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## Article

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## Portfolio management using realized covariances: Evidence from Brazil

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### Abstract

It is often argued that intraday returns can be used to construct covariance estimates that are more accurate than those based on daily returns. However, it is still unclear whether high frequency data provide more precise covariance estimates in markets more contaminated from microstructure noise such as higher bid-ask spreads and lower liquidity. We address this question by investigating the benefits of using high frequency data in the Brazilian equities market to construct optimal minimum variance portfolios. We implement alternative realized covariance estimators based on intraday returns sampled at alternative frequencies and obtain their dynamic versions using a multivariate GARCH framework. Our evidence based on a high-dimensional data set suggests that realized covariance estimators performed significantly better from an economic point of view in comparison to standard estimators based on low-frequency (close-to-close) data as they delivered less risky portfolios.

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*JEL classification:* C58; G11

*Keywords:* BEKK specification; Bootstrap test; Liquidity; Tick by tick data; Turnover

### Resumo

Argumenta-se frequentemente que retornos intradiários podem ser usados para construir estimativas de covariâncias mais precisas em relação às obtidas com retornos diários. No entanto, ainda não está claro se os dados de alta frequência fornecem estimativas de covariância mais precisas em mercados mais contaminados pelo ruído da microestrutura, como maiores spreads entre ofertas de compra e venda e baixa liquidez. Abordamos essa questão investigando os benefícios do uso de dados de alta frequência no mercado de ações brasileiro através da construção de portfólios ótimos de variância mínima. Implementamos diversos estimadores de covariâncias realizadas com base em retornos intradiários amostrados em diferentes frequências e obtemos suas versões dinâmicas usando uma estrutura GARCH multivariada. Nossa evidência baseada em um conjunto de dados de alta dimensão sugere que os estimadores de covariâncias realizadas obtiveram um desempenho significativamente melhor do ponto de vista econômico em

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comparação com os estimadores baseados em dados de baixa frequência (baseados em dados de fechamento) uma vez que geraram portfólios menos arriscados.

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*Palavras-chave:* Especificação BEKK; Bootstrap; Liquidez; Dados negócio a negócio; Turnover

## 1. Introduction

Modeling the covariation among financial time series is a fundamental task to numerous applications including derivative pricing, risk management and portfolio allocation. In this context, the search for a framework that allows a more precise estimate of the covariance matrix of financial assets returns has led to the analysis of high-frequency intraday data. The so-called realized covariance estimation initially proposed by Andersen et al. (2003) and Barndorff-Nielsen and Shephard (2004) has emerged as a viable candidate for more accurate covariance estimation. The use of intraday observations should improve estimation accuracy, however, data sampled at higher frequencies are affected by microstructure noise, which could cast doubts on the usefulness of high frequency data.

A large body of literature aims at investigating the advantages, from an economic standpoint, of using realized covariance estimators as opposed to traditional estimators based on low-frequency data. For instance, Bollerslev and Zhang (2003) demonstrate how high-frequency data may be more effectively measuring and modeling the systematic risks in factor pricing models. Fleming et al. (2003), Bandi et al. (2008), de Pooter et al. (2008), Bannouh et al. (2009), Liu (2009), Chiriac and Voev (2011), Hautsch et al. (2013), and Jin and Maheu (2013) employ alternative realized covariance estimators and conclude that the use of high-frequency data can improve the performance of mean-variance portfolios. Altogether, the empirical evidence suggests that realized covariance estimators improve volatility timing. This results in a better performance of optimal portfolios in terms of risk and/or portfolio turnover.

Even though the literature on realized volatility measures has been rapidly increasing, the empirical evidence available so far is mainly focused on developed markets (specially US), with limited attention to the applicability and potential gains from using these estimators in emerging markets. An obvious reason for this gap in the literature is the unavailability of high frequency data for these markets. This situation, however, has been changing as researchers now have more access to data from emerging markets and started addressing the benefits of using intraday information in volatility modeling. This raises the question of whether or not high frequency data also leads to more accurate estimates of the (co)variation among asset returns in less developed markets. One can argue that the increased microstructure noise coming from higher bid-ask spreads and lower liquidity in these markets might neutralize part of the gains from using high frequency data. The limited empirical evidence suggest that the use of intraday data is also beneficial in emerging markets. For instance, Bildik (2001) studies the intraday patterns of the volatility in the Istanbul stock exchange, while Kayahan et al. (2002) find that realized volatility outperforms a GARCH model in modeling the volatility of that market. Carvalho et al. (2006) and Wink Junior and Pereira (2011) construct realized volatility measures for the Brazilian equities market. The authors corroborate the evidence that realized measures provide better fit for the volatility dynamics.

One common point in the existing empirical evidence on the use of high-frequency data in emerging markets is that the authors consider only univariate realized measures, with no account on the study of multivariate realized estimators. This seems to be gap in the literature since many practical problems such as portfolio selection depends intrinsically on measuring the covariation among many assets. We fill this gap by implementing alternative realized covariance estimators in the Brazilian equities market, and check its usefulness in building optimal minimum variance portfolios. Our empirical strategy proceeds as follows. First, we use tick data of the 30 mostly traded stocks in the São Paulo stock exchange (BM&FBovespa) to build five alternative realized covariance estimators similar to those considered in de Pooter et al. (2008) and Liu (2009) and based on aggregating 5-min, 15-min, 30-min, 60-min, 90-min, and 120-min intraday returns. These estimators include not only intraday data but also the overnight volatility (Martens, 2002; Hansen and Lunde, 2005). Additionally, we also implement the realized covariance estimator proposed in Barndorff-Nielsen et al. (2011), which is the first to simultaneously address market microstructure effects and asynchronous price observations. Second, we consider a parsimonious multivariate GARCH specification to obtain a conditional dynamic version of each of the realized covariance estimators. Third, we use both unconditional and conditional

realized covariance estimators to obtain optimal minimum variance portfolios. Following [Liu \(2009\)](#), the impact of the choice of the estimation windows is also assessed in the empirical research. We formally test the differences in portfolio risk with respect to the standard covariance estimator based on low-frequency (close-to-close returns) using a bootstrap-based test proposed in [Politis and Romano \(1994\)](#). Finally, we conduct a subsample analysis in which minimum variance portfolios are obtained for two alternative data sets comprised of the 10 stocks with highest liquidity (measured in terms of trading volume) and the 10 stocks with lowest liquidity among the 30 stocks considered in the paper. In this sense, we are able to assess the robustness of our results by checking the effects of liquidity on the performance of optimal portfolios and on the choice of the most appropriate sampling frequency.

Our results suggest that the gains coming from using realized covariance estimators can be substantial. We find that realized measures do a better job in delivering minimum variance portfolios with both lower risk and lower turnover in comparison to low-frequency covariance estimators. Moreover, dynamic realized covariances modeled with the multivariate GARCH specification outperform their static counterparts. This result is robust to the choice of the estimation window and is further corroborated in the subsample analysis. This favorable evidence to the conditional estimators suggests that volatility timing can lead not only to better performance in terms of portfolio risk. Sampling frequency, however, plays an important role. We find that the 5-min sampling interval seems to be more appropriate to generate portfolios with lower portfolio risk and lower turnover. Our evidence also suggest that the use of a realized measure robust to microstructure effects and asynchronous price observations yields mixed results in terms of optimal minimum variance portfolios, since in very few instances this specification outperform the competing ones in terms of portfolio standard deviation but it delivers improved results in terms of portfolio turnover. Similar to [de Pooter et al. \(2008\)](#), we conclude that selecting the appropriate sampling frequency and size of the rolling windows appears to be fundamental in obtaining optimal portfolios with better performance, and this is also the case for two liquidity-sorted data sets considered in our subsample analysis.

The article is organized as follows. Section 2 describes the data used in the paper. Section 3 describes the unconditional and conditional realized covariance measures that are implemented in the paper. Section 4 discusses an empirical application along with the results. Finally, Section 5 concludes.

## 2. Data

The data for this study comprise of trade fills (tick by tick) for the Brazilian equities market. We select 30 assets that have the highest number of trades in the period between 2009 and 2012 resulting in a total of 631 trading days. These assets have significant liquidity and their notional volume represent a large proportion of the whole volume negotiated in the Brazilian stock market. The Brazilian equity market operates at São Paulo's BM&FBovespa stock exchange with normal<sup>1</sup> trading hours starting at 10:00 and closing at 17:00.<sup>2</sup> In the first minutes of the trading day, BMFBovespa's trading system performs an auction procedure in order to set the market opening price for all stocks. The practical effect in the data is that the first trades of the day always have the same time and price. This adds noise to the information set by artificially decreasing price volatility. In order to avoid this effect we follow a common practice in the literature and sample the prices 15 min after the opening of the market. We also adjust for daylight saving time when necessary.

[Table 1](#) reports descriptive statistics for the daily returns of the 30 stocks included in our dataset along with the ticker of each asset. Assets appear in decreasing order of trading volume. We observe that the individual assets included in the dataset have both positive and negative average returns, positive and negative skewness and excess kurtosis in the vast majority of instances.

## 3. Realized covariance estimators

The initial attempts to extend the theory of univariate realized variation to the multivariate setting were made by [Andersen et al. \(2003\)](#) and [Barndorff-Nielsen and Shephard \(2004\)](#). In this setting, realized covariances for a given day are constructed by summing intraday returns as follows. Let  $p_{t-1+ih}$  denote the  $(N \times 1)$  vector of log close transaction

<sup>1</sup> There is also the after market period that occurs after the normal hours. However, this period has substation lower volume of trades.

<sup>2</sup> This is equivalent to UTC (coordinated universal time) minus 3 (2) hours for normal (summer) time.

Table 1  
Descriptive statistics of daily returns.

Ticker	Mean (%)	Std. dev. (%)	Maximum (%)	Minimum (%)	Skewness	Kurtosis
VALE5	-0.027	2.561	11.67	-14.94	-0.300	3.771
USIM5	-0.107	3.129	15.26	-15.37	0.122	3.897
GGBR4	-0.058	3.087	16.78	-18.65	-0.145	3.647
BBDC4	-0.020	2.529	16.75	-11.66	0.010	5.049
CSNA3	-0.052	3.167	13.96	-24.15	-0.539	6.594
PETR3	-0.085	2.714	13.65	-18.10	-0.299	5.137
AMBV4	0.095	1.958	11.11	-16.45	-0.257	9.401
MMXM3	-0.103	4.656	40.18	-39.95	0.203	15.95
BBAS3	-0.026	2.743	12.74	-17.35	-0.278	4.724
CMIG4	0.001	2.310	10.95	-28.85	-2.505	28.77
ALLL11	-0.033	3.124	23.81	-28.34	-0.424	14.21
RDCD3	0.002	2.857	17.99	-16.18	-0.025	4.153
BRAP4	-0.039	2.903	15.82	-16.52	-0.183	4.058
LREN3	0.031	3.202	18.16	-16.85	0.169	3.857
CCRO3	0.057	2.367	14.03	-12.00	0.337	4.899
CYRE3	-0.046	3.924	18.81	-26.46	-0.502	6.460
ITSA4	-0.003	2.671	19.25	-14.47	0.294	7.465
TNLP4	-0.067	2.662	13.72	-13.09	0.184	5.000
GFGSA3	-0.132	4.048	18.55	-27.26	-0.471	6.003
HYPE3	-0.003	2.871	13.09	-11.67	-0.058	2.408
NATU3	0.077	2.295	12.94	-7.081	0.305	1.621
ELET3	0.015	2.299	11.99	-8.680	0.479	3.354
LAME4	-0.007	3.028	16.80	-16.48	0.096	4.414
GOLL4	-0.126	3.425	14.59	-24.02	-0.375	4.621
RSID3	-0.103	4.141	16.94	-32.29	-0.887	8.476
EMBR3	-0.056	2.529	11.44	-15.27	-0.123	4.122
MRVE3	-0.021	4.041	23.76	-27.93	-0.324	6.412
JBSS3	0.002	3.620	26.32	-22.71	0.084	5.628
PDGR3	-0.001	3.714	19.19	-21.87	-0.182	3.139
BISA3	-0.077	3.177	23.80	-16.91	0.515	6.540

The table reports descriptive statistics of daily returns for the 30 mostly traded Brazilian stocks. The sample covers the period from January 2009 and 2012 resulting in a total of 631 trading days. Assets appear in decreasing order of liquidity.

prices and let  $r_{t-1+ih,h} = p_{t-1+ih} - p_{t-1+(i-1)h}$  denote the  $(N \times 1)$  vector of returns for the  $i$ th intraday period on day  $t$ , for  $i = 2, \dots, I$ , where  $N$  is the number of stocks and  $I$  is the number of intraday interval of length  $h = 1/I$ . The return for the first intraday period,  $r_{t-1+h,h}$ , is defined as the difference between the log close and open transaction prices during that interval. The daily realized covariance matrix,  $RCOV_{t,h}$ , is defined as

$$RCOV_{t,h} = \sum_{i=2}^I r_{t-1+ih,h} r'_{t-1+ih,h}. \tag{1}$$

Barndorff-Nielsen and Shephard (2004) show that under the assumption of no market microstructure noise, the realized covariance matrix in (1) is a consistent estimator of the daily covariance matrix, i.e.

$$\sqrt{\frac{I}{h}} \left[ \text{vech}(RCOV_t) - \text{vech} \left( \int_{(i-1)h}^{ih} \Psi(u) du \right) \right] \xrightarrow{P} N(0, \Pi_t, )$$

where  $\text{vech}$  is an operator that stacks the lower triangular part of a matrix into a column vector,  $\Psi$  is the instantaneous covariance matrix, and  $\Pi_t$  is a positive definite matrix (see Barndorff-Nielsen and Shephard, 2004, for further details).

After Barndorff-Nielsen and Shephard’s seminal contribution, a large number of alternative estimators have been proposed in the literature; see, for instance, Martens (2004), Bandi and Russell (2005), Sheppard (2006), Bandi and Russell (2008), Bannouh et al. (2009), Christensen et al. (2010), Bauer and Vorkink (2011), Chiriac and Voev (2011), Noureldin et al. (2012), Hautsch et al. (2012), Fan et al. (2012b), and Jin and Maheu (2013), amongst others. In this paper, we initially follow de Pooter et al. (2008) and Liu (2009) and consider a class of realized covariance estimators

that includes not only intraday data but also the overnight volatility. In this case, the daily realized covariance matrix  $RCOV_{t,h}$  is defined as

$$RCOV_{t,h} = r_{t,c-o}r'_{t,c-o} + \sum_{i=2}^I r_{t-1+ih,h}r'_{t-1+ih,h} \tag{2}$$

where  $r_{t,c-o}$  is the  $(N \times 1)$  vector of close-to-open (overnight) returns from day  $t - 1$  (close) to day  $t$  (open). [Martens \(2002\)](#) and [Hansen and Lunde \(2005\)](#) point out that the inclusion of the overnight volatility is important because during the close-to-open hours, no data are recorded but the price of the asset will still be responding to news. We obtain realized covariances according to (2) for sampling frequencies of  $h=5, 15, 30, 60, 90, 120$  min such that in all cases the corresponding  $I$  intraday intervals completely cover the trading day. These estimators are labeled as  $RCOV5min$ ,  $RCOV15min$ ,  $RCOV30min$ ,  $RCOV60min$ ,  $RCOV90min$ , and  $RCOV120min$ , respectively. For the daily frequency, the realized covariance matrix,  $RCOVdaily$ , is defined as the outer product of the daily (close-to-close) returns, denoted by  $r_t$ , that is  $RCOVdaily_t = r_t r'_t$ .

One of the drawbacks of the standard multivariate realized estimator based on the sum of the outer product of intraday returns defined above is that under the presence of microstructure noise the realized covariation estimator in (1) is not consistent; see [Bandi and Russell \(2005\)](#). Because of that, we also consider the realized multivariate kernel (RMK) proposed by [Barndorff-Nielsen et al. \(2011\)](#), which is the first to simultaneously address market microstructure effects and asynchronous price observations while guaranteeing consistency and positive semi-definiteness of the resulting estimator. The RMK estimation is a two part process. Similar to [Hautsch et al. \(2012\)](#), the observations are synchronized via refresh time sampling (RTS). Refresh times are defined as the time it takes for all the assets in a set to trade or refresh posted prices. Once all the assets have traded, the most recent new price is used to form the RTS time scale. More formally, the first refresh time sampling point can be defined as  $\tau_1 = \max(t_1^{(1)}, \dots, t_1^{(d)})$ , and then subsequent refresh times as

$$\tau_{j+1} = \max(t_{N_{\tau_j}+1}^{(1)}, \dots, t_{N_{\tau_j}+1}^{(d)})$$

So  $\tau_1$  is the time it has taken until all the assets have been traded, i.e. all posted prices have been updated. Having defined the common time clock,  $\{\tau_j\}$ , we can now construct the vector returns series that the multivariate realized kernel will be based on. Let  $n, m \in \mathbb{N}$ , with  $n - 1 + 2m = N$ , and define the vector observations  $X_0, X_1, \dots, X_n$  as  $X_j = X(\tau_{j+m}), j = 1, 2, \dots, n - 1$ , and

$$X_0 = \frac{1}{m} \sum_{j=1}^m X(\tau_j) \text{ and } X_n = \frac{1}{m} \sum_{j=1}^m X(\tau_{N-m+j}),$$

where  $X_0$  and  $X_n$  are constructed by jittering initial and final time points. Refresh time synchronization allows us to define high frequency vector returns as  $x_j = X_{\tau_j} - X_{\tau_{j-1}}$ , where  $j = 1, 2, \dots, N$  and  $N$  is the number of refresh time observations. Finally, the RMK is defined as

$$RMK(X) = \sum_{h=-H}^H k\left(\frac{h}{H+1}\right) \Gamma_h, \tag{3}$$

where  $k(x)$  is the weight function of a Parzen kernel,  $H$  is a bandwidth parameter, and  $\Gamma_h$  is the  $h$ -th realized autocovariance matrix given by

$$\Gamma_h = \begin{cases} \sum_{j=|h|+1}^n x_j x'_{j-h}, & h \geq 0 \\ \sum_{j=|h|+1}^n x_{j-h} x'_j, & h < 0. \end{cases} \tag{4}$$

Table 2  
Mean and variance of realized variances and covariances.

	Realized variance		Realized covariance	
	Mean	Variance	Mean	Variance
RCOV5min	9.264	39.799	2.283	3.610
RCOV15min	5.756	5.272	2.265	2.874
RCOV30min	5.207	4.735	2.125	2.790
RCOV60min	4.787	4.667	2.003	2.838
RCOV90min	4.591	5.005	1.943	3.129
RCOV120min	4.387	4.999	1.869	3.244
RMK	4.713	3.539	1.619	1.760
RCOVdaily	6.445	6.994	2.860	5.092

The table reports the mean and variance of the realized variance and covariance of each of the unconditional realized covariance estimator considered in the paper.

Barndorff-Nielsen et al. (2011) show that this estimator is consistent, guaranteed to be positive semi-definite, can accommodate endogenous measurement noise and can also handle non-synchronous trading.<sup>3</sup>

In order to illustrate the characteristics of each of the daily realized covariance estimators discussed above, we report in Table 2 the mean and variance of the realized variance and covariance for each of the realized covariance estimators. We observe that the choice of the time interval for the aggregation has a clear impact in the average variance and covariance. Higher sampling intervals are associated to lower average realized variance and lower average realized covariance. This finding is consistent with the results of de Pooter et al. (2008). The explanation for the inverse association between sampling interval and average realized variance is that the bid/ask bounce of traded prices is lower for higher sampling times. This means that the autocorrelation of returns calculated based on trade prices is lower when the sampled time interval is higher, therefore decreasing the variance of the empirical returns, which in turn gives the result we see at Table 2.

### 3.1. Conditional realized covariance estimators

An important use of realized quantities is to forecast future volatilities and correlations of daily returns. The empirical application considered in this paper, based on a volatility-timing strategy and a risk measurement exercise, also depend on forecasts of future variances and covariances. Fleming (2001) and Fleming et al. (2003) provide evidence that sensible volatility timing leads to better mean-variance portfolios. In this sense, we consider the problem of forecasting the daily covariance matrix using the realized covariance estimators discussed in Section 3 as lagged explanatory variables. We follow de Pooter et al. (2008) and Barndorff-Nielsen et al. (2011) and consider a class of multivariate GARCH specifications of the form

$$r_t = \Sigma_t^{1/2} \varepsilon_t,$$

where  $\varepsilon_t \sim N(0, I)$  and  $\Sigma_t$  is a positive definite conditional covariance matrix modeled as a multivariate GARCH process. We specify  $\Sigma_t$  as a scalar BEKK process proposed by Engle and Kroner (1995), which is given by

$$\Sigma_t = C' C + \alpha r_{t-1} r'_{t-1} + \beta \Sigma_{t-1}.$$

Instead of estimating  $N(N + 1)/2$  unique elements of  $C$ , we employ the variance targeting technique as suggested in Engle and Mezrich (1996). The general idea is to estimate the intercept matrix by an auxiliary estimator that is given by

$$\hat{C}' \hat{C} = \bar{S}(1 - \alpha - \beta), \quad \bar{S} = \frac{1}{T} \sum_{t=1}^T r_t r'_t,$$

<sup>3</sup> We follow the methodology discussed in Barndorff-Nielsen et al. (2011) in order to optimize the bandwidth parameter  $H$ .

which ends up yielding the variance targeting scalar BEKK model:

$$\Sigma_t = \bar{S}(1 - \alpha - \beta) + \alpha r_{t-1} r'_{t-1} + \beta \Sigma_{t-1}, \quad (5)$$

which is covariance-stationary provided that  $\alpha + \beta < 1$ .

As suggested in [de Pooter et al. \(2008\)](#) and [Barndorff-Nielsen et al. \(2011\)](#), it is most natural to replace the daily update  $r_{t-1} r'_{t-1}$  in (5) by the daily realized covariance estimators discussed in Section 3. By doing so, we are able to define eight alternative specifications for the daily conditional covariance matrix based on intraday data:

1. RCOVdaily-BEKK

$$\Sigma_t^{daily} = \bar{S}(1 - \alpha - \beta) + \alpha RCOVdaily'_{t-1} + \beta \Sigma_{t-1}$$

2. RCOV5min-BEKK:

$$\Sigma_t^{5\min} = \bar{S}(1 - \alpha - \beta) + \alpha RCOV5min'_{t-1} + \beta \Sigma_{t-1}$$

3. RCOV15min-BEKK:

$$\Sigma_t^{15\min} = \bar{S}(1 - \alpha - \beta) + \alpha RCOV15min'_{t-1} + \beta \Sigma_{t-1}$$

4. RCOV30min-BEKK:

$$\Sigma_t^{30\min} = \bar{S}(1 - \alpha - \beta) + \alpha RCOV30min'_{t-1} + \beta \Sigma_{t-1}$$

5. RCOV60min-BEKK:

$$\Sigma_t^{60\min} = \bar{S}(1 - \alpha - \beta) + \alpha RCOV60min'_{t-1} + \beta \Sigma_{t-1}$$

6. RCOV90min-BEKK:

$$\Sigma_t^{90\min} = \bar{S}(1 - \alpha - \beta) + \alpha RCOV90min'_{t-1} + \beta \Sigma_{t-1}$$

7. RCOV120min-BEKK:

$$\Sigma_t^{120\min} = \bar{S}(1 - \alpha - \beta) + \alpha RCOV120min'_{t-1} + \beta \Sigma_{t-1}$$

8. RMK-BEKK:

$$\Sigma_t^{RMK} = \bar{S}(1 - \alpha - \beta) + \alpha RMK'_{t-1} + \beta \Sigma_{t-1}.$$

In order to estimate the parameters of each of the multivariate GARCH specifications defined above, we employ the composite likelihood (CL) method proposed by [Engle et al. \(2008\)](#). As pointed out by [Engle et al. \(2008\)](#), the



CL estimator provides more accurate parameter estimates in comparison to the standard quasi maximum likelihood estimator, specially in large problems.

#### 4. Portfolio selection using tick data

In this section we address the application of the realized covariance estimators discussed in Section 3 to portfolio selection. In particular, we consider the unconditional realized covariance estimators *RCOVdaily*, *RCOV5min*, *RCOV15min*, *RCOV30min*, *RCOV60min*, *RCOV90min*, *RCOV120min*, and *RMK* as well as their corresponding conditional versions discussed in Section 3.1, thus yielding a total of 16 alternative covariance estimators. Next we discuss the portfolio selection problem. This problem will provide an economic metric for the alternative information sets.

##### 4.1. Minimum variance portfolio construction

The mean-variance framework introduced by Markowitz (1952) is one of the milestones of modern finance theory. This approach is widely used to support managers in portfolio construction and to develop quantitative asset allocation strategies. In this framework, individuals choose their allocations in risky assets based on the trade-off between expected return and risk. In order to implement the mean-variance optimization in practice, it is common to obtain estimates of the vector of expected returns and its covariance matrix and plug these estimators in an analytical or a numerical solution to the mean-variance problem. The classic Markowitz (1952) portfolio problem is formulated as

$$\begin{aligned} & \min_{\mathbf{w}_t} \mathbf{w}'_t \Sigma_t \mathbf{w}_t \\ \text{subject to } & \mathbf{w}'_t \boldsymbol{\mu} = \mu_0 \\ & \mathbf{w}'_t \mathbf{1} = 1 \end{aligned} \tag{6}$$

where  $\mathbf{w}_t$  is the  $k \times 1$  vector of portfolio weights for time  $t$  chosen at time  $t - 1$ ,  $\Sigma_t$  is the  $N \times N$  is the conditional covariance matrix of a vector of excess returns for time  $t$ ,  $\boldsymbol{\mu}$  is the assumed vector of excess returns with respect to the risk-free asset,  $\mu_0 > 0$  is the required return, and  $\mathbf{1}$  is a conformable vector of 1's. This problem is known to be very sensitive to estimation of the mean returns (e.g. Michaud, 1989; Jagannathan and Ma, 2003). Very often, the estimation error in the mean returns degrade the overall portfolio performance and introduces an undesirable level of portfolio turnover. In fact, existing evidence suggest that the performance of optimal portfolios that do not rely on estimated mean returns is better; see, for instance, DeMiguel et al. (2013). Following Fan et al. (2012a), to avoid the difficulty associated with estimation of the expected return vector, from now on, we consider the global minimum variance portfolio (GMVP) which is the solution to:

$$\min \mathbf{w}'_t \Sigma_t \mathbf{w}_t \text{ subject to } \mathbf{w}'_t \mathbf{1} = 1. \tag{7}$$

Provided that  $\Sigma_t$  is positive definite, the solution to the GMVP problem is

$$\mathbf{w}_t = \frac{\Sigma_t^{-1} \mathbf{1}}{\mathbf{1}' \Sigma_t^{-1} \mathbf{1}}. \tag{8}$$

Finally, we consider the case in which shortsales are restricted by adding to (7) a constraint to avoid negative weights, i.e.  $w_t \geq 0$ . Previous works show that adding such a restriction can substantially improve performance, especially reducing the turnover of the portfolio, see Jagannathan and Ma (2003), among others. The solution to the shortselling constrained GMVP is obtained with numerical methods.

### Assessing minimum variance portfolio performance

The performance of the GMVP is evaluated in terms of average return ( $R_p$ ), standard deviation (volatility) of returns ( $\hat{\sigma}$ ), and turnover. These statistics are calculated as follows:

$$R_p = \frac{1}{T-1} \sum_{t=1}^{T-1} w_t' R_{t+1}$$

$$\hat{\sigma} = \sqrt{\frac{1}{T-1} \sum_{t=1}^{T-1} (w_t' R_{t+1} - R_p)^2}$$

$$\text{Turnover} = \frac{1}{T-1} \sum_{t=1}^{T-1} \sum_{j=1}^N (|w_{j,t+1} - w_{j,t}|),$$

where  $w_{j,t}$  is the weight of the asset  $j$  in the portfolio in period  $t$  before the re-balancing, while  $w_{j,t+1}$  is the desired weight of the asset  $j$  in period  $t+1$ . As pointed out by [DeMiguel et al. \(2009\)](#), the turnover as defined above, can be interpreted as the average fraction of wealth traded in each period.

In order to assess the relative performance of the covariance estimators considered in the paper, we consider as main benchmark strategy the GMVP obtained with the *RCOVdaily* covariance specification discussed in Section 3. The stationary bootstrap of [Politis and Romano \(1994\)](#) with  $B=1000$  resamples and block size  $b=5$  was used to test the statistical significance of differences between portfolio standard deviation of optimal portfolios relative to the benchmark portfolio. The methodology suggested in [Ledoit and Wolf \(2008, Note 3.2\)](#) was used to obtain  $p$ -values.

### 4.2. Implementation details

In order to implement the models, we employ a recursive estimation strategy based on a rolling window scheme. First, departing from the first  $t$  observations, all realized covariance models discussed in Section 3 are estimated and the one-step-ahead estimate of the conditional realized covariance models discussed in Section 3.1 are obtained. Next, we use these expressions to obtain optimal minimum variance portfolios. We add a new observation to the end of the sample, exclude the oldest one from it, and re-estimate all realized covariance models. This process is repeated until the end of the data set is reached. Therefore, we end up with a sample of  $T-t$  out-of-sample observations, where  $T$  is the length of the data set. In order to check the sensitivity of the results, we follow [Liu \(2009\)](#) and consider a rolling window approach with alternative sizes of the estimation window. In particular, we consider estimation windows that includes the past 18, 12, 6, and 3 months of data. In each case, the resulting realized covariance estimator is the average of the daily realized covariance over the estimation window; see [Liu \(2009, Eqs. 13–14\)](#) for details.

### 4.3. Results

In this section we discuss the main results regarding the performance of the optimal minimum variance portfolios. We report in [Table 3](#) the performance statistics for the minimum variance portfolios based on the alternative realized covariance estimators discussed in Section 3. Panels A to D of [Table 3](#) report the results considering a rolling estimation window that includes the past 18, 12, 6, and 3 months of data, respectively. The tables report the average portfolio return, portfolio standard deviation, and turnover. All figures are based on out-of-sample daily returns. Asterisks indicate that the standard deviation is statistically different at the 10% level with respect to the one obtained with benchmark unconditional covariance estimator *RCOVdaily*. The gray box indicate outperformance in the estimation window in terms of standard deviation. The transaction costs related to the re-balancing frequency might degrade the performance of the portfolios and hinder its implementation in practice. Thus, the performance of optimized portfolios is evaluated in the case of daily, weekly and monthly re-balancing frequencies. A potentially negative effect of adopting a lower re-balancing frequency is that the optimal compositions may become outdated.

When looking at the standard deviations of the minimum variance portfolios, we observe that the usage of conditional realized covariances leads to better results in terms of lower standard deviation when comparing to the unconditional case. This suggests that conditional estimators tend to improve volatility timing, and this translates to better portfolio performance. As reported in Panel A of [Table 3](#), the best overall performance in terms of portfolio standard deviation

**Table 3**  
**Out-of-sample performance of optimal minimum variance portfolios.**

Realized covariance estimator	Unconditional realized covariance									Conditional realized covariance								
	Daily rebalance			Weekly rebalance			Monthly rebalance			Daily rebalance			Weekly rebalance			Monthly rebalance		
	Mean (%)	Std dev (%)	Turnover	Mean (%)	Std dev (%)	Turnover	Mean (%)	Std dev (%)	Turnover	Mean (%)	Std dev (%)	Turnover	Mean (%)	Std dev (%)	Turnover	Mean (%)	Std dev (%)	Turnover
<i>Panel A: Past 18 months of data</i>																		
RCOVdaily	0.013	0.928	0.020	0.012	0.926	0.017	0.010	0.930	0.015	0.018	0.909*	0.151	0.024	0.911*	0.077	0.002	0.913*	0.033
RMK	0.008	0.920	0.014	0.008	0.921	0.014	0.009	0.922	0.014	0.009	0.890*	0.388	0.001	0.912*	0.132	-0.010	0.941	0.040
RCOV5min	0.012	0.939	0.019	0.016	0.949	0.018	0.000	0.940	0.017	0.023	0.869*	0.189	0.028	0.934	0.061	0.008	0.928	0.026
RCOV15min	0.014	0.925	0.017	0.013	0.925	0.015	0.013	0.928	0.014	0.011	0.904*	0.223	0.012	0.912*	0.104	0.003	0.955	0.043
RCOV30min	0.022	0.935	0.017	0.020	0.936	0.015	0.020	0.938	0.014	0.030	0.909*	0.222	0.023	0.923	0.101	0.002	0.949	0.040
RCOV60min	0.021	0.938	0.019	0.020	0.938	0.016	0.019	0.940	0.015	0.025	0.919	0.195	0.028	0.929	0.092	0.006	0.936	0.036
RCOV90min	0.025	0.949	0.019	0.023	0.949	0.016	0.024	0.953	0.014	0.027	0.918	0.178	0.027	0.925	0.085	0.017	0.931	0.036
RCOV120min	0.027	0.954	0.019	0.025	0.954	0.016	0.023	0.957	0.014	0.040	0.932	0.155	0.040	0.938	0.075	0.023	0.940	0.032
<i>Panel B: Past 12 months of data</i>																		
RCOVdaily	-0.016	1.016	0.027	-0.017	1.019	0.022	-0.021	1.030	0.018	-0.010	1.005	0.145	-0.004	1.002	0.075	-0.001	1.019*	0.032
RMK	-0.016	1.039	0.017	-0.018	1.040	0.017	-0.019	1.044	0.015	-0.008	1.036	0.400	-0.008	1.062	0.133	-0.019	1.097	0.046
RCOV5min	-0.013	1.019	0.022	-0.018	1.036	0.020	-0.021	1.042	0.018	-0.017	1.060*	0.134	-0.015	1.007*	0.054	-0.013	1.032	0.024
RCOV15min	-0.019	1.024	0.023	-0.023	1.027	0.020	-0.024	1.036	0.017	0.005	0.998*	0.207	-0.009	0.991	0.096	-0.002	1.024	0.036
RCOV30min	-0.011	1.024	0.023	-0.014	1.027	0.019	-0.013	1.034	0.016	0.012	1.000*	0.209	-0.005	1.003*	0.095	-0.002	1.037	0.038
RCOV60min	-0.013	1.020	0.026	-0.015	1.024	0.020	-0.015	1.033	0.017	0.009	1.010	0.182	-0.001	1.003*	0.087	0.001	1.024	0.035
RCOV90min	-0.008	1.030	0.027	-0.011	1.032	0.021	-0.012	1.042	0.017	0.004	1.014	0.168	-0.005	1.009	0.082	0.005	1.022	0.036
RCOV120min	-0.010	1.036	0.029	-0.013	1.039	0.022	-0.015	1.045	0.018	0.004	1.023	0.145	-0.003	1.024	0.074	-0.003	1.020	0.032
<i>Panel C: Past 6 months of data</i>																		
RCOVdaily	0.049	1.069	0.053	0.054	1.079	0.036	0.048	1.073	0.024	0.035	1.071	0.127	0.045	1.076	0.060	0.058	1.085	0.032
RMK	0.053	1.070	0.026	0.053	1.073	0.022	0.051	1.081	0.019	0.063	1.082	0.382	0.063	1.097	0.127	0.055	1.099	0.044
RCOV5min	0.063	1.055*	0.035	0.064	1.074	0.027	0.059	1.080	0.022	0.052	1.099*	0.119	0.063	1.059*	0.061	0.049	1.076	0.029
RCOV15min	0.055	1.064	0.038	0.055	1.066*	0.028	0.053	1.072	0.023	0.061	1.054*	0.200	0.055	1.060*	0.093	0.060	1.058*	0.038
RCOV30min	0.062	1.063	0.038	0.064	1.065*	0.028	0.061	1.064	0.022	0.068	1.052*	0.203	0.066	1.061*	0.089	0.055	1.072	0.038
RCOV60min	0.061	1.070	0.043	0.062	1.074	0.030	0.060	1.068	0.023	0.070	1.071	0.156	0.066	1.069	0.074	0.057	1.067	0.035
RCOV90min	0.061	1.076	0.047	0.064	1.079	0.032	0.063	1.074	0.024	0.060	1.073	0.121	0.054	1.065	0.061	0.061	1.057*	0.031
RCOV120min	0.059	1.086	0.049	0.060	1.089	0.033	0.059	1.083	0.025	0.055	1.075	0.096	0.053	1.076	0.052	0.057	1.075	0.030
<i>Panel D: Past 3 months of data</i>																		
RCOVdaily	0.045	1.175	0.108	0.054	1.186	0.065	0.046	1.222	0.039	0.043	1.181	0.143	0.058	1.187	0.071	0.049	1.230	0.041
RMK	0.064	1.148*	0.044	0.066	1.158	0.033	0.066	1.158*	0.026	0.071	1.138*	0.374	0.067	1.145*	0.115	0.077	1.141*	0.040
RCOV5min	0.073	1.107*	0.059	0.076	1.141*	0.041	0.076	1.154*	0.030	0.064	1.082*	0.131	0.072	1.142*	0.066	0.070	1.147*	0.033
RCOV15min	0.055	1.145*	0.064	0.053	1.152	0.044	0.056	1.161*	0.031	0.071	1.147*	0.166	0.068	1.152*	0.080	0.062	1.165	0.036
RCOV30min	0.062	1.140*	0.071	0.061	1.159	0.046	0.064	1.159*	0.032	0.070	1.134*	0.162	0.072	1.156*	0.078	0.062	1.165	0.036
RCOV60min	0.046	1.182	0.079	0.044	1.200	0.05	0.051	1.184	0.033	0.054	1.190	0.124	0.049	1.209	0.064	0.05	1.189	0.036
RCOV90min	0.053	1.176	0.087	0.055	1.188	0.053	0.059	1.215	0.035	0.053	1.174	0.108	0.057	1.187	0.062	0.055	1.214	0.036
RCOV120min	0.031	1.208	0.095	0.032	1.221	0.055	0.039	1.196	0.035	0.031	1.211	0.104	0.032	1.224	0.058	0.035	1.197	0.036

The table reports performance statistics for minimum variance portfolios using 30 mostly traded Brazilian stocks. The portfolios are obtained using unconditional and conditional realized covariance estimators. The rolling estimation window used to obtain the portfolios varies from 18 months (Panel A) to 3 months (Panel D). The statistics of average returns (“Mean”) and standard deviation (“Std. dev”) are based on daily returns. Optimal portfolios are re-balanced on a daily, weekly and monthly basis. Asterisks indicate that the standard deviation is statistically different at the 10% level with respect to the one obtained with benchmark unconditional covariance estimator *RCOVdaily*. The gray box indicate outperformance in the estimation window in terms of standard deviation.

across all estimation windows and re-balancing frequencies is obtained with the conditional *RCOV5min* realized covariance specification when using an estimation window of 18 months of data and daily portfolio re-balancing (0.869). This figure is statistically different with respect to the one obtained with the benchmark *RCOVdaily* unconditional estimator (0.928). We also find that lowering the portfolio re-balancing is detrimental to the performance of the minimum variance portfolios. Nevertheless, the conditional realized covariance estimators consistently outperform their unconditional counterparts in the case of daily, weekly, and monthly re-balancing frequencies.

We also see in [Table 3](#) that the overall value of the standard deviation of portfolio returns is higher as the size of the rolling windows decreases, and this holds for both conditional and unconditional realized covariance estimators. This is expected since the use of longer estimation windows tend to increase the precision of the covariance estimates, which translates to minimum variance portfolios with lower risk. Consequently, the worse performance in terms of portfolio standard deviation is obtained when using a 3-month estimation window (Panel D) in comparison to those obtained with the 18-month estimation window (Panel A).

The comparison of the results for the different sampling strategies shows that realized covariance estimators based on intraday returns sampled at the 5-min frequency seem to deliver portfolio with the lowest risk in the majority of the cases. Specifically, the conditional *RCOV5min* estimator achieved the best performance in terms of portfolio risk in all estimation windows considered.

As for the turnover measure, the results in [Table 3](#) suggest that, as expected, lowering the portfolio re-balancing frequency leads to a substantial decrease in turnover. Moreover, we also find that the turnover of portfolios obtained with high-frequency realized covariance estimators is lower with respect to those obtained with low-frequency ones in the vast majority of instances, suggesting that the use of intraday data yields not only reduces portfolio risk but also contributes to generate more stable portfolio compositions. We observe, however, that the turnover of portfolios obtained with unconditional estimators are usually lower with respect to those obtained with their conditional counterparts. For instance, the results in Panel A show that the turnover obtained when the conditional *RCOV5min* estimator is used is 0.19 under daily re-balancing whereas the same figure for the unconditional *RCOV5min* estimator is 0.02. This result indicate that the better volatility timing of conditional estimators might come at cost of a more intense trading activity. Finally, we observe that, among unconditional estimators, the best result in terms of turnover is obtained with the RMK specification, whereas among the conditional family the best result is obtained with the *RCOV5min* estimator.

In order to provide a visual inspection of our results, we plot in [Fig. 1](#) the boxplot of the minimum variance portfolio compositions obtained when the *RCOV5min* realized covariance specification is used along with a 18-month (left-hand side) and 3-month (right-hand side) estimation windows. The figure reveals that (i) changing the length of the estimation window has an impact in the resulting portfolio compositions; (ii) few assets retain the most relevant portfolio weights; (iii) many assets have portfolio weights equal to zero. This result is expected since we include a shortselling constraint to the minimum variance portfolio problem in (6). As [Jagannathan and Ma \(2003\)](#) show, included this constraint is equivalent to imposing a shrinkage to the resulting portfolio weights.

In summary, the results in [Table 3](#) leave three important messages. First, we conclude that realized covariance estimators based in high-frequency returns are able to generate minimum variance portfolios with lower risk and lower portfolio turnover with respect to those obtained with low-frequency estimators based on close-to-close returns. This is in line with existing empirical evidence such as those reported in [de Pooter et al. \(2008\)](#) and [Liu \(2009\)](#), among others. However, the sampling frequency plays an important role. We find that the realized covariance estimators based on returns sampled at the 5-min frequency usually outperform those obtained with alternative sampling strategies. Second, we find that conditional realized covariance estimators seem to do a better job in delivering minimum variance portfolios with lower risk in comparison to unconditional realized covariance estimators, although at the cost of a higher portfolio turnover. This favorable evidence to the conditional estimators suggests that volatility timing can lead to better performance in terms of portfolio risk. Third, our evidence also suggests that the use of a realized measure robust to microstructure effects and asynchronous price observations such as the RMK specification does not seem to bring much improvement in terms of optimal portfolio risk, although the results in terms of portfolio turnover obtained with this specification is promising in many instances.

#### 4.3.1. Subsample analysis

The results reported in [Table 3](#) suggest that realized covariance estimators based on intraday returns sampled at the 5-min frequency yields the best results in terms of minimum variance portfolios with lower risk. One potential concern that arises from these results is whether or not this sampling frequency is also appropriate for stocks with alternative

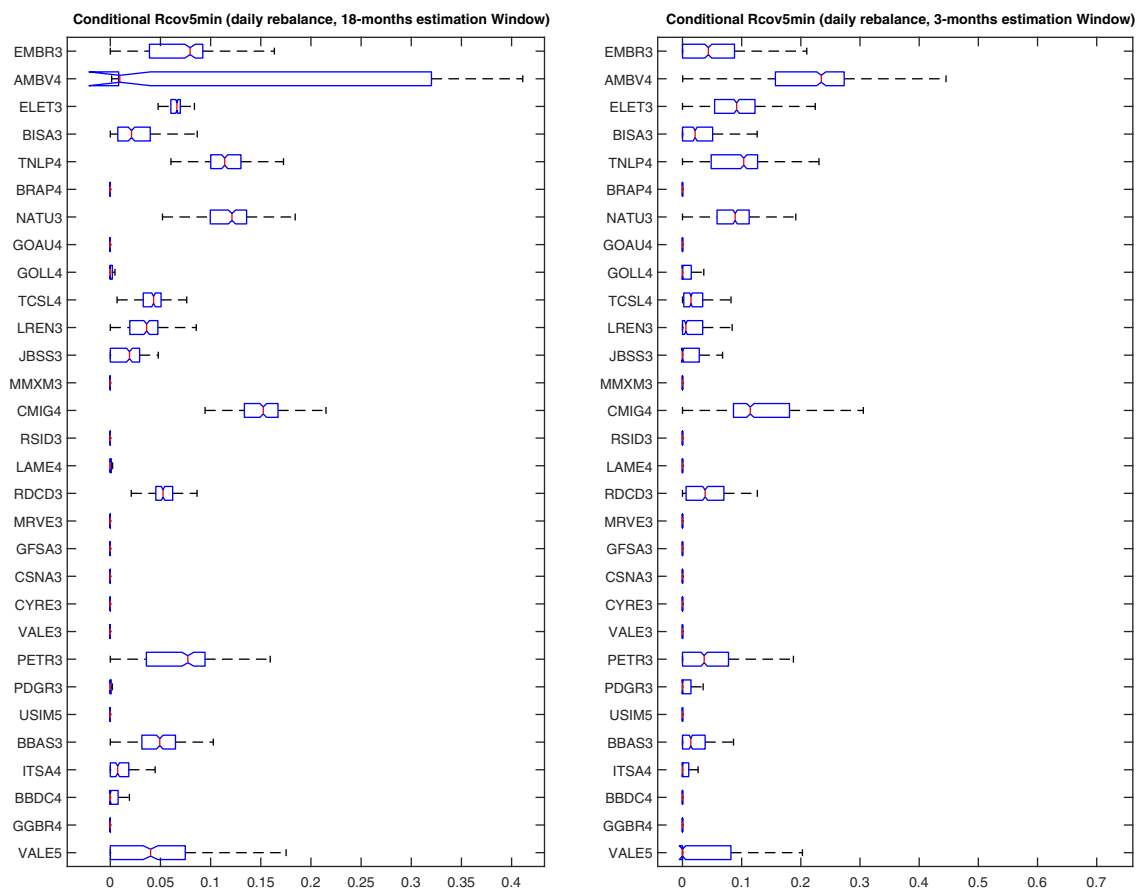


Fig. 1. Boxplot of minimum variance portfolio compositions. The figure plots the minimum variance portfolio composition obtained with the conditional *RCOV5min* realized covariance specification under a 18-month (left) and 3-month (right) estimation windows.

characteristics such as high or low liquidity (measured in terms of average trading volume). In order to assess the robustness of our results with respect to the liquidity of the underlying stocks, we run a subsample analysis and obtain minimum variance portfolios for data sets comprised of the 10 most liquid and the 10 less liquid stocks among the 30 stocks considered in the paper. By doing so, we are able to assess the effects of liquidity on the choice of the most appropriate sampling frequency.

The results of the minimum variance portfolio performance for the subsamples comprised of the 10 most liquid and the 10 less liquid stocks are reported in [Tables 4 and 5](#), respectively. The results for both subsamples corroborate to a large extent those reported for the full data set in [Table 3](#). First, minimum variance portfolios based on high-frequency realized covariance estimators outperform the ones based on low-frequency data. Second, conditional estimators outperform unconditional ones. Third, a decrease in the length of the estimation window is detrimental to the performance in terms of portfolio risk. Fourth, the 5-min sampling frequency emerges as the most appropriate in order to compute realized covariance estimators for the minimum variance portfolio problem. Therefore, we conclude from the subsample analysis that the choice of the conditional realized covariance estimator based on 5-min intraday returns is a natural “rule-of-thumb” for the portfolio selection problem in the case of the Brazilian equities market.

### 5. Concluding remarks

In this paper we look at the impact of the sampling procedure on the practical side of portfolio allocation in an emerging market. Using high-frequency (tick-by-tick) and low-frequency (close-to-close) data for the 30 mostly traded stocks in the Brazilian market, we build covariance matrices using eight alternative methods, of which seven

Table 4  
Out-of-sample performance of optimal minimum variance portfolios for the 10 most liquid stocks.

Realized covariance estimator	Unconditional realized covariance									Conditional realized covariance								
	Daily rebalance			Weekly rebalance			Monthly rebalance			Daily rebalance			Weekly rebalance			Monthly rebalance		
	Mean (%)	Std dev (%)	Turnover	Mean (%)	Std dev (%)	Turnover	Mean (%)	Std dev (%)	Turnover	Mean (%)	Std dev (%)	Turnover	Mean (%)	Std dev (%)	Turnover	Mean (%)	Std dev (%)	Turnover
<i>Panel A: Past 18 months of data</i>																		
RCOVdaily	-0.062	1.309	0.026	-0.063	1.319	0.019	-0.063	1.319	0.015	-0.046	1.242*	0.158	-0.049	1.244*	0.075	-0.044	1.269	0.035
RMK	-0.039	1.247*	0.011	-0.039	1.247*	0.011	-0.039	1.248*	0.010	-0.022	1.222*	0.359	-0.022	1.233*	0.109	-0.032	1.237*	0.032
RCOV5min	-0.034	1.292*	0.013	-0.034	1.294*	0.011	-0.036	1.293*	0.010	-0.035	1.155*	0.171	-0.033	1.200*	0.059	-0.033	1.218*	0.022
RCOV15min	-0.034	1.242*	0.014	-0.033	1.242*	0.012	-0.034	1.242*	0.011	-0.038	1.231*	0.178	-0.037	1.225*	0.083	-0.047	1.242*	0.033
RCOV30min	-0.045	1.251*	0.016	-0.045	1.251*	0.013	-0.046	1.252*	0.012	-0.040	1.233*	0.188	-0.049	1.229*	0.086	-0.066	1.249*	0.034
RCOV60min	-0.050	1.271	0.018	-0.050	1.271*	0.014	-0.054	1.273*	0.012	-0.034	1.250*	0.178	-0.031	1.250*	0.083	-0.051	1.268*	0.033
RCOV90min	-0.057	1.291	0.019	-0.056	1.291	0.014	-0.060	1.294	0.012	-0.038	1.256*	0.159	-0.034	1.262*	0.075	-0.053	1.278	0.030
RCOV120min	-0.057	1.290	0.019	-0.057	1.290	0.014	-0.062	1.293	0.012	-0.037	1.258	0.138	-0.036	1.258*	0.070	-0.063	1.274	0.029
<i>Panel B: Past 12 months of data</i>																		
RCOVdaily	-0.090	1.441	0.037	-0.093	1.443	0.026	-0.097	1.445	0.019	-0.078	1.408*	0.161	-0.081	1.409*	0.081	-0.084	1.402*	0.034
RMK	-0.063	1.424	0.013	-0.063	1.425	0.013	-0.064	1.427	0.012	-0.056	1.422*	0.385	-0.044	1.406*	0.117	-0.066	1.425*	0.040
RCOV5min	-0.065	1.408*	0.017	-0.068	1.413*	0.015	-0.069	1.416*	0.013	-0.057	1.309*	0.163	-0.062	1.386*	0.066	-0.059	1.402*	0.031
RCOV15min	-0.067	1.424	0.020	-0.068	1.425	0.017	-0.066	1.423	0.014	-0.056	1.416*	0.183	-0.055	1.414*	0.089	-0.058	1.401*	0.041
RCOV30min	-0.084	1.427	0.022	-0.084	1.429	0.019	-0.082	1.429	0.015	-0.064	1.406*	0.190	-0.061	1.414*	0.093	-0.058	1.417*	0.042
RCOV60min	-0.09	1.424	0.026	-0.090	1.427	0.02	-0.091	1.427	0.016	-0.062	1.415*	0.187	-0.06	1.414*	0.09	-0.070	1.407*	0.039
RCOV90min	-0.094	1.436	0.028	-0.095	1.439	0.021	-0.096	1.438	0.017	-0.065	1.425	0.164	-0.065	1.432	0.083	-0.084	1.422	0.039
RCOV120min	-0.093	1.432	0.028	-0.095	1.435	0.021	-0.094	1.435	0.016	-0.065	1.419*	0.147	-0.071	1.422	0.076	-0.080	1.406	0.034
<i>Panel C: Past 6 months of data</i>																		
RCOVdaily	0.065	1.432	0.038	0.067	1.436	0.028	0.061	1.427	0.020	0.067	1.434	0.116	0.074	1.410	0.054	0.067	1.431	0.029
RMK	0.059	1.422	0.020	0.060	1.420	0.019	0.059	1.425	0.017	0.071	1.428	0.366	0.073	1.401	0.116	0.104	1.406*	0.037
RCOV5min	0.057	1.414*	0.029	0.056	1.412*	0.023	0.059	1.408*	0.020	0.052	1.433	0.147	0.038	1.424	0.071	0.064	1.434	0.029
RCOV15min	0.054	1.442	0.034	0.053	1.442	0.026	0.058	1.420	0.021	0.046	1.437	0.145	0.044	1.429	0.072	0.076	1.411*	0.030
RCOV30min	0.044	1.475	0.039	0.042	1.475	0.030	0.048	1.440	0.023	0.035	1.481	0.149	0.033	1.477	0.071	0.064	1.444	0.033
RCOV60min	0.038	1.539	0.044	0.038	1.530	0.032	0.043	1.469	0.024	0.034	1.544	0.150	0.038	1.528	0.067	0.063	1.479	0.033
RCOV90min	0.025	1.609	0.047	0.021	1.604	0.034	0.035	1.495	0.025	0.022	1.612	0.114	0.018	1.609	0.059	0.049	1.501	0.031
RCOV120min	0.016	1.683	0.048	0.012	1.673	0.034	0.025	1.551	0.025	0.011	1.687	0.104	0.006	1.677	0.056	0.036	1.558	0.029
<i>Panel D: Past 3 months of data</i>																		
RCOVdaily	-0.001	1.549	0.098	0.002	1.557	0.057	-0.012	1.569	0.036	0.001	1.559	0.136	0.004	1.564	0.065	-0.010	1.572	0.038
RMK	0.023	1.514*	0.033	0.024	1.517*	0.024	0.020	1.520*	0.020	0.038	1.540	0.346	0.039	1.531*	0.104	0.044	1.541*	0.034
RCOV5min	0.027	1.505*	0.051	0.023	1.532	0.034	0.015	1.548	0.025	0.040	1.468*	0.152	0.031	1.519*	0.066	0.022	1.544*	0.033
RCOV15min	0.011	1.534	0.057	0.010	1.537	0.038	0.001	1.549	0.027	0.017	1.520*	0.143	0.019	1.524*	0.065	0.011	1.548	0.034
RCOV30min	0.007	1.540	0.066	0.009	1.547	0.042	-0.001	1.568	0.031	0.014	1.520*	0.128	0.016	1.535	0.065	0.013	1.562	0.035
RCOV60min	0.015	1.547	0.076	0.016	1.555	0.048	0.006	1.573	0.033	0.020	1.530	0.127	0.025	1.542	0.063	0.012	1.573	0.036
RCOV90min	0.008	1.562	0.083	0.009	1.570	0.051	-0.002	1.593	0.035	0.011	1.552	0.107	0.018	1.562	0.061	0.003	1.591	0.038
RCOV120min	0.007	1.547	0.088	0.009	1.554	0.054	-0.005	1.579	0.034	0.009	1.531	0.111	0.014	1.542	0.061	-0.004	1.571	0.036

The table reports performance statistics for minimum variance portfolios using 10 stocks with highest liquidity among the 30 traded stocks analyzed in the paper. The portfolios are obtained using unconditional and conditional realized covariance estimators. The rolling estimation window used to obtain the portfolios varies from 18 months (Panel A) to 3 months (Panel D). The statistics of average returns (“Mean”) and standard deviation (“Std. dev”) are based on daily returns. Optimal portfolios are re-balanced on a daily, weekly and monthly basis. Asterisks indicate that the standard deviation is statistically different at the 10% level with respect to the one obtained with benchmark unconditional covariance estimator *RCOVdaily*. The gray box indicate outperformance in the estimation window in terms of standard deviation.

**Table 5**  
**Out-of-sample performance of optimal minimum variance portfolios for the 10 less liquid stocks.**

Realized covariance estimator	Unconditional realized covariance									Conditional realized covariance								
	Daily rebalance			Weekly rebalance			Monthly rebalance			Daily rebalance			Weekly rebalance			Monthly rebalance		
	Mean (%)	Std dev (%)	Turnover	Mean (%)	Std dev (%)	Turnover	Mean (%)	Std dev (%)	Turnover	Mean (%)	Std dev (%)	Turnover	Mean (%)	Std dev (%)	Turnover	Mean (%)	Std dev (%)	Turnover
<i>Panel A: Past 18 months of data</i>																		
RCOVdaily	0.020	0.993	0.018	0.020	0.996	0.016	0.019	1.001	0.014	0.031	0.957*	0.145	0.026	0.968*	0.076	0.028	0.995	0.031
RMK	0.018	0.957*	0.012	0.018	0.958*	0.012	0.016	0.957*	0.012	0.021	0.937*	0.258	0.006	0.946*	0.093	0.021	0.978*	0.032
RCOV5min	0.010	1.002	0.017	0.013	1.013	0.016	0.006	1.009	0.016	-0.017	0.905*	0.169	-0.011	0.965*	0.075	-0.010	0.999	0.027
RCOV15min	0.029	0.973*	0.013	0.028	0.974*	0.013	0.027	0.976*	0.012	0.029	0.938*	0.167	0.021	0.942*	0.081	0.028	0.969*	0.032
RCOV30min	0.033	0.988	0.013	0.032	0.989	0.013	0.031	0.992	0.012	0.041	0.958*	0.168	0.031	0.966*	0.078	0.028	0.993	0.032
RCOV60min	0.032	0.997	0.014	0.031	0.998	0.013	0.029	1.002	0.012	0.053	0.96*	0.147	0.038	0.961*	0.075	0.028	0.995	0.029
RCOV90min	0.037	1.006	0.014	0.036	1.007	0.013	0.036	1.009	0.012	0.044	0.958*	0.162	0.039	0.963*	0.077	0.033	0.996	0.031
RCOV120min	0.037	1.007	0.015	0.035	1.009	0.014	0.034	1.011	0.012	0.057	0.968	0.15	0.040	0.983	0.072	0.026	0.981	0.029
<i>Panel B: Past 12 months of data</i>																		
RCOVdaily	-0.011	1.104	0.022	-0.010	1.107	0.020	-0.015	1.120	0.016	-0.004	1.068*	0.141	0.003	1.076*	0.075	0.012	1.100	0.033
RMK	-0.023	1.081	0.014	-0.023	1.082*	0.014	-0.025	1.086*	0.014	-0.016	1.067*	0.267	-0.009	1.070*	0.093	-0.008	1.074*	0.035
RCOV5min	-0.015	1.077*	0.018	-0.018	1.105	0.018	-0.024	1.117	0.017	-0.015	0.987*	0.153	0.005	1.057*	0.065	-0.024	1.091*	0.031
RCOV15min	-0.010	1.088	0.017	-0.010	1.093	0.016	-0.017	1.106	0.015	-0.013	1.030*	0.163	0.006	1.053*	0.081	0.010	1.058*	0.033
RCOV30min	-0.002	1.094	0.017	-0.004	1.097	0.016	-0.008	1.102	0.015	0.019	1.059*	0.175	0.017	1.078*	0.081	0.013	1.068*	0.032
RCOV60min	-0.002	1.096	0.019	-0.004	1.099	0.016	-0.011	1.112	0.015	0.024	1.054*	0.147	0.018	1.068*	0.074	-0.004	1.062*	0.031
RCOV90min	0.003	1.103	0.020	0.002	1.105	0.017	-0.004	1.118	0.015	0.017	1.060*	0.166	0.008	1.062*	0.080	-0.002	1.053*	0.033
RCOV120min	-0.004	1.103	0.021	-0.006	1.107	0.018	-0.011	1.119	0.015	0.015	1.067	0.146	0.009	1.081	0.073	-0.004	1.060*	0.031
<i>Panel C: Past 6 months of data</i>																		
RCOVdaily	0.065	1.102	0.038	0.067	1.106	0.028	0.061	1.127	0.020	0.067	1.104	0.116	0.074	1.090	0.054	0.067	1.131	0.029
RMK	0.059	1.101	0.018	0.058	1.103	0.017	0.058	1.109	0.016	0.063	1.109	0.244	0.061	1.110	0.087	0.053	1.109	0.031
RCOV5min	0.067	1.086	0.025	0.064	1.111	0.022	0.064	1.125	0.019	0.061	1.010*	0.157	0.074	1.075*	0.071	0.069	1.100	0.030
RCOV15min	0.066	1.095	0.026	0.063	1.097	0.022	0.063	1.112	0.018	0.079	1.079*	0.165	0.066	1.067*	0.073	0.069	1.090*	0.029
RCOV30min	0.074	1.094	0.027	0.071	1.096	0.022	0.071	1.103*	0.017	0.080	1.091	0.144	0.073	1.089*	0.068	0.064	1.088*	0.027
RCOV60min	0.079	1.097	0.030	0.076	1.099	0.024	0.072	1.114	0.019	0.088	1.092	0.125	0.078	1.079*	0.061	0.064	1.094*	0.027
RCOV90min	0.08	1.099	0.033	0.078	1.099	0.024	0.075	1.116	0.019	0.081	1.089	0.127	0.073	1.070*	0.064	0.059	1.089*	0.027
RCOV120min	0.076	1.109	0.035	0.073	1.111	0.026	0.066	1.130	0.020	0.073	1.101	0.107	0.066	1.087	0.054	0.059	1.108	0.026
<i>Panel D: Past 3 months of data</i>																		
RCOVdaily	0.084	1.155	0.073	0.090	1.172	0.045	0.095	1.200	0.03	0.081	1.159	0.114	0.085	1.169	0.057	0.098	1.200	0.031
RMK	0.086	1.156	0.029	0.085	1.162	0.025	0.088	1.166*	0.020	0.083	1.148	0.227	0.078	1.156	0.079	0.091	1.169*	0.032
RCOV5min	0.097	1.134*	0.040	0.094	1.160	0.031	0.091	1.164*	0.024	0.089	1.101*	0.138	0.093	1.167	0.060	0.075	1.149*	0.031
RCOV15min	0.088	1.139	0.043	0.084	1.144*	0.032	0.081	1.165*	0.024	0.098	1.142	0.120	0.090	1.132*	0.054	0.082	1.162*	0.030
RCOV30min	0.096	1.148	0.048	0.094	1.156	0.034	0.094	1.174	0.025	0.100	1.155	0.123	0.097	1.152	0.054	0.091	1.166*	0.028
RCOV60min	0.090	1.159	0.055	0.088	1.168	0.037	0.090	1.186	0.026	0.094	1.156	0.114	0.085	1.150	0.051	0.084	1.172	0.028
RCOV90min	0.084	1.174	0.059	0.083	1.182	0.039	0.092	1.234	0.028	0.080	1.156	0.119	0.077	1.148*	0.054	0.087	1.217	0.030
RCOV120min	0.079	1.159	0.063	0.077	1.170	0.041	0.083	1.210	0.029	0.081	1.162	0.103	0.075	1.164	0.051	0.080	1.213	0.029

The table reports performance statistics for minimum variance portfolios using the 10 stocks with lowest liquidity among the 30 most stocks analyzed in the paper. The portfolios are obtained using unconditional and conditional realized covariance estimators. The rolling estimation window used to obtain the portfolios varies from 18 months (Panel A) to 3 months (Panel D). The statistics of average returns (“Mean”) and standard deviation (“Std. dev”) are based on daily returns. Optimal portfolios are re-balanced on a daily, weekly and monthly basis. Asterisks indicate that the standard deviation is statistically different at the 10% level with respect to the one obtained with benchmark unconditional covariance estimator *RCOVdaily*. The gray box indicate outperformance in the estimation window in terms of standard deviation.

are based on fixed sample intervals and one is based on the dynamic sampling procedure of Barndorff-Nielsen et al. (2011). Moreover, we obtain the conditional version of each of these realized covariance estimators by employing a parsimonious multivariate GARCH specification, estimated based on rolling windows of different sizes. Using the estimated covariance matrices we assess the out of sample performance of a minimum-variance portfolio in terms of out-of-sample risk and turnover for each of the sampling methods.

The paper offers three most important messages. First, minimum variance portfolios based on realized covariance matrices that use high-frequency data achieve better performance in terms of portfolio risk and turnover in comparison to those based on covariance matrices that use low-frequency data, and this result is robust to portfolio re-balancing frequency (daily, weekly and monthly) and to the length of the rolling estimation window. Second, conditional realized covariance outperform their unconditional counterparts, suggesting that volatility timing can lead to better performance, even though this might come at the cost of a more intense trading activity as measured by the higher turnover of the conditional covariance estimators. Third, we corroborate the evidence in de Pooter et al. (2008) and find that sampling frequency plays an important role. The 5-min sampling interval seems to be more appropriate to generate portfolio with lower portfolio risk and lower turnover. Our subsample analysis reveals that the choice of this sampling interval is also robust to the level of liquidity of the underlying stocks in the portfolio.

This paper makes a strong case to market practitioners in emerging markets towards the empirical benefits of using high frequency data for building covariance matrices. The results suggest that the use of tick-by-tick data can provide lower out-of-sample portfolio risk for minimum variance portfolios. Also, our results suggest that the objective of the portfolio managers should be taken into consideration when selecting the inputs for building covariance matrices. A portfolio manager wishing to obtain optimal portfolios with lower levels of risk would be better off using longer estimation windows, daily portfolio re-balancing and adopting a conditional realized covariance matrix with intraday returns sampled at the 5-min interval.

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