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An adverse social welfare effect of a doubly gainful trade

by

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An adverse social welfare effect of a doubly gainful trade

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Abstract

Acknowledging individuals’ distaste for low relative income renders trade less appealing when trade is viewed as a technology that integrates economies by merging separate social spheres into one. We define a “trembling trade” as a situation in which gains from trade are overtaken by losses of relative income, with the result that global social welfare is reduced. A constructive example reveals that a “trembling trade” can arise even when trade is doubly gainful in that it increases the income of every individual and narrows the income gap between the trading populations.

Keywords: Gains from trade; Increase of incomes; Decrease of income gap; Integration; Change of social space; Low relative income; Social welfare

JEL classification: D31; D63; R12
1. Introduction

A fundamental insight in economics is that in comparison to a situation in which economies operate as separate autarkies, economies operating in conditions of perfect competition and free trade mutually gain from exchanging goods according to their comparative advantage.\(^1\)

Despite their embrace of free trade, ever since Ricardo’s (1817) “Principles,” economists have been aware of the need to distinguish between the gains from trade and the distributional effects of trade. Individuals in trading economies typically differ in their endowments of factors of production. Because trade alters the relative scarcity of these factors, it also alters the incomes of the individuals to different extents, possibly even in different directions (Stolper and Samuelson, 1941). In the vocabulary of the current paper, relative incomes are altered too.

It is of interest to note that even in research on a prominent form of trade, namely international trade, we observe a divide: there are researchers who are concerned about quantifying the gains from trade yet pay little attention to distributional effects (Arkolakis et al., 2012; Melitz and Trefler, 2012; Costinot and Rodríguez-Clare, 2014), and there are researchers who are concerned about distributional effects yet, in the main, leave aside the issue of welfare gains for the economy as a whole (Feenstra and Hanson, 2003; Harrison et al., 2011; Haskel et al., 2012; Costinot and Vogel, 2015). Distribution considerations can be linked with aggregate (social) welfare by acknowledging that the individuals’ wellbeing depends on the degree of inequality of the income distribution. This approach has not been pursued substantially in trade theory, although recent literature has examined the sensitivity of the welfare effects of trade to the preferences of the individuals (Mrázová and Neary, 2014; Arkolakis et al., 2015).

It is important to note right at the outset that the trade studied in this paper can, but need not, be international. As in the classic treatment of trade by Ohlin (1933), the trade that we consider is one between populations, economies, or markets in general. For example, in current times, the entities involved can be adjacent regions in different countries or two villages within a developing country, and in historical times, the entities

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\(^1\) The earliest precise statement of the gains from trade theorem is of Samuelson (1939). A follow-up treatment is in Dixit and Norman (1980).
could be different tribes or city states. In particular, the trade that we have in mind is of the type that causes the social spheres of the trading entities to merge. Recent research emphasizes that trade is the outcome of increased interaction between communities: these communities can be Indian provinces (Burgess and Donaldson, 2012), American counties (Costinot and Donaldson, 2016), or simply neighborhoods.

The constructive example developed in this paper is guided by the following reasoning. As an exchange between markets, trade affects social relations. In a variety of ways, these relations constitute a link between trade and social welfare. Trade can change social ties, broaden social horizons, erode prevailing social relations, and forge new ones. Even when trade occurs in geographical space, it leaves traces in the social sphere. In particular, the participation of populations in trade can expand their social space. This expansion may influence how the people who are affected by trade evaluate the benefits it confers. We present a framework which at its core proposes that individuals’ preferences are social in nature. This perspective naturally includes the concept of social space and the associated concepts of relative income and relative deprivation (defined in Section 2 below).

We use a social welfare function that incorporates individuals’ distaste for low relative income (concern about relative deprivation). It has already been shown that a function of this type has several desirable properties, such as superadditivity (Stark, 2013). Most importantly, the function renders analytically tractable the idea that pursuing trade broadens the comparison group of individuals and, consequently, impacts on their perceived relative income. The function is consistent with mounting empirical evidence, referred to in Section 2 below, that individuals’ wellbeing is sensitive to interpersonal comparisons and relative income. And it establishes a novel link between the distributional effects of trade on the one hand and the gains from trade on the other hand.

To the best of our knowledge, this paper is the first to weigh the two prominent gains from trade - that of increasing the income of every individual and that of narrowing the income gap between the trading entities - against individuals’ concern about low

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2 A formulation of a general social welfare function that incorporates individuals’ distaste for low relative income is provided in Stark et al. (2017b). For the purpose of the development of the constructive example presented in this paper, we use a simplified linear version of that social welfare function. In work in progress, we formulate conditions under which the main result reported here, namely the “trembling trade” conjecture, can be derived when the social welfare function takes a non-linear form.
relative income. To sharpen the focus, we make our main point starkly: we view trade as a process which brings closer together in social space populations, economies, or markets that previously were not connected. As a result of this integration, separate social spheres merge, and people’s social space and their comparators are altered. Then, even when in the wake of trade the absolute income of everyone increases and the income gap between the trading entities narrows, the perceived relative incomes of some individuals, calculated in the context of the broadened social space, may decrease. Consequently, in and by itself, the integration of social spaces may exacerbate the stress that some individuals feel from having low relative income. We measure people’s concern about low relative income by the index of “relative deprivation.” we say that an individual feels relatively deprived when his income falls behind the incomes of other individuals who constitute his comparison group. Assuming that people like high (absolute) income and dislike low relative income, we show that an increase in the aggregate stress from perceived relative deprivation brought about by trade can result in a negative overall impact of trade on (utilitarian-based) social welfare if the increase in incomes (the conventional gain from trade) is not large enough to mitigate the negative consequences of relative income loss. We refer to this situation as a “trembling trade:” the shake-up of the social spheres of the trading populations has a bigger effect on their wellbeing than the monetary gain.

In the remainder of this paper, we proceed as follows. In Section 2, we describe how trade can alter the social space of the members of the economies that embark on trading with each other, and why this change affects the gains from trade. In Section 3, we track the social welfare consequences of trade: in Subsection 3.1, we inquire how the welfare of the trading populations changes as a result of trade, assuming that the incomes of the trading populations do not overlap, and in Subsection 3.2 and in the Appendix, we inquire likewise when the incomes of the trading populations overlap. Section 4 concludes.
2. How trade alters social space, and why this change matters for the gains from trade

Trade not only expands the realm of exchange in geographical space; it also expands the social space of the individuals in the trading economies. To different extents, individuals experience a broadening of their comparison group to include individuals from the trading-partner economies.

The trade-generated expansion of social space can arise via several channels. First, purchased goods carry information about trading partners, in particular about the productivity of labor and about incomes in the partner economy. This channel operates even in the sterile world of perfect Walrasian markets. A second channel results from the interactions of people in non-Walrasian environments where trade requires social interactions aimed at matching trading partners and at mediation which, to different extents, removes the information gap between the trading economies. Third, in order to ease and intensify trade, trading partners introduce mechanisms and procedures that also reduce the social divide between them. In the context of international trade, monetary unification (an event that has occurred in Europe seven times since 1999) is one such example. Fourth, trade invites, and is often based on, exchanges of traders, trade representatives, trade delegations, and experts of various types. Presence and exposure foster comparisons. Fifth, trade is built on a study of the needs, preferences, consumption habits, and demands of the partners in trade; the expansion of commercial space brings in its wake expansion of social space. Sixth, trade is often the precursor of migration, and migrants facilitate and intensify cross-cultural awareness and inter-economy social ties.

The broadening of an individual’s social space in the wake of trade influences the individual’s wellbeing. Rich evidence from field and laboratory studies in economics, social psychology, and neuroscience confirms that individuals routinely engage in, and

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3 Stark and Wlodarczyk (2015, p. 185) argue as follows. “The introduction of a common currency is an instrument of fundamental change in economic and social relations in general, and in interpersonal comparisons of earnings, pay, and incomes in particular. Although, prior to the introduction of the euro as a common currency, individuals in specific European countries were able to compare their incomes with the incomes of individuals in other European countries, the comparison was not immediate; it required effort to convert incomes denominated in different currencies, and it was presumably not done very often. … When a single currency is introduced, the comparison environment changes, enabling, indeed inviting, simpler comparisons with others. For example, with currency unification, workers who perform the same task and who are employed by a manufacturer with plants located in different EMU countries can compare their earnings with each other directly, effortlessly, and routinely.”
are affected by, interpersonal comparisons. In particular, people are dissatisfied when their consumption or income levels are lower than those of others who constitute their comparison group. Studies that recognize such discontent are, among others, Stark and Taylor (1991), Zizzo and Oswald (2001), Luttmer (2005), Fliessbach et al. (2007), Blanchflower and Oswald (2008), Takahashi et al. (2009), Fan and Stark (2011), Stark and Fan (2011), Stark and Hyll (2011), Card et al. (2012), Stark et al. (2012), and Goerke and Pannenberg (2015). The evidence overwhelmingly supports the notion of a strong asymmetry: the comparisons that significantly affect an individual’s sense of wellbeing are the ones rendered by looking “up” the income hierarchy, whereas looking “down” does not appear to be of much consequence, or to deliver satisfaction. For example, Cohn et al. (2014) find that in choosing how much effort to exert in their work, workers respond to increased relative deprivation but not to increased “relative satisfaction.” Frey and Stutzer (2002), Walker and Smith (2002), and Stark (2013) review a large body of evidence that lends support to the “upward comparison” hypothesis.

In its February 27, 2016 issue, The Economist magazine reported the following finding of the Eurobarometer survey, which has tracked self-reported happiness for over four decades: “According to Eurostat, the EU’s statistical office, the only metric consistently correlated with European happiness is relative income. Moving one step up the income ladder increases happiness in every country in the EU.”

Particularly telling in our context is empirical evidence that perceived relative deprivation influences behavior even more significantly than absolute income. For example, in research on migration, Vernazza (2013) observes that, even though interstate migration in the US confers substantial increases in absolute income, the trigger for migration is relative deprivation (low relative income), not low absolute income. And drawing on data from the 2000 US census, Flippen (2013) reports that both blacks and whites who migrate from the North to the South have on average lower absolute incomes than their stationary northern peers, yet enjoy significantly lower relative deprivation, and

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4 Similarly illuminating is a report in the September 30, 2016 issue of the same magazine on the installation of solar panels: “One effective remedy for high installation prices may be peer pressure. Tendril, an energy-intelligence firm, crunched the numbers on solar-panel installations in San Jose, California since 2001. The company developed a machine-learning model to sort out which factors were most salient in predicting an installation, using an impressive data set that included mutual-fund investment, interest in the outdoors and “high-life behavior.” Among all these, the most likely predictor of having a solar panel was having a neighbor who had installed one. Income came in second, its predictive power only half as strong.”
that the gains from reduced relative deprivation are substantially larger for blacks than for whites.

3. The welfare implications of trade

In this section, we ask how doubly gainful trade, namely trade that increases the incomes of all the individuals in the trading populations and mitigates the income gap between the populations, affects the social welfare of the trading populations.

Let there be two populations, P1 and P2, and let each population have \( n \geq 2 \) members, where \( n \) is an integer. To begin with, we assume that the two populations are separate in the sense of having neither trade connections nor social ties with each other. We also assume that individuals from both populations derive utility from (absolute) income and disutility from low relative income. Thus, the utility of individual \( i \) who belongs to any of the two populations and who has income \( m_i \) can be represented by

\[
U_i(m_i) = (1 - \alpha)f(m_i) - \alpha RD(m_i, m_{-i}),
\]

where \( f(\cdot) \) is a strictly increasing function that converts (absolute) income into utility; \( RD(\cdot) \) is relative deprivation which measures the disutility from low relative income; \( m_{-i} \) is the set of the incomes of all the individuals in \( i \)'s comparison group (namely the group of individuals with whose incomes individual \( i \) compares his own income); \( \alpha \in (0,1) \) is the weight accorded to the disutility from low relative income; and \( 1 - \alpha \) is the weight accorded to the utility from (absolute) income. In order to concentrate on essentials, we assume that \( f(\cdot) \) is linear, meaning that \( f(m_i) = m_i \). We use a standard measure of relative deprivation: the aggregate of the excesses of the incomes of others in the individual’s comparison group, divided by the size of this group.\(^5\) (Our results will not change if (1) is “re-configured” such that \( RD(m_i, m_{-i}) \) is replaced by \( RD(m_i, m_{-i}) \equiv 1 - RD(m_i, m_{-i}) \), thereby eliminating the appearance of a negative term in

\(^5\) This measure of relative deprivation is equivalent to a measure of relative deprivation of individual \( i \) defined as the fraction of the individuals in \( i \)'s comparison group whose incomes are higher than the income of individual \( i \) times the difference between the average income of the higher income individuals and \( i \)'s income. An Online Appendix provides a brief foray into the concept of relative deprivation and a presentation of its measures.
the utility function which, to some, may appear unappealing.) Formally, in a comparison group consisting of \( l \) individuals, the relative deprivation of individual \( i \) whose income is \( m_i \) is calculated as

\[
RD(m_i, m_{-i}) = \frac{1}{l} \sum_{m_j \in m_{-i}} \max \{ m_c - m_i, 0 \}. \tag{2}
\]

Let the individuals in P1 have pre-trade incomes denoted by \( 0 \leq x_1 < \ldots < x_n \), and let the individuals in P2 have pre-trade incomes denoted by \( 0 \leq y_1 < \ldots < y_n \); the individuals are indexed according to their pre-trade incomes. We define \( x \equiv (x_1, \ldots, x_n) \) and \( y \equiv (y_1, \ldots, y_n) \).

We study doubly gainful trade, that is, as already defined, a trade that increases every individual’s income and at the same time narrows the income gap between the trading populations. To represent the first characteristic, we assume that the incomes of each of the individuals in P1 increase by \( a \), and that the incomes of each of the individuals in P2 increase by \( b \), where \( a \) and \( b \) are positive constants. Thus, \( x + a = (x_1 + a, \ldots, x_n + a) \) and \( y + b = (y_1 + b, \ldots, y_n + b) \) are the individuals’ post-trade income vectors. To represent the second characteristic, we assume that the difference between the average incomes of the populations decreases when they take up trade.

Given that prior to trade this difference is \( \left| \sum_{i=1}^{n} y_i / n - \sum_{i=1}^{n} x_i / n \right| \), and that in the wake of trade this difference is \( \left| \sum_{i=1}^{n} (y_i + b) / n - \sum_{i=1}^{n} (x_i + a) / n \right| \), we present the assumption of a reduction in the income gap between the populations as

\[
\left| \frac{\sum_{i=1}^{n} (y_i + b)}{n} - \frac{\sum_{i=1}^{n} (x_i + a)}{n} \right| < \left| \frac{\sum_{i=1}^{n} y_i}{n} - \frac{\sum_{i=1}^{n} x_i}{n} \right|. \tag{3}
\]

Substituting
\[
\sum_{i=1}^{n} x_i \over n = \bar{x} \quad \text{and} \quad \sum_{i=1}^{n} y_i \over n = \bar{y}
\]

into (3), the condition for a reduction in the income gap between the populations can be expressed as

\[
\left| (\bar{y} + b) - (\bar{x} + a) \right| < |\bar{y} - \bar{x}|.
\] (4)

Without a loss of generality, we assume that \( \bar{y} > \bar{x} \). Then, assuming that the trade-generated increases in incomes do not change the ordering of incomes between the populations, we have that \( a > b \).

Let \( SW_1^{pre} \) and \( SW_2^{pre} \) be the autarky (that is, pre-trade) levels of social welfare of populations P1 and P2, respectively, and let \( SW_1^{post} \) and \( SW_2^{post} \) be the corresponding levels of the populations’ social welfare in the wake of trade. Let the social welfare of a population be measured in a utilitarian manner, namely by the sum of the utility levels of the members of the population. Under autarky, the individuals in each population compare themselves only with members of the same population; the population that they belong to constitutes their comparison group. For the reasons detailed above, we assume that when they trade, the social space expands, extending the individuals’ comparison group into a union of the two populations. To trace the social welfare repercussions from trade, we compare the post-trade social welfare of the integrated population with the sum of the pre-trade levels of social welfare of the two populations when separate. Specifically, we ask when global social welfare will not fall in the wake of trade, namely under what conditions it will hold that \(^6\)

\[
SW_1^{pre} + SW_2^{pre} \leq SW_1^{post} + SW_2^{post}.
\] (5)

In Subsection 3.1, we analyze the social welfare implications of trade when the income ranges of the two populations do not overlap. In Subsection 3.2 and in the Appendix, we analyze these implications when the income ranges of the two populations

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\(^6\) The term “global social welfare” is used to refer either to the post-trade social welfare of the integrated population or to the sum of the pre-trade levels of social welfare of the two populations when apart.
overlap. We consider two cases of overlapping income ranges: one in which income-wise 
P1 mingles with P2, and one in which income-wise P1 lies en bloc in P2.

3.1 A change in social welfare brought about by trade between two non-overlapping 
populations

To operationalize the assumption that the incomes of the two populations do not overlap, 
we assume without loss of generality that \( x_n < y_i \); P2 is uniformly richer than P1. We 
assume that the trade-generated income increases given by \( a \) and \( b \) are such that in the 
wake of trade P2 remains uniformly richer, namely that \( x_n + a < y_i + b \). Given that both 
prior to trade and upon trade P2 is uniformly richer than P1, we have that \( \bar{y} > \bar{x} \) and that 
\( \bar{y} + b > \bar{x} + a \). Thus, the condition for a reduction in the income gap between the 
populations given by (4) requires that \( a > b \).

We introduce the following notation:

\[
V(x) = \sum_{i=1}^{n} \sum_{j=1}^{n} \max\{x_j - x_i, 0\} = \sum_{i=1}^{n} \sum_{j=i+1}^{n} (x_j - x_i), \tag{6}
\]

\[
V(y) = \sum_{j=1}^{n} \sum_{k=1}^{n} \max\{y_k - y_j, 0\} = \sum_{j=1}^{n} \sum_{k=j+1}^{n} (y_k - y_j), \tag{7}
\]

\[
W(x, y) = \sum_{i=1}^{n} \sum_{j=1}^{n} (y_j - x_i). \tag{8}
\]

Having resorted to a utilitarian-based measure of social welfare, we have that prior to 
trade, the levels of social welfare of the two populations are

\[
SW_1^{pre} = \sum_{i=1}^{n} \left[ (1 - \alpha) f(x_i) - \frac{\alpha}{n} \sum_{j=i+1}^{n} (x_j - x_i) \right] = \sum_{i=1}^{n} (1 - \alpha)x_i - \frac{\alpha}{n} \sum_{i=1}^{n} \sum_{j=i+1}^{n} (x_j - x_i) \tag{9}
\]

\[
= (1 - \alpha) \sum_{i=1}^{n} x_i - \frac{\alpha}{n} V(x),
\]

\[
SW_2^{pre} = (1 - \alpha) \sum_{j=1}^{n} y_j - \frac{\alpha}{n} V(y), \tag{10}
\]

and that in the wake of trade these levels are
Clearly, there is no $W(\cdot)$ term in (12): unlike the individuals in P1, the individuals in P2 do not experience a change in relative deprivation as a result of the expansion of social space.

Upon substituting (9) through (12) into (5), global social welfare will not fall in the wake of trade if and only if

\[
(1-\alpha)\sum_{i=1}^{n} x_i - \frac{\alpha}{n} V(x) + (1-\alpha)\sum_{j=1}^{n} y_j - \frac{\alpha}{n} V(y)
\]

\[
\leq (1-\alpha)\sum_{i=1}^{n} x_i + (1-\alpha)na - \frac{\alpha}{2n} V(x) - \frac{\alpha}{2n} W(x+a,y+b) + (1-\alpha)\sum_{j=1}^{n} y_j + (1-\alpha)nb - \frac{\alpha}{2n} V(y),
\]

which can be simplified to

\[
a + b \geq \frac{\alpha}{1 - \alpha \cdot 2n^2} [W(x+a,y+b) - V(x) - V(y)].
\]  

(13)

Given the assumption that $x_i + a < y_j + b$ for all $i, j = 1, \ldots, n$, $W(x+a,y+b)$ can be transformed into

\[
W(x+a,y+b) = \sum_{i=1}^{n} \sum_{j=1}^{n} (y_j + b - x_i - a) = \sum_{i=1}^{n} \sum_{j=1}^{n} y_j - \sum_{i=1}^{n} \sum_{j=1}^{n} x_i + \sum_{i=1}^{n} \sum_{j=1}^{n} (b-a)
\]

\[
= n \sum_{j=1}^{n} y_j - n \sum_{i=1}^{n} x_i + n^2(b-a).
\]

(14)
From substituting (14) into (13), we obtain

\[ a + b \geq \frac{\alpha}{1 - \alpha} \frac{1}{2n^2} \left[ n \sum_{j=1}^{n} y_j - n \sum_{i=1}^{n} x_i + n^2 (b - a) - V(x) - V(y) \right] . \]

Rearranging yields

\[ a \left( \frac{2}{\alpha} - 1 \right) + b \left( \frac{2}{\alpha} - 3 \right) \geq \frac{1}{n} \left[ \left( \sum_{j=1}^{n} y_j - \sum_{i=1}^{n} x_i \right) - \frac{1}{n} \left[ V(x) + V(y) \right] \right] , \]

which is the final form of the necessary and sufficient condition for doubly gainful trade not to reduce global social welfare when one population is uniformly richer than the other population. In the vocabulary of this paper, this is the condition for trade not to be trembling.

We note that the left-hand side of (15) is positive. This is so because

\[ a \left( \frac{2}{\alpha} - 1 \right) + b \left( \frac{2}{\alpha} - 3 \right) = b \left( \frac{2}{\alpha} - 1 \right) + b \left( \frac{2}{\alpha} - 3 \right) = b \left( 2 - \alpha + 2 - 3\alpha \right) = \frac{4b}{\alpha} (1 - \alpha) > 0 , \]

where the first inequality follows from the assumption that \( a > b \) and from the fact that \( 2 / \alpha > 2 > 1 \). For ease of reference, we define the right-hand side of (15) as

\[ s^* = \frac{1}{n} \left[ \left( \sum_{j=1}^{n} y_j - \sum_{i=1}^{n} x_i \right) - \frac{1}{n} \left[ V(x) + V(y) \right] \right] . \]

Can \( s^* \) be non-positive for some \( P1 \) and \( P2 \), thus guaranteeing that for these populations doubly gainful trade is always welfare-enhancing? To address this issue, we rewrite \( V(x) \), as introduced in (6), as

\[ V(x) = \sum_{i=1}^{n} (2i - n - 1)x_i , \]

which implies that

\[ \frac{1}{n} V(x) = \sum_{i=1}^{n} \left( \frac{2i - 1}{n} - 1 \right) x_i , \]

and, by analogy, we can write

\[ \frac{1}{n} V(y) = \sum_{j=1}^{n} \left( \frac{2j - 1}{n} - 1 \right) y_j . \]
Using (18), we find that
\begin{align*}
\sum_{i=1}^{n} x_i + \frac{1}{n} V(x) &= \sum_{i=1}^{n} x_i + \sum_{i=1}^{n} \left( \frac{2i-1}{n} - 1 \right) x_i = \sum_{i=1}^{n} \frac{2i-1}{n} x_i < \frac{x_n}{n} \sum_{i=1}^{n} (2i-1),
\end{align*}
where the inequality follows from the assumption that $x_1 < \ldots < x_n$. We note that the sequence $2i-1$ is arithmetic and, therefore, we have
\begin{align*}
\sum_{i=1}^{n} x_i + \frac{1}{n} V(x) &= x_n \sum_{i=1}^{n} (2i-1) = x_n \frac{1 + (2n-1)}{2} n = nx_n.
\end{align*}

On the other hand, by (19) and from the assumption that $y_1 < \ldots < y_n$, it follows that
\begin{align*}
\sum_{j=1}^{n} y_j - \frac{1}{n} V(y) &= \sum_{j=1}^{n} y_j - \sum_{j=1}^{n} \left( \frac{2j-1}{n} - 1 \right) y_j \\
&= \frac{1}{n} \sum_{j=1}^{n} [2n - (2j-1)] y_j > \frac{y_1}{n} \sum_{j=1}^{n} [2n - (2j-1)].
\end{align*}
which, on noting that the sequence $2n - (2j-1)$ is arithmetic, implies that
\begin{align*}
\sum_{j=1}^{n} y_j - \frac{1}{n} V(y) > \frac{y_1}{n} \sum_{j=1}^{n} [2n - (2j-1)] = \frac{y_1 (2n-1)+1}{2} n = ny_1.
\end{align*}

We can now substitute (21) and (23) into (16) to obtain
\begin{align*}
\text{s}^* &= \frac{1}{n} \left[ \left( \sum_{j=1}^{n} y_j - \frac{1}{n} V(y) \right) - \left( \sum_{i=1}^{n} x_i + \frac{1}{n} V(x) \right) \right] > \frac{1}{n} (ny_1 - nx_n) = (y_1 - x_n) > 0,
\end{align*}
where the second inequality in (24) follows from the assumption that P2 is uniformly richer than P1. We conclude that the condition in (15) indeed places a lower bound constraint on $a \left( \frac{2}{\alpha} - 1 \right) + b \left( \frac{2}{\alpha} - 3 \right)$ for trade to be non-trembling.

The preceding analysis can be summarized as follows.

A “trembling trade” conjecture:
Given the setting defined in this subsection, the following two statements hold true:
(i) There is a critical value, \( s^* \), such that trade between P1 and P2 does not reduce global social welfare if and only if
\[
a \left( \frac{2}{\alpha} - 1 \right) + b \left( \frac{2}{\alpha} - 3 \right) \geq s^*. \]

(ii) Because \( s^* > 0 \), trade is not welfare-enhancing for some values of trade-induced income increases \( a \) and \( b \) that narrow the income gap between P2 and P1.

To interpret (15), we present (15) jointly with (16) as
\[
a \left( \frac{2}{\alpha} - 1 \right) + b \left( \frac{2}{\alpha} - 3 \right) \geq \frac{1}{n} \left[ \left( \sum_{j=1}^{n} y_j - \sum_{i=1}^{n} x_i \right) - \frac{1}{n} [V(x) + V(y)] \right] \equiv s^*. \]

The left-hand side of the inequality above is a linear combination of the increases in incomes from trade, \( a \) and \( b \). This linear combination can be thought of as a measure of the aggregate gain from trade for two populations exhibiting an intensity \( \alpha \) of distaste for low relative income. This measure is decreasing in \( \alpha \), meaning that, other things held constant, trade is less likely to be welfare-enhancing (more likely to be trembling) for higher \( \alpha \). A higher \( \alpha \) means that a higher weight is accorded to the rising aggregate relative deprivation, rendering the gains from trade, \( a \) and \( b \), less significant for the trading populations.

In addition, we note that, other things being equal, \( s^* \) is increasing in the difference between the initial aggregate incomes of the two populations \( \left( \sum_{j=1}^{n} y_j - \sum_{i=1}^{n} x_i \right) \).

This characteristic is intuitive in that the merging of the two populations will inflict greater relative income deprivation on members of the poorer population when the difference between the initial aggregate incomes of the two populations is greater. This raises the bar for a gain in global social welfare to occur.

Also, the smaller \( s^* \) is, other factors being the same, the bigger the sum of the pre-trade levels of aggregate relative deprivation of P1 and P2 \( \left( \frac{1}{n} V(x) + \frac{1}{n} V(y) \right) \). This characteristic might appear surprising. To see where it originates from, we return to (13):
\[
a + b \geq \frac{\alpha}{1 - \alpha} \frac{1}{2n^2} [W(x + a, y + b) - V(x) - V(y)], \]
which can be rearranged into
\[ n(a + b) \geq \frac{\alpha}{1 - \alpha} \left\{ \frac{1}{2n} [W(x + a, y + b) + V(x) + V(y)] - \left[ \frac{1}{n} V(x) + \frac{1}{n} V(y) \right] \right\}. \quad (25) \]

Inequality (25) is an alternative representation of the condition for trade not to reduce global social welfare. It states that for a gain in social welfare to occur, the aggregate nominal gain from trade has to be larger than the difference between the sum of the post-trade levels of aggregate relative deprivation and the sum of the pre-trade levels of the aggregate relative deprivation, with the difference multiplied by \( \alpha / (1 - \alpha) \). Thus, other things being equal, the higher the sum of the pre-trade levels of aggregate relative deprivation, the lower the bar that the aggregate gain from trade has to surpass in order for trade not to reduce global social welfare.\(^7\)

The “trembling trade” conjecture of a global social welfare gain also survives, with some modification, when the trading populations are of unequal size. Suppose that there are \( n_1 \) individuals in population P1, and \( n_2 \) individuals in population P2. Then, as can easily be verified, the expression in (15), namely the necessary and sufficient condition for trade not to decrease social welfare, becomes

\[
\begin{align*}
\frac{a}{n_1 + n_2} \left( \frac{n_1}{n_1 + n_2} \frac{1}{\alpha} \right) + b \left( \frac{n_2}{n_1 + n_2} \frac{1}{\alpha} \right) & \geq \frac{1}{n_1 + n_2} \left[ \frac{n_1}{n_1 + n_2} \sum_{j=1}^{n_1} x_j + \frac{n_2}{n_1 + n_2} \sum_{i=1}^{n_2} x_i - \frac{1}{n_1 + n_2} (V(x) + V(y)) \right].
\end{align*}
\]

By analyzing this case in the same way as pursued in expressions (16) through (23), it can be verified that if

\[
\frac{y_i}{x_i} > \left( \frac{n_1}{n_2} \right)^2,
\]

\[ (26) \]

\(^7\)Although, strictly speaking, condition (15) is not about the minimal required level of the aggregate nominal gain from trade \( n(a + b) \), but rather about the minimal required level of the sum \( a \left( \frac{2}{\alpha} - 1 \right) + b \left( \frac{2}{\alpha} - 3 \right) \), the relationship with the pre-trade levels of aggregate relative deprivation is the same: the sign of \( [V(x) + V(y)] \) is not affected by the transformation from (13) to (25).
then the right-hand side of (15’) is positive, and trade is trembling for some values of \( a \) and \( b, \ a > b \). In particular, if \( n_1 < n_2 \), namely if the poorer population is smaller than the richer population, then condition (26) is always observed.

By assuming equal gains from trade \textit{within} populations, namely that every member of \( P_1 \) gains \( a \) and that every member of \( P_2 \) gains \( b \), we abstract from the income distribution effects of trade \textit{within} populations that have been studied in the literature referred to in Section 1. Thus, we identify an additional effect of trade on social welfare that works independently from those investigated hitherto.

### 3.2 A change in social welfare brought about by trade between two overlapping populations

Our finding that, in spite of being doubly gainful, trade may fail to improve social welfare is not specific to the case of populations that, income-wise, are non-overlapping. The same finding holds when the income ranges of the two populations overlap. In the Appendix, we consider two cases of overlapping populations: one in which the incomes of the individuals in \( P_1 \) mingle with the incomes of the individuals in \( P_2 \), and another in which the incomes of the individuals in \( P_1 \) lie en bloc between the incomes of the individuals in \( P_2 \). For such income constellations, the construction of the conditions for the necessary levels of trade-induced income increases is analogous to the constellation of two non-overlapping populations studied in Subsection 3.1. However, because this derivation is substantially more taxing, we present in the Appendix simplified cases in which the trading populations consist of two individuals each.

### 4. Discussion

Here we elaborate on the generality of our results in view of the assumptions that we made.

1. The measure of relative deprivation presented in (2) assumes that comparisons with others who are positioned to the right of the individual in the income distribution are of equal weight: the income excesses of those who are close by and the income excesses of those who are farther away are accorded equal importance. Stark et al. (2017a) question the equal weights convention. They propose a general and flexible weighting
protocol, based on the notion that the same importance need not be attached to changes in income of individuals who are placed at different distances from the individual whose relative deprivation is measured. Operationalizing the income shortfall approach via a set of axioms enables Stark et al. to obtain a class of measures that has the form of a power mean of the excesses of the incomes of others, parameterized by a positive number $p$. Replacing (2) with measures taken from this class will not qualitatively change our results as long as trade broadens the individuals’ social space.

2. There is room for an assumption that the importance that individuals accord to the disutility from low relative income, $\alpha$, is a function rather than a (constant) parameter, such that individuals who are positioned higher in the income hierarchy care more about relative income than individuals who are positioned lower down. Some empirical evidence lends support to this assumption. Research in psychology finds that the taste for more increases with having more (Piff, 2014). For example, when people become wealthier, they feel that they are entitled to have yet more wealth, and their behavior changes accordingly. Kraus et al. (2012) argue that individuals high up in the income hierarchy have greater control over their lives and enjoy more personal choices than individuals placed low in the income hierarchy. We can reason that this outcome might arise because whereas individuals lower down are mostly concerned about meeting their basic consumption needs, individuals higher up do not need to worry much about their essential needs and instead focus on status and social goals. Consequently, lower-positioned individuals are particularly concerned about their absolute income, whereas higher-positioned individuals, who recognize that their absolute income meets their basic needs, focus more strongly on comparisons with others and redirect their attention towards assessing their status and income in relation to the incomes of others. Frank (1999) notes that richer individuals expend more effort on actions that demonstrate their better situation: they spend a larger fraction of their income on costly consumer goods, showing off their better financial standing over poorer individuals. The assumed relationship between income position and concern about having low relative income also mirrors the findings of Stephens et al. (2007) that higher-positioned individuals seek to differentiate themselves from other individuals more strongly than lower-positioned individuals. Such differentiation can be achieved by advances in the income hierarchy,
which in turn strengthens the weight accorded to relative income in these individuals’ utility function. As the value of further advances increases, the desire for such advances strengthens. Adopting in our analysis the perspective that a trade that increases every individual’s income increases every individual’s $\alpha$ will strengthen our reported results, as the role of relative deprivation in the utility function will be enhanced.

3. Our measure of relative deprivation is cardinal. It is interesting to consider how our results would be affected if, instead, our measure of relative deprivation was ordinal: the incorporation of a dimension of relative income implies that income is valued in relation to the incomes of others with whom people compare themselves, but the comparison can be undertaken in terms of rank, where the preference for high rank-conferrered income is expressed as distaste for low rank in the income hierarchy. Indeed, writings in economics have long maintained that individuals have strong preference for high (social) rank, and are stressed when they have low (social) rank. Smith has remarked that “the desire of … obtaining rank among our equals, is, perhaps, the strongest of all our desires” (Smith, 1759, Part VI, Section I, Paragraph 4). There is considerable evidence from research in modern economics to the effect that the desire to escape low rank motivates workers to exert more effort (Neckermann and Frey, 2008; Kuhnen and Tymula, 2009; Duffy and Kornienko, 2010; Kosfeld and Neckermann, 2011), and students to perform better (Bandiera et al., 2009; Azmat and Iriberri, 2010).

To sharpen the implication of trade for welfare where trade is viewed as an expansion of individuals’ social space, we consider the simple case of two non-overlapping populations of two individuals each, where incomes are distinct (pairwise different), and where incomes do not change upon trade. Taking a utilitarian stance as before, we relate social welfare to the sum of ranks. In constructing a social welfare function, we assign equal weights to the utilities of all the individuals, and we assume that the individuals derive utility from income and disutility from low rank. Given that the individuals’ incomes are held constant, social welfare is maximized when the sum of the individuals’ ranks is minimized. We thus have the following definition: social welfare under rank preferences (distaste for low relative income) is the negative of the sum of the ranks of the individuals. It follows straightforwardly that the revision of social space brought about by trade unequivocally lowers global social welfare: prior to trade social
welfare in each of the two populations is \(-(1 + 2)\); following trade, global social welfare is \(-(1 + 2 + 3 + 4)\) and, obviously, \(-(1 + 2 + 3 + 4) < -(1 + 2) + [-(1 + 2)]\). Thus, our results are robust to moving from a cardinal measure of relative deprivation to a rank-based measure of relative deprivation.

4. To further hone our argument regarding the welfare consequences of trade as an expansion of social space which are adverse for the uniformly poorer population \(P_1\) and favorable for the uniformly richer population \(P_2\), we replicate the preceding setting of two non-overlapping populations of two individuals each where incomes are distinct and are held constant. We return to the use of the cardinal measure of relative deprivation (2). The welfare consequences of the integration of the two populations’ social spaces can be discerned unequivocally, and are as follows: the welfare of the individuals from the poorer population \(P_1\) is lowered because \(P_1\) members are exposed to increased relative deprivation; the welfare of the individuals from the richer population \(P_2\) is raised because \(P_2\) members are exposed to less relative deprivation, except for the richest individual; and global welfare, measured by the sum of the four levels of relative deprivation post integration, is lowered. Obviously, this superadditivity result requires a formal proof. For that, the reader is referred to Stark (2013).

5. An interesting aspect of our analysis is an implied link with policy formation. We revisit the case of non-overlapping populations, we refer to three populations \(P_1\), \(P_2\), and \(P_3\), and we assume that \(P_3\) is uniformly richer than \(P_2\), and that \(P_2\) is uniformly richer than \(P_1\). We assume further that the trade-generated income increases to members of \(P_1\) will be smaller if \(P_1\) opens up to trade with \(P_2\) than if \(P_1\) opens up to trade with \(P_3\). Suppose that \(P_1\) considers whether to begin trading with \(P_2\) or with \(P_3\). Our analysis invites caution in selecting \(P_3\) as the trading partner: in such a case, the ensuing higher income increases are delivered together with a greater increase of aggregate relative deprivation. Where there is strong distaste for low relative income (high \(\alpha\)), it will be better for \(P_1\) to trade with \(P_2\) than with \(P_3\).

5. Conclusion
The received evidence suggests that trade increases incomes, and that it often reduces the income gap between the trading populations. Although these observations lead to a
widely-held belief that trade increases the wellbeing of the trading populations, we show that social welfare does not necessarily improve; trade may be trembling. Viewing trade as a process that integrates previously unconnected economies and expands people’s social space and their comparison groups, the trade-induced integration may exacerbate people’s stress from having low relative income. Assuming that people derive utility from (absolute) income and disutility from low relative income, and resorting to a utilitarian social welfare function, we show that an increase in the aggregate stress from experiencing low relative income that is brought about by trade can result in trade having a negative overall impact on social welfare. The effect that we identify works independently from the effects of the monetary distribution within populations.
Appendix: A change in social welfare brought about by trade between two overlapping populations

Let the number of members in each of the populations P1 and P2, \( n \), be equal to two. Thus, the pre-trade incomes of the individuals in P1 are \( 0 \leq x_1 < x_2 \), and the pre-trade incomes of the individuals in P2 are \( 0 \leq y_1 < y_2 \). Prior to trade, when populations P1 and P2 are not integrated, the social welfare of each population is

\[
SW_1^{pre} = U_1(x_1) + U_2(x_2) = (1 - \alpha)x_1 - \frac{\alpha}{2}(x_2 - x_1) + (1 - \alpha)x_2, \tag{A1}
\]

and

\[
SW_2^{pre} = U_1(y_1) + U_2(y_2) = (1 - \alpha)y_1 - \frac{\alpha}{2}(y_2 - y_1) + (1 - \alpha)y_2. \tag{A2}
\]

**Example 1.** P1 mingles with P2

We take up the case in which the pre-trade incomes of the two populations are distributed as follows: \( x_1 < y_1 < x_2 < y_2 \). In the wake of trade, each of the incomes \( x_1 \) and \( x_2 \) increases by \( a \), and each of the incomes \( y_1 \) and \( y_2 \) increases by \( b \), where \( a \) and \( b \) are such that they do not change the pre-trade ordering of the incomes between the populations. Thus, the income distribution of the integrated population is given by \( x_1 + a < y_1 + b < x_2 + a < y_2 + b \). Given this income distribution between the two populations, we have that \( \bar{y} > \bar{x} \) and that \( \bar{y} + b > \bar{x} + a \). Then, it follows from (4) that in order for the income gap to be reduced when trade occurs, it has to hold that \( a > b \). The levels of the post-trade social welfare of the two populations are

\[
SW_1^{post} = (1 - \alpha)(x_1 + a)
\]

\[
-\frac{\alpha}{4} \left[ (y_1 + b) - (x_1 + a) + (x_2 + a) - (x_1 + a) + (y_2 + b) - (x_1 + a) \right], \tag{A3}
\]

\[
+ (1 - \alpha)(x_2 + a) - \frac{\alpha}{4} \left[ (y_2 + b) - (x_2 + a) \right],
\]

and
\[ SW^\text{post}_z = (1-\alpha)(y_1+b)-\frac{\alpha}{4}[(x_2+a)-(y_1+b)+(y_2+b)-(y_1+b)] + (1-\alpha)(y_2+b). \] (A4)

Global social welfare will not be reduced in the wake of trade if and only if (5) holds, which, upon substitution of (A1), (A2), (A3), and (A4), and rearrangement, becomes

\[ a\left(\frac{4}{\alpha} - 3\right) + b\left(\frac{4}{\alpha} - 5\right) \geq \frac{y_1 + y_2 - (x_1 + x_2)}{2}. \] (A5)

The left-hand side of (A5) is positive because

\[ a\left(\frac{4}{\alpha} - 3\right) + b\left(\frac{4}{\alpha} - 5\right) > b\left(\frac{4}{\alpha} - 3\right) + b\left(\frac{4}{\alpha} - 5\right) = \frac{b}{\alpha}(4-3\alpha + 4-5\alpha) = \frac{8b}{\alpha}(1-\alpha) > 0, \]

where the first inequality sign follows from the assumption that \( a > b \) and from the fact that \( 4/\alpha > 4 > 3 \). Having assumed that \( x_1 < y_1 \) and \( x_2 < y_2 \), the right-hand side of (A5) is positive, so we obtain a positive threshold value that the income gains from trade need to exceed if social welfare is to register an improvement. Consequently, if trade does not confer large enough income increases, it will fail to improve social welfare. Rewriting the numerator in the right-hand side of (A5) as \( (y_1 - x_1) + (y_2 - x_2) \) reveals that the greater the difference between the pre-trade incomes of the poorer individuals in the two populations, and the greater the difference between the pre-trade incomes of the richer individuals in the two populations, the less likely will condition (A5) be satisfied. Large discrepancies between the incomes described entail a substantial increase in relative deprivation experienced by the individuals upon integration. Consequently, trade has to confer bigger income gains to compensate the social welfare losses.

**Example 2.** P1 lies en bloc in P2
Given that the incomes of population P1 lie en bloc between the incomes of population P2, the pre-trade incomes are distributed as follows: \( y_1 < x_1 < x_2 < y_2 \). In the wake of trade and the consequent increase in the individuals’ incomes, the ordering of the
incomes between the populations is assumed not to change and, thus, the income distribution is given by \( y_1 + b < x_1 + a < x_2 + a < y_2 + b \).

Given this income distribution, the relationship between \( \bar{x} \) and \( \bar{y} \) and, at the same time, the relationship between \( \bar{x} + a \) and \( \bar{y} + b \) are not unambiguous. The sign of \( \bar{x} - \bar{y} \) depends on the exact values of the incomes, as shown by

\[
\bar{x} - \bar{y} = \frac{x_1 + x_2}{2} - \frac{y_1 + y_2}{2} = \frac{(x_1 - y_1) + (x_2 - y_2)}{2},
\]

where in the far right-hand side ratio, the term inside the first parentheses is positive and the term inside the second parentheses is negative. Consequently, as implied by (4), for the income gap between the populations to be narrowed in the wake of trade, \( b > a \) if \( x_1 - y_1 > x_2 - y_2 \), and \( a > b \) if \( x_1 - y_1 < x_2 - y_2 \). Rearrangement of these conditions leads to the following observations. When the dispersion of incomes within P2 is larger than the dispersion of incomes within P1, namely when \( y_2 - y_1 > x_2 - x_1 \), the income gap between the populations will be reduced, provided that the income gain of each individual in P2 is larger than the income gain of each individual in P1 (\( b > a \)). When the dispersion of incomes within P2 is smaller than the dispersion of incomes within P1, namely when \( y_2 - y_1 < x_2 - x_1 \), the income gap between the populations will be reduced, provided that the increase of income of each individual in P1 is larger than the increase of income of each individual in P2 (\( a > b \)).

The levels of the post-trade social welfare of the two populations are

\[
SW_{1}^{\text{post}} = (1 - \alpha)(x_1 + a) - \frac{\alpha}{4} \left[ (x_2 + a) - (x_1 + a) + (y_2 + b) - (x_1 + a) \right] + (1 - \alpha)(x_2 + a) - \frac{\alpha}{4} \left[ (y_2 + b) - (x_2 + a) \right],
\]

and

\[
SW_{2}^{\text{post}} = (1 - \alpha)(y_1 + b) - \frac{\alpha}{4} \left[ (x_1 + a) - (y_1 + b) + (x_2 + a) - (x_1 + a) + (y_2 + b) - (y_1 + b) \right] + (1 - \alpha)(y_2 + b).
\]
Substitution of (A1), (A2), (A6), and (A7) into (5) and a subsequent rearrangement of (5) reveal that in the wake of trade, global social welfare will not be lowered if and only if

\[(a + b)\left(\frac{4}{\alpha} - 4\right) \geq \frac{x_1 - y_1 + y_2 - x_2}{2}. \tag{A8}\]

Because \(\alpha < 1\), the left-hand side of (A8) is positive. And because \(y_1 < x_1\) and \(x_2 < y_2\), we once again obtain a positive threshold value for the income gains, for doubly gainful trade to confer a social welfare improvement. Rewriting the numerator in the right-hand side of (A8) as \((y_2 - y_1) - (x_2 - x_1)\) reveals that the dispersion of incomes within each population affects the threshold. Specifically, the more the dispersion of incomes within P2 exceeds the dispersion of incomes within P1, the harder it is for the condition to be met, namely the harder it is for the post-trade social welfare level not to decline. When the dispersion of incomes within P2 exceeds by a large amount the dispersion of incomes within P1, the two trading populations are more different with respect to income distribution and, thus, integration inflicts greater stress resulting from heightened relative deprivation.
References


