section.

3 Estimated Effects of Inflation Targeting

3.1 Interest Rate Channels

It will first be useful to outline the various channels through which interest rates affect output in the U.S. part of the MC model. Consider a decrease in the U.S. short term interest rate, say a policy change by the Fed. This decreases long term interest rates through estimated term structure equations. Interest rates appear as explanatory variables in the consumption, residential investment, and nonresidential fixed investment equations, all with negative coefficient estimates. In addition, decreases in interest rates have a positive effect on the change in stock prices through an estimated capital gains and losses equation, which has a positive effect on household wealth. This in turn has a positive effect on consumption because wealth appears as an explanatory variable in the consumption equations. Also, a decrease in U.S. interest rates (relative to other countries’ interest rates) leads to a depreciation of the U.S. dollar through estimated exchange rate equations.\(^\text{12}\)

\(^{12}\)A relative interest rate variable appears in the exchange rate equations for Canada, Japan, the United Kingdom, and Germany (Euroland after 1999). (All exchange rate equations are relative to the U.S. dollar.)
<table>
<thead>
<tr>
<th>Property</th>
<th>NK Model</th>
<th>MC Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intertemporal optimization?</td>
<td>Yes.</td>
<td>Yes.</td>
</tr>
<tr>
<td>Rational expectations?</td>
<td>Yes.</td>
<td>No.</td>
</tr>
<tr>
<td>Imperfect competition?</td>
<td>Yes.</td>
<td>Yes.</td>
</tr>
<tr>
<td>Costly price adjustment?</td>
<td>Yes.</td>
<td>No.</td>
</tr>
<tr>
<td>Estimation</td>
<td>Parameters of the theoretical model are calibrated or estimated.</td>
<td>The theoretical model is used to guide the specification of the econometric model, which is then estimated. No calibration for econometric model.</td>
</tr>
<tr>
<td>Demand disaggregation.</td>
<td>One aggregate demand equation.</td>
<td>Three consumption equations: services, nondurables, durables; three investment equations: nonresidential fixed, residential, inventory; import demand equation.</td>
</tr>
<tr>
<td>Foreign sector?</td>
<td>Usually not.</td>
<td>Yes.</td>
</tr>
<tr>
<td>Stock effects?</td>
<td>No.</td>
<td>Yes, on durable consumption, residential investment, nonresidential fixed investment, inventory investment.</td>
</tr>
<tr>
<td>Wealth effects?</td>
<td>No.</td>
<td>Yes, on the three categories of consumption.</td>
</tr>
<tr>
<td>Wage equation?</td>
<td>Usually not.</td>
<td>Yes, separately estimated wage and price equations.</td>
</tr>
<tr>
<td>Real versus nominal interest rate effects.</td>
<td>Real effects imposed.</td>
<td>Tested, where nominal interest rates generally dominate.</td>
</tr>
<tr>
<td>Effects of a positive price shock with the nominal interest rate held constant.</td>
<td>Explosive or indeterminate.</td>
<td>Contractary.</td>
</tr>
<tr>
<td>Lucas critique a problem?</td>
<td>No.</td>
<td>Not under the assumptions about expectations.</td>
</tr>
<tr>
<td>Long run tradeoff between inflation and output?</td>
<td>No.</td>
<td>Lack of tradeoff not tested because of limited data; see last paragraph in Section 3.2. Relationship likely to be nonlinear.</td>
</tr>
<tr>
<td>Accuracy.</td>
<td>See Table 1.</td>
<td>See Table 1.</td>
</tr>
</tbody>
</table>
Other things being equal, this depreciation is expansionary because U.S. exports rise and U.S. imports fall. A decrease in interest rates thus has a positive effect on aggregate demand through these channels.\footnote{There is one effect that works in the opposite direction. An decrease in interest rates decreases household interest income, which has a negative effect on household expenditures through a disposable income variable in the household expenditure equations. This effect is, however, smaller than the positive effects, and so the net effect of an interest rate decrease is positive.}

### 3.2 The U.S. Price Equation

It will next be useful to outline the main price equation in the U.S. part of the MC model. In this equation the log of the price level (the private nonfarm price deflator) is regressed on a constant, the lagged logged price level, the log of the wage rate, the log of the import price deflator, the unemployment rate, and the time trend. The coefficient estimates are presented in Table 3. The cost variables are the wage rate and the import price deflator, and the demand variable is the unemployment rate. The time trend is added to pick up trend effects on the price level not captured by the other variables. Adding the time trend to this equation is like adding a constant term to an equation specified using the inflation rate rather than the price level.

This equation does well in various chi-squared tests—reported in Table A 10, p. 206, in Fair (2004), with updated results on the website. No significant improvement in fit occurs when 1) the logged price level lagged twice, the log of the wage rate lagged once, the log of the import price deflator lagged once, and the unemployment rate lagged once are added as explanatory variables, 2) the equation is estimated under the assumption of fourth order serial correlation of the error term,
Table 3  
U.S. Price Equation  
LHS Variable is log $PF$

<table>
<thead>
<tr>
<th>RHS Variable</th>
<th>Coef.</th>
<th>t-stat.</th>
</tr>
</thead>
<tbody>
<tr>
<td>cnst</td>
<td>-0.036</td>
<td>-3.21</td>
</tr>
<tr>
<td>log $PF_{-1}$</td>
<td>0.881</td>
<td>92.56</td>
</tr>
<tr>
<td>log $W$</td>
<td>0.040</td>
<td>3.36</td>
</tr>
<tr>
<td>log $PIM$</td>
<td>0.050</td>
<td>21.23</td>
</tr>
<tr>
<td>$UR/100$</td>
<td>-0.177</td>
<td>-7.40</td>
</tr>
<tr>
<td>time trend</td>
<td>0.00032</td>
<td>9.88</td>
</tr>
<tr>
<td>SE</td>
<td>0.00343</td>
<td></td>
</tr>
</tbody>
</table>

- $PF =$ private nonfarm price deflator.  
- $W =$ nominal wage rate adjusted for labor productivity.  
- $PIM =$ import price deflator.  
- $UR =$ unemployment rate.  
- Estimation method: 2SLS.

3) the log of the wage rate led once is added, 4) the log of the wage rate led four times is added, 5) the log of the wage rate led eight times is added, and 6) an output gap variable is added. When the output gap variable is added, the unemployment rate retains its significance, and so it dominates the output gap as an explanatory variable.

If the wage rate variable were dropped from the equation in Table 3 and the equation were specified as an inflation equation rather than a price-level equation, the coefficient on log $PF_{-1}$ would be one. In addition, if lagged inflation were added as an explanatory variable to the inflation equation, this would introduce log $PF_{-2}$ with restrictions on the coefficients of both log $PF_{-1}$ and log $PF_{-2}$. These restrictions were tested in Fair (2000) and updated to other countries in
Chapter 4 in Fair (2004). They were rejected for the United States and generally rejected for the other countries. They suggest that the price equation should be specified in terms of price levels rather than inflation rates or changes in inflation rates. Using changes in inflation rates is off by two derivatives!

The wage equation in the U.S. part of the MC model has $\log W$ on the left hand side and on the right hand side: the constant, $\log W_{-1}$, $\log PF$, $\log PF_{-1}$, and the time trend. The price and wage equations are identified because $\log PIM$ and $UR$ are excluded from the wage equation, and $\log W_{-1}$ is excluded from the price equation. In the estimation of the wage equation a long run restriction was imposed regarding the real wage, which is that the derived real wage equation does not have on the right hand side the price level separately or the wage rate separately. This restriction is not rejected by the data. The price and wage equations were tested in Fair (2000) and (2004, Chapter 4) against standard NAIRU equations, and they lead to considerably more accurate price level and inflation predictions. This is consistent with the rejection of the NAIRU dynamics mentioned above.

A long run property of the price and wage equations is the following. If, say, the unemployment rate is permanently decreased by one percentage point, the price level is permanently higher, but the inflation rate converges back to its initial value. There is no permanent effect on the inflation rate. The evidence in favor of this property is the lack of rejection of the restrictions discussed above.

Regarding this long run property, it is obviously not sensible to think that the unemployment rate can be driven to zero with no permanent effect on the inflation rate. The problem in my view with the specification in Table 3 (or with specifications in terms of inflation rates or changes in inflation rates) is the linearity
assumption regarding the effect of the unemployment rate or measures of the output gap on the price level (or the inflation rate or the change in the inflation rate). At low levels of the unemployment rate, this effect is likely to be nonlinear. I have tried for both the United States and other countries to pick up nonlinear effects, but there appear to be too few times in which the unemployment rate is very low (or the output gap very small) to allow sensible estimates to be obtained. This does not mean, however, that the true functional form is linear, only that the data are insufficient for estimating the true functional form. What this means regarding the MC model is that one should not run experiments in which unemployment rates or output gaps are driven to historically low levels. Price-level or inflation-rate equations are unlikely to be reliable in these cases. Because of this, an effort has been made in the experiments below to stay around historical values.

3.3 The U.S. Interest Rate Rule

The final equation to discuss is the U.S. estimated interest rate rule. This rule was first estimated and added to my U.S. model in 1978—Fair (1978). This is the first instance that such as rule was added to a model, but the rules themselves go back to Dewald and Johnson (1963). This was long before the rules came to be called “Taylor rules;” they should really be called “Dewald-Johnson rules.” The estimated rule is presented in Table 4.

The left hand side variable is the three-month Treasury bill rate (RS), which is taken as the control variable of the Fed.14 The Fed is estimated to respond to

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14 The actual control variable is the federal funds rate, but this rate and RS are so highly correlated that it makes little difference which is used.
### Table 4
**U.S. Interest Rate Rule**

**LHS Variable is RS**

<table>
<thead>
<tr>
<th>RHS Variable</th>
<th>Coef.</th>
<th>t-stat.</th>
</tr>
</thead>
<tbody>
<tr>
<td>const</td>
<td>0.774</td>
<td>5.23</td>
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<tr>
<td>$RS_{-1}$</td>
<td>0.922</td>
<td>53.30</td>
</tr>
<tr>
<td>$PD$</td>
<td>0.071</td>
<td>4.18</td>
</tr>
<tr>
<td>$UR$</td>
<td>-0.125</td>
<td>-4.22</td>
</tr>
<tr>
<td>$\Delta UR$</td>
<td>-0.761</td>
<td>-6.02</td>
</tr>
<tr>
<td>$M1_{-1}$</td>
<td>0.012</td>
<td>2.30</td>
</tr>
<tr>
<td>$D794823 \cdot M1_{-1}$</td>
<td>0.215</td>
<td>9.73</td>
</tr>
<tr>
<td>$\Delta RS_{-1}$</td>
<td>0.228</td>
<td>4.24</td>
</tr>
<tr>
<td>$\Delta RS_{-2}$</td>
<td>-0.332</td>
<td>-6.76</td>
</tr>
<tr>
<td>SE</td>
<td>0.463</td>
<td></td>
</tr>
</tbody>
</table>

Stability test, 1954.1-1979.3 versus 1982.4-2006.1:

Wald statistic is 15.33 (8 degrees of freedom, $p$-value = .0531.)

- $RS$ = three-month Treasury bill rate.
- $PD$ = price deflator for domestic sales.
- $UR$ = unemployment rate.
- $M1$ = money supply.
- $D794823$ = dummy variable that is 1 between 1979:4 and 1982:3 and 0 otherwise.
- A dot over a variable means percentage change at an annual rate.
- Estimation method: 2SLS.

Inflation,\(^{15}\) the unemployment rate, the change in the unemployment rate, and the lagged growth of the money supply. The lagged values of $RS$ are meant to soak

\(^{15}\)Note in Table 4 that the Fed is taken to respond to changes in $PD$, the price deflator for domestic sales, not $PF$, the private nonfarm price deflator. $PD$, contrary to $PF$, includes import prices and excludes export prices. It is close in concept to the consumer price index. Better results are obtained using $PD$ rather than $PF$ in the interest rate rule. The exact definitions of $PD$ and $PF$ are in Fair (2004) and on the website.
up the dynamics, which are estimated to be fairly complicated. Between 1979:4 and 1982:3 (to be called the “early Volcker” period) the Fed, according to its own announcements, operated under a procedure that focused more on monetary aggregates than was the case before (or that was the case subsequently). This behavioral change was handled in the specification by adding a variable that is the lagged growth of the money supply multiplied by a dummy variable that is one in the early Volcker period and zero otherwise. As can be seen in Table 4, the coefficient estimate for the lagged money supply growth is about 20 times larger in the early Volcker period than otherwise. This way of accounting for the Fed policy shift does not, of course, capture the richness of the change in behavior, but at least it seems to capture some of the change.

The equation in Table 4 does well in various chi-squared tests (reported in Table A30, p. 216, in Fair (2004), with updated results on the website). No significant improvement in fit occurs when 1) RS lagged four times, the inflation rate lagged once, the unemployment rate lagged twice, and the percentage growth in the money supply lagged twice are added as explanatory variables, 2) the equation is estimated under the assumption of fourth order serial correlation of the error term, 3) the inflation rate and the unemployment rate led once are added, 4) the inflation rate and the unemployment rate led four times are added, 5) the inflation rate and the unemployment rate led eight times are added, and 6) and 7) two measures of expected future inflation are added.

The stability test listed at the bottom of Table 4 is of the hypothesis that the coefficients of the rule are the same before the early Volcker period as after. Much of the literature is of the view that the Fed behaved differently in the two periods,
but the hypothesis of stability is not rejected at the 5 percent level: the p-value is .0531. These results thus suggest that the Fed changed its behavior (as it announced it did) in the early Volcker period, but then went back to its earlier behavior after that.

The equation in Table 4 is taken in the experiments below as the rule that the Fed has followed in the past. It will provide a basis of comparison for other rules and procedures. Note that this rule is taken to be positive, not normative. It is the rule the Fed is estimated to have followed (aside from the early Volcker years), not a rule that is necessarily optimal.

3.4 Effects of a Decrease in $RS$ in the MC Model

The period examined is 1994:1–1998:4, 20 quarters, although, as discussed in footnote 20, the results are not very sensitive to the use of different periods. The first experiment is simply to examine the effects of a change in $RS$ in the model. The estimated residuals from all the stochastic equations are first added to the model and taken to be exogenous. This means that when the model is solved using the actual values of all exogenous variables, a perfect tracking solution is obtained. The base path is thus just the historical path.\(^{16}\) The interest rate rule in Table 4 is dropped from the model, and $RS$ is decreased by one percentage point from its historical value for each quarter. The model is then solved. The difference between the predicted value of each variable and each period from this solution

\(^{16}\)Regarding the above discussion of the price equation, this use of the estimated residuals is a way of keeping the model close to the historical values and thus away from very low values of the unemployment rate.
and its base (actual) value is the estimated effect of the interest rate change.

Selected results are presented in Table 5. The decrease in $RS$ led to an increase in output, a decrease in the unemployment rate, and an increase in inflation and the price level. The dollar depreciated relative to the Japanese and German currencies (as well as those of other countries) because of the relative interest rate effect in the exchange rate equations. ($RS$ fell relative to the interest rates of the other countries.) This resulted in an increase in the U.S. price of imports ($PIM$) and U.S. real exports ($EX$). The increase in $PIM$ has a positive effect on $PF$ (through the equation in Table 3), and the increase in $EX$ has a positive effect on output. $PF$ rises both from the increase in $PIM$ and the decrease in the unemployment rate. $PD$, the price deflator for domestic sales, rises slightly more than $PF$ because it is inclusive of import prices and $PIM$ has risen because of the depreciation of the dollar.\(^{17}\) As a rough rule of thumb, after two or three years the effect of a one percentage point fall in $RS$ is for output to be about .6 percent higher, the unemployment rate about .3 percentage points lower, and inflation about .3 percentage points higher. The effects then diminish after that.

### 3.5 Results for a Price Shock and a Demand Shock

The second experiment examines the effects of a positive shock to the price equation in Table 3. Again, the first step is to add the estimated residuals to all the stochastic equations and take them as exogenous. The constant term in the price equation was then increased by 0.005 (0.50 percentage points) from its estimated value. The

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\(^{17}\) $PF$ is essentially the price deflator for domestic output. It is affected by $PIM$ through the equation in Table 3, but the goods relevant for $PF$ are domestically produced goods only.
Table 5
Effects of a Decrease in RS in the MC Model

<table>
<thead>
<tr>
<th>Variable</th>
<th>1</th>
<th>4</th>
<th>8</th>
<th>12</th>
<th>16</th>
<th>20</th>
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</thead>
<tbody>
<tr>
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<td>-1.00</td>
<td>-1.00</td>
<td>-1.00</td>
<td>-1.00</td>
<td>-1.00</td>
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<tr>
<td>PD</td>
<td>.05</td>
<td>.22</td>
<td>.53</td>
<td>.84</td>
<td>1.10</td>
<td>1.29</td>
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<tr>
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<td>.98</td>
<td>1.18</td>
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<td>.30</td>
<td>.29</td>
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<td>.16</td>
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<tr>
<td>Y</td>
<td>.05</td>
<td>.36</td>
<td>.59</td>
<td>.63</td>
<td>.57</td>
<td>.49</td>
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<tr>
<td>UR</td>
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<td>-.13</td>
<td>-.26</td>
<td>-.29</td>
<td>-.26</td>
<td>-.20</td>
</tr>
<tr>
<td>EJA</td>
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<td>-.83</td>
<td>-1.23</td>
<td>-1.51</td>
<td>-1.74</td>
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<tr>
<td>EGE</td>
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<td>-2.95</td>
<td>-3.51</td>
<td>-3.96</td>
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<td>1.09</td>
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<td>.50</td>
<td>.72</td>
<td>.94</td>
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</tbody>
</table>

- RS = three-month Treasury bill rate.
- PD = price deflator for domestic sales.
- PF = private nonfarm price deflator.
- Y = private real output.
- UR = unemployment rate.
- EJA = Japanese exchange rate relative to U.S. dollar; a decrease is a depreciation of the dollar.
- EGE = German exchange rate relative to U.S. dollar; a decrease is a depreciation of the dollar.
- PIM = U.S. import price deflator.
- EX = real value of U.S. exports.
- A dot over a variable means percentage change at an annual rate.

The model was then solved under various assumptions about monetary policy. Selected results are presented in Table 6.
Table 6
Effects of a Positive Price Shock

<table>
<thead>
<tr>
<th>Changes from Base Values</th>
<th>Quarters Ahead</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
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<td>Sum</td>
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<td>.59</td>
<td>.37</td>
<td>.20</td>
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</tr>
<tr>
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<td>-.79</td>
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<td>-.98</td>
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</tr>
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<td>.66</td>
<td>.74</td>
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<td>Case 5: Price-level Rule</td>
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</tr>
</tbody>
</table>

- Estimated Rule: equation in Table 4.
- Inflation Rule .2838: equation in Table 4 with .2838 coefficient on $\dot{PD}$ and zero coefficients on $UR, \Delta UR, M_{1-1}$, and $D794823 \cdot M_{1-1}$.
- Inflation Rule .5676: equation in Table 4 with .5676 coefficient on $\dot{PD}$ and zero coefficients on $UR, \Delta UR, M_{1-1}$, and $D794823 \cdot M_{1-1}$.
- Price-level Rule: equation in Table 4 with $\log(\frac{PD}{PD^*})$ replacing $\dot{PD}$ with a coefficient of 25 and zero coefficients on $UR, \Delta UR, M_{1-1}$, and $D794823 \cdot M_{1-1}$, where $PD^*$ is the target level of $PD$.
- For notation see notes to Table 5.
Five cases are considered. For the first $RS$ is exogenous (the interest rate rule dropped). For the second the estimated interest rate rule in Table 4 is used. For the third and fourth cases the coefficients on $UR$, $\Delta UR$, $\dot{M} 1_{-1}$, and $D 794823 \cdot \dot{M} 1_{-1}$ in the interest rate rule are taken to be zero and the coefficient on inflation, $PD$, is increased. In the third case it is quadrupled to .2838, and in the fourth case it is doubled from this value to .5676. The fifth case is like the third and fourth cases except that the inflation variable is replaced by a price-level variable, the deviation of $PD$ from a target value, $PD^*$. A coefficient of 25 is used for this variable. The number for output under the “Sum” column is the percentage cumulative loss of output over the 20 quarters. The number for the unemployment rate under this column is the average increase over the 20 quarters.

For all the experiments in this paper in which interest rate rules were used, $RS$ was never allowed to be less than 0.5. If a rule called for a smaller value than 0.5, 0.5 was used. For the optimal control experiments below, $RS$ was constrained to be 0.5 or larger by the specification of the loss function.

The results in Table 6 are fairly easy to explain. Remember from Section 2 that a positive price shock in the MC model is contractionary (and inflationary). The Fed faces an unpleasant tradeoff. In case 1, where the Fed does nothing, the price level ($PD$) is higher by 4.22 percent after 20 quarters and the cumulative output loss is 0.98 percent. In case 2, where the Fed behaves according to the estimated rule, it is interesting that the Fed does very little. $RS$ does not change much; the price level is higher by 4.19 percent after 20 quarters; and the cumulative output

\[ ^{18}\text{Because the base path is just the historical path (because of the use of the estimated residuals), the target value of } PD \text{ for each quarter is just its historical value.} \]
loss is 1.01 percent. If the estimated rule has adequately captured historic Fed behavior, it says that the Fed responds to a bad price shock by not changing the interest rate much and thus accepting some increase in inflation and the price level and some loss of output.

Cases 3 and 4 show the tradeoff from using a rule that weights only inflation. In both cases the interest rate is increased substantially. In case 3 the price level is down to 2.54 percent higher after 20 quarters and the cumulative output loss is 1.60 percent. In case 4 the price level is down further to 1.80 percent higher after 20 quarters and the cumulative output loss is 1.90 percent. Inflation obviously comes down much faster in cases 3 and 4 than in cases 1 and 2.

Case 5, which uses the price-level rule, has a much higher interest rate at the end and much lower output. At the end of the 20 quarters the price level is down to an increase of only .78 percent, but the decrease in output is 3.90 percent and the increase in the unemployment rate is 1.74 percentage points. The increase in $RS$ is 3.62 percentage points. The price-level rule also has the feature of getting started slowly relative to the rules that use the inflation rate. This reflects the fact that the price shock leads to an immediate large change in the inflation rate but a more slowly increasing price level. In general the price-level rule would appear to be dominated by the inflation rules regarding responses to price shocks.

The third experiment examines the effects of a positive demand shock. For this experiment the constant terms in two of the U.S. consumption equations were increased. Otherwise, the same procedures were followed for this experiment as were followed for the second one. Selected results are presented in Table 7.
Table 7
Effects of a Positive Demand Shock

<table>
<thead>
<tr>
<th>Variable</th>
<th>1</th>
<th>4</th>
<th>8</th>
<th>12</th>
<th>16</th>
<th>20</th>
<th>Sum</th>
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<td>Case 1:</td>
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<td></td>
<td></td>
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<tr>
<td>PD</td>
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<td>.72</td>
<td>1.17</td>
<td>1.50</td>
<td>1.67</td>
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</tr>
<tr>
<td>PD</td>
<td>.13</td>
<td>.38</td>
<td>.46</td>
<td>.39</td>
<td>.23</td>
<td>.10</td>
<td></td>
</tr>
<tr>
<td>Y</td>
<td>.30</td>
<td>1.25</td>
<td>1.78</td>
<td>1.76</td>
<td>1.56</td>
<td>1.37</td>
<td>1.41</td>
</tr>
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<td>UR</td>
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<td>-.44</td>
<td>-.74</td>
<td>-.73</td>
<td>-.59</td>
<td>-.42</td>
<td>-.56</td>
</tr>
<tr>
<td>RS</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Case 2:</td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PD</td>
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<td>.21</td>
<td>.51</td>
<td>.72</td>
<td>.84</td>
<td>.87</td>
<td></td>
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<tr>
<td>PD</td>
<td>.10</td>
<td>.30</td>
<td>.29</td>
<td>.19</td>
<td>.07</td>
<td>.01</td>
<td></td>
</tr>
<tr>
<td>Y</td>
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<td>1.17</td>
<td>1.49</td>
<td>1.33</td>
<td>1.13</td>
<td>1.02</td>
<td>1.12</td>
</tr>
<tr>
<td>UR</td>
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<td>-.42</td>
<td>-.63</td>
<td>-.54</td>
<td>-.39</td>
<td>-.27</td>
<td>-.43</td>
</tr>
<tr>
<td>RS</td>
<td>.06</td>
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<td>.73</td>
<td>.75</td>
<td>.67</td>
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</tr>
<tr>
<td>Case 3:</td>
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<td></td>
<td></td>
</tr>
<tr>
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<td>.86</td>
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<tr>
<td>PD</td>
<td>.11</td>
<td>.34</td>
<td>.34</td>
<td>.23</td>
<td>.08</td>
<td>.01</td>
<td></td>
</tr>
<tr>
<td>Y</td>
<td>.30</td>
<td>1.21</td>
<td>1.59</td>
<td>1.44</td>
<td>1.20</td>
<td>1.06</td>
<td>1.19</td>
</tr>
<tr>
<td>UR</td>
<td>-.06</td>
<td>-.43</td>
<td>-.67</td>
<td>-.59</td>
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<td>-.46</td>
</tr>
<tr>
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<td>.03</td>
<td>.25</td>
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<td>.63</td>
<td>.58</td>
<td>.46</td>
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</tr>
<tr>
<td>Case 4:</td>
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<td></td>
<td></td>
</tr>
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<td>.67</td>
<td>.73</td>
<td>.73</td>
<td></td>
</tr>
<tr>
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<td>.26</td>
<td>.15</td>
<td>.03</td>
<td>.01</td>
<td></td>
</tr>
<tr>
<td>Y</td>
<td>.29</td>
<td>1.17</td>
<td>1.46</td>
<td>1.25</td>
<td>1.03</td>
<td>.95</td>
<td>1.07</td>
</tr>
<tr>
<td>UR</td>
<td>-.06</td>
<td>-.42</td>
<td>-.62</td>
<td>-.51</td>
<td>-.34</td>
<td>-.24</td>
<td>-.41</td>
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<tr>
<td>RS</td>
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<td>.94</td>
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<td>.59</td>
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<tr>
<td>Case 5:</td>
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<td></td>
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<td></td>
<td></td>
</tr>
<tr>
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<td>.63</td>
<td>.87</td>
<td>.83</td>
<td>.58</td>
<td></td>
</tr>
<tr>
<td>PD</td>
<td>.12</td>
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<td>.36</td>
<td>.16</td>
<td>-.11</td>
<td>-.27</td>
<td></td>
</tr>
<tr>
<td>Y</td>
<td>.30</td>
<td>1.24</td>
<td>1.65</td>
<td>1.38</td>
<td>.91</td>
<td>.53</td>
<td>1.07</td>
</tr>
<tr>
<td>UR</td>
<td>-.06</td>
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<td>-.58</td>
<td>-.31</td>
<td>-.06</td>
<td>-.42</td>
</tr>
<tr>
<td>RS</td>
<td>.01</td>
<td>.13</td>
<td>.53</td>
<td>1.05</td>
<td>1.46</td>
<td>1.61</td>
<td></td>
</tr>
</tbody>
</table>

- For notation see notes to Table 5.

For this experiment the estimated rule and the inflation rules respond similarly—cases 2, 3, and 4. The interest rate is increased, which lowers both
output and the price level relative to case 1 of no interest rate change. Comparing
the estimated rule to say, inflation rule .2858, the differences are small, and one
could conclude that moving from current Fed behavior to behavior in which only
inflation is in the rule makes little difference. This, of course, is not true for the
price shock in Table 6, and so the consequences of changing rules depends on
the type of shock. Regarding case 5, the price-level rule, there is a slower initial
response and then larger effects at the end.

3.6 Stochastic Simulation Results

The shocks in Tables 6 and 7 are just made up shocks. A more general way of ex-
amining the consequences of using different interest rate rules is to use historically
estimated residuals and stochastic simulation. There are 328 stochastic equations
in the MC model, 182 quarterly and 146 annual. There is an estimated error term
for each of these equations for each period. Although the equations do not all
have the same estimation period, the period 1977–2004 is common to all equa-
tions. There are thus available 28 vectors of annual error terms and 112 vectors
of quarterly error terms. These vectors are taken as estimates of the economic
shocks, and they are drawn in the manner discussed below. Since these vectors are
vectors of the historical shocks, they pick up the historical correlations of the error
terms. If, for example, shocks in two consumption equations are highly positively
correlated, the error terms in the two equations will tend to be high together or low

The base path is again taken to be the historical path, which is obtained by added
the estimated residuals to all the equations and taking them to be exogenous. Thus, for all the stochastic simulations the estimated residuals are added to the model and the draws are around these residuals. Each trial for the stochastic simulation is a dynamic deterministic simulation for 1994:1–1998:4 using a particular draw of the error terms. For each of the five years for a given trial an integer is drawn between 1 and 28 with probability 1/28 for each integer. This draw determines which of the 28 vectors of annual error terms is used for that year. The four vectors of quarterly error terms used are the four that correspond to that year. Each trial is thus based on drawing five integers. The solution of the model for this trial is an estimate of what the world economy would have been like had the particular drawn error terms actually occurred. (Remember that the drawn error terms are on top of the historical residuals for 1994:1–1998:4, which are always used.) The number of trials taken is 1000, so 1000 world economic outcomes for 1994:1–1998:4 are available for analysis.

The historical residuals are added to whatever interest rate rule is used, but no errors are drawn for it. Adding the historical residuals means that when the model inclusive of the rule is solved with no errors for any equation drawn, a perfect tracking solution results. Not drawing errors for the rule means that the Fed does not behave randomly but simply follows the rule.

Let \( y_t^j \) be the predicted value of endogenous variable \( y \) for quarter \( t \) on trial \( j \), and let \( y_t^* \) be the base (actual) value. How best to summarize the 1000 × 20 values of \( y_t^j \)? One possibility for a variability measure is to compute the variability of \( y_t^j \)

\[ \text{Each of the rules has a different set of estimated residuals associated with it because the predicted values from the rules differ.} \]
around \( y_t^* \) for each \( t \): 
\[
(1/J) \sum_{j=1}^{J} (y_t^j - y_t^*)^2,
\]
where \( J \) is the total number of trials.\(^7\)

The problem with this measure, however, is that there are 20 values per variable, which makes summary difficult. A more useful measure is the following. Let \( L^j \) be:
\[
L^j = \frac{1}{T} \sum_{i=1}^{T} (y_t^j - y_t^*)^2
\]
(1)

where \( T \) is the length of the simulation period (20). Then the measure is
\[
L = \frac{1}{J} \sum_{j=1}^{J} L^j
\]
(2)

\( L \) is a measure of the deviation of the variable from its base values over the whole period. It is not an estimated variance, just a summary measure of variability.

Selected results are presented in the first five rows in Table 8: values of \( L \) are presented for five variables. Comparing rows 1 and 2, the estimated rule does a fairly good job in lowering the values of \( L \). \( L \) for \( PD \) falls from 4.69 to 3.08, and \( L \) for \( Y \) falls from 2.88 to 2.22. \( L \) increases for \( RS \) from zero to .97.\(^{20}\) In row 3 inflation rule .2838 lowers \( L \) for \( PD \) more (to 2.53) at a cost of higher values of \( L \) for \( Y \) (2.55) and \( RS \) (1.59) compared to the estimated rule. The results in rows 2 and 3, for the estimated rule and inflation rule .2858, are not as similar as they are for the demand shock in Table 7, but they are more similar than for the price shock in Table 6. This is as expected since the errors used for the stochastic-simulation

\(^{20}\)When the experiment in row 2 is done for the 2000:1–2004:4 period (instead of 1994:1–1998:4), the values of \( L \) are: 3.86 for \( PD \), 2.90 for \( PD \), 2.08 for \( Y \), .58 for \( UR \), and .64 for \( RS \). In this later period the actual values of \( RS \) are on average smaller, and in the experiment there are more times when the 0.5 constraint for the minimum value of \( RS \) is binding. This is the main reason that \( L \) for \( RS \) is lower in the later period: the Fed has less room to maneuver. (Remember that the base path for an experiment is just the historical path.) This constraint results in somewhat higher values of \( L \) for \( PD \) and \( PD \) in the later period, but in general the results are fairly close.
Table 8  
Variability Estimates: Values of $L$  

<table>
<thead>
<tr>
<th>MC Model</th>
<th>$PD$</th>
<th>$PD$</th>
<th>$Y$</th>
<th>$UR$</th>
<th>$RS$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. No rule ($RS$ exogenous)</td>
<td>4.69</td>
<td>2.88</td>
<td>.80</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>2. Estimated rule</td>
<td>3.08</td>
<td>2.45</td>
<td>2.22</td>
<td>.59</td>
<td>.97</td>
</tr>
<tr>
<td>3. Inflation Rule .2838</td>
<td>2.53</td>
<td>2.33</td>
<td>2.55</td>
<td>.67</td>
<td>1.59</td>
</tr>
<tr>
<td>4. Inflation Rule .5676</td>
<td>1.63</td>
<td>2.03</td>
<td>2.56</td>
<td>.63</td>
<td>4.04</td>
</tr>
<tr>
<td>5. Price-level Rule</td>
<td>2.60</td>
<td>2.83</td>
<td>3.21</td>
<td>.78</td>
<td>3.61</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>US(EX,PIM) Model</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>6. No rule ($RS$ exogenous)</td>
<td>4.48</td>
<td>2.58</td>
<td>3.29</td>
<td>.97</td>
<td>0</td>
</tr>
<tr>
<td>7. Estimated rule</td>
<td>3.66</td>
<td>2.44</td>
<td>2.50</td>
<td>.72</td>
<td>.87</td>
</tr>
<tr>
<td>8. Optimal ($\lambda_1 = 1.5, \lambda_2 = 1.5$)</td>
<td>3.72</td>
<td>2.40</td>
<td>2.46</td>
<td>.72</td>
<td>.98</td>
</tr>
<tr>
<td>9. Optimal ($\lambda_1 = 0.0, \lambda_2 = 3.0$)</td>
<td>3.04</td>
<td>2.25</td>
<td>2.97</td>
<td>.86</td>
<td>.99</td>
</tr>
<tr>
<td>10. Optimal ($\lambda_1 = 0.0, \lambda_2 = 1.0)^a$</td>
<td>2.29</td>
<td>2.64</td>
<td>4.32</td>
<td>1.15</td>
<td>3.33</td>
</tr>
</tbody>
</table>

$^a$Price-level loss function; see text

- See notes to Tables 5 and 6.

draws consist of both demand and price shocks. The draws are, of course, more representative of actual shocks than are the shocks used in Tables 6 and 7.

The second inflation rule is more extreme than the first (row 4 versus row 3). $L$ for $PD$ falls to 1.63, but $L$ for $RS$ is now 4.04. It is interesting in this case that the cost of lowering $L$ for $PD$ is added variability of $RS$, not of $Y$ and $UR$. Other things being equal, lowering the variability of $PD$ in the MC model lowers the variability of $Y$, and this affect dampens the effects that work in the opposite direction, which arise from the higher variability of $RS$.

Again, the price-level rule (row 5) is not very good. Comparing rows 3 and 5, the price level rule has about the same value of $L$ for $PD$, but much larger values
for $Y$ and $RS$.

### 3.7 Optimal Control Results

Optimal control techniques are the obvious ones to use in evaluating inflation targeting, and so the most weight should probably be placed on the following results. The optimal control methodology requires that a loss function be postulated for the Fed. Assume that the loss for quarter $t$ is:

$$H_t = \lambda_1 (UR_t - UR^*_t)^2 + \lambda_2 (P^t_t - P^*_t)^2 + 9.0 (\Delta RS_t - \Delta RS^*_t)^2$$

$$+ 1.0/(RS_t - 0.499) + 1.0/(16.001 - RS_t)$$

(3)

where * denotes a base value. $\lambda_1$ is the weight on unemployment deviations, and $\lambda_2$ is the weight on inflation deviations. The last two terms in (3) insure that the optimal values of $RS$ will be between 0.5 and 16.0. The middle term penalizes changes in $RS$; more will be said about it below. As was done for the other experiments, the estimated residuals are first added to the stochastic equations and taken to be exogenous. The base path is then the historical path, and the target values in (3) are the historical values.

Assume that the control period of interest is 1 through $T$, where in the present case 1 is 1994:1 and $T$ is 1998:4. Although this is the control period of interest, in order not to have to assume that life ends in $T$, the control problem should be thought of as one of minimizing the expected value of $\sum_{t=1}^{T+n} H_t$, where $n$ is chosen to be large enough to avoid unusual end-of-horizon effects near $T$. The overall control problem should thus be thought of as choosing values of $RS$ that minimize the expected value of $\sum_{t=1}^{T+n} H_t$ subject to the model used.
If the model used is linear and the loss function quadratic, it is possible to derive analytically optimal feedback equations for the control variables. In general, however, optimal feedback equations cannot be derived for nonlinear models or for loss functions with nonlinear constraints on the instruments, and a numerical procedure must be used. The following procedure was used for the present results. It is based on a sequence of solutions of deterministic control problems, one sequence per trial.

Recall what a trial for the stochastic simulation is. A trial is a set of draws of 20 vectors of error terms, one vector per quarter. Given this set, the model is solved dynamically for the 20 quarters using an interest rate rule (or no rule). This entire procedure is then repeated the chosen number of trials, at which time the summary statistics are computed. As will now be discussed, each trial for the optimal control procedure requires that 20 deterministic control problems be solved.

For purposes of solving the control problems, the Fed is assumed to know the model (its structure and coefficient estimates) and the exogenous variables, both past and future. The Fed is assumed not to know the future values of any endogenous variable or any error draw when solving the control problems. The Fed is assumed to know the error draws for the first quarter for each solution. This is consistent with the use of the above rules, where the error draws for the quarter are used when solving the model with the rule.

The procedure for solving the overall control problem is as follows.

1. Draw a vector of errors for quarter 1, and add these errors to the equations. Take the errors for quarters 2 through $k$ to be their historical values (no draws), where $k$ is defined shortly. Choose values of $RS$ for quarters 1 through $k$ that minimize $\sum_{t=1}^{k} H_t$ subject to the model as just described.
This is just a deterministic optimal control problem, which can be solved, for example, by the method in Fair (1974b).\textsuperscript{21} Let $RS_1^*$ denote the optimal value of $RS$ for quarter 1 that results from this solution. The value of $k$ should be chosen to be large enough so that making it larger has a negligible effect on $RS_1^*$. (This value can be chosen ahead of time by experimentation.) $RS_1^*$ is a value that the Fed could have computed at the beginning of quarter 1 (assuming the model and exogenous variables were known) having knowledge of the error draws for quarter 1, but not for future quarters.

2. Record the solution values from the model for quarter 1 using $RS_1^*$ and the error draws. These solution values are what the model estimates would have occurred in quarter 1 had the Fed chosen $RS_1^*$ and had the error terms been as drawn.

3. Repeat steps 1 and 2 for the control problem beginning in quarter 2, then for the control problem beginning in quarter 3, and so on through the control problem beginning in quarter $T$. For an arbitrary beginning quarter $s$, use the solution values of all endogenous variables for quarters $s-1$ and back, as well as the values of $RS_s^{*}$ and back.

4. Steps 1 through 3 constitute one trial, i.e., one set of $T$ drawn vectors of errors. Do these steps again for another set of $T$ drawn vectors. Keep doing this until the specified number of trials has been completed.

The solution values of the endogenous variables carried along for a given trial from quarter to quarter in the above procedure are estimates of what the economy would have been like had the Fed chosen $RS_1^*, ..., RS_T^*$ and the error terms been as drawn.

The optimal control procedure is too costly in terms of computer time to be able to be used for the MC model, and for this work the U.S. subset of the model was used, denoted US(EX,PIM). This model is exactly the same as the model for the United States in the MC model except for the treatment of U.S. exports

\textsuperscript{21}This method sets up the problem as an unconstrained nonlinear optimization problem and uses an optimization algorithm like DFP to find the optimum.
(EX) and the U.S. price of imports (PIM). These two variables change when RS changes—primarily because the value of the dollar changes—and the effects of RS on EX and PIM were approximated in the following way.

First, \( \log EX_t - \alpha_1 RS_t \) was regressed on a constant, \( t \), \( \log EX_{t-1} \), \( \log EX_{t-2} \), \( \log EX_{t-3} \), and \( \log EX_{t-4} \), and \( \log PIM_t - \alpha_2 RS_t \) was regressed on a constant, \( t \), \( \log PIM_{t-1} \), \( \log PIM_{t-2} \), \( \log PIM_{t-3} \), and \( \log PIM_{t-4} \). Second, these two equations were added to the US(EX,PIM) model for particular values of \( \alpha_1 \) and \( \alpha_2 \), and an experiment was run in which the estimated interest rate rule of the Fed was dropped and RS was decreased by one percentage point. This was done many times for different values of \( \alpha_1 \) and \( \alpha_2 \). The final values of \( \alpha_1 \) and \( \alpha_2 \) chosen were ones whose experimental results most closely matched the results for the same experiment using the complete MC model. The final values chosen were -.0004 and -.0007 respectively. Third, the experiment in row 2 of Table 8 was run for the US(EX,PIM) model (with the EX and PIM equations added) and with the estimated errors from the EX and PIM equations being used in the drawing of the errors. When an error for the EX equation was drawn, it was multiplied by \( \beta_1 \), and when an error for the PIM equation was drawn, it was multiplied by \( \beta_2 \). The experiment was run many times for different values of \( \beta_1 \) and \( \beta_2 \), and the final values chosen were ones that led to results similar to those in the row 2 of Table 8. The values were \( \beta_1 = .4 \) and \( \beta_2 = .75 \). The results using these values are in row 7 of Table 8. The chosen values of \( \alpha_1 \), \( \alpha_2 \), \( \beta_1 \), and \( \beta_2 \) were then used for the experiments in rows 8–10.

Because of computational costs, 100 rather than 1000 trials were used for the optimal control experiments. The results are presented in rows 6-10 in Table 8.
Each experiment in a row uses the same sets of error draws, which lessens stochastic simulation error across experiments, although these sets of error draws are different from those used for the experiments in rows 1–5. Rows 6 and 7 are equivalent to rows 1 and 2: no rule and the estimated rule, respectively. Comparing these rows, the same pattern holds for both the overall MC model and the US(EX,PIM) model, namely that the estimated rule substantially lowers the variability of both PD and Y.

Row 8 uses equal weights on unemployment and inflation in the loss function. The values of \( \lambda_1 \) and \( \lambda_2 \) of 1.5 were chosen after some experimentation—using the coefficient of 9 on the middle term in equation (3)—to have the value of \( L \) for \( RS \) to be similar to its value when the estimated rule is used. The value of \( L \) is .98, which is close to .87 for the estimated rule. The aim is to constrain the optimal control procedure from variations in \( RS \) much different from what the Fed is estimated to have done historically (aside from the early Volcker period). The results using the estimated rule in row 7 and the equally-weighted optimal control procedure in row 8 are quite similar. In other words, the estimated rule is consistent with the Fed solving an optimal control problem with equal weights on unemployment and inflation.

Row 9 uses a zero weight on unemployment and a weight of 3.0 on inflation. Again, the weight of 3.0 led the value of \( L \) for \( RS \) (.99) being close to the value using the estimated rule. Comparing row 9 to row 7, \( L \) for \( PD \) has fallen from 3.66 to 3.04 at a cost of \( L \) rising for \( Y \) from 2.50 to 2.97. For the inflation rate (\( \dot{PD} \)) \( L \) falls from 2.44 to 2.25, and for the unemployment rate \( L \) rises from .72 to .86.
For the results in row 10 the inflation term in (3) was replaced by
\[ \lambda_2[(PD_t - PD_t^*)/PD_t^*]^2 \]
where \( PD^* \) is the target (historical) value. The results in row 10 are not as good as those in row 9. The value of \( L \) for \( PD \) is lower, but all the other values are larger. These poor results are consistent with the poor results in row 5 in Table 8 and in case 5 in Tables 6 and 7.

4 Conclusion

One obvious conclusion from the results in Section 3 is that price-level targeting is not a good idea. This is contrary to the conclusion of Cecchetti and Kim (2005), who argue that price-level targeting is less risky than inflation targeting. This conclusion is thus obviously model dependent. Cecchetti and Kim use a very simple model, and their conclusion is obviously not robust to the use of a model like the MC model.

Another conclusion is that there is clearly some output cost to inflation targeting. In terms of variability, rows 8 and 9 in Table 8 show that lowering the variability of the price level by 18 percent (from 3.72 to 3.04) results in an increase in the variability of output of 21 percent (from 2.46 to 2.97). The variability of the unemployment rate increases by 19 percent (from .72 to .86).

Tables 6 and 7 show that price shocks make more of a difference than demand shocks in terms of the output costs of inflation targeting. For the price shock in Table 6 the inflation targeting rule with a weight of .2838 compared to the estimated
rule lowered the increase in the price level by 40 percent after 20 quarters (from 4.20 to 2.54) and increased the cumulative output loss from 1.01 percent to 1.60 percent. For the demand shock in Table 7, on the other hand, the results for these two rules are very similar.

Regarding the estimated rule in Table 4, rows 7 and 8 in Table 8 show that the rule is consistent with the Fed weighting unemployment and inflation equally. Rows 7 and 9 show that the rule is not consistent with weighting only inflation. The results using the estimated rule in Tables 6, 7, and 8 show that the rule does a fairly good job in responding to shocks in that the variability of both the price level and output is substantially lowered relative to the case of no rule. And in Table 6 it is interesting that when there is a bad price shock, where the price level increases and output falls, the estimated rule suggests that the Fed essentially splits the difference and does nothing. This, of course, is not the case under the inflation targeting rules.

Finally, given that the results in Table 8 are based on historical shocks, it is clear that whatever policy the Fed follows, considerable variability is left. The Fed's power is limited. This is clear from the results in Table 5, where the effects of a change in $\pi_s$ on the price level and output are moderate.

Overall, the results in Section 3 suggest that the Fed should continue to behave as it has in the past.

One caveat regarding the present results concerns the price and wage equations in the MC model. Remember that by adding the estimated residuals to the stochastic equations and thus taking the base path to be the historical path, the MC model has in effect been steered away from very low values of the unemployment rate. The
assumption in the price equation in Table 3 the unemployment rate has a linear
effect on the price level is not likely to hold at very low values of the unemployment
rate. Thus the above conclusions about tradeoffs between price level or inflation
variability and output variability are not likely to pertain to cases of very low
unemployment rates.

Another caveat concerns the treatment of expectations in the MC model. Say
that at the beginning of some quarter the Fed announced that it was switching
to inflation targeting. This announcement would have no immediate effects in
the MC model. The effects come when the Fed actually changes interest rates.
For example, if this new policy led the Fed to raise interest rates sharply in the
current quarter and the next few quarters, this would have large effects in the
MC model in the current and future quarters as the higher interest rates came
about. But there is no change from the announcement alone and no change in the
parameters of the estimated decision equations. Agents don’t know the model,
including the rule of the Fed, and they make their decisions based in effect on
adaptive expectations. As discussed in Section 2.3, tests of the rational expectations
hypothesis have generally not been supportive, and it may be that the assumption
of adaptive expectations is a reasonable approximation of how expectations are
actually formed. The key question for purposes of this paper is whether the adaptive
expectations assumption is a good approximation if the Fed explicitly announces
a policy change—a new rule. It is hard to test this because there have been so
few announced rule changes. The results in this paper are thus based on the
assumption that agents react to policy-variable changes as they take place but
not to pure announcements. If research on NK models, which are based on the
assumption of rational expectations, progresses to the point where the models are good approximations of the economy, it will be interesting to see if the present results are substantially changed.
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Further information


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