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Article

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Effects of the taxation on inheritance in a microfounded model of growth and post-Keynesian distribution with overlapping generations and life cycle

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Abstract

This paper extends the model of overlapping generations with heterogeneous agents, allowing both classes (capitalist and worker) to hold a positive intergenerational capital stock. The main results were:

- (i) equilibrium interest rates that maximize the consumption and savings plans of working and capitalist classes were positively affected by taxation;
- (ii) an increase in taxation and therefore, in transfers to the working class raises the participation of intergenerational inheritance of this class in the total capital stock;

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- (iii) the taxation affects the distribution of wealth between the classes because it increases the participation of the working class in the total capital stock of the economy.

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Keywords: Intertemporal consumption; Post-Keynesian; Government transfers

Resumo

Este artigo estende o modelo de gerações sobrepostas com agentes heterogêneos, permitindo que ambas as classes (capitalista e trabalhador) mantenham um estoque de capital intergeracional positivo. Os principais resultados foram:

- (i) as taxas de juros de equilíbrio que maximizam os planos de consumo e poupança das classes operária e capitalista foram positivamente afetadas pela tributação;
- (ii) aumento da tributação e, portanto, nas transferências para a classe trabalhadora, eleva a participação da herança intergeracional desta classe no capital social total;
- (iii) a tributação afeta a distribuição de riqueza entre as classes porque aumenta a participação da classe trabalhadora no capital total da economia.

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Palavras-chave: Consumo intertemporal; Pós-keynesianos; Transferência do governo

1. Introduction

Since the overcoming of the razor's edge problem⁵ proposed by Kaldor (1955), post-Keynesian theorists have been working on long-term macroeconomic problems. However, unlike the Solow solution (Solow, 1956), the model proposed by Kaldor (1955) necessarily depends on there being at least two income classes, namely, capital and labor. This dependence results from the flexibility of the propensity to save that is no longer an exogenous constant and becomes dependent on the distribution of income between capital and labor, which hypothetically, will have a different propensity to save. Thus, if the propensity to save part of the profit is greater than the propensity to save part of the wages, there will be a distribution of income between capital and labor that will cause the average propensity to save in economics solve the problem raised by Harrod (1939) and Domar (1946).

In the same research line, Pasinetti (1962) proposes a new model with the existence of two social classes, the capitalists and the workers. This change comes from the observation that individuals must have one only propensity to save, regardless of whether the source of their income is derived from capital or labor. Although Pasinetti (1962) works with the definition of personal income distribution (between capitalists and workers) in place of functional income distribution (between capital and labor) as Kaldor (1955), his work resulted in the same solution to the problem of “razor's edge” and reached the same equation for the rate of profit: (1) $P/K = g_n/s_c$, where P is the mass of profit, K the capital stock of the economy, g_n the natural growth, and s_c the propensity to save of the profit (in Kaldor) or of the capitalist (in Pasinetti).

Eq. (1), known as the Pasinetti Theorem or Cambridge equation shows that the rate of profit depends on the natural growth rate of the product and the capitalist class's savings behavior, and it has not, at first, any relation to the form of the function production (or marginal productivity of capital) and the behavior of workers.

⁵ The problem of the “razor's edge,” developed by Harrod (1939) and Domar (1946) says that full employment of the factors is only possible if the workers growth rate is equal to the ratio of the propensity to save of the economy and the technological coefficient where the technological coefficient is the capital/product ratio.

Two important issues related to the macroeconomic model of Kaldor–Pasinetti concern, first, to the implications of the introduction of the government on determining the long-term interest rate (Cambridge equation) and personal income distribution and, second, the compatibility of macro model results (with and without government) with the Orthodox microfoundations. To deal with the first problem, [Steedman \(1973\)](#), [Dalziel \(1989\)](#) and [Pasinetti \(1989a,b\)](#) insert the government activity in the Kaldor–Pasinetti model assuming that the government collects direct and indirect taxes, and makes transfers to the workers. In opposition to [Steedman \(1973\)](#), [Dalziel \(1989\)](#) and [Pasinetti \(1989a,b\)](#) admit that the government's savings is not necessarily equal to zero. The results found by [Steedman \(1973\)](#) and [Pasinetti \(1989a,b\)](#) show that both the long-term interest rate of the economy, determined by the Cambridge equation, as the share of income profits come to depend positively on tax profits (t_p). To [Dalziel \(1989\)](#), the government's insertion does not change the interest rate nor does it affect the functional income distribution. Although there is a disagreement over the effects of the taxation of the profits, the fundamental results of the Kaldor–Pasinetti model are kept in all these works, that is, the interest rate and income distribution depend fundamentally on the natural growth rate (g_n) and the propensity to save of the capitalists (s_c).

Regarding the second issue, [Baranzini \(1991\)](#) proposed a microfoundation of the model of growth and income distribution of Kaldor–Pasinetti from the use of utility functions of two representative agents, one a capitalist and the other a worker. In this model, [Baranzini \(1991\)](#) uses economies where individuals save for the life cycle and may save for their descendants (the next generation). Thus, by capitalists and workers maximizing intergenerational utility functions with life cycle, [Baranzini \(1991\)](#) comes to a series of new equilibrium results. Particularly in the discrete version of the model, the equilibrium capital ratio belonging to each class is obtained and it is possible to calculate the total capital stock of the economy. In addition, the propensity to save of the capitalists and workers are no longer exogenous as in the Kaldor–Pasinetti model and they begin to depend on the parameters of the individuals' preferences. An important result is the attainment of the equilibrium interest rate, as well as in the original models of Kaldor–Pasinetti, independently of any technology represented by a production function.

[Teixeira et al. \(2002\)](#) try to show that the model of Kaldor–Pasinetti with government can be supported by Orthodox microfoundations. To do so, they analyze a model of overlapping generations with heterogeneous agents of [Baranzini \(1991\)](#), allowing the government to tax the inheritance at the beginning of each capitalist generation and make transfers to the workers. As in [Steedman \(1973\)](#), the authors assume that the government maintains the budget balanced in each instant of time, so that the revenue from the taxation of the capitalist's inheritance is fully passed on to the worker. Two results of this study should be highlighted:

- (i) as in the macro model of Kaldor–Pasinetti, the interest rate that keeps the economy on a balanced growing trend (steady state) depends positively on the natural rate of the output growth and the taxation, and does not depend on the behavior of the working class and/or the technology;
- (ii) the taxation of the capitalist's inheritance decreases its share in the capital stock of the economy and therefore, changes the income distribution in favor of the worker.

This work follows the line of the [Teixeira et al. \(2002\)](#) since it pursues to conceive orthodox microfoundations to the macroeconomic model of Kaldor–Pasinetti with government. This work aims to extend the overlapping generations with heterogeneous agents and government model to allow both classes (capitalist and worker) to keep a positive intergenerational stock and discusses the conditions of existence of the two classes in equilibrium. Additionally, it checks the impact of taxation on the equilibrium interest rate and the distribution of wealth between the classes.

To achieve the outlined objectives, this work is divided into three sections, besides this introduction. Section 2 presents the overlapping generations with government basic model ([Teixeira et al., 2002](#)). In Section 3, the hypotheses and the main results of these basic model extensions are discussed. The conclusions of the study are presented in Section 4.

2. A model of overlapping generations with heterogeneous agents and government

In this section, the structure of a base model with government transfers outlined in the article of the [Teixeira et al. \(2002\)](#) is presented. The government is supposed to interfere in the economy only using a direct tax on the inheritance at the beginning of each capitalist generation, which is fully passed on to the workers. There is no kind of cost for the existence of this transfer.

The main assumptions of the model are:

- (i) *demographic*: individuals live only two periods of equal intervals and there is no uncertainty about the date of retirement at the end of the first period, and the date of death at the end of the second period. At the end of the first period, each person has $1 + g$ children;
- (ii) *income*: both workers and capitalists are born at the end of period $t - 1$. The capitalists will receive an amount of B_{t-1} as heritage immediately at birth and therefore, they will have a yield equal to rB_{t-1} (where r is the interest rate). The workers will receive a salary equal to W_t in the first period of their lives. To simplify, it is also assumed that retirees do not receive pensions.
- (iii) *plans of consumption and accumulation*: capitalists and workers save in order to maximize the present value of its consumption C_t and C_{t+1} . The capitalists add to their utility function what they leave in inheritance to their children. The capital stock of the economy is formed of inter-generational inheritance of the capitalists and the life cycle savings of workers and capitalists.
- (iv) *utility function*: the shape of the utility function is: $V(c_t) = (1/a)(C_t)^a$, where the elasticity “ a ” is a positive constant. There is also, as it will be seen, a utility discount rate (b) to the inheritances of the capitalists.

The capitalist will maximize its utility function taking into account their preferred intertemporal consumption, as well as their willingness to leave legacy to their descendants. However, these preferences will be subject now to a new budget constraint. The capitalists have to solve the following maximization problem:

$$\begin{aligned} \text{Max } V(C_t^c, C_{t+1}^c, B_t) &= \text{Max} \frac{1}{a} \left[(C_t^c)^a + \frac{(C_{t+1}^c)^a}{1 + \sigma} + \frac{1 + g}{1 + b} B_t^a \right] \\ \text{subject to : } (1 - t_b)(1 + r)B_{t-1} &= C_t^c + \frac{C_{t+1}^c}{1 + r} + (1 + g)B_t. \end{aligned} \quad (2.1)$$

where t_b is the taxation of inheritance at the beginning of every capitalist generation and σ is a coefficient of time preference for consumption. It is assumed that $0 < t_b < 1$.

The workers are also facing a problem of intertemporal maximization:

$$\begin{aligned} \text{Max } V(C_t^w, C_{t+1}^w) &= \text{Max} \frac{1}{a} \left[(C_t^w)^a + \frac{(C_{t+1}^w)^a}{1 + \sigma} \right] \\ \text{subject to : } t_b(1 + r)B_{t-1} + W_t &= C_t^w + \frac{C_{t+1}^w}{1 + r} \end{aligned} \quad (2.2)$$

where $t_b(1 + r)B_{t-1}$ is the government transfer supposed to be equal to the total tax collected on inheritance.

The first order conditions of optimization problems imply in:

$$C_{t+1}^{w,c} = C_t^{w,c}(1 + \sigma)^{1/(a-1)}(1 + r)^{1/(1-a)} \quad (2.3)$$

$$C_t^c = (1 + b)^{1/(1-a)} B_t \quad \text{or} \quad B_t = (1 + b)^{1/(a-1)} C_t^c \quad (2.4)$$

$$C_{t+1}^c = (1 + b)^{1/(1-a)} B_t(1 + r)^{1/(1-a)}(1 + \sigma)^{1/(a-1)}. \quad (2.5)$$

Using (2.4) and replacing C_{t+1}^c in the restriction (2.1):

$$(1 - t_b)(1 + r)B_{t-1} = C_t^c[1 + (1 + r)^{a/(1-a)}(1 + \sigma)^{1/(a-1)} + (1 + g)(1 + b)^{1/(a-1)}]. \quad (2.6)$$

Using (2.3), we may replace C_{t+1}^w in the restriction of the problem (2.2), obtaining:

$$W_t + t_b(1 + r)B_{t-1} = C_t^w[1 + (1 + r)^{a/(1-a)}(1 + \sigma)^{1/(a-1)}]. \quad (2.7)$$

From relations (2.4) and (2.6),⁶ assuming that there is no technological progress and therefore, the per capita growth rate of the capital stock in steady state is zero, and assuming $a = 0$, it follows that:

$$r^* = \frac{1+b}{1-t_b} + \frac{1+b}{(1+\sigma)(1-t_b)} + \frac{g}{1-t_b} + \frac{t_b}{1-t_b} \quad (2.8)$$

which is the equilibrium interest rate from which the utility of the capitalists is maximized and, at the same time, it ensures a balanced growth of the capital stock of this class. From (2.8), it is verified that the equilibrium interest rate does not depend on a specific form of the production function nor on the value of the capital-labor ratio. In this sense, the outcome of the Kaldor–Pasinetti Cambridge theorem is supported by this model with orthodox microfoundations.

The relationship between the equilibrium interest rate and the tax rate is convex, as may be seen in Fig. 1, shown in Appendix. Another relevant aspect about the equilibrium interest rate concerns the influence of the parameters. The elevation of the discount rate for inheritance causes an increase in the rate of interest higher than the increase in growth rate of the population, assuming same variation for both parameters. This indicates that the reduction of the importance given by the agent to the endowment left as inheritance influences significantly more the interest rate than the increase of the population growth.

The impact of taxation on the equilibrium interest rate is given by:

$$\frac{\partial r^*}{\partial t_b} = \frac{1+b}{(1-t_b)^2} + \frac{1+b}{(1+a)(1-t_b)} + \frac{g}{(1-t_b)^2} + \frac{1}{(1-t_b)^2} > 0 \quad (2.9)$$

This result holds symmetry with Steedman's (1973) and Pasinetti's (1989a,b), in that the equilibrium interest rate depends positively on the taxation t_b .

In equilibrium, the capital stock of the economy at time t is given by:

$$K_t = B_{t-1} + S_t^c + S_t^w. \quad (2.10)$$

Assuming that the number of workers and capitalists (who are in the first period of their lives) is equal to 1 and using (2.4), it follows in equilibrium:

$$S_t^c = B_{t-1}[(1-t_b)r^* - 1 - b] \quad (2.11)$$

From (2.7), we obtain:

$$S_t^w = K_t^w = \frac{1}{2+\sigma} [W_t + t_b(1+r^*)B_{t-1}] \quad (2.12)$$

where $W = Y - P$ and assuming that in the long term $P/K = r^*$, we have:

$$K_t^w = K \left(\frac{Y}{K} - r^* \right) (2+\sigma)^{-1} + [t_b(1+r^*)B_{t-1}](2+\sigma)^{-1} \quad (2.13)$$

Inserting (2.11) and (2.13) in (2.10), it is verified that the total capital in equilibrium will be:

$$K^* = \frac{(2+\sigma)(1-t_b)r^* - (2+\sigma)b + t_b(1+r^*)}{(2+\sigma) - \left(\frac{Y}{K} - r^*\right)} B^* \quad (2.14)$$

From (2.14) we can get a sense of the importance of inheritance, obtaining the same aspect ratio on the total capital:

$$\left(\frac{B}{K} \right)^* = \frac{(2+\sigma) - \left(\frac{Y}{K} - r^*\right)}{(2+\sigma)(r^* - t_b r^* - b) + t_b(1+r^*)} > 0 \quad (2.15)$$

Admitting that $r^*(1-t_b) > b$ and $(Y/K - r^*) < 1$. Differentiating $(B/K)^*$ with respect to t_b :

$$\frac{\partial(B/K)^*}{\partial t_b} = \frac{-1 + (1+\sigma)r^*}{[(2+\sigma)(r^* - t_b r^* - b) + t_b(1+r^*)]^2} < 0 \quad (2.16)$$

This result is not surprising, because it is expected that if the appearance of a tax decreases $(B/K)^*$, marginal increases in it also exert negative influence on B/K .

⁶ Reminding that, in equilibrium, $B_{t-1} = B_t = B^*$.

The effect of government transfers on the distribution of the capital of the capitalist class in relation to the total capital of the economy will be verified now. From (2.13):

$$\left(\frac{K_c}{K}\right)^* = 1 + \frac{r^* - \frac{Y}{K}}{2 + \sigma} - \frac{t_b(1 + r^*)(2 + \sigma + r^* - \frac{Y}{K})}{(2 + \sigma)[(2 + \sigma)(r^* - t_b r^* - b) + t_b(1 + r^*)]}. \quad (2.17)$$

Differentiating $(K_c/K)^*$ with respect to t_b :

$$\begin{aligned} \frac{\partial \left(\frac{K_c}{K}\right)}{\partial t_b} = & - \frac{\left[\left(1 + r^* + \frac{\partial r^*}{\partial t_b}\right)(2 + \sigma + r^* - \frac{Y}{K}) + (1 + r^*)\frac{\partial r^*}{\partial t_b}\right] \{(2 + \sigma)[(2 + \sigma)(r^* - t_b r^* - b) + (1 + r^*)t_b]\}}{\{(2 + \sigma)[(2 + \sigma)(r^* - t_b r^* - b) + t_b(1 + r^*)]\}^2} \\ & \times \frac{-\left\{(2 + \sigma)\left[(2 + \sigma)\left(\frac{\partial r^*}{\partial t_b}(1 - t_b) - r^*\right) + 1 + t_b \frac{\partial r^*}{\partial t_b} + r^*\right]\right\} \{t_b(1 + r^*)(2 + \sigma + r^* - \frac{Y}{K})\}}{\{(2 + \sigma)[(2 + \sigma)(r^* - t_b r^* - b) + t_b(1 + r^*)]\}^2} < 0 \end{aligned} \quad (2.18)$$

which translates into a lower portion of capital for the capitalists in relation to the total capital of the economy. Thus, we demonstrate that taxation and consequently income transfer to the working class affects the distribution of wealth between the classes because it reduces the share of the capitalist class in the total capital stock of the economy.

3. The model with government transfers and both classes ensuring intergenerational stock capital

If the workers will leave inheritance at the end of period t , then the maximization problem (2.2) will need to incorporate the reason inheritance b_w , the inheritance received at the end of period $t - 1$ (B_{t-1}^w) and allowed to t (B_t^w).

The maximization problem of the worker is now:

$$V(C_t^w, C_{t+1}^w, B_t^w) = \text{Max} \frac{1}{a} \left[(C_t^w)^a + \frac{(C_{t+1}^w)^a}{1 + \sigma} + \frac{1 + g}{1 + b_w} (B_t^w)^a \right] \quad (3.1)$$

$$\text{subject : } t_b(1 + r)B_{t-1}^c + W_t + (1 + r)B_{t-1}^w = C_t^w + \frac{C_{t+1}^w}{1 + r} + (1 + g)B_t^w$$

where $t_b(1 + r)B_{t-1}^c$ is the government transfers equal to the tax collected on the inheritance.

From the first order conditions, we have:

$$C_{t+1}^w = C_t^w(1 + r)^{1/(1-a)}(1 + \sigma)^{1/(a-1)} \quad (3.2)$$

$$B_t^w = (1 + b_w)^{1/(a-1)} C_t^w \quad (3.3)$$

Leaving the right side of the budget constraint of the workers due to C_t^w and parameters:

$$t_b(1 + r)B_{t-1}^c + W_t + (1 + r)B_{t-1}^w = C_t^w \left[1 + (1 + r)^{a/(1-a)}(1 + \sigma)^{1/(a-1)} + (1 + g)(1 + b_w)^{1/(a-1)} \right] \quad (3.4)$$

Inserting (3.3) in (3.4):

$$t_b(1 + r)B_{t-1}^c + W_t + (1 + r)B_{t-1}^w = B_t^w(1 + b_w)^{1/(1-a)} \left[1 + (1 + r)^{a/(1-a)}(1 + \sigma)^{1/(a-1)} + (1 + g)(1 + b_w)^{1/(a-1)} \right]$$

Assuming the absence of technological progress, it follows that at the equilibrium $\dot{k}_w = (\dot{k}_c = \dot{k}) = 0$, implying that:

$$B_{t-1}^w = B_t^w = B^* \quad (3.5)$$

Knowing that $W = Y - r$, supposing $a = 0$ and using (3.4), it follows that the equilibrium interest rate is given by:

$$r_w^* = \frac{\frac{Y}{K} + t_b \frac{B_{t-1}^c}{K} - \frac{B^w}{K} \left[g + (1 + b_w) \frac{2 + \sigma}{1 + \sigma} \right]}{1 - \frac{B^w}{K} - t_b \frac{B_{t-1}^c}{K}} \quad (3.6)$$

Notice that the interest rate arising from the behavior of the capitalists is given by:

$$r_c^* = \frac{1+b_c}{1-t_b} + \frac{1+b_c}{(1+\sigma)(1-t_b)} + \frac{g}{1-t_b} + \frac{t_b}{1-t_b}. \quad (3.7)$$

Eqs. (3.6) and (3.7) indicate that interest rates that maximize the plans of consumption and savings of the working and the capitalist classes cannot be equal. This means that one of the classes can require a different rate of accumulation than the other, causing the class with the lowest rate to disappear from the model. In this case, it would go back to the neoclassical model with only one type of representative agent.

Differentiating r_w^* and r_c^* with respect to t_b , we have:

$$\frac{\partial r_w^*}{\partial t_b} = \frac{\frac{B_{t-1}^c}{K} \left(1 - \frac{B^w}{K} - t_b \frac{B_{t-1}^c}{K}\right) + \frac{B_{t-1}^c}{K} \left\{ \frac{Y}{K} + t_b \frac{B_{t-1}^c}{K} - \frac{B^w}{K} \left[g + (1+b_w) \frac{(2+\sigma)}{(1+\sigma)} \right] \right\}}{\left(1 - \frac{B^w}{K} - t_b \frac{B_{t-1}^c}{K}\right)^2} > 0 \quad (3.8)$$

$$\frac{\partial r_c^*}{\partial t_b} = \frac{(1+b_c)}{(1+t_b)^2} + \frac{(1+\sigma)(1+b_c)}{[(1+\sigma)(1-t_b)]^2} + \frac{g}{(1-t_b)^2} + \frac{1}{(1-t_b)^2} > 0 \quad (3.9)$$

The relationships (3.8) and (3.9) show an interesting result compared to the model [Steedman \(1973\)](#). As stated earlier in this paper, by using the same variables of the original models of Kaldor–Pasinetti, it turns out that the introduction of a direct tax on the profit mass will cause the profit rate of the economy (in the long run, supposedly equal to the equilibrium interest rate) to increase its value. The results, now with microfoundations, indicate that in a situation where the capitalist and working classes have their consumption/savings plans with optimized lifecycle and inheritance, they should necessarily follow a relationship of equilibrium, in which the interest rate is positively related to the interest rate.

[Steedman \(1973\)](#) also showed that net interest rate of the taxes is exactly equal to the result of [Pasinetti \(1962\)](#). In this model, it can be observed from Eq. (3.7), that:

$$\frac{r^*}{1-t_b} - t = g + (1+b_t) \frac{(2+\sigma)}{(1+\sigma)}. \quad (3.10)$$

The right side of (3.10) is exactly equal to the interest rate in the model without taxation, found by [Baranzini \(1991\)](#).

A solution that can ensure an equilibrium trajectory with positive capital stock for both classes can be a structure where there is only one equilibrium interest rate in the economy, but with inheritance rates necessarily different ($b_c \neq b_w$).

Making $r_w^* = r_c^* = r^*$, it follows that:

$$b_c = -b_w \frac{(1-t_b) \frac{B^w}{K}}{1 - \frac{B^w}{K} - t_b \frac{B_{t-1}^c}{K}} + \frac{\left[\frac{Y}{K} + t_b \frac{B_{t-1}^c}{K} - g \frac{B^w}{K} \right] \frac{(1+\sigma)(1-t_b)}{(2+\sigma)} - (1-t_b) \frac{B^w}{K}}{1 - \frac{B^w}{K} - t_b \frac{B_{t-1}^c}{K}} - \left[(g+t_b) \frac{(1+\sigma)}{(2+\sigma)} \right] - 1 \quad (3.11)$$

The relation (3.11) yields the value of b_c , the discount rate of the capitalist inheritance, in which the two interest rates are equal. As it can be seen, this value b_c is a function of all model parameters (the population growth rate g , the consumption discount rate σ , the product/capital ratio), the taxation t_b , the participation of intergenerational inheritance of the workers in the total capital stock (B^w/K), the discount rate of the workers' inheritance b_w and the ratio B_{t-1}^c/K .

By performing a simulation exercise based on the values contained in [Baranzini \(1991\)](#), it is noticed that the condition for equal rate between the interest rate is unlikely to be met. By assuming a population growth rate of 0.01, $b_c = 0.042$, $b_w = 0.08$, $\sigma = 0.11$, $t_b = 0.2$ and $Y/K = 0.3$, the relation between the share of the intergenerational capital of the working class and the share of intergenerational capital of the capitalists in the total capital ($t-1$) is shown as inappropriate.⁷ In this situation, the share of intergenerational capital of the working class should be above the unit for any value of B^c/K . When b_c is raised to 0.01, it is required that B^w/K is raised even more, so the equality between the

⁷ Fig. 2 presented in [Appendix](#).

interest rate can be checked. It was verified that the relationship between B^w/K and B^c/K in $(t-1)$ assumes a plausible behavior when b_w and b_c gain values between 0.4 and 0.6 with $b_w < b_c$. In this case, shown in Fig. 2(C) presented in Appendix, the correlation between B^w/K and B^c_{t-1}/K is negative, i.e., when the intergenerational capital stock of the capitalist class is low on $(t-1)$, a considerable part of the total capital of the economy should be represented by the accumulation performed by the workers in order to bequeath capital.

Another possible scenario is the acceptance of high values for b_c and b_w . Taking $b_c = 1$ and $b_w = 5$, it is observed that the relation between B^w/K and B^c_{t-1}/K becomes positive, although B^w/K shows low values for any value of B^c_{t-1}/K .

The simulations indicated that the equality between the equilibrium interest rates of the worker and the capitalist accepts low values, below the unit, to b_w and b_c only when $b_w < b_c$. However, it is known that such a condition leads the economy to dual equilibrium configuration, or anti-Pasinetti (Samuelson and Modigliani, 1966). In contrast, it was observed that $b_c < b_w$ is compatible with the equality between the interest rates when the parameters assume high values, indicating unwillingness of the agents to transfer capital to the next generation. Thus, we conclude that equality between the equilibrium interest rate in both classes may be considered unlikely. From the relation between the inheritance given by (3.11), one can get the share of intergenerational capital belonging to workers (and therefore, to the capitalists) isolating B^w/K :

$$\left(\frac{B^w}{K}\right)^* = \frac{\{[(1+g)(1+\sigma)(2+\sigma)^{-1} + (1+b_c)]t_b \frac{B^c_{t-1}}{K} + [(1-t_b)\frac{Y}{K} - (g+t_b)]\frac{(1+\sigma)}{(2+\sigma)} - (1+b_c)\}}{b_w(1-t_b) - [b_c + (1 + (1+g)\frac{(1+\sigma)}{(2+\sigma))}t_b]}. \quad (3.12)$$

If it is assumed that $b_w(1-t_b) > [b_c + (1 + (1+g)(1+\sigma)(2+\sigma)^{-1})t_b]$, then the workers will maintain a positive participation of the intergenerational inheritance in the total capital stock in the economy only when:

$$b_c < \frac{[(1+g)\frac{(1+\sigma)}{(2+\sigma)} + 1]t_b \frac{B^c_{t-1}}{K} + [(1-t_b)\frac{Y}{K} - (g+t_b)]\frac{(1+\sigma)}{(2+\sigma)} - 1}{1 - t_b \frac{B^c_{t-1}}{K}} = A^* \quad (3.13')$$

$$b_w > \frac{A^* \left(1 - t_b \frac{B^c_{t-1}}{K}\right) + [1 + (1+g)\frac{(1+\sigma)}{(2+\sigma)}]t_b + t_b b_c \frac{B^c_{t-1}}{K}}{1 - t_b} \quad (3.13'')$$

The conditions (3.13') and (3.13'') show that the existence of two classes in the model requires an even greater difference between the motives inheritance than in the model without government.⁸ This is due to the fact that the transfer of the capitalists to the workers imposes a greater effort to the capitalist class, who must now replace the inheritance confiscated by the government. It can be expected, therefore, that in an economy in which the desire of the capitalists in transmitting inheritance is not so high, the introduction of transfers causes this class to disappear from the system.

Differentiating $(B/K)^*$ with respect to t_b :

$$\begin{aligned} \frac{\partial(B^w/K)^*}{\partial t_b} &= \frac{\left\{[(1+g)(1+\sigma)(2+\sigma)^{-1} + (1+b_c)]\frac{B^c_{t-1}}{K} - \left[\frac{Y}{K} + 1\right]\frac{(1+\sigma)}{(2+\sigma)}\right\}\{b_w(1-t_b) - [b_c + (1 + (1+g)\frac{(1+\sigma)}{(2+\sigma))}t_b]\}}{\left\{b_w(1-t_b) - [b_c + (1 + (1+g)\frac{(1+\sigma)}{(2+\sigma))}t_b]\right\}^2} \\ &+ \frac{\left\{[(1+g)(1+\sigma)(2+\sigma)^{-1} + (1+b_c)]t_b \frac{B^c_{t-1}}{K} + [(1-t_b)\frac{Y}{K} - (g+t_b)]\frac{(1+\sigma)}{(2+\sigma)} - (1+b_c)\right\}\{b_w + (1 + (1+g)\frac{(1+\sigma)}{(2+\sigma))}\}}{\left\{b_w(1-t_b) - [b_c + (1 + (1+g)\frac{(1+\sigma)}{(2+\sigma))}t_b]\right\}^2} > 0 \end{aligned} \quad (3.14)$$

assuming that $b_w(1-t_b) > [b_c + (1 + (1+g)(1+\sigma)(2+\sigma)^{-1})t_b]$. Thus, it can be observed that an increase in taxation t_b and consequently, in the transfers to the working class raises the participation of intergenerational inheritance of this class in the total capital stock of the economy.

⁸ In the model without government transfers presented by Baranzini (1991), the condition of existence for the two classes is: $b_c < ((Y/K) - g)(1+\sigma)/(2+\sigma) - 1 = A^* < b_w$.

In order to analyze the importance of the capital stock of the capitalist and under what circumstances it is positive in equilibrium, the value of the share of the capital stock will derive of the other class in the capital stock of the economy. From Eqs. (3.2)–(3.4), the capital stock of the working class can be defined as the sum of its intergenerational assets over the assets of the life cycle, i.e.:

$$\begin{aligned} K_t^* &= B^w + S^w = B^w + t_b(1+r^*)B_{t-1}^c + r^*B^w + W - C^w \\ &= [(1+r^*)B^w + t_b(1+r^*)B_{t-1}^c + (Y/K) - r^*] \frac{1+b_w+(1+g)(1+\sigma)}{(1+b_w)(2+\sigma)+(1+g)(1+\sigma)} \end{aligned} \quad (3.15)$$

where r^* is given by (3.7). Thus, in equilibrium, we have:

$$\left(\frac{K^w}{K}\right)^* = \left[(1+r^*)\left(\frac{B^w}{K}\right)^* + t_b(1+r^*)\frac{B_{t-1}^c}{K} + \frac{Y}{K} - r^*\right] \frac{(1+b_w)+(1+g)(1+\sigma)}{(1+b_w)(2+\sigma)+(1+g)(1+\sigma)} \quad (3.16)$$

where $(B^w/K)^*$ is given by (3.12). Thus, the capital share of the capitalist (K^c/K) will be positive, if the right side of (3.13) is smaller than 1. This will occur when:

$$b_c > \frac{\left\{ \left[(1+g)\frac{(1+\sigma)}{(2+\sigma)} + 1 \right] t_b \frac{B_{t-1}^c}{K} + \left[(1-t_b)\frac{Y}{K} - (g+t_b) \right] \frac{(1+\sigma)}{(2+\sigma)} - 1 \right\} \left[\frac{(1+b_w)}{(1+\sigma)} + (1+g) \right] - b_w(1+t_b) + \left[1 + (1+g)\frac{(1+\sigma)}{(2+\sigma)} \right] t_b}{\left(1 - t_b \frac{B_{t-1}^c}{K} \right) \left[\frac{(1+b_w)}{(1+\sigma)} + (1+g) \right] - 1} \quad (3.17)$$

So when (3.17) is satisfied, there is $K^c/K > 0$ and the capitalist exists in equilibrium with a positive share of the total intergenerational inheritance (this is because the sine qua non condition necessary for the existence of the capitalist in the model is that B^c is a positive value).

Finally, the effect of taxation on the distribution of capital on the two classes, including both intergenerational assets and the life cycle will be verified. From (3.16) it follows that:

$$\begin{aligned} \frac{\partial \left(\frac{K^w}{K}\right)^*}{\partial t_b} &= \left\{ \frac{\partial \left(\frac{B^w}{K}\right)^*}{\partial t_b} r^* + (1+r^*)\frac{B_{t-1}^c}{K} - \left[\left(1 - t_b \frac{B_{t-1}^c}{K} - \left(\frac{B^w}{K}\right)^* \right) \frac{\partial r^*}{\partial t_b} \right] \right\} \\ &\quad \frac{(1+b_w)+(1+g)(1+\sigma)}{(1+b_w)(2+\sigma)+(1+g)(1+\sigma)} > 0 \end{aligned} \quad (3.18)$$

Similarly to the simpler case studied in Section 2, it turns out again that taxation affects the distribution of wealth between the classes since it increases the participation of the working class in the total capital stock of the economy.

4. Conclusions

This work shows a number of interesting results. It was found that in the model with government and inheritance only for capitalists, the net interest rate is equal to the model without government. With the hypothesis inheritance for both classes, the optimal interest rate for the workers will not necessarily be equal to the capitalist class. However, one of the possible requirements for this model to continue with heterogeneous subjects is the maintenance of a single interest rate. In this case, it is also necessary that there is a strong willingness of the capitalists to leave assets for their descendants.

It is worth noting that the savings/consumption plan of the working class is also essential in this model. Whether the provision of this class in leaving inheritance to their descendants is very high, i.e., if it is too low so that the relation (3.13'') is no longer respected, there would be, in some way, the return of the dual problem, in that the workers' frugality behavior leads to the disappearance of the capitalist class.

Other results found were:

- (i) an increase in taxes and consequently transfers to the working class raises the participation of intergenerational inheritance of this class in the total capital stock;

- (ii) the taxation affects the distribution of wealth between the classes because it increases the participation of the working class in the total capital stock of the economy.

It is highlighted that this work aims to contribute to the post-Keynesian and neoclassical literature, as it incorporates representative agents in heterogeneous classes, showing new results for both schools. A possible extension of this work is to check the effects of taxation on inheritances with the hypothesis of a single rate of inheritance and two possible equilibrium interest rates. This hypothesis can be interesting because it assumes that the preferences of the capitalists and the workers are under parameters of equal values, but getting differentiated remuneration for its capital stocks. This result may illustrate economies with imperfect capital market.

Appendix.

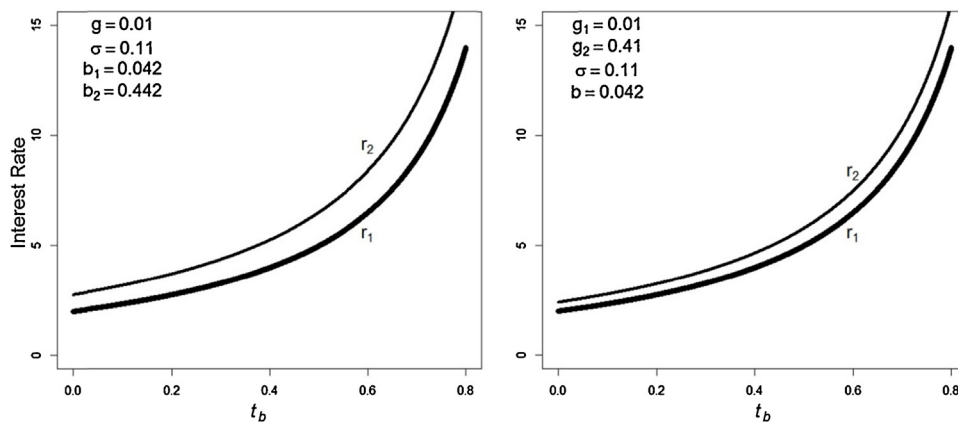


Fig. 1. Equilibrium interest rate depending on the tax rate.

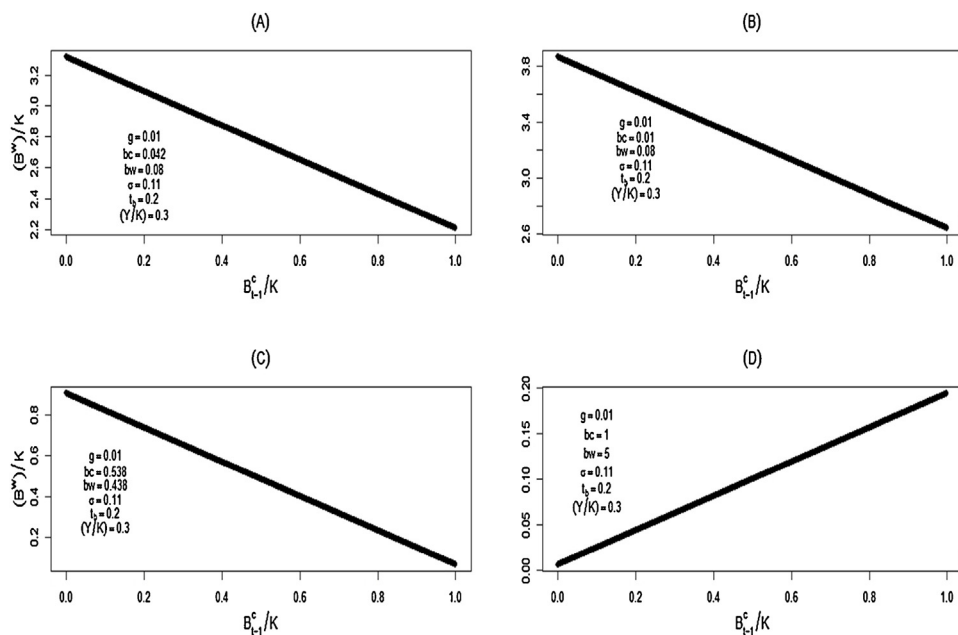


Fig. 2. Condition for $r_w^* = r_c^* = r^*$.

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