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On social preferences and the intensity of risk aversion

by

Oded Stark
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Abstract

We study the relative risk aversion of an individual with particular social preferences: his wellbeing is influenced by his relative wealth, and by how concerned he is about having low relative wealth. Holding constant the individual’s absolute wealth, we obtain two results. First, if the individual’s level of concern about low relative wealth does not change, the individual becomes more risk averse when he rises in the wealth hierarchy. Second, if the individual’s level of concern about low relative wealth intensifies when he rises in the wealth hierarchy and if, in precise sense, this intensification is strong enough, then the individual becomes less risk averse: the individual’s desire to advance further in the wealth hierarchy is more important to him than possibly missing out on a better rank.

Keywords: Relative risk aversion; Wealth rank; Concern about low relative wealth

JEL classification: D31; D81; G11
1. Introduction

Pratt (1964) and Arrow (1965, 1970) introduced measures of risk aversion, drawing the profession’s attention to the need to study the relationship between wealth and attitudes towards risk: “the behavior of these measures as wealth varies is of the greatest importance for prediction of economic reactions in the presence of uncertainty” (Arrow, 1970, p. 35). Arrow and Pratt hypothesized that relative risk aversion increases with wealth.

Although many subsequent studies, based both on laboratory data (for example, Holt and Laury, 2002) and on field data (for example, Szpiro, 1983, and Eisenhauer and Halek, 1999), lend support to the hypothesis of increasing relative risk aversion, a considerable amount of empirical work fails to align with Arrow and Pratt’s hypothesis. Several researchers find that relative risk aversion decreases with wealth (for example, Cohn et al., 1975, and Bellante and Saba, 1986, using field data; Levy, 1994, using laboratory data); that relative risk aversion is constant regardless of wealth (for example, Szpiro, 1986, and Chiappori and Paiella, 2011, based on field data); or that the relationship between wealth and relative risk aversion is non-linear (for example, Morin and Suarez, 1983, and Halek and Eisenhauer, 2001, who use field data, find that relative risk aversion increases with wealth for low levels of wealth and decreases with wealth for high levels of wealth; the first of these studies concludes that for the richest, the pattern of decreasing relative risk aversion is so weak that relative risk aversion can be considered constant). One reason for the apparent divergence could be that in the received studies, the “behavior” of the measure of relative risk aversion is related to changes in absolute wealth, neglecting to account for changes in relative wealth; implicitly, relative wealth is kept constant. In seeking to deepen our understanding of the impact of wealth on attitudes towards risk, in this paper we relate the varying “behavior” of the measure of relative risk aversion not to variation in absolute wealth, which we keep constant, but to variation in relative wealth and, in particular, to variation in the level of concern about having low relative wealth.

The perception that differences in risk preferences are attributable to relative wealth is supported by research in development economics (Thai villages) which finds
that these differences “turn out not to be related to wealth,” and that “there is no correlation of risk aversion with wealth, and so the more risk-tolerant are not necessarily more wealthy” (Townsend, 2016, p. 207).

While there is some acknowledgement in the received literature of a link between relative wealth / status and risk-taking / gambling behavior, the received writings do not follow the track that we pursue in this paper. Most notably, Gregory (1980) and Robson (1992) allude to a link between relative wealth and gambling behavior, remarking that the incorporation of relative wealth / status in an individual’s utility function can explain the Friedman and Savage (1948) paradox of seemingly inconsistent risk-taking behavior of an individual following a change in his wealth. However, Gregory does not specify a link between relative wealth and any concrete measure of risk aversion. Robson investigates connections between wealth distributions and fair gambles, yet he too does not link status with any measure of risk aversion. Robson expands the individual’s utility function to include a status term based on the individual’s rank in the wealth distribution. There are, though, two notable differences between Robson’s approach and ours: one difference relates to the rank character of Robson’s measure, the other - to the direction of wealth-related comparisons. First, in Robson’s model, an increase in the wealth of individuals to the right of individual i in the ascendingly-ordered wealth distribution which occurs while i’s rank remains unchanged does not change i’s status and, consequently, i’s utility is not affected. However, the relative wealth deprivation of individual i does change (it increases) when the wealth of individuals to his right increases (even when i’s rank does not change) which, in turn, impinges on i’s utility. (As explained and defined in the next section, we use the received cardinal index of relative deprivation of individual i as a measure of the individual’s relative wealth deprivation.) Second, incorporating rank in the preferences can be interpreted as looking towards both the poorer and richer individuals when evaluating utility, whereas the measure of relative wealth deprivation that we use assumes that comparisons are upward (or that the comparisons that behaviorally matter are upward). This difference in approach to the inclusion of interpersonal comparisons in the utility function is substantial. Our approach is in line with the overwhelming weight of the received evidence, which supports the notion of a strong asymmetry: the comparisons that significantly affect an individual’s sense of
wellbeing are the ones rendered by looking “up” the hierarchy, whereas looking “down” does not appear to be of much consequence. For example, Cohn et al. (2014) find that in deciding how hard to work, workers respond to increased relative deprivation but not to increased “relative satisfaction.” Frey and Stutzer (2002), Walker and Smith (2002), and Stark (2013) review a large body of evidence that lends support to the “upward comparison” hypothesis.

It is worth emphasizing that an important difference between Robson’s approach and ours is that, whereas rank is an ordinal measure, our measure of relative wealth deprivation is cardinal. To underscore the importance of this distinction, we note that in our setting, holding an individual’s wealth constant, and holding his rank as well as the ranks of all other individuals in the wealth hierarchy constant, the individual’s relative risk aversion will vary when the wealth of any individual higher up in the hierarchy changes. Drawing on well-established measures of relative deprivation and relative risk aversion, we conduct a systematic analysis of how a cardinally-measured concern about relative wealth affects relative risk aversion. We do this without limiting ourselves to specific distributions of wealth. Therefore, our approach is somewhat more general than the approaches of Gregory or Robson, who both use specific distributions of wealth of the population (Gregory - a gamma distribution; Robson, in the main - a uniform distribution).

Our approach to the social context in which preferences towards risk taking are formed differs from an approach taken in the psychology literature with regard to the influence of group affiliation on risk-taking behavior. That literature presents approaches such as a “risky shift” (Stoner, 1961), where the risk tolerance in the decision-making of a group is higher than the average risk tolerance of the individual members of the group, and a “cautious shift” (Wallach et al., 1963), where a decreased propensity to take risks is attributed to responsibility for the wellbeing of the group. In our approach, it is not group membership per se but, rather, the position in the group’s wealth hierarchy that shapes risk preferences.

In Stark and Zawojaska (2015), a causal link is established between an individual’s degree of concern about his relative wealth and his relative risk aversion, yielding the
result that individuals who assign greater importance to their relative wealth exhibit weaker relative risk aversion. This result is obtained while keeping the individual’s wealth constant, both in absolute terms and in relative terms, and while assuming that the individual’s distaste for low relative wealth is exogenous, in particular in the sense of not being affected by the individual’s position in the wealth hierarchy. In the current paper we inquire how relative risk aversion is modified when we relax these assumptions, namely when, at the same time, we allow the individual’s relative wealth to change, and the intensity of his concern about having low relative wealth to change monotonically with his rank in the wealth hierarchy; we assume that when an individual advances in the wealth hierarchy, his distaste for low relative wealth intensifies.

The assumption that individuals who are positioned higher in the wealth hierarchy care more about relative wealth than individuals who are positioned lower down is supported by empirical evidence. Research in psychology finds that the taste for more increases with having more (Piff, 2014). For example, when people become wealthier, they feel that they are entitled to have more wealth, and their behavior changes accordingly. Kraus et al. (2012) find that individuals who are placed high in the wealth distribution have greater control over their lives and enjoy more personal choices than individuals placed low in the wealth distribution. We can reason that this outcome might arise because whereas individuals lower down are mostly concerned about meeting their basic consumption needs, individuals higher up do not need to worry much about their essential needs and, instead, they focus on their status and goals. Consequently, lower-ranked individuals are particularly concerned about their absolute wealth, whereas higher-ranked individuals who recognize that their absolute wealth meets their basic needs, focus more strongly on comparisons with others, and redirect their attention towards assessing their status and wealth in relation to the wealth of others. Frank (1999) notes that in comparison with individuals placed low in the wealth hierarchy, individuals placed high expend more effort on actions that demonstrate their better situation: higher ranked individuals spend a larger fraction of their income on costly consumer goods, showing off their better financial standing over lower ranked individuals. The assumed relationship between wealth rank and concern about having low relative wealth also mirrors the findings of Stephens et al. (2007) that higher-ranked individuals seek to
differentiate themselves from other individuals more strongly than lower-ranked individuals. Such differentiation can be achieved by advances in the wealth hierarchy, which in turn strengthens the weight accorded to relative wealth in these individuals’ utility function. As the value of further advances increases, the desire for such advances strengthens.

We structure our argument as follows. As a base for comparison, we show, using standard measures of relative wealth, that when an individual’s concern about having low relative wealth does not change as he advances in the wealth hierarchy while his absolute wealth is held constant, the individual becomes more risk averse. In the base case, the individual’s distaste for low relative wealth is assumed to be exogenous and constant (that is, it is not affected by his position in the wealth hierarchy), and his intensified risk aversion reflects his desire to preserve his superior wealth rank. However, when the intensity of the individual’s concern about having low relative wealth increases as he rises in the wealth hierarchy, the individual may become less risk averse; the individual’s desire to advance further in the wealth hierarchy then outdoes the possible forfeiting of the better rank. Put differently, when an advance in the wealth hierarchy strengthens the taste for further advancement, the temptation to take risks may gain strength too.

Section 2 forges a link between distaste for low relative wealth and relative risk aversion. Section 3 shows how relative risk aversion changes when an individual rises in the wealth hierarchy, keeping constant both his absolute wealth and the intensity of his concern about having low relative wealth. We find that under a specified condition that is fulfilled by the standard measures of relative deprivation, a rise in rank leads to an increase in the individual’s relative risk aversion. Section 4 explores the effect on relative risk aversion of allowing the intensity of concern about having low relative wealth to increase as the individual rises in the wealth hierarchy. We find that when the increase in the individual’s concern about having low relative wealth is sufficiently strong, the individual’s relative risk aversion decreases with his rise in rank, a result that is the opposite of the one obtained in Section 3. Section 5 provides a graphical intuition for the results reported in Sections 3 and 4. Section 6 concludes. In the Appendix we conduct a robustness check of the findings reported in Sections 3 and 4. Replacing the assumption that utility depends linearly on relative deprivation with the assumption of a (small)
degree of concavity of the utility function with respect to relative deprivation, we show that the two main results reported in Sections 3 and 4 continue to hold.

2. Linking distaste for low relative wealth with relative risk aversion

Consider a population $P$ of $n \geq 3$ individuals, where $n$ is an integer, with a vector $\mathbf{w} = (w_1, \ldots, w_n)$ of wealth levels such that $0 < w_1 < w_2 < \ldots < w_n$. Let the utility function of individual $i \in P$ be $u_i(w_i, RD_i)$, where $w_i$ is the wealth level of individual $i$, and $RD_i$ is the relative deprivation of individual $i$, which we use as a measure of the individual’s relative wealth. We assume that $u_i(w_i, RD_i)$ is twice continuously differentiable, strictly increasing in $w_i$, and strictly decreasing in $RD_i$. Specifically, we let

$$u_i(w_i, RD_i) = (1 - \beta_i) f(w_i) - \beta_i RD_i(w),$$

(1)

where (positive) $f(w_i)$, a twice continuously differentiable, strictly increasing, and strictly concave function, describes the preferences towards one’s own wealth; $\beta_i \in (0,1)$ is a measure of the intensity of individual $i$’s concern about having low relative wealth; and $RD_i(w)$ is twice continuously differentiable in some neighborhood of $w_i$ and strictly decreasing in $w_i$, and $RD_i(w_n) = 0$. The assumptions concerning $RD_i(w)$ imply that for relatively deprived individuals, namely for individuals who experience low relative wealth (these are individuals who have another individual or other individuals in $P$ richer than themselves),

$$\frac{\partial RD_i(w)}{\partial w_i} = RD_i'(w) < 0$$

(an increase in the absolute wealth of a relatively deprived individual $i$ lowers his relative deprivation). We also assume that for relatively deprived individuals, $\frac{\Delta RD_i'(w)}{\Delta i} > 0$ which, because $RD_i'(w) < 0$, means that the change in the individual’s relative deprivation arising from an increase in his wealth is smaller when the individual’s rank in the wealth hierarchy is higher. Put differently, in terms of a reduction in relative deprivation, richer individuals will be less affected by a given wealth gain than less rich individuals. We consider this assumption to be quite natural (recall the literature references provided in the Introduction).
Our measure of the individual’s relative deprivation is cardinal: it is sensitive to changes in the wealth levels of the individuals who are higher up in the wealth distribution even if the changes do not translate into revisions of the individual’s ordinal rank. For example, in wealth distribution (10, 20), the ordinal measure of relative deprivation of the individual whose wealth is 10 is the same (second) as in wealth distribution (10, 11), whereas the cardinal measure is not the same. A rationale, background, and applications of the cardinal measure are provided in Stark (2013).

For the sake of brevity, we denote by \( u_i(w_i) \) the utility of individual \( i \) as defined in (1). The first derivative of \( u_i(w_i) \) with respect to \( w_i \) is then
\[
    u'_i(w_i) = (1 - \beta_i)f'(w_i) - \beta_iRD'_i(w),
\]
and the second derivative of \( u_i(w_i) \) with respect to \( w_i \) is
\[
    u''_i(w_i) = (1 - \beta_i)f''(w_i) - \beta_iRD''_i(w),
\]
using the convention of a prime and of a double prime to represent first and second derivatives, respectively.

We measure the relative risk aversion of individual \( i \) by the Arrow-Pratt coefficient \( r_i \) (Pratt, 1964; Arrow, 1965, 1970), that is, by
\[
    r_i \equiv \frac{-w_iu''_i(w_i)}{u'_i(w_i)},
\]
which is well-defined in some neighborhood of \( w_i \).

Using (2) and (3), \( r_i \) takes the form
\[
    r_i = \frac{-w_i[(1 - \beta_i)f''(w_i) - \beta_iRD''_i(w)]}{(1 - \beta_i)f'(w_i) - \beta_iRD'_i(w)}.
\]
3. An improved position in the wealth distribution with no (absolute) wealth gain

We consider a change in the levels of wealth among the members of population $P$ such that an individual who, to begin with, had wealth $w_i^1$ and held a rank other than the top one advances by one rank, namely advances from rank $i \in \{1, \ldots, n-1\}$ to rank $i+1$, and does so while retaining his wealth level.\(^1\) We refer to this individual as “individual $\omega$."

We represent the change in the wealth hierarchy in $P$ as a change in the ordered vector of the wealth levels from $w^1 = (w_1^1, \ldots, w_n^1)$ to $w^2 = (w_1^2, \ldots, w_n^2)$ such that the wealth level of individual $\omega$ in the two wealth distributions is held the same, namely $w_i^1 = w_i^2$. The said gain in rank can arise from a sufficient decrease in the wealth of one of the individuals who, to begin with, had a rank higher than the rank of individual $\omega$ (that is, a rank higher than rank $i$) and who, subsequently, ends up positioned below individual $\omega$ in the wealth hierarchy.

We assume that the following decompositions can be made:

$$RD_{i+1}(w^2) = RD_i(w^1) + \frac{\Delta RD_i(w^1)}{\Delta i} \quad \text{and} \quad RD_{i+1}(w^2) = RD_i(w^1) + \frac{\Delta RD_i(w^1)}{\Delta i},$$

namely that the change in the derivatives of relative deprivation arising from a change in the wealth distribution is additive. This assumption is not overly restrictive because it holds for the standard measures of relative deprivation used in the received literature (as, for example, in Stark, 2013; these measures are presented between equations (4) and (5) below). For more complex general measures, the assumption can be perceived as a first order approximation.

In this section we assume that population $P$ is homogenous and static, in the sense that the preferences for low relative wealth are the same for all the individuals belonging to $P$, and that these preferences do not depend on the individual’s position in the wealth hierarchy, namely we assume that $\beta_1 = \beta_2 = \ldots = \beta_n = \beta$. How is the relative risk aversion of individual $\omega$ affected by the change in his position in the wealth hierarchy?

We denote by $\Delta R_{i,i+1}$ the change in $\omega$’s relative risk aversion after he advances from the

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\(^1\) To ease the derivations, we consider a gain of a single position in the wealth hierarchy. However, our conclusions would not change if we were to examine a gain by $l > 1$ positions, namely if an individual were to advance from rank $j \in \{1, \ldots, n-l\}$ to rank $j+l$. 

8
ith position to the \((i + 1)\)th position, while both his wealth level and the intensity of his concern about having low relative wealth are held constant. We note that

\[
\Delta r_{i,i+1} = r_{i+1} - r_i
\]

\[
= -w_i^2 \left[ (1 - \beta) f''(w_i^2) - \beta \Delta \text{RD}_i^2(w_i^2) \right] - \frac{w_i \left[ (1 - \beta) f''(w_i^2) - \beta \Delta \text{RD}_i^2(w_i^2) \right]}{(1 - \beta) f'(w_i^2) - \beta \Delta \text{RD}_i'(w_i^2)} - \frac{\Delta \text{RD}_i'(w_i^2)}{\Delta i} \frac{(1 - \beta) f''(w_i^2) - \beta \Delta \text{RD}_i'(w_i^2)}{(1 - \beta) f'(w_i^2) - \beta \Delta \text{RD}_i'(w_i^2)} - \frac{\Delta \text{RD}_i'(w_i^2)}{\Delta i} \frac{(1 - \beta) f''(w_i^2) - \beta \Delta \text{RD}_i'(w_i^2)}{(1 - \beta) f'(w_i^2) - \beta \Delta \text{RD}_i'(w_i^2)}
\]

\[
= \{ (1 - \beta) f''(w_i^2) - \beta \Delta \text{RD}_i'(w_i^2) \} \left[ (1 - \beta) f''(w_i^2) - \beta \Delta \text{RD}_i'(w_i^2) \right] - \frac{\Delta \text{RD}_i'(w_i^2)}{\Delta i} \frac{(1 - \beta) f''(w_i^2) - \beta \Delta \text{RD}_i'(w_i^2)}{(1 - \beta) f'(w_i^2) - \beta \Delta \text{RD}_i'(w_i^2)} - \frac{\Delta \text{RD}_i'(w_i^2)}{\Delta i} \frac{(1 - \beta) f''(w_i^2) - \beta \Delta \text{RD}_i'(w_i^2)}{(1 - \beta) f'(w_i^2) - \beta \Delta \text{RD}_i'(w_i^2)}
\]

\[
= \frac{\Delta \text{RD}_i'(w_i^2)}{\Delta i} \frac{(1 - \beta) f''(w_i^2) - \beta \Delta \text{RD}_i'(w_i^2)}{(1 - \beta) f'(w_i^2) - \beta \Delta \text{RD}_i'(w_i^2)} - \frac{\Delta \text{RD}_i'(w_i^2)}{\Delta i} \frac{(1 - \beta) f''(w_i^2) - \beta \Delta \text{RD}_i'(w_i^2)}{(1 - \beta) f'(w_i^2) - \beta \Delta \text{RD}_i'(w_i^2)}
\]

In the last line of (4), the term \(u_i'(w_i^2)u_i'(w_i^2)\) in the denominator is clearly positive, and the term \(-w_i\beta\) in the numerator is obviously negative. Thus, the sign of \(\Delta r_{i,i+1}\) depends on the sign of the numerator term \(\frac{\Delta \text{RD}_i'(w_i^2)}{\Delta i} \frac{(1 - \beta) f''(w_i^2) - \beta \Delta \text{RD}_i'(w_i^2)}{(1 - \beta) f'(w_i^2) - \beta \Delta \text{RD}_i'(w_i^2)}\). For the standard measures of relative deprivation defined (as, for example, in Stark, 2013) as \(\text{RD}_{1,i}(w) = \frac{1}{n} \sum_{k=1}^{n} \max\{w_k - w_i, 0\}\) and \(\text{RD}_{2,i}(w) = \max\{\bar{W} - w_i, 0\}\) where \(\bar{W}\) denotes the average wealth of population \(P\), we have that \(\text{RD}_{i,i}(w) = 0\) and, therefore, (3) can be simplified to \(u_i''(w_i) = (1 - \beta_i)f''(w_i)\).\(^2\) Then, for the bracketed term in the numerator in the last line of (4), we get that

\[
\frac{\Delta \text{RD}_i'(w_i^2)}{\Delta i} \frac{(1 - \beta) f''(w_i^2) - \beta \Delta \text{RD}_i'(w_i^2)}{(1 - \beta) f'(w_i^2) - \beta \Delta \text{RD}_i'(w_i^2)} < 0,\]

\(^2\) Because \(\text{RD}_{2,i}(w)\) is not differentiable at \(w_i = \bar{W}\), we confine the analysis to wealth levels \(w_i\) such that \(w_i < \bar{W}\).
where the inequality sign in (5) follows from the strict concavity of $f(w_i)$. As a consequence, $\Delta r_{i,+1}$ in (4) is positive: the relative risk aversion of individual $\omega$ increases when his position in the wealth distribution improves. That an individual who attains a better position in terms of relative wealth becomes more relatively risk averse can be reasoned by the possible negative consequence of undertaking risky decisions, as these decisions can jeopardize the individual’s valued gain of an improved position in the wealth distribution. Thus, intensified risk aversion reflects the individual’s desire to preserve the better wealth rank that he has achieved. Conversely, that an individual who loses rank in the wake of a revision in the wealth distribution becomes less risk averse can be explained by acknowledging a desire to escape the unwarranted loss of position; taking risks provides this individual with an opportunity to gain a swift improvement in his position in the wealth hierarchy. In essence, this reasoning resembles an argument of Banerjee and Newman (1994, p. 211) that “the poor are closer to the lower bound on their utility than the rest of the population. Consequently, threats of punishment work less well against the poor than against others: the poor behave as if they have nothing to lose.”

4. An improved position in the wealth distribution with no (absolute) wealth gain, accompanied by intensified concern about having low relative wealth

We now relax the assumption that the individuals’ preferences towards relative wealth, represented by the $\beta$ coefficients assigned to relative deprivation in their utility functions, are homogenous and static. The assumption that the intensity of an individual’s distaste for being relatively deprived does not change when he changes his position in the wealth hierarchy is debatable if we consider it plausible, as reasoned in the Introduction, that the importance that individuals attach to relative wealth is position-sensitive in the sense that individuals higher up in the wealth distribution care more about relative wealth than those lower down.

We thus ask whether the positive sign of (4) continues to hold when the $\beta$ weight changes, specifically when it increases monotonically with rank. We find that for individual $\omega$ (as defined in Section 3), the link exhibited in (4), namely that lowered
relative deprivation arising from improved wealth rank increases relative risk aversion, might be reversed.

We consider individual $\omega$ who advances in the wealth hierarchy from the $i$th rank to the $(i+1)$th rank, while his wealth level remains intact (that is, $w_{i+1} = w_i$) and, who upon gaining a higher position in the wealth hierarchy, increases the weight that he accords to low relative wealth from $\beta_i$ to $\beta_{i+1}$. How does $\omega$’s relative risk aversion change? Rewriting $\Delta r_{i,i+1}$ as a function of the weights, namely expressing $\Delta r_{i,i+1}$ as

$$
\Delta r_{i,i+1}(\beta_i, \beta_{i+1}) = r_{i+1} - r_i
$$

$$
= \frac{w_i^2}{(1 - \beta_{i+1})f'(w_i)} \left[ (1 - \beta_{i+1})f''(w_i) - \beta_{i+1}RD_i'(w_i) \right] - \frac{w_i^2}{(1 - \beta_i)f'(w_i)} \left[ (1 - \beta_i)f''(w_i) - \beta_iRD_i'(w_i) \right] - w_i^2 \left[ RD_i'(w_i) + \frac{\Delta RD_i'(w_i)}{\Delta i} \right]
$$

$$
= \frac{w_i^2}{(1 - \beta_{i+1})f'(w_i)} \left[ (1 - \beta_{i+1})f''(w_i) - \beta_{i+1}RD_i'(w_i) \right] - \frac{w_i^2}{(1 - \beta_i)f'(w_i)} \left[ (1 - \beta_i)f''(w_i) - \beta_iRD_i'(w_i) \right] - w_i^2 \left[ RD_i'(w_i) + \frac{\Delta RD_i'(w_i)}{\Delta i} \right]
$$

$$
= \frac{w_i^2}{u_{i+1}(w_i)u_i(w_i)} \left[ \frac{\Delta RD_i'(w_i)}{\Delta i} u_i'(w_i) - \frac{\Delta RD_i'(w_i)}{\Delta i} u_i'(w_i) \right]
$$

$$
+ \frac{\beta_{i+1} - \beta_i}{\beta_{i+1}} \left[ RD_i'(w_i) f''(w_i) - RD_i'(w_i) f'(w_i) \right].
$$

The denominator of the first term in the last part of (6) (namely $u_{i+1}(w_i)u_i'(w_i)$) is positive, and the numerator of the first term in the last part of (6) (namely $-w_i^2$) is obviously negative. To find out the sign of (6), the sign of the expression inside the curly brackets in the last part of (6) needs to be determined. This expression is similar to the equivalent one in the last line of (4), with the difference being the added term $\frac{\beta_{i+1} - \beta_i}{\beta_{i+1}} \left[ RD_i'(w_i) f''(w_i) - RD_i'(w_i) f'(w_i) \right]$ in (6). If $RD_i'(w_i) = 0$, which is the case for

3 As in the comment made in footnote 1, our conclusions in this section will not change if we consider an individual who advances by $l$ positions in the wealth hierarchy.
the standard measures of relative deprivation, namely when \( RD_i(w^i) = RD_{i,j}(w^i) \) and when \( RD_i(w^i) = RD_{2,j}(w^i) \) as presented in the preceding section, then
\[
\frac{\beta_{i+1} - \beta_i}{\beta_{i+1}} \cdot RD_i'(w^i) f''(w^i) \]
will be positive. Therefore, if the difference \( \beta_{i+1} - \beta_i \) is large enough (namely if the weight accorded to relative deprivation is highly responsive to a change in rank), \( \Delta r_{i,i+1}(\beta_i, \beta_{i+1}) \) in (6) will be negative.

To gain further insight into this constellation, we refer once again to the specific measure of relative deprivation \( RD_{i,i}(w) = \frac{1}{n} \sum_{k=1}^{n} \max\{w_k - w_i, 0\} \). For this measure we have that \( RD_{i,i}'(w) = -\frac{n-i}{n} \), that \( RD_{i,i}'(w) = 0 \), that \( \frac{\Delta RD_{i,i}'(w)}{\Delta i} = \frac{1}{n} \), and that
\[
\frac{\Delta RD_{i,i}''(w)}{\Delta i} = 0. \]
Then, (6) simplifies into
\[
\Delta r_{i,i+1}(\beta_i, \beta_{i+1}) = -w_i^j \beta_{i+1} \left[ \frac{1}{n} \left( 1 - \beta_i \right) f''(w^i) \right] - \frac{\beta_{i+1} - \beta_i}{\beta_{i+1}} \left( -\frac{n-i}{n} \right) \left( 1 - \beta_i \right) f'(w^i) \beta_i \left( -\frac{n-i}{n} \right)
\]
\[
= -w_i^j \left[- f''(w^i) \right] \frac{1}{n} \left( (n-i)(\beta_{i+1} - \beta_i) - \beta_{i+1}(1-\beta_i) \right) \left( 1 - \beta_i \right) f'(w^i) + \beta_i \frac{n-i}{n} \left( 1 - \beta_i \right) f'(w^i) \beta_i \frac{n-i}{n} \]
\[
\text{(7)}
\]
Looking at the last line of (7), we note that given the obvious signs of the denominator (where \( n-i-1 > 0 \)) and of the first three terms in the numerator (namely the terms \( -w_i^j \left[- f''(w^i) \right] \frac{1}{n} \)), the sign of \( \Delta r_{i,i+1}(\beta_i, \beta_{i+1}) \) is determined by the sign of the last term in the numerator (namely by the sign of \( (n-i)(\beta_{i+1} - \beta_i) - \beta_{i+1}(1-\beta_i) \)). Thus, the move to a position at which concern about having low relative wealth is more intense will lower relative risk aversion, namely it will result in \( \Delta r_{i,i+1}(\beta_i, \beta_{i+1}) < 0 \), if and only if
\[
(n-i)(\beta_{i+1} - \beta_i) - \beta_{i+1}(1-\beta_i) > 0,
\]
which is equivalent to stating that \( \Delta r_{i,i+1}(\beta_i, \beta_{i+1}) < 0 \) if and only if

\[
\beta_{i+1} > \frac{(n-i)\beta_i}{n-i-(1-\beta_i)} \equiv \beta^*_{i+1}(\beta_i).
\]

(8)

By assumption, \( \beta_{i+1} \) must fall between 0 and 1. And, for sure, \( \beta^*_{i+1}(\beta_i) \) is positive and, what is more, we have that \( \beta^*_{i+1}(\beta_i) > \beta_i \) because \( \frac{n-i}{n-i-(1-\beta_i)} > 1 \). Consequently, if there exists \( \beta^*_{i+1}(\beta_i) \) that is less than 1, then the relative risk aversion of an individual who becomes increasingly concerned about having low relative wealth as he advances in the wealth hierarchy can decrease. We note that \( \beta^*_{i+1}(\beta_i) < 1 \) if and only if

\[
\frac{(n-i)\beta_i}{n-i-(1-\beta_i)} < 1,
\]

which is equivalent to \( 0 < (1-\beta_i)(n-i-1) \). Because \((1-\beta_i)\) and \((n-i-1)\) are both positive, the condition \( \beta^*_{i+1}(\beta_i) < 1 \) indeed holds.

This discussion reveals that there can exist \( \beta_{i+1} \in (\beta^*_{i+1}(\beta_i),1) \) such that the relative risk aversion of individual \( \omega \) decreases as he gains in rank. Therefore, starting with some \( \beta_1 < 1 \), we can construct a sequence of the \( \beta \) weights such that \( \beta_{i+1} > \beta^*_{i+1}(\beta_i) \) while \( \beta_{i+1} < 1 \) for \( i \in \{2, \ldots, n-2\} \), which represents the progression of the increasing weights assigned to relative wealth of an individual who starts at the bottom of the wealth hierarchy and subsequently advances in the wealth hierarchy, one step at a time, while his own (absolute) wealth does not change.

To complete the characterization of the change in relative risk aversion when an increase in rank is accompanied by an increase in the \( \beta \) weight, we note that for \( i = n-1 \),
\[
\Delta r_{n-1,n}(\beta_{n-1}, \beta_n) = r_n - r_{n-1} \\
= \frac{-w_n^2(1 - \beta_n) f''(w_n^2)}{(1 - \beta_n) f'(w_n^2)} - \frac{-w_{n-1}^2(1 - \beta_{n-1}) f''(w_{n-1}^2)}{(1 - \beta_{n-1}) f'(w_{n-1}^2)} + \frac{\beta_{n-1}}{n} \\
= \frac{-w_{n-1}^2 f'(w_{n-1})}{f'(w_{n-1})} - \frac{-w_{n-1}^2(1 - \beta_{n-1}) f''(w_{n-1}^2)}{(1 - \beta_{n-1}) f'(w_{n-1}) + \frac{\beta_{n-1}}{n}} \\
= \frac{-w_{n-1}^2(1 - \beta_{n-1}) f''(w_{n-1}^2)}{(1 - \beta_{n-1}) f'(w_{n-1})} - \frac{-w_{n-1}^2(1 - \beta_{n-1}) f''(w_{n-1}^2)}{(1 - \beta_{n-1}) f'(w_{n-1}) + \frac{\beta_{n-1}}{n}} > 0.
\]

Thus, regardless of the change in the \( \beta \) weight from \( \beta_{n-1} \) to \( \beta_n \), the relative risk aversion of an individual who advances to the top of the wealth hierarchy certainly increases. After all, for an individual who has reached the top there is nothing to be gained in terms of wealth rank, so for such an individual, it is not reasonable to expect an increasing willingness to take risks in order to improve his position in the wealth hierarchy.

We illustrate our finding with the help of an example. In Figure 1 we depict a series \( \beta_i \) satisfying condition (8), where \( \beta_1 = 0.1 \), and \( \beta_i = \beta_i^\ast (\beta_{i-1}) + 0.01 \) for \( i = 2, \ldots, 19 \), and \( n = 20 \). In Figure 2 we plot the relative risk aversion, \( r_i \), of individual \( \omega \) with preferences towards absolute wealth described by the logarithmic function \( f(w) = \ln w \), with a constant level of wealth \( w \) set at 10, with \( \beta_1 = 0.1 \), and with \( \beta_i = \beta_i^\ast (\beta_{i-1}) + 0.01 \) for \( i = 2, \ldots, 19 \). We observe that as the individual gains rank and attaches increasing weight to his distaste for having low relative wealth, the individual’s relative risk aversion coefficient decreases.
Figure 1. The progression of $\beta_i$ where $\beta_1 = 0.1$, and $\beta_i = \beta_i^{(1)} (\beta_{i-1}) + 0.01$ for $i = 2, \ldots, 19$.

Note: $\beta_i$ is a measure of the intensity of individual $i$’s concern about having low relative wealth.
Figure 2. The progression of $r_i$ where $w = 10$, $\beta_i = 0.1$, and $\beta_i = \beta_i^* (\beta_{i-1}) + 0.01$ for $i = 2, \ldots, 19$.

Note: $r_i$ is the Arrow-Pratt coefficient of relative risk aversion.

5. A graphical intuition for the results reported in Sections 3 and 4

Going back to the initial representation and notation of the utility function in Section 2, the utility function $u_i(w_i, RD_i)$ in (1) is a weighted average of two increasing functions, $f(w_i)$ and $-RD_i$, with weights equal to $(1 - \beta_i)$ and $\beta_i$, respectively. The standard measures of relative deprivation assume that the function $RD_i$ is linear in some neighborhood of $w_i$. Given this assumption, Figure 3 presents the two components of the utility function in some neighborhood of $w_i$: 
Figure 3. A graphical representation of the components of the function $u_i(w_i, RD_i)$.

Note: $f(w_i)$ describes the preferences of individual $i$ towards his own wealth; $RD_i$ is the relative deprivation of individual $i$; $w_i/w_j$ is the wealth level of individual $i$.

Two observations merit comment.

First, the coefficient of relative risk aversion $r_i = -\frac{w_iu_i'(w_i)}{u_i(w_i)}$ depends on the steepness of the utility function (the denominator of the coefficient) and on the concavity of the utility function (the numerator of the coefficient). The steeper the linear component of the utility function, the bigger the denominator of the coefficient. If the linear component is steep, then for a given marginal change of $w_i$, we will have a bigger change in utility. When the concavity of the utility function (the numerator of the coefficient) is held constant, the change in the coefficient of relative risk aversion will be determined entirely by the influence of the linear component in the denominator of the coefficient. To illustrate this, we draw on the specific formulation of the relative deprivation measure introduced in Section 3, namely on
\( RD_i = RD_{i,i}(w) = \frac{1}{n} \sum_{k=1}^{n} \max \{w_k - w_i, 0\} \). For this measure, the utility function (1) can be decomposed as follows:

\[
u(w_i) = (1 - \beta_i) f(w_i) - \beta_i \frac{1}{n} \sum_{k=1}^{n} \max \{w_k - w_i, 0\} = (1 - \beta_i) f(w_i) + \beta_i \frac{n - i}{n} w_i - \beta_i \frac{1}{n} \sum_{k=1}^{n} w_k,
\]

where \( \frac{n - i}{n} \) is the slope of the linear component (and where \( \frac{1}{n} \sum_{k=1}^{n} w_k \) is a constant).

Because \( RD''_{i,i}(w) = 0 \) (the relative deprivation component does not influence the concavity of the utility function), the coefficient of relative risk aversion can then be written as

\[
r_i = \frac{-w_i(1 - \beta_i) f''(w_i)}{(1 - \beta_i) f'(w_i) + \beta_i \frac{n - i}{n}}, \quad (9)
\]

and it is evident that the coefficient decreases with the slope \( \frac{n - i}{n} \).

Second, the greater the weight of the linear component of the utility function, the steeper the function (the bigger the denominator of the coefficient), implying a lower level of relative risk aversion. This observation can be confirmed by inspecting the first derivative of (9) with respect to \( \beta_i \), given by

\[
\frac{\partial r_i}{\partial \beta_i} = \frac{w_i f''(w_i) \frac{n - i}{n}}{\left[ (1 - \beta_i) f'(w_i) + \beta_i \frac{n - i}{n} \right]^2} < 0,
\]

where the inequality sign is due to the concavity of \( f(w_i) \).

In light of these two observations, we can offer the following intuition for the results obtained in Sections 3 and 4.

The result reported in Section 3 rests on the following observation: when the individual’s rank improves while the individual’s absolute wealth does not change, the linear component in Figure 1 becomes flatter (the red line \( -RD'_i \)' in Figure 4). This can
be easily seen, as the slope is $\frac{n-i}{n}$. If $-RD_i$ becomes flatter then, in accordance with the first observation above, relative risk aversion increases, namely we get the result reported in Section 3.

![Figure 4](image)

Figure 4. A representation of the result of Section 3.

Note: $f(w_i)$ describes the preferences of individual $i$ towards his own wealth; $RD_i$ is the relative deprivation of individual $i$; $w_i/w_i$ is the wealth level of individual $i$.

However, according to the second observation, if the $\beta_i$ weight increases, then relative risk aversion is reduced. So it is intuitive that despite the linear component $-RD_i$ becoming flatter, relative risk aversion will be lower if at the same time there is a sufficient increase in $\beta_i$, namely we have the result reported in Section 4.

In the Appendix we ask how robust the results reported in Sections 3 and 4 are if the assumption that the utility of individual $i$ depends linearly on a measure of his relative deprivation (as per equation (1)) is relaxed. We find that under conditions that are similar to the ones provided in the main text, the two main results continue to hold: when the
preferences towards low relative wealth do not vary with rank, a gain in rank leads to an increase in relative risk aversion, and when the preferences towards low relative wealth increase sufficiently strongly with higher rank, a gain in rank leads to a decrease in relative risk aversion.

6. Conclusions

This paper presents a finding from an ongoing research program aimed at deepening our understanding of the social-psychological foundations of risk-taking behavior, particularly in the context of varying relative wealth. The importance of working out changes in attitude towards risk as wealth, measured both in absolute and in relative terms, changes is quite clear, as it has wide-ranging applications in situations of uncertainty: the optimal tailoring of financial products, ranging from stock market portfolios through retirement packages to the appetite for risk, is just one of many examples. Whereas the effect of absolute wealth on attitudes towards risk has been broadly examined in the received literature (recalling the references in the Introduction), considerably less attention has been paid to the role of relative wealth in risk-taking behavior and, to the best of our knowledge, none to the role that individuals accord to relative wealth in their preference functions. We highlight the importance of these two roles.

An increase in an individual’s relative wealth can arise when he gains (absolute) wealth, and / or when others lose wealth. In the former case, there is a compound change in the wealth of the individual as he experiences both an absolute wealth gain and a relative wealth gain. In order to confine our analysis purely to a relative wealth gain, we considered the case in which the change in the individual’s position in the wealth hierarchy arises from a fall in the wealth of another individual (other individuals). We showed that the individual’s response in terms of relative risk aversion to an improvement in his relative wealth, keeping his absolute wealth intact, depends on whether or not there is a concomitant change in the weight that the individual assigns to his relative wealth. Specifically, when this weight is constant, a gain in rank (a gain in relative wealth) increases relative risk aversion (Section 3). However, when the weight
assigned to relative wealth intensifies with relative wealth, a gain in rank (a gain in relative wealth) can reduce relative risk aversion (Section 4).

It is worth adding that because the results and proofs that pertain to relative risk aversion analogously apply to absolute risk aversion, our analysis is relevant to risk-taking behavior in general.

The idea that concern about experiencing relative deprivation / low relative wealth maps onto risk-taking behavior in a systematic and predictable manner deserves empirical attention. The implications and applications of the link between relative wealth and risk aversion are many. For example, in setting premiums, insurers may want not only to find out the wealth of would-be clients, but also to ascertain the extent of their concern about having low relative wealth, and assess whether the intensity of that concern is likely to increase if their would-be clients experience gains in terms of their placement in the wealth hierarchy.

Although throughout we have referred to the preferences of individuals, the approach that we have taken could apply also to firms. For example, firms that are concerned about their market share are, in a sense, similar to individuals who are concerned about their relative deprivation. A stronger concern about market share could induce firms to take riskier decisions which, when bad outcomes materialize, could lead to bankruptcy. Thus, we could forge a behavioral link between a tendency for firms to fall into bankruptcy and their anxiety about their low market share. This line of reasoning could be extended. Firms are run by managers, and managers care about their prestige, reputation, and social standing. “[M]anagers seek to avoid financial distress and bankruptcy. … But they are not risk averse in the ordinary sense either. To them, a risky project is not necessarily one with a large variance. It is a project with a high chance of forcing the firm into financial difficulties. Their behavior will reflect this concern” (Rose-Ackerman, 1991, p. 279). If bankruptcy implies liquidation, and liquidation results in financial ruin and managers losing their prestige, variance (in the standard mean-variance sense) is not an adequate term to feature in managers’ utility functions. Our approach suggests an alternative ingredient. Furthermore, managers may well differ in the extent to which they fear losing prestige and erosion of their standing in the corporate world.
Managers with a high level of concern about their relative wealth will be more likely to undertake risky projects, and managers with a higher level of concern about their relative wealth will be more likely to undertake riskier projects. We can then conceptually settle the tension that arises from “some claim that the threat of bankruptcy generates caution, while others claim that it produces gambling behavior” (Rose-Ackerman, 1991, p. 283). Because a higher incidence of managerial risk taking correlates with a higher incidence of bankruptcy, we will have in place a causal relationship between managers’ concern about their social standing, lowered risk aversion, and the occurrence of bankruptcies.

There are large data sets on risk taking by firms, and there is even an attempt to link the incidence of such behavior with cultural factors and with variation in these factors across countries (Mihet, 2012). It will be intriguing to study whether in a culture that assigns more weight to social standing and status riskier behavior by firms is more prevalent.
Appendix: Robustness of the two key results in case of an alternative specification of the utility function

In the main text of the paper we assume that the utility of individual \( i \) depends linearly on a measure of his relative deprivation (as per equation (1)). As a robustness test, we now relax this assumption, asking whether the results obtained in the main text withstand replacement of that functional form with the following specification of the utility function of individual \( i \):

\[
u_i(w_i, RD) = (1 - \beta_i) f(w_i) - \beta_i [RD_{i,i}(w)]^\alpha ,
\]

where \( RD_{i,i}(w) = \frac{1}{n} \sum_{k=i+1}^{n} \max\{w_k - w_i, 0\} \), as introduced in Section 3, and \( \alpha \in (1, 2) \). By assuming this range for the parameter \( \alpha \), we introduce a (small) degree of concavity of the utility function with respect to relative deprivation. It is helpful to rewrite \( RD_{i,i}(w) \) in a slightly different form:

\[
RD_{i,i}(w) = \frac{n-i}{n} \frac{1}{n-i} \sum_{k=i+1}^{n} (w_k - w_i) = \frac{n-i}{n} \left( \frac{\sum_{k=i+1}^{n} w_k}{n-i} - w_i \right) = \frac{n-i}{n} (w_i - w_i),
\]

where \( w_i = \frac{1}{n-i} \sum_{k=i+1}^{n} w_k \) is the average wealth of the individuals who are richer than individual \( i \) (these individuals are positioned to the right of individual \( i \) in the wealth distribution).

The relative risk aversion coefficient, calculated in the same way as the one employed in Section 2 of the main text of the paper, takes the following functional form

\[
r_i(w_i) = \frac{-w_i \left[ (1 - \beta_i) f''(w_i) - \beta_i \alpha (\alpha - 1) \left( \frac{n-i}{n} \right)^\alpha (w_i - w_i)^{\alpha-2} \right]}{(1 - \beta_i) f'(w_i) + \beta_i \alpha \left( \frac{n-i}{n} \right)^\alpha (w_i - w_i)^{\alpha-1}}.
\]
A1. An improved position in the wealth distribution with no (absolute) wealth gain

As in Section 3 we assume initially that \( \beta_1 = \beta_2 = \ldots = \beta_n = \beta \); that \( w_i \) (the wealth level of the individual) stays constant; and that \( w_i \) stays constant as well. We investigate the sign of the derivative of \( r_i(w_i) \) with respect to \( \frac{n-i}{n} \), which for a large \( n \) approaches a continuous variable. Here, an improved position in the wealth hierarchy reduces \( \frac{n-i}{n} \), and a worsened position increases \( \frac{n-i}{n} \). We have that

\[
\frac{\partial r_i(w_i)}{\partial \frac{n-i}{n}} = \frac{w_i \beta_i \alpha^2 \left( \frac{n-i}{n} \right)^{\alpha-1} (w_i - w_i)^{\alpha-1}}{\left[ (1 - \beta_i) f'(w_i) + \beta_i \alpha \left( \frac{n-i}{n} \right)^{\alpha} (w_i - w_i)^{\alpha-1} \right]^2}
\]

\[
\times \left[ \frac{\alpha-1}{w_i - w_i} \left( (1 - \beta_i) f'(w_i) + \beta_i \alpha \left( \frac{n-i}{n} \right)^{\alpha} (w_i - w_i)^{\alpha-1} \right) \right]
\]

\[
+(1 - \beta_i) f''(w_i) - \beta_i \alpha (\alpha - 1) \left( \frac{n-i}{n} \right)^{\alpha} (w_i - w_i)^{\alpha-2}
\]

\[
= \frac{w_i (1 - \beta_i) \beta \alpha^2 \left( \frac{n-i}{n} \right)^{\alpha-1} (w_i - w_i)^{\alpha-1}}{\left[ (1 - \beta_i) f'(w_i) + \beta_i \alpha \left( \frac{n-i}{n} \right)^{\alpha} (w_i - w_i)^{\alpha-1} \right]^2} \left[ \frac{\alpha-1}{w_i - w_i} f'(w_i) + f''(w_i) \right].
\]

Because the fraction preceding \( \frac{\alpha-1}{w_i - w_i} f'(w_i) + f''(w_i) \) is positive, the sign of \( \frac{\partial r_i(w_i)}{\partial \frac{n-i}{n}} \) depends on the sign of \( \frac{\alpha-1}{w_i - w_i} f'(w_i) + f''(w_i) \). We will then have that \( \frac{\partial r_i(w_i)}{\partial \frac{n-i}{n}} < 0 \) if

\[
\frac{\alpha-1}{w_i - w_i} f'(w_i) + f''(w_i) < 0.
\]

Thus, if \( \alpha \) is close enough (from above) to 1 (recalling that
\( \alpha \in (1, 2) \), then \( \frac{\partial r_i(w_i)}{\partial \frac{n-i}{n}} < 0 \). With \( \frac{n-i}{n} \) depending inversely on the rank \( i \), we thus conclude that a gain in rank leads to an increase in relative risk aversion.

A2. An improved position in the wealth distribution with no (absolute) wealth gain, accompanied by intensified concern about having low relative wealth

We now depart from the assumption \( \beta_1 = \beta_2 = \ldots = \beta_n = \beta \) and, instead, assume that the \( \beta \) weight increases monotonically with rank, implying that for \( \beta_i = \beta \left( \frac{n-i}{n} \right) = \beta(x) \),

\[
\frac{\partial \beta(x)}{\partial x} = \beta'(x) < 0.
\]

The derivative of \( r_i(w_i) \) with respect to \( \frac{n-i}{n} \) is

\[
\frac{\partial r_i(w_i)}{\partial \frac{n-i}{n}} = \frac{w_i \alpha \left( \frac{n-i}{n} \right)^{\alpha-1} \left( w_i - w_i \right)^{\alpha-1} \left( \frac{\alpha - 1}{w_i - w_i} f'(w_i) + f''(w_i) \right)}{\left[ 1 - \beta \left( \frac{n-i}{n} \right) \right] f'(w_i) + \beta \left( \frac{n-i}{n} \right) \alpha \left( \frac{n-i}{n} \right)^{\alpha-1} \left( w_i - w_i \right)^{\alpha-1}}^{\alpha-1} \times \left[ 1 - \beta \left( \frac{n-i}{n} \right) \right] \beta \left( \frac{n-i}{n} \right) \alpha + \beta' \left( \frac{n-i}{n} \right) \frac{n-i}{n}.
\]

Bringing over the condition \( \frac{\alpha - 1}{w_i - w_i} f'(w_i) + f''(w_i) < 0 \) from Sub-section A1, we note that because the sign of the fraction preceding

\[
\left( 1 - \beta \left( \frac{n-i}{n} \right) \right) \beta \left( \frac{n-i}{n} \right) \alpha + \beta' \left( \frac{n-i}{n} \right) \frac{n-i}{n}
\]

is negative, the sign of \( \frac{\partial r_i(w_i)}{\partial \frac{n-i}{n}} \) depends on the sign of \( \left( 1 - \beta \left( \frac{n-i}{n} \right) \right) \beta \left( \frac{n-i}{n} \right) \alpha + \beta' \left( \frac{n-i}{n} \right) \frac{n-i}{n} \). If the sign of this latter term is negative, then \( \frac{\partial r_i(w_i)}{\partial \frac{n-i}{n}} \) will be positive. Requiring that

\[
\left( 1 - \beta \left( \frac{n-i}{n} \right) \right) \beta \left( \frac{n-i}{n} \right) \alpha + \beta' \left( \frac{n-i}{n} \right) \frac{n-i}{n} < 0
\]

is equivalent to requiring that
\[ \beta\left(\frac{n-i}{n}\right) < \left(1 - \beta\left(\frac{n-i}{n}\right)\right) \beta\left(\frac{n-i}{n}\right) \alpha \frac{n}{n-i}. \]  

(A1)

We note that although \( \left(1 - \beta\left(\frac{n-i}{n}\right)\right) \beta\left(\frac{n-i}{n}\right) \alpha < 0.5, \) we also have that \( \frac{n}{n-i} > 1. \)

Therefore, the absolute value of the right-hand side of (A1) can be large. Nevertheless, if concern about relative deprivation decreases sufficiently with \( \frac{n-i}{n} \), then \( \frac{\partial r_i(w_i)}{\partial \frac{n-i}{n}} > 0. \)

And, as already noted, because \( \frac{n-i}{n} \) depends inversely on rank \( i \), it follows that if (A1) is satisfied, a gain in rank will decrease relative risk aversion, a result that is the opposite of the one obtained in the preceding Sub-section A1.

The results reported in this Appendix are thus quite similar to the results reported in the main text of the paper. Analogously to Section 3 where we found that \( \frac{\Delta RD_i^\prime(w_i^\prime)}{\Delta i} u_i^\prime(w_i^\prime) - \frac{\Delta RD_i^\prime\prime(w_i^\prime)}{\Delta i} u_i^\prime(w_i^\prime) < 0 \) in the numerator of (4) is the condition needed to ensure that a gain in rank leads to an increase in relative risk aversion, here too we obtain a single condition \( \frac{\alpha - 1}{w_i - w_i^\prime} f^\prime(w_i) + f^\prime\prime(w_i) < 0 \), which depends on the shape of the function \( f\left(w_i\right) \) and on the shape of the measure of relative deprivation as expressed by the parameter \( \alpha \).

---

\(^4\) This upper bound arises from the fact that the maximum of a function \( g(x) = (1-x)x \) is achieved at \( x = 0.5 \), with \( g(0.5) = 0.25 \). Because \( \alpha \in (1, 2) \), then \( g(x)\alpha = (1-x)x\alpha < 0.5. \)
References


