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Article

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Bubble, weak and strong hyperinflation: Theory and empirical evidence

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Abstract

This paper presents a theoretical framework that allows a taxonomy of hyperinflation, namely: (i) bubble, (ii) weak and (iii) strong hyperinflation. The inflation tax revenue curve is used to characterize each type of hyperinflation and we use this curve to test them. The bubble and strong hyperinflation hypotheses are rejected using Brazilian data. The weak hyperinflation hypothesis is not rejected and the economy could have been on the ‘wrong’ side of the Laffer curve during hyperinflation. This outcome, contrary to conventional wisdom, is predicted by this hypothesis, which presents a solution to an old puzzle of the hyperinflation literature.

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Keywords: Hyperinflation; Fiscal crisis; Intertemporal budget constraint; Inflation tax revenue

Resumo

Este trabalho apresenta um arcabouço teórico que permite uma taxonomia da hiperinflação. Ela é classificada em três tipos: (i) bolha; (ii) fraca; e (iii) forte. A curva do imposto inflacionário é usada para identificar cada tipo e para testá-las. As hipóteses de hiperinflação forte e de bolha são rejeitadas com dados da economia brasileira. A hipótese de hiperinflação fraca não é rejeitada e a economia poderia estar no “lado errado” da curva de Laffer. Esta ocorrência, uma possibilidade da hipótese de hiperinflação fraca, soluciona um quebra-cabeça antigo da literatura de hiperinflação.

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Palavras-chave: Hiperinflação; Crise fiscal; Restrição orçamentária intertemporal; Curva do imposto inflacionário

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1. Introduction

The monthly (continuous) inflation rate that maximizes the inflation tax revenue varies widely (from 18.3% to 143%), according to semi-elasticity estimates for the German hyperinflation made by several authors (Cagan, 1956; Barro, 1970; Frenkel, 1977; Sargent, 1977; Goodfriend, 1982; Burmeister and Wall, 1987; Christiano, 1987; Casella, 1989; Taylor, 1991; Engsted, 1993; Imrohoroglu, 1993; Michael et al., 1994). Those estimates lead one to conclude that during hyperinflation the German government could have obtained more tax revenue with lower inflation rates. This is an old puzzle of the hyperinflation literature raised by Cagan's (1956) seminal paper, namely: that only an economy with an irrational Government could operate on the 'wrong' side of the Laffer curve, since the Government could collect more tax with lower inflation rates. The weak hyperinflation hypothesis presented in this paper is consistent with the economy being on the 'wrong' side of the Laffer curve, for some time, during hyperinflation, even when agents are rational. Indeed, this "puzzling" outcome is predicted by the weak hyperinflation hypothesis.

In contrast to the traditional theories of hyperinflation, e.g. Sargent and Wallace (1987) and Bruno and Fischer (1990) – which assume a constant fiscal deficit – the driving force to cause hyperinflation in our framework is an increasing fiscal deficit being financed by money (see Barbosa et al., 2006). We understand that this assumption is crucial from an empirical viewpoint since, to our best knowledge, non-constant, increasing fiscal deficits have been ubiquitous in actual hyperinflation episodes.¹ The increasing deficit reaches a point where the intertemporal government budget constraint is not sustainable anymore. This point characterizes the start of the hyperinflation.² The process lasts at most a time span before the fiscal policy collapses.

In this intertemporal framework the particular hyperinflation path depends on the functional form of the inflation tax revenue curve, which is a key tool to discriminate among different hypotheses. There are three possibilities. If inflation tax revenue increases when the rate of inflation increases, the real quantity of money tends to zero as time approaches the end of hyperinflation. We call this a strong hyperinflation. When the inflation tax revenue ends up decreasing as the rate of inflation increases, the real quantity of money is still positive at the end of the hyperinflation. We name this a weak hyperinflation. Finally, hyperinflation can also be caused by a bubble, a possibility that can only occur if the inflation tax revenue increases as the rate of inflation increases.

Besides the theoretical framework, this paper departs from other papers in the literature in several ways. First, it follows a different empirical strategy and tests hyperinflation hypotheses estimating the inflation tax revenue curve (inflation tax curve, for short) directly, which can be used to discriminate among different hypotheses. Second, the inflation tax curve functional form used encompasses several specifications as particular cases, making inference more reliable. Thus, this approach allows one to test whether or not the demand for money specification used by Cagan is appropriate. Third, the inflation tax revenue data refer to the 1947–2003 period.³ This period includes the Brazilian hyperinflation that lasted a very long period, starting in the first half of the 1980s and ended in 1994, with the Real Plan. Therefore, in contrast to other empirical studies, which use very small samples covering only hyperinflation periods, the sample here covers almost half a century, in which both inflation and the inflation tax revenue showed great variability.⁴

The empirical evidence shows that the hypothesis that inflation tax revenue is an increasing function of the inflation rate is rejected for Brazil. Thus, both the bubble and the strong hyperinflation hypotheses are rejected, while the weak hyperinflation hypothesis is not. This result supports the claim that hyperinflation was caused by fundamentals – via increasing deficits – in Brazil, as well as in many other countries, rather than by a spontaneous combustion as in a bubble, the workhorse hypothesis widely used in the literature. Moreover, the empirical evidence for a weak-type

¹ See, for example, Cukierman (1988, pp. 20 and 47) and Morales (1988, p. 313) for the German and Bolivian experiences, respectively. Morales states that "The public sector deficits were increasingly financed by the central bank by the building up of arrears on the public external and internal debt." For Argentina, Machinea and Fanelli (1988, p. 111) state that "... the fiscal deficit never fell below 5% of GDP, and in some cases it even exceeded 15% of GDP...".

² In our framework the constant deficit enters as a particular case. This framework has also the advantage of providing a technical and precise definition of hyperinflation (see Barbosa et al., 2006), in contrast to the usual *ad hoc* definition, due to Cagan, which considers hyperinflation monthly inflation rates above 50%.

³ The annual inflation tax revenue data comes from Cysne and Lisboa (2004).

⁴ This could matter empirically, since low degrees of freedom might be one important factor behind the wide range of empirical estimates shown above, given that German hyperinflation lasted only a few months.

hyperinflation in Brazil is consistent with the hypothesis that the Brazilian economy could have been on the ‘wrong’ side of the Laffer curve during hyperinflation.

The paper is organized as follows: Section 2 presents our hyperinflation model; Section 3 lays out a functional form for the inflation tax curve, which encompasses several specifications and presents graphical evidence on the link between the inflation rate and the inflation tax revenue for Brazil; Section 4 tests the hyperinflation model presented in this paper by estimating the inflation tax curve using cointegration techniques and Section 5 concludes.

2. Hyperinflation model

This section presents an extension of the rational expectations fiscal crisis model, due to [Barbosa et al. \(2006\)](#). The basic hypothesis of this as well as other hyperinflation models are that money finances the fiscal deficit. Thus, the government flow budget constraint is given by:

$$\frac{\dot{M}}{P} = f \quad (1)$$

A dot represents a time derivative. Money creation (\dot{M}) depends on the price level (P) and the fiscal deficit (f). The real seigniorage (\dot{M}/P) is equal to the fiscal deficit. Taking the time derivative of real cash balance ($m = M/P$) this flow budget constraint can be written as:

$$\dot{m} = f - \pi m = f - \tau(m)$$

where $\tau(m) = \pi m$, the inflation tax revenue, is a function of the real cash balance, the tax base. The inflation rate π is the tax rate. The flow budget constraint states that the change in tax base is equal to the difference between real seigniorage and the inflation tax revenue.

This flow budget constraint can be written as a government stock budget constraint by using standard procedure. First, multiply both sides of the flow constraint by $\exp(-rx)$, where r is the real rate of interest. Then, by taking into account that

$$\frac{d}{dx} m e^{-rx} = \dot{m} e^{-rx} - r m e^{-rx} = [\dot{m} - r m] e^{-rx}$$

we can write

$$d m e^{-rx} = [f - s(m)] e^{-rx} dx \quad (3)$$

where $s(m) = r m + \tau(m)$ is a measure of the value of the services provided by money, which is equivalent to the user cost, or rental price, of capital, and can be defined as the user cost of money. By integrating this expression from the present (t) to the future (T) we obtain the intertemporal budget constraint for the government:

$$m(t) = m(T) e^{-r(T-t)} + \int_t^T e^{-r(x-t)} [s(m) - f] dx \quad (4)$$

The real quantity of money at time t is equal to the present value of the real quantity of money at time T plus the present value of the surplus of the user cost of money over the public deficit. This surplus is similar to the primary fiscal surplus in the analysis of public debt sustainability. We use this analogy to name it the monetary surplus. Eq. (4) states that people are willing to hold money issued by the central bank as long as the value of the services provided by it is greater, in present value terms, than the resources transferred to the government to finance the public deficit.

This intertemporal budget constraint can be given an interpretation that is very useful to understand hyperinflation. Money to have value, in this economic policy regime, has to be backed up by a monetary surplus (in present value terms). Otherwise, the intertemporal budget would not be sustainable. This fact would generate a flight of money because people would get rid of it. Thus, hyperinflation occurs when the intertemporal budget becomes unsustainable and it is not related to the flow budget constraint, i.e. the current deficit, as assumed by previous theories of hyperinflation ([Sargent and Wallace, 1987](#); [Kiguel, 1989](#); [Marcet and Sargent, 1989](#); [Bruno and Fischer, 1990](#); [Marcet and Nicolini, 2003](#); [Sargent et al., 2009](#)). The sequence of current and future monetary surplus is the fundamental that yields hyperinflation in the fiscal crisis model.

The intertemporal government budget constraint is not sustainable when the size of the fiscal deficit to be financed by money increases in such a way that it would overtake, at some time in the future, the value of the services provided by money. The monetary surplus would become negative. The fiscal crisis assumes that the fiscal deficit to be financed by money increases through time:

$$f = f(t), \quad f' > 0, \quad f \leq g, \quad \lim_{t \rightarrow \infty} f(t) = \bar{f} > \bar{\tau} \quad (5)$$

The fiscal deficit can be at most equal to the level of government expenditures g and the last inequality says that as time passes, the fiscal deficit to be financed by money becomes larger than the maximum amount of inflation tax revenue that can be collected from society.

We have to take into consideration two cases that can happen in a fiscal crisis environment. In the first, the demand for money is inelastic with respect to the interest rate and the value of the services provided by money increases when the real quantity of money decreases: $s'(m) < 0$. If the hyperinflation lasts until the last moment, when the real quantity of money is equal to zero, the fiscal deficit is equal to $f(t_h)$.⁵ The intertemporal budget constraint at time t is given by:

$$m(t) = \int_t^{t_h} e^{-r(x-t)} [s(m) - f] dx$$

In this case the inflation rate is unbounded and the real quantity of money goes to zero. This corresponds to a strong hyperinflation.

In the second case, the demand for money is non-inelastic with respect to the interest rate and $s'(m) > 0$ as $m \rightarrow 0$. At the end of the hyperinflation the real quantity of money is still positive and the intertemporal budget constraint at time t is given by:

$$m(t) = m(t_h) e^{-r(t_h-t)} + \int_t^{t_h} e^{-r(x-t)} [s(m) - f] dx \quad (7)$$

We define this second case as a weak hyperinflation. Both types of hyperinflation can be easily understood by using phase diagrams of the differential equation for the real quantity of money, $\dot{m} = f(t) - \tau(m)$. These phase diagrams are different from the standard phase diagrams because in our model the differential equation is non-autonomous, i.e., it does depend on time.

We have two cases: (i) strong hyperinflation: $\lim_{m \rightarrow 0+} \tau(m) > 0$ and (ii) weak hyperinflation; $\lim_{m \rightarrow 0+} \tau(m) = 0$.⁶ Fig. 1 shows the strong hyperinflation case. The fiscal deficit starts at $f(0)$ and the real quantity of money is equal to $m(0)$. The fiscal deficit increases and in the last moment it reaches $f(t_h)$ when the real quantity of money is equal to zero. The hyperinflation path is HH , as indicated by the arrows that point out the fiscal deficit sliding up the inflation tax curve.

Fig. 2 shows the phase diagram of a weak hyperinflation since the rate of inflation has an upper limit. The fiscal deficit starts at $f(0)$ and the real quantity of money is equal to $m(0)$. The hyperinflation path is HH , showing the fiscal deficit sliding up and then down the Laffer curve.⁷ If the hyperinflation lasts until the last moment, when the real quantity of money is equal to $m(t_h)$ the fiscal deficit is equal to $f(t_h)$. In the hyperinflation experiences Cagan analysed inflation tax revenue decreased after inflation rate had exceeded the inflation tax maximizing inflation rate. The weak hyperinflation path is consistent with this fact, as can be seen in Fig. 2. Furthermore, the fiscal crisis model provides a rationale for this outcome, since inflation and the real quantity of money will reach the “wrong” side of the Laffer curve.

Financing an increasing level of government expenditures through money issue, which characterizes a fiscal crisis and introducing rational expectations, can yield several outcomes depending upon the shape of the demand for money

⁵ The fiscal deficit at the beginning of the fiscal crisis is $f(0)$ and t_h represents the time that the hyperinflation may last. Thus $f(t_h)$ is the fiscal deficit at the end of hyperinflation. The initial real quantity of money, for both cases (Figs. 1 and 2) satisfies the inequality: $\dot{m} = f(0) - \tau(m(0)) < 0$. For more details see Barbosa et al. (2006).

⁶ Since $\tau(m) = s(m) - rm$, $\tau'(m) < 0$ when $s'(m) < 0$. Thus, in the strong hyperinflation case the inflation tax revenue increases when real cash balance decreases. In the weak inflation case the demand for money is non-inelastic. Thus, $s(m) \rightarrow 0$ when $m \rightarrow 0$. Therefore, the inflation tax revenue goes to zero when the real quantity of money goes to zero.

⁷ The fiscal crisis provides a rationale for the following statement made by Sargent and Wallace (1987, p. 194): “. . . hyperinflation has been characterized by a process of sliding down the slippery side of the Laffer curve.”

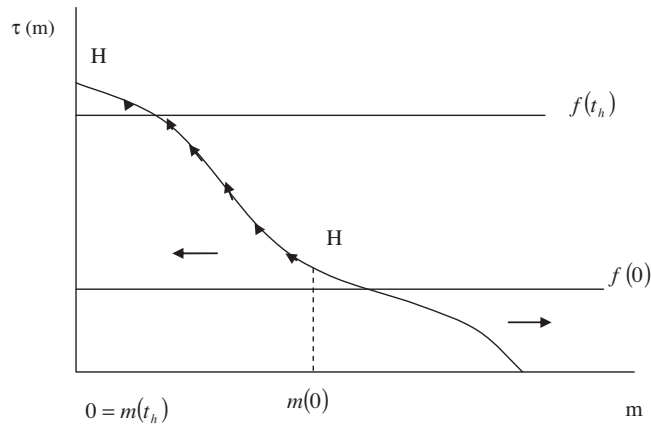


Fig. 1. Strong hyperinflation.

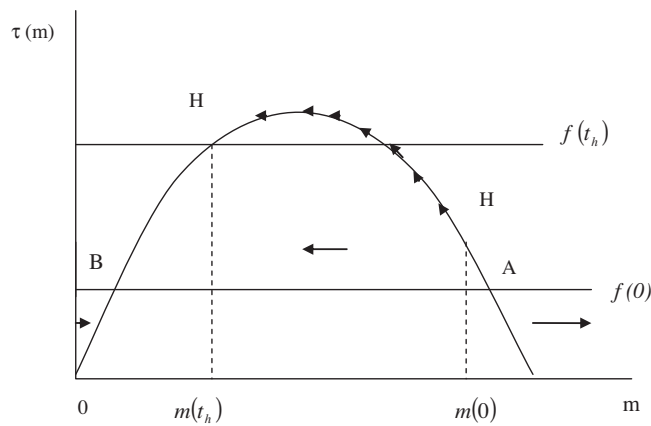


Fig. 2. Weak hyperinflation summing up the model predictions.

function. When the demand for money is non-inelastic the model generates a weak hyperinflation. Before the unfolding of the fiscal crisis the economy is in the low inflation equilibrium. Once the fiscal crisis is in motion, this policy yields a dynamic path that takes the economy from the low inflation equilibrium towards the slippery side of the Laffer curve. When the demand for money is inelastic, the model yields a strong hyperinflation as the rate of inflation goes to infinite. The driving force behind both the weak and the strong hyperinflation is the fiscal crisis. When the demand for money is inelastic there is also the possibility of a hyperinflation bubble, as indicated by the horizontal arrows in Fig. 1.

Most specifications to test hyperinflation bubbles, including the seminal paper by Flood and Garber (1980), have used an inappropriate theoretical framework. The price solutions are obtained by solving the money demand equation forward recursively, and the fundamental price solution depends on the expected sequence of current and future money supply. In such a setup there is a bubble solution, since they do not take into account the government budget constraint. However, when the budget constraint is taking into consideration, due to the fact that money is being issued to finance the fiscal deficit, there is no bubble solution for Cagan’s money demand rational expectation specification. The horizontal arrow away from the origin indicates that a bubble is not feasible when money is non-inelastic, as shown in Fig. 2.

We may conclude that a bubble or a strong hyperinflation may occur only if money is inelastic; a weak hyperinflation may occur only if money is non-inelastic. Thus, the inflation tax curve can be used as a device to discriminate among hyperinflation hypotheses.

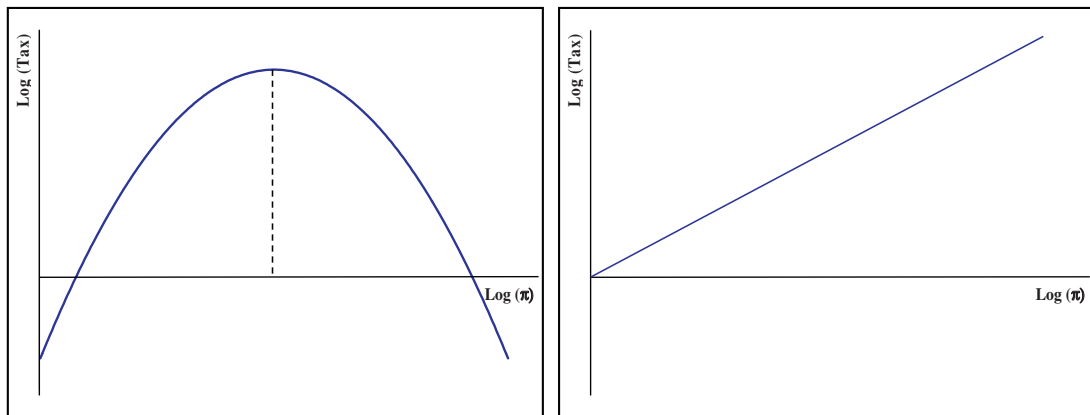


Fig. 3. Two cases for the inflation tax curve.

3. Inflation tax curve: specification and first look at the Brazilian data

The inflation tax revenue (τ) equals the tax rate (π) times the tax base (m). That is: $\tau = \pi m$. Both the inflation tax revenue and the real quantity of money are defined in relation to real GDP, assuming an income elasticity of money equal to one. It is more convenient to write the inflation tax revenue in logarithmic form:

$$\log \tau = \log \pi + \log m \quad (8)$$

Note that the specification of Eq. (8) depends on the demand for money functional form. The two specifications below correspond, respectively, to the semi-logarithmic and logarithmic cases. In the first case the semi-elasticity is constant and the absolute value of the real demand for money inflation elasticity $|\eta|$ is proportional to the inflation rate. In the second case the elasticity is constant.

$$\log m = k_1 - \alpha\pi, \quad |\eta| = \alpha\pi, \quad \alpha > 0 \quad (9)$$

$$\log m = k_2 - \beta \log \pi, \quad |\eta| = \beta < 1 \quad (10)$$

Hence the inflation tax revenue functional forms for each case are as follows:

$$\log \tau = k_1 + \log \pi - \alpha\pi \quad (11)$$

$$\log \tau = k_2 + (1 - \beta) \log \pi \quad (12)$$

Fig. 3a shows the inflation tax curve produced by Eq. (11), in which the semi-elasticity is constant. That curve has a maximum for a given inflation rate, that is, the inflation tax revenue initially increases with inflation and after a certain rate it begins to decrease. Fig. 3b shows the inflation tax curve yielded by Eq. (12), in which the demand for money curve has a logarithmic specification. The curve is a straight line, that is, the inflation tax revenue increases as the inflation rate increases. In this case money is essential since the elasticity of the demand for money with respect to the inflation rate is always less than one (Barbosa and Cunha, 2003; Barbosa et al., 2006).

Note that the two functional forms of the inflation tax curve are obtained as particular cases of the following function:

$$\log \tau = a_0 + a_1 \log \pi - a_2\pi \quad (13)$$

The two particular cases are as follows: (a) constant semi-elasticity: $a_1 = 1$, $a_2 > 0$; constant elasticity: $a_2 = 0$. The functional form (13) also encompasses other possibilities that are not restricted to the above two cases. The inflation tax elasticity (ε) with respect to the inflation rate is given by:

$$\varepsilon = \frac{\partial \log \tau}{\partial \log \pi} = a_1 - a_2\pi \quad (14)$$

This elasticity may be either negative or positive, depending upon the tax curve parameters and the inflation rate. That is, the functional form (13) is flexible enough to allow the data to reveal the shape of the inflation tax curve.

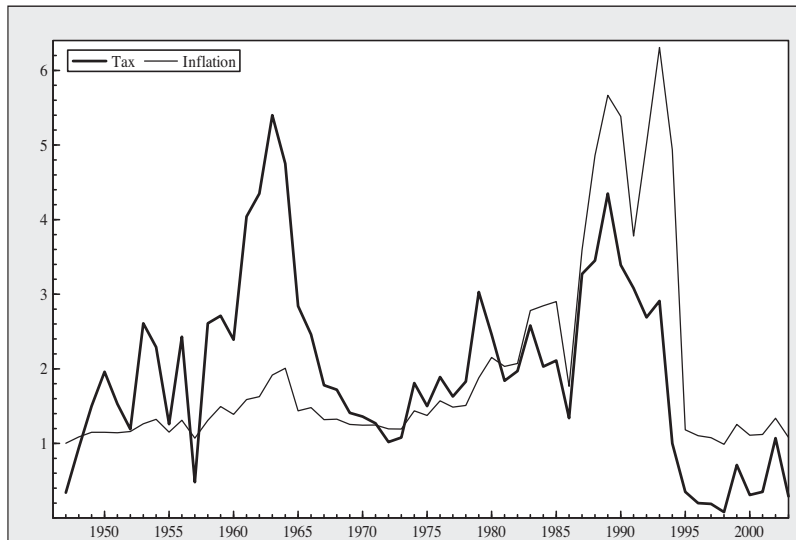


Fig. 4. The inflation rate and the inflation tax revenue in Brazil. (Note: The series are adjusted by their sample means for maximum fit.)

The inflation tax revenue was an important source of government financing in Brazil up to 1994, when the monetary policy regime changed and the Central Bank began to have inflation control as its goal. During the period 1994–1999 the Brazilian Central Bank adopted a system of administered exchange rate in order to curb inflation, and since 1999 it has been operating under an inflation targeting framework.

Fig. 4 shows how the inflation tax revenue and the (continuous) inflation rate evolved in Brazil during the period 1947–2003. The former increased from 1947 to mid-1960s, when it began to decrease until the beginning of the 1970s. Thereafter a new upward phase began, which ended with the Real Plan in 1994. Note that while the inflation tax revenue peaked during the 1960s, inflation peaked during the 1990s. Indeed, inflation was substantially higher during the latter period, when hyperinflation was going on. This evidence strongly suggests the occurrence of important financial innovations during the sample period, which sharply decreased the base of the inflation tax for a given inflation rate.

Although unobservable, financial innovation is widely recognized in the literature as being an essential factor behind the demand for money. Indeed, it was the key factor behind the instability of the demand for money after the 1970s, causing the break between money and income and leading many central banks to give up money as an operational target for monetary policy. Hence, if one wants to estimate a stable inflation tax curve during a long period, then a proxy for financial innovation is needed.

Fig. 5 displays four different scatter plots between the (continuous) inflation rate (x -axis) and the associated inflation tax revenue (y -axis), according to the way each variable is measured. Note that the two graphs in the left half side do not show any obvious relationship between the two variables. However, the graphs placed on the right half side are more revealing concerning the shape of the inflation tax curve. In both cases inflation is measured in log terms and a clear positive relationship arises. The double-log specification seems to provide a slightly better fit, although it also suggests the possibility of a non-linear relationship.

4. Empirical results

Table 1 shows the ADF unit root test results for the inflation tax revenue, the inflation rate (measured in continuous terms) and associated transformations. The tests on the level of the variables do not reject the hypothesis of a unit root in all cases. However, the null is rejected when they are expressed in first differences, which means that all variables seem to be $I(1)$. This result opens the possibility of estimating the inflation tax curve using a cointegration framework.

In order to uncover the format of the inflation tax curve, a general-to-specific model selection strategy is used (see Hendry, 1995). The general unrestricted equilibrium correction model is as follows:

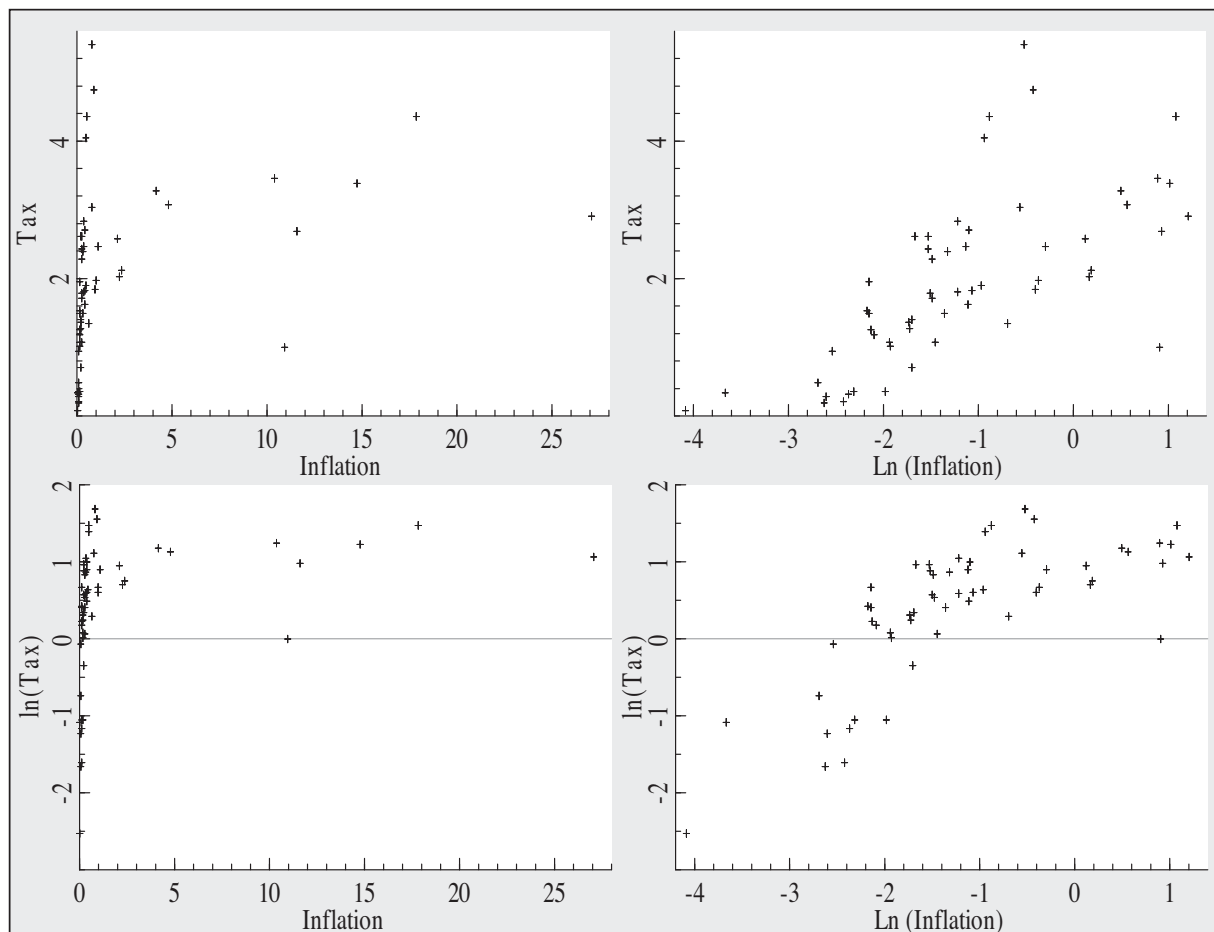


Fig. 5. Inflation tax curve: preliminary evidence.

Table 1
ADF unit root tests.^a

Variable	$\hat{\beta}$	t_{ADF}	Lags	δ_{t-test}	δ_{t-prob}
Levels					
Tax	0.73	-2.81	0	-	-
π	0.85	-1.61	2	-2.00	0.05
$\ln(\pi)$	0.76	-2.17	1	-2.13	0.04
$\ln(Tax)$	0.81	-2.17	0	-	-
First differences					
ΔTax	-0.24	-8.9**	0	-	-
$\Delta \pi$	0.14	-6.11**	0	-	-
$\Delta \ln(Tax)$	-0.41	-10.52**	0	-	-
$\Delta \ln(\pi)$	-0.25	-8.45**	0	-	-

^a The test equation is given by $y_t = c + \alpha t + (\beta - 1)y_{t-1} + \sum_{i=1}^2 \delta_i y_{t-i} + \varepsilon_t$. The lag column indicates the number of lagged differences, while δ_{t-test} stands for the t -value of the highest lag difference.

* Significant level of 5% is expressed.

** Significant level of 10% is expressed.

Table 2
Tax revenue maximizing inflation rates.

Model	Semi-elasticity (α)	Continuous rate (100/ α)	Discrete rate
Model 1	0.29	344%	3.030%
Model 2 (UC smooth)	0.27	374%	4.137%
Model 3 (UC St. level/slope)	0.29	345%	3.055%

$$\Delta \ln Tax_t = \alpha_0 + (\alpha_1 T + \alpha_2 ST_{xx} + \dots + \alpha_k ST_{xx}) + \beta_0 \ln Tax_{t-1} + \beta_1 \ln \pi_{t-1} + \beta_2 \pi_{t-1} + \sum_{i=1}^n \delta_i \Delta \ln Tax_{t-i} + \sum_{i=0}^n \gamma_i \Delta \ln \pi_{t-i} + \sum_{i=0}^n \lambda_i \Delta \pi_{t-i} + \varepsilon_t \tag{15}$$

where n is usually set at 2 and ST_{xx} stands for a specific split trend. For example, ST_{70} indicates a time trend beginning in 1970.

Note that the resulting split trend is an essential part of the model, since it acts as a proxy for (persistent) financial innovation, an unobservable variable according to economic theory, as argued in the last section.⁸ Indeed, as Fig. 4 strongly suggests, financial innovation was substantial during the period under analysis and it is a key factor in the inflation tax curve, given that it decreases the tax base for a given inflation rate. Therefore, it is likely to cause a structural break in the link between the inflation rate and the inflation tax revenue. Its exclusion from the model would yield a misspecification.⁹

Finally, note that (15) encompasses the possibility of a split trend with several breaks. This possibility makes theoretical sense since financial innovation could have evolved at different paces during the sample. For instance, the higher the inflation rate, the bigger the incentive for agents to come up with more money saving innovations.

Eq. (16) shows the final selected specification (called Model 1 in Table 2). The variables are cointegrated, as indicates the significant lagged inflation tax revenue coefficient, suggesting a long-run relation among them.¹⁰ The model passes all diagnostic tests. Moreover, recursive estimates as well as recursive Chow tests show that parameters are stable and no obvious structural break is found. Those are significant results not only due to the long sample involved, but also due to the fact that during this period the Brazilian economy experienced significant changes and was subjected to large and frequent economic shocks, including several stabilization plans, most of them with heterodox features.

$$\Delta \ln Tax_t = \begin{matrix} 3.40 \\ (12.62) \end{matrix} - \begin{matrix} 0.03 \\ (-6.54) \end{matrix} T - \begin{matrix} 0.05 \\ (-4.35) \end{matrix} ST_{72} - \begin{matrix} 0.10 \\ (-4.41) \end{matrix} ST_{80} + \begin{matrix} 0.62 \\ (5.81) \end{matrix} ST_{86} - \begin{matrix} 0.49 \\ (-4.90) \end{matrix} ST_{87} + \begin{matrix} 0.18 \\ (6.06) \end{matrix} ST_{95} - \begin{matrix} 1.14 \\ (-9.21) \end{matrix} D_{94} + \begin{matrix} 0.80 \\ (5.97) \end{matrix} D_{98} - \begin{matrix} 1.26 \\ (-14.04) \end{matrix} \ln Tax_{t-1} + \begin{matrix} 1.34 \\ (12.47) \end{matrix} \ln \pi_{t-1} - \begin{matrix} 0.371 \\ (-4.65) \end{matrix} \pi_{t-1} + \begin{matrix} 1.21 \\ (26.36) \end{matrix} \Delta \ln \pi_t - \begin{matrix} 0.42 \\ (-6.89) \end{matrix} \Delta \pi_t \tag{16}$$

$T=52$ (1952–2003); $\hat{\sigma} = 9.93\%$; $R^2=0.98$; $DW=2.04$; $AR\ 1-2: F(2,36)=0.32$ (0.73); $ARCH\ 1-1: F(1,36)=0.65$ (0.42); $Hetero: F(23, 14)=0.32$ (0.99); $Normality: \chi^2(2)=1.59$ (0.45); $RESET: F(1, 37)=0.07$ (0.79).

Long-run elasticity: $\ln \pi = 1.06$; Long-run semi-elasticity: $\pi = 0.29$.

The model contains two dummies, which are linked to specific economic events. The first refers to the year 1994, when inflation dropped sharply due to the Real Plan. The second dummy refers to 1998, when inflation reached its lowest record level (1.7%) in the sample period, the year just before the floating of the currency in 1999, which was followed by an increase in inflation.

⁸ For the sake of rigour, it should be pointed out that theoretically the trend picks other factors that influence the desire to hold money for a given inflation rate, whether those are due to financial innovations or not (e.g. changes in taxes). Nonetheless, for simplicity, from now on we should refer to it as a proxy of financial innovation.

⁹ Indeed, from the outset it should be called to attention that cointegration was found only in those specifications where we allowed for financial innovation.

¹⁰ Note that it is very unlikely the existence of simultaneity in both cases. Moreover, the strategy used here – where the long run solution and the short run dynamics are estimated at the same time – has the advantage of dealing with the large finite-sample biases found in practice when the Engle–Granger method is used, despite super-consistency. For Monte Carlo evidence on the large bias in the estimation of the static long run solution see Banerjee et al. (1986). See also Banerjee et al. (1993).

The functional form given by (9) is not rejected, so that the money inelasticity hypothesis is rejected. Note also that the hypothesis of an elasticity equal to one ($a_1 = 1$, Eq. (13)) could not be rejected. The value of the coefficient attached to the level of inflation (the semi-elasticity parameter) is significant.¹¹

Table 2 shows the implied tax revenue maximizing inflation rate, which seems to lie around 350% in continuous terms.¹² This translates into discrete rates a little bit above 3000% on an annual basis. That level is above the maximum calendar-year inflation rate reached during the sample, which took place in 1993, when inflation reached 2708%, according to the IGP–DI price index. However, it is well below the twelve-month rates observed in several months, such as those from February 1990 to August 1990 and February 1994 to July 1994. The highest rate in those two periods occurred in April 1990 (6602%) and June 1994 (5153%). The tax revenue maximizing inflation rate implied by the model is lower than those actually observed during the worst months of hyperinflation. Thus, the Government was on the decreasing part of the inflation tax curve. This fact is consistent with the weak hyperinflation hypothesis. Thus, the Government did not maximize the inflation tax revenue during hyperinflation.

Although the use of split trends provided a simple and flexible framework within which persistent structural changes could be modelled, and the resulting model is coherent with the data, one could argue that a more flexible strategy is to use the unobserved components (UC) framework and estimate a stochastic trend instead. Moreover, it could provide a robustness test for our empirical evidence. Thus, two kinds of stochastic trends were estimated using the Kalman Filter. In the first case one assumes that financial innovation can be modelled as a smooth trend, which makes sense since it should evolve like a diffusion process over time. In the second, one assumes a more flexible specification where both the level and the slope are allowed to evolve stochastically.

Using a general-to-specific model selection strategy two equilibrium correction specifications were selected. Eq. (17) – labelled Model 2 in Table 2 – refers to a specification with stochastic trend and a fixed level, while Eq. (18) – labelled Model 3 – represents specification where the level is allowed to vary stochastically.

$$\begin{aligned} \Delta \ln Tax_t = & 1.84\mu_T - 1.11 D_{94} + 0.66 D_{98} - 1.25 \ln Tax_{t-1} + 1.21 \ln \pi_{t-1} - 0.33 \pi_{t-1} \\ & (7.03) \quad (-7.56) \quad (4.14) \quad (-13.02) \quad (9.07) \quad (-2.91) \\ & + 1.13 \Delta \ln \pi_t - 0.40 \Delta \pi_t \\ & (17.62) \quad (-5.25) \end{aligned} \quad (17)$$

$T = 52$ (1952–2003); $\hat{\sigma} = 15.46\%$; $R^2 = 0.98$; $DW = 1.72$; $Q(7, 6) = 10.91$ (0.09); $H(16) = 0.55$ (0.88); Normality: $\chi_{DH}^2(2) = 1.34$ (0.51).

Long-run elasticity: $\ln \pi = 0.97$; Long-run semi-elasticity: $\pi = 0.27$

$$\begin{aligned} \Delta \ln Tax_t = & 1.93\mu_T - 1.08 D_{94} + 0.65 D_{98} - 1.28 \ln Tax_{t-1} + 1.26 \ln \pi_{t-1} \\ & (7.06) \quad (-7.52) \quad (4.14) \quad (-13.32) \quad (9.17) \\ & - 0.37 \pi_{t-1} + 1.14 \Delta \ln \pi_t - 0.41 \Delta \pi_t \\ & (-3.09) \quad (17.63) \quad (-5.18) \end{aligned} \quad (18)$$

$T = 52$ (1952–2003); $\hat{\sigma} = 14.71\%$; $R^2 = 0.98$; $DW = 1.79$; $Q(8, 6) = 5.50$ (0.48); $H(16) = 0.72$ (0.74); Normality: $\chi_{DH}^2(2) = 0.61$ (0.74).

Long-run elasticity: $\ln \pi = 0.98$; Long-run semi-elasticity, $\pi = 0.29$.

The symbol μ_T stands for the value of the stochastic trend at the end of the sample. $Q(p, q)$ is the Box–Ljung statistic for residual autocorrelation based on the first p autocorrelations. $H(h)$ is a heteroscedasticity test and $\chi_{DH}^2(2)$ is a normality test based on the Bowman–Shenton statistic with a correction due to Doornik and Henrik (1994) (see Koopman et al., 2000 for further details).

As before, both models pass all diagnostic tests. Indeed, they are very similar to the one obtained earlier, despite being estimated independently. Cointegration is found once again, suggesting a long-run relation among the variables. Likewise, the OLS case, the relevant inflation tax revenue functional form is given by Eq. (9) and, therefore, the money inelasticity hypothesis is rejected once again. Moreover, the elasticity of the inflation tax revenue with respect to inflation is around one as before, and the value of inflation semi-elasticity is practically the same as that obtained from Model 1. Table 2 gives the associated tax revenue maximizing inflation levels.

¹¹ However, note that small variations in the semi-elasticity could mean large discrepancies in the associated discrete inflation rates. Therefore, inferences regarding the tax maximizing inflation rate based solely on the difference between coefficients could be misleading.

¹² Although the semi-elasticity is expressed with two decimal places, the implied inflation rates shown in Table 2 were calculated with four decimal places, since small changes in the former lead to big changes in the latter.

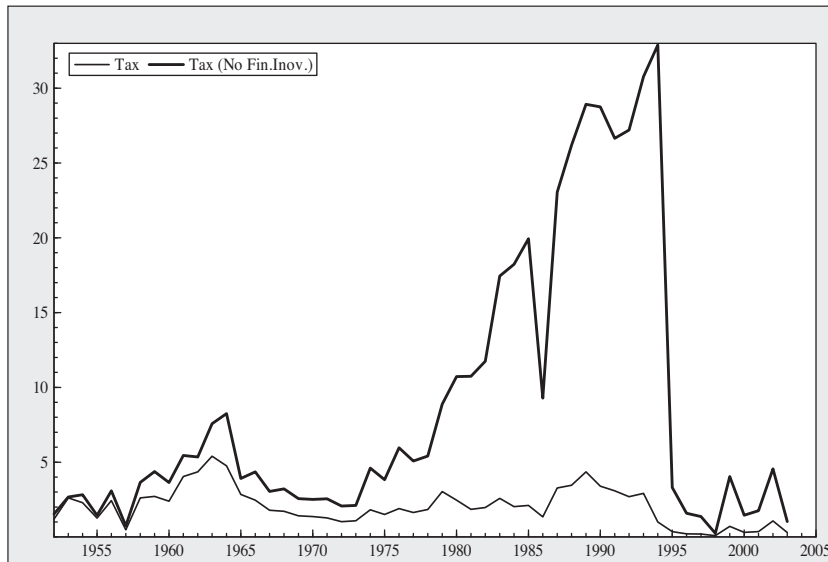


Fig. 6. Inflation tax revenue: with and without financial innovation.

It should be pointed out that not only both specifications produce stochastic trends with virtually the same shape – although Model’s 3 trend is less smoother than Model 2 – but their shape is very similar to what was obtained before. The standard error of specification (16) is smaller than those of specifications (17) and (18), suggesting that the simpler OLS method does a better job in modelling the inflation tax than the UC framework. More importantly, the results are robust to the choice of how to model financial innovation.

One interesting exercise is to reckon the inflation tax revenue that would have been collected by the Brazilian government if there had been no financial innovations. The result is shown in Fig. 6, according to the predicted values of Model 1, when financial innovation variables are set equal to zero, and assuming that the beginning of the sample financial innovation was zero, which seems a reasonable hypothesis to pin down its level. This figure also displays the actual inflation tax revenue collected by the government, so that it can be seen that financial innovation made a huge difference for the inflation tax revenue accrued.

5. Conclusion

This paper uses a methodology that provides a richer and encompassing framework within which one can analyse the hyperinflation phenomenon, and presents a solution to an old puzzle that has not been solved by previous works on hyperinflation: the fact that the economy during some time lies on the ‘wrong’ side of the Laffer curve, even when expectations are formed rationally. As we show, this fact needs not stem from irrational behaviour but rather is consistent with the weak hyperinflation hypothesis, which is the outcome of an increasing fiscal deficit financed by money plus a non-monotonic inflation tax curve. Hence, the statement usually made that the government could have obtained more tax revenue with lower inflation rates during hyperinflation, is not correct when those conditions are observed. This fact is the very outcome of the dynamics of the fiscal crisis that yields the hyperinflation path.

Most specifications to test hyperinflation bubbles, following the footsteps of Flood and Garber (1980), have used an inappropriate theoretical framework to build their tests.¹³ The inflation tax curve used in this paper provides a very simple and sound way to test hyperinflation bubbles.

The empirical evidence presented in this paper is consistent with the weak hyperinflation hypothesis for the Brazilian economy, which could have been on the wrong side of the Laffer curve for some period during hyperinflation. The hypothesis that the Brazilian hyperinflation could have been caused by either a bubble or a strong-type hyperinflation is rejected.

¹³ See, for example, Hooker (2000).

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